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# Control of Towing Kites under Uncertainty using Robust Economic Nonlinear Model Predictive Control

Sergio Lucia and Sebastian Engell

**Abstract**—In the last years, the development of kites as a new approach to use the wind energy to produce electricity or to pull boats in order to save fuel has received attention from both the industry and academia. In this study, we propose a nonlinear model predictive control (NMPC) approach to control a towing kite based on an economic objective function in contrast to the tracking of a predefined trajectory, as it is usually done. We use a simple model of a kite that has been recently developed. The use of a simple model usually comes at a price of higher model inaccuracy. To cope with the uncertainties in the model as well as with external disturbances we propose the use of multi-stage nonlinear model predictive control. Simulation results show that the use of the multi-stage NMPC approach avoids the violation of constraints and achieves a better performance than standard NMPC under the presence of strong uncertainties.

## I. INTRODUCTION

In the search for new renewable energy sources, kites have emerged as a new possibility to take advantage of the constant and high wind speeds that occur at high altitudes, avoiding the problems of growing size and mass of standard wind generators [1]. In order to make use of the big aerodynamic force that acts on a kite flying at high altitude, the kite is attached to a cable which transfers the aerodynamic force to the ground. This force can then be used directly to pull boats [2] or can be converted into electricity using a suitable generator [3]. This topic has raised attention not only in the academia but also in the industry as can be seen by the number of companies (SkySails ([4]), EnerKite [5], KITEng [6] among others) that exploit high altitude energy generation via kites in different forms.

The academic community has focused on one of the big challenges of this technology, the difficult control problem that has to be solved to attain optimal operation under the presence of several and strong uncertainties such as wind speed or uncertain model parameters. Different approaches have been presented in the literature during the last years. Many of them are based on the separation of the control problem into two different parts. The approach proposed in [2] exploits the structure of the system to propose a simple cascade controller. The control approach presented in [7] uses three nested control loops with simple controllers. Another control strategy proposed in the literature is to first generate an optimal trajectory of the kite and then to implement a path following controller that tracks the pre-calculated trajectory in the presence of disturbances. Disturbances can also be taken into account in the computation of

the trajectories by using robust optimization [8] or Real Time Optimization (RTO) techniques to update the trajectory based on the available measurements [9]. Different algorithms have been used for the tracking of the trajectory, in particular, nonlinear model predictive control [10] [11].

This paper focuses on the control of kites that are used to pull a ship via nonlinear model predictive control using as the cost function directly the thrust generated by the kite within the prediction horizon without a fixed shape of the trajectory as in [12]. In the controller, we use the simple kite model proposed in [2] with the modification introduced in [9]. The use of a simple model usually implies a higher uncertainty in the predictions that are based on the model due to the assumptions made. More importantly, some of the parameters are not exactly known because the necessary wind tunnel tests are too expensive from a practical point of view, as explained in [2], and there are strong external disturbances that act on the system (in particular wind turbulences). As a novel contribution, in this work we show that the use of standard NMPC with large model errors can fail in the control of kites and we propose the use of a multi-stage NMPC scheme that has been proved to have good results in the field of process control [13], [14] to overcome the problems caused by the presence of the uncertainty in the model. Simulation results show that multi-stage NMPC is able to fulfill the constraints for all the different values of the parameters that are uncertain and in addition it exhibits a better average performance than standard NMPC. We also analyze the quality of the solution with a finite prediction horizon compared to the solution of the periodic optimal problem. To guarantee closed-loop stability of economically optimizing MPC with finite horizon is challenging and first results have been published in recent years [15], [16]. In this application study, we cope with the stability issue by choosing a sufficiently long prediction horizon which was validated in simulations. The focus of this paper is to demonstrate the potential of economically optimizing multi-stage NMPC for a challenging nonlinear application example.

The paper is structured as follows. Section 2 describes the model of the kite and Section 3 presents a comparison of the periodic optimal control problem and the solution of the receding horizon approach with a finite prediction horizon when there are no uncertainties. Section 4 explains the multi-stage NMPC approach and Section 5 contains the main results of the paper, a comparison of several standard and multi-stage NMPC controllers under the influence of large uncertainties. Finally, Section 6 concludes the paper.

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## II. KITE MODEL AND CONTROL PROBLEM

Different kite models have been proposed in the literature ranging from detailed multi-body dynamics models [17] to simpler models that are more suitable for online optimizing controllers ([11], [12], [2]). In this work, we use the model proposed in [2] that consists of only 3 differential equations. A detailed explanation of the underlying assumptions and derivations is out of the scope of this paper and the reader is referred to [2] and [9]. Based upon the modeling assumptions, the dynamics of the kite can be described by the following set of ordinary differential equations:

$$\dot{\theta} = \frac{v_a}{L}(\cos \psi - \frac{\tan \theta}{E}), \quad (1a)$$

$$\dot{\phi} = -\frac{v_a}{L \sin \theta} \sin \psi, \quad (1b)$$

$$\dot{\psi} = \frac{v_a}{L} \tilde{u} + \dot{\phi} \cos \theta, \quad (1c)$$

where

$$v_a = v_0 E \cos \theta, \quad (1d)$$

$$E = E_0 - \tilde{c} \tilde{u}^2, \quad (1e)$$

$$P_D = \rho v_0^2 / 2 \quad (1f)$$

$$T_F = P_D A \cos^2 \theta (E + 1) \sqrt{E^2 + 1} \cdot (\cos \theta \cos \beta + \sin \theta \sin \beta \sin \phi). \quad (1g)$$

The model is formulated in a spherical coordinate system. The states are the zenith angle  $\theta$  (angle between wind and tether), the azimuth angle  $\phi$  (angle between the vertical and the plane whose normal is the wind direction) and the angle  $\psi$ , which denotes the orientation of the kite. The parameter  $E$  is an aerodynamic coefficient called the glide ratio,  $v_0$  is the apparent wind speed referenced to the boat,  $\rho$  is the air density,  $A$  is the area of the kite,  $L$  is the length of the tether, and  $\beta$  is related to the angle between the directions of the boat and of the wind. The obtained thrust is denoted by  $T_F$ ,  $\tilde{u}$  is the steering deflection command which is used as a control input and  $\tilde{c}$  is a correction coefficient. The model presented in [2] has been slightly modified in [9] by adding a varying glide ratio  $E$  according to (1e) instead of choosing a constant one. The motivation behind this modification is that it was observed in experiments that steering deflection commands ( $\tilde{u}$ ) reduce the tension of the tether, especially for large values of the control input. Thus (1e) reduces the glide ratio  $E$  for steering deflections.  $\tilde{c}$  is a constant (uncertain) coefficient and the glide ratio in the absence of steering deflections is equal to  $E_0$  [9]. The parameters values assumed in this work are given in Table I and are the same as those used in [9].

The goal of the controller is to maximize the obtained average thrust  $T_F$ . The input is constrained to be between given bounds and the height of the kite should be higher than  $h_{\min}$  because of safety reasons. The height can be calculated as a function of the states variables so that the constraint can be formulated as  $L \sin \theta \cos \phi \geq h_{\min}$ .

TABLE I  
PARAMETER VALUES OF THE KITE MODEL

Parameter	Value	Units
$L$	400	m
$A$	300	m <sup>2</sup>
$h_{\min}$	100	m
$\rho$	1	kg/m <sup>3</sup>
$\beta$	0	degrees
$v_0$	10	m/s
$\tilde{c}$	0.028	-
$E_0$	5	-
$u_{\min}$	-10	-
$u_{\max}$	10	-

## III. PERIODIC OPTIMAL CONTROL VS. RECEDING HORIZON CONTROL

Several control approaches proposed in the literature (see e.g. [8] and [9]) are based on the formulation and solution of a periodic optimal control problem for the maximum generation of thrust (or energy) and an additional path tracking controller that takes care of following the previously obtained trajectory. Using a piece-wise constant control input over each sampling time  $k$ , the periodic optimal control problem can be written as follows:

$$\max_{x(\cdot) \tilde{u}(\cdot), N_p} \frac{1}{N_p} \sum_{k=0}^{N_p-1} T_F \quad (2a)$$

subject to:

$$\text{model in (1)}, \quad (2b)$$

$$x(N_p) = x(0), \quad (2c)$$

$$u_{\min} \leq \tilde{u}(k) \leq u_{\max}, \quad \forall k = 0..N_p - 1, \quad (2d)$$

$$L \sin(\theta(k)) \cos(\phi(k)) \geq h_{\min}, \quad \forall k = 0..N_p, \quad (2e)$$

where  $N_p$  is the final sampling time and  $x$  denotes the state vector  $x = [\theta, \phi, \psi]^T$ . The problem has to be solved subject to the input constraints (2d) and state constraints (2e). The periodicity is imposed via (2c). The solution of this optimal control problem for the parameters shown in Table I is an *eight-like* trajectory that is shown in Fig. 1. The obtained trajectory is different from the one shown in [9] and it obtains a higher average thrust (256.18 kN vs. 252.21 kN).

Using the pre-calculated trajectory, a tracking controller that follows the path and satisfies the constraints under uncertainty is needed. This is a non-trivial task that strongly affects the overall performance of the system (see [9]). In addition, under the presence of uncertainties, the original trajectory may be suboptimal, or even infeasible. For these reasons, we propose to use a receding horizon control approach that directly controls the kite by maximizing the obtained thrust over a finite prediction horizon. The optimization problem solved at each sampling time can be written as:

$$\max_{x(\cdot) \tilde{u}(\cdot)} \frac{1}{N_p} \sum_{k=0}^{N_p-1} T_F \quad (3a)$$

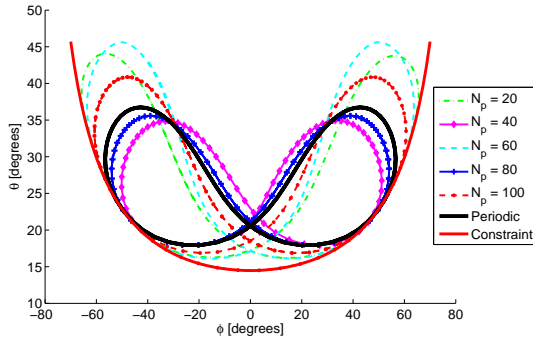


Fig. 1. Trajectory obtained for nominal NMPC with different prediction horizons and for the solution of the periodic optimal control problem with exact plant model and with economic cost function.

subject to:

$$\text{model in (1),} \quad (3b)$$

$$u_{\min} \leq \tilde{u}(k) \leq u_{\max}, \quad \forall k = 0..N_p - 1, \quad (3c)$$

$$L \sin(\theta(k)) \cos(\phi(k)) \geq h_{\min}, \quad \forall k = 0..N_p.$$

In this case, a fixed prediction horizon  $N_p$  is chosen a priori and the periodicity constraint is removed. If there are no uncertainties, this strategy also leads to a periodic trajectory. The obtained trajectories for different prediction horizons  $N_p$  can be seen in Fig. 1. It can be seen that they depend on the chosen prediction horizon. The main reason is the so-called procrastination effect that is inherent in any receding horizon approach with finite horizon. Due to the finite prediction horizon of the controller, it delays actions that do not increase the value of the cost function until in the predicted behavior they lead to a better cost in the future. If the controller realizes this soon, the trajectories are similar to the ones obtained from solving the periodic control problem. If the controller realizes it late, the kite flies along the path constraint resulting in bigger trajectories (see Fig. 1) that achieve a lower average thrust. Due to the periodicity of the problem, the procrastination effect has a stronger impact for prediction horizons which differ much from the length of the optimal periodic orbit or half its value (because of the symmetry of the orbit), i.e. are different from  $N_p = 80$  or  $N_p = 40$ . This explains the non-monotonic behavior of the cost with respect to the length of the prediction horizon that can be seen in Table II.

TABLE II

PERFORMANCE COMPARISON FOR NOMINAL NMPC WITH ECONOMIC COST FUNCTION FOR DIFFERENT PREDICTION HORIZONS WITH THE SOLUTION OF THE PERIODIC OPTIMAL CONTROL PROBLEM

Prediction horizon	Average thrust per period [kN]
20	243.11
40	250.21
60	241.24
80	255.10
100	252.35
Periodic OCP	256.18

For all the results presented in this paper, we solve the optimal control problems using a simultaneous approach, i.e., we discretize both the states and the control inputs using orthogonal collocation. The implementation is done using CasADi [18], which provides an easy and very efficient implementation of the different algorithms. First and second order exact derivatives are provided by CasADi using Automatic Differentiation (AD) and the resulting Nonlinear Programming Problems are solved using IPOPT [19]. After calculating the optimal control input, the real system is simulated using the SUNDIALS [20] interface of CasADi with a high accuracy.

#### IV. MULTI-STAGE NMPC

Multi-stage NMPC [13] is a robust NMPC approach that is based on modeling the uncertainty by a tree of discrete scenarios (see Fig. 2). The tree structure makes it possible to take into account explicitly future control inputs and disturbances. In this way, future control inputs depend on the value of the previous realization of the uncertainty, acting as recourse variables that counteract the effect of the uncertainty. Therefore, multi-stage NMPC is a closed-loop NMPC approach [21] in contrast to typical open-loop min-max approaches that optimize over a sequence of control inputs checking the constraints for all the cases of the uncertainty. This reduces the conservativeness significantly compared to open-loop min-max NMPC [22]. A similar approach has been proposed for linear MPC in [23]. Other approaches that achieve some degree of feedback in the predictions are tube-based methods [24] or min-max based on affine control policies [25].

The main assumption of the multi-stage approach is that the uncertainty can be described by a set of discrete scenarios. In the case that the uncertainty is discrete valued, multi-stage NMPC computes the optimal feedback policy and it is therefore the best solution possible of the robust NMPC problem with finite horizon with guaranteed constraint satisfaction. In the case that the uncertainty does not take only discrete values that are contained in the scenario tree, multi-stage NMPC computes an approximation that can be very good if a suitable scenario tree is chosen as has been shown in simulation studies [26], [13], [14]. While for general nonlinear systems constraint satisfaction is not guaranteed for values that are not explicitly in the tree, very often the value of parameters that produce the worst-case scenario are on the boundaries of the parameter interval [27]. Therefore, a suitable scenario tree should include as scenarios the extreme values of all the parameters. It is then clear that the main challenge of the approach is that the size of the resulting optimization problem grows exponentially with the number of uncertainties and with the prediction horizon. A simple strategy to avoid the exponential growth of the scenario tree with the prediction horizon is to assume that the uncertainty remains constant after a certain point in time (called robust horizon) as can be seen in Fig. 2. See [13] for a more detailed explanation of this concept. The scenario tree setting assumes a discrete-time formulation of an uncertain nonlinear system

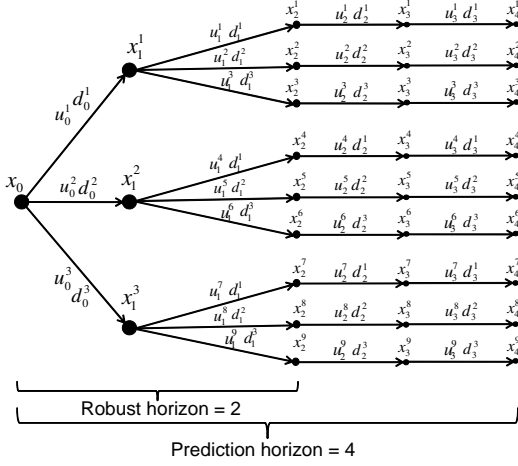


Fig. 2. Scenario tree representation of the uncertainty evolution for multi-stage NMPC.

that can be written as:

$$x_{k+1}^j = f(x_k^{p(j)}, u_k^j, d_k^{r(j)}), \quad (4)$$

where each state vector  $x_{k+1}^j$  at stage  $k+1$  and position  $j$  in the scenario tree depends on the parent state  $x_k^{p(j)}$  at stage  $k$ , the control input  $u_k^j$  and the corresponding realization  $r$  of the uncertainty  $d_k^{r(j)}$  (for example in Fig. 2,  $x_2^6 = f(x_1^2, u_1^6, d_1^3)$ ). The uncertainty at stage  $k$  is defined by  $d_k^{r(j)} \in \{d_k^1, d_k^2, \dots, d_k^s\}$  for  $s$  different possible values of the uncertainty. We define the set of occurring indices  $(j, k)$  in the scenario tree as  $I$ .  $S_i$  denotes scenario number  $i$ , where a scenario is defined as the path from the root node  $x_0$  until each one of the leaf nodes.

The optimization problem that has to be solved at each sampling instant can be written as:

$$\min_{x_k^j, u_k^j \forall (j,k) \in I} \tilde{J}(x_{k+1}^j, u_k^j) \quad (5a)$$

subject to:

$$x_{k+1}^j = f(x_k^{p(j)}, u_k^j, d_k^{r(j)}), \quad \forall (j, k+1) \in I, \quad (5b)$$

$$x_k^j \in \mathbb{X}, \quad \forall (j, k) \in I, \quad (5c)$$

$$u_k^j \in \mathbb{U}, \quad \forall (j, k) \in I, \quad (5d)$$

$$u_k^j = u_k^l \text{ if } x_k^{p(j)} = x_k^{p(l)}, \quad \forall (j, k), (l, k) \in I, \quad (5e)$$

where  $\tilde{J}(x_{k+1}^j, u_k^j) = \sum_{i=1}^N (\omega_i J_i(x_{k+1}^j, u_k^j))$ . The probability of each scenario  $S_i$  is denoted as  $\omega_i$ . If no probability is known the scenarios are assumed to be equally probable. If some information about the uncertainty can be obtained online, the probabilities can be adapted in order to increase the performance, as shown in [13]. There are  $N$  different scenarios and the cost of each of them is denoted by  $J_i(x_{k+1}^j, u_k^j)$  and can be written as

$$J_i(x_{k+1}^j, u_k^j) = \sum_{k=0}^{N_p-1} L(x_{k+1}^j, u_k^j), \quad \forall x_{k+1}^j, u_k^j \in S_i, \quad (6)$$

where  $L(x_{k+1}^j, u_k^j)$  is the stage cost. In order to represent the real-time decision problem correctly, the control inputs must not anticipate the realization of the uncertainty. This is modeled by the non-anticipativity constraints in (5e) that force all the control inputs  $u_k^j$  that branch at the same parent node  $x_k^{p(j)}$  to be the same.

## V. STANDARD NMPC VS. MULTI-STAGE NMPC

In this section we evaluate the performance of standard NMPC and multi-stage NMPC when controlling the kite under the presence of strong uncertainties in some of the parameters of the model. We assume that the aerodynamic coefficient  $E_0$  is uncertain and can take values in the interval  $E_0 \in [3, 7]$ . The coefficient in the glide ratio  $\tilde{c}$  is also uncertain and can vary between the values  $\tilde{c} \in [0.005, 0.04]$ . Finally, we consider a wind evolution described by  $v_0 = 10 + 5 \sin(0.1 \cdot 2\pi t)$  where  $t$  is the current time. The sampling time is taken as  $T_s = 0.3$  s. In order to have a better comparison of the approaches we use different parameters in the nominal model and different prediction horizons to analyze the effects that this choice has in the final performance. A summary of the controllers chosen can be seen in Table III.

TABLE III  
DESIGN OF THE DIFFERENT CONTROLLERS

Value used in the model				
Controller	$E_0$	$\tilde{c}$	$v_0$	$N_p$
<b>Standard 1</b>	5	0.028	10	40
<b>Standard 2</b>	3	0.005	10	40
<b>Standard 3</b>	5	0.028	10	60
<b>Multi-stage 1</b>	{3,7}	{0.005,0.04}	{5,15}	40
<b>Multi-stage 2</b>	{3,7}	{0.005,0.04}	{5,15}	60

For the design of the multi-stage NMPC controllers, we consider as possible scenarios the combinations of the maximum and minimum values of the uncertainties. We use a robust horizon = 1, i.e. the branching of the scenario tree occurs only in the first stage, and afterwards we consider the uncertainty to be constant, which makes the size of the problem grow only linearly with the prediction horizon from the second stage on. Thus the total number of scenarios for the multi-stage NMPC is eight. The optimization problem solved for all the controllers is the same as presented in (3) formulated in the multi-stage setting (5) and with a fixed initial condition of  $x_{\text{init}} = [20, 30, 0]^T$ , where the angles are given in degrees.

Table IV shows the average thrust over 150 s obtained by applying the different algorithms to the kite problem for different values of the uncertain parameters ( $E_0$ ,  $\tilde{c}$ ) and for a time varying wind speed  $v_0$ . It can be seen that all the standard NMPC controllers violate the height constraint and result in infeasible optimization problems for some scenarios (denoted as inf. in the table). If the optimization problem is infeasible and the obtained control input is applied to the plant the resulting behavior of the kite might be dangerous as seen in Fig. 3 and should be avoided. The orientation of

TABLE IV  
PERFORMANCE COMPARISON FOR STANDARD AND MULTI-STAGE NMPC UNDER VARYING WIND SPEEDS  $v_0$  AND FOR DIFFERENT VALUES OF THE  
UNCERTAIN PARAMETERS

Parameter value		Average thrust over 150 s [kN]				
$E[-]$	$\tilde{c}[-]$	Standard 1	Standard 2	Standard 3	Multi-stage 1	Multi-stage 2
5.0	0.005	332.54	370.86	305.28	340.91	365.44
5.0	0.028	275.96	219.44	265.60	278.74	277.51
5.0	0.04	242.03	139.93	252.43	245.44	237.94
3.0	0.005	141.14	146.51	126.58	143.99	145.52
3.0	0.028	inf.	58.13	inf.	98.31	99.89
3.0	0.04	inf.	inf.	inf.	80.89	75.29
7.0	0.005	641.93	700.10	556.93	643.36	654.46
7.0	0.028	547.95	481.55	515.61	553.28	544.02
7.0	0.04	502.89	375.05	483.10	505.26	510.65
Average performance [kN] <sup>1</sup>		383.49	347.54	357.93	387.28	390.79

the kite ( $\psi$ ), the control input  $\tilde{u}$ , the obtained thrust  $T_F$  and the varying wind speed  $v_0$  used can be seen in Fig. 4 for the controllers Standard 2 and Multi-stage 1, with  $E = 7$  and  $\tilde{c} = 0.028$ . In contrast to standard NMPC, multi-stage NMPC satisfies the constraints for all the cases of the uncertainty, including also those that are not explicitly included in the scenario tree. The infeasible optimization problems can be avoided by using soft constraints but the use of multi-stage NMPC not only avoids the violation of the constraints but also increases the average performance over all the scenarios. The gain on the performance is small ( $\sim 1\%$ ) for a good choice of the nominal parameters and of the prediction horizon (Standard 1) but it is bigger for other choices of the nominal parameters in the standard controller (Standard 2,  $\sim 11\%$  gain) or for a wrong choice of the prediction horizon (Standard 3,  $\sim 9\%$  gain). We consider wrong choices of the prediction horizon those which lead to a big influence of the procrastination effect. Interestingly, the use of a different prediction horizon for multi-stage controllers has a small influence on the performance. The main reason for that is that having a higher wind speed in the model is similar to having a longer prediction horizon because the cycles can be completed faster. Since multi-stage NMPC includes several wind speeds (and several other values of the parameters) in the predictions, at the time that the procrastination effect would affect standard NMPC, the predictions of the other scenarios in multi-stage NMPC lead the kite to stay in trajectories that are good on the average (trajectory similar to the periodic solution in Fig. 1), mitigating the procrastination effect described in the previous sections that results in sub-optimal (in average) cycles as the ones for  $N_p = 60$  in Fig. 1 for the controller Standard 3.

The average computation times per NMPC iteration using a prediction horizon  $N_p = 40$  are 0.060 s for standard NMPC and 0.9 s for multi-stage NMPC. While the standard NMPC problem is solved fast enough for a real-time

<sup>1</sup>In order to have a direct performance comparison between the different controllers the average performance is calculated as the average thrust for all feasible scenarios only, i.e. the scenarios with ( $E = 3.0$ ,  $\tilde{c} = 0.028$ ) and ( $E = 3.0$ ,  $\tilde{c} = 0.04$ ) are excluded from this calculation for all the controllers.

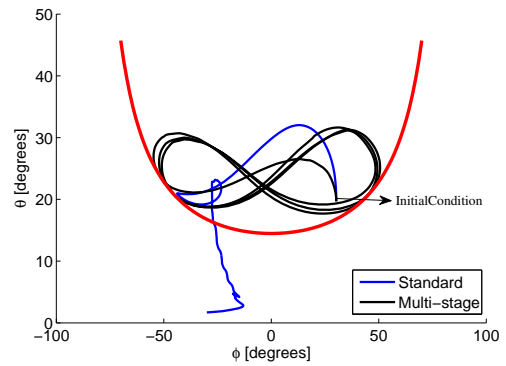


Fig. 3. Trajectory obtained for Standard 1 and Multi-stage 1 NMPC for  $E = 3$ ,  $\tilde{c} = 0.04$  and varying wind speeds for 150 seconds (sixth row in Table IV).

implementation, the computation times obtained for multi-stage NMPC with the current implementation are not short enough. The generation of tailored C-code and the use of the real-time iteration scheme [28] have a potential to decrease the computation time dramatically and thus would enable a real-time implementation of the proposed controller.

## VI. CONCLUSION

This paper presented a control strategy for towing kites using nonlinear model predictive control based on a simple model proposed recently in the literature. The assumptions made in the modeling, together with the fact that is not possible to determine some critical parameters with high accuracy and that kites are subject to strong external disturbances such as varying wind conditions make it necessary to take uncertainties explicitly into account. Tracking a pre-calculated trajectory can be sub-optimal or even infeasible under the presence of strong uncertainties. Therefore, an economic cost function that maximizes directly the thrust (energy) produced by the kite is used. Instead of using a standard NMPC approach, multi-stage NMPC is proposed in order to deal with the uncertainties. Simulation results illustrate that under the presence of parametric uncertainties



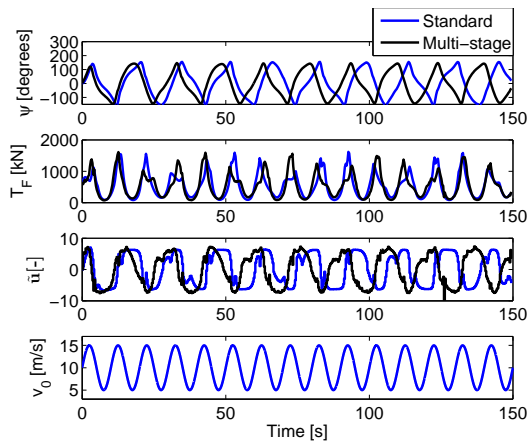


Fig. 4. State  $\psi$ , thrust ( $T_F$ ), control input ( $\tilde{u}$ ) and wind speed  $v_0$  for Standard 2 and Multi-stage 1 for  $E = 7$ ,  $\tilde{c} = 0.028$  (eighth row in Table IV).

and time-varying disturbances, the simple use of standard NMPC leads to infeasible problems or suboptimal solutions, while multi-stage NMPC is able to satisfy the constraints for all the scenarios achieving a better average performance regardless of the nominal model or of the prediction horizon used in the standard NMPC controller. The drawback of the approach is a higher computational effort.

Future work includes a comparison of the performance of different RTO techniques such as the one included in [9] with multi-stage NMPC and with techniques without trajectory pre-calculation [7]. In addition, the NMPC controllers using a simple model presented in this paper will be tested using a more detailed model as a simulation of the real kite.

## VII. ACKNOWLEDGEMENTS

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