Problem Set 4

Quantitative Economics, Fall 2024

January 10, 2025

This problem set consists of two problems. Submit your solutions until Feb. 5th 11:59 PM. You can work in teams of up to three students.

Problem 1: Root-finding in the Neoclassical Growth Model

Recall the Bellman equation for the Neoclassical Growth Model:

$$V(k) = \max_{k' \in [0, f(k)+(1-\delta)k]} u\big(f(k) + (1-\delta)\,k - k'\big) + \beta V\big(k'\big).$$

Here for simplicity we assume there are no shocks to the production function. The parameters are $\beta \in (0,1)$, $\delta \in (0,1)$, $u(\cdot)$ is the period utility function, $f(\cdot)$ is the production function.

First order condition with respect to k' is

$$u'\big(f(k) + (1-\delta)\,k - k'\big) = \beta V'\big(k'\big).$$

We can thus find the optimal k' for a given k by solving the equation above.

Suppose that we work with a grid of points $\{k_i\}_{i=1}^n$ and we want to find the optimal k' for each k_i. We also have derivatives of $V(k)$, $V'(k)$ for each k_i. We do not know $V'(k)$ for an arbitrary k, though. We will need to interpolate $V'(k)$.

Your task is to write a function that takes the following inputs: a tuple `model` containing the parameters of the model as well as functions f and u (and their derivatives), a vector of $\{k_i\}_{i=1}^n$, a vector of $\{V'(k_i)\}_{i=1}^n$. The function should return a vector of $\{k'(k_i)\}_{i=1}^n$, where $k'(k_i)$ is the optimal k' for k_i.

Instead of passing a tuple you can pass a structure and use the `Parameters.jl` package.

Assume that $u(\cdot)$ is the CRRA utility function, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, where $\gamma > 0$ is the coefficient of relative risk aversion. Assume that $f(k) = k^\alpha$, where $\alpha \in (0,1)$. Use the following parameters: $\alpha = 0.3$, $\beta = 0.96$, $\delta = 0.05$, $\gamma = 2$. Use the following grid of points: $\{k_i\}_{i=1}^n = \{...\}$.

1. AAA

Feel free to embed this function in a full program that solves the Neoclassical Growth Model. This is not required for the problem set. You will need to think how to link the function you wrote with the Bellman equation.