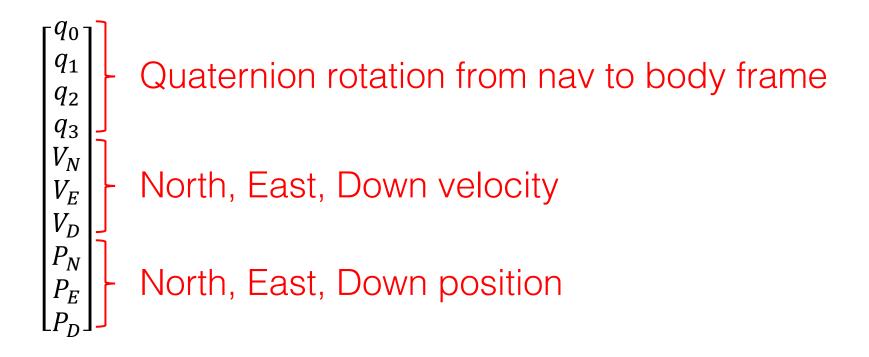
Filter Overview

- 24 State EKF
 - Quaternions (Q0,...,Q3)
 - Velocity (NED)
 - Position (NED)
 - Gyro delta angle bias vector (XYZ)
 - Accelerometer bias (XYZ)
 - Earth magnetic field vector (NED)
 - Magnetometer bias errors (XYZ)
 - Wind Velocity (NE)
- Uses the following sensors
 - Inertial Measurement Unit angular rates and specific forces
 - GPS position (in local NED frame)
 - GPS velocity (in local NED frame)
 - Pressure altitude
 - 3-axis magnetometer

 Pose information is captured in the first 10 states which use a dynamic process model that defines the movement of the body frame (XYZ RH axis system) in a navigation inertial reference frame (North, East, Down)



• The first four states are the quaternions that define the angular position of the XYZ body frame relative to NED navigation frame.

$$\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

• The rotation matrix from body to navigation frame is given by:

$$[T]_B^N = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1 \cdot q_2 - q_0 \cdot q_3) & 2(q_1 \cdot q_3 + q_0 \cdot q_2) \\ 2(q_1 \cdot q_2 + q_0 \cdot q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2 \cdot q_3 - q_0 \cdot q_1) \\ 2(q_1 \cdot q_3 - q_0 \cdot q_2) & 2(q_2 \cdot q_3 + q_0 \cdot q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

• Where the rotation from navigation to **body** frame is required, the transpose will be used and is denoted by $[T]_N^B$

• The truth delta angles are calculated from the IMU measurements and delta angle bias states Δ_{ang_bias}

$$\Delta_{ang_meas} = \begin{bmatrix} \Delta_{ang_x} \\ \Delta_{ang_y} \\ \Delta_{ang_z} \end{bmatrix} = \int_{t_k}^{t_{k+1}} \omega \cdot dt$$

$$\Delta_{ang_bias} = \begin{bmatrix} \Delta_{ang_bias_x} \\ \Delta_{ang_bias_y} \\ \Delta_{ang_bias_z} \end{bmatrix} \text{ Delta angle bias states }$$

$$\Delta_{ang_truth} = \Delta_{ang_meas} - \Delta_{ang_bias}$$

• The truth delta velocities are calculated from the IMU measurements and delta velocity states Δ_{vel_bias}

$$\Delta_{vel_meas} = \begin{bmatrix} \Delta_{vel_x} \\ \Delta_{vel_y} \\ \Delta_{vel_z} \end{bmatrix}$$
 Delta angle IMU measurements

$$\Delta_{vel_bias} = \begin{bmatrix} \Delta_{vel_bias_x} \\ \Delta_{vel_bias_y} \\ \Delta_{vel_bias_z} \end{bmatrix} \text{ Delta angle bias states}$$

$$\Delta_{vel_truth} = \Delta_{vel_meas} - \Delta_{vel_bias}$$

• The quaternion Δ_{quat} that defines the rotation from the quaternion at frame k to k+1 is calculated from the truth delta angle Δ_{ang_truth} using a small angle approximation. The inertial navigation uses the exact method.

$$\Delta_{quat} = \begin{bmatrix} \Delta q_0 \\ \Delta q_1 \\ \Delta q_2 \\ \Delta q_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\Delta_{ang_truth_x}} \\ \frac{\Delta_{ang_truth_y}}{2} \\ \frac{\Delta_{ang_truth_z}}{2} \end{bmatrix}$$

• The quaternion product rule is used to rotate the quaternion state forward by the delta quaternion Δ_{quat} from frame k to k+1.

$$\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}_{k+1} = \begin{bmatrix} q_0 \Delta q_0 - q_1 \Delta q_1 - q_2 \Delta q_2 - q_3 \Delta q_3 \\ q_0 \Delta q_1 + \Delta q_0 q_1 + q_2 \Delta q_3 - \Delta q_2 q_3 \\ q_0 \Delta q_2 + \Delta q_0 q_2 - q_1 \Delta q_3 + \Delta q_1 q_3 \\ q_0 \Delta q_3 + \Delta q_0 q_3 + q_1 \Delta q_2 - \Delta q_1 q_2 \end{bmatrix}$$

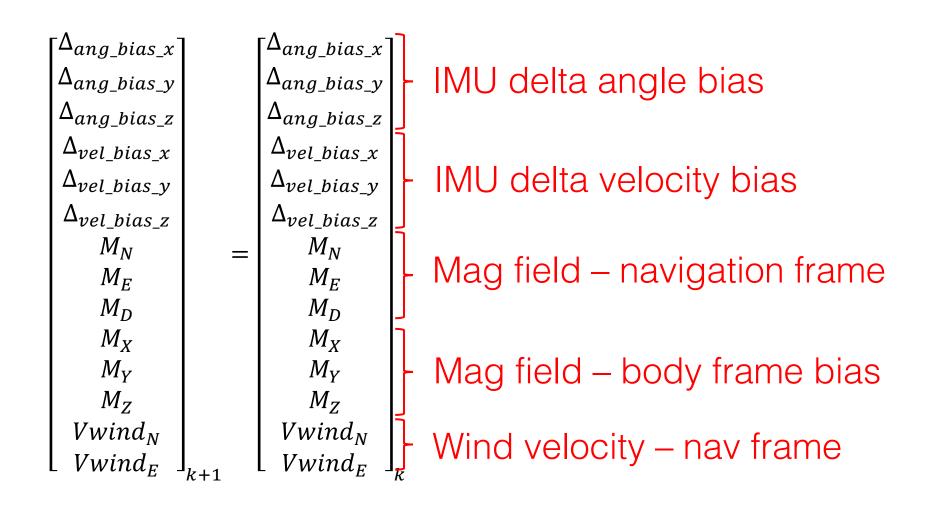
 The truth delta velocity vector is rotated from body frame to earth frame and gravity is subtracted to calculate the change in velocity states from frame k to k+1

$$\begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix}_{k+1} = \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix}_k + [T]_B^N \cdot \Delta_{vel_truth} + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \cdot \Delta t$$

 The position states are updated using Euler integration (the inertial navigation uses a more accurate trapezoidal integration method)

$$\begin{bmatrix} P_N \\ P_E \\ P_D \end{bmatrix}_{k+1} = \begin{bmatrix} P_N \\ P_E \\ P_D \end{bmatrix}_k + \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix}_k \cdot \Delta t$$

 The IMU sensor bias, magnetic field and wind states all use a static process model



- The GPS position, Baro height and GPS velocity involve direct observation of states, so the observation model is trivial.
- The magnetomer is assumed to be aligned with the body frame and experiences a magnetic field vector which is the sum of a navigation frame filed rotted into body frame and a body frame fixed field.

$$\begin{bmatrix} M_X \\ M_Y \\ M_Z \end{bmatrix}_{meas} = [T]_N^B \cdot \begin{bmatrix} M_N \\ M_E \\ M_D \end{bmatrix} + \begin{bmatrix} M_X \\ M_Y \\ M_Z \end{bmatrix}_{bias}$$

We can also use the earth frame magnetic field declination as an observation

$$\psi_{DECLINATION} = \tan^{-1} \left(\frac{M_E}{M_N} \right)$$

This can be used to prevent unwanted yaw rotation of the earth field estimates during periods when heading is poorly observable.

 We can also use the rotation matrix elements to provide a direct yaw observation model using either a 321 or 312 Euler sequence

$$\psi_{321} = \tan^{-1} \left(\frac{2(q_1 \cdot q_2 + q_0 \cdot q_3)}{q_0^2 + q_1^2 - q_2^2 - q_3^2} \right)$$

$$\psi_{312} = \tan^{-1} \left(\frac{-2(q_1 \cdot q_2 - q_0 \cdot q_3)}{q_0^2 - q_1^2 + q_2^2 - q_3^2} \right)$$

By selecting the appropriate transformation, a direct heading measurement can be used that avoids gimbal lock.

• The optical flow observation equation assumes a sensor aligned with the Z body frame at a distance R from a stationary scene in the navigation frame.

$$\begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix} = [T]_N^B \cdot \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix}$$

$$\begin{bmatrix} LOS_X \\ LOS_Y \end{bmatrix}_{meas} = \begin{bmatrix} \frac{-V_Y}{R} \\ \frac{V_X}{R} \end{bmatrix}$$

• The visual odometry observation equation assumes measurement of velocity states rotated into the body frame:

$$\begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix}_{meas} = [T]_N^B \cdot \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix}$$

 The airspeed observation equation assumes a sensor that measures the magnitude of velocity relative to the wind field:

$$\begin{bmatrix} Vrel_{N} \\ Vrel_{E} \\ Vrel_{D} \end{bmatrix} = \begin{bmatrix} V_{N} \\ V_{E} \\ V_{D} \end{bmatrix} - \begin{bmatrix} Vwind_{N} \\ Vwind_{E} \\ 0 \end{bmatrix}$$

$$TAS_{meas} = \sqrt{(Vrel_{N}^{2} + Vrel_{E}^{2} + Vrel_{D}^{2})}$$