5.

Define $\varepsilon = X - E[X]$

a. What is $E[\varepsilon]$, use linearity of expectation

$$E[\varepsilon] = E[X - E[X]] = E[X] - E[E[X]]$$

But $E[X]$ is a constant so

$$E[\varepsilon] = E[X] - E[X] = 0$$

b. What is $V[\varepsilon]$?

Since adding/subtracting a constant doesn't change variance

$$V[\varepsilon] = V[X - E[X]] = V[X]$$

So, $V[\varepsilon] = V[X] = \sigma^2_X$

So, we can write $X = E[X] + \varepsilon$, $E[\varepsilon] = 0$, $V[\varepsilon] = \sigma^2_X$

. standardize X

$$X = E[X] + \sigma_X \varepsilon \Rightarrow \varepsilon = \frac{X - E[X]}{\sigma_X}$$

then $E\left(\frac{X - E[X]}{\sigma_X}\right) = 0$ (mean)

and $V\left(\frac{X - E[X]}{\sigma_X}\right) = \frac{1}{\sigma^2_X} V(X) = 1$ (variance)

. Now replace $E[X]$ w/ $X\beta \rightarrow$ regression

Let $E[X]$ be replace w/ $E[Y|X] = X\beta$

then

$$\boxed{Y = X\beta + \varepsilon}$$

6. Taylor Series

Show that

$$E[\hat{f}_{x,h}(x)] = \frac{F(x+h) - F(x-h)}{2h} = f(x) + O(h^2)$$

using Taylor expansions $f(x+h)$ and $f(x-h)$

a. Subtract: $F(x+h) - F(x-h)$

cancels to $O(h^3)$ [order stays]

$$F(x+h) - F(x-h) = \left[\cancel{F(x)} + hf(x) + \cancel{\frac{h^2}{2} f'(x)} + \cancel{O(h^3)}\right]$$
$$- \left[\cancel{F(x)} - hf(x) + \cancel{\frac{h^2}{2} f'(x)} - \cancel{O(h^3)}\right]$$

actually keep

then,

$$F(x+h) - F(x-h) = 2hf(x) + O(h^3)$$

then divide by $2h$

$$\frac{\cancel{2h}f(x) + O(\cancel{h^3})^{h^2}}{\cancel{2h}} = f(x) + O(h^2)$$

So, $E[\hat{f}_{x,h}(x)] = f(x) + O(h^2)$

and the bias is $O(h^2)$