Problem 2

2, b.

Show $E[a+bX] = a + bE[X]$.

we know $E[Y] = \sum_y y\,P(Y=y)$

then $E[a+bX] = \sum_x (a+bX)\,P(X=x)$ distribute

$$= \sum_x a\,P(X=x) + \sum_x bX\,P(X=x)$$ remove constant from summation

$$= a\sum_x P(X=x) + b\sum_x X\,P(X=x)$$

and we know $\sum_x P(X=x) = 1$ b/c the total prob over all possible outcomes is 1. So,

$$= a(1) + b\sum_x X\,P(X=x)$$

and we know $\sum_x X\,P(X=x) = E[X]$. So,

$$= a\cancel{(1)} + bE[X] = a + bE[X]$$

therefore,

$$E[a+bX] = a + bE[X] \quad //$$

Problem 2

2c.

Use a Bournulli random variable such that there are only two possible outcomes.

$$X = \begin{cases} 1, & \text{with probability } p = \frac{1}{2} \\ 0, & \text{with probability } (p-1) = \frac{1}{2} \end{cases}$$

And take the function

$$v(y) = y^2$$

First, compute $v(E[X])$

$$E[X] = \frac{1}{2}$$

$$v(E[X]) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Second, compute $E(v[X])$

since X is bournulli, $X = 0, 1$ so $X^2 = X$.

So, $$E[v(X)] = E[X] = \frac{1}{2}$$

Then we have

$$v(E[X]) = v\left(\frac{1}{2}\right) = \frac{1}{4} \neq E[v(X)] = E[X] = \frac{1}{2}$$

$$\frac{1}{4} \neq \frac{1}{2}, \text{ so } v(E[X]) \neq E[v(X)]$$

Problem 3

3.

Full likelyhood:

For n patients

$$L(p) = \prod_{i=1}^{n} p^{y_i}(1-p)^{1-y_i}.$$

Let

$$S = \sum_{i=1}^{n} y_i \quad (\text{\# of patients who got radiation})$$

then

$$L(p) = p^{S}(1-p)^{n-S}$$

Take logs:

$$\ell(p) = S\log p + (n-S)\log(1-p)$$

Now find MLE:

$$\frac{d\ell}{dp} = \frac{S}{p} - \frac{n-S}{1-p} = 0$$

Solve for p:

$$\frac{d\ell}{dp} = \frac{S}{p} - \frac{n-S}{1-p} = 0$$

solving for p:

$$\frac{S}{p} = \frac{n-S}{1-p} \Rightarrow S(1-p) = p(n-S)$$

$$\Rightarrow S = pn \Rightarrow \hat{p} = \frac{S}{n},$$

where S = # of patients who receive radiation

n is totall # of patients