# Monte Carlo Method Exercises

**1)**

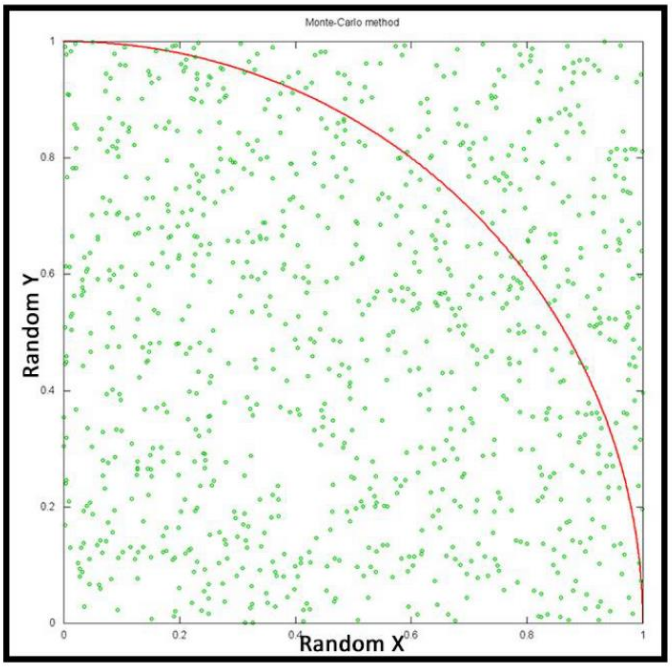
Radius of Circle = 1 Unit
Area of Quarter Circle = PI * R * R / 4
= PI * 1 * 1 / 4
= PI / 4 sq. units

Side of Square, a = 1 Unit
Area of Square = a * a
= 1 sq. unit

Probablity of Random Point, R(x,y) being inside the circle and square P=
Area of Quarter Circle / Area of Square
= PI / 4.

For N samples, M points lie within the circle and the square, and N-M points lie outside the circle but inside the square.

Using the probablitly, M for N samples
=> M = N*PI/4
=> PI = 4*M/N




**2)**

There are 150 children trying to escape from school by jumping over a wall. The heights of the children are distributed randomly between 100 cm and 170 cm. A child can jump to the other side (outer world) if (s)he is at least 120 cm tall. Write a Monte Carlo code to find the number of the children that managed to escape.

Then add to the code a teacher that catches a student with a possibility of 20%.

**3)**

A soccer team wins a game with a possibility of 40%, loses with a possibility of 50% and a draw occurs with a possibility of 10%. A win brings 3 points, a draw brings 1 point and no point is earned on a loss.

Write a Monte Carlo code to simulate 34 matches (one season) for this team and find the total points of the team after one season.

**4)**

There are 16 holes on a square (placed like the elements of a 4x4 matrix). Every second, a ball is released from a random hole and you try to hit it with a stick.

You also pick the hole you will hit randomly. Therefore, you hit the ball if the both random choices are the same.

You gain 1 TL if you hit a ball and lose 0.5 TL if you miss one. You have 20 TL initially and you play the game for five minutes.

Write a code to find your final money.

**5)**

Turkey is located between 36-42° in latitude and between 26-45° in longitude.

The annual probability of an astroid impact for the whole earth is 0.2% and the probability is equal for the all points on the earth.

Latitude values change between [-90,90] and longitude values change between [-180,180] (all real values, not integers).

Start the Monte Carlo simulation from this year and stop when an astroid impact occurs in Turkey. Print the final year and the latitude and longitude values on the screen.

**6)**

A group of scientists visit monkeys all around the world and try them with a typewriter. They expect them to write your name by pressing the keys. This work is done randomly as the subject of the research is a monkey.

They visit a monkey and if it does not succeed after pressing enough keys to write your name (for ali, it is 3 and for ayse it is 4), they pass to another.

What is the number of monkeys that they need to try to get the name “ali” typed correctly? Assume the typewriter has 27 keys and the monkeys are equally literate.

**7)**

There are 7 notes:

A (La - 1), B (Si - 2), C (Do - 3), D (Re - 4), E (Mi - 5), F (Fa - 6), G (Sol - 7).

Define a very simple piano having only these 7 keys (no black keys, either).

C-major (Do major) chord has these sounds: C (Do - 3), E (Mi - 5), G (Sol - 7).

You hire a monkey to play the C-major chord: Your monkey plays three notes together randomly. If the chord is C-major, it stops playing. If the chord is not C-major, it goes on playing. Find the number of tries that your monkey needs to play a C-chord.

**Notice that** the order of playing sounds is **not** important. If you select three random numbers (n1,n2,n3), then their order is not important. For example, (n1=3, n2=5, n3=7) is as correct as (n1=7, n2=3, n3=5).

**8)**

A student needs to get 65 points over 100 questions in a test to pass. Questions have equal grades as 1 and each question has five choices: (a, b, c, d, e).

There is no penalty for a wrong answer.

The student does not want to study and s/he always answers the questions **in a random way.**

The student can take the test once a year and the correct answers of the test and her/his random answers change each year.

Start from this year and find the year that the immortal student passed the test.

**Note** [You do **not** have to use this!]**:**
You can produce a single random number in the closed range **[1,n]** using the MATLAB command **randi(n)**. To produce a(k,m) matrix you can use **randi(n,[k,m])**.

**9)**

A student needs to get 65 points over 100 questions in a test to pass. Questions have equal grades as 1 and each question has five choices: (a, b, c, d, e).

There is no penalty for a wrong answer.

The student does not want to study and s/he always answers the questions **by choosing "b" for each question.**

The student can take the test once a year and the correct answers of the test change each year.

In some years, the student tries to cheat (with a 30% possibility) and the possibility that s/he is seen by the observer is 15%. If the student is seen, s/he is expelled from the school. If not seen, s/he can solve 10 questions correctly.

Start from this year and find the year that the immortal student passed the test or expelled from the school.

**10)**

Tyrion and Jon are playing a game using a dice of six sides with the numbers **{-4,-3,-1,1,2,4}** written on the sides. Each roll the dice three times and add the results. The aim is to get **zero** and who gets zero **first** wins the game. Jon starts the game.

*For example,*
Jon:-4+1+4=1, Tyrion:-1+2+4=5, Jon:-3-4-1=-8,
Tyrion:-3+1+1=1, Jon:-4+2+2=0 and Jon wins.

Simulate this game **until** Tyrion or Jon wins. Print the **name** of the winner on the screen.

**HINT:**
**i)** The MATLAB command **randi(n)** returns a random integer number on the interval [1,n] where n is an integer number.
**ii)** The MATLAB command **mod(x,y)** returns the remainder of the division x/y.

**11)**

We will produce water ($H_2O$) molecules using hydrogen and oxygen atoms.

Suppose that we have 10000 hydrogen (H) and 10000 oxygen (O) atoms in a small box.

Two hydrogen and one oxygen atoms are needed to form a water molecule. The probability of "forming a water molecule per time step" is given by the ratio of the number of hydrogen molecules to oxygen molecules in the box.

Plot the numbers of H, O and $H_2O$ substances versus time step all in the same graph.

**12)**

A drunken man wants to go home. Now, he is in front of the bar and his home is at the opposite side of a circular street. Suppose he is standing at the $50^{th}$ meter of the street and you can denote the location of his home as at the $100^{th}$ or the $0^{th}$ meter of the street.

He is so drunken that he steps forward with a possibility of 50% and another 50% backwards. The step size of the man is 1 meter.

Write a Monte-Carlo random walk simulation and find how many steps should the drunken man walk to get his home. Plot his position versus steps ( $i^{th}$ position - $i^{th}$ step ).

**13)**

A particle sits at the origin of a two-dimensional box extending from -10 to +10 in both dimensions (a square).

Every second, the particle changes its x and y positions by random step-sizes between -1 and 1 in both directions.

The particle will eventually diffuse out of the box.

Determine the number of seconds that the particle is inside the box before exiting.

**14)**

A soldier is walking on a minefield with a mine detector. He can walk in four directions: North (or 1), south (or 2), east (or 3) and west (or 4). He picks a direction randomly and walks if this direction is clear (He can forget his choice and pick the same direction again).

There is only one path that he can walk: [1,3,2,3,3,1,2,2,4,4,2,4,4,4,1,1,4,4,2,3,3,3,1,1,3].

He can pick and test one direction in one minute and he has 60 minutes to escape.

Write a code to find the number of steps he needed to escape if he managed. If he could not escape in 60 minutes, print "helicopter on the way" to rescue him.

**15)**

Simulate the radioactive decay of Thulium-167 isotope using a Monte Carlo scheme:

The decay rate per atom is lambda = 0.0008675.

Start with N=100 atoms, enter a **while** loop, select a random number x between 0 and 1,
if x<lambda, reduce one atom ( N(step) = N(step-1) – 1 ), (if not, N(step) = N(step-1) ).
continue the loop until all atoms decay.

Plot the graph " time step vs. number of atoms".

**16)**

Rıfkı is 25 years old. He lives in Mecidiyekoy and works in Gayrettepe. Each day he either walks or takes the subway to work (50% each). He works seven days a week.

If he walks, there is a 0.005% possibility that he dies along his way. If he takes the subway, this possibility is 0.002%.

If he walks, he meets and communicates with a woman with a possibility of 1%. If he takes the subway this possibility is 0.5%.

If he meets a woman, there is a 3% possibility that they like each other and get married.

Write a program to simulate Rıfkı's life and find his age when he either dies or gets married. (One year=365 days)

**17)**

The "Monty Hall problem" is a probability puzzle based on the American television game show *Let's Make a Deal* and named after its original host, Monty Hall.

The problem can be stated as the following:

Suppose you are on a game show and you are given the choice of three doors:
Behind one door is a car; behind the others, goats.

You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat.

He then says to you, "Do you want to pick door No. 2?"

**Is it to your advantage to switch your choice to win the car?**

Simulate 10000 games to give an answer to this problem.

**18)**

Two players are playing a game:

- The game involves a path with 100 steps with 3 holes which are located randomly (the first step, namely path(1) and/or the last step, namely path(100) cannot be a hole). The locations of the holes remain the same throughout the game. (You do not need to use vectors. However, you can use vectors if you wish.)

- The game finishes when one of the players completes the path. (If the current position is larger than or equal to 100.)

- Initially, Player 1 and Player 2 are both in path(1).

- Player 1 starts the game.

- The player rolls a dice with sides [1,2,3,4,5,6] and moves according to the result. (e.g. if the current position is 50 and the dice is 3, the player moves to the position 50+3=53.)

- If the player's current position is a hole, s/he restarts the game from the beginning. (Her/his location is changed to path(1) but the game is not finished yet.)

- After Player 1, Player 2 rolls the same dice and plays accordingly with the same rules, on the same path. (The locations of Player 1 and Player 2 can coincide.)

- After Player 2, Player 1 plays her/his move by rolling the dice, and so on.

- When a player reaches path(100) or further, the game finishes.

Simulate this game and print the winner on the screen.

**19)**

We will try to simulate a small company that produces and sells toy rabbits. The capital of the company is $10^6$ TL (initial money). The variables are,

- The price needed to produce one toy. (prodpripertoy)
- The selling price of one toy. (sellpripertoy)
- The location rank of the toy store section. (Better location sells more but the store rent is higher) (storeloc)

| Store rank | Rent/month (TL) | Customers/day |
|---|---|---|
| 1 | 100 TL | 2000 |
| 2 | 75 TL | 1000 |
| 3 | 50 TL | 500 |

- Number of stores (some are in good and some are in bad locations) (storenum)
- You can take one month as 30 days if you need to calculate the rent per day.
- Advertisement encourages the customers to buy toys but costs 1/10 of the store rent per month for a store. Take it on a daily basis. (advperday)
- Customers tend to buy toys more for some special occasions (five times more than a usual day) such as the New Year's Eve [put 10 days for shopping, from Dec. 21 (Day 355) to Dec. 31 (Day 365)] or St. Valentine's Day [from Feb. 4 (Day 35) to Feb. 14 (Day 45)]. (dayimportance = 1 or 5)
- The only thing that you will randomly choose is the mind of the customer and compare it to a demand rate that you will choose according to the variables above. (For example,

advperday increases the sells but sellpripertoy reduces. You are free to choose any rate you want. It is, of course also related to dayimportance)

- If you generate numbers between 0 and 1 as the customer's mind, be careful not to exceed 1 as the upper limit of demand. Demand should always be less than 1 as 1 means evey customer would buy a toy rabbit.

Guess some values for the variables above yourself (not randomly), simulate one year for this company and plot the graph of profit per day versus days (profit=income-expenses). After one year, change your guessed values if the capital is not increased.