Executive Education | POST GRADUATION

DATA SCIENCE FOR FINANCE

Fixed Income Securities

WINTER 2024

Group Assignment

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1. Consider a government **Capital Indexed Inflation Linked Bond** (ILB)with term sheet:

Notional amount	25000
Coupon Type	Fixed
Coupon rate	6.75%
Coupon frequency	Semi-annual
Currency	USD
Issue Date	31/7/2020
Maturity Date	21/7/2025
Trade Date	18/09/2020
Settlement Lag	T+1
Day Count	ACT/ACT
Inflation Reference Index	US Consumer Price Index
Inflation Reference Index Level at issue	237.14365
Inflation Reference Index Level at Settlement	251.14721

Assume the CPI index I_t follows a log-normal model (geometric Brownian motion), i.e.,

$$dI_t \,/\, I_t = \mu d_t + \sigma dW_t$$

where W_t is a Wiener process, μ the constant drift, and $\sigma > 0$ is the diffusion coefficient with estimates $\hat{\mu} = 0.05321$ and $\hat{\sigma} = 0.06358$. Assume there is no inflation indexation lag.

Assume that the issuers yield curve on the valuation date is given by the Nelson–Siegel-Svensson zero-coupon rate function parameters,

β_0	β_1	β_2	β_3	τ_1	τ_2
5.9%	-1.6%	-0.5%	1%	5	0.5

Tasks:

a) Compute the accrued interest.

Accrued Interest: $267.05

The Accrued interest (AI) is computed as

In our Capital Indexed Inflation Linked Bond's term sheet there's no specification of when is the next coupon date. We thus assume that interest began to accrue from the issue date. Thus:

1. Since the trade date happens before there is a coupon payment, *u* is the no. of days from the date when interest begins to accrue, which is July 31st 2020.

2. The trade date was September 18th 2020, a Friday. However, since the Settlement Lag is T+1, the settlement happened only on September 21st, Monday. Hence, 52 days.

3. Since coupon frequency is equal to 2, this means that the coupons are paid every 6 months. The specific day in which that happens, would be dependent on the maturity date. Since our bond matures July 21st 2025, we would expect that the coupons would happen on both January 21st and July 21st of each year, unless this day would fall in a weekend or a bank holiday, where a convention would have to be followed, like Following Business Day or Modified Following Business Day conventions. In our case this means that the number of days since the last and next coupon would be counted from the bond issuance to the first coupon date. Thus, 174 days.

4. The inflation index at settlement is 251.147210 while 237.143650 at issue.

5. The notional amount is $25 000, while the coupon rate is 6.75%.

The accrued interest is thus: $267.05.

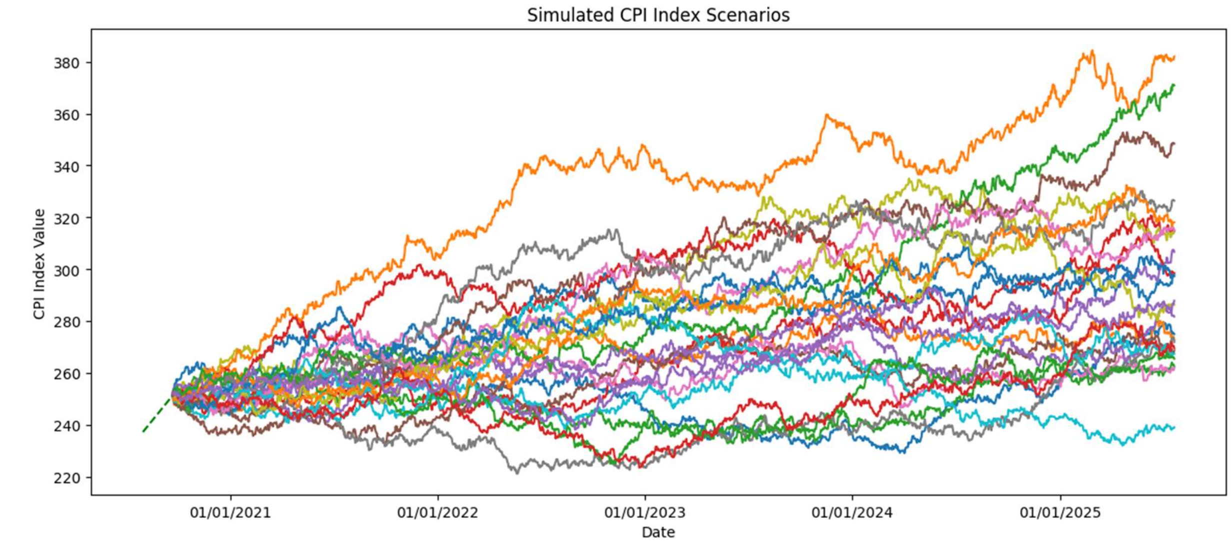
b) Simulate 10000 scenarios for the inflation rate curve and CPI index.

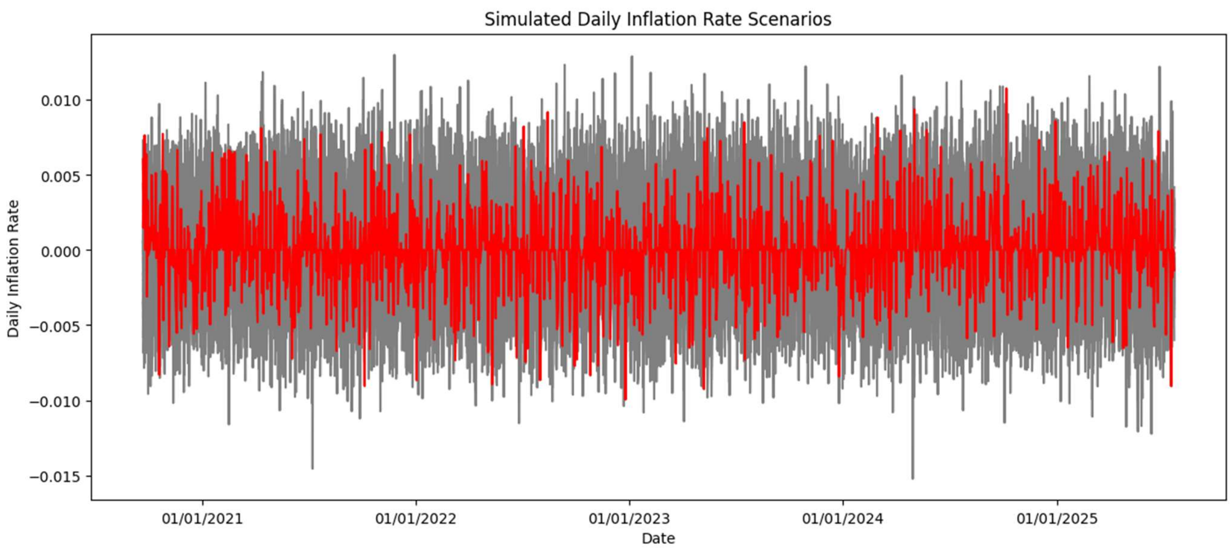
The simulation of the Consumer Price Index (CPI) employs a Geometric Brownian Motion (GBM) model, a stochastic process commonly used in financial mathematics for asset price modeling.

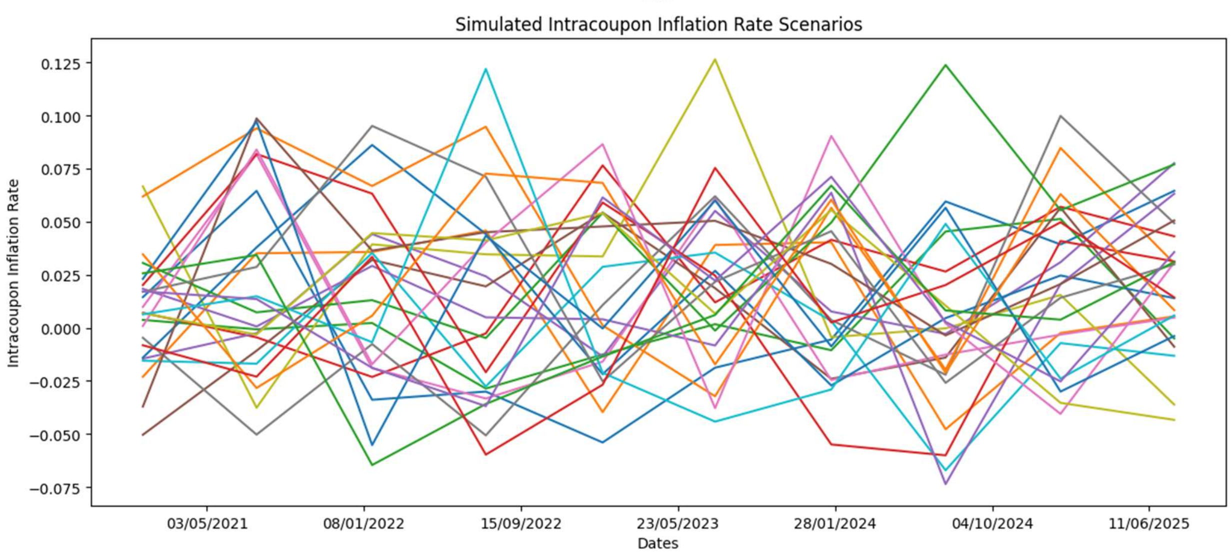
In our simulation, 10,000 scenarios are generated over a total of 4 x 365 + 366 days. The initial CPI value is set at 237.14365, and the estimated parameters for drift (μ) and volatility (σ) are 0.05321 and 0.06358, respectively.

Each scenario involves simulating the CPI index's path using the discretized version of the GBM formula.

The inflation rate for each day is then computed as:







This model enables the creation of numerous potential future paths for the CPI, providing insights into the potential variability and trends in inflation rates over time.

c) For each scenario, calculate the ILB cash flows and estimate their fair value.

To determine the cash flows and estimate the fair value of an Inflation-Linked Bond (ILB) under various inflation scenarios, we simulate the CPI index. Subsequently, we calculate the cash flows based on coupon payments and principal repayment at maturity. These cash flows are then discounted to their present value using the Nelson-Siegel-Svensson (NSS) model, reflecting the time value of money.

The adjusted notional for each coupon period is computed by multiplying the notional amount by the ratio of the CPI at the given period to the initial CPI. The fair value (FV) is obtained by summing the discounted future cash flows, computed through continuous compounding.

The NSS yield curve, which provides discount rates for each time period, is defined by a formula involving NSS parameters ($\beta 0$, $\beta 1$, $\beta 2$, $\beta 3$) and time decay factors ($\tau 1$, $\tau 2$). The final cash flow, which includes the adjusted principal repayment, is calculated on the last day of the simulation.

For each scenario, the fair value is the sum of the discounted cash flows, including the final principal repayment. This process is repeated across 10,000 inflation scenarios, resulting in a distribution of potential fair values for the ILB. Analyzing this distribution allows us to understand how the bond's value may be affected by changes in inflation expectations.

Below is the result of the Python code:

	Present Value
0	23579.619353
1	23395.796670
2	22565.993758
3	26807.547186
4	26139.677491
...	...
9995	20740.023024
9996	27063.425951
9997	25120.549320
9998	21312.727982
9999	28851.396712

	21/01/2021	21/07/2021	21/01/2022	21/07/2022	23/01/2023	21/07/2023	22/01/2024	22/07/2024	21/01/2025	21/07/2025
0	171.184282	179.632194	163.026560	163.678317	159.633498	165.575261	167.815049	178.790866	175.373229	27547.790171
1	164.857982	174.668602	174.778153	176.479616	162.045832	175.327688	175.547787	160.683506	168.330957	27309.239243
2	169.376703	168.627201	169.126054	163.919252	166.623671	169.033316	166.964801	176.399191	177.399809	26287.396022
3	169.940674	167.986720	164.841791	168.301577	181.656274	170.767623	175.739994	173.217569	178.368682	31525.838866
4	166.296174	168.317163	176.195564	172.839667	166.471451	178.040708	170.042873	168.257825	173.963354	30707.365178
...	...	...	...	...	...	...	...	...	...	...
9995	167.661132	172.372781	170.422546	172.757637	164.791169	175.490146	171.530986	156.800833	159.219233	24036.293079
9996	175.253865	168.451148	175.502357	180.162641	172.308237	161.439925	174.918645	177.156815	171.273385	31834.960483
9997	169.804845	174.182556	182.979556	168.837813	176.157568	165.777384	166.167013	170.651126	160.915214	29445.404015
9998	170.177229	181.547467	158.997453	165.202328	166.894412	168.238996	170.318205	172.086296	170.381506	24734.550597
9999	174.904529	179.756065	185.226611	176.250476	177.357802	159.887337	179.925400	169.079931	174.502423	34029.778730

d) Estimate and analyse the inflation linked bond price distribution, including interest rate and inflation risk measures.

To assess and analyze the distribution of prices for inflation-linked bonds (ILBs), we have computed the present values for each of the 10,000 simulated scenarios. These present values encapsulate both interest rate and inflation risks, reflecting changes in the consumer price index (CPI) and shifts in the yield curve.

The statistical measures derived from the present value distribution include:

1. **Mean**: The average of all present values, offering a measure of central tendency.

2. **Median**: The middle value of the present value distribution, indicating the point at which half the observations fall above and below.

3. **Standard Deviation**: A measure of the dispersion of present values around the mean.

4. **Variance**: The square of the standard deviation, providing an additional measure of dispersion.

5. **Skewness**: A gauge of the asymmetry of the present value distribution. Skewness close to zero suggests a symmetrical distribution.

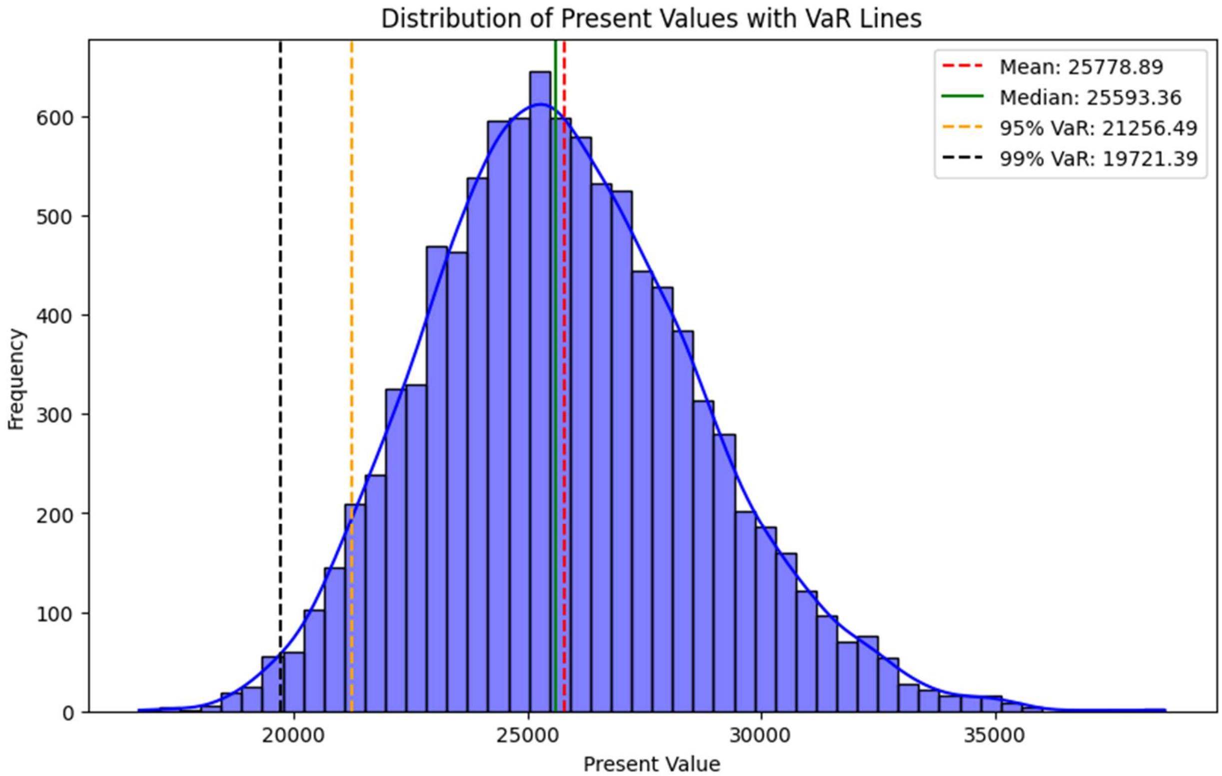
6. **Kurtosis**: A measure of the 'tailedness' of the distribution. Negative kurtosis indicates a distribution with lighter tails and a flatter peak than the normal distribution.

Risk measures, such as Value at Risk (VaR) and Expected Shortfall (ES), offer insights into the potential loss in ILB value:

- **Value at Risk (VaR):** Estimates the maximum loss over a specified time period with a given confidence interval. For example, a 95% VaR of $20,414 suggests a 5% probability that the ILB could experience a loss exceeding $20,414.

- **Expected Shortfall (ES)**: Represents the average loss in value when the loss surpasses the VaR threshold. It provides an estimate of the expected loss on the worst days.

Visual representation of the present value distribution is conveyed through a histogram, with overlaid VaR lines. This visualization offers insights into the distribution and risk measures. The mean and median provide a central tendency perspective, while the VaR lines serve as a visual indicator of potential downside risk.



2. An asset manager holds the following portfolio of fixed-rate Treasury bonds (deliveringannual coupons, with a face value).

Bond	Maturity	Coupon rate (%)	Quantity
1	01/12/2025	4	10000
2	04/12/2026	7.75	250000
3	06/12/2027	4	50000
4	10/12/2028	7	100000
5	03/12/2029	5.75	10000
6	09/12/2030	5.5	200000
7	06/12/2032	4	15000
8	03/12/2035	4.75	10000
9	03/12/2030	4.5	30000
10	04/12/2045	5	75000
11	04/12/2050	4.5	100000
12	01/12/2051	4	10000
13	07/12/2052	5	10000

He wants to hedge it against yield curve shifts. Assume the spot market yield curve on the valuation date 09/02/2022 is well described by the Nelson–Siegel-Svensson (NSS) parameters:

β_0	β_1	β_2	β_3	τ_1	τ_2
5.9%	-1.6%	-0.5%	1%	5	0.5

He selected the following annual coupon paying Treasury bonds (with a €100 face value) as hedging instruments:

Hedging asset	Coupon rate (%)	Maturity
H1	4.5	12/04/2026
H2	5	28/12/2032
H3	6	06/05/2035
H4	6	10/10/2040
H5	6.5	10/10/2051

Tasks:

a) Compute the level, slope and curvature durations and $durations of target portfolio.

Fixed Income Portfolio Duration Measures

Effectively managing a fixed income portfolio necessitates a profound comprehension of its sensitivity to interest rate fluctuations, encapsulated in various duration measures: Dollar Duration, Level Duration, Slope Duration, and Curvature Duration.

1. **Dollar Duration**: Dollar Duration serves as a crucial metric, quantifying the portfolio's value change for a 1 basis point shift in interest rates. It expresses interest rate risk in monetary terms and is calculated as the product of Modified Duration and Present Value.

2. **Level Duration (Macaulay Duration):** Resembling traditional Macaulay Duration, Level Duration gauges the portfolio's susceptibility to parallel shifts in the yield curve. It represents the weighted average time until cash flows are received and is notably influenced by the $\beta 0$ parameter of the Nelson-Siegel-Svensson (NSS) model.

3. **Slope Duration**: Slope Duration assesses the portfolio's responsiveness to changes in the slope of the yield curve. It is instrumental in understanding the disparate movements between short-term and long-term interest rates and is impacted by the $\beta 1$ parameter of the NSS model.

4. **Curvature Duration**: Focusing on the portfolio's sensitivity to alterations in the curvature of the yield curve, Curvature Duration is essential for comprehending the bond's price sensitivity to nonlinear shifts in the yield curve. This duration is notably influenced by the $\beta 2$ and $\beta 3$ parameters of the NSS model.

Nelson-Siegel-Svensson Model: The NSS model, integral to these computations, is defined by a formula capturing the yield curve's dynamics.

Calculating Portfolio Durations involves weighting each bond's duration by its market value within the portfolio. This methodology incorporates individual bond sensitivities to the yield curve's level, slope, and curvature, as per the NSS model.

Target Portfolio Total Durations:

	Standard Durations	Dollar-Durations Sensitivities
level-beta0	7.585480	-748.972934
slope-beta1	3.399951	-346.114050
curvature-beta2	1.918902	-191.156437
curvature-beta3	0.485627	-49.951813

Per each bond in the portfolio:

Bond	Maturity	Coupon rate (%)	Quantity	Bond Price	level-beta0 Duration	level-beta0 Sensitivity	slope-beta1 Duration	slope-beta1 Sensitivity	curvature-beta2 Duration	curvature-beta2 Sensitivity	curvature-beta3 Duration	curvature-beta3 Sensitivity
1	01/12/2025	4.00	10000	96.741840	3.770583	-364.773162	2.621103	-253.570334	0.884976	-85.614198	0.488620	-47.269958
2	04/12/2026	7.75	250000	112.078569	4.370446	-489.833335	2.839292	-318.223827	1.112573	-124.695553	0.482933	-54.126416
3	06/12/2027	4.00	50000	94.818814	5.432823	-515.133810	3.236954	-306.924136	1.492629	-141.529338	0.489662	-46.429136
4	10/12/2028	7.00	100000	111.184284	5.833387	-648.580915	3.295956	-366.458550	1.637634	-182.079150	0.484613	-53.881402
5	03/12/2029	5.75	10000	104.097349	6.656565	-692.930724	3.505734	-364.937585	1.914584	-199.303089	0.486503	-50.643718
6	09/12/2030	5.50	200000	102.416585	7.346916	-752.446042	3.638698	-372.663029	2.130130	-218.160676	0.486879	-49.864472
7	06/12/2032	4.00	15000	89.786788	8.969759	-805.365860	3.912919	-351.328394	2.588189	-232.385218	0.489115	-43.916077
8	03/12/2035	4.75	10000	94.346398	10.378161	-979.142143	3.983792	-375.856441	2.850590	-268.942872	0.487699	-46.012635
9	03/12/2030	4.50	30000	95.321640	7.546806	-719.373944	3.703367	-353.010994	2.197324	-209.452487	0.488465	-46.561329
10	04/12/2045	5.00	75000	93.343343	13.964373	-1303.481225	4.046650	-377.727857	3.157800	-294.759653	0.486912	-45.450025
11	04/12/2050	4.50	100000	84.936321	15.420546	-1309.764482	4.063741	-345.159184	3.212397	-272.849145	0.487055	-41.368679
12	01/12/2051	4.00	10000	77.327677	15.994706	-1236.833436	4.086219	-315.977792	3.256331	-251.804532	0.487361	-37.686521
13	07/12/2052	5.00	10000	91.767240	15.450991	-1417.894754	4.038619	-370.612931	3.170777	-290.973409	0.486688	-44.661973

In summary, a comprehensive understanding of these duration measures is indispensable for effective fixed income portfolio management. They collectively offer a nuanced perspective on the portfolio's interest rate risk, forming the foundation for robust hedging strategies.

b) Compute the level, slope and curvature durations and $durations of the hedging assets.

Computing Durations and Sensitivities of Hedging Assets

For fixed income portfolio management, it's critical to understand not only the portfolio's sensitivity to interest rate movements but also the characteristics of the hedging assets. This is achieved by computing level, slope, and curvature durations, as well as their sensitivities for each hedging asset.

1. **Level, Slope, and Curvature Durations for Hedging Assets:** These durations are calculated for each hedging instrument in the portfolio. They represent the sensitivity of the hedging assets to various changes in the yield curve: parallel shifts (level), steepening or flattening (slope), and changes in curvature (curvature).

Level Duration (Beta0): Reflects sensitivity to parallel shifts in the yield curve.

Slope Duration (Beta1): Measures sensitivity to changes in the slope of the yield curve.

Curvature Duration (Beta2 and Beta3): Indicates sensitivity to changes in the curvature of the yield curve.

2. **Sensitivities for Hedging Assets**: Similar to the portfolio's sensitivities, each hedging instrument's sensitivities to the level, slope, and curvature of the yield curve are calculated. These sensitivities help in understanding the impact of interest rate changes on the hedging instruments.

3. **Practical Application**: By analyzing these durations and sensitivities, one can effectively strategize the hedging process against interest rate movements. This involves aligning the portfolio's interest rate risk profile with that of the hedging instruments.

4. **Computational Approach**: Using the Nelson-Siegel-Svensson (NSS) model, the sensitivity of each hedging instrument to changes in interest rates is computed. This involves detailed calculations of the bond's price, its durations, and sensitivities based on the NSS model parameters.

Hedging Asset	Maturity	Coupon rate (%)	Bond Price	level-beta0 Duration	level-beta0 Sensitivity	slope-beta1 Duration	slope-beta1 Sensitivity	curvature-beta2 Duration	curvature-beta2 Sensitivity	curvature-beta3 Duration	curvature-beta3 Sensitivity
H1	12/04/2026	4.5	98.519792	3.746564	-369.110696	2.607257	-256.866378	0.877536	-86.454640	0.487586	-48.036885
H2	28/12/2032	5.0	98.049020	8.676050	-850.678215	3.835802	-376.096588	2.495753	-244.706135	0.487540	-47.802864
H3	06/05/2035	6.0	106.512956	9.471809	-1008.870331	3.867106	-411.896844	2.646052	-281.838808	0.486237	-51.790502
H4	10/10/2040	6.0	106.689206	11.962751	-1276.296357	3.979428	-424.561965	2.986689	-318.647488	0.486259	-51.878627
H5	10/10/2051	6.5	113.899468	14.548919	-1657.114117	3.993167	-454.819602	3.085286	-351.412409	0.486057	-55.361594

The computed durations and sensitivities for the hedging assets provide valuable insights for managing interest rate risk in a fixed income portfolio. It enables the development of robust hedging strategies to mitigate potential risks arising from interest rate fluctuations.

c) Estimate the holdings of the hedging portfolio assuming the hedger wants to implement a self-financing (full) hedging strategy.

Estimating Holdings for a Self-Financing Hedging Strategy

In fixed income portfolio management, implementing a self-financing (full) hedging strategy involves precisely determining the holdings of hedging instruments. The objective is to align the hedging portfolio's interest rate risk profile with that of the target portfolio, thereby mitigating potential interest rate risks.

1. **Self-Financing Hedging Strategy** A self-financing hedging strategy aims to fully offset the interest rate risk in the target portfolio without additional funding. This involves calculating the necessary quantities of each hedging instrument to match the duration and sensitivity profiles of the target portfolio.

2. **Computational Approach:** To estimate the holdings of the hedging portfolio, a system of linear equations is set up and solved. This system represents the relationship between the durations and sensitivities of the hedging instruments and the target portfolio.

Matrix A: Represents the sensitivities of the hedging instruments to the level, slope, and curvature of the yield curve, along with their prices.

Vector B: Represents the target portfolio's sensitivities to these factors and its market value.

Solution (w): The solution vector 'w' provides the required holdings in each hedging instrument to achieve the desired hedging effect.

3. **Solving the System of Equations** The system of equations is solved using matrix algebra, specifically the numpy.linalg.solve method in Python. This yields the exact quantities of each hedging instrument needed for the self-financing hedging strategy.

4. **Verifying Solution Correctness** The correctness of the solution can be verified by ensuring that the calculated holdings align the duration and sensitivity profiles of the hedging portfolio with those of the target portfolio. The solution should also respect the self-financing constraint, implying that the sum of the products of the quantities and prices of the hedging instruments equals the market value of the target portfolio.

Successfully implementing a self-financing hedging strategy requires precise calculations and a thorough understanding of the interest rate risk profiles of both the target and hedging portfolios. The computed holdings enable the portfolio manager to effectively hedge against interest rate risks without additional funding.

d) Assume that immediately after the hedging strategy was established, the yield curve changed and is now given by the following set of NSS parameters:

β_0	β_1	β_2	β_3	τ_1	τ_2
6.5%	-1.0%	0.1%	2%	5	0.5

i. Estimate the impact of this shift in the yield curve on the Target Portfolio assuming no hedging strategy had been implemented. Discuss the results.

In this section, we assess the impact of a yield curve shift on the target bond portfolio in the absence of a hedging strategy. The new yield curve is characterized by a set of Nelson-Siegel-Svensson (NSS) parameters, representing an altered interest rate environment. The key steps involved in this evaluation are outlined below:

1. **Update Bond Prices**: Employ the updated NSS parameters to recompute the bond prices within the target portfolio. This process entails revising the spot rate calculation for each bond using the NSS model, considering their unique maturity and coupon rate.

2. **Calculate Value Changes**: Determine the change in value for each bond by calculating the difference between the new and original bond prices and multiplying it by the quantity of each bond held in the portfolio.

3. **Aggregate Impact**: Sum up the individual value changes across all bonds in the portfolio to estimate the overall impact.

The calculated total impact provides insights into the portfolio's sensitivity to the specified yield curve shift. A negative value indicates a loss resulting from rising interest rates, while a positive value signifies a gain in a falling rate environment.

Percentage Change in Total Portfolio Value (No Hedging): 3801.79%

This analysis is crucial for comprehending the inherent interest rate risk present in the portfolio, emphasizing the significance of a hedging strategy in mitigating such risks.

ii. Estimate the impact of this change in the yield curve on the global portfolio (target bond portfolio plus hedging instruments) and discuss the performance of the hedging strategy.

This section assesses the overall performance of the hedging strategy by examining its effectiveness against the same yield curve shift as in part d) i). The global portfolio includes both the target bond portfolio and the hedging instruments. The approach is similar to that in d) i), but applied to the combined portfolio:

1. **Update Bond Prices for Hedging Instruments**: Recalculate bond prices for the hedging instruments using the new NSS parameters. This mirrors the process in the target portfolio, but applies to the hedging instruments.

2. **Calculate Value Changes for Both Portfolios**: Determine the change in value for each instrument in both the target and hedging portfolios. This involves the difference in bond prices (new price minus original price), multiplied by the respective quantities. For hedging instruments, the quantities are as determined in the hedging strategy.

3. **Combined Portfolio Impact**: Sum the value changes from both the target and hedging portfolios to find the total impact on the global portfolio.

The overall impact on the global portfolio provides insight into the efficacy of the hedging strategy. An effective hedge would minimize the portfolio's sensitivity to the yield curve shift, reducing the total impact compared to the target portfolio alone. This result helps gauge the resilience of the hedging strategy under the new interest rate conditions, highlighting its role in risk management.

Total Impact on Global Portfolio (With Hedging): $183,906,942,331.65