Numerical Analysis Homework 2

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1 Apply Gaussian and Jaboci's method (50 points)

Program Usage:

$.$/solver -i system.txt -m GESP/JCB

1.1 Use Gaussian Elimination with Scaled Partial Pivoting

1.1.1 Question 6.2-1c

$$2x_1 - 3x_2 + 2x_3 = 5 \tag{1}$$
$$-4x_1 + 2x_2 - 6x_3 = 14 \tag{2}$$
$$2x_1 + 2x_2 + 4x_3 = 8 \tag{3}$$

$$\left[\ A\mid b\ \right]^{(1)} = \left[\begin{array}{ccc|c} 2 & -3 & 2 & 5 \\ -4 & 2 & -6 & 14 \\ 2 & 2 & 4 & 8 \end{array}\right]$$

Find Max values for each lines:

$$S_i = max_{i=1}^n \|a_{ij}\| \tag{4}$$

$$S_1 = max\{\|2\|, \|-3\|, \|2\|\} = 3 \tag{5}$$
$$S_2 = max\{\|-4\|, \|2\|, \|-6\|\} = 6 \tag{6}$$
$$S_3 = max\{\|2\|, \|2\|, \|4\|\} = 3 \tag{7}$$

Now, find scaled partial pivots of first column to check if need interchanges

$$max_{i=1}^n \{\frac{\|a_{ij}\|}{S_i}\} \tag{8}$$

$$\frac{\|a_{11}\|}{S_1} = 0.666667 \tag{9}$$

$$\frac{\|a_{21}\|}{S_2} = 0.666667 \tag{10}$$

$$\frac{\|a_{31}\|}{S_3} = 0.500000 \tag{11}$$

Maximum pivot index=1. There is no row interchange Now calculate m_{21} and m_{31} to reduce rows

$$m_{21} = \frac{a_{21}}{a_{11}} = -2 \tag{12}$$

$$m_{31} = \frac{a_{31}}{a_{11}} = 1 \tag{13}$$

Apply this operations on matrix:

$$E_2 - m_{21} * E_1 -> E_2 \tag{14}$$
$$E_3 - m_{31} * E_1 -> E_3 \tag{15}$$

$$\left[\ A\mid b\ \right]^{(2)} = \left[\begin{array}{ccc|c} 2 & -3 & 2 & 5 \\ 0 & -4 & -2 & 24 \\ 0 & 5 & 2 & 3 \end{array}\right]$$

Now, find scaled partial pivots of second column to check if need interchanges

$$\frac{\|a_{22}\|}{S_2} = 0.666667 \tag{16}$$

$$\frac{\|a_{32}\|}{S_3} = 1.250000 \tag{17}$$

Maximum pivot index=3 and pivot=1.25. Change row 3 with 2.

$$\left[\ A\mid b\ \right]^{(2)} = \left[\begin{array}{ccc|c} 2 & -3 & 2 & 5 \\ 0 & 5 & 2 & 3 \\ 0 & -4 & -2 & 24 \end{array}\right]$$

Now calculate m_{32} to reduce row

$$m_{32} = \frac{a_{32}}{a_{22}} = -2 \tag{18}$$

Apply this operations on matrix:

$$E_3 - m_{32} * E_2 -> E_3 \tag{19}$$

$$[\ A \mid b\ ]^{(3)} = \begin{bmatrix} 2 & -3 & 2 & 5 \\ 0 & 5 & 2 & 3 \\ 0 & 0 & -0.4 & 26.4 \end{bmatrix}$$

Solve matrix with backward substitution.
Results:

$$\begin{bmatrix} X \end{bmatrix} = \begin{bmatrix} 109 & 27 & -66 \end{bmatrix}^t$$

1.1.2   Question 7.3.1-a

$$3x_1 - 1x_2 + 1x_3 = 1 \tag{20}$$
$$3x_1 + 6x_2 + 2x_3 = 0 \tag{21}$$
$$3x_1 + 3x_2 + 7x_3 = 4 \tag{22}$$

$$[\ A \mid b\ ]^{(1)} = \begin{bmatrix} 3 & -1 & 1 & 1 \\ 3 & 6 & 2 & 0 \\ 3 & 3 & 7 & 4 \end{bmatrix}$$

Find Max values for each lines:

$$S_i = max_{i=1}^{n} \|a_{ij}\| \tag{23}$$

$$S_1 = max\{\|3\|, \|-1\|, \|1\|\} = 3 \tag{24}$$
$$S_2 = max\{\|3\|, \|6\|, \|2\|\} = 6 \tag{25}$$
$$S_3 = max\{\|3\|, \|3\|, \|7\|\} = 7 \tag{26}$$

Now, find scaled partial pivots of first column to check if need interchanges

$$max_{i=1}^{n}\{\frac{\|a_{ij}\|}{S_i}\} \tag{27}$$

$$\frac{\|a_{11}\|}{S_1} = 1.000000 \tag{28}$$

$$\frac{\|a_{21}\|}{S_2} = 0.500000 \tag{29}$$

$$\frac{\|a_{31}\|}{S_3} = 0.428571 \tag{30}$$

Maximum pivot index=1 and pivot=1.0. There is no row interchange Now calculate m_{21} and m_{31} to reduce rows

$$m_{21} = \frac{a_{21}}{a_{11}} = 1 \tag{31}$$

$$m_{31} = \frac{a_{31}}{a_{11}} = 1 \tag{32}$$

Apply this operations on matrix:

$$E_2 - m_{21} * E_1 -> E_2 \tag{33}$$
$$E_3 - m_{31} * E_1 -> E_3 \tag{34}$$

$$[\ A \mid b\ ]^{(2)} = \begin{bmatrix} 3 & -1 & 1 & 1 \\ 0 & 7 & 1 & -1 \\ 0 & 4 & 6 & 3 \end{bmatrix}$$

Now, find scaled partial pivots of second column to check if need interchanges

$$\frac{\|a_{22}\|}{S_2} = 1.166667 \tag{35}$$

$$\frac{\|a_{32}\|}{S_3} = 0.571429 \tag{36}$$

Maximum pivot index=2 and pivot=1.166667. No Row Change
Now calculate m_{32} to reduce row

$$m_{32} = \frac{a_{32}}{a_{22}} = 0.571429 \tag{37}$$

Apply this operations on matrix:

$$E_3 - m_{32} * E_2 -> E_3 \tag{38}$$

$$[\ A \mid b\ ]^{(3)} = \begin{bmatrix} 3 & -1 & 1 & 1 \\ 0 & 7 & 1 & -1 \\ 0 & 0 & 5.429 & 3.571 \end{bmatrix}$$

Solve matrix with backward substitution.
Results:

$$\begin{bmatrix} X \end{bmatrix} = \begin{bmatrix} 0..351 & -0.2368 & 0.6579 \end{bmatrix}^t$$

1.1.3 Question 7.3.1-b

$$10x_1 - x_2 + 0 = 9 \tag{39}$$
$$-x_1 + 10x_2 - 2x_3 = 7 \tag{40}$$
$$0 - 2x_2 + 10x_3 = 6 \tag{41}$$

$$\begin{bmatrix} A \mid b \end{bmatrix}^{(1)} = \begin{bmatrix} 10 & -1 & 0 & | & 9 \\ -1 & 10 & -2 & | & 7 \\ 0 & -2 & 10 & | & 6 \end{bmatrix}$$

Find Max values for each lines:

$$S_i = max_{i=1}^n \|a_{ij}\| \tag{42}$$

$$S_1 = max\{\|10\|, \|-1\|, \|0\|\} = 10 \tag{43}$$
$$S_2 = max\{\|-1\|, \|10\|, \|-2\|\} = 10 \tag{44}$$
$$S_3 = max\{\|0\|, \|-2\|, \|10\|\} = 10 \tag{45}$$

Now, find scaled partial pivots of first column to check if need interchanges

$$max_{i=1}^n \{\frac{\|a_{ij}\|}{S_i}\} \tag{46}$$

$$\frac{\|a_{11}\|}{S_1} = 1.000000 \tag{47}$$

$$\frac{\|a_{21}\|}{S_2} = 0.100000 \tag{48}$$

$$\frac{\|a_{31}\|}{S_3} = 0.000000 \tag{49}$$

Maximum pivot index=1 and pivot=1.0. There is no row interchange.
Now calculate m_{21} and m_{31} to reduce rows

$$m_{21} = \frac{a_{21}}{a_{11}} = -0.1 \tag{50}$$

$$m_{31} = \frac{a_{31}}{a_{11}} = 0 \tag{51}$$

Apply this operations on matrix:

$$E_2 - m_{21} * E_1 -> E_2 \tag{52}$$
$$E_3 - m_{31} * E_1 -> E_3 \tag{53}$$

$$\begin{bmatrix} A \mid b \end{bmatrix}^{(2)} = \begin{bmatrix} 10 & -1 & 0 & | & 9 \\ 0 & 9.9 & -2 & | & 7.9 \\ 0 & -2 & 10 & | & 6 \end{bmatrix}$$

Now, find scaled partial pivots of second column to check if need interchanges

$$\frac{\|a_{22}\|}{S_2} = 0.990000 \tag{54}$$

$$\frac{\|a_{32}\|}{S_3} = 0.200000 \tag{55}$$

Maximum pivot index=2 and pivot=0.990000. No Row Change
Now calculate m_{32} to reduce row

$$m_{32} = \frac{a_{32}}{a_{22}} = 0.202020 \tag{56}$$

Apply this operations on matrix:

$$E_3 - m_{32} * E_2 -> E_3 \tag{57}$$

$$\begin{bmatrix} A \mid b \end{bmatrix}^{(3)} = \begin{bmatrix} 10 & -1 & 0 & | & 9 \\ 0 & 9.9 & -2 & | & 7.9 \\ 0 & 0 & 9.596 & | & 7.596 \end{bmatrix}$$

Solve matrix with backward substitution.
Results:

$$\begin{bmatrix} X \end{bmatrix} = \begin{bmatrix} 0.9958 & 0.9579 & 0.7916 \end{bmatrix}^t$$