Solution:

Squaring: We first show how to use the calculator to calculate x^2 for any given x. If $x = 0$ or $x = 1$, then $x^2 = x$ so no computation is needed. Otherwise, observe that

$$\frac{1}{x-1} - \frac{1}{x} = \frac{1}{x^2 - x}$$

so

$$\left(\frac{1}{\dfrac{1}{x-1} - \dfrac{1}{x}} \right) + x = x^2.$$

Halving: Given any $x \neq 0$, we can use the calculator to compute $\dfrac{x}{2}$ by observing

$$\frac{1}{\dfrac{1}{x} + \dfrac{1}{x}} = \frac{x}{2}$$

Products: Now given x and y, we can compute xy using the calculator. We have

$$xy = \left(\frac{x+y}{2}\right)^2 - \left(\frac{x-y}{2}\right)^2$$

and the terms on the left hand sides can be computed by first determining each of $x \pm y$, halving (which can be done as shown above), squaring each (which can be done as shown above), and adding the results. We must, however, be careful that none of the operations involve taking a quotient by 0. This amounts to determining what to do when $x + y, x - y \in \{0, 1\}$. If $x \pm y = 0$, then $xy = x^2$ or $-x^2$, each of which can be taking care of by squaring (and potentially negating afterward). If $x + y = 1$, then $xy = x(1 - x)$ which we can compute as in the discussion on squaring. If $x + y = -1$, then $xy = -x(1 + x)$, so we can again apply the squaring technique (unless $x = -1$ in which case the product is 0 anyway).

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