Numerical Analysis Homework 1

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1 Q1: Section 2.1 exercise 6

```
/usr/bin/python3.5 /home/hmenn/Workspace/MAT214_NumericalAnalysis_2017/HW1/main.py 1 2 RELATIVE_ERROR 1E-5
Epsilon ϵ:  1e-05 || Stopping Criteria:  RELATIVE_ERROR
Step:   1 | Absolute Error: 1.5000000  | Relative Error: 1.0000000  | pn: 1.5000000  | f(pn): 0.0183109
Step:   2 | Absolute Error: 0.2500000  | Relative Error: 0.1428571  | pn: 1.7500000  | f(pn): -0.5046027
Step:   3 | Absolute Error: 0.1250000  | Relative Error: 0.0769231  | pn: 1.6250000  | f(pn): -0.2034190
Step:   4 | Absolute Error: 0.0625000  | Relative Error: 0.0400000  | pn: 1.5625000  | f(pn): -0.0832332
Step:   5 | Absolute Error: 0.0312500  | Relative Error: 0.0204082  | pn: 1.5312500  | f(pn): -0.0302032
Step:   6 | Absolute Error: 0.0156250  | Relative Error: 0.0103093  | pn: 1.5156250  | f(pn): -0.0053904
Step:   7 | Absolute Error: 0.0078125  | Relative Error: 0.0051813  | pn: 1.5078125  | f(pn): 0.0065981
Step:   8 | Absolute Error: 0.0039062  | Relative Error: 0.0025840  | pn: 1.5117188  | f(pn): 0.0006384
Step:   9 | Absolute Error: 0.0019531  | Relative Error: 0.0012903  | pn: 1.5136719  | f(pn): -0.0023673
Step:  10 | Absolute Error: 0.0009766  | Relative Error: 0.0006456  | pn: 1.5126953  | f(pn): -0.0008623
Step:  11 | Absolute Error: 0.0004883  | Relative Error: 0.0003229  | pn: 1.5122070  | f(pn): -0.0001114
Step:  12 | Absolute Error: 0.0002441  | Relative Error: 0.0001615  | pn: 1.5119629  | f(pn): 0.0002637
Step:  13 | Absolute Error: 0.0001221  | Relative Error: 0.0000807  | pn: 1.5120850  | f(pn): 0.0000762
Step:  14 | Absolute Error: 0.0000610  | Relative Error: 0.0000404  | pn: 1.5121460  | f(pn): -0.0000176
Step:  15 | Absolute Error: 0.0000305  | Relative Error: 0.0000202  | pn: 1.5121155  | f(pn): 0.0000293
Step:  16 | Absolute Error: 0.0000153  | Relative Error: 0.0000101  | pn: 1.5121307  | f(pn): 0.0000059
Step:  17 | Absolute Error: 0.0000076  | Relative Error: 0.0000050  | pn: 1.5121384  | f(pn): -0.0000059

Step: 17 Approximate root: 1.51213837
Theoretically required number of iterations: 16.60964047
```

Figure 1: a

There is no valid root between 0 and 1. If we draw pilot, we will see root between 1-2.

```
/usr/bin/python3.5 /home/hmenn/Workspace/MAT214_NumericalAnalysis_2017/HW1/main.py 0 1 RELATIVE_ERROR 1E-5
Epsilon ϵ:  1e-05 || Stopping Criteria:  RELATIVE_ERROR
Step:   1 f(a) and f(b) must have different signs
There is no Approximate root

Process finished with exit code 0
```

Figure 2: b

```
/usr/bin/python3.5 /home/hmenn/Workspace/MAT214_NumericalAnalysis_2017/HW1/main.py 1 2 RELATIVE_ERROR 1E-5
Epsilon ϵ:  1e-05 || A: 1.0  B: 2.0  || Stopping Criteria:  RELATIVE_ERROR
Step:   1 | Absolute Error: 1.5000000  | Relative Error: 1.0000000  | pn: 1.5000000  | f(pn): -0.1554651
Step:   2 | Absolute Error: 0.2500000  | Relative Error: 0.2000000  | pn: 1.2500000  | f(pn): 0.3393564
Step:   3 | Absolute Error: 0.1250000  | Relative Error: 0.0909091  | pn: 1.3750000  | f(pn): 0.0721713
Step:   4 | Absolute Error: 0.0625000  | Relative Error: 0.0434783  | pn: 1.4375000  | f(pn): -0.0464992
Step:   5 | Absolute Error: 0.0312500  | Relative Error: 0.0222222  | pn: 1.4062500  | f(pn): 0.0116125
Step:   6 | Absolute Error: 0.0156250  | Relative Error: 0.0109890  | pn: 1.4218750  | f(pn): -0.0177479
Step:   7 | Absolute Error: 0.0078125  | Relative Error: 0.0055249  | pn: 1.4140625  | f(pn): -0.0031440
Step:   8 | Absolute Error: 0.0039062  | Relative Error: 0.0027701  | pn: 1.4101562  | f(pn): 0.0042151
Step:   9 | Absolute Error: 0.0019531  | Relative Error: 0.0013831  | pn: 1.4121094  | f(pn): 0.0005308
Step:  10 | Absolute Error: 0.0009766  | Relative Error: 0.0006911  | pn: 1.4130859  | f(pn): -0.0013078
Step:  11 | Absolute Error: 0.0004883  | Relative Error: 0.0003457  | pn: 1.4125977  | f(pn): -0.0003888
Step:  12 | Absolute Error: 0.0002441  | Relative Error: 0.0001729  | pn: 1.4123535  | f(pn): 0.0000709
Step:  13 | Absolute Error: 0.0001221  | Relative Error: 0.0000864  | pn: 1.4124756  | f(pn): -0.0001590
Step:  14 | Absolute Error: 0.0000610  | Relative Error: 0.0000432  | pn: 1.4124146  | f(pn): -0.0000440
Step:  15 | Absolute Error: 0.0000305  | Relative Error: 0.0000216  | pn: 1.4123840  | f(pn): 0.0000134
Step:  16 | Absolute Error: 0.0000153  | Relative Error: 0.0000108  | pn: 1.4123993  | f(pn): -0.0000153
Step:  17 | Absolute Error: 0.0000076  | Relative Error: 0.0000054  | pn: 1.4123917  | f(pn): -0.0000009

Step: 17 Approximate root: 1.41239166
Theoretically required number of iterations: 16.60964047

Process finished with exit code 0
```

Figure 3: c1

```
/usr/bin/python3.5 /home/hmenn/Workspace/MAT214_NumericalAnalysis_2017/HW1/main.py 2 4 RELATIVE_ERROR 1E-5
Epsilon ϵ:  1e-05 || A: 2.0  B: 4.0  || Stopping Criteria:  RELATIVE_ERROR
Step:   1 | Absolute Error: 3.0000000  | Relative Error: 1.0000000  | pn: 3.0000000  | f(pn): -0.0986123
Step:   2 | Absolute Error: 0.5000000  | Relative Error: 0.1428571  | pn: 3.5000000  | f(pn): 0.9972370
Step:   3 | Absolute Error: 0.2500000  | Relative Error: 0.0769231  | pn: 3.2500000  | f(pn): 0.3838450
Step:   4 | Absolute Error: 0.1250000  | Relative Error: 0.0400000  | pn: 3.1250000  | f(pn): 0.1261907
Step:   5 | Absolute Error: 0.0625000  | Relative Error: 0.0204082  | pn: 3.0625000  | f(pn): 0.0096747
Step:   6 | Absolute Error: 0.0312500  | Relative Error: 0.0103093  | pn: 3.0312500  | f(pn): -0.0454985
Step:   7 | Absolute Error: 0.0156250  | Relative Error: 0.0051282  | pn: 3.0468750  | f(pn): -0.0181692
Step:   8 | Absolute Error: 0.0078125  | Relative Error: 0.0025575  | pn: 3.0546875  | f(pn): -0.0043116
Step:   9 | Absolute Error: 0.0039062  | Relative Error: 0.0012771  | pn: 3.0585938  | f(pn): 0.0026655
Step:  10 | Absolute Error: 0.0019531  | Relative Error: 0.0006390  | pn: 3.0566406  | f(pn): -0.0008271
Step:  11 | Absolute Error: 0.0009766  | Relative Error: 0.0003194  | pn: 3.0576172  | f(pn): 0.0009182
Step:  12 | Absolute Error: 0.0004883  | Relative Error: 0.0001597  | pn: 3.0571289  | f(pn): 0.0000453
Step:  13 | Absolute Error: 0.0002441  | Relative Error: 0.0000799  | pn: 3.0568848  | f(pn): -0.0003909
Step:  14 | Absolute Error: 0.0001221  | Relative Error: 0.0000399  | pn: 3.0570068  | f(pn): -0.0001728
Step:  15 | Absolute Error: 0.0000610  | Relative Error: 0.0000200  | pn: 3.0570679  | f(pn): -0.0000638
Step:  16 | Absolute Error: 0.0000305  | Relative Error: 0.0000100  | pn: 3.0570984  | f(pn): -0.0000092

Step: 16 Approximate root: 3.05709839
Theoretically required number of iterations: 17.60964047

Process finished with exit code 0
```

Figure 4: c2

```
/usr/bin/python3.5 /home/hmenn/Workspace/MAT214_NumericalAnalysis_2017/HW1/main.py 0 0.5 RELATIVE_ERROR 1E-5
Epsilon ϵ:  1e-05 || A: 0.0  B: 0.5  || Stopping Criteria:  RELATIVE_ERROR
Step:   1 | Absolute Error: 0.2500000  | Relative Error: 1.0000000  | pn: 0.2500000  | f(pn): -0.1642136
Step:   2 | Absolute Error: 0.1250000  | Relative Error: 1.0000000  | pn: 0.1250000  | f(pn): 0.3596331
Step:   3 | Absolute Error: 0.0625000  | Relative Error: 0.3333333  | pn: 0.1875000  | f(pn): 0.0763595
Step:   4 | Absolute Error: 0.0312500  | Relative Error: 0.1428571  | pn: 0.2187500  | f(pn): -0.0500366
Step:   5 | Absolute Error: 0.0156250  | Relative Error: 0.0769231  | pn: 0.2031250  | f(pn): 0.0117264
Step:   6 | Absolute Error: 0.0078125  | Relative Error: 0.0370370  | pn: 0.2109375  | f(pn): -0.0195257
Step:   7 | Absolute Error: 0.0039062  | Relative Error: 0.0188679  | pn: 0.2070312  | f(pn): -0.0039908
Step:   8 | Absolute Error: 0.0019531  | Relative Error: 0.0095238  | pn: 0.2050781  | f(pn): 0.0038452
Step:   9 | Absolute Error: 0.0009766  | Relative Error: 0.0047393  | pn: 0.2060547  | f(pn): -0.0000785
Step:  10 | Absolute Error: 0.0004883  | Relative Error: 0.0023753  | pn: 0.2055664  | f(pn): 0.0018819
Step:  11 | Absolute Error: 0.0002441  | Relative Error: 0.0011862  | pn: 0.2058105  | f(pn): 0.0009013
Step:  12 | Absolute Error: 0.0001221  | Relative Error: 0.0005928  | pn: 0.2059326  | f(pn): 0.0004113
Step:  13 | Absolute Error: 0.0000610  | Relative Error: 0.0002963  | pn: 0.2059937  | f(pn): 0.0001664
Step:  14 | Absolute Error: 0.0000305  | Relative Error: 0.0001481  | pn: 0.2060242  | f(pn): 0.0000439
Step:  15 | Absolute Error: 0.0000153  | Relative Error: 0.0000741  | pn: 0.2060394  | f(pn): -0.0000173
Step:  16 | Absolute Error: 0.0000076  | Relative Error: 0.0000370  | pn: 0.2060318  | f(pn): 0.0000133
Step:  17 | Absolute Error: 0.0000038  | Relative Error: 0.0000185  | pn: 0.2060356  | f(pn): -0.0000020
Step:  18 | Absolute Error: 0.0000019  | Relative Error: 0.0000093  | pn: 0.2060337  | f(pn): 0.0000057

Step: 18 Approximate root: 0.20603371
Theoretically required number of iterations: 15.60964047

Process finished with exit code 0
```

Figure 5: d1

```
/usr/bin/python3.5 /home/hmenn/Workspace/MAT214_NumericalAnalysis_2017/HW1/main.py 0.5 1 RELATIVE_ERROR 1E-5
Epsilon ϵ:  1e-05 || A: 0.5  B: 1.0  || Stopping Criteria:  RELATIVE_ERROR
Step:   1 | Absolute Error: 0.7500000  | Relative Error: 1.0000000  | pn: 0.7500000  | f(pn): 0.3357864
Step:   2 | Absolute Error: 0.1250000  | Relative Error: 0.2000000  | pn: 0.6250000  | f(pn): -0.2227591
Step:   3 | Absolute Error: 0.0625000  | Relative Error: 0.0909091  | pn: 0.6875000  | f(pn): 0.0245608
Step:   4 | Absolute Error: 0.0312500  | Relative Error: 0.0476190  | pn: 0.6562500  | f(pn): -0.1075925
Step:   5 | Absolute Error: 0.0156250  | Relative Error: 0.0232558  | pn: 0.6718750  | f(pn): -0.0435822
Step:   6 | Absolute Error: 0.0078125  | Relative Error: 0.0114943  | pn: 0.6796875  | f(pn): -0.0100196
Step:   7 | Absolute Error: 0.0039062  | Relative Error: 0.0057143  | pn: 0.6835938  | f(pn): 0.0071443
Step:   8 | Absolute Error: 0.0019531  | Relative Error: 0.0028653  | pn: 0.6816406  | f(pn): -0.0014693
Step:   9 | Absolute Error: 0.0009766  | Relative Error: 0.0014306  | pn: 0.6826172  | f(pn): 0.0028296
Step:  10 | Absolute Error: 0.0004883  | Relative Error: 0.0007158  | pn: 0.6821289  | f(pn): 0.0006782
Step:  11 | Absolute Error: 0.0002441  | Relative Error: 0.0003580  | pn: 0.6818848  | f(pn): -0.0003961
Step:  12 | Absolute Error: 0.0001221  | Relative Error: 0.0001790  | pn: 0.6820068  | f(pn): 0.0001409
Step:  13 | Absolute Error: 0.0000610  | Relative Error: 0.0000895  | pn: 0.6819458  | f(pn): -0.0001276
Step:  14 | Absolute Error: 0.0000305  | Relative Error: 0.0000447  | pn: 0.6819763  | f(pn): 0.0000066
Step:  15 | Absolute Error: 0.0000153  | Relative Error: 0.0000224  | pn: 0.6819611  | f(pn): -0.0000605
Step:  16 | Absolute Error: 0.0000076  | Relative Error: 0.0000112  | pn: 0.6819687  | f(pn): -0.0000269
Step:  17 | Absolute Error: 0.0000038  | Relative Error: 0.0000056  | pn: 0.6819725  | f(pn): -0.0000101

Step: 17 Approximate root: 0.68197250
Theoretically required number of iterations: 15.60964047

Process finished with exit code 0
```

Figure 6: d2

2 Q2: Section 2.2 exercise 5

If g is defined on [a, b] and g(p) = p for some p ∈ [a, b], then the function g is said to have the fixed point p in [a, b].

Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^4 - 3 * x^2 - 3 = 0$ on $[1, 2]$. Use $p_0 = 1$.

2.1 Answer

Firstly, manipulate function to obtain $x = g(x)$

$$g(x) = x^4 - 3 * x^2 - 3 = 0 \tag{1}$$
$$x^4 = 3 * x^2 + 3 \tag{2}$$
$$x = (3 * x^2 + 3)^{\frac{1}{4}} = g(x) \tag{3}$$

Let's substitute $p_0 = 1$ in function to calculate $p_1 =?$

$$p_1 = g(p_0) \tag{4}$$
$$p_1 = (3 * 1^2 + 3)^{\frac{1}{4}} \tag{5}$$
$$p_1 = 1.5650846 \tag{6}$$

Next step, find absolute difference between two points(p_0 and p_1) to make a comparison with $\varepsilon = 10^{-2}$

if $|p_1 - p_0| < \epsilon$ return p_1
else set $p_0 = p_1$ and go step 4

$$|p_1 - p_0| = 1.5650846 - 1.0 = 0.5650846 \tag{7}$$
$$|p_1 - p_0| > \epsilon = 10^{-2} \tag{8}$$

Set p_0 to p_1 and go step. Calculate all steps until find a good approximate value.

step(i)	$p_(i-1)$	p_i	$\lvert p_0 - p_i \rvert$
1	1.0000000	1.5650846	0.5650846
2	1.5650846	1.7935729	0.2284883
3	1.7935729	1.8859437	0.0923709
4	1.8859437	1.9228478	0.0369041
5	1.9228478	1.9375075	0.0146597
6	1.9375075	1.9433169	0.0058094

Result: $p_6 = 1.9433169$

Theoretically number of iteration:

$$|p - p_n| \leq \frac{k^n}{1-k} * |p_1 - p_0| \tag{9}$$
$$g'(x) \approx k = ((3 * x^2 + 3)^{\frac{1}{4}})' \tag{10}$$
$$g'(x) \approx k = (1.5 * x * (3 * x^2 + 3))^{\frac{-3}{4}} \tag{11}$$
$$g'(1) \approx k = 0.391 \tag{12}$$
$$|p - p_n| \leq \frac{(0.391)^n}{1 - 0.391} * |1.5650846 - 1| \tag{13}$$
$$\leq \frac{(0.391)^n}{0.609} * |0.5650846| \tag{14}$$
$$(0.391)^n * 0,92788 < 10^{-2} \tag{15}$$
$$n \approx 6 \tag{16}$$

3 Q3: Section 2.3 exercise 4

Let $f(x) = -x^3 - cos(x)$. With $p_0 = -1$ and $p_1 = 0$, find p_3.
a. Use the Secant method. b. Use the method of False Position.

3.1 a. Secant Method

$f(0) = -1, f(-1) = 0.4596976$

$$p_n = p_{n-1} - \frac{f(p_{n-1})(p_{n-1} - p_{n-2})}{f(p_{n-1}) - f(p_{n-2})} \tag{17}$$
$$p_3 = p_2 - \frac{f(p_2)(p_2 - p_1)}{f(p_2) - f(p_1)} \tag{18}$$
$$p_2 = p_1 - \frac{f(p_1)(p_1 - p_0)}{f(p_1) - f(p_0)} \tag{19}$$
$$p_2 = 0 - \frac{f(0)(0 - (-1))}{f(0) - f(-1)} \tag{20}$$
$$p_2 = \frac{1}{-1 - (0.4596976)} \tag{21}$$
$$p_2 = -0.6850733 \tag{22}$$
$$\tag{23}$$

Substitute p_2 to step(18)

$$p_3 = (-0.6850733) - \frac{f(-0.6850733)(-0.6850733 - 0)}{f(-0.6850733) - f(0)} \quad (24)$$

$$p_3 = -1.2520764 \quad (25)$$

3.2 b. The Method of False Position

$p_0 = -1, p_1 = 0, f(0) = -1, f(-1) = 0.4596976$

$$p_n = p_{n-1} - q_{n-1}\frac{p_{n-1} - p_{n-2}}{f(p_{n-1}) - f(p_{n-2}))} \quad (26)$$

$$p_3 = p_2 - q_2\frac{p_2 - p_1}{f(p_2) - f(p_1))} \quad (27)$$

$$p_2 = p_1 - q_1\frac{p_1 - p_0}{f(p_1) - f(p_0))} \quad (28)$$

$$q_1 = f(p_1) \quad (29)$$

$$p_2 = 0 - f(p_1)\frac{0 - (-1)}{f(0) - f(-1)} \quad (30)$$

$$p_2 = -0.68507335 \quad (31)$$

$$(32)$$

Substitute p_2 to step(27)

$$p_3 = p_2 - f(p_2)\frac{p_2 - p_1}{f(p_2) - f(p_1))} \quad (33)$$

$$p_3 = -0.841355125 \quad (34)$$

4 Q3: Section 2.3 exercise 5

Use Newton's method to find solutions accurate to within 10^{-4} for the following problems.
Newton's Method

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})} \quad (35)$$

4.1 a. $f(x) = x^3 - 2 * x^2 - 5 = 0, [1, 4]$

$$f'(x) = 3 * x^2 - 4 * x \quad (36)$$

Let's take $p_0 = 2$ and $\epsilon = 10^{-2}$

$$p_1 = p_0 - \frac{f(p_0)}{f'(p_o)} \quad (37)$$

$$p_1 = 2 - \frac{f(2)}{f'(2)} \quad (38)$$

$$p_1 = 3.2500000 \quad (39)$$

$$|p_1 - p_0| = 1.250000 > \epsilon \quad (40)$$

p_1 has not enough approximate value. Calculate p_2

$$p_2 = p_1 - \frac{f(p_1)}{f'(p_1)} \quad (41)$$

$$p_2 = 3.25 - \frac{f(3.25)}{f'(3.25)} \quad (42)$$

$$p_2 = 2.8110368 \quad (43)$$

$$|p_2 - p_1| = 0.4389632 > \epsilon \quad (44)$$

p_2 has not enough approximate value. Calculate p_3

$$p_3 = p_2 - \frac{f(p_2)}{f'(p_2)} \quad (45)$$

$$p_3 = 2.8110368 - \frac{f(2.8110368)}{f'(2.8110368)} \quad (46)$$

$$p_3 = 2.6979895 \quad (47)$$

$$|p_3 - p_2| = 0.1130473 > \epsilon \quad (48)$$

p_3 has not enough approximate value. Calculate p_4

$$p_4 = p_3 - \frac{f(p_3)}{f'(p_3)} \quad (49)$$

$$p_4 = 2.6979895 - \frac{f(2.6979895)}{f'(2.6979895)} \quad (50)$$

$$p_4 = 2.6906772 \quad (51)$$

$$|p_4 - p_3| = 0.0073123 > \epsilon \quad (52)$$

p_4 has not enough approximate value. Calculate p_5

$$p_5 = p_4 - \frac{f(p_4)}{f'(p_4)} \tag{53}$$
$$p_5 = 2.6906772 - \frac{f(2.6906772)}{f'(2.6906772)} \tag{54}$$
$$p_5 = 2.6906474 \tag{55}$$
$$|p_5 - p_4| = 0.0000297 < \epsilon \tag{56}$$

Answer is $p_5 : 2.6906474$ and $1 \leq p_5 \leq 4$. Founded in 5.step

4.2 b. $f(x) = x^3 + 2 * x^2 - 1 = 0, [-3, -2]$

$$f'(x) = 3 * x^2 + 4 * x \tag{57}$$

Let's take $p_0 = -3$ and $\epsilon = 10^{-2}$

$$p_1 = p_0 - \frac{f(p_0)}{f'(p_o)} \tag{58}$$
$$p_1 = -3 - \frac{f(-3)}{f'(-3)} \tag{59}$$
$$p_1 = -2.8888889 \tag{60}$$
$$|p_1 - p_0| = 0.1111111 > \epsilon \tag{61}$$

p_1 has not enough approximate value. Calculate p_2

$$p_2 = p_1 - \frac{f(p_1)}{f'(p_1)} \tag{62}$$
$$p_2 = -2.8888889 - \frac{f(-2.8888889)}{f'(-2.8888889)} \tag{63}$$
$$p_2 = -2.8794516 \tag{64}$$
$$|p_2 - p_1| = 0.0094373 > \epsilon \tag{65}$$

p_2 has not enough approximate value. Calculate p_3

$$p_3 = p_2 - \frac{f(p_2)}{f'(p_2)} \tag{66}$$
$$p_3 = -2.8794516 - \frac{f(-2.8794516)}{f'(-2.8794516)} \tag{67}$$
$$p_3 = -2.8793852 \tag{68}$$
$$|p_3 - p_2| = 0.0000663 < \epsilon \tag{69}$$

Answer is $p_3 : -2.8793852$ and $-3 \leq p_3 \leq -2$. Founded in 3.step

4.3 c. $f(x) = x - cos(x) = 0, [0, \frac{\pi}{2}]$

$$f'(x) = 1 + sin(x) \tag{70}$$

Let's take $p_0 = 0$ and $\epsilon = 10^{-2}$

$$p_1 = p_0 - \frac{f(p_0)}{f'(p_o)} \tag{71}$$
$$p_1 = 0 - \frac{f(0)}{f'(0)} \tag{72}$$
$$p_1 = 1.0000000 \tag{73}$$
$$|p_1 - p_0| = 1.0000000 > \epsilon \tag{74}$$

p_1 has not enough approximate value. Calculate p_2

$$p_2 = p_1 - \frac{f(p_1)}{f'(p_1)} \tag{75}$$
$$p_2 = 1.0000000 - \frac{f(1.0000000)}{f'(1.0000000)} \tag{76}$$
$$p_2 = 0.7503639 \tag{77}$$
$$|p_2 - p_1| = 0.2496361 > \epsilon \tag{78}$$

p_2 has not enough approximate value. Calculate p_3

$$p_3 = p_2 - \frac{f(p_2)}{f'(p_2)} \tag{79}$$
$$p_3 = 0.7503639 - \frac{f(0.7503639)}{f'(0.7503639)} \tag{80}$$
$$p_3 = 0.7391129 \tag{81}$$
$$|p_3 - p_2| = 0.0112510 > \epsilon \tag{82}$$

p_3 has not enough approximate value. Calculate p_4

$$p_4 = p_3 - \frac{f(p_3)}{f'(p_3)} \tag{83}$$

$$p_4 = 0.739112 - \frac{f(0.739112)}{f'(0.739112)} \tag{84}$$

$$p_4 = 0.7390851 \tag{85}$$

$$|p_4 - p_3| = 0.0000278 < \epsilon \tag{86}$$

Answer is $p_4 : 0.7390851$ and $0 \leq p_4 \leq \frac{\pi}{2}$. Founded in 4.step

4.4 d. $f(x) = x - 0.8 - 0.2 * sin(x) = 0, [0, \frac{\pi}{2}]$

$$f'(x) = 1 - 0.2 * cos(x) \tag{87}$$

Let's take $p_0 = 0$ and $\epsilon = 10^{-2}$

$$p_1 = p_0 - \frac{f(p_0)}{f'(p_o)} \tag{88}$$

$$p_1 = 0 - \frac{f(0)}{f'(0)} \tag{89}$$

$$p_1 = 1.0000000 \tag{90}$$

$$|p_1 - p_0| = 1.0000000 > \epsilon \tag{91}$$

p_1 has not enough approximate value. Calculate p_2

$$p_2 = p_1 - \frac{f(p_1)}{f'(p_1)} \tag{92}$$

$$p_2 = 1.0000000 - \frac{f(1.0000000)}{f'(1.0000000)} \tag{93}$$

$$p_2 = 0.9644530 \tag{94}$$

$$|p_2 - p_1| = 0.0355470 > \epsilon \tag{95}$$

p_2 has not enough approximate value. Calculate p_3

$$p_3 = p_2 - \frac{f(p_2)}{f'(p_2)} \tag{96}$$

$$p_3 = 0.9644530 - \frac{f(0.9644530)}{f'(0.9644530)} \tag{97}$$

$$p_3 = 0.9643339 \tag{98}$$

$$|p_3 - p_2| = 0.0001191 < \epsilon \tag{99}$$

Answer is $p_3 : 0.9643339$ and $0 \leq p_3 \leq \frac{\pi}{2}$. Founded in 3.step