Numerical Analysis HomeWork 1

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1 Question 2 from Section 2.2 Q5

If g is defined on [a, b] and g(p) = p for some p ∈ [a, b], then the function g is said to have the fixed point p in [a, b].

Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^4 - 3 * x^2 - 3 = 0$ on $[1, 2]$. Use $p_0 = 1$.

1.1 Answer

Firstly, manipulate function to obtain $x = g(x)$

$$g(x) = x^4 - 3 * x^2 - 3 = 0 \tag{1}$$
$$x^4 = 3 * x^2 + 3 \tag{2}$$
$$x = (3 * x^2 + 3)^{\frac{1}{4}} = g(x) \tag{3}$$

Let's substitute $p_0 = 1$ in function to calculate $p_1 = ?$

$$p_1 = g(p_0) \tag{4}$$
$$p_1 = (3 * 1^2 + 3)^{\frac{1}{4}} \tag{5}$$
$$p_1 = 1.5650846 \tag{6}$$

Next step, find absolute difference between two points(p_0 and p_1) to make a comparison with $\varepsilon = 10^{-2}$

if $|p_1 - p_0| < \epsilon$ return p_1
else set $p_0 = p_1$ and go step 4

$$|p_1 - p_0| = 1.5650846 - 1.0 = 0.5650846 \tag{7}$$
$$|p_1 - p_0| > \epsilon = 10^{-2} \tag{8}$$

Set p_0 to p_1 and go step. Calculate all steps until find a good approximate value.

step(i)	$p_(i-1)$	p_i	$\|p_0 - p_i\|$
1	1.0000000	1.5650846	0.5650846
2	1.5650846	1.7935729	0.2284883
3	1.7935729	1.8859437	0.0923709
4	1.8859437	1.9228478	0.0369041
5	1.9228478	1.9375075	0.0146597
6	1.9375075	1.9433169	0.0058094

Result: $p_6 = 1.9433169$

Theorically number of iteration:

2 Question 3 from Section 2.3 Q4

Let $f(x) = -x^3 - cos(x)$. With $p_0 = -1$ and $p_1 = 0$, find p_3.
a. Use the Secant method. b. Use the method of False Position.

2.1 a. Secant Method

$f(0) = -1, f(-1) = 0.4596976$

$$p_n = p_{n-1} - \frac{f(p_{n-1})(p_{n-1} - p_{n-2})}{f(p_{n-1}) - f(p_{n-2})} \tag{9}$$

$$p_3 = p_2 - \frac{f(p_2)(p_2 - p_1)}{f(p_2) - f(p_1)} \tag{10}$$

$$p_2 = p_1 - \frac{f(p_1)(p_1 - p_0)}{f(p_1) - f(p_0)} \tag{11}$$

$$p_2 = 0 - \frac{f(0)(0 - (-1))}{f(0) - f(-1)} \tag{12}$$

$$p_2 = \frac{1}{-1 - (0.4596976)} \tag{13}$$

$$p_2 = -0.6850733 \tag{14}$$

$$\tag{15}$$

Substitute p_2 to step(10)

$$p_3 = (-0.6850733) - \frac{f(-0.6850733)(-0.6850733 - 0)}{f(-0.6850733) - f(0)} \tag{16}$$

$$p_3 = -1.2520764 \tag{17}$$

2.2 a. The Method of False Position

3 Question 4 from section 2.3 Q5

Use Newton's method to find solutions accurate to within 10^{-4} for the following problems.

Newton's Method

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})} \tag{18}$$

3.1 a. $f(x) = x^3 - 2*x^2 - 5 = 0, [1, 4]$

$$f'(x) = 3*x^2 - 4*x \tag{19}$$

Let's take $p_0 = 2$ and $\epsilon = 10^{-2}$

$$p_1 = p_0 - \frac{f(p_0)}{f'(p_o)} \tag{20}$$

$$p_1 = 2 - \frac{f(2)}{f'(2)} \tag{21}$$

$$p_1 = 3.2500000 \tag{22}$$

$$|p_1 - p_0| = 1.250000 > \epsilon \tag{23}$$

p_1 has not enough approximate value. Calculate p_2

$$p_2 = p_1 - \frac{f(p_1)}{f'(p_1)} \tag{24}$$

$$p_2 = 3.25 - \frac{f(3.25)}{f'(3.25)} \tag{25}$$

$$p_2 = 2.8110368 \tag{26}$$

$$|p_2 - p_1| = 0.4389632 > \epsilon \tag{27}$$

p_2 has not enough approximate value. Calculate p_3

$$p_3 = p_2 - \frac{f(p_2)}{f'(p_2)} \tag{28}$$

$$p_3 = 2.8110368 - \frac{f(2.8110368)}{f'(2.8110368)} \tag{29}$$

$$p_3 = 2.6979895 \tag{30}$$

$$|p_3 - p_2| = 0.1130473 > \epsilon \tag{31}$$

p_3 has not enough approximate value. Calculate p_4

$$p_4 = p_3 - \frac{f(p_3)}{f'(p_3)} \tag{32}$$

$$p_4 = 2.6979895 - \frac{f(2.6979895)}{f'(2.6979895)} \tag{33}$$

$$p_4 = 2.6906772 \tag{34}$$

$$|p_4 - p_3| = 0.0073123 > \epsilon \tag{35}$$

p_4 has not enough approximate value. Calculate p_5

$$p_5 = p_4 - \frac{f(p_4)}{f'(p_4)} \tag{36}$$

$$p_5 = 2.6906772 - \frac{f(2.6906772)}{f'(2.6906772)} \tag{37}$$

$$p_5 = 2.6906474 \tag{38}$$

$$|p_5 - p_4| = 0.0000297 < \epsilon \tag{39}$$

Answer is $p_5 : 2.6906474$ and $1 \leq p_5 \leq 4$. Founded in 5.step

3.2 b. $f(x) = x^3 + 2 * x^2 - 1 = 0, [-3, -2]$

$$f'(x) = 3 * x^2 + 4 * x \tag{40}$$

Let's take $p_0 = -3$ and $\epsilon = 10^{-2}$

$$p_1 = p_0 - \frac{f(p_0)}{f'(p_o)} \tag{41}$$

$$p_1 = -3 - \frac{f(-3)}{f'(-3)} \tag{42}$$

$$p_1 = -2.8888889 \tag{43}$$

$$|p_1 - p_0| = 0.1111111 > \epsilon \tag{44}$$

p_1 has not enough approximate value. Calculate p_2

$$p_2 = p_1 - \frac{f(p_1)}{f'(p_1)} \tag{45}$$

$$p_2 = -2.8888889 - \frac{f(-2.8888889)}{f'(-2.8888889)} \tag{46}$$

$$p_2 = -2.8794516 \tag{47}$$

$$|p_2 - p_1| = 0.0094373 > \epsilon \tag{48}$$

p_2 has not enough approximate value. Calculate p_3

$$p_3 = p_2 - \frac{f(p_2)}{f'(p_2)} \tag{49}$$

$$p_3 = -2.8794516 - \frac{f(-2.8794516)}{f'(-2.8794516)} \tag{50}$$

$$p_3 = -2.8793852 \tag{51}$$

$$|p_3 - p_2| = 0.0000663 < \epsilon \tag{52}$$

Answer is $p_3 : -2.8793852$ and $-3 \leq p_3 \leq -2$. Founded in 3.step

3.3 c. $f(x) = x - cos(x) = 0, [0, \frac{\pi}{2}]$

$$f'(x) = 1 + sin(x) \tag{53}$$

Let's take $p_0 = 0$ and $\epsilon = 10^{-2}$

$$p_1 = p_0 - \frac{f(p_0)}{f'(p_o)} \tag{54}$$

$$p_1 = 0 - \frac{f(0)}{f'(0)} \tag{55}$$

$$p_1 = 1.0000000 \tag{56}$$

$$|p_1 - p_0| = 1.0000000 > \epsilon \tag{57}$$

p_1 has not enough approximate value. Calculate p_2

$$p_2 = p_1 - \frac{f(p_1)}{f'(p_1)} \tag{58}$$

$$p_2 = 1.0000000 - \frac{f(1.0000000)}{f'(1.0000000)} \tag{59}$$

$$p_2 = 0.7503639 \tag{60}$$

$$|p_2 - p_1| = 0.2496361 > \epsilon \tag{61}$$

p_2 has not enough approximate value. Calculate p_3

$$p_3 = p_2 - \frac{f(p_2)}{f'(p_2)} \tag{62}$$

$$p_3 = 0.7503639 - \frac{f(0.7503639)}{f'(0.7503639)} \tag{63}$$

$$p_3 = 0.7391129 \tag{64}$$

$$|p_3 - p_2| = 0.0112510 > \epsilon \tag{65}$$

p_3 has not enough approximate value. Calculate p_4

$$p_4 = p_3 - \frac{f(p_3)}{f'(p_3)} \tag{66}$$

$$p_4 = 0.739112 - \frac{f(0.739112)}{f'(0.739112)} \tag{67}$$

$$p_4 = 0.7390851 \tag{68}$$

$$|p_4 - p_3| = 0.0000278 < \epsilon \tag{69}$$

Answer is $p_4 : 0.7390851$ and $0 \leq p_4 \leq \frac{\pi}{2}$. Founded in 4.step

3.4 d. $f(x) = x - 0.8 - 0.2 * sin(x) = 0, [0, \frac{\pi}{2}]$

$$f'(x) = 1 - 0.2 * cos(x) \tag{70}$$

Let's take $p_0 = 0$ and $\epsilon = 10^{-2}$

$$p_1 = p_0 - \frac{f(p_0)}{f'(p_o)} \tag{71}$$
$$p_1 = 0 - \frac{f(0)}{f'(0)} \tag{72}$$
$$p_1 = 1.0000000 \tag{73}$$
$$|p_1 - p_0| = 1.0000000 > \epsilon \tag{74}$$

p_1 has not enough approximate value. Calculate p_2

$$p_2 = p_1 - \frac{f(p_1)}{f'(p_1)} \tag{75}$$
$$p_2 = 1.0000000 - \frac{f(1.0000000)}{f'(1.0000000)} \tag{76}$$
$$p_2 = 0.9644530 \tag{77}$$
$$|p_2 - p_1| = 0.0355470 > \epsilon \tag{78}$$

p_2 has not enough approximate value. Calculate p_3

$$p_3 = p_2 - \frac{f(p_2)}{f'(p_2)} \tag{79}$$
$$p_3 = 0.9644530 - \frac{f(0.9644530)}{f'(0.9644530)} \tag{80}$$
$$p_3 = 0.9643339 \tag{81}$$
$$|p_3 - p_2| = 0.0001191 < \epsilon \tag{82}$$

Answer is $p_3 : 0.9643339$ and $0 \leq p_3 \leq \frac{\pi}{2}$. Founded in 3.step