

Feasible Agile Maneuver Identification and Generation Algorithms on Multi Modal Control Framework

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Multi Modal Control Framework for Agile Maneuvering UCAVs, consists of decomposing the arbitrary maneuvers into a sequence of maneuver modes and maneuver parameters (modal inputs), where each mode can be locally controlled via an assigned controller, thus resulting on a switched control system. Two algorithms are presented for this framework in this paper, to work with a path planner. First algorithm called “Maneuver Identification Algorithm”, takes a well defined maneuver and converts into the modal sequence in the proposed motion language without checking its feasibility. Second algorithm “Feasible Maneuver Generation” algorithm, takes a flight trajectory in 3D plane and imposes the sequential and envelope constraints (via a mode transition table and agility metrics) to create a feasible maneuver sequence. Both algorithms are tested via agile combat maneuvers.

I. Introduction

Generation and control of agile maneuver profiles, began to study heavily especially in the last decade due to fact that most Unmanned Combat Air Vehicles (UCAVs) are in threat for limited maneuvering capability in complex battlefield scenarios. In this sense, agility is referred as being able to execute controllable maneuvers under high “g” forces on complex flight trajectories, very much like piloted fighter aircrafts do. It is of critical importance to develop autonomous UCAV systems, which can plan and execute agile maneuvers in full flight envelope in order to avoid the threats in battlefield. In this study, focus will be on maneuver planning part on the multi modal control framework.

Multi Modal control framework was first proposed by authors in²³, which basically consists of decomposing the arbitrary maneuvers into set of maneuver modes and associated maneuver parameters. The main aim of the work was to help to reduce complexity of the both planning and control part. Complexity of Path/Motion planning part has been reduced by reducing the dimension of the problem (modal sequence has strictly lower dimension than state space description) and control part was relaxed by designing specific controllers for each mode and switch between them, in order track maneuver mode sequence instead of designing a single controller for maneuver tracking over full flight envelope. General approach of multi modal control framework; such as inspiration from aerobatics and combat maneuvers, main challenges in decomposing and generating maneuver sequences, switched control system and conversion of the system into hybrid system form, was discussed in²⁴. This paper extends the decomposition and generation ideas of that paper by developing structures for feasible maneuver generation algorithm.

Motion planning problem for aerospace vehicles are complicated by the fact that, planners based on optimal performance begins to fail in means of computation, when one takes into account of constraints related with dynamical equations of aircraft. Due to fact that, aircrafts state space is at least 12 dimensional, input-state search becomes too complicated; therefore such planners are only successful for vehicles with small state space dimensions¹². To reduce the complexity of this problem, motion description languages and quantized control concept have been adopted into motion planning⁵. Motion description languages, makes use of classified combination of simplified

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control laws to track generalized outputs. Most of these languages are strongly connected with the concept of hybrid systems, which in general, classifies the motion by using discrete states which switches in between according to input and state information and each discrete state having its own continuous dynamics. A subclass of such languages which is based on classification of behavior (or reaction) of the dynamic systems, has been successfully adapted for non-holonomic robotic systems⁶ More recently, closed loop hybrid control systems were developed based on linear temporal logic for the same purpose by⁷ For aerospace vehicles, a hybrid model for aircraft traffic management was developed in⁸ Study showed that, hybrid system representation gives opportunity to calculate reachable sets of the system and design hybrid control laws to drive the system to safe states⁹. Frazzoli⁹ suggested a maneuver automaton, which uses a number of feasible system trajectories to represent the building blocks of the motion plan of the aircraft, and a trajectory based (based on maneuver regulation principle) control system which asymptotically regulates the actual trajectory to the trajectory generated by maneuver automaton. However, motion plans and controllable trajectories are restricted to the library of the maneuver automaton. Such libraries can be built by using interpolation between feasible trajectories¹⁰.¹¹ extended this system for online planning of feasible trajectories in partially unknown environments by using receding horizon iterations.

Description of aircraft dynamics from hybrid system point of view has been studied previously in^{3,4,13}. These works have been successful in using the advantages of hybrid system methodology in control of both single and multiple aircrafts. However, these approaches did not include the full flight envelope dynamics of the aircraft. Specifically, both mode selection and controller design is strictly based on selected maneuvers; therefore controllability is limited^{3,4,13} to these predefined trajectories. In our work, we make use of parameterized sub maneuvers which builds up complex maneuver sequences. We show that it is possible to cover almost any arbitrary maneuver and the entire flight envelope by this approach.

Main contribution of this paper is a new perspective on maneuver/motion planning algorithms, which doesn't require any pre-build maneuver libraries and relies on the parameterized modal decomposition of arbitrary flight maneuvers. This planning method can be used on any flight trajectory over full flight envelope and due to fact that algorithms take in to account of envelope and continuity constraints on maneuvers; once a maneuver profile is created it is guaranteed that is trackable by the switched control system, since every mode is locally controllable.

Paper is organized as follows; firstly concept of maneuver modes and multi modal control framework is reviewed. Then structure of the maneuver identification algorithm is developed on the next section. Fourth section, deals with explanation of feasibility constraints and structure of the feasible maneuver generation algorithm based on these constraints. Paper ends with an agile combat maneuver example.

II. A Brief Review of Maneuver Modes, Modal Inputs and Multi Modal Control Framework

Concept of maneuver modes and modal inputs originally appeared on²³. Basically, main idea is to divide an arbitrary flight maneuver into smaller maneuver segments (called maneuver modes) and associated maneuver parameters (called modal inputs). If the maneuver modes are found properly, one can describe any maneuver by giving the maneuver mode sequence. This idea makes use of the fact that, 12 states of the conventional aircraft are not independent during all maneuvers and one does not need to give all the state trajectory of the aircraft to define a maneuver.

A simple example is given below inspired by aerobatics. One can describe Immelman turn (which is also a popular combat maneuver) by dividing the maneuver into segments like "Level Flight", "Loop in the Longitudinal Plane", "Roll around Stability Axis"... etc. By giving the associated maneuver parameters for each mode (such as the radius of the loop), and time duration of each mode; conservative state space trajectory approach is replaced by the lower dimensional maneuver mode sequence.

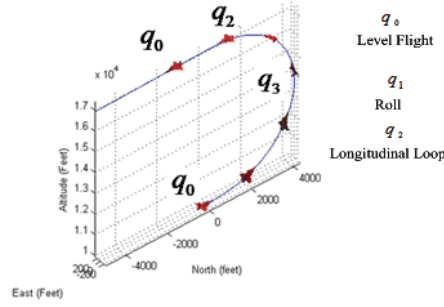


Figure 1 - Immelmann Turn decomposed into maneuver modes

A. Maneuver Modes and Modal Inputs

Complete list of modes and their modal inputs along with state constraints on each mode was given in ²⁵. We review this table below since the same modes will be used during design of the algorithms. Note that for 6 DOF flight state space variables are chosen as:

$$X = [V_T \quad \alpha \quad \beta \quad \phi \quad \theta \quad \psi \quad P \quad Q \quad R \quad n_p \quad e_p \quad h]^T \quad (1)$$

Where, V_T is the total speed, α and β are aerodynamic angles, angle of attack and sideslip angle respectively. $\Phi = [\phi \quad \theta \quad \psi]^T$ is 3-2-1 Euler angle set (due singularity of Euler angles at 90 degrees of pitch angle they may seem not convenient for agile flight, however they are still used to represent the maneuvers due to their straightforward descriptive nature, singularities can be avoided by switching them with Quaternions during integration of the differential equations on controller design). $\omega = [P \quad Q \quad R]^T$ is the angular velocity vector in the body axes, and $\rho = [n_p \quad e_p \quad h]^T$ is the set of Cartesian coordinates. Flight path Euler angles (or wind axis angles) are denoted with Φ_w . For the sake of simplicity, we assume that all flight maneuvers are coordinated (that is zero sideslip angle), but this framework can be easily extended to un-coordinated maneuvers by expanding the table 1 with extra modes where $\beta \neq 0$. Also note that, safety mode on table 1 is an artificial mode which serves as an emergency break for the control framework, in the case that aircraft goes out of domains of a particular mode or becomes unstable, it recovers the aircraft by setting it back to level flight.

Table 1 – Flight Modes and Modal Inputs

	Mode	State Constraints	Modal Inputs
q_0	Level Flight	$\dot{h} = 0, (\dot{\phi}, \dot{\theta}, \dot{\psi}) = 0$	V_T, α
q_1	Climb/Descent	$(\dot{\phi}, \dot{\theta}, \dot{\psi}) = 0$	$V_T, (\dot{h}, \theta_w)$
q_2	Roll	$(\dot{\theta}, \dot{\psi}) = 0, \beta = 0$	$V_T, \int P_w dt$
q_3	Longitudinal Loop	$(\dot{\phi}, \dot{\psi}) = 0,$	$(V_T, r_{loop}), \dot{\theta}$
q_4	Lateral Loop	$\dot{h} = 0, (\dot{\phi}, \dot{\theta}) = 0$	$(V_T, r_{loop}), \dot{\psi}$
q_5	3D Mode	$\{ \}$	V_T, P, Q, R $V_T, \phi_w, \theta_w, \psi_w$
q_6	Safety	$\{ \}$	$\{0, 1\}$

So via table 1 state trajectory $X(t)$ is replaced by the triplet $(q_i, \sigma_i, \tau_i), i = 1, 2, \dots, N$, where q_i is the i^{th} maneuver mode, σ_i is the set of modal input values associated with i^{th} mode, and τ_i is the time duration of the i^{th} mode. N is the number of maneuver modes in total. This triplet is abbreviated as simply “modal sequence”.

B. Multi Modal Control Framework

Other main advantage of maneuver decomposition methodology, other than reduction of the order of the problem, is; it gives opportunity to design specific controllers for each mode of the system. This task is natural to the system, because each set of modal inputs also serve as a reference output profile for a tracking controller. It is also obvious that if a successful controller for (and possibly nonlinear controller due to coupled nonlinear dynamics of agile maneuvers) each mode is designed, one can gain control over full flight maneuver sequence by switching the controllers. For the assignment of such a switched controller family see ²⁵, and for design of an actual system based on Higher Order Sliding Modes see ²⁴.

However to ensure that controllers are capable of tracking the maneuver, one must guarantee that maneuvers are feasible in the sense that they are executable by a piloted system, thus satisfying the saturation envelopes. An additional criterion for switching stability is the smooth connection of each mode to another in terms of kinematic parameters, if they are not; discontinuous jumps in output profiles while switching the control system can result in degradation of tracking performance or even instability.

Feasibility is a center issue in this paper and is treated in section IV, where feasible maneuver generation algorithm ensures that generated tracking output profiles are trackable by the low-level controllers.

Main aim of this paper is to generate feasible maneuvers based on this maneuver description methodology. However, before starting on this task it must be shown that, method is capable of replacing the state space trajectory description. Therefore an algorithm which converts the given flight trajectory and attitude variables (kinematic part of the state space) of the aircraft in to a modal sequence must be developed first. In the next section structure of this algorithm is shown and an example is provided.

III. Maneuver Identification Algorithm

To strengthen the ideas provided in section II, an algorithm which converts the given state space trajectory of the aircraft into a modal sequence is discussed in this section. Easiest way to describe a maneuver in state space is to give the flight trajectory plus attitude on flight trajectory. This kind of description is not often used by path planners, which can give the flight trajectory in a discrete form with constant time step, but not the attitude. So this algorithm will convert a well described maneuver into a modal sequence without checking the feasibility of the trajectory.

This algorithm assumes that a feasible kinematic state space trajectory of the maneuver is given (feasible in the sense that maneuver is executable by the aircraft, so all of the saturation and flight envelope constraints are met), and by using the state constraints on each mode in table 1, identifies the flight segments and extracts modal input from each of them.

In this section, firstly trajectory variables are transformed into wind axes, since they are more appropriate for identifying the modes, then how each mode can be identified via its state constraints is described. Then a general algorithm for converting the state trajectory is constructed and explained. Section ends with an aerobatic maneuver example.

A. Transformation to Wind Axes

In general, state space trajectory consists of 12 states that are given in eq. 1. As it is explained in the introduction of this section, we will assume that flight trajectory $(n_p(t), e_p(t), h(t))$, and attitude of the aircraft $(\theta(t), \psi(t))$ is given via a path planner algorithm or extracted from flight data. The Euler roll angle is usually

not given, because it is strongly connected to other equations and giving all six kinematic variables often results in infeasible flight maneuvers. Therefore we have to find roll angle time history from the given variables.

First, velocity variables are extracted from trajectory data in wind axes via eq. 2.

$$\begin{aligned} \psi_w(t) &= \tan^{-1} \left(\frac{\dot{e}_p}{\dot{n}_p} \right) \\ \begin{bmatrix} \dot{n}_p \\ \dot{e}_p \\ \dot{h} \end{bmatrix} &= V_T \begin{bmatrix} \cos \theta_w \cos \psi_w \\ \cos \theta_w \sin \psi_w \\ \sin \theta_w \end{bmatrix} \Rightarrow \theta_w(t) = \tan^{-1} \left(\frac{\dot{h}}{\dot{e}_p} \sin \psi_w \right) \\ V_T(t) &= \sqrt{\dot{n}_p^2 + \dot{e}_p^2 + \dot{h}^2} \end{aligned} \quad (2)$$

After solving the wind axes velocity variables, we can easily find the angle of attack and roll Euler angle from kinematic equations. We follow the work of ²⁶ here. For this purpose let the orthogonal transformation matrix be presented by $R(\cdot) \in SO(3)$, by using the transformation equations from inertial, wind and body axes we write;

$$R(-\beta, \alpha, 0).R(\psi_w, \theta_w, \phi_w) = R(\psi, \theta, \phi) \quad (3)$$

By setting sideslip angle zero, and comparing two sides of these equations which do not contain the variable ϕ_w , we write the equations in closed form;

$$\begin{aligned} \alpha &= f_\alpha(\psi_w, \theta_w, \psi, \theta) \\ \phi &= f_\phi(\psi_w, \theta_w, \psi, \theta) \end{aligned} \quad (4)$$

Now that we have angle of attack and all of the attitude angles, we can start to identify maneuver segments.

B. Identifying the maneuver segments

Simplest way to identify the maneuver segments is to discretize the flight trajectory check the state constraints between each adjacent waypoints. Once the particular set of waypoints is identified as a maneuver mode, set of modal inputs associated with maneuver mode can be found using the data obtained from flight trajectory. In this section, identification technique for each mode is presented.

Wind Axes Euler Angles Identification Table

Since the wind axes Euler angles are obtained from eq. 2, it is possible to label each mode based on the time history of these angles. This is due to fact that, flight path is bended and twisted according to these angles independent of the velocity on the flight path. Since most of the maneuver modes are identified due to shape of the flight path, this is a very natural way for identifying them. Table 2 shows how wind axes Euler angles evolve during each maneuver mode.

Table 2 - Wind Axes Euler Angles Identification

Mode / Angle	θ_w	$\dot{\theta}_w$	ψ_w	$\dot{\psi}_w$
Level Flight	0	0	C	0
Climb / Descent	C	0	C	0
Roll	C	0	C	0
Lon. Loop	T	T	C	0
Lat. Loop	C	0	T	T
3D Mode	T	T	T	T

On the table “C” means constant and “T” means time varying. Table is self-explanatory; for example in level flight, flight path angle (or wind axis pitch angle) must be zero so that aircrafts altitude doesn’t change, while heading can take any value as long as it doesn’t vary with time (i.e. zero derivative). This straightforward logic is applicable to every mode on the table. Only exception is the roll mode, which also requires to check the Euler Roll Angle history (if Wind axes Euler angles are constant and the Euler Roll angle is time varying, mode is identified as “roll mode”). Therefore one can decompose the maneuver into waypoints and check the obtained wind axes Euler angles to identify each mode via table 2. Next step is to obtain modal inputs for each mode.

Obtaining the Modal Inputs

After decomposing the trajectory and labeling each mode from table 2, we have to extract the modal inputs associated with each mode. Total velocity is a modal input for every mode and it is available from eq. 2. For level flight and Climb/Descent mode, everything needed is already available on table 2. (Climbing/Diving rate). In roll mode, only desired roll angle displacement (value of the integral associated with roll mode on table 1) is needed, which can be obtained from Euler roll angle time history. Loop modes require the body Euler angle rates as modal input, which can be obtained from $\theta(t)$ and $\psi(t)$ for Longitudinal Loop and Lateral Loop respectively. Things are a bit more complicated in 3D mode, because it is required to extract the angular body rates from a given 3D trajectory and attitude data. Since all the body Euler angles are available, kinematical equation for Euler angles can be solved inversely to acquire the angular rates, but this is not convenient since equations have singular points and requires manipulating trigonometric equations. More elegant approach would be converting the Euler angles to Quaternions, and solve the algebraic, singularity free Quaternion kinematical equation to obtain body angular rates. Eq. 5 gives the well known formula for converting Euler angles to Quaternions and eq. 6 shows the kinematical Quaternion equation which has to be solved inversely in order to obtain the angular rates.

$$Q = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} \pm \left(C \frac{\phi}{2} C \frac{\theta}{2} C \frac{\psi}{2} + S \frac{\phi}{2} S \frac{\theta}{2} S \frac{\psi}{2} \right) \\ \pm \left(S \frac{\phi}{2} C \frac{\theta}{2} C \frac{\psi}{2} - C \frac{\phi}{2} S \frac{\theta}{2} S \frac{\psi}{2} \right) \\ \pm \left(C \frac{\phi}{2} S \frac{\theta}{2} C \frac{\psi}{2} + S \frac{\phi}{2} C \frac{\theta}{2} S \frac{\psi}{2} \right) \\ \pm \left(C \frac{\phi}{2} C \frac{\theta}{2} S \frac{\psi}{2} - S \frac{\phi}{2} S \frac{\theta}{2} C \frac{\psi}{2} \right) \end{bmatrix} S : Sin, C : Cos \quad (5)$$

$$\dot{q} = \begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & -P & -Q & R \\ P & 0 & R & -Q \\ Q & -R & 0 & P \\ R & Q & -P & 0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \quad (6)$$

C. Structure of the Algorithm

Now we are ready to write down the algorithm for, decomposing the given feasible flight trajectory in maneuver mode sequence, thus identifying the maneuver in the proposed motion language. Algorithm 1 (Maneuver Identification /Decomposition algorithm), takes the flight trajectory and Euler pitch and yaw angle time history as input. In the first and second step, algorithm computes the rest of the state space variables by the equations given in this section. In the third step flight path is divided in to waypoints with constant time step (so flight path is “sampled” in a sense). This allows us to check each flight segment step by step and compare the wind axes Euler angles of each waypoint. This task is done at fifth step in the algorithm, and the mode is labeled after finding its name on the table 2. In sixth step modal input set is obtained from state space trajectories or kinematic equations given in sub subsection “Obtaining Modal Inputs”. After labeling and finding modal input tasks, total time duration is calculated from summing the constant time step intervals. Then these tasks are repeated for next maneuver mode, until the flight trajectory is complete. Output of the algorithm is the modal sequence which has total N pieces.

Output of the algorithm is shown below on a Cuban Eight example, which also appeared in

5.

Algorithm I: Maneuver Identification / Decomposition	
Input	: Flight Trajectory $(n_p(t), e_p(t), h(t))$ and attitude angles $(\theta(t), \psi(t))$
Output:	Modal Sequence $(q_i, \sigma_i, \tau_i), i = 1, 2, \dots, N$
1:	Solve the velocity variables form eq. 2
2:	Solve the angle of attack and roll angle history from eq. 4
3:	Discretize the flight trajectory, with constant time step Δt
4:	Repeat
	Repeat
5:	Check $(\theta_{w_{j+1}} - \theta_{w_j})$ and $(\psi_{w_{j+1}} - \psi_{w_j})$, Label The mode according to table 2.
6:	Switch $q_i \leftarrow$ “Mode Label”
	Case “Level Flight” $\sigma_i \leftarrow \{ \text{Obtain From } V_T(t) \}$
	Case “Climb / Descent” $\sigma_i \leftarrow \{ \text{Obtain From } V_T(t), \theta_w(t) \}$
	Case “Roll Mode” $\sigma_i \leftarrow \{ \text{Obtain From } V_T(t), \phi(t) \}$
	Case “Lon. Loop” $\sigma_i \leftarrow \{ \text{Obtain From } V_T(t), \theta(t) \}$
	Case “Lat. Loop” $\sigma_i \leftarrow \{ \text{Obtain From } V_T(t), \psi(t) \}$
	Case “3D Mode” $\sigma_i \leftarrow \{ \text{Obtain From } V_T(t), \text{eqs. 5 and 6} \}$
	Until Next Mode
7:	Calculate duration of the mode $\tau_i \leftarrow \sum \Delta t_i$
	Until Trajectory is complete $i = N$

Maneuver Identification Example - Cuban Eight

Cuban eight is a popular aerobatics maneuver, which consists of connected loops and rolls in order to draw an “eight” figure in the 2D plane. Maneuver can be broken down into maneuver mode and modal input sequence as shown by fig. 2 and table 3.

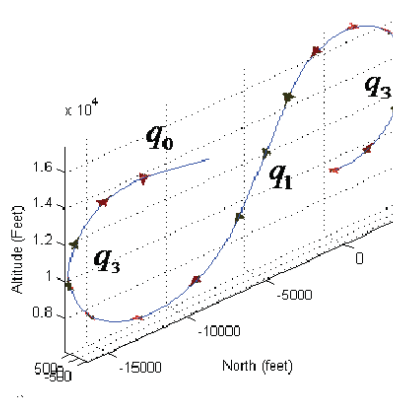


Figure 2 - Cuban Eight

Table 3 – Modal Decomposition of Cuban Eight

Maneuver Mode	Modal Inputs	Time Intervals (sec)
q_3	$r_{loop} = 3000 \text{ ft}, \dot{\theta} = 225/5 \text{ deg} \cdot s^{-1}$	$[0, 5]$
q_1	$V_T = 400 \text{ ft} / s, \theta_w = 45 \text{ deg}$	$[5, 7]$
q_3	$r_{loop} = 3000 \text{ ft}, \dot{\theta} = 225/5 \text{ deg} \cdot s^{-1}$	$[7, 12]$
q_0	$V_T = 400 \text{ ft} / s, \alpha = 2 \text{ deg}$	$[12, 14]$

Advantage of multimodal state framework is easily seen; instead of giving 12 state histories for this 14 seconds lasting maneuver, all the information is contained in table 3 with only 4 maneuver modes and modal inputs.

In this section maneuver identification algorithm was developed and tested on an agile flight maneuver. Only direct use of the algorithm is possible if the path/motion planner guarantees that the generated flight trajectory is feasible. Then this algorithm neglects checking feasibility and concentrates on decomposing the given maneuver into language of maneuver modes and modal inputs. After this decomposition multi-modal control approach can be used on the system to track the maneuver.

However in general case, path planner do not focus on feasibility but only on finding a path that connects initial and final waypoints while avoiding forbidden areas (such as obstacles) in the environment. Again in general they will not give the attitude of the aircraft but only the flight trajectory in 3D plane possibly without any time constraint. Therefore the algorithm must find the feasible sequence of maneuver modes and modal inputs for this trajectory as well as the time duration (therefore velocity) for each mode. Next section is dedicated to design of such an algorithm called “Feasible Maneuver Generation Algorithm”, which approaches the feasibility problem via “sequential and envelope constraints, and generates the feasible maneuver sequence by the help of searching tables and graphs based on the modal approach of the proposed language.

IV. Feasible Maneuver Generation Algorithm

This section focuses on creating a feasible maneuver sequence from a given flight trajectory. So, when integrated with a path planner, this algorithm will solve the motion/planning problem in multi modal control framework. Feasibility in this section is divided into two classes²⁵; feasibility of the mode sequence (sequential constraints) and feasibility of modal inputs (envelope constraints). If the modal sequence satisfies these constraints it would be

accepted as a feasible maneuver. Thus, algorithm created in this section consists of the expansion of maneuver identification algorithm from previous section with feasibility checks of the constraints above.

Firstly, physical nature of these two types of constraints along with the proposed solution methods for dealing with these constraints will be presented. Next, general structure of the algorithm is discussed. Finally, an example agile trajectory taken from a path planner will be converted to a feasible maneuver sequence.

A. Sequential Constrains and Mode Transition Table

Multi modal control system is based on the successful execution of one mode after another, but this doesn't mean that mode sequence is arbitrary. We can simply cast the mode compatibility condition as; final states of the first maneuver mode must intersect with the second modes initial conditions ²¹. This is valid for kinematical variables, since most of the dynamic variables (i.e. modal inputs) are subject to envelope constrains rather than sequential which will be discussed in the next section.

For the trajectory variables n_p, e_p, h there is no problem, since the trajectory given by path planner is already connected. Challenging part is the attitude compatibility problem, because attitude is usually unnatural to flight path in agile maneuvers. Therefore, we have to check if the attitude of each mode is compatible with each other. This problem mostly rises on body roll angle and pitch angle. For example for a lateral loop, aircraft is assumed to have an initial roll angle required for that turn, however in level flight wings are kept at zero roll angle. So transition for level flight to lateral loop needs a roll mode in between to connect each modes attitude. Analogously, longitudinal loop mode can be used for same purpose to set to correct pitch angle when two modes pitch attitude is not compatible with each other. Based on this discussion, a mode transition table was prepared (table 4), which explicitly shows which mode are directly compatible with each other and which needs a change of attitude between them.

Table 4 – Mode Transition Table

δ_{ij}	q_0	q_1	q_2	q_3	q_4	q_5	q_6
q_0	1	θ^*	1	1	ϕ^*	θ^*, ϕ^*	1
q_1	θ^*	θ^*	1	θ^*	θ^*, ϕ^*	θ^*, ϕ^*	1
q_2	1	θ^*	1	θ^*	1	θ^*, ϕ^*	1
q_3	1	1	1	1	θ^*, ϕ^*	θ^*, ϕ^*	1
q_4	ϕ^*	θ^*, ϕ^*	1	θ^*, ϕ^*	ϕ^*	θ^*, ϕ^*	1
q_5	θ^*, ϕ^*	θ^*, ϕ^*	1	θ^*, ϕ^*	θ^*, ϕ^*	θ^*, ϕ^*	1
q_6	1	0	0	0	0	0	0

Inspection of the table shows that roll mode and longitudinal loop mode acts as a transition mode between the others because they can create the desired shift in roll and pitch angles. Thus one can construct maneuvers by using level and climbing flights along with loops and connect them using rolling and pitching. It is also noted that safety mode is accessible all the time, once it is activated it automatically regulates back to level flight, so all other transitions from this mode is blocked. It must be also noted that table relaxes the checking attitude continuity between each mode by just checking the necessary angles.

B. Envelope Constraints and Agility Metrics

Second types of constraints are risen from physical limitations of the aircraft, i.e. envelope constraints. Due to capability of engine and control surfaces of the aircraft, as well as the aerodynamic limits of the aircraft, there is a certain limit on the max. aircraft velocity, max. climb rate etc. However these boundaries are not constant, they vary continuously with the aircrafts angle of attack, altitude etc.

These constraints can be embedded into multimodal control framework as limits on modal inputs. Then feasible maneuver generation algorithm can select the modal inputs while ensuring they are in operation limits. This method however, does not reflect the true agility characteristics of the fighter aircraft. Limits imposed on modal inputs (as well as envelope limits) shows only potential agility of the aircraft and often limited to trimmed maneuvers, they do not show the transient capabilities of the aircraft. Therefore, a more elegant solution would be searching the modal input parameters not in their original envelope, but in another norm, which could show the agile capability of the aircraft.

A similar problem, in fighter aircraft technology development was studied during 1970s. Aircraft designers wanted the design and compare fighter aircrafts in new metrics, because existing metrics (such as wing loading to thrust to weight ratio) was incapable of showing aircrafts transient performance, and often shadowed the maneuver capability of aircraft. Such metrics were found, tested on the simulations on various NASA reports^{15,16,27,28,31}.

In this subsection, we take the advantage of multimodal control framework and develop separate agility metrics for each mode. Most of these metrics were taken from the NASA reports above. All of the metrics were evaluated on a high fidelity F-16 aircraft simulation¹⁸. Detailed description of the metrics and simulation results are contained below.

C. Selected Agility Metrics

Some of the metrics were chosen and inspected in previous work²⁵ but as the research progressed, these metrics were replaced by more convenient metrics. Metric graphs were produced by nonlinear 6 DOF F-16 simulations via feedback control loops for various angle of attack and Mach numbers for various altitudes. Example graphs given at the end of the subsection are for sea level.

Level and Climbing Flight:

For level and climbing/diving flight total speed and acceleration capability is the most important parameter. Maximum and minimum achievable speed depends heavily on available power and thrust. Selected agility metric is power onset/loss parameter which can be written as:

$$\dot{P}_s = \frac{d}{dt} \left(\frac{V_T (T - D)}{W} \right) \quad (7)$$

Where T is thrust, D is drag and W is weight. This agility metric quantifies maximum thrust and drag of the aircraft, which determines the total speed and acceleration capability. This metric is plotted against Mach number and angle of attack, in a 3D region for F-16 in fig. 3.

Roll Mode:

For rolling motion either average roll rate or time to go through certain roll displacement can be used. Second one is more convenient as it gives transient performance more clearly. For specific angle 90 degrees can be used, because most of the rolling maneuvers consists of rolling aircraft to side (knife edge), or inverting it (180 degrees). Therefore selected agility metric is

$$TR_{90} = \text{Time To Go Through 90 degrees roll angle} \quad (8)$$

3D plot of this metric is shown in fig. 4.

Longitudinal Loop:

For pitch up/down motion, very simple and convenient metric is average pitch rate which can be written as

$$Q_{avg} = \frac{\int_{t_1}^{t_2} Q dt}{t_2 - t_1} \quad (9)$$

Plot of this metric is shown in fig. 5.

Lateral Loop:

For turning performance load factor and turning radius is chosen as a predominant factor, by using the simple kinematic equation ¹⁹:

$$r_{loop} = \frac{V_T^2}{g(n^2 - 1)} \quad (10)$$

3D Mode:

In 3D mode both rolling and pitching motion becomes dominant; therefore we seek a metric which can combine these two properties. An appropriate metric is loaded roll which is given by the formula

$$PN = p_w N_{z,w} \quad (11)$$

This is simply the product of roll rate in wind axes and normal acceleration in wind axes. This metric belongs to torsional agility and combines the rolling motion of the aircraft with bending of the flight path.

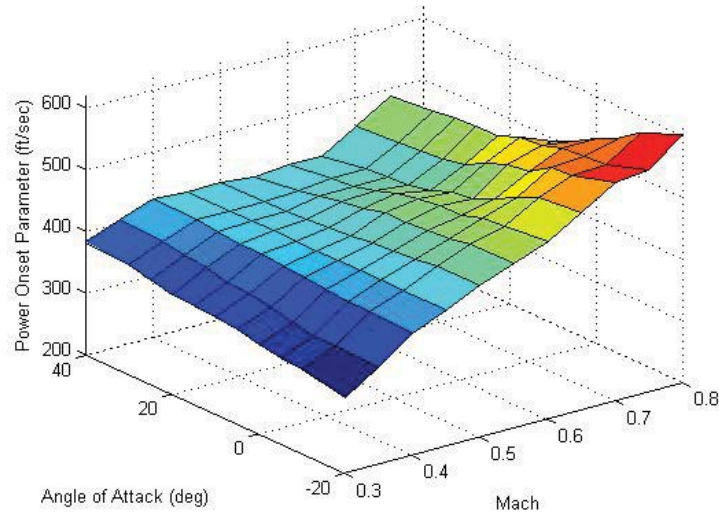


Figure 3 - Power Onset Parameter Metric for sea level

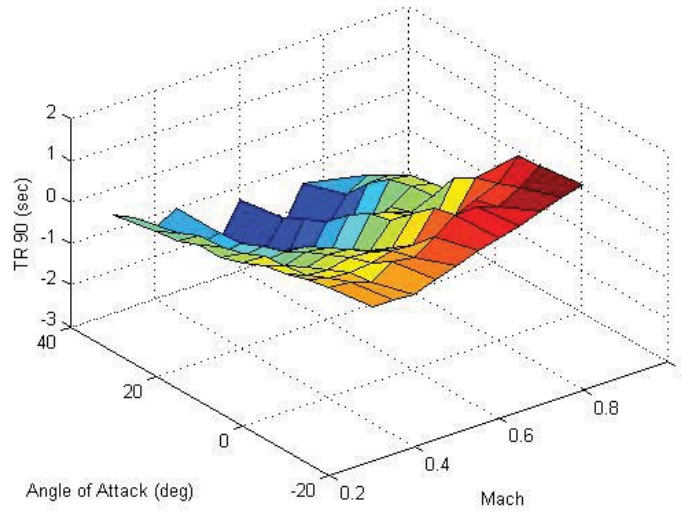


Figure 4 - Time to Capture 90 Deg Roll Angle Metric for sea level

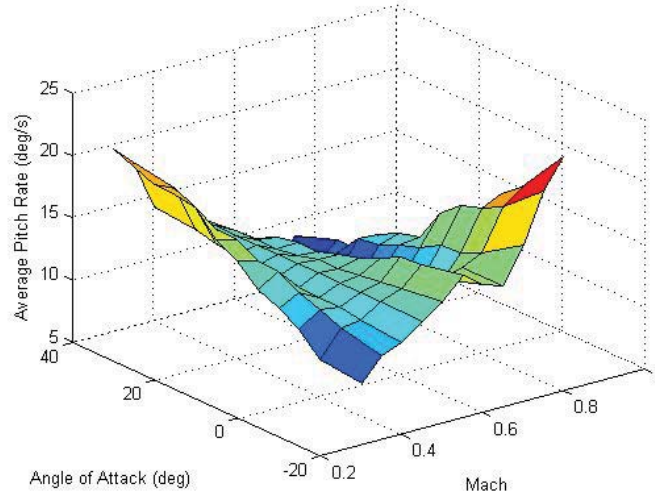


Figure 5 - Average Pitch Rate Metric for sea level

D. Structure of the Algorithm

General structure of the algorithm is explained step by step below;

Determining Rough Maneuver Sequence - (Line 1)

Initially, path planning algorithm provides the flight trajectory history, $n_p(t), e_p(t), h(t)$. From this data it is easy to recover the velocity variables from eq. 2. Since mode labeling action only depends on wind Axes Euler Angles (via table 2), it is possible to recover the mode sequence and time durations for each mode. However complete modal sequence is incomplete because modal input sequence cannot be recovered without attitude history. In the next step each mode will be analyzed via agility metric graphs, and modal inputs will be recovered from flight equations or graphs.

Recovering Modal Inputs from Agility Metrics - (Line 2)

Since we have decomposed the flight trajectory into mode sequence, we can determine the feasibility of each mode and recover its modal input parameters (and quantify its agility). Very similar to Maneuver identification algorithm, this process is different for each mode. For example in level flight since the velocity is known (so is the Mach number), only free variable on 3D agility metric plot is angle of attack. Since searching for a certain parameter or optimization process could take time and complicate the real time implementability of the framework, free parameter angle of attack is chosen randomly in the feasible interval, but with more weight on a certain subinterval (this method is very similar to kinodynamic path planner appeared on ²³ where modal inputs are directly selected on this methodology). If the corresponding velocity is not in the feasible interval in the graph, or results in angle of attack with unacceptable values, flight paths time can be scaled to change the velocity variable (for detailed discussion on time scaling of flight paths see²⁹).

For other modes, process is similar; once a desired velocity and angle of attack couple is found from agility metric graph, modal inputs are recovered from flight equations 3-6 and agility metric equations 7-11. Also when modal inputs of the i^{th} mode is recovered, if the $(i+1)^{th}$ mode is sharing the same modal inputs, initial values of the σ_{i+1} are chosen same as σ_i to maintain continuity.

When both total velocity and angle of attack is recovered, next step is to compute the attitude angles. In this step problem is to solve eq. 4, which contains two equations but three unknowns (Euler angles), this is solved by setting $\psi = \psi_w$ (which is not a bad approximation for coordinated flight), then solving the equations. Authors are still working on this part of the algorithm to get more efficient results.

Although the attitude history is obtained, due to modal decomposition at first step, attitude history is discontinuous between mode transitions, this problem will be solved by connecting them via mode transition table at the next step.

Connecting the Maneuver modes (Satisfying the sequential constraints) – (Line 3)

Now the flight path is decomposed into modes and modal inputs (which are feasible in the sense that they fall into feasible region of agility metric graphs) but modes must satisfy the mode compatibility condition which was discussed on 4.1. Flight path is already given connected via path planner but attitude must be checked via mode transition table (table 4). At this step, mode transitions which satisfy the table are neglected and the modes which require attitude tweaking (in either roll or pitch angle) is checked to be compatible. If they are not, additional translational modes (roll mode and longitudinal loop) are inserted between these modes to connect the attitudes of each mode. These attitude changes are made quickly as possible to not to change shape of the flight trajectory, if these transitions make dramatic changes on the flight trajectory, re-planning of the trajectory may be required to make sure that path avoids the obstacles. This interaction between the multimodal framework and path planning is discussed on ³⁰.

Recovering The Feasible Modal Sequence –(line 4)

After checking the mode transition table, and finding the appropriate attitude changes for sequential feasibility, these transition modes are added to original mode sequence and final modal sequence is recovered. This modal sequence is feasible in the sense that it satisfies the envelope and sequential constraints. An example for an agile combat maneuver is provided to show the capability of the algorithm.

Algorithm II: Feasible Maneuver Generation		
Input	Flight Trajectory $(n_p(t), e_p(t), h(t))$	
Output:	Feasible	Modal Sequence
	$(q_i, \sigma_i, \tau_i), i = 1, 2, \dots, N$	
1:	Run Maneuver Identification Algorithm (Algorithm I) up to line 5 and recover q_i	
2:	Repeat <div style="margin-left: 20px;"> Switch “Mode Label” <div style="margin-left: 20px;"> Case “Mode Label” Check Agility Metric Graph, recover α Recover Modal inputs σ_i through flight And agility metric equations. Adjust σ_i such that that it is compatible with σ_{i-1} </div> </div> Until Trajectory is complete	
3:	Repeat <div style="margin-left: 20px;"> Check Mode Transition between q_{i-1} And q_i, through table 4. <div style="margin-left: 20px;"> Case 1 Do Nothing Case θ^* Insert “Longitudinal Loop” for pitch regulation Case ϕ^* Insert “Roll Mode” for Roll pitch regulation Case θ^*, ϕ^* Insert both “Longitudinal Loop” And “Roll Mode” </div> </div> Until Trajectory is complete	
4:	Gather feasible modal sequence by combining results of 2 and 3	

E. Application to Agile Combat Maneuver

The maneuver is inspired from the book ²⁰ which displays the agile maneuvering capability of an aircraft, through connecting loops and rolls which is very common in fighter combat. Maneuver is similar to High Yo-Yo maneuver where aircraft gains altitude in order to make a steady fast on the target get behind it to enter into firing range.

Trajectory generated by the path planner, shown on figure 6. This trajectory is entered as input the Algorithm 2. By going through the steps of the algorithm, feasible modal sequence is created, which can be found on table 5. It is revealed that, maneuver (on figure 6) starts with a level flight, then continues with a loop in the longitudinal plane, then moves onto on arc in 3D plane, it is seen that start of the arc and end the loop had different roll angles, so this is compensated by adding a roll mode between these two. After 3D mode aircraft begins to dive, and finally enters into

another arc. Another roll mode followed by a mini longitudinal loop is inserted here in order to satisfy sequential constraints.

Time history of fey modal inputs selected on agility metric criteria and kinematic parameters are shown in figure 7 (red square highlight the modal inputs for each mode). This feasible modal sequence profile, actually acts as a reference profile for the switched control system (multimodal control framework) introduced in section II.

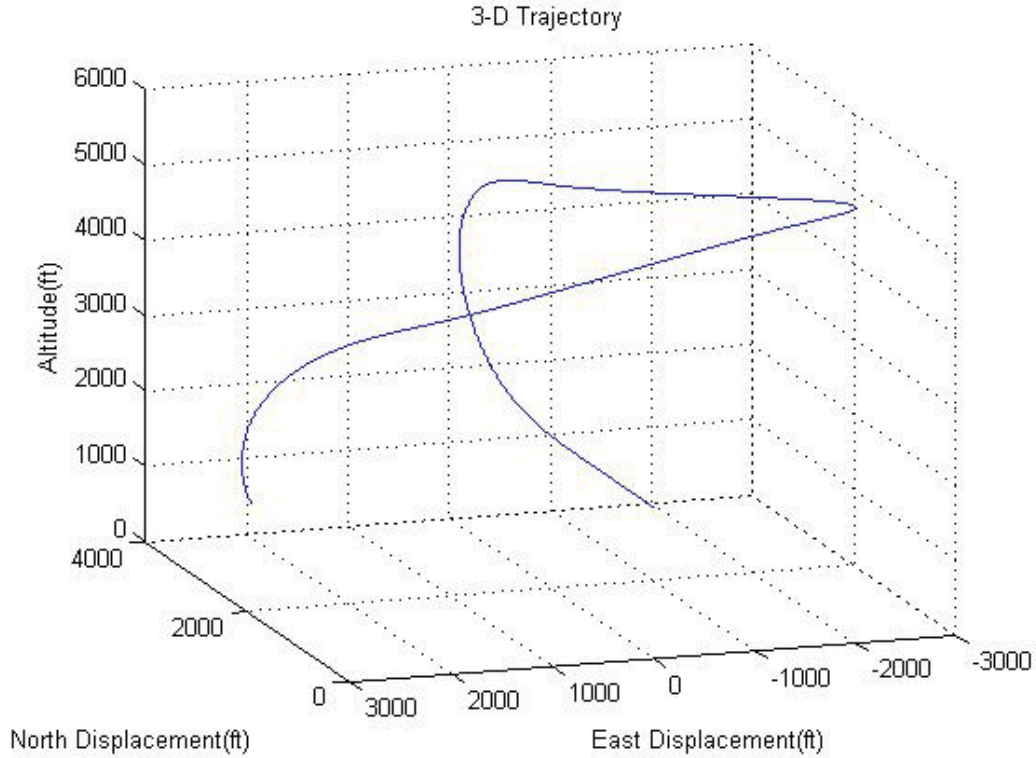


Figure 6 - Trajectory Generated by Path Planner

Table 5 – Modal Decomposition of Agile Combat Maneuver from Algorithm 2

Mode	Label	Time Intervals (sec)
q_0	Level Flight	$[0, 3]$
q_3	Longitudinal Loop	$[3, 9.7]$
q_2	Roll Mode (Transition)	$[9.7, 11.7]$
q_5	3D Mode	$[11.7, 22.8]$
q_1	Dive	$[22.8, 25.4]$
q_2	Roll Mode (Transition)	$[25.4, 26.6]$
q_3	Longitudinal Loop (Transition)	$[26.6, 27.8]$
q_5	3D Mode	$[27.8, 32]$

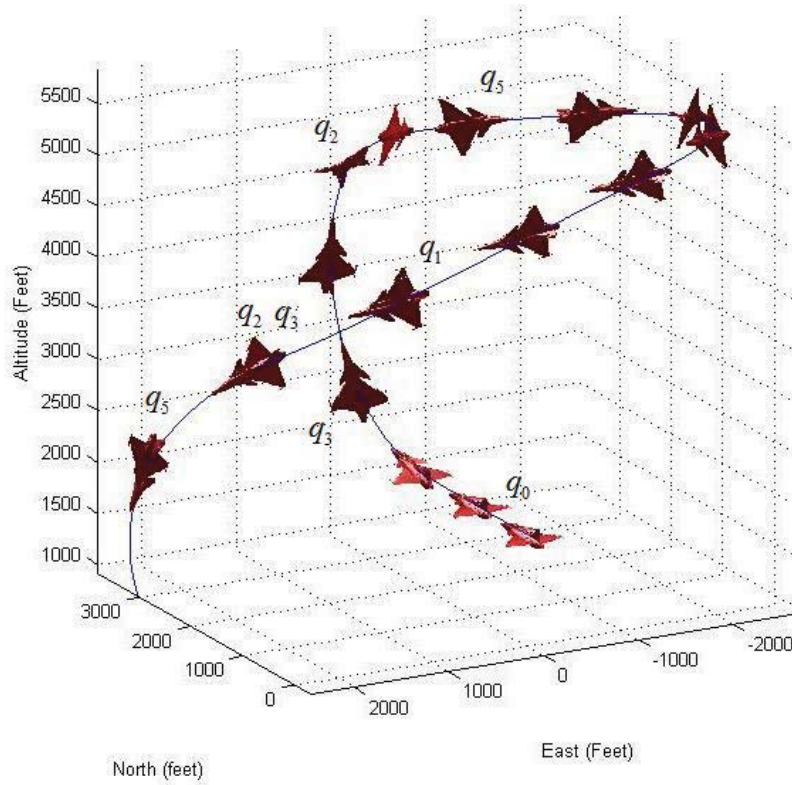


Figure 6 - Agile Combat Maneuver

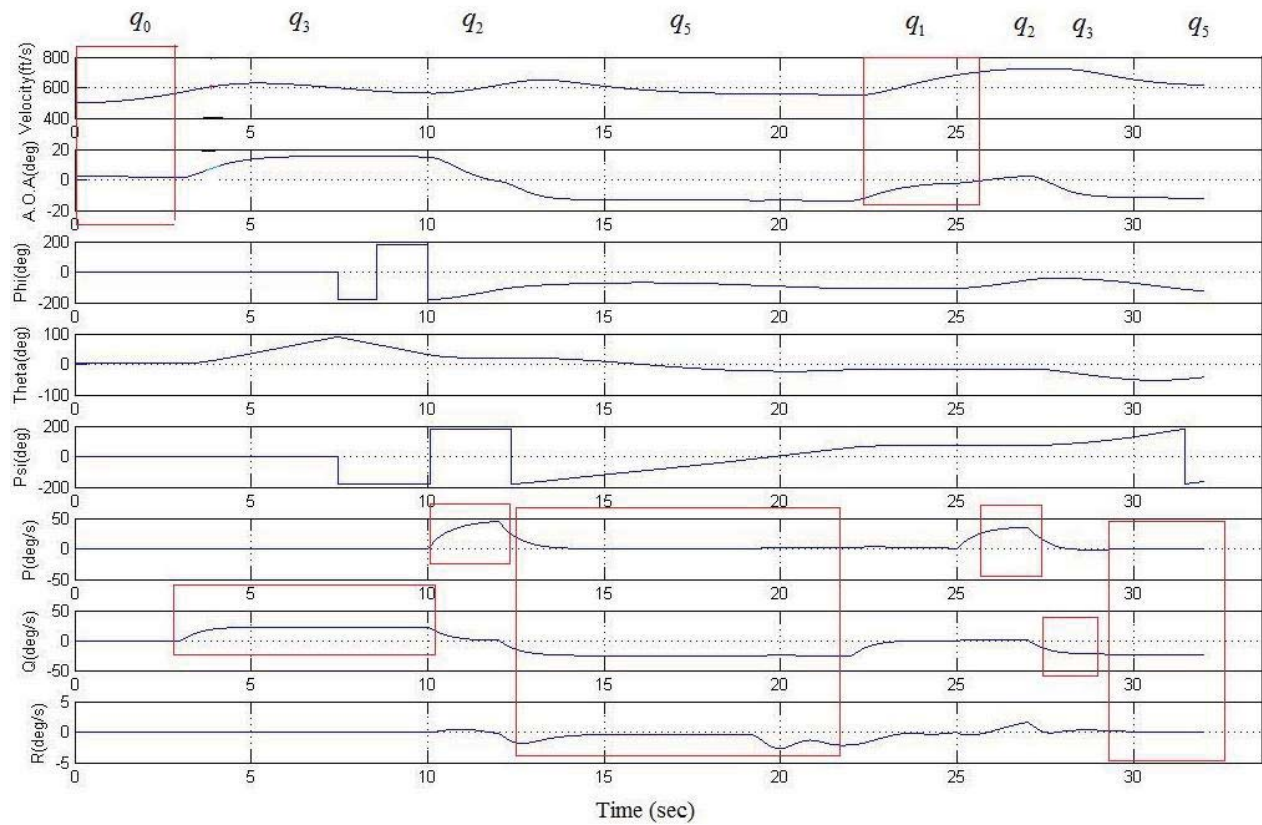


Figure 7 - State and Modal Input History

V. Conclusions

On this study, maneuver identification and generation algorithms based on the multi modal control framework were developed. Identification algorithm showed the capability of the proposed modal decomposition methodology and the fact that state space description is indeed replaceable by the modal sequence. This algorithm also helped to develop the more critical algorithm “Feasible Maneuver Generation Algorithm”. Simulation results of the algorithm showed that, when integrated with a path planner, overall system is capable of generating a continuous (in the sense that it satisfies sequential constraints), feasible (in the sense that modal input satisfies envelope constraints via agility metrics) modal sequence, which can be controlled globally via switching control system (multi modal control framework).

Future research consists of exploring the hybrid system properties of the framework and interaction between the maneuver and path planning layer for healthier obstacle avoidance in complex environments.

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