Multiparty session types

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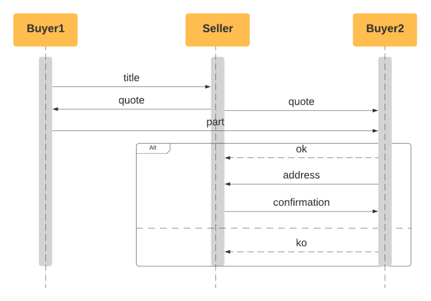
Multiparty session types[1]

- Extension of binary session types to multiparty sessions
- Asynchronous communications
- Interactions are abstracted as a global scenario, namely, Global types
  - specify dependencies and causal chains of multiparty asynchronous interactions

[1] Honda, K., Yoshida, N., & Carbone, M. Multiparty asynchronous session types. POPL 2008

Global Graph (Choreography)

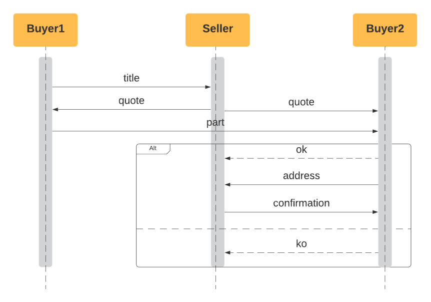
Two Buyers Protocol



$$\mathtt{TBProt} = \begin{array}{l} \mathtt{b1}{\to}\mathtt{s} : \langle \mathtt{string} \rangle . \\ \mathtt{s}{\to}\mathtt{b1} : \langle \mathtt{float} \rangle . \\ \mathtt{s}{\to}\mathtt{b2} : \langle \mathtt{float} \rangle . \\ \mathtt{b1}{\to}\mathtt{b2} : \langle \mathtt{float} \rangle . \\ \mathtt{b2}{\to}\mathtt{s} : \left\{ \begin{array}{l} ok : \mathtt{b2}{\to}\mathtt{s} : \langle \mathtt{string} \rangle . \mathtt{s}{\to}\mathtt{b2} : \langle \mathtt{string} \rangle . \mathsf{end}, \\ ko : \mathsf{end} \end{array} \right\} \end{array}$$

Local Types

Two Buyers Protocol



$$\texttt{Buyer1} = \; !\texttt{string}.?\texttt{float}.!\texttt{float}.\texttt{end}$$

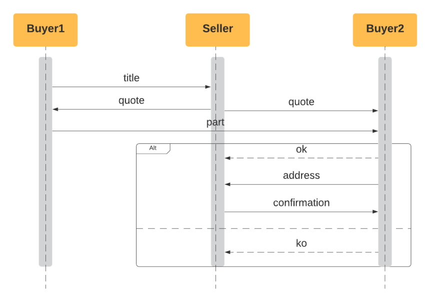
- The first message ($\texttt{string}$) is for the Seller and the second one ($\texttt{float}$) is for Buyer2? Information absent in the local type
- There are alternatives:
  - *a là* communicating machines: one channel for each pair in each direction

    $$\texttt{Buyer1} = b_1 s!\langle\texttt{string}\rangle.sb_1?\langle\texttt{float}\rangle.b_1 b_2!\langle\texttt{float}\rangle.\texttt{end}$$

  - Decorated global graphs

Global Graph (Choreography)

Two Buyers Protocol



$$
\begin{aligned}
\mathtt{TBProt} \; = \; & \mathtt{b1}\!\to\!\mathtt{s} : x\,\langle \mathtt{string}\rangle. \\
& \mathtt{s}\!\to\!\mathtt{b1} : y\,\langle \mathtt{float}\rangle. \\
& \mathtt{s}\!\to\!\mathtt{b2} : z_1\,\langle \mathtt{float}\rangle. \\
& \mathtt{b1}\!\to\!\mathtt{b2} : z_2\,\langle \mathtt{float}\rangle. \\
& \mathtt{b2}\!\to\!\mathtt{s} : x \left\{ \begin{array}{l} ok : \mathtt{b2}\!\to\!\mathtt{s} : x\,\langle \mathtt{string}\rangle.\mathtt{s}\!\to\!\mathtt{b2} : z_1\,\langle \mathtt{string}\rangle.\mathtt{end}, \\ ko : \mathtt{end} \end{array} \right\}
\end{aligned}
$$

$$
\mathtt{Buyer1} \; = x!\langle \mathtt{string}\rangle.y?\langle \mathtt{float}\rangle.z_2!\langle \mathtt{float}\rangle.\mathtt{end}
$$

First-order, finite MST

Syntax

$$
\begin{array}{rcll}
\eta & ::= & \mathsf{p}{\to}\mathsf{q} : x & \text{action} \\
\mathsf{G} & ::= & \eta\langle\tilde{\mathsf{S}}\rangle.\mathsf{G} & \text{interaction} \\
& \mid & \eta\{l_j : \mathsf{G}_j\}_{j \in J} & \text{branch} \\
& \mid & \mathsf{G} \mid \mathsf{G} & \text{parallel} \\
& \mid & \mathsf{end} & \text{termination} \\
\mathsf{S} & ::= & \mathtt{int} \mid \mathtt{unit} \mid \mathtt{bool} \mid \ldots & \text{basic sorts}
\end{array}
$$

- p, r, ... : participants (also roles)
- x, y, ...: communication channels
- l, ... : labels
- $\tilde{\_}$: tuples

Realizations

Example

$$
\begin{aligned}
\mathtt{G} &= \mathtt{p}{\to}\mathtt{q} : x \, \langle \mathtt{int} \rangle . \mathtt{p}{\to}\mathtt{r} : y \, \langle \mathtt{bool} \rangle . \mathtt{end} \\
P_p &= x\,!\,1\,.\,y\,!\,\mathtt{true}.\mathbf{0} \\
P_q &= x\,?(i).\mathbf{0} \\
P_r &= y\,?(j).\mathbf{0}
\end{aligned}
$$

Realizations

Example

$$
\begin{aligned}
\mathtt{G} &= \mathtt{p}{\to}\mathtt{q} : x\,\langle \mathtt{int} \rangle.\mathtt{p}{\to}\mathtt{r} : x\,\langle \mathtt{bool} \rangle.\mathtt{end} \\
P_p &= x\,\mathtt{!}1.x\,\mathtt{!}\mathtt{true}.\mathbf{0} \\
P_q &= x\,\mathtt{?}(i).\mathbf{0} \\
P_r &= x\,\mathtt{?}(j).\mathbf{0}
\end{aligned}
$$

Realizations

Example

$$\begin{aligned}
\mathtt{G} &= \mathtt{p}{\to}\mathtt{q} : x\,\langle\mathtt{int}\rangle.\mathtt{p}{\to}\mathtt{r} : x\,\langle\mathtt{bool}\rangle.\mathtt{end} \\
P_p &= x\,!\,1.x\,!\,\mathtt{true}.\mathbf{0} \\
P_q &= x\,?(i).\mathbf{0} \\
P_r &= x\,?(j).\mathbf{0}
\end{aligned}$$

We cannot ensure that P_q gets 1 and P_r gets `true` (race on x)
Hence, `G` is bad (Output-to-Output bad)

Realizations

Example

$$
\begin{aligned}
\mathtt{G} &= \mathtt{p}{\to}\mathtt{q} : x\,\langle\mathtt{int}\rangle.\mathtt{r}{\to}\mathtt{q} : y\,\langle\mathtt{bool}\rangle.\mathtt{end} \\
P_p &= x\,!\,1.\mathbf{0} \\
P_q &= x\,?(i).y\,?(j).\mathbf{0} \\
P_r &= y\,!\,\mathtt{true}.\mathbf{0}
\end{aligned}
$$

Realizations

Example

$$\begin{aligned}
\mathtt{G} &= \mathtt{p}{\to}\mathtt{q} : x\,\langle\mathtt{int}\rangle\mathtt{.}\,\mathtt{r}{\to}\mathtt{q} : y\,\langle\mathtt{bool}\rangle\mathtt{.end} \\
P_p &= x\,\mathtt{!}1\mathtt{.}\mathbf{0} \\
P_q &= x\,\mathtt{?}(i)\mathtt{.}\,y\,\mathtt{?}(j)\mathtt{.}\mathbf{0} \\
P_r &= y\,\mathtt{!}\mathtt{true}\mathtt{.}\mathbf{0}
\end{aligned}$$

No races on channels x and y

Hence, $\mathtt{G}$ is good (Input-to-Input good)

Realizations

Example

$$\begin{aligned}
\mathtt{G} &= \mathtt{p}{\to}\mathtt{q} : x\,\langle \mathtt{int}\rangle.\mathtt{r}{\to}\mathtt{q} : x\,\langle \mathtt{bool}\rangle.\mathtt{end} \\
P_p &= x\,\mathtt{!}\,\mathtt{1}.\mathbf{0} \\
P_q &= x\,\mathtt{?}(i).x\,\mathtt{?}(j).\mathbf{0} \\
P_r &= x\,\mathtt{!}\,\mathtt{true}.\mathbf{0}
\end{aligned}$$

Realizations

Example

$$\begin{aligned}
\mathtt{G} &= \mathtt{p}{\to}\mathtt{q} : x\,\langle \mathtt{int}\rangle.\mathtt{r}{\to}\mathtt{q} : x\,\langle \mathtt{bool}\rangle.\mathtt{end}\\
P_p &= x\,\mathtt{!}1.\mathbf{0}\\
P_q &= x\,\mathtt{?}(i).x\,\mathtt{?}(j).\mathbf{0}\\
P_r &= x\,\mathtt{!}\mathtt{true}.\mathbf{0}
\end{aligned}$$

Race on x

Hence, $\mathtt{G}$ is bad (Input-to-Input bad)

Realizations

Example

$$\begin{aligned}
\mathtt{G} &= \mathsf{p}{\to}\mathsf{q} : x\,\langle\mathtt{int}\rangle.\mathsf{q}{\to}\mathsf{r} : x\,\langle\mathtt{int}\rangle.\mathtt{end} \\
P_p &= x\,\mathtt{!}\,1.\mathbf{0} \\
P_q &= x\mathtt{?}(i).x\,\mathtt{!}\,i.\mathbf{0} \\
P_r &= x\mathtt{?}(i).\mathbf{0}
\end{aligned}$$

Realizations

Example

$$\begin{aligned}
\mathtt{G} &= \mathtt{p}{\rightarrow}\mathtt{q} : x\,\langle\mathtt{int}\rangle.\mathtt{q}{\rightarrow}\mathtt{r} : x\,\langle\mathtt{int}\rangle.\mathtt{end}\\
P_p &= x\,!\,1.\mathbf{0}\\
P_q &= x?(i).x\,!\,i.\mathbf{0}\\
P_r &= x?(i).\mathbf{0}
\end{aligned}$$

Race on x
Hence, $\mathtt{G}$ is bad (Input-to-Output bad)

Realizations

Example

$$\begin{aligned}
\mathtt{G} &= \mathtt{p}{\to}\mathtt{q} : x\,\langle\mathtt{int}\rangle.\mathtt{q}{\to}\mathtt{r} : y\,\langle\mathtt{int}\rangle.\mathtt{end} \\
P_p &= x\,!\,1.\mathbf{0} \\
P_q &= x?(i).y\,!\,i.\mathbf{0} \\
P_r &= y?(i).\mathbf{0}
\end{aligned}$$

Realizations

Example

$$
\begin{aligned}
\mathtt{G} &= \mathtt{p}\to\mathtt{q} : x\,\langle\mathtt{int}\rangle.\mathtt{q}\to\mathtt{r} : y\,\langle\mathtt{int}\rangle.\mathtt{end} \\
P_p &= x!1.\mathbf{0} \\
P_q &= x?(i).y!i.\mathbf{0} \\
P_r &= y?(i).\mathbf{0}
\end{aligned}
$$

No Races on x and y

Hence, $\mathtt{G}$ is good (Input-to-Output good)

Realizations

Example

$$\begin{aligned} \mathtt{G} &= \mathtt{p}{\to}\mathtt{q} : x\,\langle \mathtt{int} \rangle.\mathtt{p}{\to}\mathtt{q} : x\,\langle \mathtt{bool} \rangle.\mathtt{end} \\ P_p &= x!1.x!\mathtt{true}.\mathbf{0} \\ P_q &= x?(i).y?(j).\mathbf{0} \end{aligned}$$

Realizations

Example

$$\begin{aligned}\mathtt{G} &= \mathtt{p}{\to}\mathtt{q} : x\,\langle\mathtt{int}\rangle.\mathtt{p}{\to}\mathtt{q} : x\,\langle\mathtt{bool}\rangle.\mathtt{end}\\ P_p &= x!1.x!\mathtt{true}.\mathbf{0}\\ P_q &= x?(i).y?(j).\mathbf{0}\end{aligned}$$

No Races on x
Hence, $\mathtt{G}$ is good (Input-to-Input, Output-to-Output good)

Realizations

Example

$$
\begin{aligned}
\mathtt{G} &= \mathtt{p}{\to}\mathtt{q} : x\,\langle \mathtt{int}\rangle.\mathtt{s}{\to}\mathtt{r} : y\,\langle \mathtt{bool}\rangle.\mathtt{p}{\to}\mathtt{r} : x\,\langle \mathtt{bool}\rangle.\mathtt{end} \\
P_p &= x\mathtt{!}1.x\mathtt{!}\mathtt{true}.\mathbf{0} \\
P_q &= x\mathtt{?}(i).\mathbf{0} \\
P_r &= y\mathtt{?}(i).x\mathtt{?}(j).\mathbf{0} \\
P_s &= y\mathtt{!}\mathtt{true}.\mathbf{0}
\end{aligned}
$$

Realizations

Example

$$
\begin{aligned}
\mathtt{G} &= \mathtt{p}{\rightarrow}\mathtt{q} : x\,\langle \mathtt{int}\rangle.\mathtt{s}{\rightarrow}\mathtt{r} : y\,\langle \mathtt{bool}\rangle.\mathtt{p}{\rightarrow}\mathtt{r} : x\,\langle \mathtt{bool}\rangle.\mathtt{end} \\
P_p &= x!1.x!\mathtt{true}.\mathbf{0} \\
P_q &= x?(i).\mathbf{0} \\
P_r &= y?(i).x?(j).\mathbf{0} \\
P_s &= y!\mathtt{true}.\mathbf{0}
\end{aligned}
$$

Races on x
Hence, $\mathtt{G}$ is bad

Realizations

Example

$$
\begin{aligned}
\mathtt{G} &= \mathtt{p}{\to}\mathtt{q} : x\,\langle\mathtt{int}\rangle.\mathtt{q}{\to}\mathtt{r} : y\,\langle\mathtt{bool}\rangle.\mathtt{p}{\to}\mathtt{r} : x\,\langle\mathtt{bool}\rangle.\mathtt{end} \\
P_p &= x!1.x!\mathtt{true}.\mathbf{0} \\
P_q &= x?(i).y!\mathtt{true}.\mathbf{0} \\
P_r &= y?(i).x?(j).\mathbf{0}
\end{aligned}
$$

Realizations

Example

$$\begin{aligned}
\mathtt{G} &= \mathtt{p}\to\mathtt{q} : x\,\langle \mathtt{int}\rangle.\mathtt{q}\to\mathtt{r} : y\,\langle \mathtt{bool}\rangle.\mathtt{p}\to\mathtt{r} : x\,\langle \mathtt{bool}\rangle.\mathtt{end}\\
P_p &= x!1.x!\mathtt{true}.\mathbf{0}\\
P_q &= x?(i).y!\mathtt{true}.\mathbf{0}\\
P_r &= y?(i).x?(j).\mathbf{0}
\end{aligned}$$

No races on x and y
Hence, G is good

This notion is formalised as LINEARITY (we postpone its definition)

Local types

Syntax

$\mathsf{T} ::=$		$x?\langle\tilde{\mathsf{S}}\rangle.\mathsf{T}$	receive
	$\mid$	$x!\langle\tilde{\mathsf{S}}\rangle.\mathsf{T}$	send
	$\mid$	$x \oplus \{l_i : \mathsf{T}_i\}_{i\in I}$	select
	$\mid$	$x\&\{l_i : \mathsf{T}_i\}_{i\in I}$	branch
	$\mid$	$\mathtt{end}$	termination
$\mathsf{S} ::=$		$\mathtt{int} \mid \mathtt{unit} \mid \mathtt{bool} \mid \ldots$	basic sorts

Projection

Definition

$$
\mathsf{G}\restriction \mathsf{p} = \begin{cases}
x\,!\langle\tilde{\mathsf{S}}\rangle.\mathsf{G}'\restriction\mathsf{p} & \text{if } \mathsf{G} = \mathsf{p}\to\mathsf{q} : x\,\langle\tilde{\mathsf{S}}\rangle.\mathsf{G}' \textit{ and } \mathsf{p} \neq \mathsf{q} \\
x\,?\langle\tilde{\mathsf{S}}\rangle.\mathsf{G}'\restriction\mathsf{p} & \text{if } \mathsf{G} = \mathsf{q}\to\mathsf{p} : x\,\langle\tilde{\mathsf{S}}\rangle.\mathsf{G}' \textit{ and } p \neq \mathsf{q} \\
\mathsf{G}'\restriction\mathsf{p} & \text{if } \mathsf{G} = \mathsf{q}\to\mathsf{r} : x\,\langle\tilde{\mathsf{S}}\rangle.\mathsf{G}' \textit{ and } \mathsf{p} \neq \mathsf{q} \neq \mathsf{r} \\
x \oplus \{l_i : \mathsf{G}_i\restriction\mathsf{p}\}_{i\in I} & \text{if } \mathsf{G} = \mathsf{p}\to\mathsf{q} : x\{l_i : \mathsf{G}_i\}_{i\in I} \textit{ and } \mathsf{p} \neq \mathsf{q} \\
x\&\{l_i : \mathsf{G}_i\restriction\mathsf{p}\}_{i\in I} & \text{if } \mathsf{G} = \mathsf{q}\to\mathsf{p} : x\{l_i : \mathsf{G}_i\}_{i\in I} \textit{ and } \mathsf{p} \neq \mathsf{q} \\
\mathsf{G}_1\restriction\mathsf{p} & \text{if } \mathsf{G} = \mathsf{q}\to\mathsf{r} : x\{l_i : \mathsf{G}_i\}_{i\in I} \textit{ and } \mathsf{p} \neq \mathsf{q} \neq \mathsf{r} \textit{ and} \\
& \qquad \forall i,j.\mathsf{G}_i\restriction\mathsf{p} = \mathsf{G}_j\restriction\mathsf{p} \\
\mathsf{G}_i\restriction\mathsf{p} & \text{if } \mathsf{G} = \mathsf{G}_1 \mid \mathsf{G}_2 \textit{ and } \mathsf{p} \in \mathsf{G}_i \textit{ and } \mathsf{p} \notin \mathsf{G}_j \textit{ and} \\
& \qquad i \neq j \in \{1,2\} \\
\mathsf{end} & \text{if } \mathsf{G} = \mathsf{G}_1 \mid \mathsf{G}_2 \textit{ and } \mathsf{p} \notin \mathsf{G}_1 \textit{ and } \mathsf{p} \notin \mathsf{G}_2
\end{cases}
$$