Q1. Let us denote by x the event that a car is of brand x, and by R the event that a car needs repair during its first year of purchase, then (1)

(a) $P(R) = P(A\cdot R) + P(B,R) + P(C,R)$
$= P(R|A)P(A) + P(R|B)P(B) + P(R|C)P(C)$ (1)
$= 0.05 \times 20\% + 0.1 \times 30\% + 0.15 \times 50\% = 11.5\%$ (1)

(b) $P(A|R) = \dfrac{P(A.R)}{P(R)} = \dfrac{P(R|A)\cdot P(A)}{P(R)} = \dfrac{0.05 \times 20\%}{11.5\%} = 8.7\%$ (1) (1)

Q2. (a) $K \times (0.5 - (-0.5)) \times \frac{1}{2} = 1$ (3)

$\therefore K = 2$

only has answer → 1.

get answer, ~~not directly~~ has a little bit calculation → (2)

(b). $P(X > 0.25) = 1 - F_X(0.25) = 1 - 0.875 = 12.5\%$. (3)

(c). $P(X>0 | X<0.25) = \dfrac{P(X>0, X<0.25)}{P(X<0.25)} = \dfrac{F_X(0.25) - F_X(0)}{1 - P(X>0.25)}$ (3)

$= \dfrac{0.875 - 0.5}{0.875} = 42.86\%$.

(d) first find CDF : $F_X(x|X>0) = P(X \le x | X>0) = \dfrac{P(X\le x, X>0)}{P(X>0)}$. (3)

For $P(X \le x, X>0) = F_X(x) - F_X(0) = \begin{cases} 0 & , x \le 0 \\ F_X(x) - \frac{1}{2} & , x > 0. \end{cases}$

and $P(X>0) = \frac{1}{2}$.

thus $F_X(x|X>0) = \begin{cases} 0 & , x \le 0 \\ 2F_X(x) - 1 & , x > 0. \end{cases}$

$f_X(x|X>0)$ is the differentiating of $F_X(x|X>0)$

thus $f_X(x|X>0) = \begin{cases} 0 & , x \le 0. \\ 2(-4x+2) & , x>0 \end{cases} = \begin{cases} 0 & , x \le 0 \\ -8x+4 & , x>0. \end{cases}$

(e). $E[X|X>0] = \int_{-\infty}^{\infty} x f_X(x|X>0)\,dx = \int_{-\infty}^{\infty} x(-8x+4)\,dx$ (3)

$= \int_0^{0.5} (-8x^2 + 4x)\,dx = \left[-\frac{8}{3}x^3 + 2x^2\right]_0^{0.5} = 0.167$.

Summative Assignment.

① ② ③

Q3 ② 0|00|1|01|010|0100|11|0101|000|01010|111|10|001|101|100|0011|1010|11

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

18 = 7

0		(index, appedex)	Code	this can be dropped if needed
1	0	0,0	~~0~~0 10	1 bit
2	00	1,0		
3	1	0,1	001	2 bit.
4	01	1,1	011	
5	010	4,0	1000	3 bit
6	0100	5,0.	1010	
7	11	3,1	0111	
8	0101	5,1	1011	
9	000	2,0	00100	4 bit.
10	01010	8,0	10000	
11	111	7,1	01111	
12	10	3,0	00110	
13	001	2,1	00101	
14	101	12,1	11001	
15	100	12,0	11000	
16	0011	13,1	11011	
17	1010	14,0	011100	5 bit
18	11	7	00111 X	
		4.	4	

* If someone has used 5 bits fixed length for index, that is fine and acceptable as well.

Encoded Sequence is

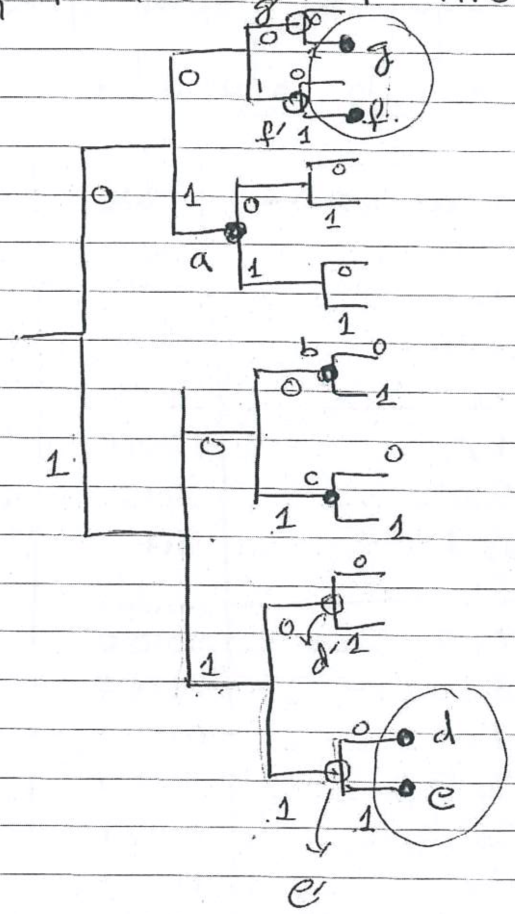
0010 001 011 1000 1010 . ...

If first zero is not written the answer is still correct.

Q4

	a	b	c	d	e	f	g	
{	01	100	101	1110	1111	0011	0001	}.
				↑ 110.	↑ 111	↓ 001	↓ 000	

0 ↑
1 ↓



① ② ③

Q5. (CH1): Capacity of this channel :
(15) 5

$$C_1 = \max_{P(x)} [H(Y) - H(Y|X)]. \quad 1$$

~~H(Y)~~ let q be the probability of the input symbol 0.
therefore $1-q$ is for symbol 1.

$$H(Y) = -P(Y=0)\log(P(Y=0)) - P(Y=1)\log(P(Y=1))$$
$$= -q\log q - (1-q)\log(1-q)$$
$$H(Y|X) = -\sum_x \sum_Y P(y|x)P(x)\log(P(y|x)) = 0. \quad 2$$

to find the max : here $C_1 = \max_{P(x)} [H(Y) - H(Y|X)] = \ldots$ 1

$\frac{\partial C_1}{\partial q} = 0$, where we got $q = \frac{1}{2}$ 1 and for $P(0) = P(1) = \frac{1}{2}$,

We have $C_1 = 1$

Q5 Cont.

(CH2): let q be the probability of the input symbol 0, and thus $1-q$ the prob of symbol 1.

(10)

$$\cancel{H(Y|X)}\ P(Y=0) = q \cdot 1 + 0.5(1-q) = 0.5 + 0.5q$$

$$P(Y=1) = 0.5(1-q) = 0.5 - 0.5q \quad (2)$$

$$H(Y|X) = \sum_x p(x) H(Y|X=x)$$

$$= q H(Y|X=0) + (1-q) H(Y|X=1)$$

$$= (1-q) H(Y|X=1) = 1-q \quad (2)$$

$$C_2 = \max_q [H(Y) - (1-q)].$$

$$H(Y) = -(0.5+0.5q)\log(0.5+0.5q) - (0.5-0.5q)\log(0.5-0.5q) \quad (1)$$

to achieve max: then $C_2 = \ldots\ldots$ (1)

$$\frac{\partial C_2}{\partial q} = 0 = 1 - \left[0.5\log_2(0.5+0.5q) + (0.5+0.5q)\frac{0.5}{0.5+0.5q} \cdot \frac{1}{\ln 2}\right]$$

$$- \left[-0.5\log_2(0.5-0.5q) + (0.5-0.5q)\frac{-0.5}{0.5-0.5q}\frac{1}{\ln 2}\right]$$

$$= 1 + 0.5\log_2(0.5-0.5q) - 0.5\log_2(0.5+0.5q)$$

(3)

Therefore $\log_2 \frac{0.5-0.5q}{0.5+0.5q} = -2 \Rightarrow q = \frac{3}{5}$. (1)

thus $C_2 = \cancel{k}\ 0.3219$.

10

Q6:

Since the rate of transmission is $R = 10^5$ bits/sec, the bit interval T_b is 10^{-5} sec. The probability of error in a binary PAM system is

$$P(e) = Q\left[\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right] \quad 3$$

where the bit energy is $\mathcal{E}_b = A^2T_b$. With $P(e) = P_2 = 10^{-6}$, we obtain

$$\sqrt{\frac{2\mathcal{E}_b}{N_0}} = 4.75 \Longrightarrow \mathcal{E}_b = \frac{4.75^2 N_0}{2} = 0.112813$$

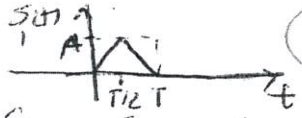
4

Thus

$$A^2T_b = 0.112813 \Longrightarrow A = \sqrt{0.112813 \times 10^5} = 106.21 \quad 3$$

$$Q(x) = 10^{-6} \Rightarrow x = ?$$

(15)



a) (3) $\mathcal{E}_b = \mathcal{E}_{s_1} = \mathcal{E}_{s_2} = \int_{-\infty}^{\infty} s_1(t)^2 dt = 2\int_0^{T/2} (\frac{2A}{T} t)^2 dt = \frac{1}{3} A^2 T$

b) (2)

$$\underset{-\sqrt{\mathcal{E}_b}}{\overset{s_2}{\circ}} \quad\text{———}\quad \underset{0}{\cdot} \quad\text{———}\quad \underset{\sqrt{\mathcal{E}_b}}{\overset{s_1}{\circ}} \longrightarrow$$

(3) c)

$$r(t) \longrightarrow \boxed{h(t) = s_1(T-t)} \longrightarrow \text{(switch)} \longrightarrow \text{to detector}$$

Sample at T

(2) d)

$$\tau_h = \frac{N_0}{4\sqrt{\mathcal{E}_b}} \cdot \ln \frac{1-p}{p} = \frac{N_0}{4A\sqrt{T/3}} \cdot \ln \frac{1-p}{p}$$

e)

(5)

$$P_b = p \times P(e|s_1) + (1-p)\, P(e|s_2) \quad (1)$$

$$P(e|s_1) = P(r < \tau_h | s_1)$$

$$= \int_{-\infty}^{\tau_h} f_R(r|s_1)\,dr = \int_{-\infty}^{\tau_h} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r-\sqrt{\mathcal{E}_b})^2}{N_0}} dr$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\sqrt{\frac{2}{N_0}}(\tau_h - \sqrt{\mathcal{E}_b})} e^{-x^2/2} \cdot dx$$

$$= \Phi\left[\sqrt{\frac{2}{N_0}}(\tau_h - \sqrt{\mathcal{E}_b})\right]$$

(2)

Similarly: $P(e|s_2) = \Phi\left[\sqrt{\frac{2}{N_0}}(\tau_h + \sqrt{\mathcal{E}_b})\right]$ (2)

$P_b = \text{..................}$

Q8. (a).

KARMA –

K	A	R	M	A
3	1	5	4	2
T	R	A	N	S
P	O	S	I	T
I	O	N	C	I
P	H	E	R	C
H	A	N	G	E
S	T	H	E	O
R	D	E	R	O
F	T	H	E	L
E	T	T	E	R
S.				

A	A	K	M	R
1	2	3	4	5
R	S	T	N	A
O	T	P	I	S
O	I	I	C	N
H	C	P	R	E
A	E	H	G	N
T	O	S	E	H
D	O	R	R	E
T	L	F	E	H
T	R	E	E	T.
		S.		

ROOHATDTTSTICEOOLR

TPIPHSRFESNICRGE

REEASNENHEHT.

(b) please see the lecture notes.

(c). DES : $2^{56} \approx 7\times10^{16}$ Keys, $7\times10^{16} / (10^{12}\times60\times60) = 20$ hours.

3DES: $2^{112} \approx 5\times10^{33}$ Keys, $5\times10^{33} / (10^{12}\times60\times60) = 1.4\times10^{18}$ hours.

Q9. Max Round Trip Delay $= 2\times\frac{1}{200000} = 10\,\mu S$.

max frame size: $10\times10^{-6}\times10^{9}$ bits $= 1250$ bytes $= 100000$ bits

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Q10. (a) I_1

(b) IO.

(c) R_2

(d) R_1

(e) R_2