

Alias-Free ViT:

Fractional Shift Invariance via Linear Attention



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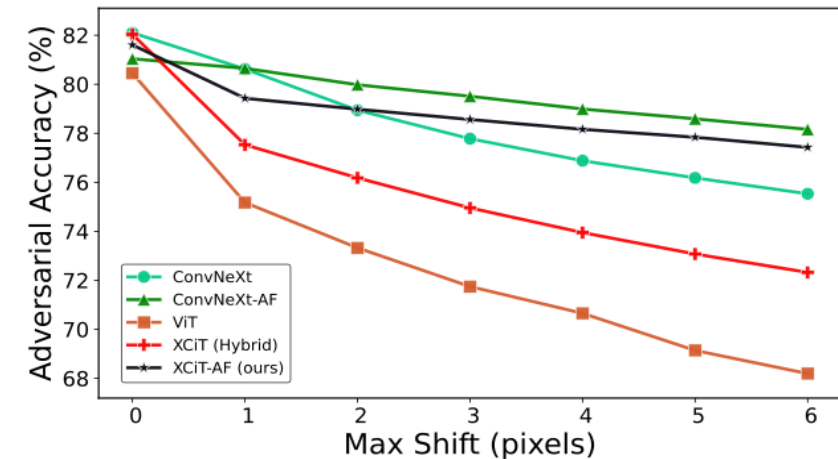
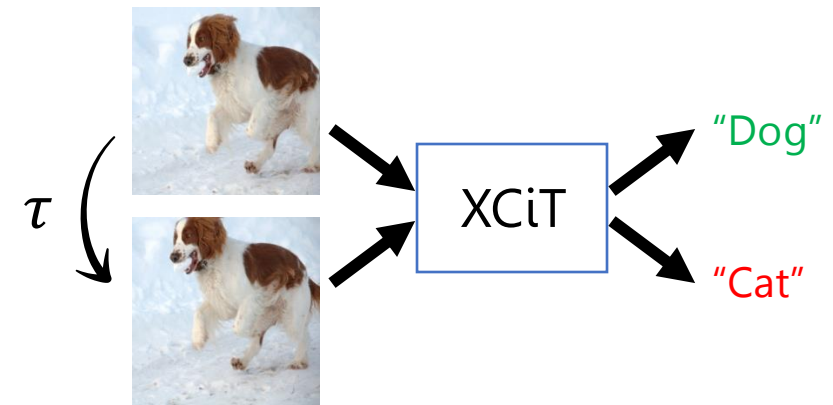


Daniel Soudry



Motivation: built-in prior

- Shift-invariance
 - A key prior assumption for image classifiers
 - Motivating the first convolutional models
- However, modern **Vision Transformers (ViTs)** are very sensitive to small image translations
- **Convnets** are less sensitive to image translations, but still not perfectly robust
 - Aliasing in pooling and nonlinearities break shift-equivariance¹
 - Alias-Free convnets² have provable shift-invariance



Goal: Use Alias-Free framework to construct shift-invariant ViT

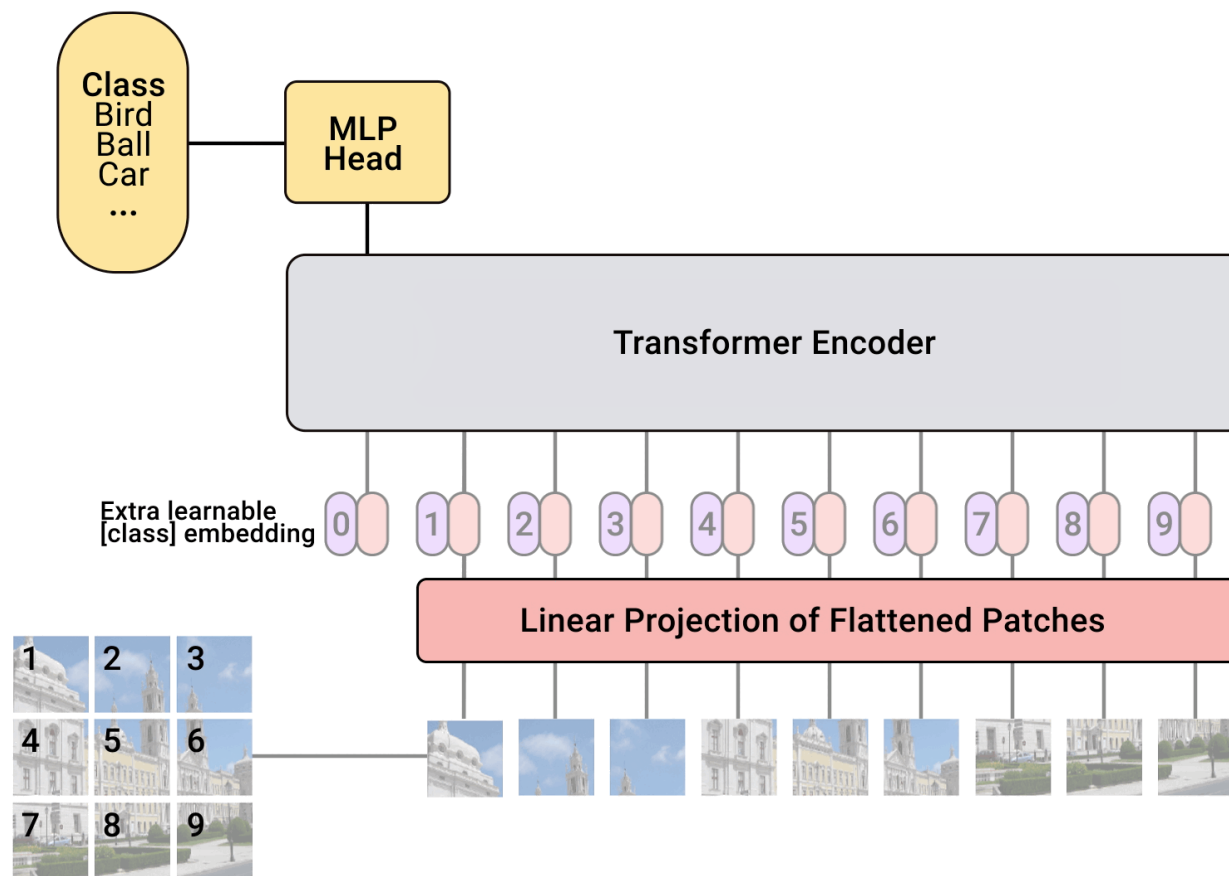
[1] Making convolutional networks shift-invariant again. Zhang, ICML 2019

[2] Alias-Free Convnets: Fractional Shift Invariance via Polynomial Activations. Michaeli et al, CVPR 2023

Contribution

- We present a class of Shift-equivariant attention layers (SEA)
 - Includes linear attention and cross-covariance attention
- We design an Alias-Free Vision Transformer (AFT)
 - Using cross-covariance attention and Alias-free nonlinearities
 - Competitive in image classification
- We improve translation robustness
 - ~99% consistency for fractional-cyclic shifts
 - Significant improvement in adversarial robustness to practical translation-attacks

Background: Vision Transformers¹



Shift-Invariant ViT

Shift-Invariant ViT – Patch Embedding

Observation:

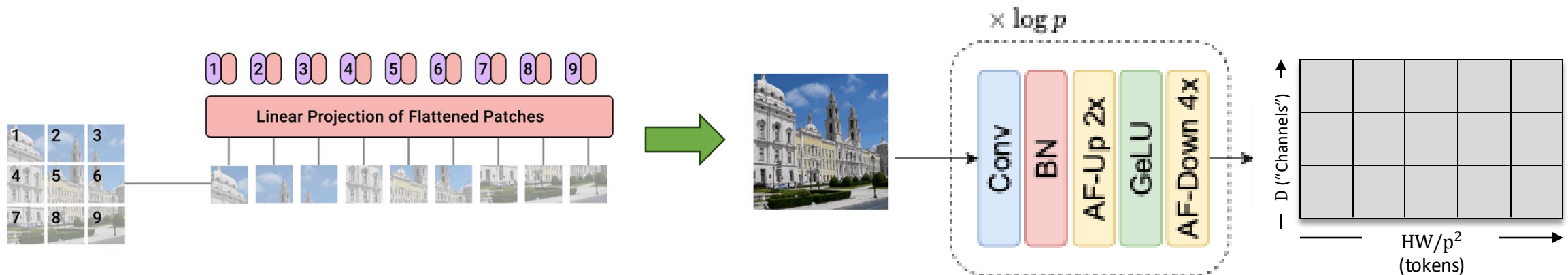
Patch-embedding \leftrightarrow Strided convolution

Aliasing in PE and POE break shift-equivariance

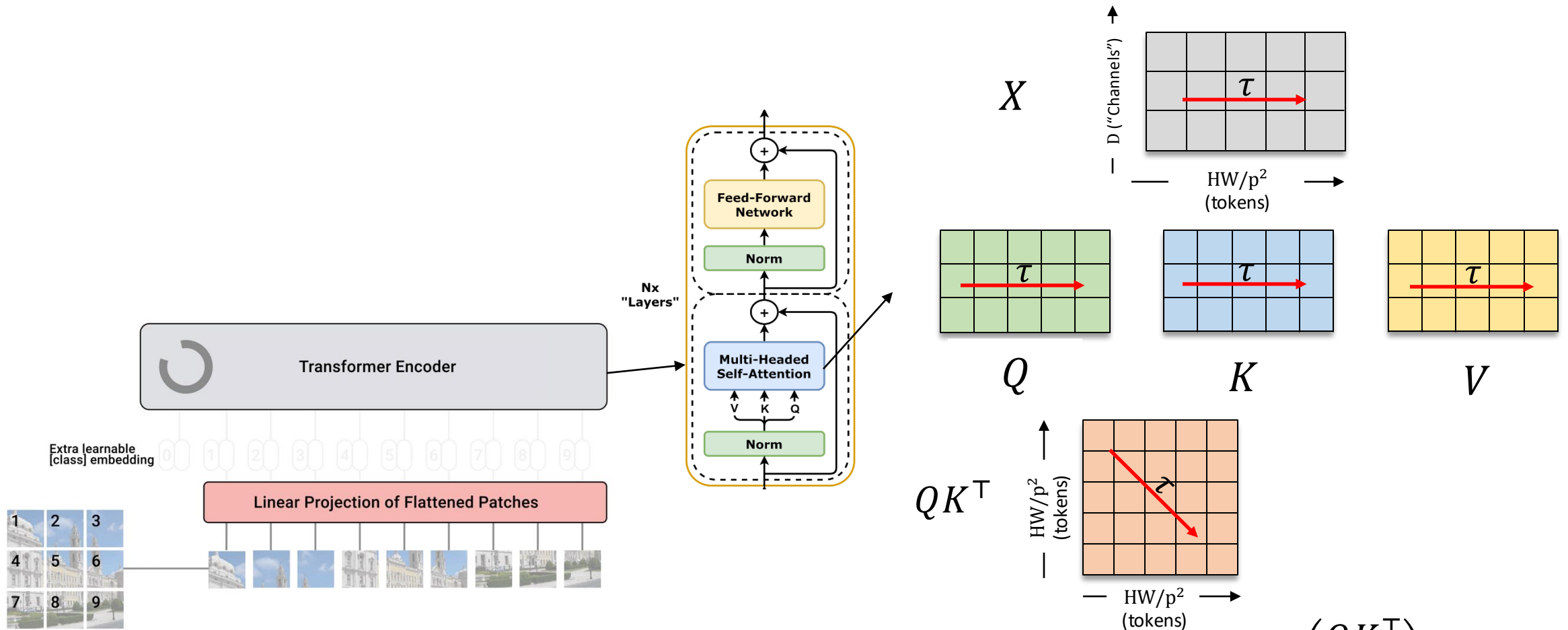
Solution:

Alias-Free Convolutional Patch Embedding

- Gradual, alias-free downsampling
- Alias-free nonlinearities
- No positional encoding



Shift-Invariant ViT – Attention



$$SA(X) = \text{softmax} \left(\frac{QK^T}{\sqrt{D}} \right) V$$

Shift-Invariant ViT – Attention

Proposition 1:

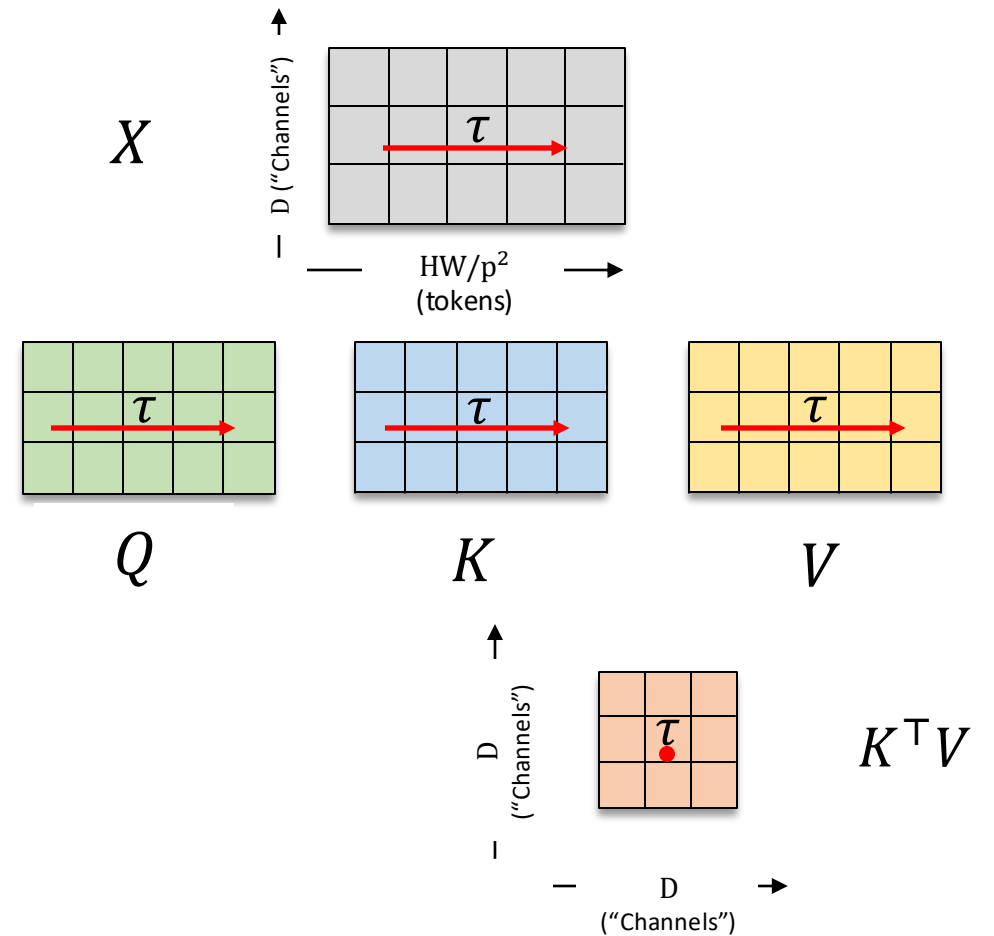
Q, K, V are shift-equivariant

Proposition 2:

$K^\top V$ is shift-invariant

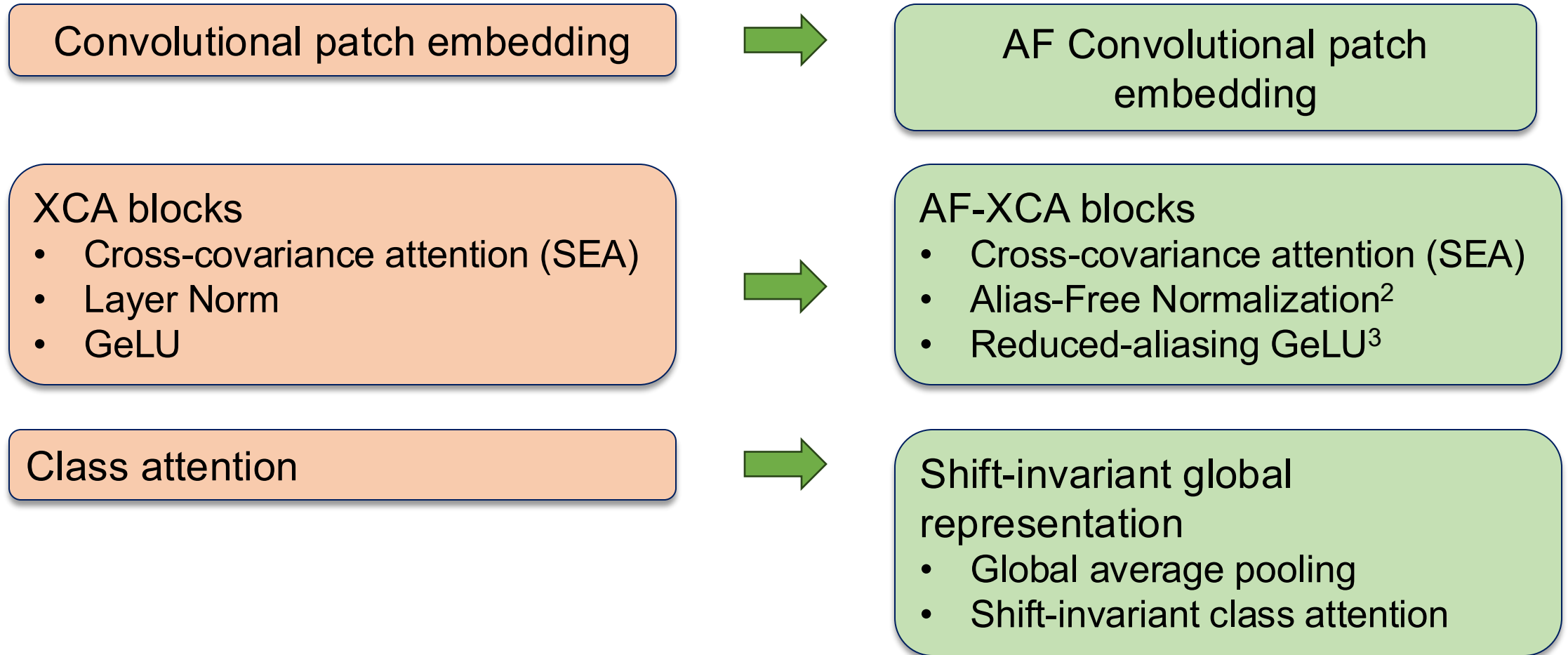
Proposition 3:

$SEA(X) = Q f(K^\top V)$ is shift-equivariant



$$SEA(X) = Q f(K^\top V)$$

Alias-Free XCiT¹



[1] XCiT: Cross-covariance image transformers. Ali et al, NeurIPS 2021

[2] Alias-Free Convnets: Fractional Shift Invariance via Polynomial Activations. Michaeli et al, CVPR 2023

[3] Alias-Free Latent Diffusion Models: Improving Fractional Shift Equivariance of Diffusion Latent Space. Zhou et al, CVPR 2025

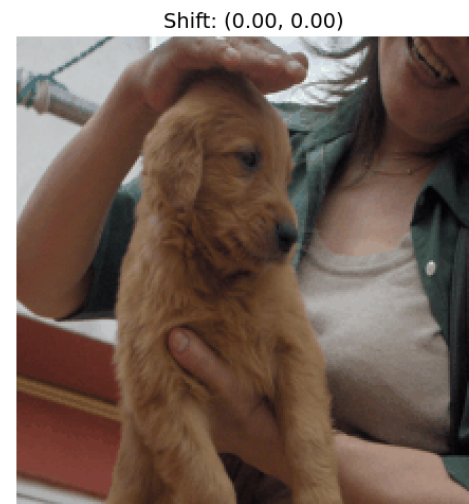
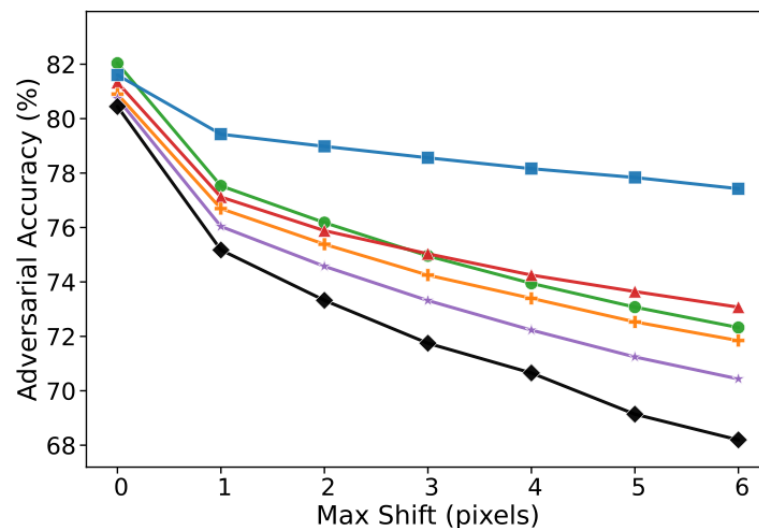
Results: ImageNet accuracy and consistency

- Accuracy on par with baseline
- ~ 99% consistency to cyclic integer and fractional shifts

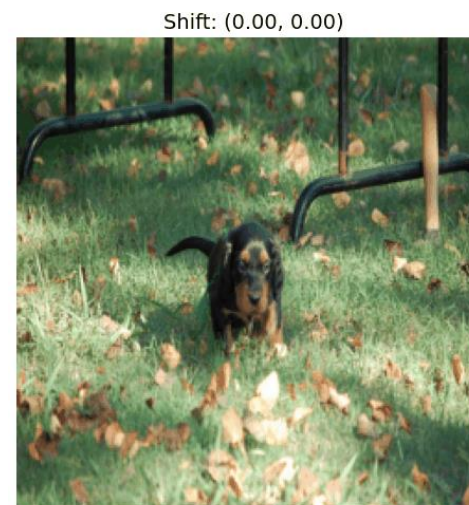
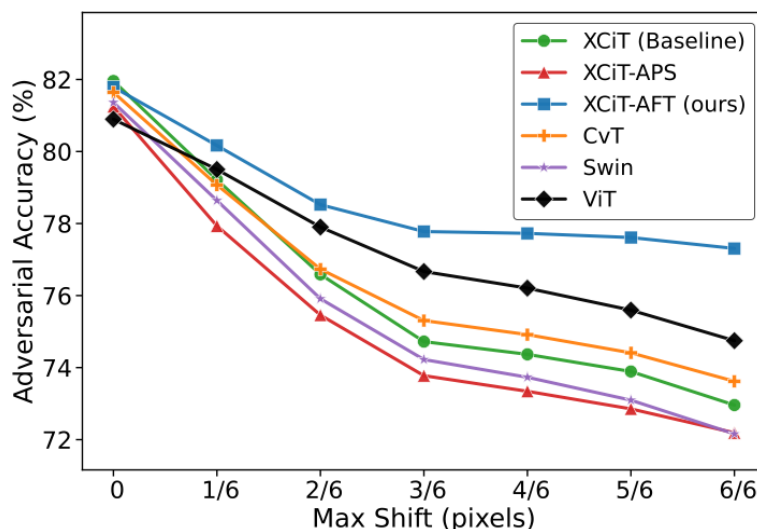
	Model	Test Accuracy	Integer shift consistency	Half-pixel shift consistency
XCiT-Nano	Baseline	70.4	83.7	82.0
	APS	68.4	100.0	87.5
	AF (ours)	70.5	99.2	98.7
XCiT-Small	Baseline	82.0	91.4	89.8
	APS	81.3	100.0	94.0
	AF (ours)	81.8	99.5	99.4

Results: Practical translations

- Crop-shifts:
 - Imitating camera translations
- Bilinear-interpolation fractional shifts:
 - Imitating small, sub-pixel translations



Target: Golden retriever
Baseline: Golden retriever
APS: Golden retriever
AF: Golden retriever



Target: Gordon setter
Baseline: Gordon setter
APS: Gordon setter
AF: Gordon setter

Summary

- Problem & goal
 - ViTs are very sensitive to image translation comparing to Convnets
 - Build alias-free shift-equivariant transformer encoder
- Approach
 - Shift-Equivariant Attention (SEA): $SEA(X) = Q \cdot f(K^T V)$
 - Includes linear and cross-covariance attention
 - Alias-Free ViT (AFT):
 - Shift-Equivariant Attention
 - Alias-free patch embedding, activations, and normalization
- Results
 - Competitive ImageNet accuracy
 - ~99% consistency under fractional cyclic shifts
 - Stronger robustness to realistic translations (crop / sub-pixel)