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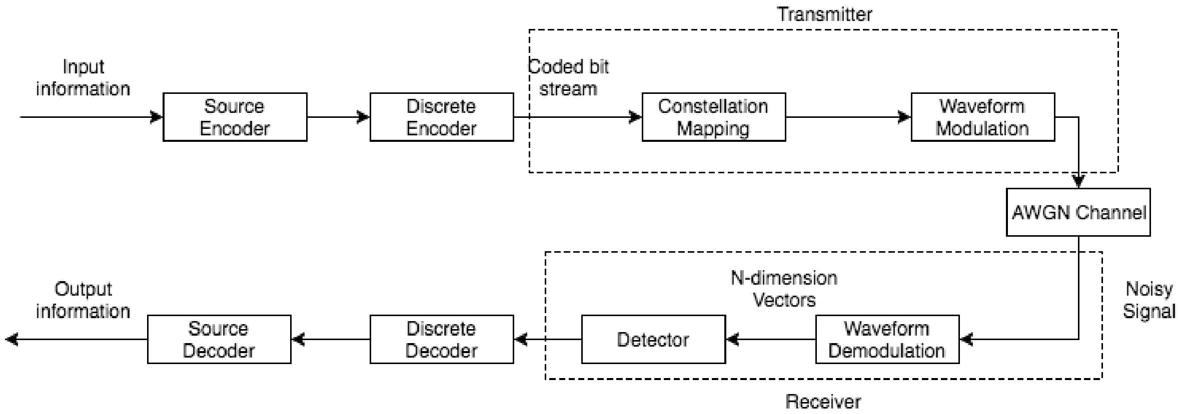
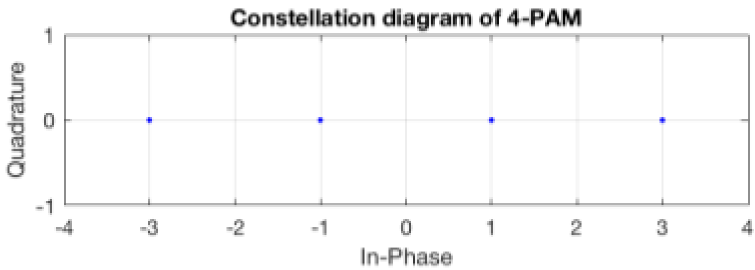
EEE2040 Communication and Network Mini-Project Report

1. Draw the block diagram of the transmitter and receiver.



2. Plot the constellation diagram of the system.



3. Describe the function of the optimum detectors of this system. Your description must explain the decision criterion for this particular system.

Optimum detectors are used to determine the signal level of a constellation after the signal passes through a noisy channel.
Using Bayes' rule, we can say the probability of a symbol in the system is

$$P(S_m|R=r) = \frac{f_R(r|S_m)P(S_m)}{f_R(r)}$$

where $f_R(r|S_m)$ is the likelihood function the symbol;
$P(S_m)$ is the priori probability of m-th signal;
$f_R(r)$ is the probability density function of the observed vector.
Since $f_R(r)$ is independent of the symbol, the decision criterion of a system is $f_R(r|S_m)P(S_m)$.
In this system, we are using 4-Pulse Amplitude Modulation(4-PAM).
Given that the symbols in 4-PAM are equally probable, $P(S_m) = \frac{1}{4}$.
The probability density function of noise in additive white Gaussian noise channel (AWGN) is $f(x) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{x^2}{2\sigma_n^2}}$.

So, the likelihood function for symbols is $f_R(r|S_m) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{\left(r-\sqrt{\varepsilon_S}\right)^2}{2\sigma_n^2}}$.

Therefore, the decision criterion for 4-PAM is $\boxed{f_R(r|S_m)P(S_m) = \frac{1}{4}\frac{1}{\sqrt{2\pi}\sigma_n}e^{-\frac{\left(r-\sqrt{\varepsilon_S}\right)^2}{2\sigma_n^2}}}$.

4. Obtain the average SNR per bit as a function of *d* and σ.

The average energy per symbol, ε_s, as a function of d and σ, is given by

$$\varepsilon_s = \frac{1}{M}\sum_{i=1}^{4}\varepsilon_{s_i} = \frac{2}{4}(d^2 + (3d)^2) = 5d^2$$

where:
M is the number of symbols of the system;
d is the distance parameter.
By definition,

$$\because\ \sigma^2 = \frac{N_0}{2},$$

$$\therefore \frac{\varepsilon_s}{N_0} = \frac{\varepsilon_s}{2\sigma^2} = \frac{5d^2}{2\sigma^2}$$

For 4-PAM,

$$\varepsilon_s = \log_2 4 \times \varepsilon_b = 2\varepsilon_b$$

$$\therefore \frac{\varepsilon_s}{N_0} = \frac{2\varepsilon_b}{N_0}$$

where $\frac{\varepsilon_b}{N_0}$ is the average SNR per bit.
Combining the equations,

$$\frac{2\varepsilon_b}{N_0} = \frac{5d^2}{2\sigma^2}$$

$$\boxed{\frac{\varepsilon_b}{N_0} = \frac{5d^2}{4\sigma^2}}$$

5. Obtain the theoretical formula for probability of bit error for this system as the function of the average SNR per bit. This must be a closed form equation.

Firstly, we define the midpoint of two symbols as detection thresholds, i.e. -2d, 0, 2d. The likelihood functions of the system for each symbol are given as the following:

$$f_R(r|S_0) = f_R(r|S_3) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r-3d)^2}{N_0}} = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r+3d)^2}{N_0}}$$

$$f_R(r|S_1) = f_R(r|S_2) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r-d)^2}{N_0}} = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r+d)^2}{N_0}}$$

We can say the above two sets of probabilities are equal due to symmetry in PAM. Similarly, we can find the probabilities of errors for each symbol with symmetry:

$$P(e_s|S_0) = P(e_s|S_3) = P(r \geq 2d \mid S_3)$$

$$= \int_{-\infty}^{2d} f_R(r|S_3)dr$$

$$= \int_{-\infty}^{2d} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r-3d)^2}{N_0}} dr$$

Substitute $x = \frac{r-3d}{\sqrt{\frac{N_0}{2}}}$,

$$\int_{-\infty}^{2d} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r-3d)^2}{N_0}} dr = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\sqrt{\frac{2d^2}{N_0}}} e^{-\frac{x^2}{2}} dx$$

$$= Q\left(\sqrt{\frac{2d^2}{N_0}}\right)$$

where Q-function is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-x} e^{-\frac{u^2}{2}} du = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{u^2}{2}} du$$

Since plain MATLAB code does not contain the Q-function, we must convert it into a complementary error function. The Q-function, in terms of complementary error function, is $Q(x) = \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}}\right)$. We can then express the Q-function for S_0, S_3 as

$$Q\left(\sqrt{\frac{2d^2}{N_0}}\right) = \frac{1}{2} erfc\left(\sqrt{\frac{d^2}{N_0}}\right)$$

For S_1, S_2, we have

$$P(e_s|S_1) = P(e_s|S_2) = P(r < 2d, r \geq 0|S_2)$$

$$= \int_{-\infty}^{0} f_R(r|S_2)dr + \int_{2d}^{\infty} f_R(r|S_2)dr$$

$$= \int_{-\infty}^{0} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r-d)^2}{N_0}} dr + \int_{2d}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r-d)^2}{N_0}} dr$$

Substitute $x = \frac{r-d}{\sqrt{\frac{N_0}{2}}}$,

$$\int_{-\infty}^{0} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r-d)^2}{N_0}} dr + \int_{2d}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r-d)^2}{N_0}} dr$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\sqrt{\frac{2d^2}{N_0}}} e^{-\frac{x^2}{2}} dx + \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{2d^2}{N_0}}}^{\infty} e^{-\frac{x^2}{2}} dx$$

$$= 2Q\left(\sqrt{\frac{2d^2}{N_0}}\right)$$

$$= erfc\left(\sqrt{\frac{d^2}{N_0}}\right)$$

The total probability of symbol error is simply the mean of sum of symbols errors. Therefore, we can calculate it by

$$P(e_s) = \frac{1}{4} \sum_{m=0}^{3} P(e|S_m)$$

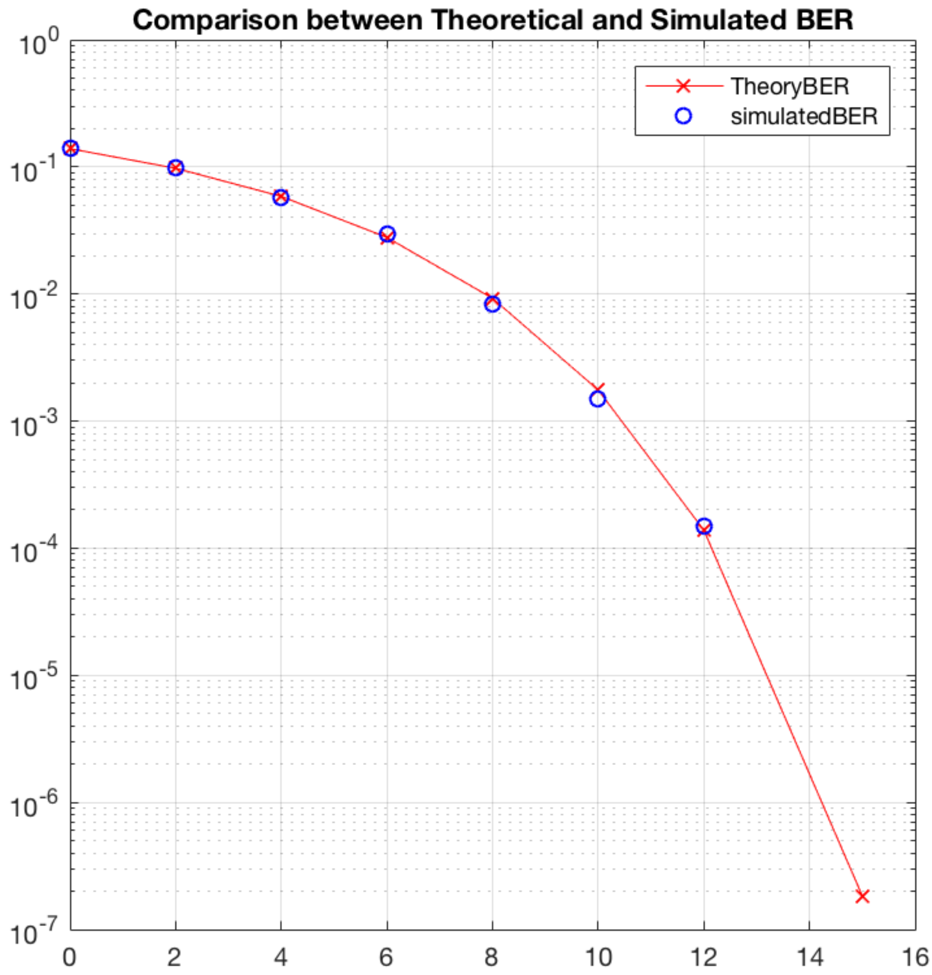
$$= \frac{1}{4}\left(\frac{2}{2} erfc\left(\sqrt{\frac{d^2}{N_0}}\right) + 2erfc\left(\sqrt{\frac{d^2}{N_0}}\right)\right)$$

$$= \frac{3}{4} erfc\left(\sqrt{\frac{d^2}{N_0}}\right)$$

The total probability of bit error, as we can deduce from the solution for question 3, is the total probability of symbol error divided by the number of bits per symbol. We can then conclude that the probability of bit error is

$$\boxed{P(e_b) = \frac{3}{8} erfc\left(\sqrt{\frac{d^2}{N_0}}\right) = \frac{3}{8} erfc\left(\sqrt{\frac{2\varepsilon_b}{5N_0}}\right)}$$

6. Run your simulation for 10,000 transmissions for the average SNR per bit equal to the following values (0, 2, 4, 6, 8, 10, 12, 15) dB. For each value of the average SNR, compute the probability of error from the equation that you obtained in Question 5 and the simulations. Then, plot both results on a single plot using MATLAB's plot function. Use a solid line for the theoretical values and markers 'o' for the simulation values.



There is no simulated BER at 15dB since no error is detected. It follows the bit error rate of 10^{-6} as suggested by the theoretical BER since sample size of 10^4 transmissions cannot simulate an error rate which is 100 times smaller. A total transmission of at least 10^6 times is required to accurately simulate the bit error rate at 15dB.

7. Print and attach the source code as the last section of your report.

```
clear, close all
clc

% Declare predefined variables for AWGN channel and 4-PAM
N = 10^4;
d = 1;
variance = [1.25 0.789 0.498 0.314 0.198 0.125 0.0789 0.0395];
L = 2;
numSymbols = 4;
Es = abs(2*(d^2 + (3*d)^2))/4;
EbN0 = (5/4)*(d^2./variance);
EbN0db = (10.*log10(EbN0));

% Allocate dynamic arrays for number of bit errors and simulated BER
numbitErr = zeros(1, length(variance));
simErrBER = zeros(1, length(variance));

for varIndex = 1:length(variance)
  % Generate equally probable bits and map them into different signal levels
  for i = 1:N
    RandomBits = randi([0,1], 1, 2);
    if RandomBits == [0 0]
      Tx(:,i) = -3*d;
    elseif RandomBits == [0 1]
      Tx(:,i) = -d;
    elseif RandomBits == [1 1]
      Tx(:,i) = d;
    elseif RandomBits == [1 0]
      Tx(:,i) = 3*d;
    end
  end

  % Pass the transmitted symbols into AWGN channel
  awgnNoise = randn(1, N) * sqrt(variance(1,varIndex));
  NoisySignal = Tx  + awgnNoise;

  % Optimum detector implementation: Using mid-point of symbol as
  % detection criterion
  Rx(find(NoisySignal >= 2*d)) = 3*d;
  Rx(find(NoisySignal < 2*d & NoisySignal >= 0)) = d;
  Rx(find(NoisySignal < 0 & NoisySignal >= -2*d)) = -d;
  Rx(find(NoisySignal < -2*d)) = -3*d;

  % Calculate number of bit errors
  for i = 1:N
    if (abs((Rx(:,i)-Tx(:,i))) == 2)
      numbitErr(varIndex) = (numbitErr(varIndex) + 1);
    elseif (abs((Rx(:,i)-Tx(:,i))) == 4)
      numbitErr(varIndex) = (numbitErr(varIndex) + 2);
    end
  end
end
```

```
% Plotting constellation diagram of 4-PAM
% x = linspace(-3*d, 3*d, 4);
% scatterplot(x);
% grid on
% axis([-4 4 -1 1]);
% title('Constellation diagram of 4-PAM');

% Number of bits = 2 * Number of symbols
simErrBER = numbitErr/(L*N);
% theoryBER = 0.75*erfc(sqrt(2*EbN0/5))*(1/L); % Alternative expression for
% theoretical BER
theoryBER = 0.75*erfc(sqrt(d^2./(2.*variance)))*(1/L)

% Plot both theoretical BER and simulated BER onto the same graph
semilogy(EbN0db, theoryBER, 'rx-');
hold on
semilogy(EbN0db, simErrBER, 'bo');
grid on
legend('TheoryBER', 'simulatedBER')
title('Comparison between Theoretical and Simulated BER');
```