Prob: Predict $\hat{y}(z)$ at z using MSE

$$MSE(\hat{y}(z)) = \frac{1}{N}\sum_{i=1}^{N} \frac{1}{h}k\left(\frac{z-x_i}{h}\right)(y_i - \hat{y}(z))^2$$

Take derivative wrt $\hat{y}(z)$ and set equal to 0

let $w = \frac{1}{h}k\left(\frac{z-x_i}{h}\right)$ [weight for point i]

then,

$$MSE(\hat{y}(z)) = \frac{1}{N}\sum_{i=1}^{N} w_i (y_i - \hat{y}(z))^2$$

differentiate wrt $\hat{y}(z)$,

$$\frac{d}{d\hat{y}(z)} MSE(\hat{y}(z)) = \cancel{\frac{1}{N}}\sum_{i=1}^{N} w_i \cdot \cancel{2}(\hat{y}(z) - y_i) = 0$$

$$\sum_{i=1}^{N} w_i (\hat{y}(z) - y_i) = 0$$

isolate $\hat{y}(z)$,

$$\Rightarrow \sum_{i=1}^{N} w_i \cdot \hat{y}(z) - \sum_{i=1}^{N} w_i y_i = 0$$

$$\hat{y}(z)\sum_{i=1}^{N} w_i = \sum_{i=1}^{N} w_i y_i$$

$$\hat{y}(z) = \frac{\sum_{i=1}^{N} w_i y_i}{\sum_{i=1}^{N} w_i}$$

Sub back in $w_i = \frac{1}{h} k\left(\frac{z - x_i}{h}\right)$

$$\tilde{y}(z) = \frac{\sum_{i=1}^{N} \frac{1}{h} k\left(\frac{z - x_i}{h}\right) y_i}{\sum_{i=1}^{N} \frac{1}{h} k\left(\frac{z - x_i}{h}\right)} \quad //$$

Thus is the LCLS estimator