## HAETAE: Rejecting on Hyperballs

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## **HAETAE**

- Digital signature scheme, submitted to KpqC competition.
- Secure against quantum attacks
  - based on lattice hard problems, MLWE and MSIS
  - follows Fiat-Shamir with aborts framework, secure in QROM
- Goal:

#### Push Fiat-Shamir Signatures to the Limits!

Scheme	LvI.	Sig.	vk	ConstT.	Maskable
Falcon-512	1	666B	897B	✓ [Por19]	✗ [Pre23]
Dilithium-2	2	2,420B	1,312B	√ [DKL <sup>+</sup> 18a]	√ [MGTF19]
HAETAE-120	2	1,463B	992B	<b>√</b>	✓

Table: NIST security level, signature size, verification key size, and implementation security, with respect to constant-time and masking of selected signature schemes.

## **HAETAE**

- Simple but short
  - simpler than Falcon<sup>1</sup> & shorter than Dilithium<sup>1</sup>
  - optimal rejection rate with simple rejection condition
- Design rationale: We combine the recent approaches,
  - Fiat-Shamir with Aborts framework
  - Bimodal rejection sampling
  - randomness sampling from Hyperball distribution

### with the NEW techniques.

- secret key rejection sampling: efficient and easily maskable
- verification key truncation: in bimodal setting
- signature compression: in hyperball setting
- discretized hyperball sampling: a fixed-point implementation

<sup>&</sup>lt;sup>1</sup>NIST 2022 PQC signature standards

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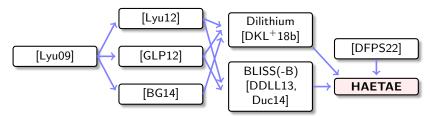
### 2. Rejection Sampling

- What is "Rejection Sampling?"
- Rejection sampling in FS signatures
- Rejection sampling in HAETAE

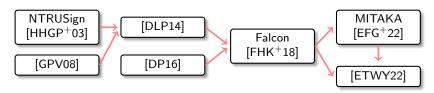
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## Lattice-based signatures

#### Fiat-Shamir with Aborts



### Hash-and-Sign



## Fiat-Shamir with Aborts

From an interactive identification protocol, FS transform provides a non-interactive ID protocol, say signature. E.g. Schnorr ID protocol  $\stackrel{FS}{\longrightarrow}$  Schnorr signature.

### Basic "Fiat-Shamir with aborts" framework [Lyu09, Lyu12]

KeyGen: output (sk = s, vk = A), where  $t = As \mod q$  and s is short.

 $\begin{aligned} \operatorname{Sign}(\mathsf{sk} = \mathbf{s}, \ m) : \ \text{for short } \mathbf{y}, \ \text{compute } c = H(\mathbf{A}\mathbf{y} \bmod q, \ m) \ \text{and} \\ \mathbf{z} = \mathbf{y} + c\mathbf{s}, \ \text{then output } (c, \ \mathbf{z}) \ \text{via rejection sampling}. \end{aligned}$ 

Verify(vk =  $\mathbf{A}$ , m): check  $c = H(\mathbf{Az} - c\mathbf{t} \mod q, m)$  and  $\mathbf{z}$  is short.

#### Correctness:

- First,  $\bf y$  and  $\bf s$  are short. Since  $c=H(\cdot)$  is binary,  $c{\bf s}$  is also short. Thus,  ${\bf z}={\bf y}+c{\bf s}$  is short.
- It holds that  $Az ct = A(y + cs) ct = Ay \mod q$  since  $As = t \mod q$ .

## Fiat-Shamir with Aborts

#### Basic "Fiat-Shamir with aborts" framework [Lyu09, Lyu12]

Sign(sk = s, m): for short y, compute  $c = H(\mathbf{Ay} \mod q, m)$  and  $\mathbf{z} = \mathbf{y} + c\mathbf{s}$ , then output  $(c, \mathbf{z})$  via rejection sampling.

### Security:

- In the interactive setting, the signature  $\mathbf{z} = \mathbf{y} + c\mathbf{s}$  can leak information about  $\mathbf{s}$  if  $\|\mathbf{y}\|$  is small. To avoid this, the noise flooding technique is generally used: setting  $\|\mathbf{y}\| \approx 2^B \cdot \|c\mathbf{s}\|$  for B bit security.
- But using noise flooding makes the signature sizes much larger.
- "Aborting", or "rejection sampling", makes it possible to have a signature distribution independent of the secret, during the FS transforms.

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## Rejection sampling

- Rejection sampling is a widely studied and used, folklore technique from probabilities<sup>2</sup>.
- In general, the signing procedure is given as:
  - 1  $\mathbf{v} \leftarrow Q_0$
  - $c \leftarrow H(\mathbf{A}\mathbf{y}, m)$
  - 3  $\mathbf{z} \leftarrow \mathbf{y} + c\mathbf{s}$
  - 4 with probability  $\min\Big(1, \frac{P(c,\mathbf{z})}{M\cdot Q(c,\mathbf{z})}\Big)$ , return  $\sigma=(c,\mathbf{z})$
  - 5 if it is not returned, go to step 1

where Q is the probability distribution of  $(c, \mathbf{z})$ .

• Assuming  $R_{\infty}(P\|Q) \leq M$  for some M>0, the distribution of the signature in step 3  $(\sigma \sim Q)$ , turns into a distribution independent of s  $(\sigma \sim P)$ .

<sup>&</sup>lt;sup>2</sup> Julein Devevey, On Rejection Sampling in Lyubashevsky's Signature Scheme, Journées Codage et Cryptographie — Hendaye, 2022.

Rejection sampling strategy can be rewritten as:

Given access to  $X_1, X_2, \cdots \xleftarrow{i.i.d.} Q$ , it is a family of randomized algorithms

$$\mathcal{A}_i : \mathsf{supp}(Q)^i \to [i] \cup \{\bot\},$$

finding the smallest  $i^*$  such that  $X_{i^*}$  is distributed following P, by defining

$$\mathcal{A}_i: (X_1, \cdots, X_i) \mapsto \left\{ egin{array}{l} i ext{ with prob.} & rac{P(X_i)}{R_{\infty}(P||Q) \cdot Q(X_i)}, \\ ot & ext{otherwise}, \end{array} 
ight.$$

from  $i=1,\cdots$ , which ends if  $\mathcal{A}_i \to i (=i^*)$ , then finally outputs  $X_{i^*}$ .

Cf. Short recap on Rényi divergence:  ${}^3$ for  $supp(P) \subseteq supp(Q)$ ,

$$R_{\infty}(P||Q) := \sup_{x \in \text{supp}(P)} P(x)/Q(x).$$

<sup>&</sup>lt;sup>3</sup>We can also consider  $supp(P) \not\subseteq supp(Q)$ , say smooth Rényi, but not here.

• Running time: the expected run-time is  $\mathbb{E}[i^*]$  since it ends when  $\mathcal{A}_i$  outputs i. A quick computation shows  $\mathbb{E}[i^*] = R_{\infty}(P||Q)$ :

$$\begin{split} \Pr[\mathcal{A}_i \to i] &= \sum_{x_i} Q(x_i) \cdot \frac{P(x_i)}{R_{\infty}(P\|Q) \cdot Q(x_i)} = R_{\infty}(P\|Q)^{-1} (\mathsf{let}, = p), \\ \mathbb{E}[i^*] &= \sum_{i \geq 1} i \cdot \Pr[i^* = i] \\ &= \sum_{i \geq 1} i \cdot \Pr[(\mathcal{A}_1, \cdots, \mathcal{A}_{i-1} \to \bot) \wedge (\mathcal{A}_i \to i)] \\ &= \sum_{i \geq 1} i \cdot p \cdot (1 - p)^{i-1} = p^{-1} = R_{\infty}(P\|Q). \end{split}$$

• Distribution of final output  $X_{i^*}$ : the probability density function of the final output becomes P:

$$\begin{aligned} \mathsf{pdf}[X_{i^*} = x] &= \sum_{i \geq 1} \Pr[\mathcal{A}_1, \cdots, \mathcal{A}_{i-1} \to \bot] \cdot \Pr[(\mathcal{A}_i \to i) \land (X_i = x)] \\ &= \sum_{i \geq 1} (1 - p)^{i-1} \cdot Q(x) \cdot \frac{P(x)}{R_{\infty}(P \parallel Q) \cdot Q(x)} \\ &= P(x) \cdot \sum_{i \geq 1} p(1 - p)^{i-1} = P(x). \end{aligned}$$

So far, the transcripts (the final output) and the run-time (the number of iterations) of the rejection sampling strategy and that of the following algorithm are indistinguishable:

Given access to  $X \leftarrow P$ , it samples  $X \leftarrow P$ , and outputs X with probability  $R_{\infty}(P||Q)^{-1}$ , else re-sample it and repeat.

- run-time:  $R_{\infty}(P\|Q)$ ,
- final output:  $X \leftarrow P$ .

### Three simple facts:

- the same thing holds in the continuous domain,
- the Rényi divergence in the denominator can be replaced by M>0 such that  $R_{\infty}(P\|Q) \leq M$ ,
- more analysis is needed if we set a bound on  $i^*$ , say **bounded rejection**.

Hence, if  $R_{\infty}(P||Q) \leq M < \infty$ , the following two games are indistinguishable:

$\mathcal{A}^{real}$ :	$\mathcal{A}^{ideal}$ :
1: $\mathbf{x} \leftarrow Q$	1: $\mathbf{x} \leftarrow P$
2: Return $\mathbf{x}$ with probability $\frac{P(\mathbf{x})}{M \cdot Q(\mathbf{x})}$	2: Return ${f x}$ with probability ${1\over M}$
3: Else repeat 1–2	3: Else repeat 1–2

### Imperfect rejection:

- Similar thing holds also for  $M \approx R_{\infty}(P\|Q)$  or for smooth-Rényi divergence, i.e., when  $\operatorname{supp}(P) \not\subseteq \operatorname{supp}(Q)$ , with some statistical distance between the outputs.
- Since the fraction could have a value larger than 1, it should be replaced by  $\min\left(\frac{P(\mathbf{x})}{M\cdot Q(\mathbf{x})},1\right)$ .

Cf. HAETAE uses the perfect, unbounded rejection.

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## Rejection sampling in FS signatures

- The **FS signatures** are commonly given as follows:
  - 1  $\mathbf{y} \leftarrow Q_0$
  - $\mathbf{2} \ c \leftarrow H(\mathbf{A}\mathbf{y}, m)$
  - 3  $\mathbf{z} \leftarrow \mathbf{y} + c\mathbf{s}$
  - 4 with probability  $\min\left(1, \frac{P(c, \mathbf{z})}{M \cdot Q(c, \mathbf{z})}\right)$ , return  $\sigma = (c, \mathbf{z})$ , else go to step 1
- The ideal signing can be given as:
  - 1  $c \leftarrow U(\mathcal{C})$
  - $\mathbf{z} \leftarrow P^z$
  - 3 with probability 1/M, return  $(c, \mathbf{z})$ , else go to step 1
- In the simulation-based proofs, the hash can be reprogrammed, and the challenge sampling can be treated as  $c \leftarrow U(\mathcal{C})$ .
- Then, it can be seen as  $Q = Q_{cs} \otimes U(\mathcal{C})$  and  $P = P^z \otimes U(\mathcal{C})$ .
- Then, the real and ideal signing algorithms are indistinguishable.

## Rejection sampling in FS signatures

- The **FS** signatures are commonly given as follows:
  - 1  $\mathbf{y} \leftarrow Q_0$
  - 2  $c \leftarrow H(\mathbf{A}\mathbf{y}, m)$
  - 3  $\mathbf{z} \leftarrow \mathbf{y} + c\mathbf{s}$
  - 4 with probability  $\min\Big(1, \frac{P(c,\mathbf{z})}{M \cdot Q(c,\mathbf{z})}\Big)$ , return  $\sigma = (c,\mathbf{z})$ , else go to step 1
- The ideal signing can be given as:
  - 1  $c \leftarrow U(\mathcal{C})$
  - $\mathbf{z} \leftarrow P^z$
  - 3 with probability 1/M, return  $(c, \mathbf{z})$ , else go to step 1
- Remark 1. The aborted transcripts can even be simulated [DFPS23].

Remark 2. The rewinding and reprogramming can not be directly treated in the QROM (see [DFPS23] for the recently corrected proof of [KLS18] using remark 1).

## Rejection sampling in FS signatures

One important thing in practice is accepting a signature with probability  $\frac{P(c,\mathbf{z})}{M\cdot Q(c,\mathbf{z})} = \frac{P^z(\mathbf{z})}{M\cdot Q_{cs}(\mathbf{z})}$ , which is also a challenging point.

• In [Lyu09] and Dilithium [DKL<sup>+</sup>18b], the uniform distributions in hypercubes are used both for  $Q_0$  and  $P^z$ , making it

$$\frac{P(c,\mathbf{z})}{M\cdot Q(c,\mathbf{z})} = \frac{\frac{1}{|I|^n}\cdot \chi(\mathbf{z}\in I^n)}{M\cdot \frac{1}{|J|^n}\cdot \chi(\mathbf{z}\in (J^n+c\mathbf{s}))} = \left\{ \begin{array}{l} 1 & \text{if } \mathbf{z}\in I^n\cap (J^n+c\mathbf{s}) \\ 0 & \text{otherwise} \end{array} \right.,$$

where I and J are appropriate intervals, and  $\chi$  is a characteristic function.

• In [Lyu12] and Bliss [DDLL13]<sup>4</sup>, the n-dimensional discrete Gaussian distributions are used. As a result, aborting the signature with Gaussian probability makes it hard to implement [EFGT17].

In fact, a bit different due to bimodal distribution

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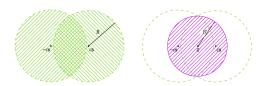
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## Hyperball bimodal rejection sampling

In HAETAE, we instead, use  $uniform\ hyperball\ distribution\ for\ sampling\ y$  following [DFPS22];

- $Q_{cs}$  becomes a uniform distribution over a union of hyperballs with an intersection,  $\mathcal{HB}_{-cs}(B) \cup \mathcal{HB}_{cs}(B)$ ,
- P becomes a hyperball uniform distribution,  $\mathcal{HB}_{-cs}(B')$ ,

as shown below.



Distribution of  $Q_{cs}$  and P.

Remark. The purple hyperball should be included in **every**  $\mathcal{HB}_{-cs}(B) \cup \mathcal{HB}_{cs}(B)$  for the perfect rejection.

## Hyperball bimodal rejection sampling

The use of hyperball distribution makes it possible

- ullet to exploit optimal rejection rate,  $\mathbb{E}[i^*]$ ,
- ullet to reduce signature sizes,  $\mathbb{E}[\|\mathbf{x}\|]$ ,





Figure: Distribution of P and Q

### and use the bimodal approach [DDLL13];

- for more compact signature sizes,
- but with a simpler rejection condition, which leads to the easier implementation of secure rejection.

## Hyperball bimodal rejection sampling: detailed analysis

The distributions can be expressed as follows:

• 
$$Q_{c\mathbf{s}}(\mathbf{z}) = \frac{1}{2} \cdot \frac{1}{\text{vol}(\mathcal{HB}(B))} \cdot \chi(\|\mathbf{z} - c\mathbf{s}\| < B) + \frac{1}{2} \cdot \frac{1}{\text{vol}(\mathcal{HB}(B))} \cdot \chi(\|\mathbf{z} + c\mathbf{s}\| < B),$$

• 
$$P(\mathbf{z}) = \frac{1}{\text{vol}(\mathcal{HB}(B))} \cdot \chi(\|\mathbf{z}\| < B').$$

This leads to

$$\begin{split} \frac{P(\mathbf{z})}{M \cdot Q_{c\mathbf{s}}(\mathbf{z})} &= \frac{\chi(\|\mathbf{z}\| < B')}{\chi(\|\mathbf{z} - c\mathbf{s}\| < B) + \chi(\|\mathbf{z} + c\mathbf{s}\| < B)} \\ &= \begin{cases} 0 & \text{if } \mathbf{z} \notin \mathcal{HB}(B'), \\ 1/2 & \text{if } \mathbf{z} \in \mathcal{HB}(B') \cap \mathcal{HB}_{c\mathbf{s}}(B) \cap \mathcal{HB}_{-c\mathbf{s}}(B), \\ 1 & \text{if } \mathbf{z} \in \mathcal{HB}(B') \setminus (\mathcal{HB}(B, c\mathbf{s}) \cap \mathcal{HB}(B, -c\mathbf{s})) \end{cases} \end{split}$$

for some M > 0.

# Hyperball bimodal rejection sampling

That is, we return  $\mathbf{x} = (c, \mathbf{z})$  with probability

- 0: if  $\|\mathbf{z}\| \ge B'$ ,
- 1/2: else if  $\|\mathbf{z} c\mathbf{s}\| < B$  and  $\|\mathbf{z} + c\mathbf{s}\| < B$ ,
- 1: otherwise.

Since  $\mathbf{z} = \mathbf{y} + (-1)^b c\mathbf{s}$ , we can do this without using  $\mathbf{s}$ ,

ng s, 0 1

- if  $\|\mathbf{z}\| \geq B'$ , reject,
- else if  $||2\mathbf{z} \mathbf{y}|| < B$ , reject with probability 1/2,
- otherwise, accept,

resulting in a signature, distributed uniform in a hyperball  $\mathcal{HB}(B')$ .

 $<sup>^{5}\{\</sup>mathbf{z}\pm c\mathbf{s}\} = \{\mathbf{y}, 2\mathbf{z} - \mathbf{y}\} \text{ and always } \|\mathbf{y}\| < B.$ 

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## **Updates**

After submitting to KpqC Round 1, we had many further improvements, consisting of

- Missing parts inclusion: rANS encoding, rejection sampling for secret key sampling,
- $\bullet$  New compressions: public key truncation and updated signature (especially the hint vector h) compression,
- New secret key rejection: security was underestimated due to a non-tight bound for  $\|c\mathbf{s}\|$ ,
- Fully discretized hyperball: bound the statistical distance between 'continuous' and 'discretized' hyperballs and their effects on security,
- and some minor updates, adapted from Dilithium and others.

Considering the above changes, we update the parameters and implementation.

## **Updates**

### Implementation:

- Fixed-Point and Constant-Time<sup>6</sup>,
- Easily Maskable!: detailed analysis is given in ia.cr/2023/624, and the masked implementation is ongoing,

#### Sizes and Performance:

		Sizes (bytes)		Cycles (med)		
Param. set	LvI.	Sig.	vk	KeyGen	Sign	Verify
HAETAE-120/Dilithium-2	2	60%	76%	408%	548%	106%
HAETAE-180/Dilithium-3	3	71%	75%	383%	484%	123%
HAETAE-260/Dilithium-5	5	63%	80%	181%	363%	94%
Falcon-512/HAETAE-120	1/2	46%	90%	3,885%	277%	27%
Falcon-1024/HAETAE-260	5	44%	86%	9,110%	423%	25%

Table: Relative comparison between HAETAE, Dilithium, and Falcon using their constant-time reference implementation<sup>7</sup>.

<sup>&</sup>lt;sup>6</sup>available at HAETAE website: kpqc.cryptolab.co.kr.

<sup>&</sup>lt;sup>7</sup>not yet optimized, yet ongoing with some basic optimizations.

Thanks!

Any question?

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