HAETAE: Bridging Algebraic Number Theory to Post-Quantum Digital Signatures

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2024 Algebra Camp
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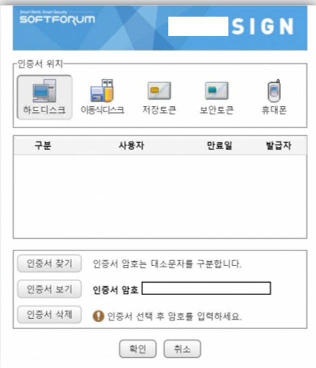
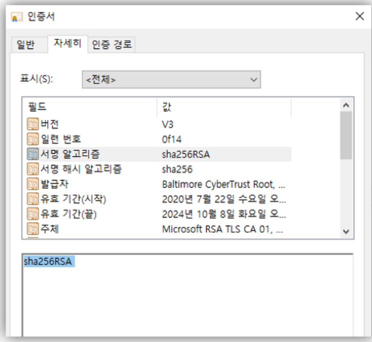
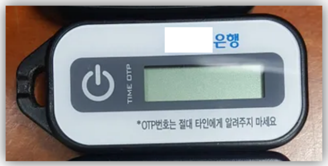
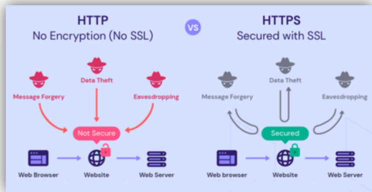
   - Hyperball bimodal rejection sampling
   - Comparison to SotA lattice signatures
   - Current status

What is Cryptography?

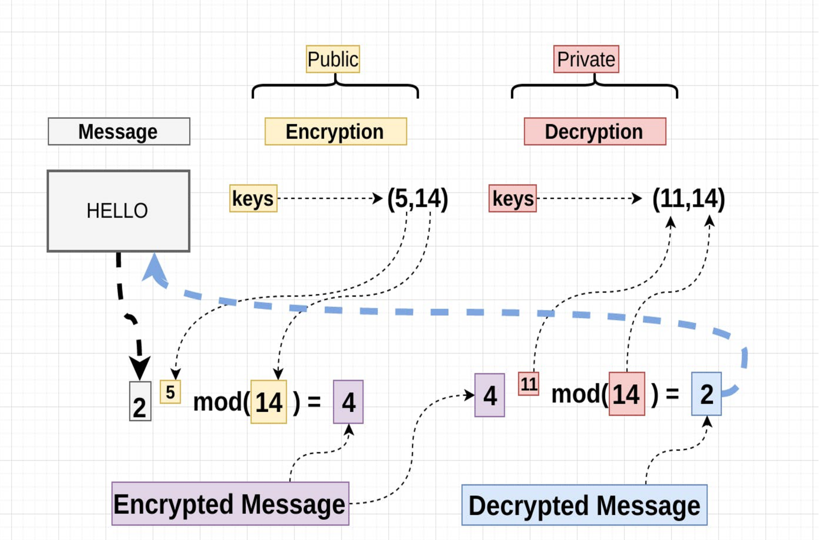






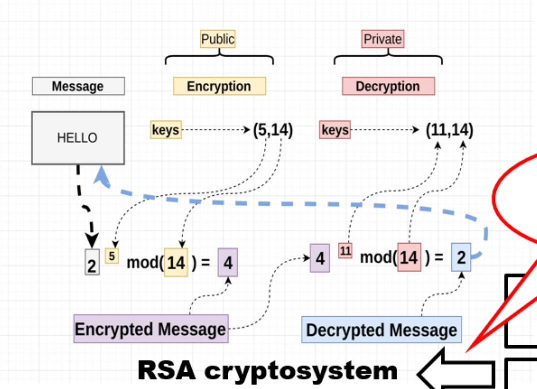


What is Cryptography?



RSA cryptosystem

What is Cryptography?



Reduction!

Integer Factorization problem

RSA problem: *for given* $a, e, n \in \mathbb{Z}$, ***find*** $b \in \mathbb{Z}$ ***satisfying*** $a \equiv b^e \bmod n$.

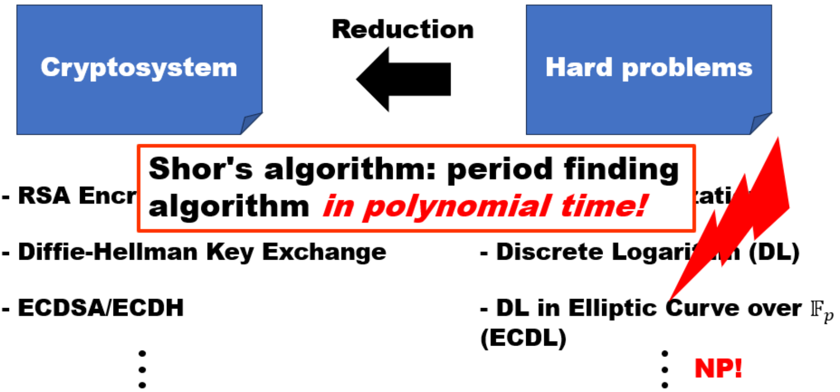
What is Cryptography?

Cryptosystem $\xleftarrow{\textbf{Reduction}}$ **Hard problems**

- RSA Encryption/Signature	**- Integer Factorization**
- Diffie-Hellman Key Exchange	**- Discrete Logarithm (DL)**
- ECDSA/ECDH	**- DL in Elliptic Curve over $\mathbb{F}_p$ (ECDL)**
$\vdots$	$\vdots$ **NP!**

Post-Quntum Cryptography

However, including the quantum algorithms...



Post-Quntum Cryptography

Post-Quantum Cryptography

Reduction

Hard problems (even) against Quantum Algorithms

- Lattice-based cryptography

- Code-based cryptography

⋮

- Shortest/Closest Vector Problem (SVP/CVP)

- Syndrome Decoding Problem (SDP)

⋮

NP-hard!*

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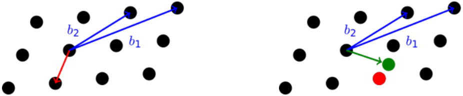
Lattice hard problems

Useful hard problems:

- LWE, Ring-LWE, and Module-LWE
- SIS, Ring-SIS, and Module-SIS

NP-hard problems:

- Shortest Vector Problem (SVP)
- Closet Vector Problem (CVP)



SVP and CVP in dimension two.

Reductions:

$$\text{Schemes} \Leftarrow \text{Useful hard problems} \Leftarrow \text{NP-hard problems}$$

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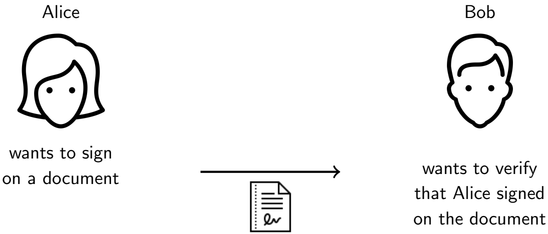
Digital signatures

Conventional signatures work as:



Digital signatures

Conventional signatures work as:

Alice

wants to sign
on a document

Bob

wants to verify
that Alice signed
on the document

Digital signatures work as:

$(\mathsf{sk}, \mathsf{vk}) \leftarrow \mathsf{KeyGen}$ and broadcast vk

Alice (knows sk)

signature $\sigma \leftarrow \mathsf{Sign}(\mathsf{sk}, m)$

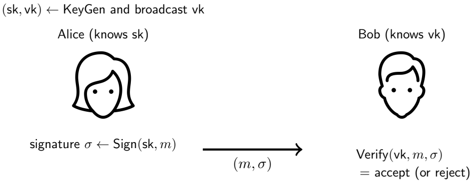
(m, σ)

Bob (knows vk)

$\mathsf{Verify}(\mathsf{vk}, m, \sigma)$
$=$ accept (or reject)

Digital signatures

Digital signatures work as:



Necessary properties:

- **Correctness**:

$$\mathsf{Verify}(\mathsf{vk}, m, \mathsf{Sign}(\mathsf{sk}, m)) = \mathsf{accept}$$

- **Unforgeability**: No one else than Alice can make a new signature. More formally,

for a given verification key and some message-signature pairs, no adversary can forge a new valid signature.

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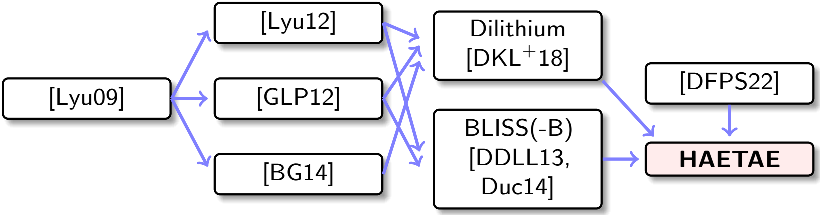
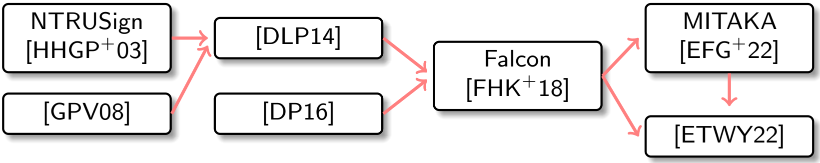
Lattice-based signatures

Fiat-Shamir with abort



Hash-and-Sign



Lattice-based signatures

Fiat-Shamir with abort:

Key: ($\mathbf{s}$: small secret, $\mathbf{t} = \mathbf{A}\mathbf{s} \bmod q$: public)

Sign: $(c = H(\mathbf{A}\mathbf{y} \bmod q,\ m),\ \mathbf{z} = \mathbf{y} + c\mathbf{s})$ for short $\mathbf{y}$, with rejection sampling

Verify: check whether $c = H(\mathbf{A}\mathbf{z} - c\mathbf{t} \bmod q, m)$ and $\mathbf{z}$ is short.

Correctness of FSwA:

- $\mathbf{y}$, $\mathbf{s}$: short, and $c = H(\cdot)$: binary $\Rightarrow c\mathbf{s}$: short. $\Rightarrow \mathbf{z} = \mathbf{y} + c\mathbf{s}$: short.
- $\mathbf{A}\mathbf{z} - c\mathbf{t} = \mathbf{A}(\mathbf{y} + c\mathbf{s}) - c\mathbf{t} = \mathbf{A}\mathbf{y} + c(\mathbf{A}\mathbf{s} - \mathbf{t}) = \mathbf{A}\mathbf{y} \bmod q$.

Lattice-based signatures

Fiat-Shamir with abort:

Key: ($\mathbf{s}$: small secret, $\mathbf{t} = \mathbf{A}\mathbf{s} \bmod q$: public)

Sign: $(c = H(\mathbf{A}\mathbf{y} \bmod q,\ m),\ \mathbf{z} = \mathbf{y} + c\mathbf{s})$ for short $\mathbf{y}$, with rejection sampling

Verify: check whether $c = H(\mathbf{A}\mathbf{z} - c\mathbf{t} \bmod q, m)$ and $\mathbf{z}$ is short.

Unforgeability of FSwA:

- **key** is secure $\Leftarrow$ Module-LWE,
- signature $(c, \mathbf{z})$ **do not leak the secret** $\mathbf{s}$ due to rejection sampling,
- **no new signatures** can be sampled **without** $\mathbf{s}$ $\Leftarrow$ Module-SIS,

even in the use of quantum algorithms.

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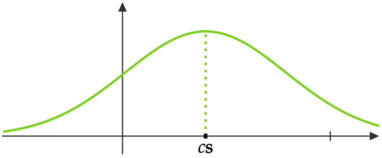
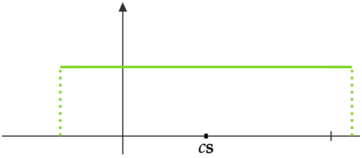
Rejection sampling

Leakage from $(c, \mathbf{z} = \mathbf{y} + c\mathbf{s})$?

With ∞ pairs of $(c, \mathbf{z} = \mathbf{y} + c\mathbf{s})$, we can collect $\mathbf{z}$ for the same c:



$y \leftarrow \mathcal{N}(0, \sigma^2)$



$y \leftarrow U[-a, a]$

$\Rightarrow$ Recover $\mathbf{s}$ from $c\mathbf{s}$.

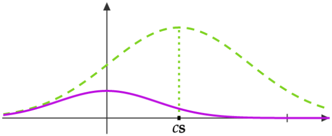
Rejection sampling

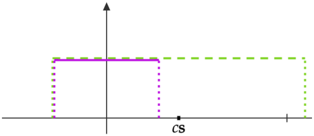
Rejection sampling

$$D_{\text{source}} = \{(c, \mathbf{z})\} \xrightarrow[\text{prob. } p(c,\mathbf{z})]{\text{reject with}} D_{\text{target}}$$

distribution of $(c, \mathbf{z})$, possibly leak $\mathbf{s}$

new distribution, independent of $\mathbf{s}$





$y \leftarrow \mathcal{N}(0, \sigma^2)$ $y \leftarrow U[-a, a]$

Rejection sampling

The **FSwA signatures** are commonly given as follows:

1. $\mathbf{y} \leftarrow D_0$
2. $c \leftarrow H(\mathbf{A}\mathbf{y}, m)$
3. $\mathbf{z} \leftarrow \mathbf{y} + c\mathbf{s}$
4. with probability $\frac{p_{\text{target}}(c,\mathbf{z})}{M \cdot p_{\text{source}}(c,\mathbf{z})}$, return $\sigma = (c, \mathbf{z})$, else go to step 1

M: bounding factor for the probability to be ≤ 1.

Final distribution $\sim D_{\text{target}}$.

Run-time $\propto M$.

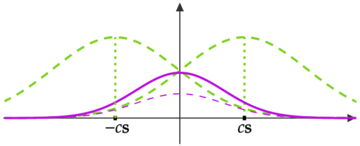
Bimodal rejection sampling

Run-time $\propto M$ ($\approx$ green area / purple area).

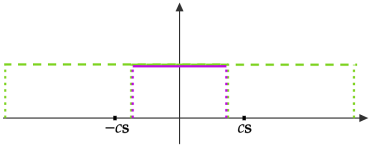
To decrease M, [DDLL13] uses

$$\mathbf{z} = \mathbf{y} + (-1)^b c\mathbf{s} \bmod 2q$$

instead of $\mathbf{z} = \mathbf{y} + c\mathbf{s} \bmod q$:



$y \leftarrow \mathcal{N}(0, \sigma^2)$



$y \leftarrow U[-a, a]$

Note, no change for the uniform case.

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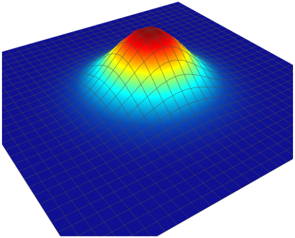
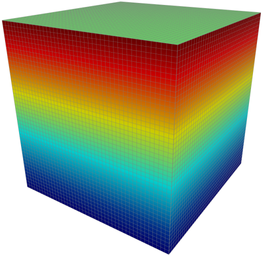
Hyperball bimodal rejection sampling

Previously, the randomness $\mathbf{y}$ was chosen from either discrete Gaussian or uniform hypercube[1].



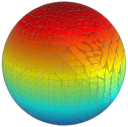


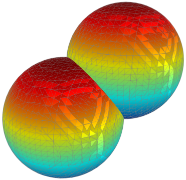
[1] The vectors $\mathbf{y}$ and $\mathbf{z}$ are high-dimensional vectors, so uniform in an interval is indeed a uniform hypercube.

Hyperball bimodal rejection sampling

We, instead, use **uniform hyperball** distribution for sampling $\mathbf{y}$ [DFPS22];

- to exploit optimal M,
- to reduce signature and verification key sizes,



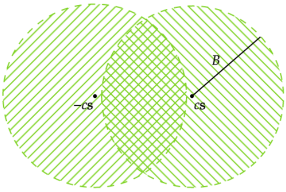


based on the **bimodal approach** [DDLL13].

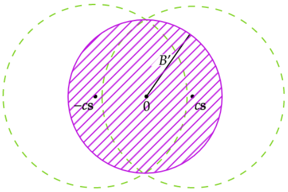
Hyperball bimodal rejection sampling

We reject $(c, \mathbf{z}) \sim D_\mathsf{s}$ (with p.d.f. p_s) to a target distribution D_t (with p.d.f. p_t), where

- p_s: uniform in hyperballs of radii B centered at $\pm c\mathbf{s}$
  - union of two large balls
- p_t: uniform in a smaller hyperball of radii B' centered at zero
  - a smaller ball in the middle



p_s



p_t

Hyperball bimodal rejection sampling

- $p_{\mathsf{s}}(\mathbf{x}) = \frac{1}{2\cdot\mathsf{vol}(\mathcal{B}(B))} \cdot \chi_{\|\mathbf{z}-c\mathbf{s}\|<B} + \frac{1}{2\cdot\mathsf{vol}(\mathcal{B}(B))} \cdot \chi_{\|\mathbf{z}+c\mathbf{s}\|<B}$,
- $p_{\mathsf{t}}(\mathbf{x}) = \frac{1}{\mathsf{vol}(\mathcal{B}(B))} \cdot \chi_{\|\mathbf{z}\|<B'}$.

$$\Rightarrow p(\mathbf{x}) = \frac{p_{\mathsf{t}}(\mathbf{x})}{M \cdot p_{\mathsf{s}}(\mathbf{x})} = \frac{\chi_{\|\mathbf{z}\|<B'}}{\chi_{\|\mathbf{z}-c\mathbf{s}\|<B} + \chi_{\|\mathbf{z}+c\mathbf{s}\|<B}}$$

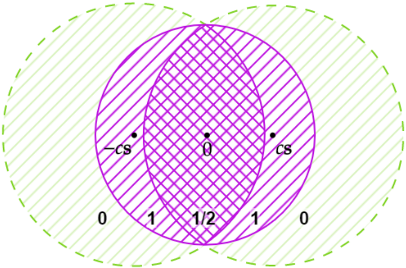
$$= \begin{array}{cl} 0 & \text{if } \mathbf{z} \notin \mathcal{B}(B'), \\ 1/2 & \text{if } \mathbf{z} \in \mathcal{B}(B') \cap \mathcal{B}(B, c\mathbf{s}) \cap \mathcal{B}(B, -c\mathbf{s}), \\ 1 & \text{if } \mathbf{z} \in \mathcal{B}(B') \setminus (\mathcal{B}(B, c\mathbf{s}) \cap \mathcal{B}(B, -c\mathbf{s})), \end{array}$$

for some $M > 0$.

Hyperball bimodal rejection sampling

That is, we return $\mathbf{x} = (c, \mathbf{z})$ with probability

- 0: if $\|\mathbf{z}\| \geq B'$,
- 1/2: else if $\|\mathbf{z} - c\mathbf{s}\| < B$ and $\|\mathbf{z} + c\mathbf{s}\| < B$,
- 1: otherwise.



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Comparison to SotA lattice signatures.

For 120-bit classical security. Sizes are in bytes.

Scheme	*sig*	*vk*	KeyGen	Sign	
				sampling	rejection
Dilithium-2	2420	1312	fast	Hypercube	$\|\cdot\|_\infty < B$
Bliss-1024[2]	1700	1792	fast	dGaussian at 0	reject with prob. f(sk, Sig)
HAETAE120	1468	1056	fast	dHyperball at 0	$\|\cdot\|_2 < B$
Mitaka-512[3]	713	896	slow	dGaussian at 0 & intGaussian at $H(m)$	none
Falcon-512	666	897	slow	dGaussian at $H(m)$	none

Table: Comparison between different lattice-based signature schemes.

[2] modified Bliss (to ≥ 120 bit-security) in Dilithium paper.

[3] Mitaka-512 has 102 bits of security

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Current Status

NIST PQC

- Competition for USA standard PQC schemes.
- HAETAE is one of the candidates in *Additional Signatures* track.

KPQC

- Competition for Korean standard PQC schemes.
- HAETAE is advanced to Round 2, one of four candidates in *Digital Signatures* track.

Thank you!

Any question?

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HAETAE description (high-level)

KeyGen(1^λ)

1: $\mathbf{A}_{\text{gen}} \leftarrow \mathcal{R}_q^{k\times(\ell-1)}$ and $(\mathbf{s}_{\text{gen}}, \mathbf{e}_{\text{gen}}) \leftarrow S_\eta^{\ell-1} \times S_\eta^k$
2: $\mathbf{b} = \mathbf{A}_{\text{gen}} \cdot \mathbf{s}_{\text{gen}} + \mathbf{e}_{\text{gen}} \in \mathcal{R}_q^k$
3: $\mathbf{A} = (-2\mathbf{b} + q\mathbf{j} \mid 2\mathbf{A}_{\text{gen}} \mid 2\mathbf{Id}_k) \bmod 2q$ and write $\mathbf{A} = (\mathbf{A}_1 \mid 2\mathbf{Id}_k)$
4: $\mathbf{s} = (1, \mathbf{s}_{\text{gen}}, \mathbf{e}_{\text{gen}})$
5: **if** $\sigma_{\max}(\text{rot}(\mathbf{s}_{\text{gen}})) > \gamma$, then restart
6: Return $\text{sk} = \mathbf{s}$, $\text{vk} = \mathbf{A}$

Sign(sk, M)

1: $\mathbf{y} \leftarrow U(\mathcal{B}_{(1/N)\mathcal{R},(k+\ell)}(B))$
2: $c = H(\text{HighBits}_{2q}^{\text{hint}}(\mathbf{A}\lfloor \mathbf{y} \rceil, \alpha), \text{LSB}(\lfloor y_0 \rceil), M) \in \mathcal{R}_2$
3: $\mathbf{z} = (\mathbf{z}_1, \mathbf{z}_2) = \mathbf{y} + (-1)^b c \cdot \mathbf{s}$ for $b \leftarrow U(\{0,1\})$
4: $\mathbf{h} = \text{HighBits}_{2q}^{\text{hint}}(\mathbf{A}\lfloor \mathbf{z} \rceil - qc\mathbf{j}, \alpha) - \text{HighBits}_{2q}^{\text{hint}}(\mathbf{A}_1 \lfloor \mathbf{z}_1 \rceil - qc\mathbf{j}, \alpha) \bmod^+ \frac{2(q-1)}{\alpha}$
5: **if** $\|\mathbf{z}\|_2 \geq B'$, then restart
6: **if** $\|2\mathbf{z} - \mathbf{y}\|_2 < B$, then restart with probability $1/2$
7: Return $\sigma = (\text{Encode}(\text{HighBits}(\lfloor \mathbf{z}_1 \rceil, a)), \text{LowBits}(\lfloor \mathbf{z}_1 \rceil, a), \text{Encode}(\mathbf{h}), c)$

Verify$(\text{vk}, M, \sigma = (x, \mathbf{v}, h, c))$

1: $\tilde{\mathbf{z}}_1 = \text{Decode}(x) \cdot a + \mathbf{v}$ and $\tilde{\mathbf{h}} = \text{Decode}(h)$
2: $\mathbf{w} = \tilde{\mathbf{h}} + \text{HighBits}_{2q}^{\text{hint}}(\mathbf{A}_1 \tilde{\mathbf{z}}_1 - qc\mathbf{j}, \alpha) \bmod^+ \frac{2(q-1)}{\alpha}$
3: $w' = \text{LSB}(\tilde{z}_0 - c)$
4: $\tilde{\mathbf{z}}_2 = \left[\mathbf{w} \cdot \alpha + w'\mathbf{j} - (\mathbf{A}_1 \tilde{z}_1 - qc\mathbf{j})\right]/2 \bmod^{\pm} q$
5: $\tilde{\mathbf{z}} = (\tilde{\mathbf{z}}_1, \tilde{\mathbf{z}}_2)$
6: Return $(\ c = H(\mathbf{w}, w', M)\ ) \wedge (\ \|\tilde{\mathbf{z}}\| < B''\ )$