

# HAETAE

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February 27, 2024  
2024 KpqC Winter Camp



**HAETAE**  
HEAAN  
CRYPTO LAB

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## 2. Preliminaries:

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- Lattice hard problems
- Lattice-based signatures
  - Bimodal rejection sampling

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- Hyperball bimodal rejection sampling
- Comparison to SotA lattice signatures

## 4. Changes after Round 1

# HAETAE

- Digital signature scheme
- Secure against **quantum attacks!**
  - based on **lattice hard problems** MLWE and MSIS
  - follows **Fiat-Shamir with aborts** framework, secure in QROM
- Simple but **short!**
  - simpler than Falcon<sup>1</sup> & shorter than Dilithium<sup>1</sup>
  - optimal rejection rate with simple rejection condition
- Design rationale
  - **Bimodal** rejection sampling
  - **Hyperball** distribution
- Candidate in *KpqC 2nd round & NIST PQC Additional Signatures*<sup>2</sup>

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<sup>1</sup>NIST 2022 PQC signature standards

<sup>2</sup>NIST's on-ramp PQC signature competition, from 2023.



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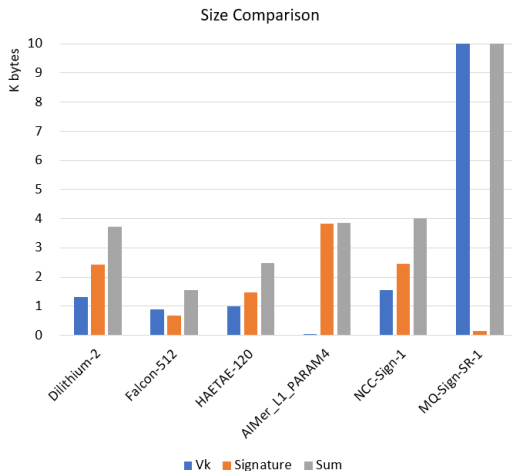


Figure: KpqC round 2, signature schemes

# HAETAE



## 40 submissions

- **Code-based**
  - Enhanced pqsigRM
  - FuLeeca
  - LESS
  - MEDS
  - Wave
- **Isogenies**
  - SQISign
- **Lattices**
  - EHT
  - EagleSign
  - HAETAE
  - HAWK
  - HuFu
  - Raccoon
  - Squirrels
- **MPC-in-the-Head**
  - CROSS
  - MIRA
  - MQOM
  - MiRitH
  - PERK
  - RYDE
  - SDitH
- **Symmetric**
  - AIMer
  - Ascon-Sign
  - FAEST
  - SPHINCS-alpha
- **Multivariate**
  - 3WISE
  - Biscuit
  - DME-Sign
  - HPPC
  - MAYO
  - PROV
  - QR-UOV
  - SNOVA
  - TUOV
  - UOV
  - VOX
- **Other**
  - ALTEQ
  - KAZ-Sign
  - PREON
  - Xifrat1-Sign.I
  - eMLE-Sig 2.0

Public - PQShield / Cloudflare - CC-BY

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Slide from <https://datatracker.ietf.org/meeting/117/materials/>

slides-117-tls-new-post-quantum-signature-algorithms-on-the-horizon-00



# HAETAE



## 40 submissions: the first eliminations (July 19<sup>th</sup>)

- **Code-based**
  - Enhanced pqsigRM
  - ~~Fulcrum~~
  - LESS
  - MEDS ⚠
  - Wave
- **Isogenies**
  - SQIsign
- **Lattices**
  - EHT
  - ~~EagleSign~~
  - HAETAE
  - HAWK
  - HuFu
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- **Multivariate**
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  - ~~Biscuit ?~~
  - DME-Sign
  - ~~HPPE~~
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  - ~~Xifrat1 Sign.1~~
  - ~~eMLE Sig 2.0~~

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Slide from <https://datatracker.ietf.org/meeting/117/materials/>

slides-117-tls-new-post-quantum-signature-algorithms-on-the-horizon-00

# HAETAE



## Submissions: verification < 5ms

- **Code-based**
  - Enhanced pqsigRM
  - ← LESS
  - ← Wave
- **Isogenies**
  - ← SQIsign
- **Lattices**
  - EHT
  - HAETAE
  - HAWK
  - HuFu
  - Raccoon
  - Squirrels
- **MPC-in-the-Head**
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  - ← MIRA
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  - DME-Sign
  - MAYO
  - ← PROV
  - ← QR-UV
  - ← SNOVA
  - TUOV
  - UOV
  - VOX
- **Other**
  - ← PREON

Note: based on current, often not exactly optimized, performance metrics.

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Slide from <https://datatracker.ietf.org/meeting/117/materials/>

slides-117-tls-new-post-quantum-signature-algorithms-on-the-horizon-00

# HAETAE



## Submissions: signature < 3000 bytes

- Code-based
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- Lattices
  - EHT
  - HAETAE
  - HAWK
  - HuFu
  - Raccoon
  - Squirrels
- MPC-in-the-Head
  - CROSS
  - MQOM
  - MiRiTH
  - PERK
  - RYDE
  - SDiTH
- Multivariate
  - DME-Sign
  - MAYO
  - TUOV
  - UOV
  - VOX
- Symmetric
  - A1Mer
  - Ascon Sign
  - FAEST
  - SPHINCS alpha

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Slide from <https://datatracker.ietf.org/meeting/117/materials/>

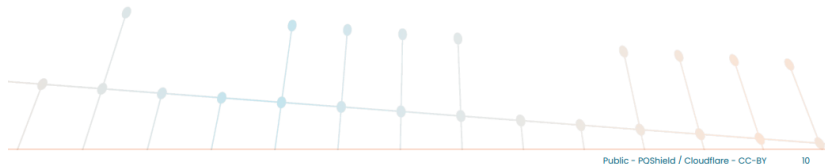
slides-117-tls-new-post-quantum-signature-algorithms-on-the-horizon-00

# HAETAE

## Certificate usage: public key + sig < 4 KB (Dilithium)



- Code-based
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  - EHT
  - HAETAE
  - HAWK
  - HuFu
  - Squirrels
- Multivariate
  - DME-Sign
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# Digital signatures

## Conventional signatures:



Some images are from [https://kr.freepik.com/search?format=search&last\\_filter=type&last\\_value=icon&query=magnifier&selection=1&type=icon](https://kr.freepik.com/search?format=search&last_filter=type&last_value=icon&query=magnifier&selection=1&type=icon)

# Digital signatures

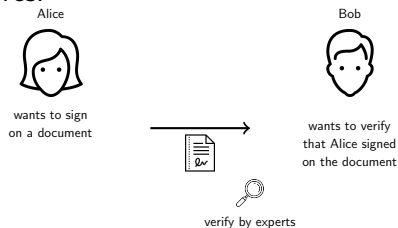
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# Digital signatures

## Conventional signatures:



## Digital signatures:

$(sk, vk) \leftarrow \text{KeyGen}$  and broadcast  $vk$

Alice (knows  $sk$ )



signature  $\sigma \leftarrow \text{Sign}(sk, m)$

Bob (knows  $vk$ )



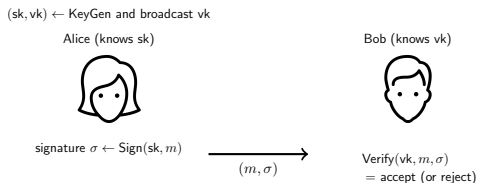
$(m, \sigma)$

$\text{Verify}(vk, m, \sigma)$   
= accept (or reject)



# Digital signatures

## Digital signatures:



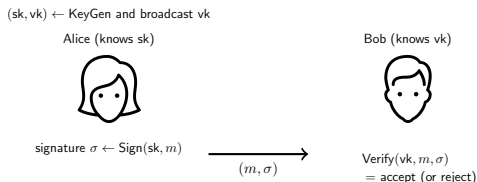
Anyone (who can access  $vk$ ) can verify that  $(m, \sigma)$  is from Alice or not!

**Correctness:**  $\text{Verify}(vk, m, \text{Sign}(sk, m)) = \text{accept}$

**Unforgeability:** No one but Alice can make a new signature.

# Digital signatures

## Digital signatures:



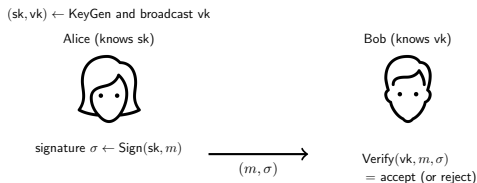
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# Digital signatures

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# Lattice hard problems

Lattice-based cryptography is ... are currently important candidates for post-quantum cryptography.

- *Wikipedia* -

Lattice-based cryptography bases its security on lattice hard problems, which have strong theoretical backgrounds:

- SVP and  $\text{GapSVP}_\lambda$ : NP-hard! [Ajt96, HR07]
- Worst-case to average-case **reductions** [Ajt96]
- Useful hard problems: NTRU, LWE, SIS, MLWE, MSIS, etc

# Lattice hard problems

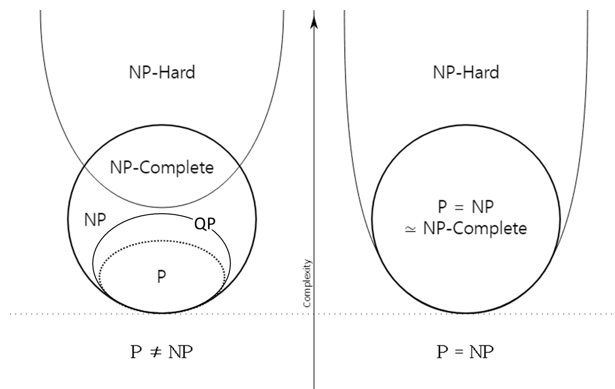
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# Lattice hard problems



**Figure:** Category of hard problems when  $P \neq NP$  and  $P = NP$ .

No proofs for Quantum Poly (QP), but is believed to be separated to NP-Hard problems.

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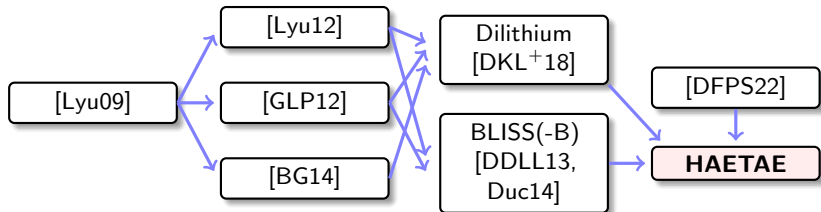
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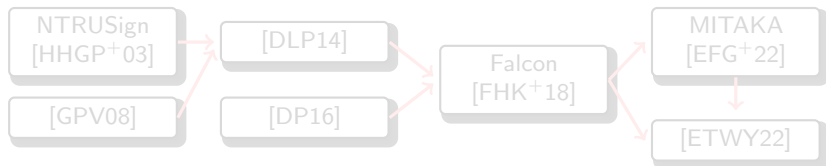


# Lattice-based signatures

## Fiat-Shamir with abort

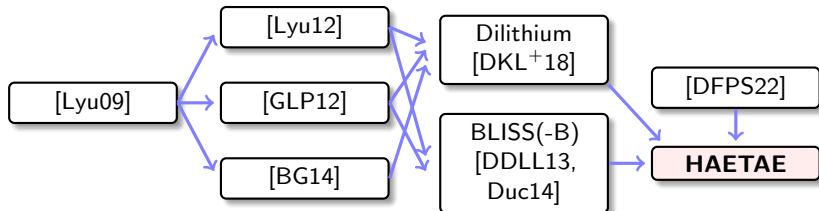


## Hash-and-Sign

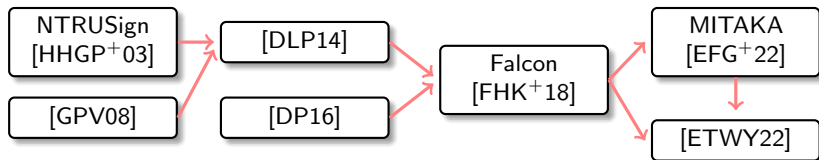


# Lattice-based signatures

## Fiat-Shamir with abort



## Hash-and-Sign



# Lattice-based signatures

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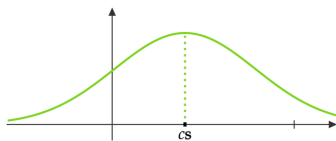
For secret  $s$ , random  $y$ ,  $c$ , signature  $\sigma = (c, \mathbf{z} = \mathbf{y} + c\mathbf{s})$

$$\left( \begin{bmatrix} 8 \end{bmatrix}, \begin{bmatrix} 9 \\ -9 \\ 6 \\ 2 \\ 1 \\ 11 \\ 8 \\ -9 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ -1 \\ -2 \\ 2 \\ 1 \\ 3 \\ 0 \\ -1 \end{bmatrix} + \begin{bmatrix} 8 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

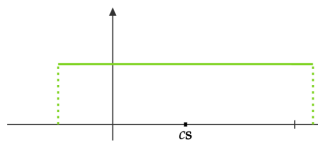
# Lattice-based signatures

## Leakage from $(c, \mathbf{z} = \mathbf{y} + c\mathbf{s})$ ?

(High-level) With  $\infty$  pairs of  $(c, \mathbf{z} = \mathbf{y} + c\mathbf{s})$ , we may collect  $\mathbf{z}$  for same  $c$ :



$$(y \leftarrow \mathcal{N}(0, \sigma^2))$$



$$(y \leftarrow U[-a, a])$$

$\Rightarrow$  Recover  $\mathbf{s}$  from  $c\mathbf{s}$ .

How to make it safe?

$$(c, \mathbf{z} = \mathbf{y} + c\mathbf{z}) \xrightarrow[\text{Rejection Sampling}]{\text{several trials}} \sigma = (c, \mathbf{z} = \mathbf{y} + c\mathbf{z})$$

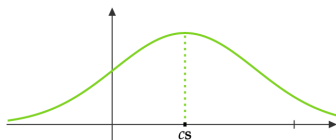
not safe

safe

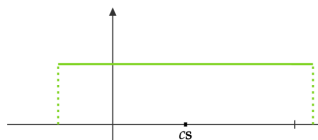
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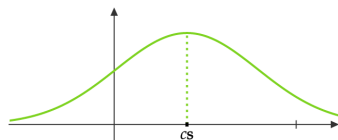
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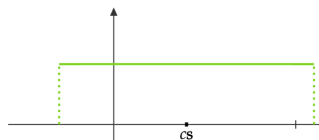
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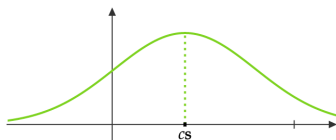
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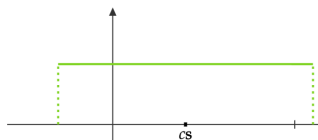
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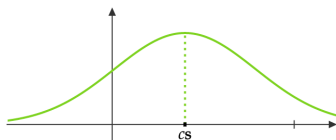
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not safe safe

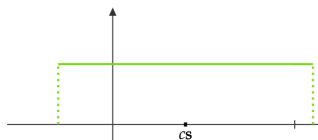
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 (c, \mathbf{z} = \mathbf{y} + c\mathbf{z}) & \xrightarrow[\text{Rejection Sampling}]{\text{several trials}} & \sigma = (c, \mathbf{z} = \mathbf{y} + c\mathbf{z}) \\
 \text{not safe} & & \text{safe}
 \end{array}$$



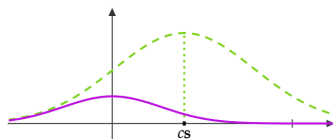
# Rejection sampling

## Rejection sampling

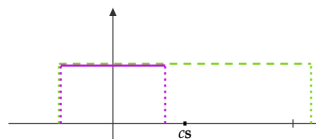
$$D_{\text{source}} = \{(c, \mathbf{z})\} \xrightarrow[\text{prob. } p(c, \mathbf{z})]{\text{reject with}} D_{\text{target}}$$

distribution of  $(c, \mathbf{z})$ ,  
possibly leak  $s$

new distribution,  
independent of  $s$



$$y \leftarrow \mathcal{N}(0, \sigma^2)$$



$$y \leftarrow U[-a, a]$$

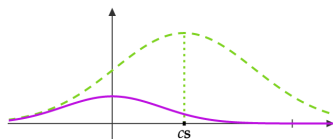
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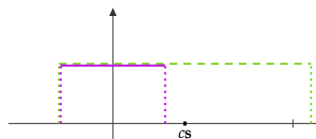
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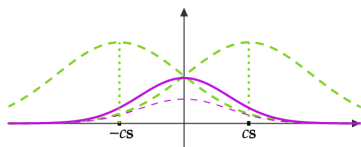
# Bimodal rejection sampling

Run-time  $\propto M$  ( $\approx$  green area / purple area).

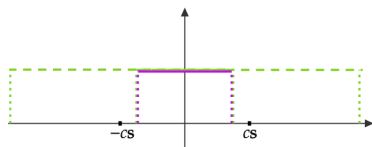
To decrease  $M$ , [DDL13] uses

$$\mathbf{z} = \mathbf{y} + (-1)^b cs$$

instead of  $\mathbf{z} = \mathbf{y} + cs$ :



$$y \leftarrow \mathcal{N}(0, \sigma^2)$$



$$y \leftarrow U[-a, a]$$

Note, no change for the uniform case.

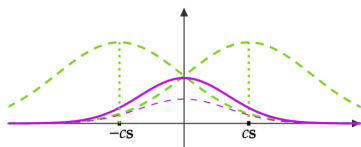
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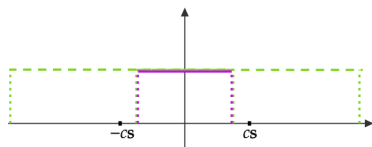
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$$\mathbf{z} = \mathbf{y} + (-1)^b cs$$

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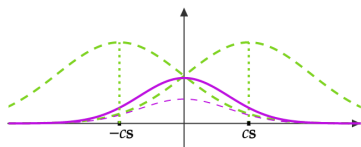
# Bimodal rejection sampling

Run-time  $\propto M$  ( $\approx$  green area / purple area).

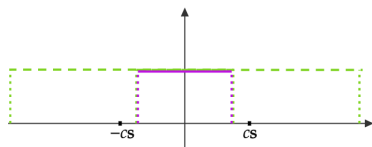
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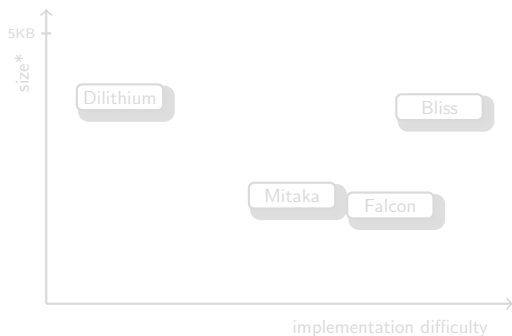
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For e.g., for  $\approx 120$  bits security<sup>45</sup>,



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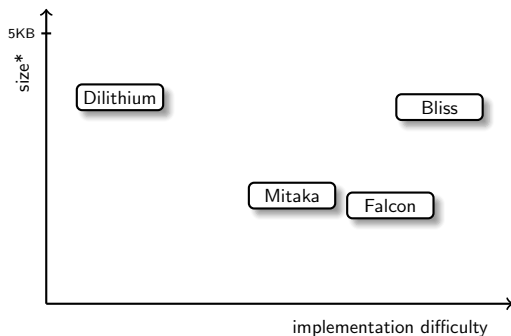
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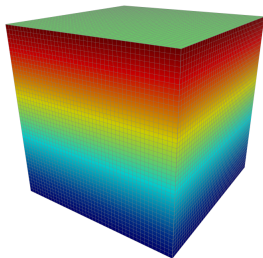
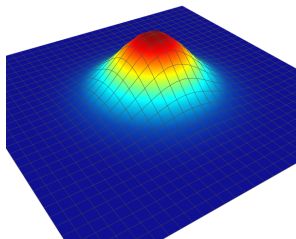
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## 4. Changes after Round 1



# Hyperball bimodal rejection sampling

Previously, the randomness  $\mathbf{y}$  was chosen from either discrete Gaussian or uniform hypercube<sup>6</sup>.



---

<sup>6</sup>The vectors  $\mathbf{y}$  and  $\mathbf{z}$  are high-dimensional vectors, so uniform in an interval is indeed a uniform hypercube.

# Hyperball bimodal rejection sampling

We, instead, use **uniform hyperball** distribution for sampling  $y$  [DFPS22];

- to exploit optimal  $M$ ,
- to reduce signature and verification key sizes,



based on the **bimodal approach** [DDLL13].

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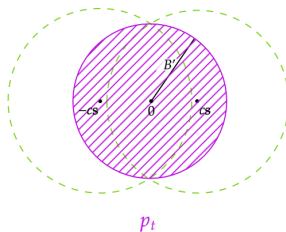
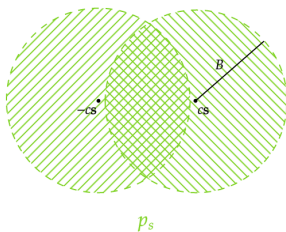


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We reject  $(c, \mathbf{z}) \sim D_s$  (with p.d.f.  $p_s$ ) to a target distribution  $D_t$  (with p.d.f.  $p_t$ ), where

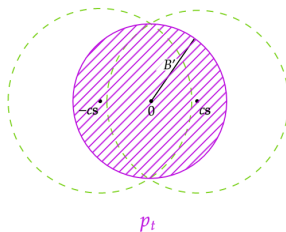
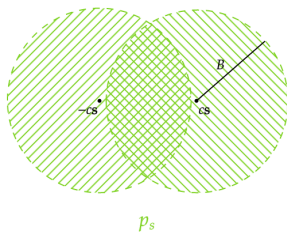
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# Hyperball bimodal rejection sampling

- $p_s(\mathbf{x}) = \frac{1}{2 \cdot \text{vol}(\mathcal{B}(B))} \cdot \chi_{\|\mathbf{z}-c\mathbf{s}\| < B} + \frac{1}{2 \cdot \text{vol}(\mathcal{B}(B))} \cdot \chi_{\|\mathbf{z}+c\mathbf{s}\| < B},$
- $p_t(\mathbf{x}) = \frac{1}{\text{vol}(\mathcal{B}(B'))} \cdot \chi_{\|\mathbf{z}\| < B'}.$

$$\Rightarrow p(\mathbf{x}) = \frac{p_t(\mathbf{x})}{M \cdot p_s(\mathbf{x})} = \frac{\chi_{\|\mathbf{z}\| < B'}}{\chi_{\|\mathbf{z}-c\mathbf{s}\| < B} + \chi_{\|\mathbf{z}+c\mathbf{s}\| < B}}$$

$$= \begin{cases} 0 & \text{if } \mathbf{z} \notin \mathcal{B}(B'), \\ 1/2 & \text{if } \mathbf{z} \in \mathcal{B}(B') \cap \mathcal{B}(B, c\mathbf{s}) \cap \mathcal{B}(B, -c\mathbf{s}), \\ 1 & \text{if } \mathbf{z} \in \mathcal{B}(B') \setminus (\mathcal{B}(B, c\mathbf{s}) \cap \mathcal{B}(B, -c\mathbf{s})), \end{cases}$$

for some  $M > 0$ .

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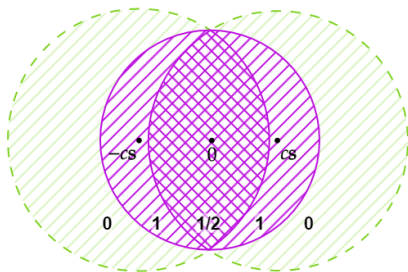
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# Hyperball bimodal rejection sampling

That is, we return  $\mathbf{x} = (c, \mathbf{z})$  with probability

- 0: if  $\|\mathbf{z}\| \geq B'$ ,
- $1/2$ : else if  $\|\mathbf{z} - c\mathbf{s}\| < B$  and  $\|\mathbf{z} + c\mathbf{s}\| < B$ ,
- 1: otherwise.





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# Comparison to SotA lattice signatures.

For 120-bit classical security. Sizes are in bytes.

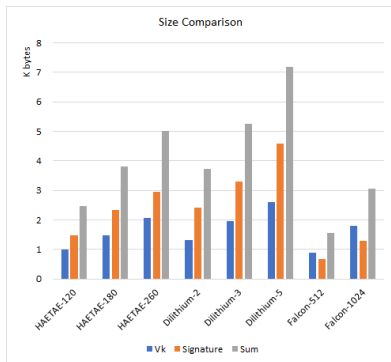
Scheme	<i>sig</i>	<i>vk</i>	KeyGen	Sign	
				sampling	rejection
Dilithium-2	2420	1312	fast	Hypercube	$\ \cdot\ _\infty < B$
Bliss-1024 <sup>7</sup>	1700	1792	fast	dGaussian at 0	reject with prob. $f(\text{sk}, \text{Sig})$
HAETAE120	1468	1056	fast	dHyperball at 0	$\ \cdot\ _2 < B$
Mitaka-512 <sup>8</sup>	713	896	slow	dGaussian at 0 & intGaussian at $H(m)$	none
Falcon-512	666	897	slow	dGaussian at $H(m)$	none

Table: Comparison between different lattice-based signature schemes.

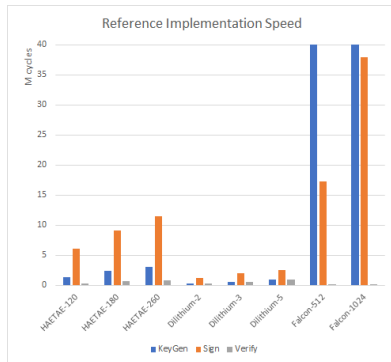
<sup>7</sup>modified Bliss (to  $\geq 120$  bit-security) in Dilithium paper.

<sup>8</sup>Mitaka-512 has 102 bits of security

# Numbers - Updated Reference Implementation

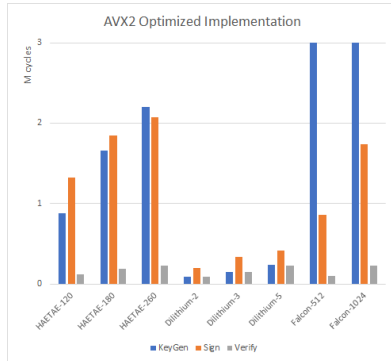
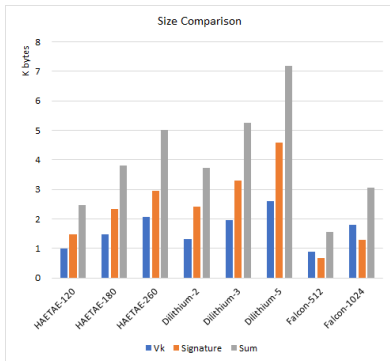


Size



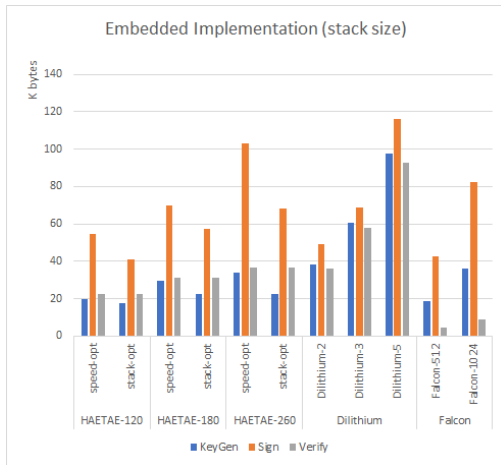
Performance

# Numbers - AVX2 optimized Implementation



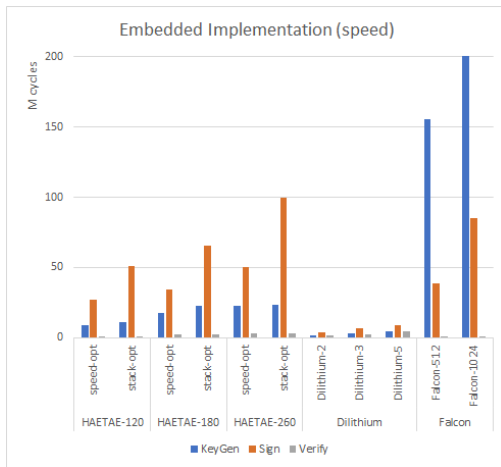
# Numbers - Embedded Implementation on Cortex-M4

Stack-size of HAETAE and others on Cortex-M4.



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Speed of HAETAE and others on Cortex-M4.



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### May, 2023 (v1.0)

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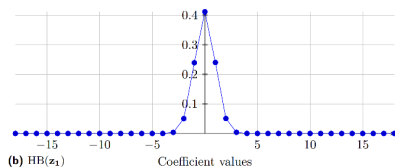
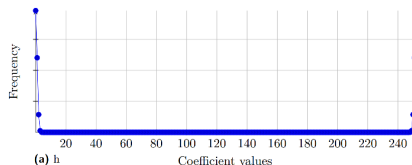
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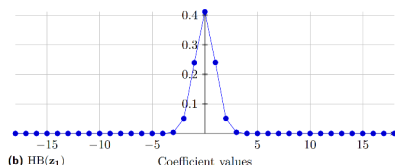
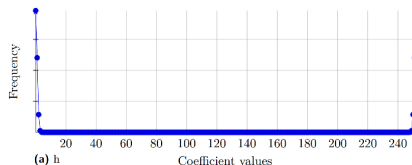
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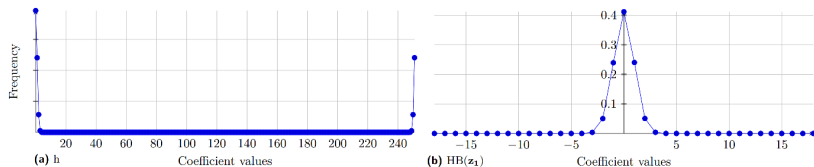
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# Thanks!

Check <http://kpgc.cryptolab.co.kr/haetae!>

Check <https://github.com/mupq/pqm4> for the *embedded code*!

## Any question?



# References I

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# HAETAE description (high-level)

## KeyGen( $1^\lambda$ )

- 1:  $\mathbf{A}_{\text{gen}} \leftarrow \mathcal{R}_q^{k \times (\ell-1)}$  and  $(\mathbf{s}_{\text{gen}}, \mathbf{e}_{\text{gen}}) \leftarrow S_\eta^{\ell-1} \times S_\eta^k$
- 2:  $\mathbf{b} = \mathbf{A}_{\text{gen}} \cdot \mathbf{s}_{\text{gen}} + \mathbf{e}_{\text{gen}} \in \mathcal{R}_q^k$
- 3:  $\mathbf{A} = (-2\mathbf{b} + q\mathbf{j} \mid 2\mathbf{A}_{\text{gen}} \mid 2\mathbf{Id}_k) \bmod 2q$  and write  $\mathbf{A} = (\mathbf{A}_1 \mid 2\mathbf{Id}_k)$
- 4:  $\mathbf{s} = (1, \mathbf{s}_{\text{gen}}, \mathbf{e}_{\text{gen}})$
- 5: **if**  $\sigma_{\max}(\text{rot}(\mathbf{s}_{\text{gen}})) > \gamma$ , **then restart**
- 6: **Return**  $\text{sk} = \mathbf{s}, \text{vk} = \mathbf{A}$

## Sign( $\text{sk}, M$ )

- 1:  $\mathbf{y} \leftarrow U(\mathcal{B}_{(1/N)\mathcal{R}, (k+\ell)}(B))$
- 2:  $c = H(\text{HighBits}_{2q}^{\text{hint}}(\mathbf{A} \lfloor \mathbf{y} \rfloor, \alpha), \text{LSB}(\lfloor y_0 \rfloor), M) \in \mathcal{R}_2$
- 3:  $\mathbf{z} = (\mathbf{z}_1, \mathbf{z}_2) = \mathbf{y} + (-1)^b c \cdot \mathbf{s}$  **for**  $b \leftarrow U(\{0, 1\})$
- 4:  $\mathbf{h} = \text{HighBits}_{2q}^{\text{hint}}(\mathbf{A} \lfloor \mathbf{z} \rfloor - qc\mathbf{j}, \alpha) - \text{HighBits}_{2q}^{\text{hint}}(\mathbf{A}_1 \lfloor \mathbf{z}_1 \rfloor - qc\mathbf{j}, \alpha) \bmod^+ \frac{2(q-1)}{\alpha}$
- 5: **if**  $\|\mathbf{z}\|_2 \geq B'$ , **then restart**
- 6: **if**  $\|2\mathbf{z} - \mathbf{y}\|_2 < B$ , **then restart with probability**  $1/2$
- 7: **Return**  $\sigma = (\text{Encode}(\text{HighBits}(\lfloor \mathbf{z}_1 \rfloor, a)), \text{LowBits}(\lfloor \mathbf{z}_1 \rfloor, a), \text{Encode}(\mathbf{h}), c)$

## Verify( $\text{vk}, M, \sigma = (x, \mathbf{v}, h, c)$ )

- 1:  $\tilde{\mathbf{z}}_1 = \text{Decode}(x) \cdot a + \mathbf{v}$  and  $\tilde{\mathbf{h}} = \text{Decode}(h)$
- 2:  $\mathbf{w} = \tilde{\mathbf{h}} + \text{HighBits}_{2q}^{\text{hint}}(\mathbf{A}_1 \tilde{\mathbf{z}}_1 - qc\mathbf{j}, \alpha) \bmod^+ \frac{2(q-1)}{\alpha}$
- 3:  $w' = \text{LSB}(\tilde{z}_0 - c)$
- 4:  $\tilde{\mathbf{z}}_2 = [\mathbf{w} \cdot \alpha + w' \mathbf{j} - (\mathbf{A}_1 \tilde{\mathbf{z}}_1 - qc\mathbf{j})] / 2 \bmod^\pm q$
- 5:  $\tilde{\mathbf{z}} = (\tilde{\mathbf{z}}_1, \tilde{\mathbf{z}}_2)$
- 6: **Return**  $(c = H(\mathbf{w}, w', M)) \wedge (\|\tilde{\mathbf{z}}\| < B'')$