DATA SCIENCE FOR FINANCE
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Report

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Introduction

In the realm of finance, the valuation of interest rate swaps is a fundamental task for financial analysts, risk managers, and institutional investors. This report delves into the comprehensive analysis of a 25-year fixed rate receiver interest rate swap (IRS), with a particular focus on constructing a yield curve through various interpolation methods, calculating accrued interests, determining market values, and assessing the contract's net present value. Furthermore, the report extends to estimate the swap's par rate and its sensitivity to interest rate fluctuations, quantified by the IRS Greeks: PV01, DV01, and Gamma. These measures are crucial for understanding the contract's inherent interest rate risk and for making informed hedging and investment decisions. The analyses are conducted in the context of the current economic climate, characterized by the EURIBOR 6-month rate, and are visualized with precision to facilitate a clear understanding of the swap's valuation and risk profile.

1 Yield Curve Interpolation

The yield curve is a graphical representation that shows the relationship between the interest rates (often referred to as the yield) and the time to maturity of debt securities. It is a crucial tool used by economists and investors to gauge the economic situation and to make investment decisions.

In practice, the yield curve does not provide interest rates for every possible maturity. Therefore, we employ interpolation techniques to estimate rates for maturities where market data is not available. Here we have used several interpolation methods:

- **Linear Interpolation**: This method assumes that the change between two points is linear and does not account for any possible curvature between them. Mathematically, the linear interpolation for a value y at point x between two known points (x_1, y_1) and (x_2, y_2) is given by the equation:

$$y = y_1 + \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

- **Quadratic and Cubic Splines**: These methods fit polynomials of second or third degree, respectively, between each pair of data points. They are more flexible than linear interpolation and can better capture the curvature of the yield curve.
- **Piecewise Polynomial Interpolation (PCHIP)**: This method ensures that the interpolation is both smooth and monotonic, thereby preserving the shape of the data and avoiding the oscillations that can occur with high-degree polynomial fits.
- **Akima Interpolation**: This is a type of spline interpolation that is less sensitive to large variations in the slope of the data points. It is particularly useful when the data set contains outliers or abrupt changes in the derivative.

We have calculated these interpolated rates for a dense set of points along the period range to create a smooth curve that extends from the shortest to the longest maturities in our data set.

The following plot shows the original market data points and the yield curves obtained from each interpolation method. This visual comparison helps us to choose the most suitable interpolation for our analysis based on how well it fits the market rates and its behavior between data points.

In the following questions, I decided to use the cubic spline.

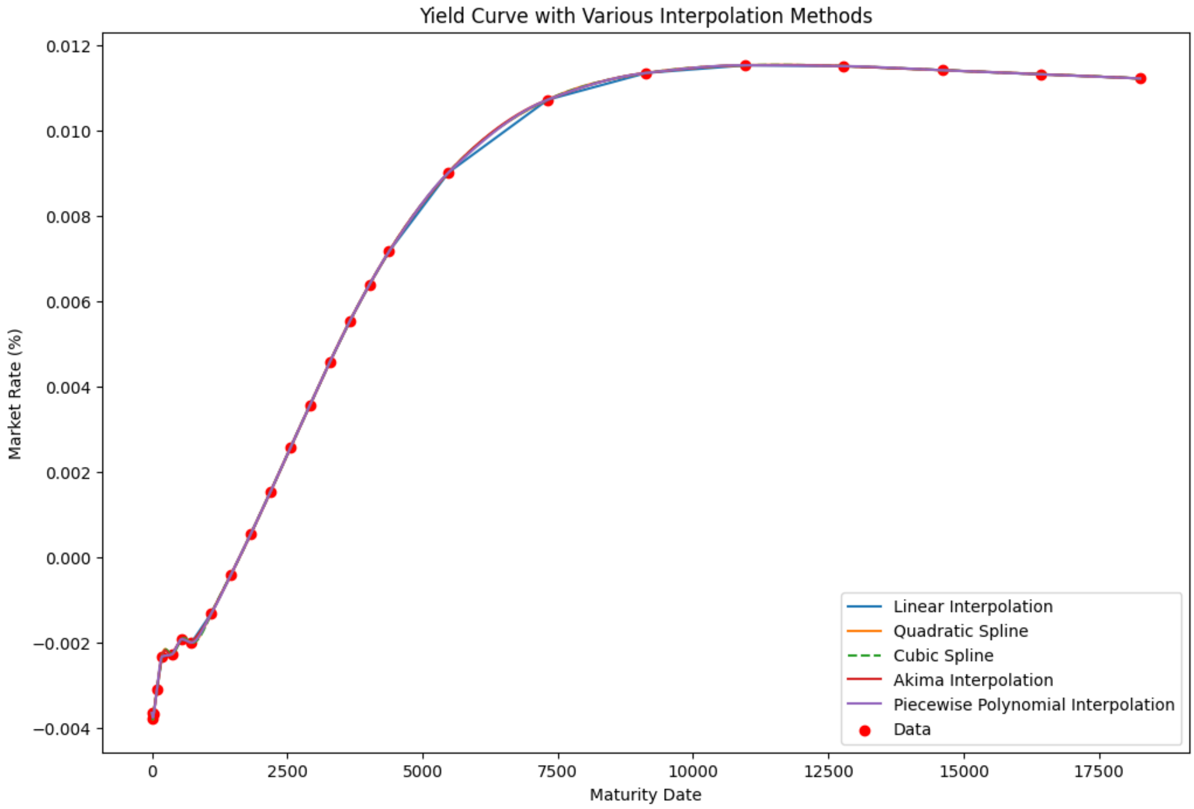

Figure 1: Yield Curve with Various Interpolation Methods

2 Accrued Interest Computation

Accrued interest is the interest that accumulates on a bond or other fixed-income security between the payment periods. It represents the amount of interest that has been earned but not yet paid to the bondholder. The calculation of accrued interest is essential for a fair assessment of the current value of a financial instrument, as it accounts for earned income that has not yet been realized.

For the **fixed leg** of an interest rate swap (IRS), the accrued interest is calculated using the formula:

$$\text{Accrued Interest} = \frac{\text{Notional Amount} \times \text{Coupon Rate} \times \text{Day Count}}{360} \tag{1}$$

where:

- The **Notional Amount** is the principal amount on which the interest payments are based.
- The **Coupon Rate** is the annual interest rate paid by the security.
- The **Day Count** is the number of days since the last payment.

For the **floating leg**, the calculation is similar, but it also includes the **Spread** and the **Current Reset Rate**. The formula is:

$$\text{Accrued Interest} = \frac{\text{Notional Amount} \times (\text{Current Reset Rate} + \text{Spread}) \times \text{Day Count}}{360} \tag{2}$$

In both cases, a 360-day year is assumed, following the day count convention specified in the contract details.

Using the provided code, the accrued interest for both legs of the contract was computed as follows:

- **Fixed Leg Accrued Interest**: €1412.42
- **Floating Leg Accrued Interest**: €-11603.33

The negative value for the floating leg indicates that the accrued interest is working against the holder of the floating leg, reflective of the specifics of the interest rate environment or contract terms at the valuation date.

3 Clean (principal) and Dirty Market Value

To determine the market value of a swap, we need to calculate the present values (PV) of both the fixed and floating legs, and then adjust for accrued interest to find the dirty and clean market values. Since this is a Euro-denominated swap, all values are expressed in EUR.

Present Value of Floating Leg

The floating leg's cashflows depend on the forward rates, which reflect the expected future EURIBOR rates determined from the yield curve. The cashflows are calculated as follows:

$$\text{Cashflow}_{\text{floating}} = \text{Notional} \times \text{Forward Rate} \times \frac{\text{Day Count}}{360} \tag{3}$$

Each cashflow is discounted to its present value using the discount factor derived from the yield curve:

$$\text{PV}_{\text{floating}} = \sum \frac{\text{Cashflow}_{\text{floating}}}{(1 + \frac{\text{Discount Rate}}{2})^{2\times\text{Time to Payment}}} \tag{4}$$

Where Time to Payment is the time in years from the valuation date to each cash flow payment date.

Present Value of Fixed Leg

For the fixed leg, the cashflows are based on the agreed fixed coupon rate. The cashflows for the fixed leg are given by:

$$\text{Cashflow}_{\text{fixed}} = \text{Notional} \times \text{Coupon Rate} \tag{5}$$

These cashflows are also discounted back to present value using discount factors calculated from the yield curve:

$$\text{PV}_{\text{fixed}} = \sum \frac{\text{Cashflow}_{\text{fixed}}}{(1 + \text{Discount Rate})^{\text{Time to Payment}}} \tag{6}$$

Clean and Dirty Market Value

The *dirty market value* of the swap is the net present value of the fixed leg minus the net present value of the floating leg, adjusted for accrued interest:

$$\text{Dirty Market Value} = \text{PV}_{\text{fixed}} - \text{PV}_{\text{floating}} + \text{Accrued Interest}_{\text{fixed}} + \text{Accrued Interest}_{\text{floating}} \tag{7}$$

The *clean market value* is the dirty market value minus the accrued interest:

$$\text{Clean Market Value} = \text{Dirty Market Value} - \text{Accrued Interest}_{\text{fixed}} - \text{Accrued Interest}_{\text{floating}} \tag{8}$$

Using the code provided, the calculated market values of the swap contract in EUR were:

- **Clean Market Value**: €8,205,734.83
- **Dirty Market Value**: €8,195,543.91

These market values represent the economic worth of the swap to the involved parties on the valuation date, with the clean market value providing a standardized measure for comparison across different swaps.

4 Net Present Value Computation

The Net Present Value (NPV) of a swap contract is a financial metric that represents the difference in value between the fixed leg and the floating leg of the swap. It reflects the current worth of all future cash flows that each party expects to receive or pay. In this context, the NPV helps us understand the value of the swap from one party's perspective at the valuation date.

The NPV is calculated by subtracting the present value (PV) of payments to be made from the PV of payments to be received. For a fixed rate receiver swap, the NPV formula is given by:

$$\text{NPV} = \text{PV}_{\text{fixed leg}} - \text{PV}_{\text{floating leg}} \tag{9}$$

Where:

- $\text{PV}_{\text{fixed leg}}$ is the present value of the fixed leg cash flows, and
- $\text{PV}_{\text{floating leg}}$ is the present value of the floating leg cash flows.

In the given Euro-denominated swap contract, the calculated NPV is:

$$\text{Net Present Value of the Swap Contract} = 8205734.83 \text{ EUR} \tag{10}$$

This value signifies the economic benefit that the fixed rate receiver gains from the swap contract at the valuation date. If the NPV is positive, the contract is favorable to the fixed rate receiver. Conversely, a negative NPV would indicate a benefit to the floating rate payer.

5 Swap par Rate Estimation

The swap par rate is the fixed interest rate at which the present value of the fixed leg cash flows equals the present value of the floating leg cash flows. In essence, it is the rate that makes the swap's net present value (NPV) equal to zero at inception. This rate is essential for swap pricing and comparative analysis.

The par rate can be determined using the following iterative method, typically employing Newton-Raphson numerical technique for finding roots:

1. Calculate the present value of the fixed leg for a given swap rate.
2. Adjust the swap rate such that the difference between the present value of the fixed leg and the floating leg is zero.

Mathematically, the par rate (r_{par}) can be found by solving the equation:

$$\sum_{i=1}^{n} (\text{Notional} \times r_{\text{par}} \times DF_i) = \text{PV}_{\text{floating leg}} \tag{11}$$

where:

- DF_i is the discount factor for the i-th payment date,
- Notional is the notional amount of the swap,
- $\text{PV}_{\text{floating leg}}$ is the present value of the floating leg.

By applying this process to the provided market data and discount factors, the swap par rate is calculated to be:

$$\text{Swap Par Rate} = -0.069920 \tag{12}$$

This negative par rate indicates that, given the current market conditions and discount factors, the fixed rate that would make the swap value zero is negative. This outcome is reflective of the prevailing low or negative interest rate environment in the Eurozone at the time of valuation.

6 IRS Greeks Estimation

Interest rate swaps (IRS) Greeks are measures of the sensitivity of the swap's value to changes in underlying interest rate movements. Specifically, the following are calculated:

PV01 (Present Value of a Basis Point) PV01 measures the change in the swap's value for a one basis point (0.01%) parallel shift in the yield curve. It is a first-order (linear) measure of interest rate risk. The formula for PV01 is:

$$PV01 = \frac{PV_{\text{up}} - PV_{\text{down}}}{2} \tag{13}$$

where PV_{up} and PV_{down} are the present values of the swap after the yield curve is shifted up and down by one basis point, respectively.

DV01 (Dollar Value of a 01) DV01, also known as "dollar duration," represents the monetary change in the swap's value per one basis point change in interest rates. It is equivalent to PV01 when the notional amount is in the same currency as the calculated PV01.

Gamma Gamma measures the curvature of the swap's value in relation to interest rate movements and is a second-order (quadratic) risk measure. It indicates the rate of change of PV01 as the yield changes and is computed as:

$$\Gamma = \frac{PV01_{\text{up shift}} - PV01}{\text{shift size}} \tag{14}$$

A higher Gamma indicates a larger change in PV01 for interest rate movements, signifying greater convexity and thus, more pronounced non-linear interest rate risk.

Discussion The IRS Greeks provide valuable insights into the contract's sensitivity to interest rate changes. A positive PV01 implies that the swap's value increases as rates fall, and vice versa. A large absolute Gamma value suggests that small changes in interest rates could lead to significant changes in the swap's market value, indicating higher interest rate risk.

In the present calculation, the swap exhibits a PV01 of 218736.69 EUR, meaning that for each basis point increase in interest rates, the swap's value decreases by this amount. The Gamma value of -1095915879.71 indicates a high sensitivity of PV01 to interest rate changes, reflecting significant interest rate risk for this IRS contract.

Conclusion

Throughout this analysis, various financial modeling techniques and computational methods were employed to evaluate key aspects of an interest rate swap (IRS) contract. I built a complete yield curve using interpolation, calculated accrued interests, determined the swap's market value, estimated the net present value, and analyzed the swap par rate. Additionally, there was an assessment the interest rate risk by estimating IRS Greeks such as PV01, DV01, and Gamma.

While the computations provided valuable insights, some of the results, particularly the negative swap par rate and the high Gamma value, suggest an atypical market situation or possibly indicate the need for a review of the logic embedded in our code. These unusual outcomes underline the importance of not only relying on computational models but also integrating market intuition and thorough checks for potential anomalies or errors in the implementation.

In conclusion, while the models and calculations offer essential tools for swap valuation and risk assessment, they should be complemented with a critical review of the underlying assumptions, market conditions, and the consistency of the results with economic logic.

References

Lectures from Fixed Income