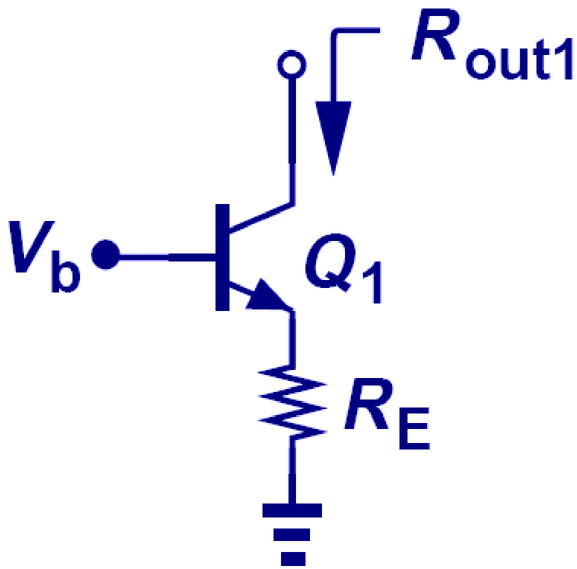
Fundamentals of Microelectronics II

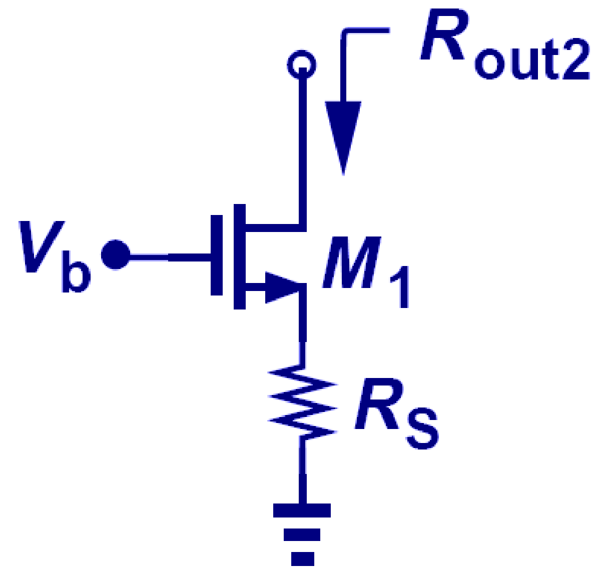
- **CH9 Cascode Stages and Current Mirrors**
- **CH10 Differential Amplifiers**
- **CH11 Frequency Response**
- **CH12 Feedback**

Chapter 9 Cascode Stages and Current Mirrors

- 9.1 Cascode Stage
- 9.2 Current Mirrors

Boosted Output Impedances

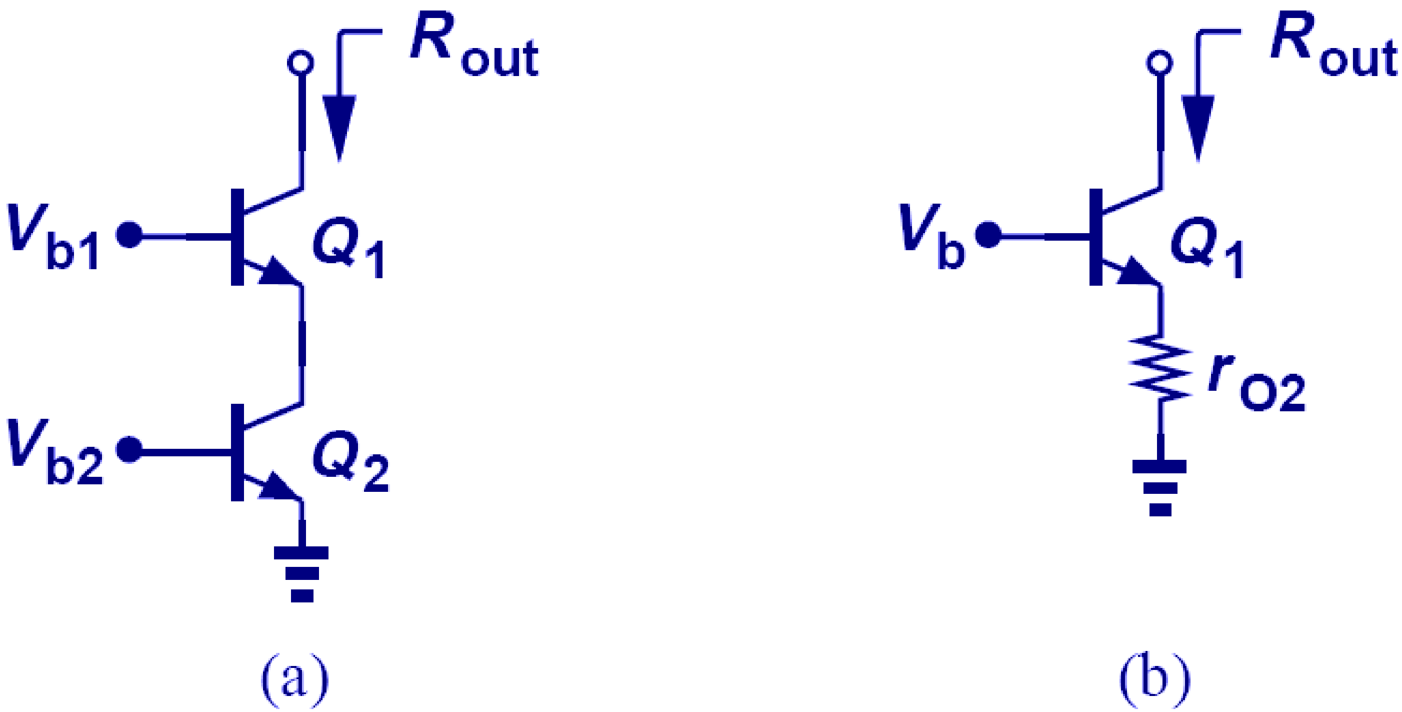




$$R_{out1} = [1 + g_m(R_E \parallel r_\pi)]r_O + R_E \parallel r_\pi$$

$$R_{out2} = (1 + g_m R_S)r_O + R_S$$

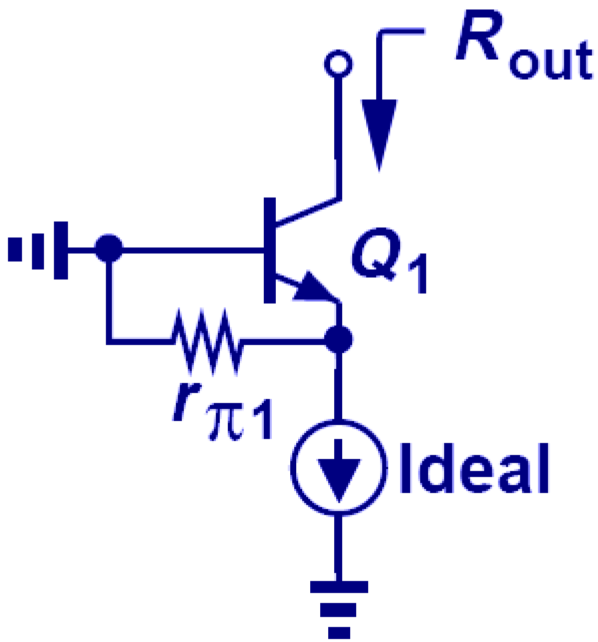
Bipolar Cascode Stage



$$R_{out} = [1 + g_m(r_{O2} \parallel r_{\pi 1})]r_{O1} + r_{O2} \parallel r_{\pi 1}$$

$$R_{out} \approx g_{m1} r_{O1}\left(r_{O2} \parallel r_{\pi 1}\right)$$

Maximum Bipolar Cascode Output Impedance

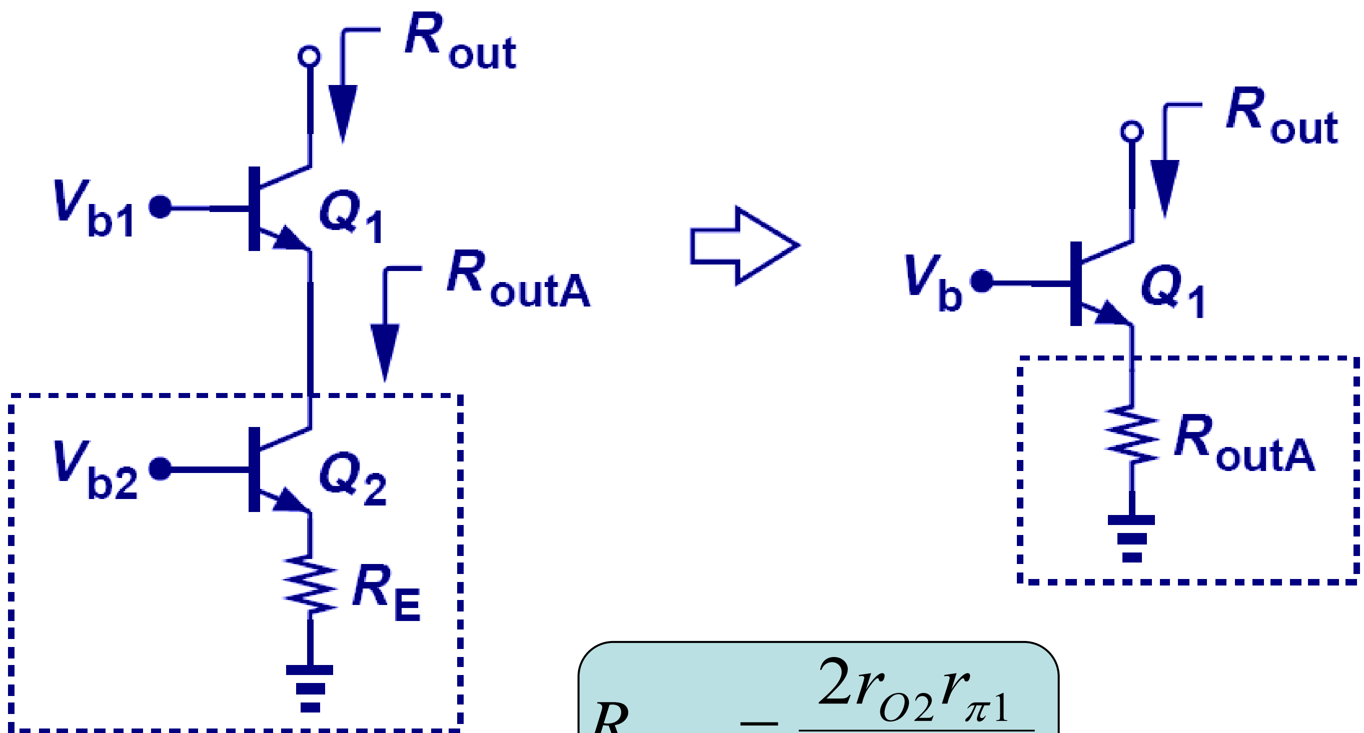


$$R_{out,\max} \approx g_{m1} r_{O1} r_{\pi 1}$$

$$R_{out,\max} \approx \beta_1 r_{O1}$$

- **The maximum output impedance of a bipolar cascode is bounded by the ever-present r_π between emitter and ground of Q_1.**

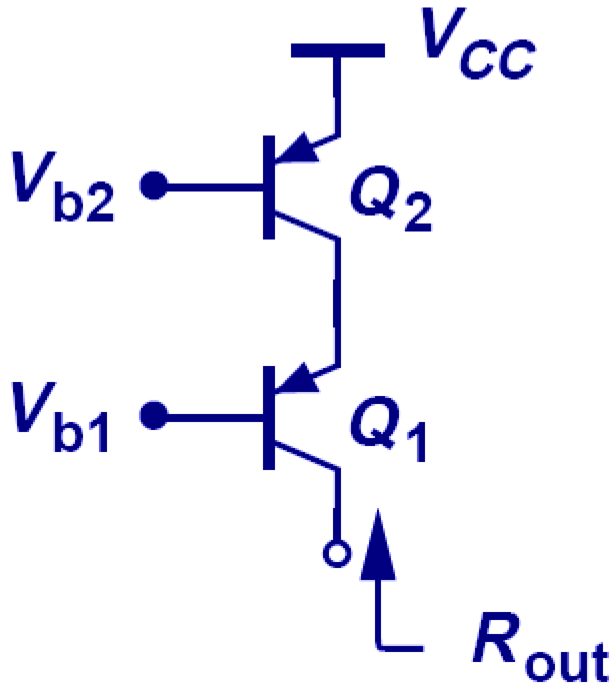
Example: Output Impedance



$$R_{outA} = \frac{2r_{O2}r_{\pi 1}}{r_{\pi 1} - r_{O2}}$$

- **Typically r_π is smaller than r_O, so in general it is impossible to double the output impedance by degenerating Q_2 with a resistor.**

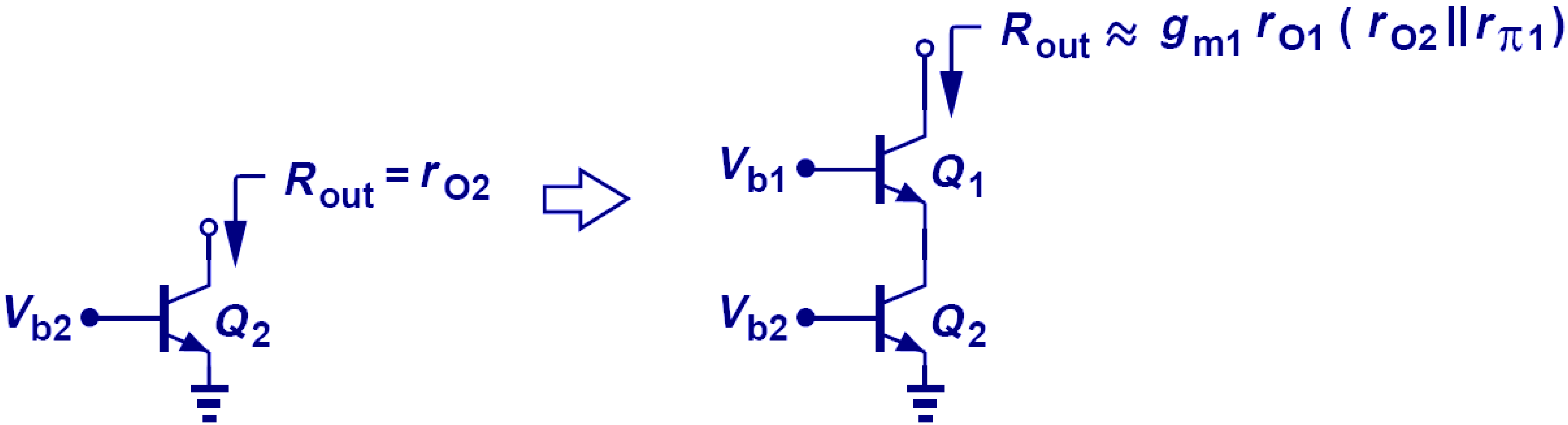
PNP Cascode Stage



$$R_{out} = [1 + g_m (r_{O2} \| r_{\pi 1})] r_{O1} + r_{O2} \| r_{\pi 1}$$

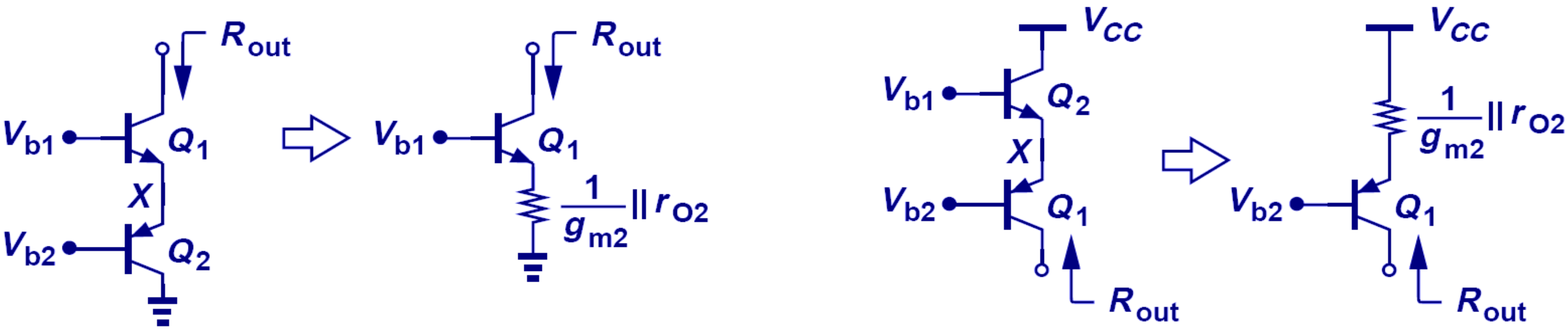
$$R_{out} \approx g_{m1} r_{O1} \left(r_{O2} \| r_{\pi 1} \right)$$

Another Interpretation of Bipolar Cascode



- **Instead of treating cascode as Q_2 degenerating Q_1, we can also think of it as Q_1 stacking on top of Q_2 (current source) to boost Q_2's output impedance.**

False Cascodes



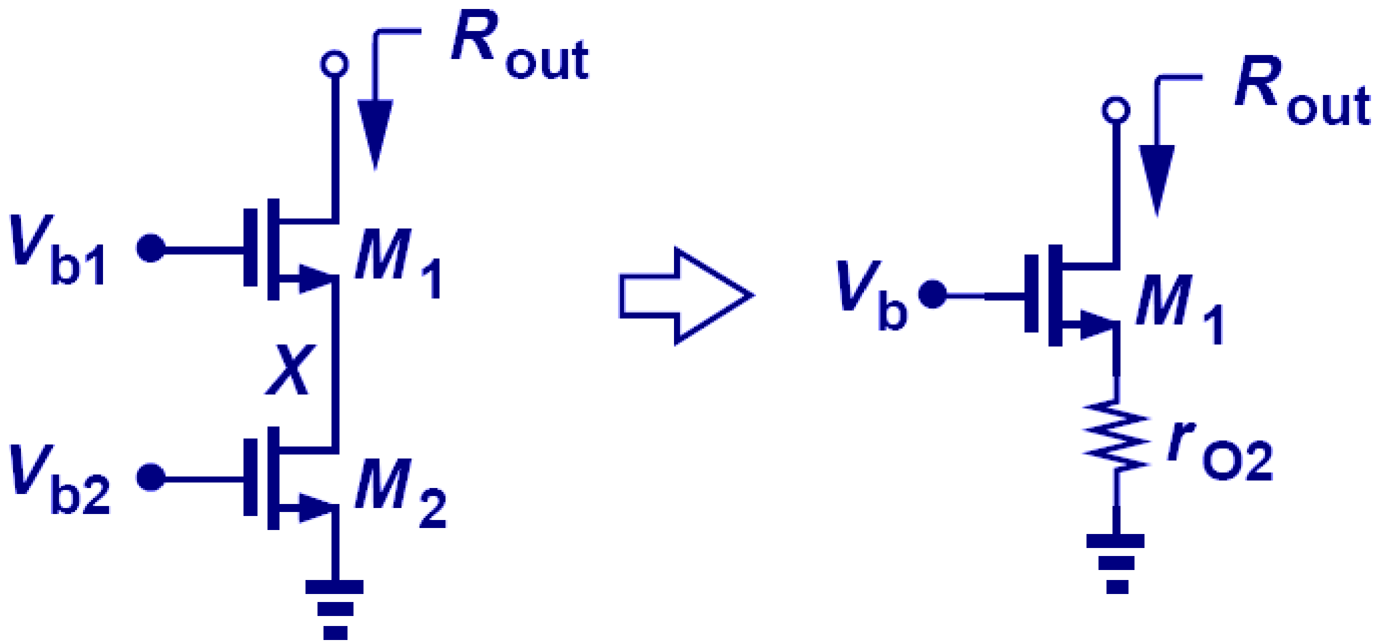
(a) (b)

$$R_{out} = \left[1 + g_{m1}\left(\frac{1}{g_{m2}} \| r_{O2} \| r_{\pi 1}\right)\right] r_{O1} + \frac{1}{g_{m2}} \| r_{O2} \| r_{\pi 1}$$

$$R_{out} \approx \left(1 + \frac{g_{m1}}{g_{m2}}\right) r_{O1} + \frac{1}{g_{m2}} \approx 2 r_{O1}$$

- When the emitter of Q_1 is connected to the emitter of Q_2, it's no longer a cascode since Q_2 becomes a diode-connected device instead of a current source.

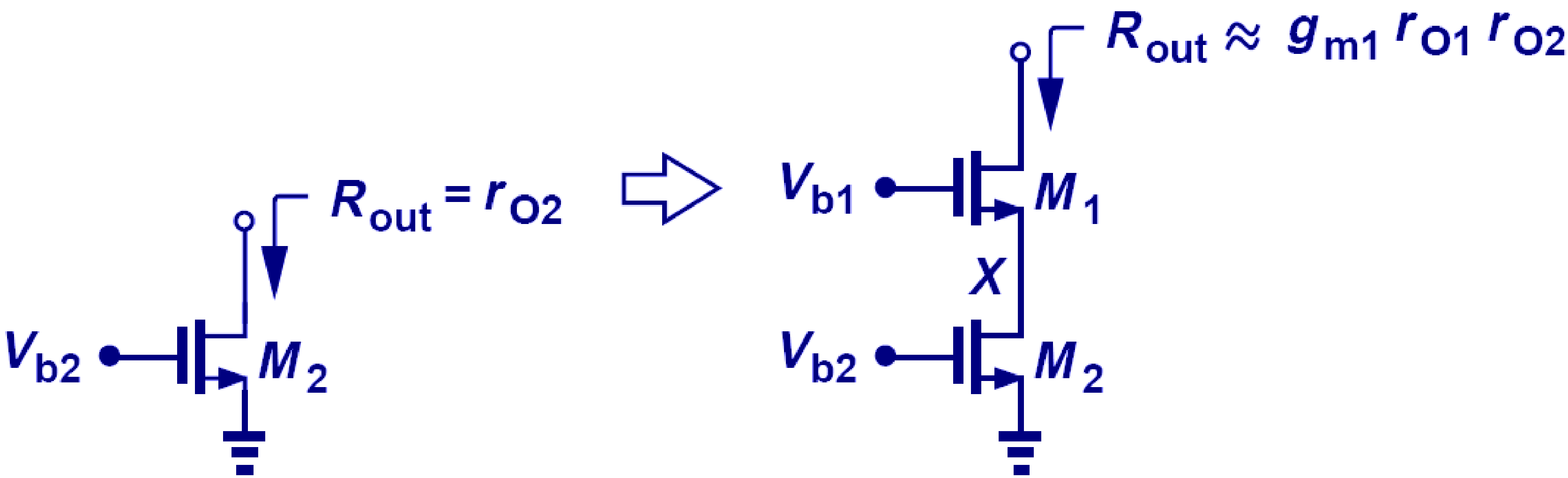
MOS Cascode Stage



$$R_{out} = (1 + g_{m1} r_{O2}) r_{O1} + r_{O2}$$

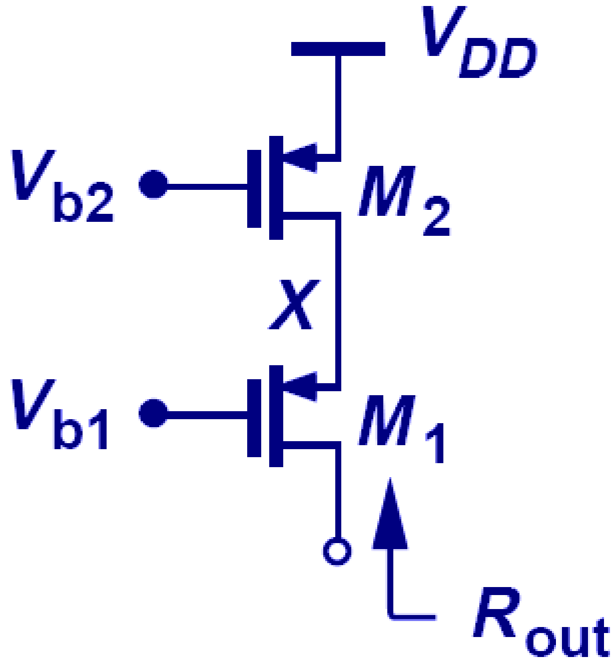
$$R_{out} \approx g_{m1} r_{O1} r_{O2}$$

Another Interpretation of MOS Cascode



- Similar to its bipolar counterpart, MOS cascode can be thought of as stacking a transistor on top of a current source.
- Unlike bipolar cascode, the output impedance is not limited by β.

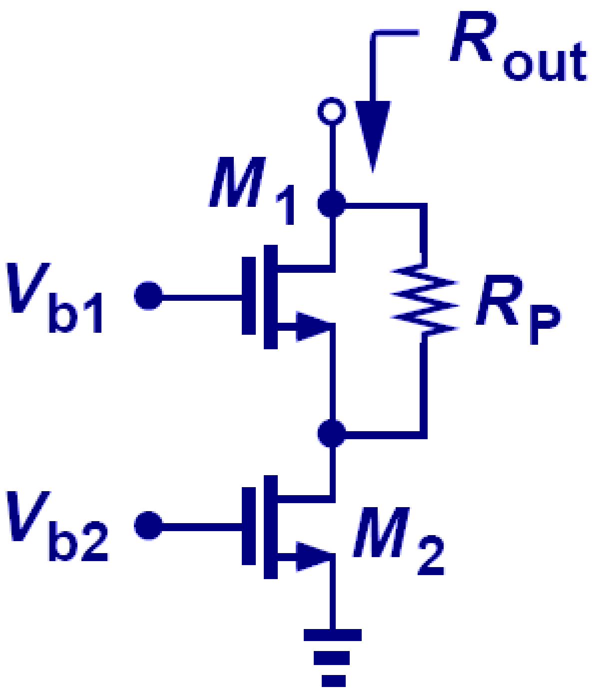
PMOS Cascode Stage



$$R_{out} = (1 + g_{m1} r_{O2}) r_{O1} + r_{O2}$$

$$R_{out} \approx g_{m1} r_{O1} r_{O2}$$

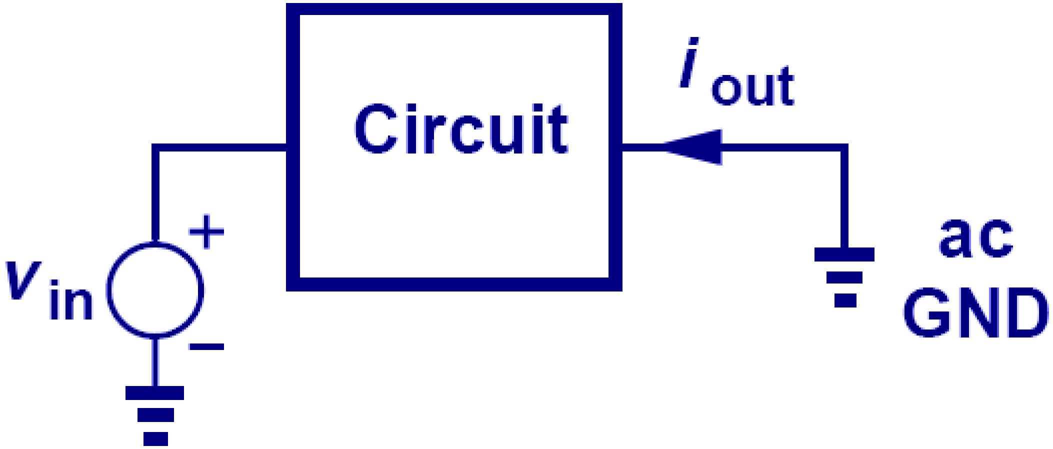
Example: Parasitic Resistance



$$R_{out} = (1 + g_{m1} r_{O2})(r_{O1} \| R_P) + r_{O2}$$

- R_P will lower the output impedance, since its parallel combination with r_{O1} will always be lower than r_{O1}.

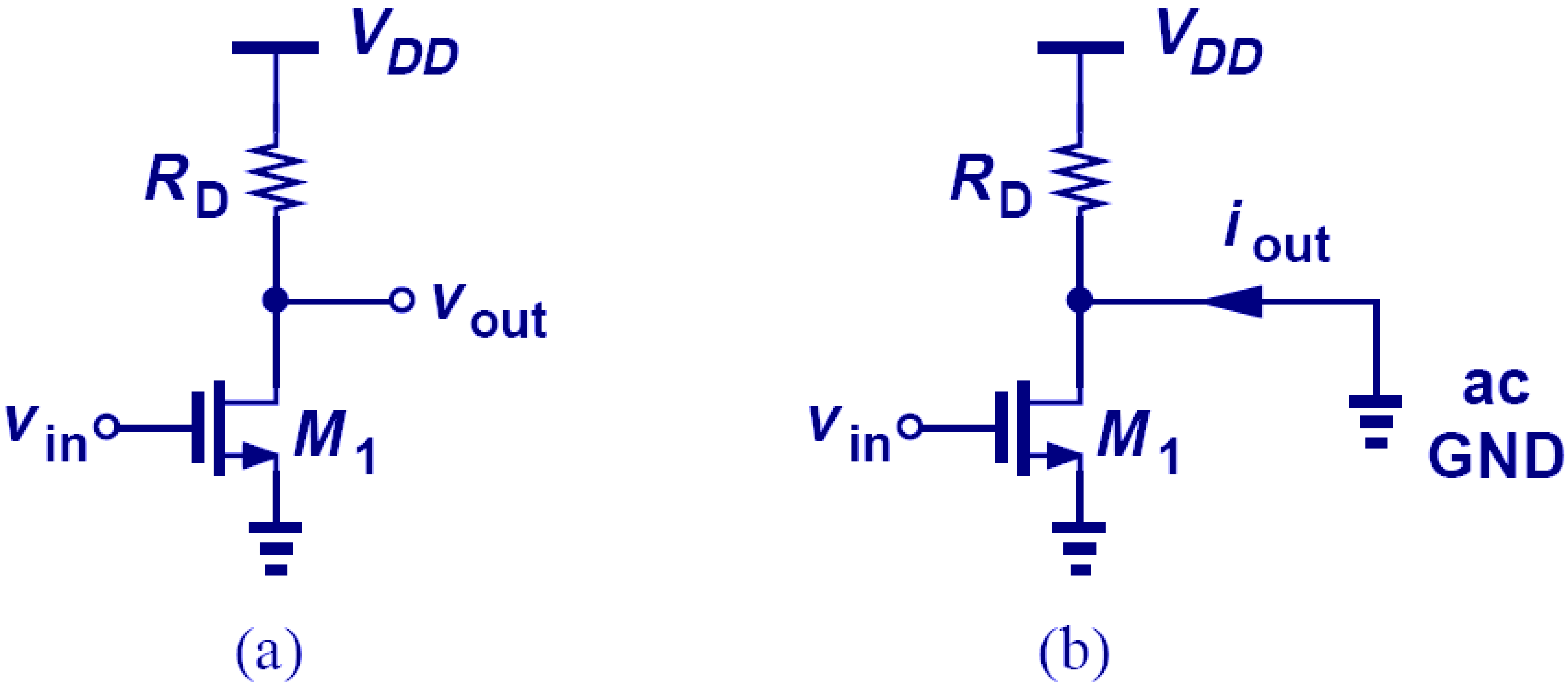
Short-Circuit Transconductance



$$G_m = \left.\frac{i_{out}}{v_{in}}\right|_{v_{out}=0}$$

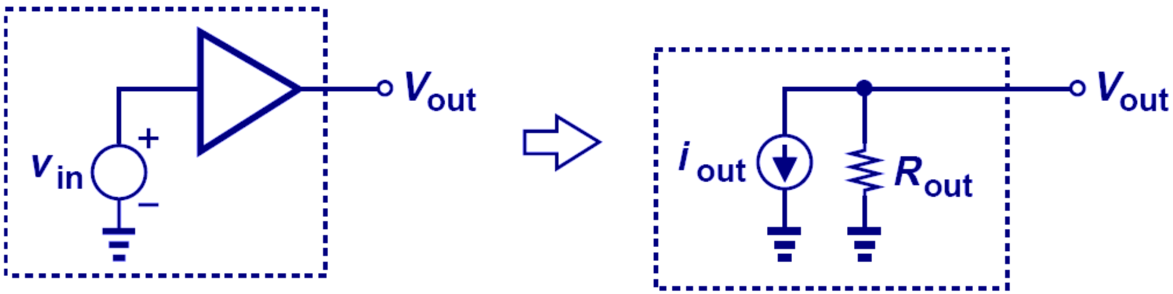
- The short-circuit transconductance of a circuit measures its strength in converting input voltage to output current.

Transconductance Example

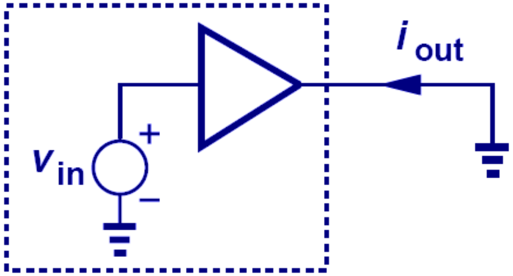


$$G_m = g_{m1}$$

Derivation of Voltage Gain



(a)



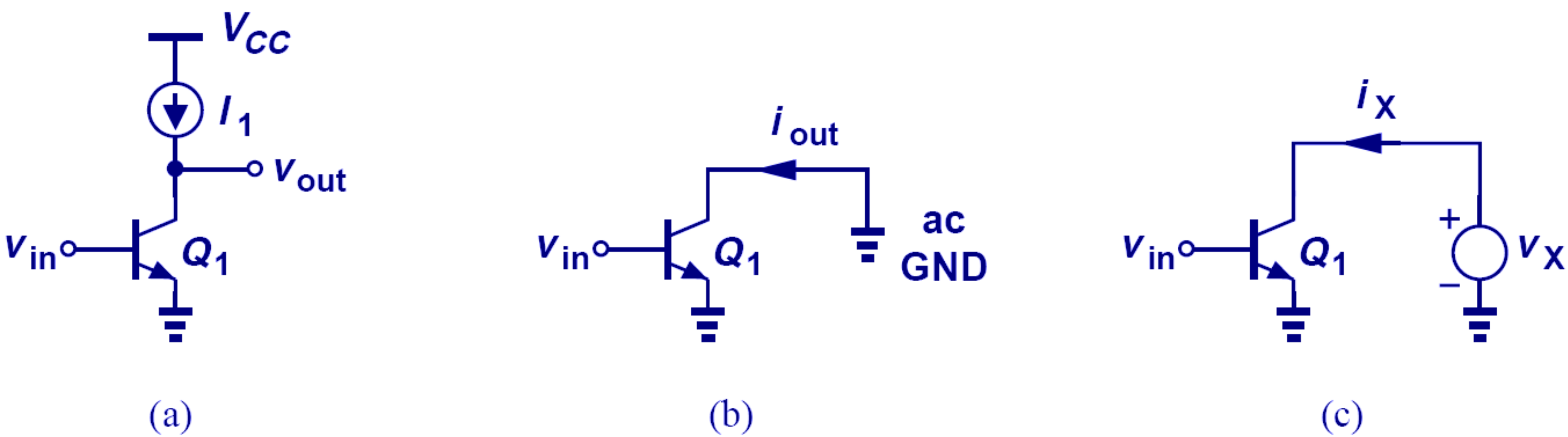
(b)

$$v_{out} = -i_{out} R_{out} = -G_m v_{in} R_{out}$$

$$v_{out}/v_{in} = -G_m R_{out}$$

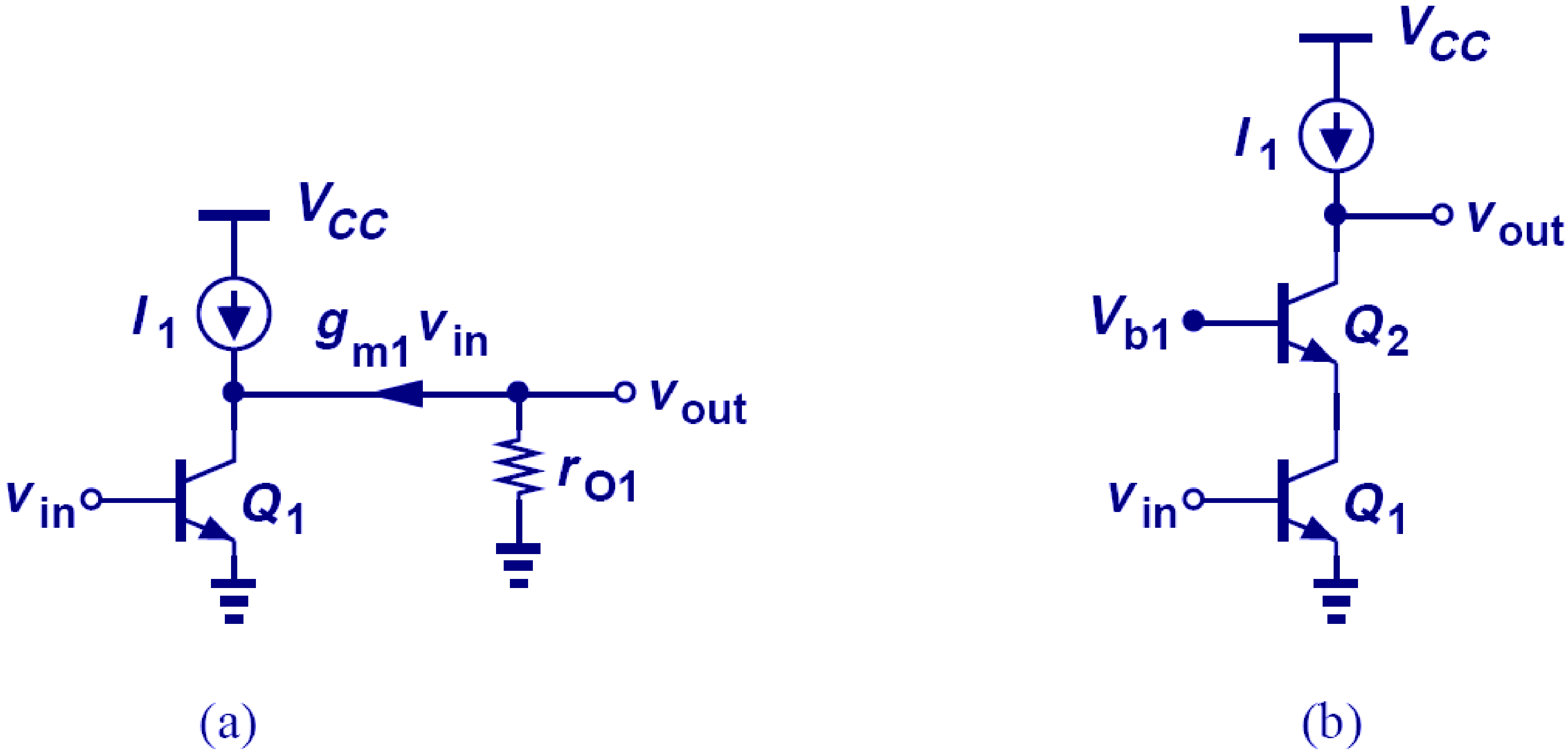
- **By representing a linear circuit with its Norton equivalent, the relationship between V_{out} and V_{in} can be expressed by the product of G_m and R_{out}.**

Example: Voltage Gain



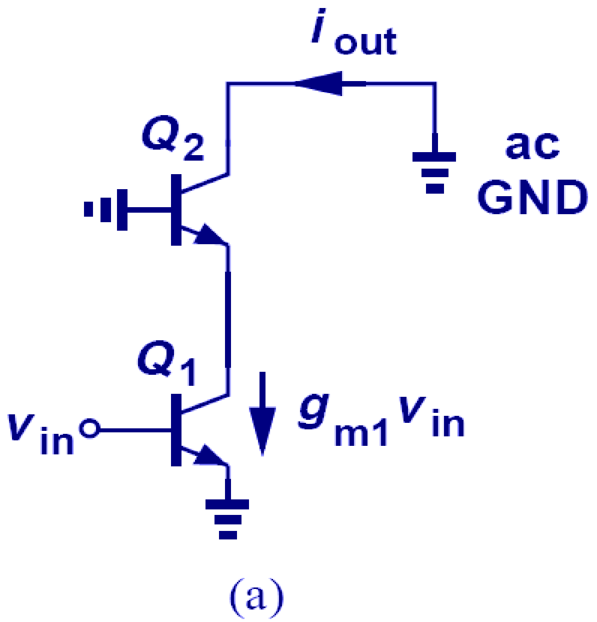
$$A_v = -g_{m1} r_{O1}$$

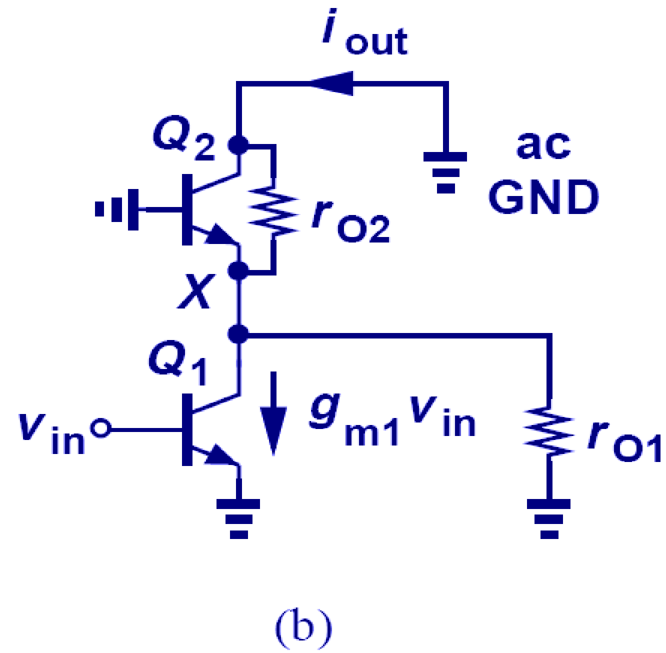
Comparison between Bipolar Cascode and CE Stage



- Since the output impedance of bipolar cascode is higher than that of the CE stage, we would expect its voltage gain to be higher as well.

Voltage Gain of Bipolar Cascode Amplifier



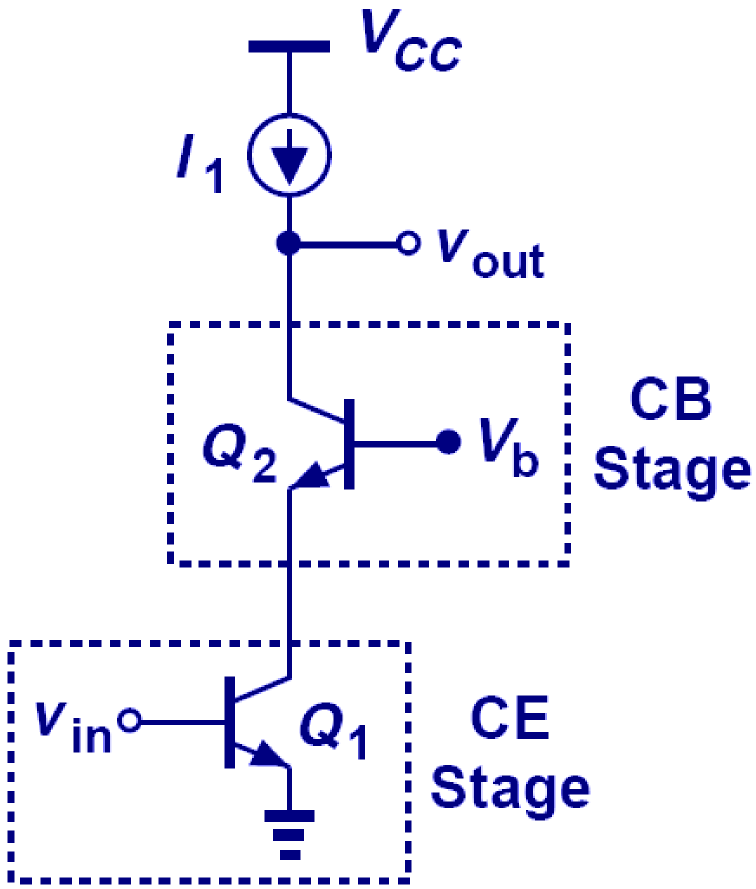


$$G_m \approx g_{m1}$$

$$A_v \approx -g_{m1} r_{O1} g_{m1} (r_{O1} \| r_{\pi 2})$$

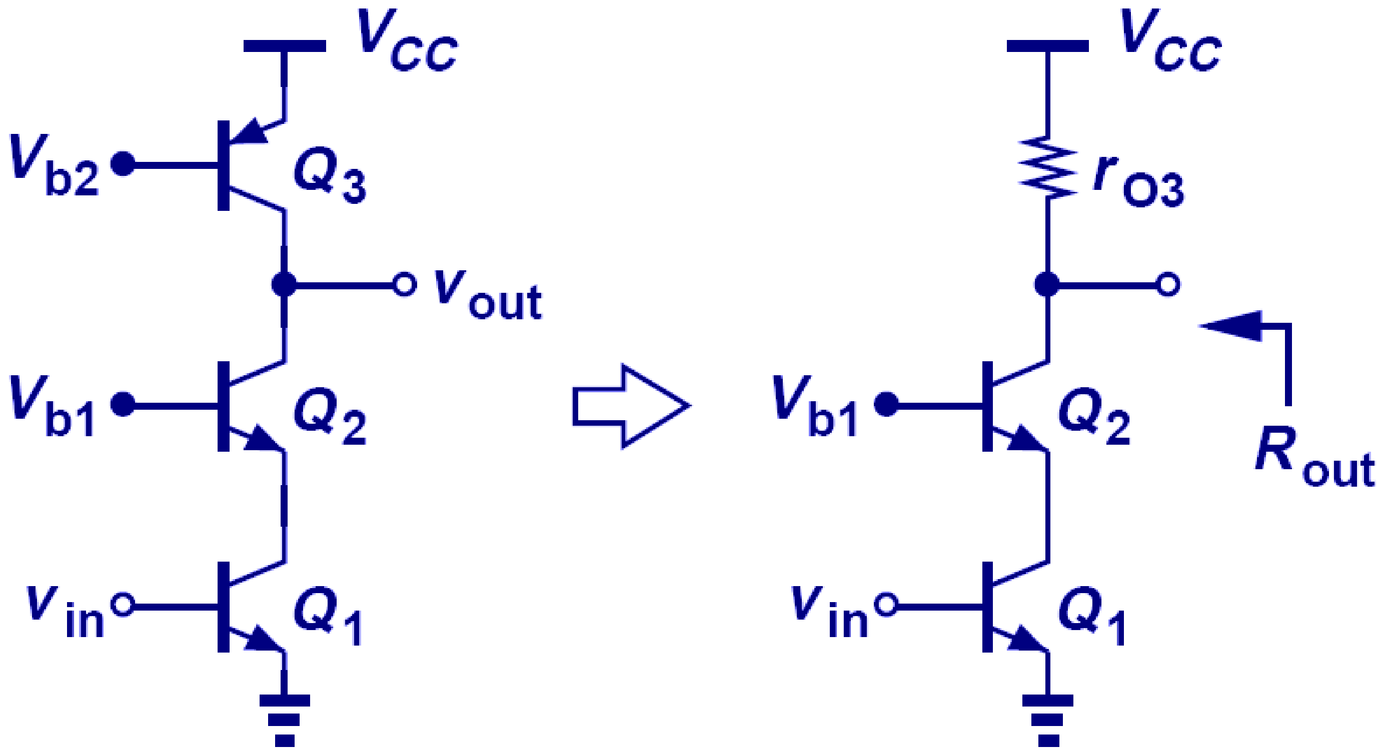
- Since r_O is much larger than $1/g_m$, most of $I_{C,Q1}$ flows into the diode-connected Q_2. Using R_{out} as before, A_V is easily calculated.

Alternate View of Cascode Amplifier



- A bipolar cascode amplifier is also a CE stage in series with a CB stage.

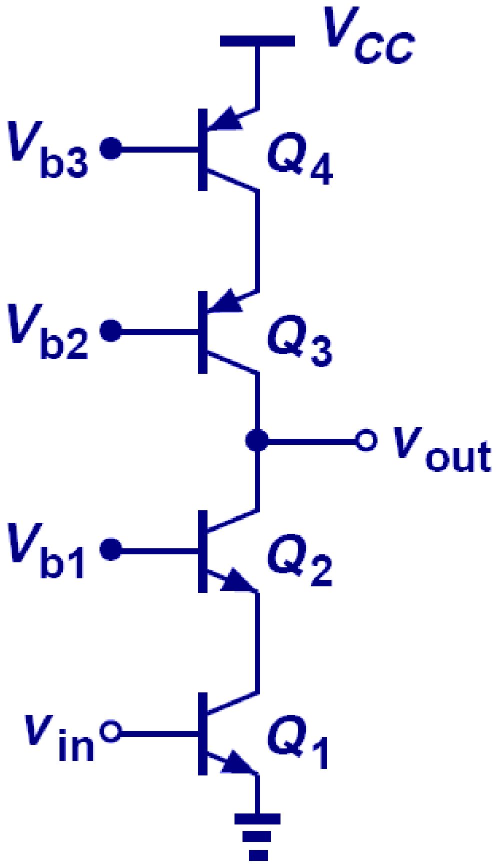
Practical Cascode Stage



$$R_{out} \approx r_{O3} \parallel g_{m2} r_{O2}(r_{O1} \parallel r_{\pi 2})$$

- Since no current source can be ideal, the output impedance drops.

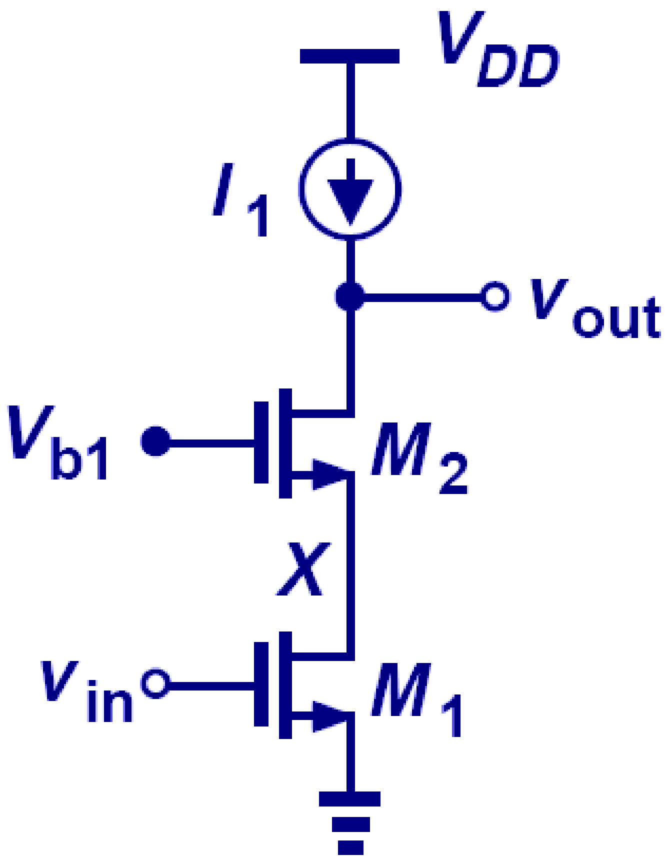
Improved Cascode Stage



$$R_{out} \approx g_{m3} r_{O3} (r_{O4} \parallel r_{\pi 3}) \parallel g_{m2} r_{O2} (r_{O1} \parallel r_{\pi 2})$$

- In order to preserve the high output impedance, a cascode PNP current source is used.

MOS Cascode Amplifier

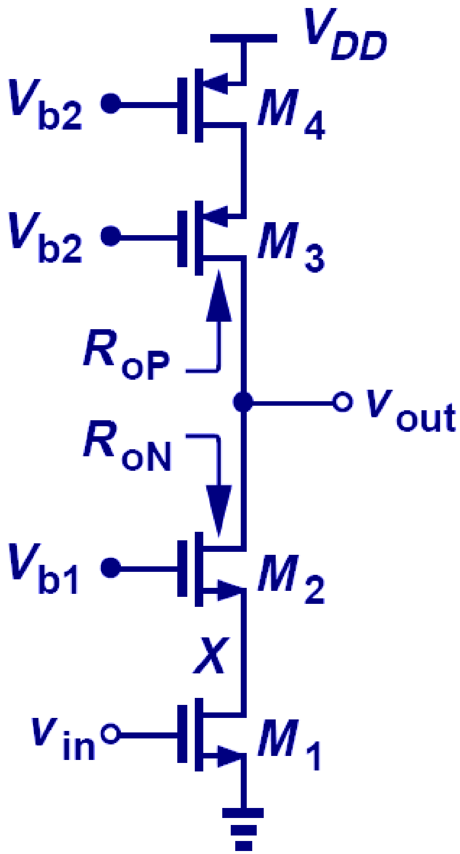


$$A_v = -G_m R_{out}$$

$$A_v \approx -g_{m1}\left[(1 + g_{m2} r_{O2}) r_{O1} + r_{O2}\right]$$

$$A_v \approx -g_{m1} r_{O1} g_{m2} r_{O2}$$

Improved MOS Cascode Amplifier



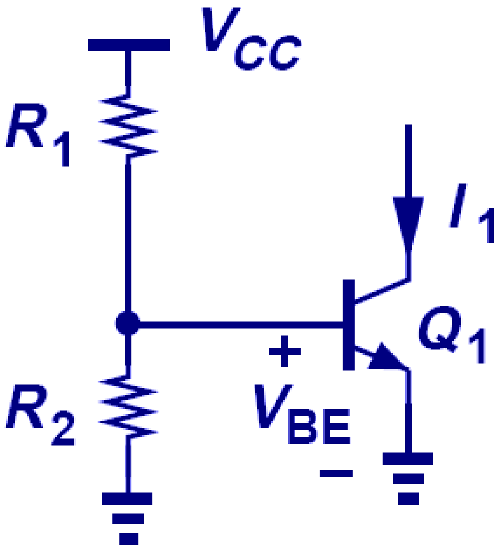
$$R_{on} \approx g_{m2} r_{O2} r_{O1}$$

$$R_{op} \approx g_{m3} r_{O3} r_{O4}$$

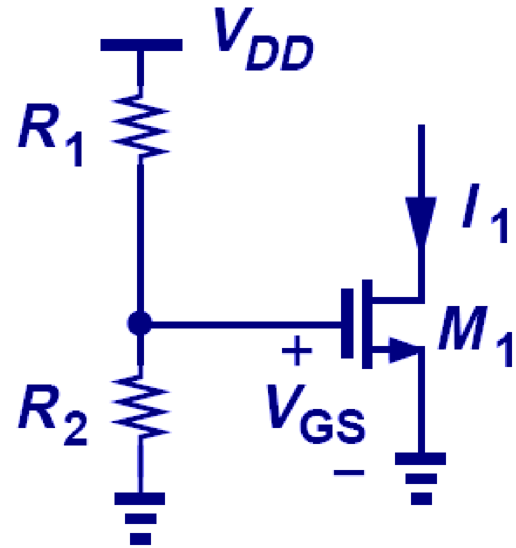
$$R_{out} = R_{on} \parallel R_{op}$$

- **Similar to its bipolar counterpart, the output impedance of a MOS cascode amplifier can be improved by using a PMOS cascode current source.**

Temperature and Supply Dependence of Bias Current



(a)



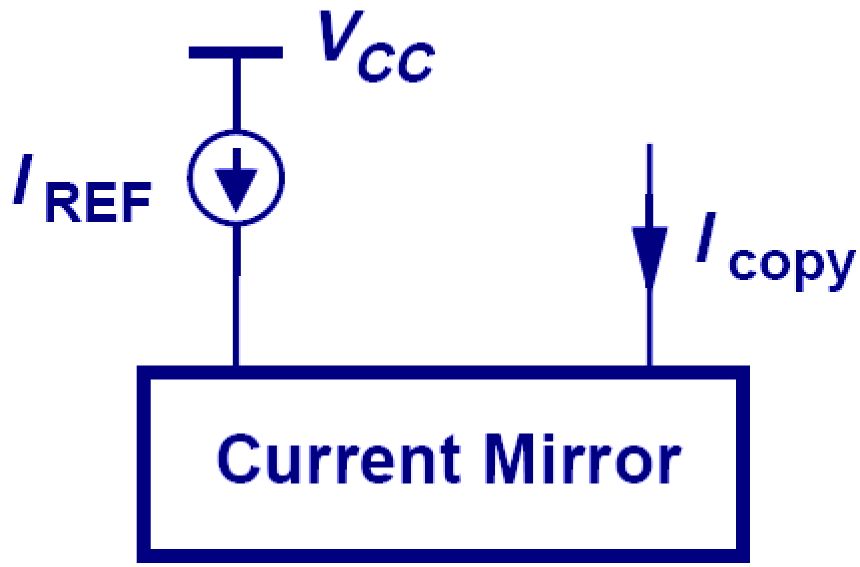
(b)

$$R_2 V_{CC}/(R_1+R_2) = V_T \ln(I_1/I_S)$$

$$I_1 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(\frac{R_2}{R_1+R_2}V_{DD} - V_{TH}\right)^2$$

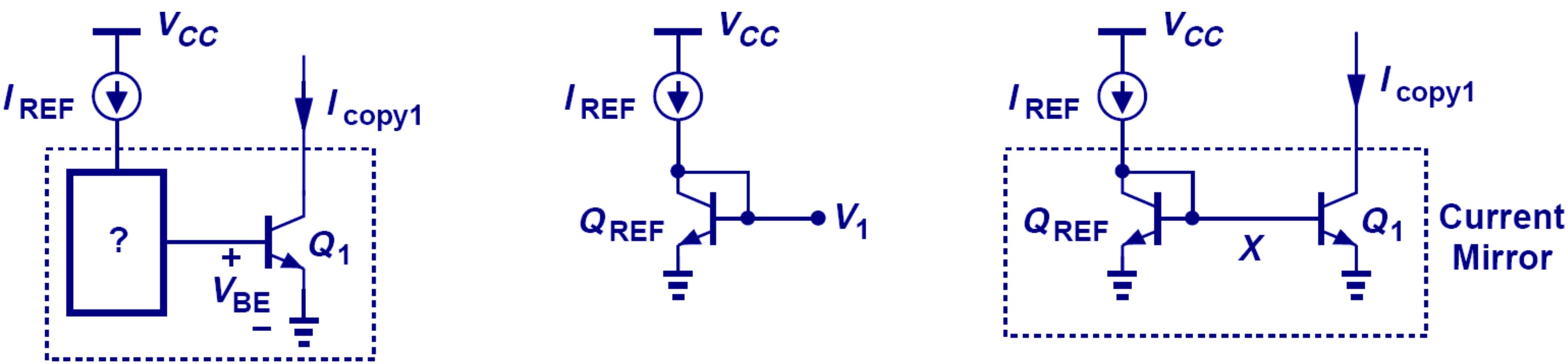
- Since V_T, I_S, μ_n, and V_{TH} all depend on temperature, I_1 for both bipolar and MOS depends on temperature and supply.

Concept of Current Mirror



- **The motivation behind a current mirror is to sense the current from a "golden current source" and duplicate this "golden current" to other locations.**

Bipolar Current Mirror Circuitry

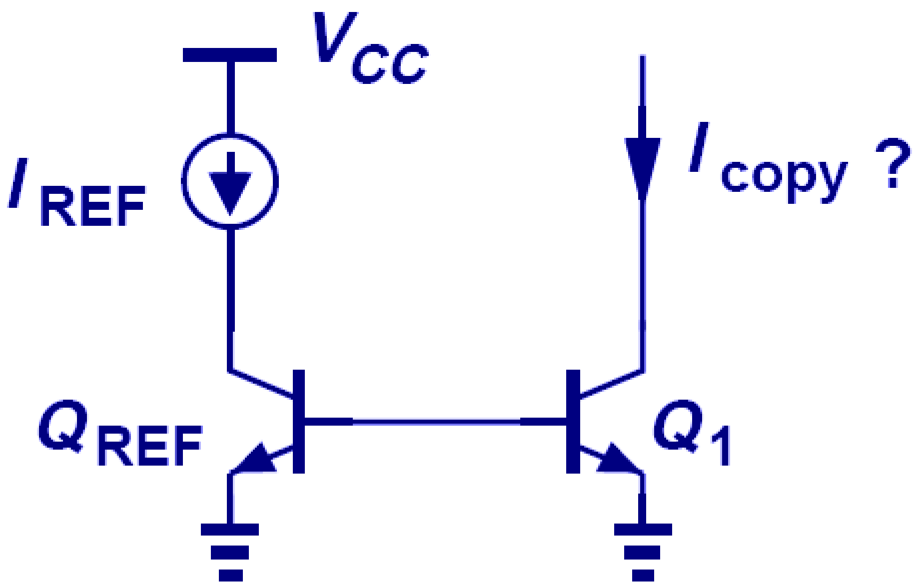


(a) (b) (c)

$$I_{copy} = \frac{I_{S1}}{I_{S,REF}} I_{REF}$$

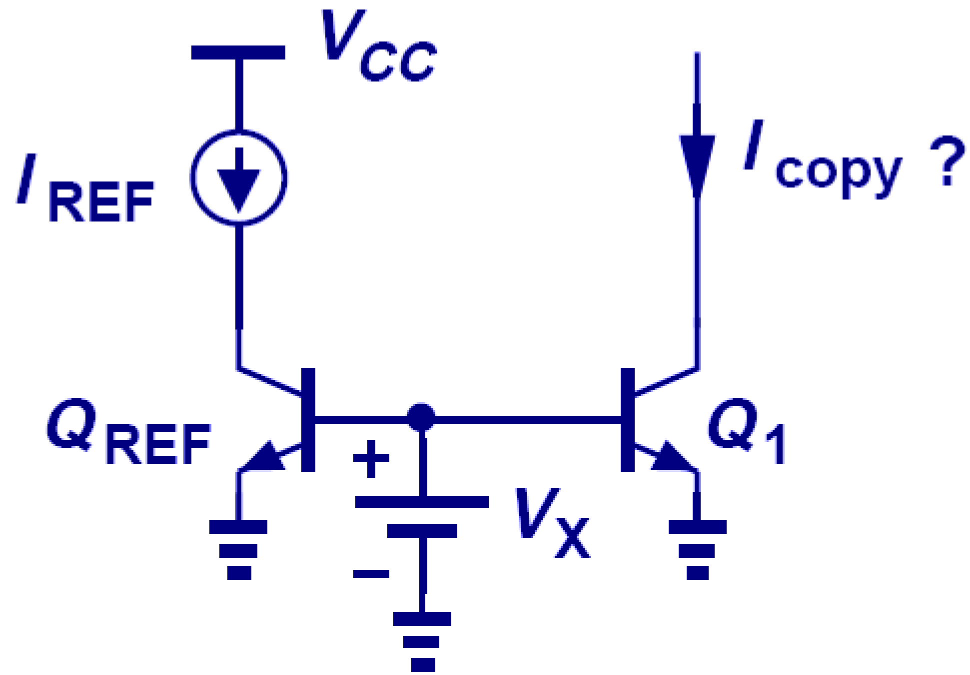
- The diode-connected Q_{REF} produces an output voltage V_1 that forces $I_{copy1} = I_{REF}$, if $Q_1 = Q_{REF}$.

Bad Current Mirror Example I



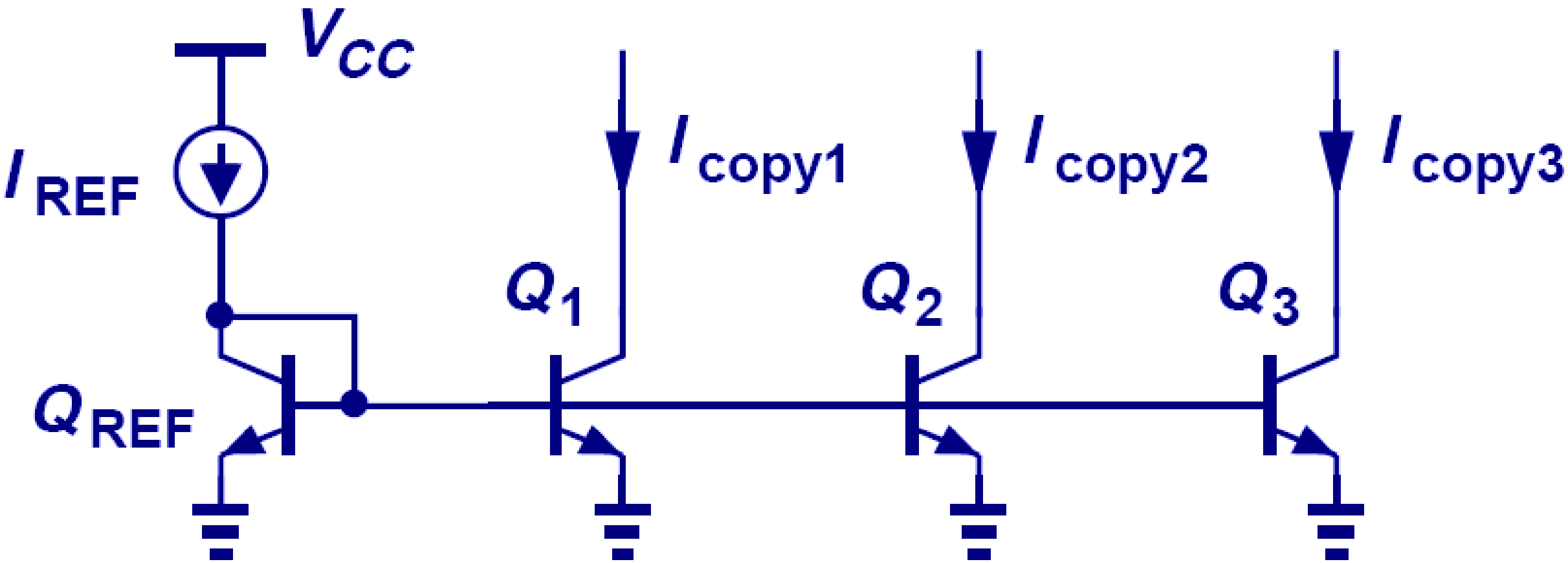
- **Without shorting the collector and base of Q_{REF} together, there will not be a path for the base currents to flow, therefore, I_{copy} is zero.**

Bad Current Mirror Example II



- Although a path for base currents exists, this technique of biasing is no better than resistive divider.

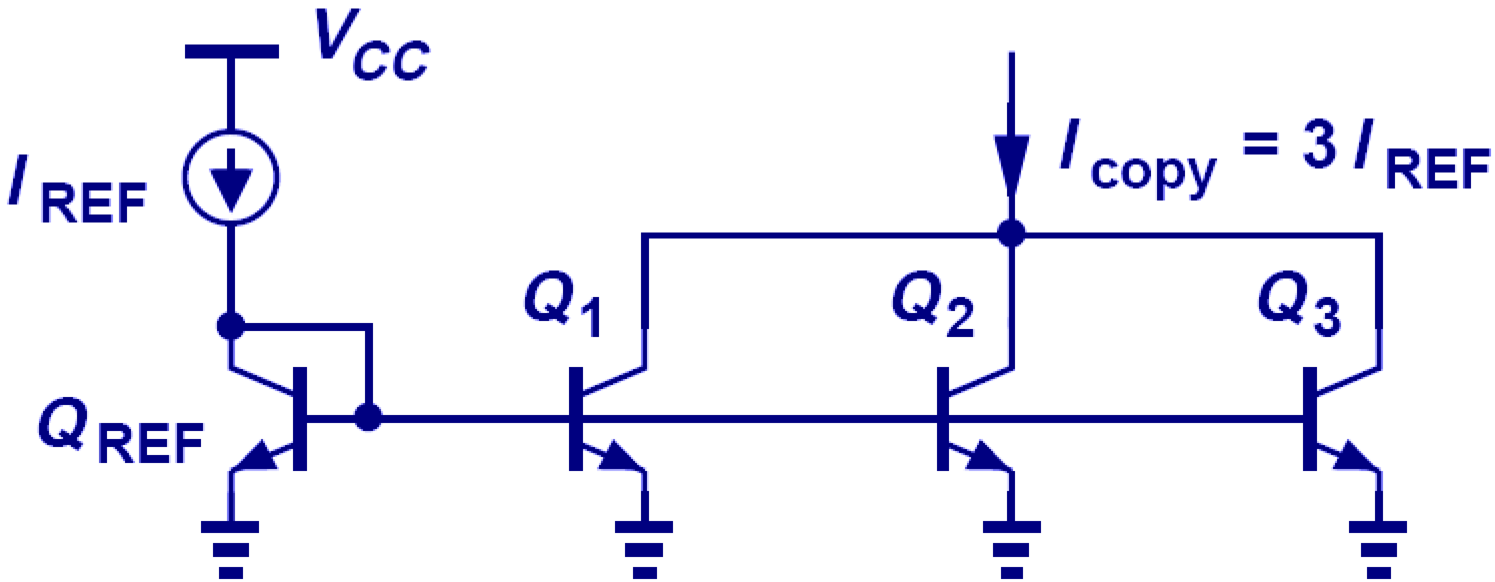
Multiple Copies of I_{REF}



$$I_{copy,j} = \frac{I_{S,j}}{I_{S,REF}} I_{REF}$$

- Multiple copies of I_{REF} can be generated at different locations by simply applying the idea of current mirror to more transistors.

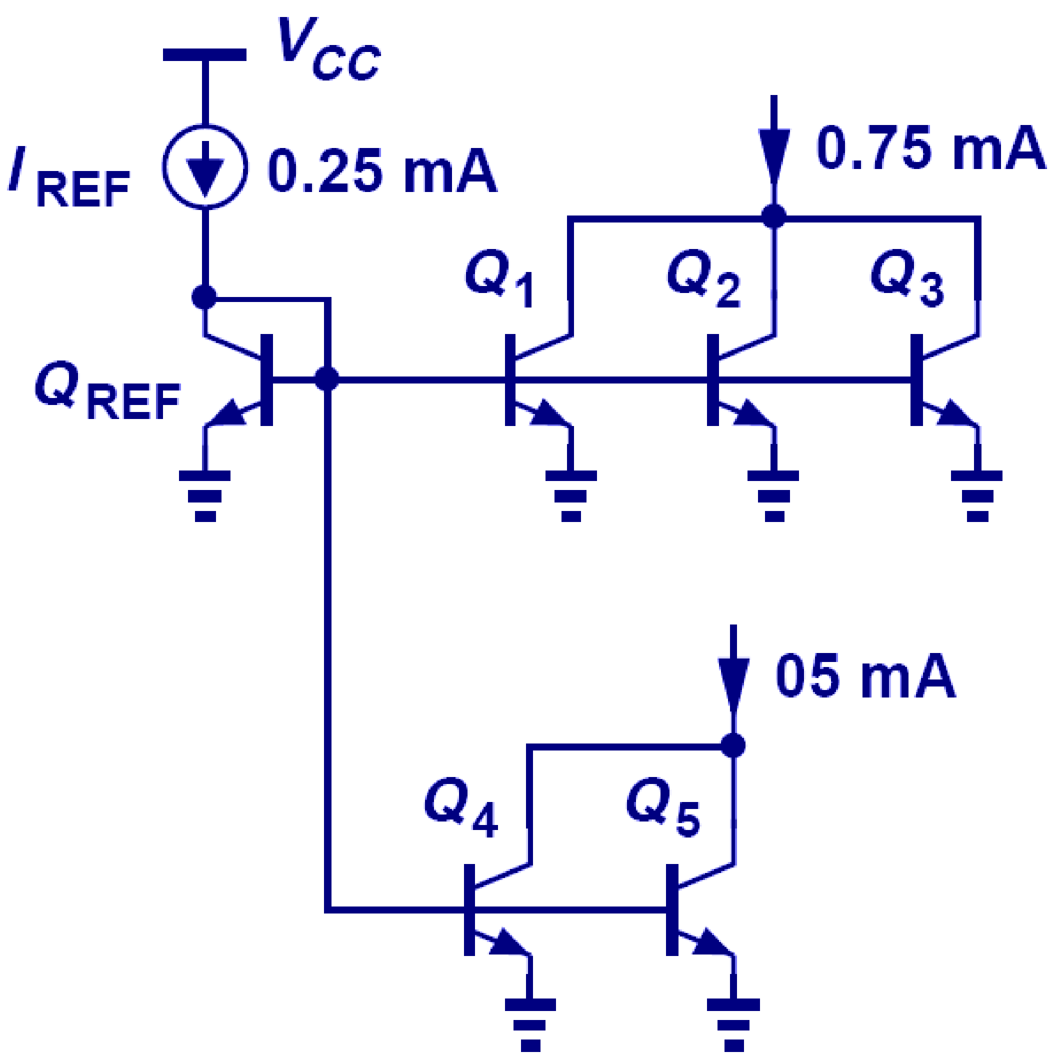
Current Scaling



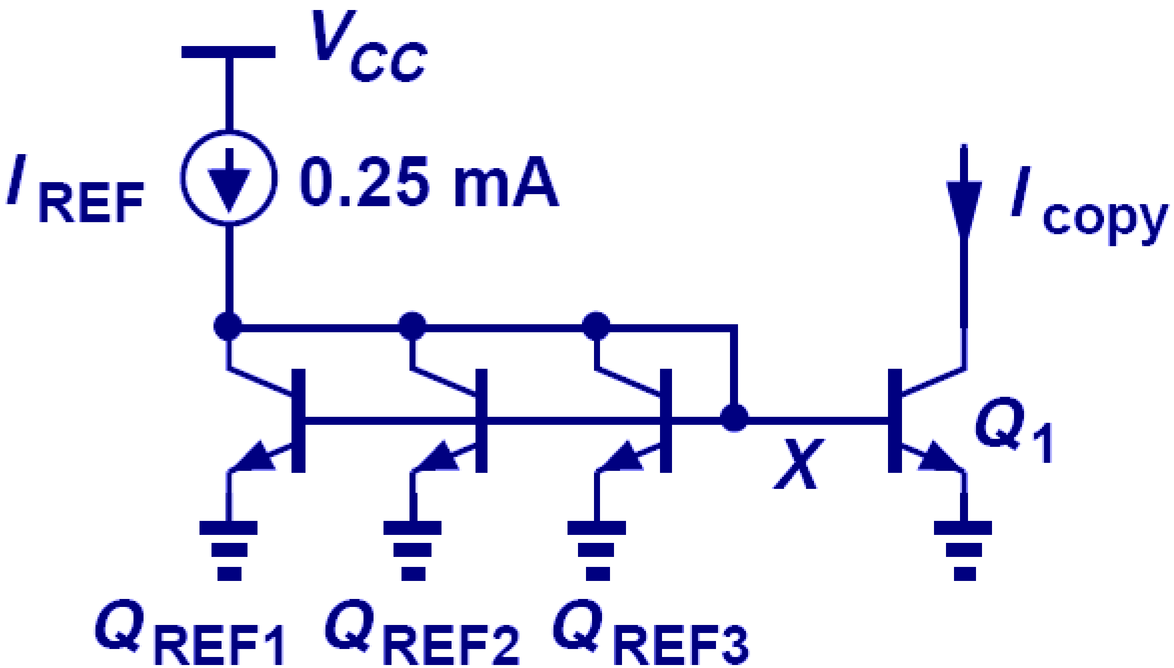
$$I_{copy,j} = nI_{REF}$$

- By scaling the emitter area of Q_j n times with respect to Q_{REF}, $I_{copy,j}$ is also n times larger than I_{REF}. This is equivalent to placing n unit-size transistors in parallel.

Example: Scaled Current



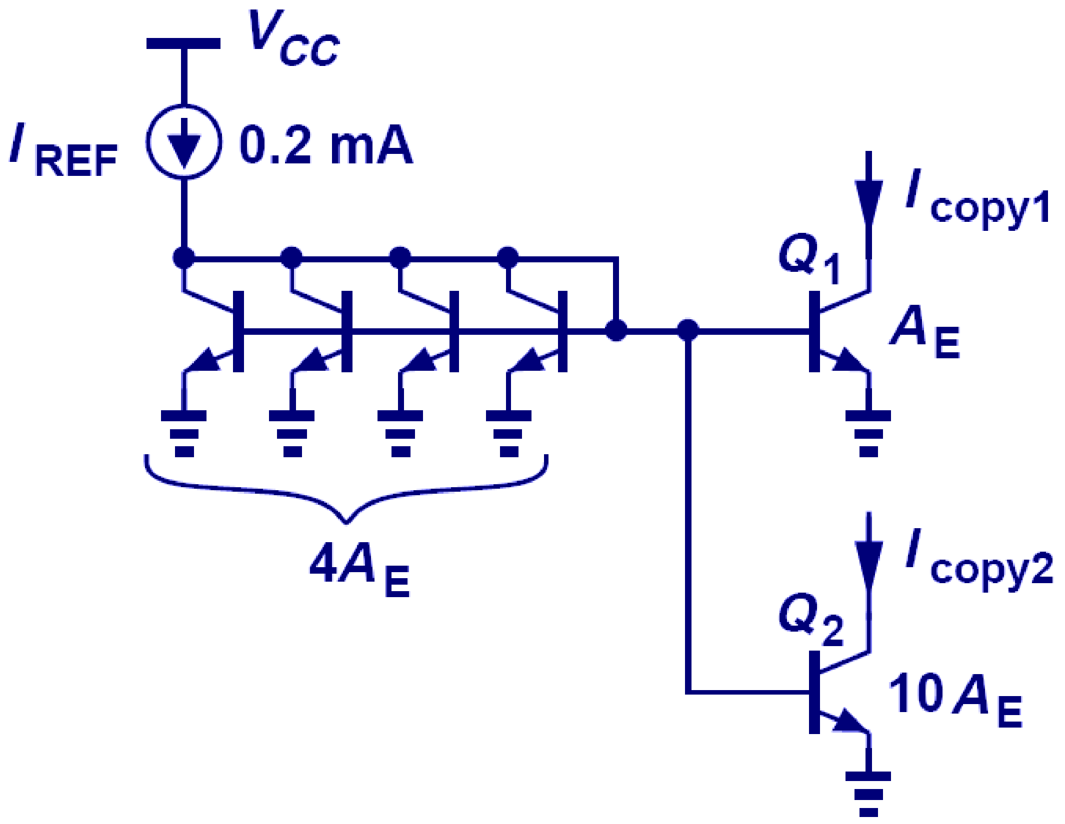
Fractional Scaling



$$I_{copy} = \frac{1}{3} I_{REF}$$

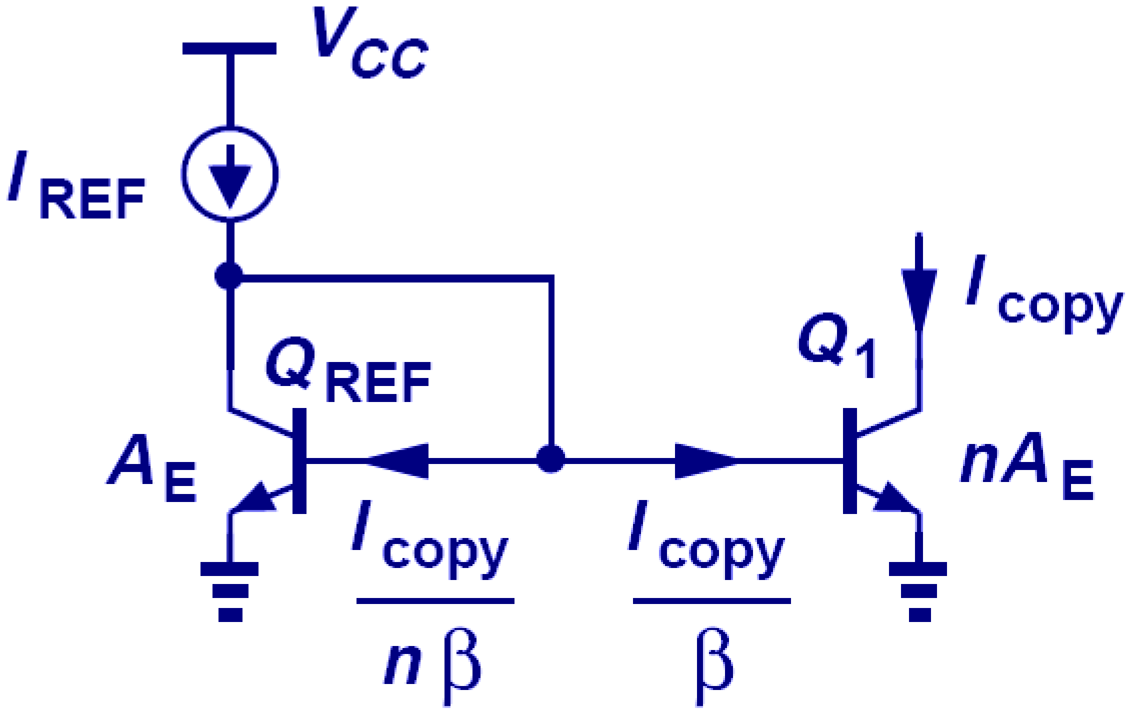
- A fraction of I_{REF} can be created on Q_1 by scaling up the emitter area of Q_{REF}.

Example: Different Mirroring Ratio



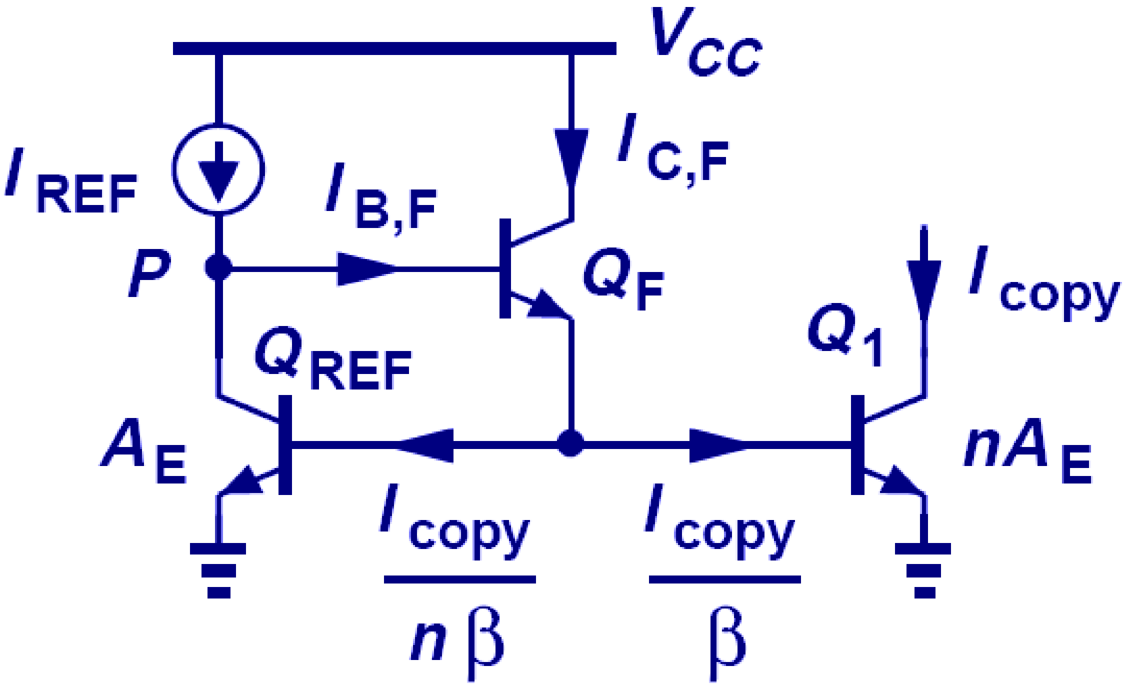
- Using the idea of current scaling and fractional scaling, I_{copy2} is 0.5mA and I_{copy1} is 0.05mA respectively. All coming from a source of 0.2mA.

Mirroring Error Due to Base Currents



$$I_{copy} = \frac{nI_{REF}}{1 + \frac{1}{\beta}(n+1)}$$

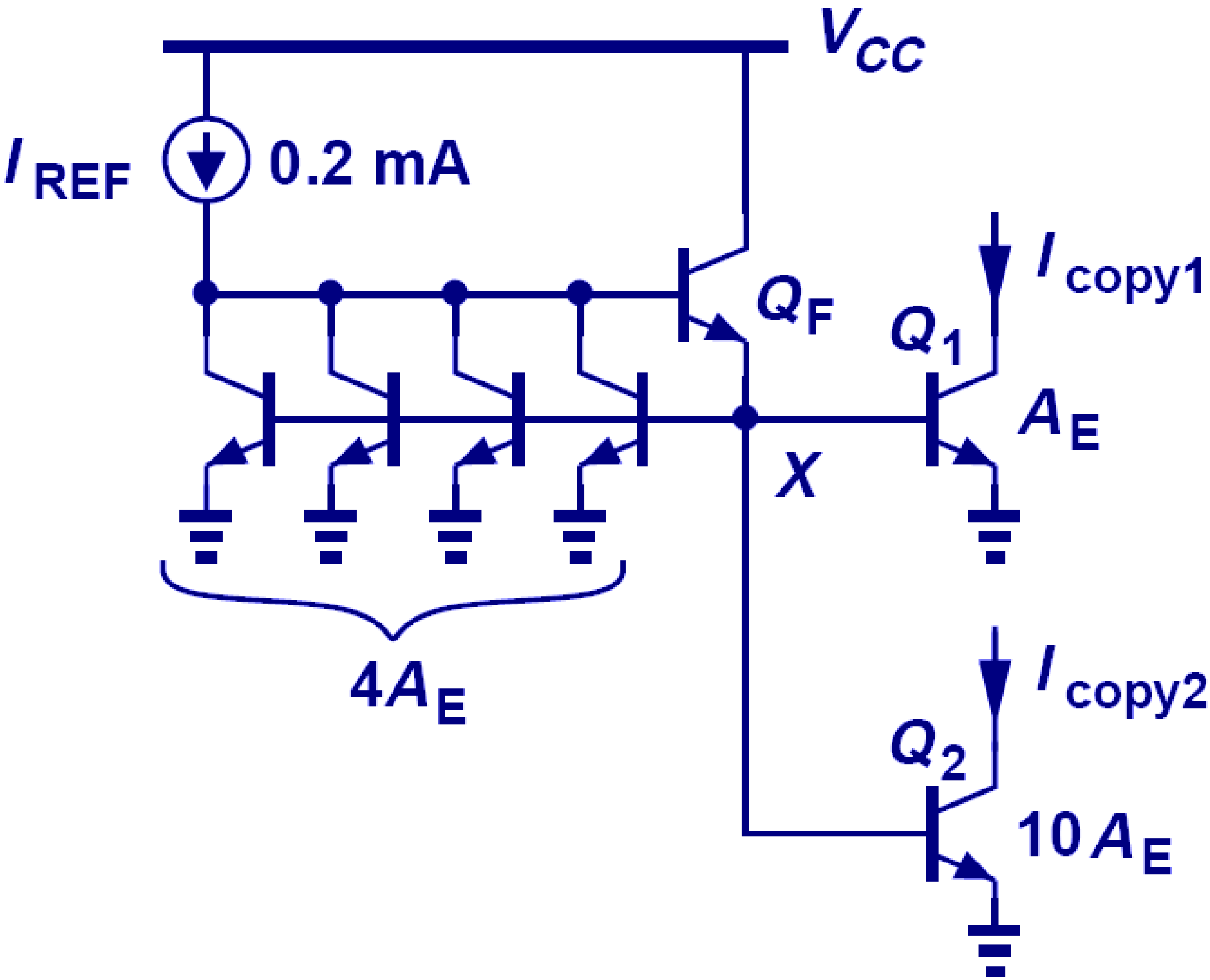
Improved Mirroring Accuracy



$$I_{copy} = \frac{nI_{REF}}{1 + \frac{1}{\beta^2}(n+1)}$$

- Because of Q_F, the base currents of Q_{REF} and Q_1 are mostly supplied by Q_F rather than I_{REF}. Mirroring error is reduced β times.

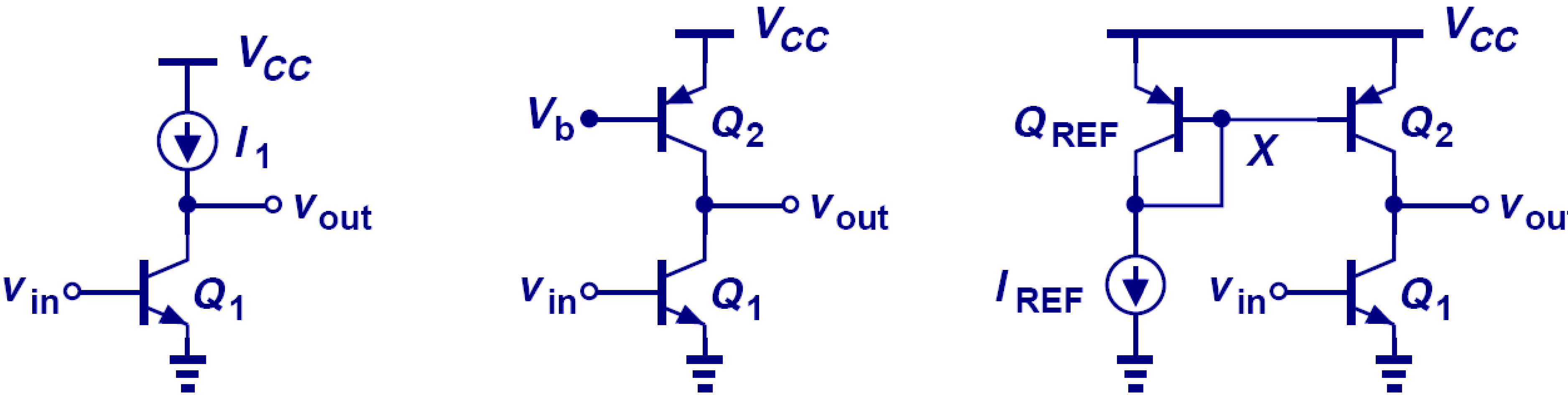
Example: Different Mirroring Ratio Accuracy



$$I_{copy1} = \frac{I_{REF}}{4 + \frac{15}{\beta^2}}$$

$$I_{copy2} = \frac{10 I_{REF}}{4 + \frac{15}{\beta^2}}$$

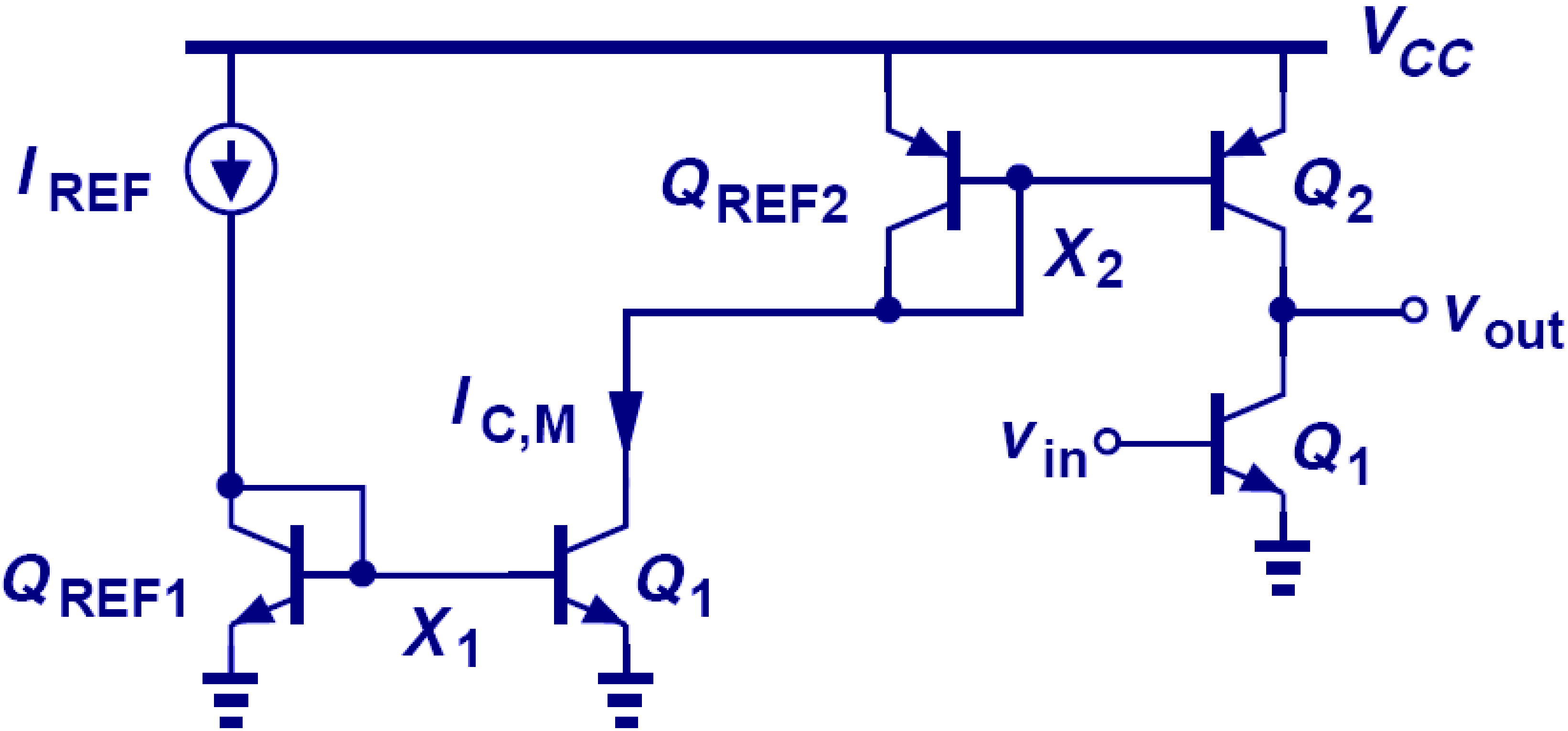
PNP Current Mirror



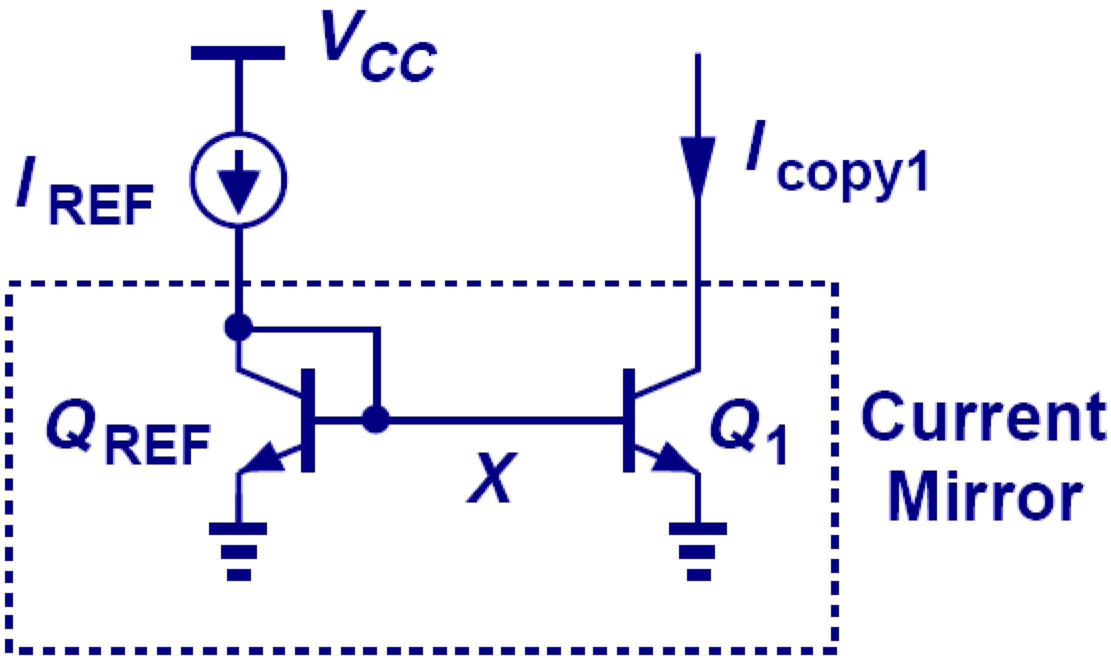
(a) (b) (c)

- PNP current mirror is used as a current source load to an NPN amplifier stage.

Generation of I_{REF} for PNP Current Mirror

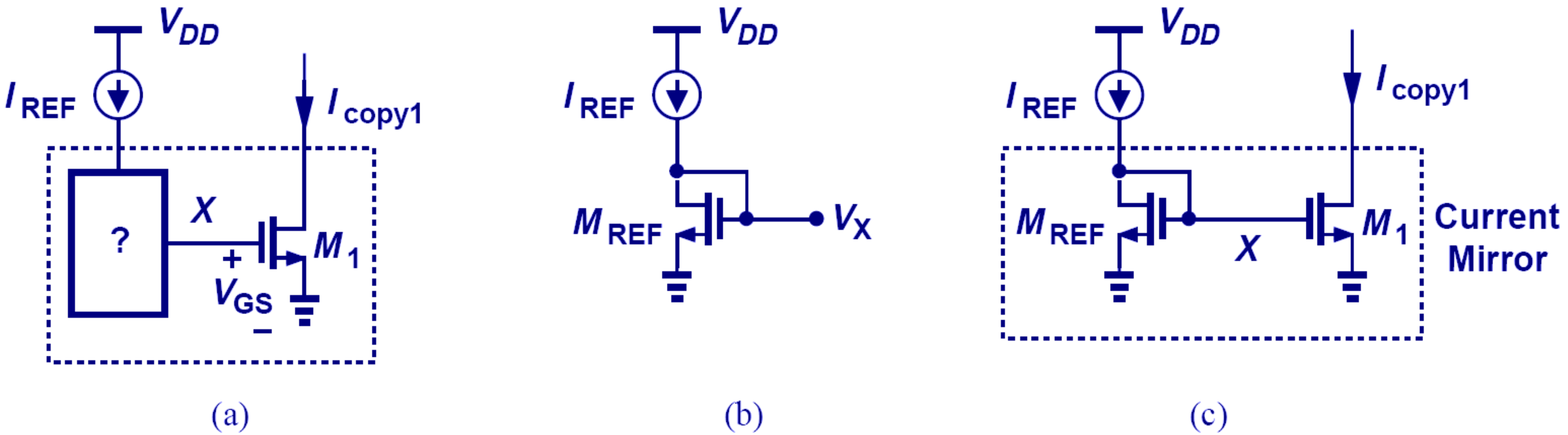


Example: Current Mirror with Discrete Devices



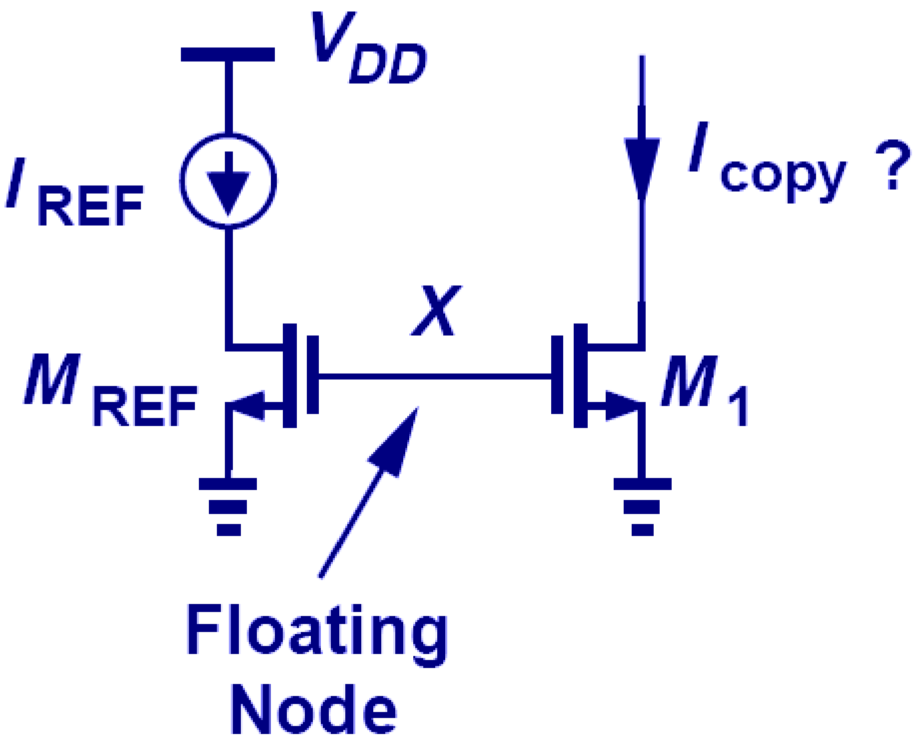
- Let Q_{REF} and Q_1 be discrete NPN devices. I_{REF} and I_{copy1} can vary in large magnitude due to I_S mismatch.

MOS Current Mirror



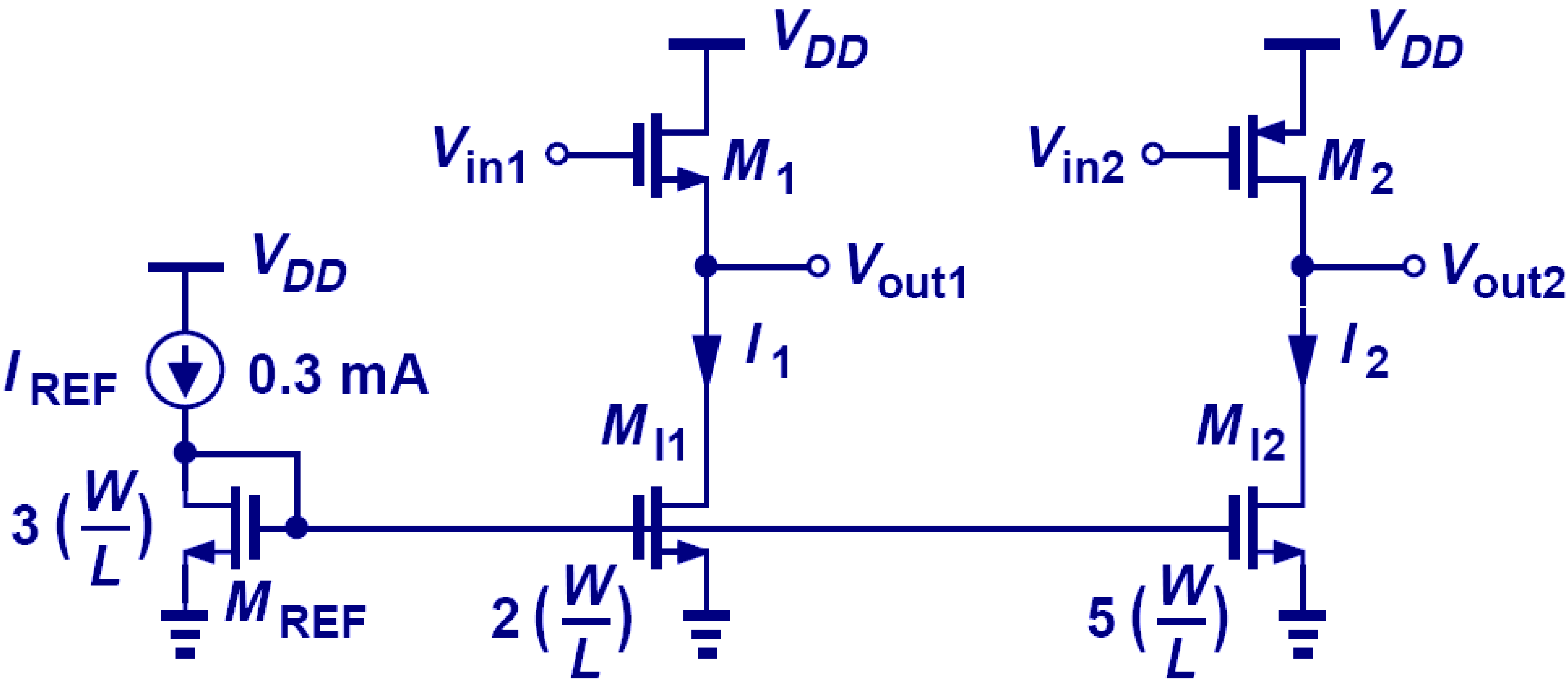
- The same concept of current mirror can be applied to MOS transistors as well.

Bad MOS Current Mirror Example



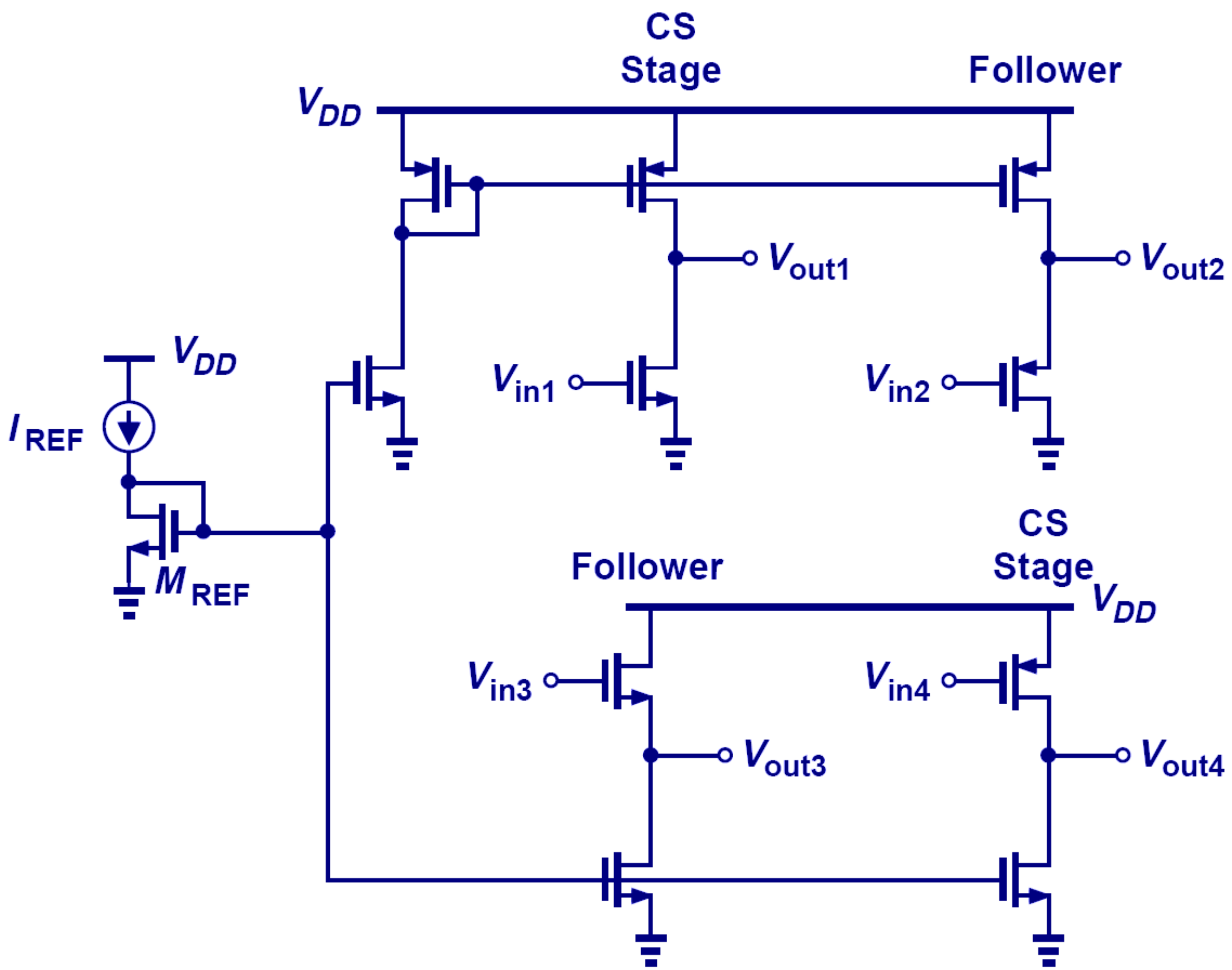
- This is not a current mirror since the relationship between V_X and I_{REF} is not clearly defined.
- The only way to clearly define V_X with I_{REF} is to use a diode-connected MOS since it provides square-law I-V relationship.

Example: Current Scaling



- **Similar to their bipolar counterpart, MOS current mirrors can also scale I_{REF} up or down (I_1 = 0.2mA, I_2 = 0.5mA).**

CMOS Current Mirror

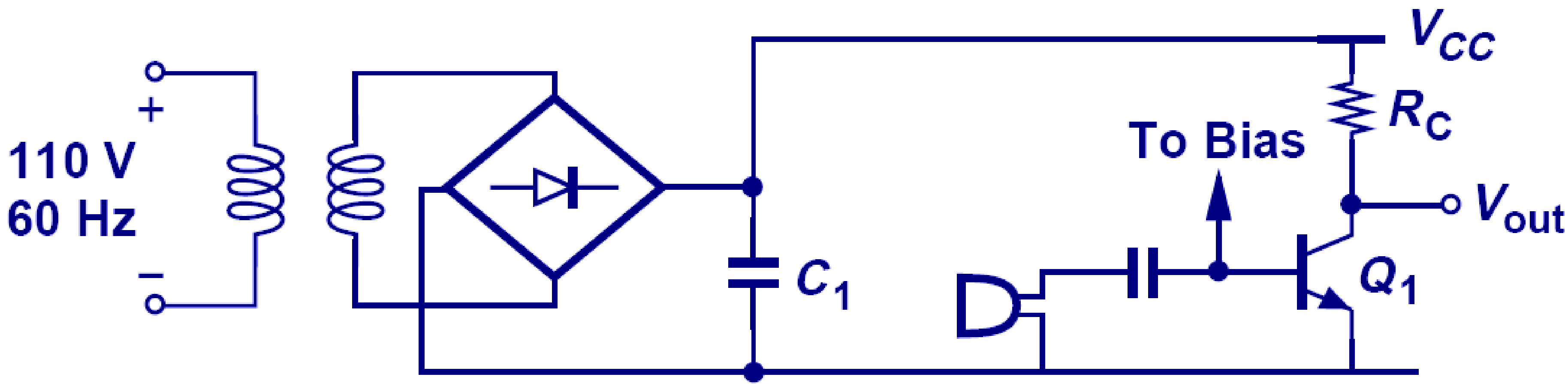


- **The idea of combining NMOS and PMOS to produce CMOS current mirror is shown above.**

Chapter 10 Differential Amplifiers

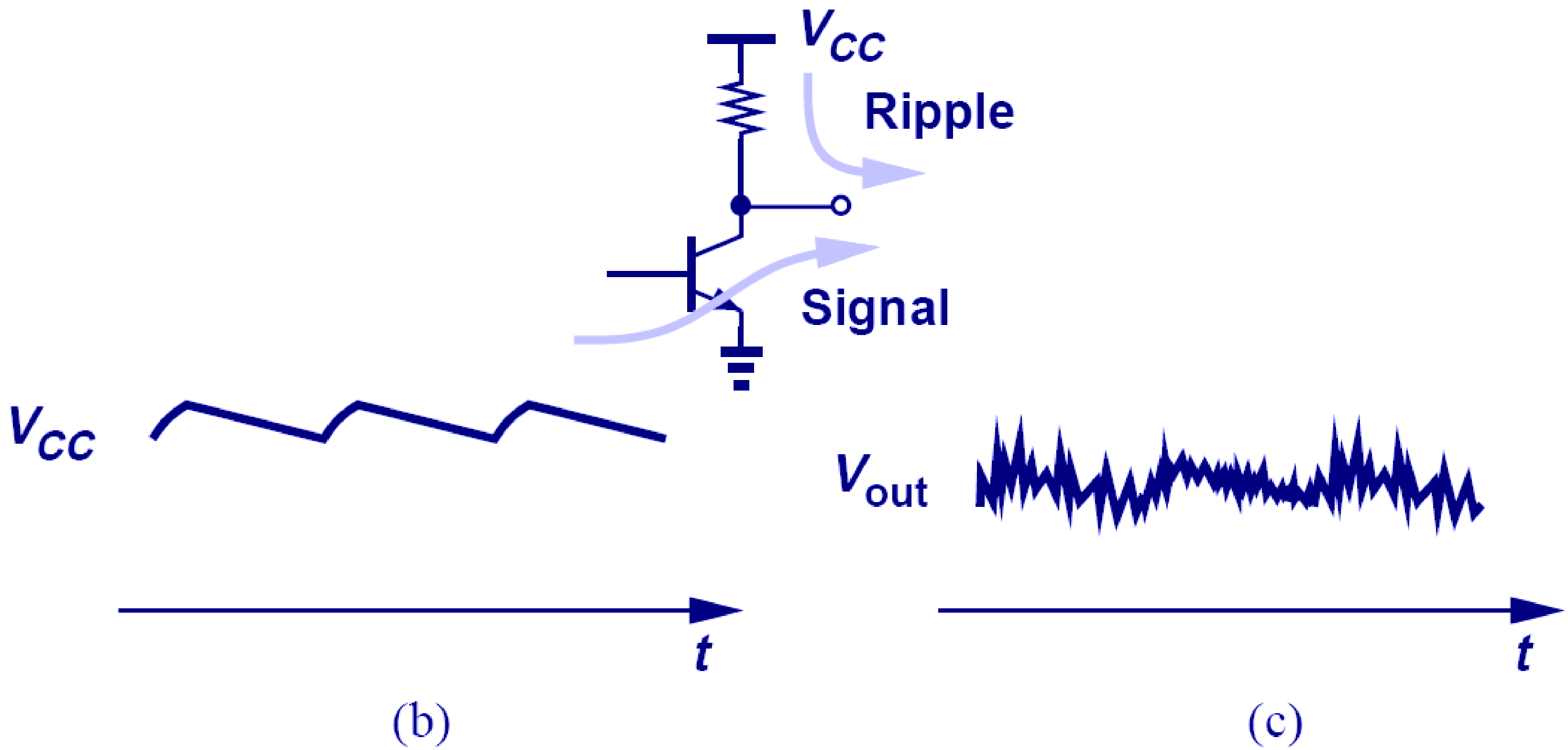
- 10.1 General Considerations
- 10.2 Bipolar Differential Pair
- 10.3 MOS Differential Pair
- 10.4 Cascode Differential Amplifiers
- 10.5 Common-Mode Rejection
- 10.6 Differential Pair with Active Load

Audio Amplifier Example



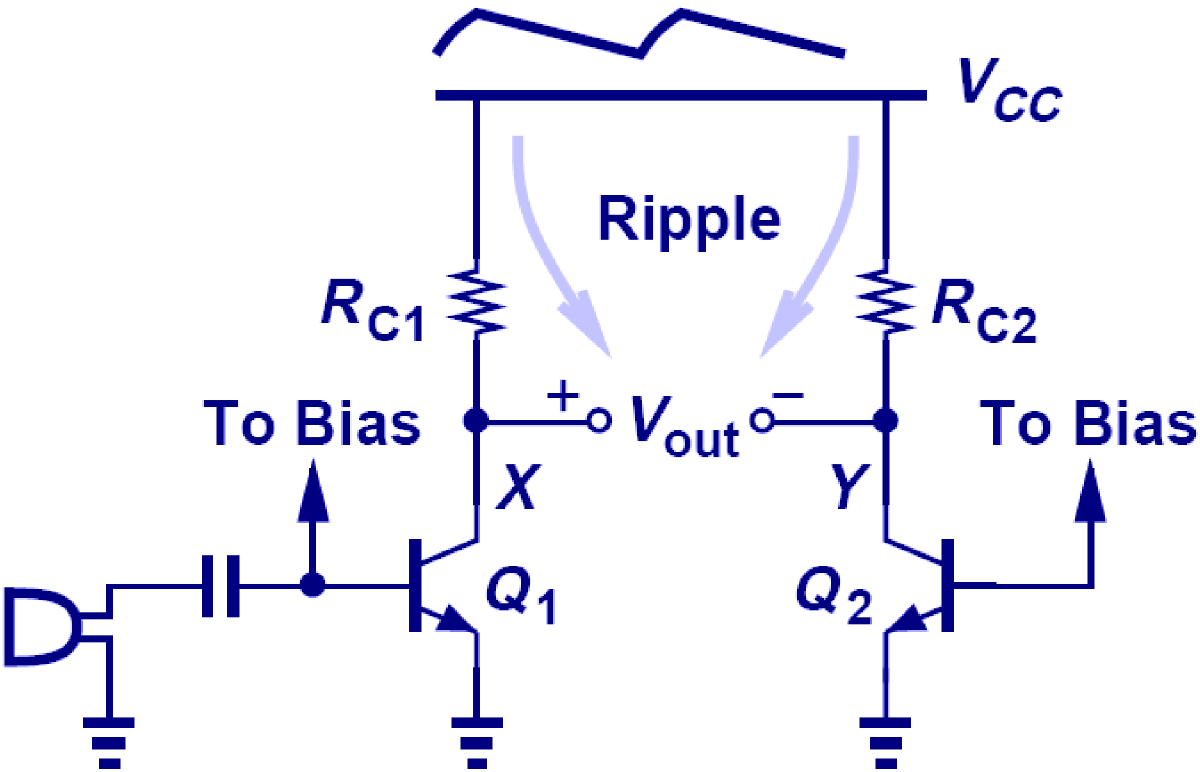
- An audio amplifier is constructed above that takes on a rectified AC voltage as its supply and amplifies an audio signal from a microphone.

“Humming” Noise in Audio Amplifier Example



- However, V_{CC} contains a ripple from rectification that leaks to the output and is perceived as a “humming” noise by the user.

Supply Ripple Rejection



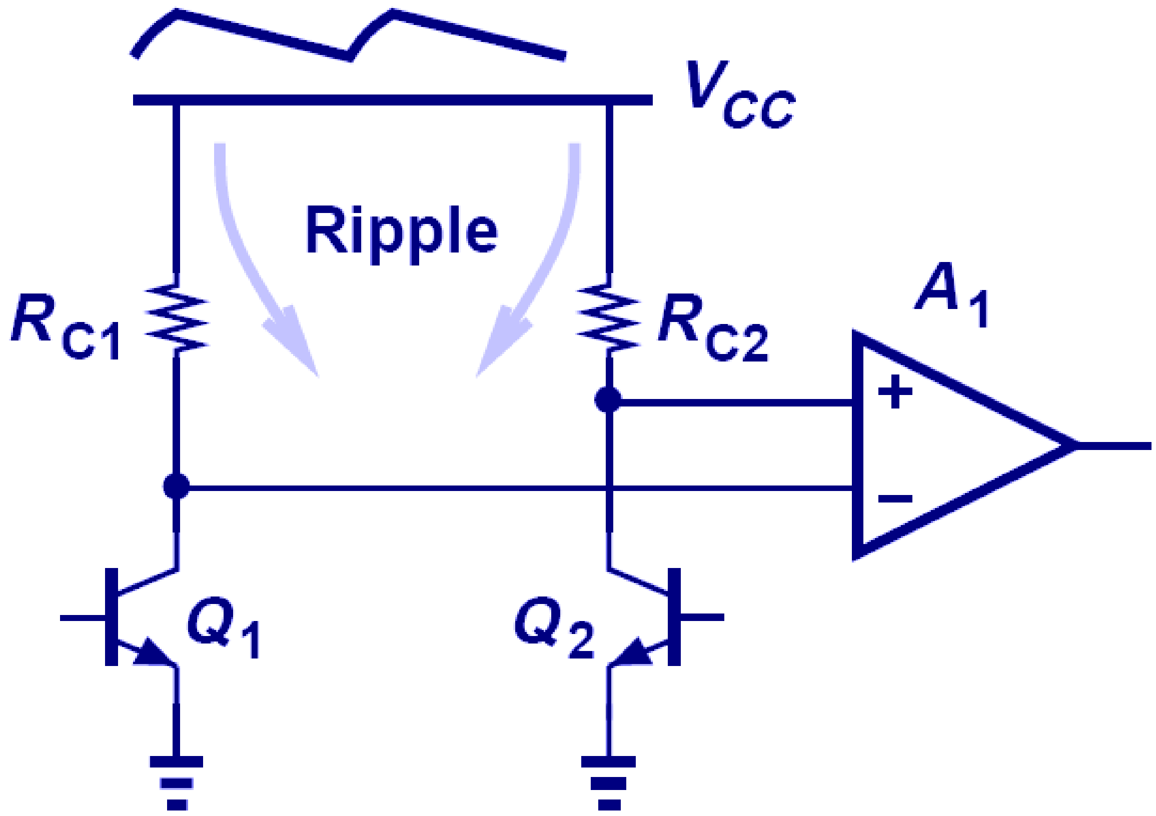
$$v_X = A_v v_{in} + v_r$$

$$v_Y = v_r$$

$$v_X - v_Y = A_v v_{in}$$

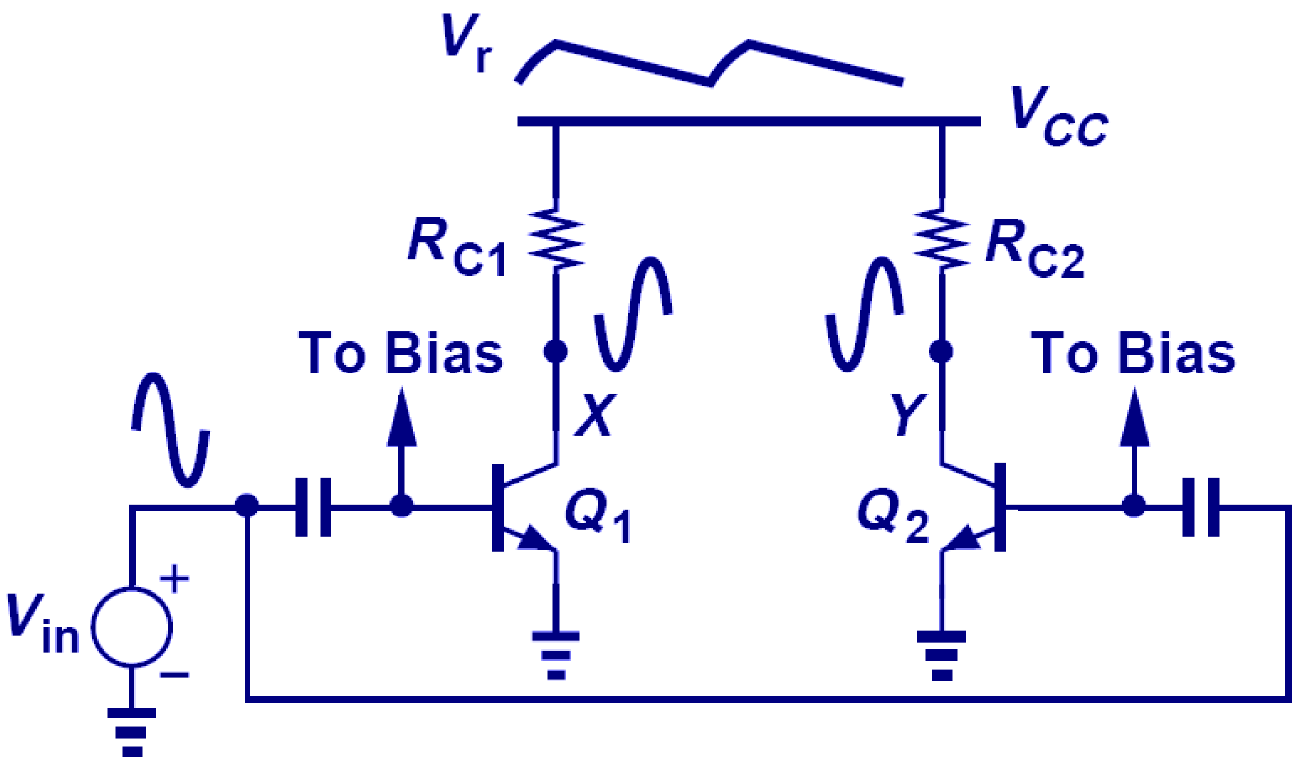
- Since both node X and Y contain the ripple, their difference will be free of ripple.

Ripple-Free Differential Output



- Since the signal is taken as a difference between two nodes, an amplifier that senses differential signals is needed.

Common Inputs to Differential Amplifier



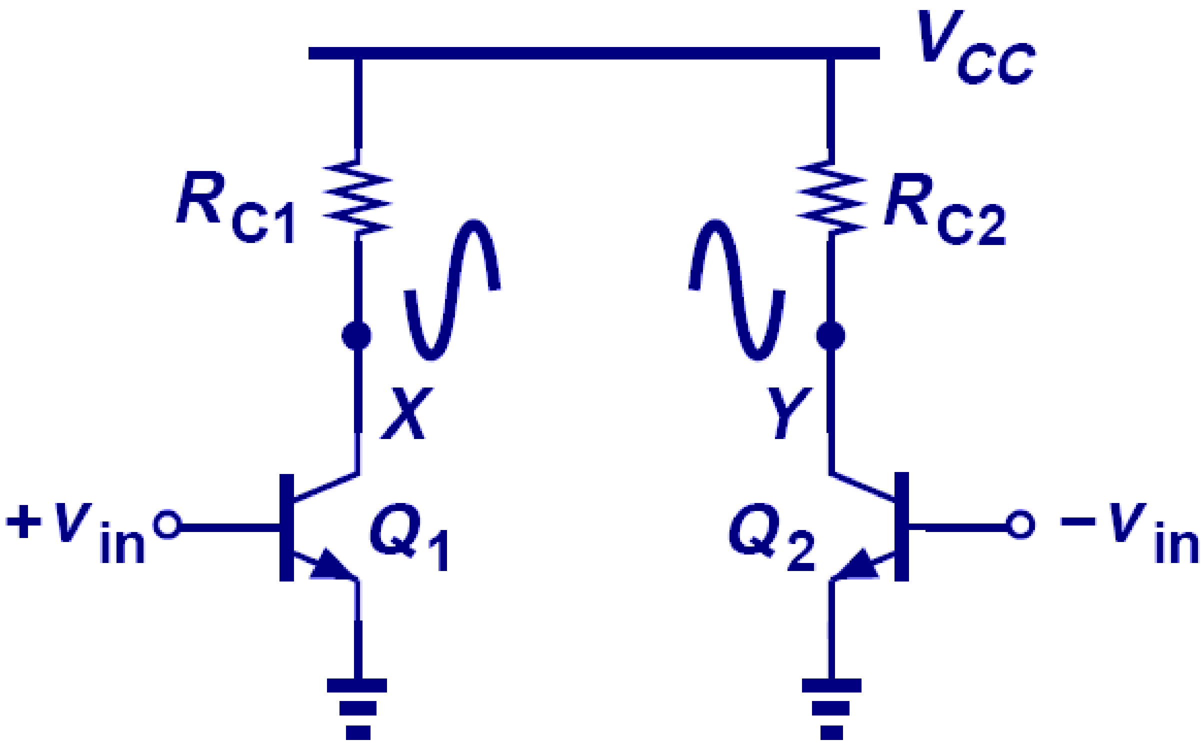
$$v_X = A_v v_{in} + v_r$$

$$v_Y = A_v v_{in} + v_r$$

$$v_X - v_Y = 0$$

- **Signals cannot be applied in phase to the inputs of a differential amplifier, since the outputs will also be in phase, producing zero differential output.**

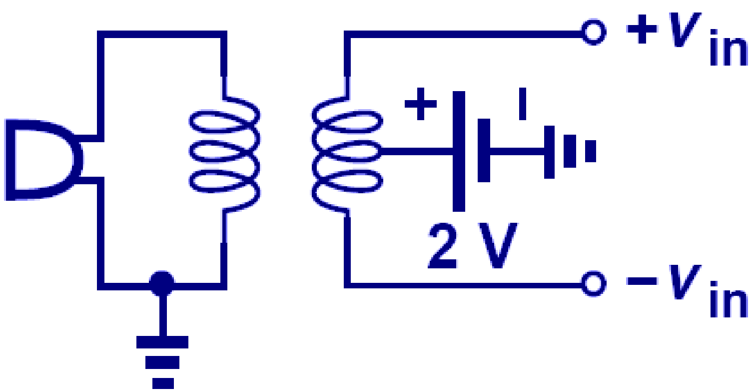
Differential Inputs to Differential Amplifier

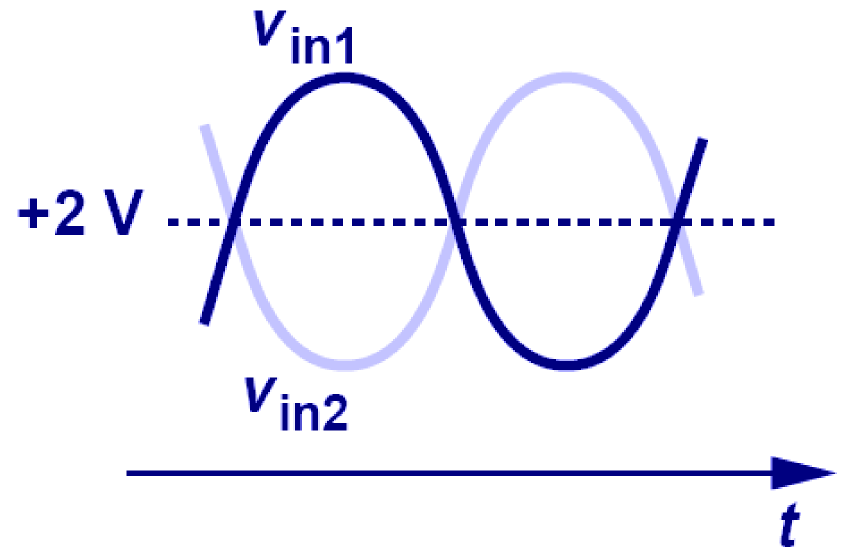


$$v_X = A_v v_{in} + v_r$$
$$v_Y = -A_v v_{in} + v_r$$
$$v_X - v_Y = 2A_v v_{in}$$

- **When the inputs are applied differentially, the outputs are 180° out of phase; enhancing each other when sensed differentially.**

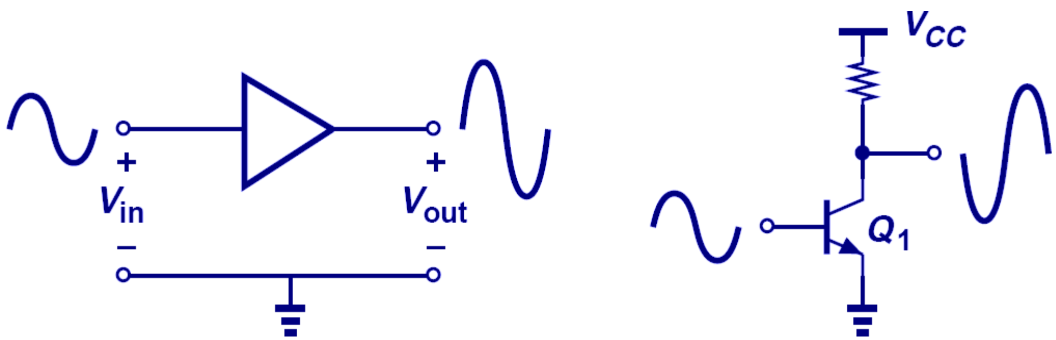
Differential Signals



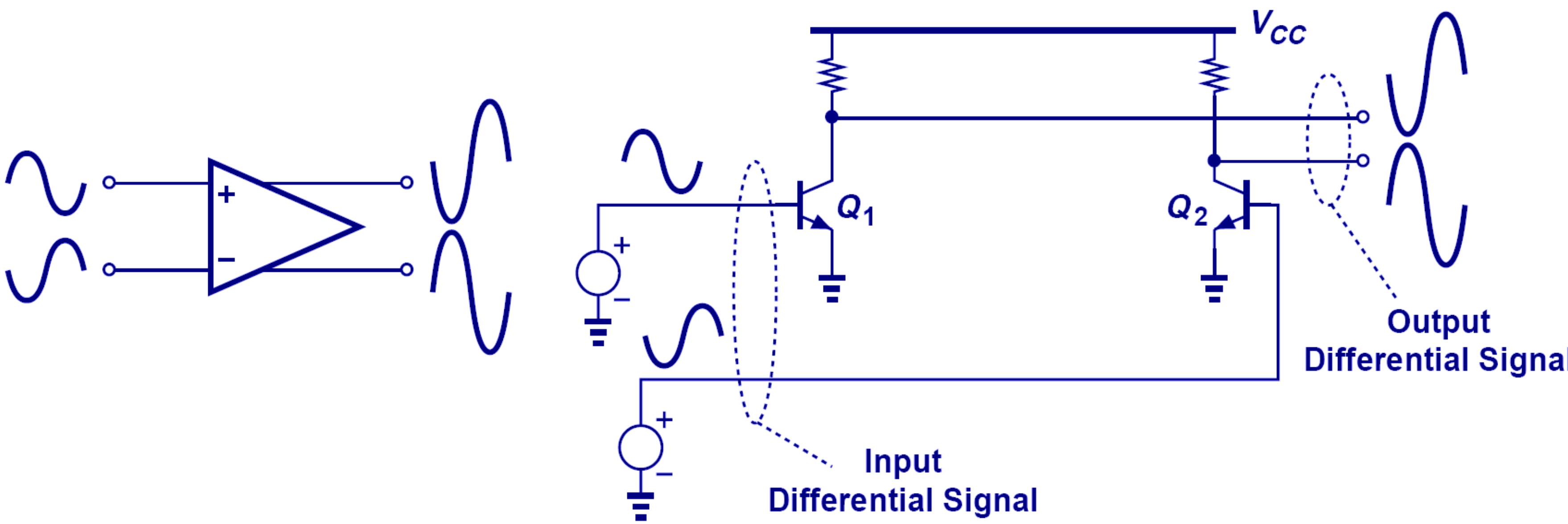


- A pair of differential signals can be generated, among other ways, by a transformer.
- Differential signals have the property that they share the same average value to ground and are equal in magnitude but opposite in phase.

Single-ended vs. Differential Signals

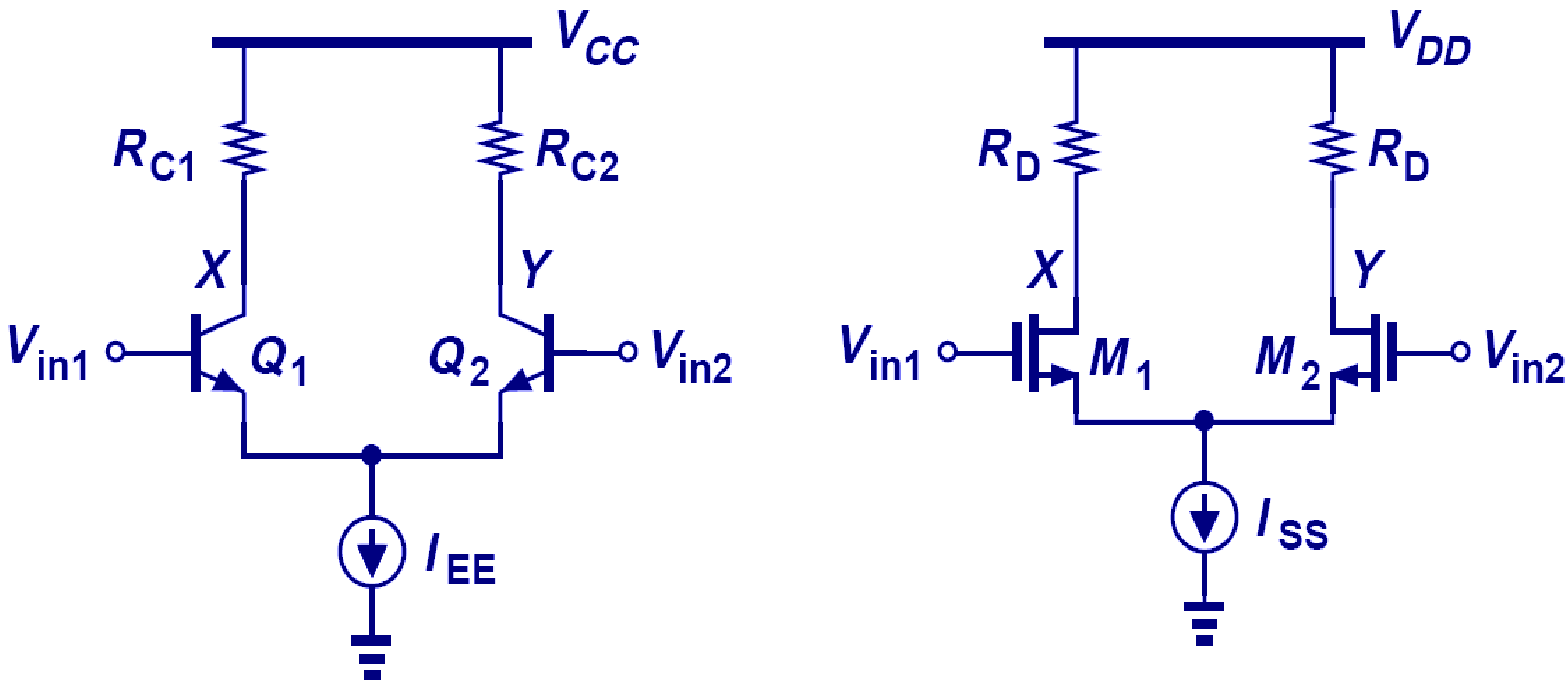


(a)



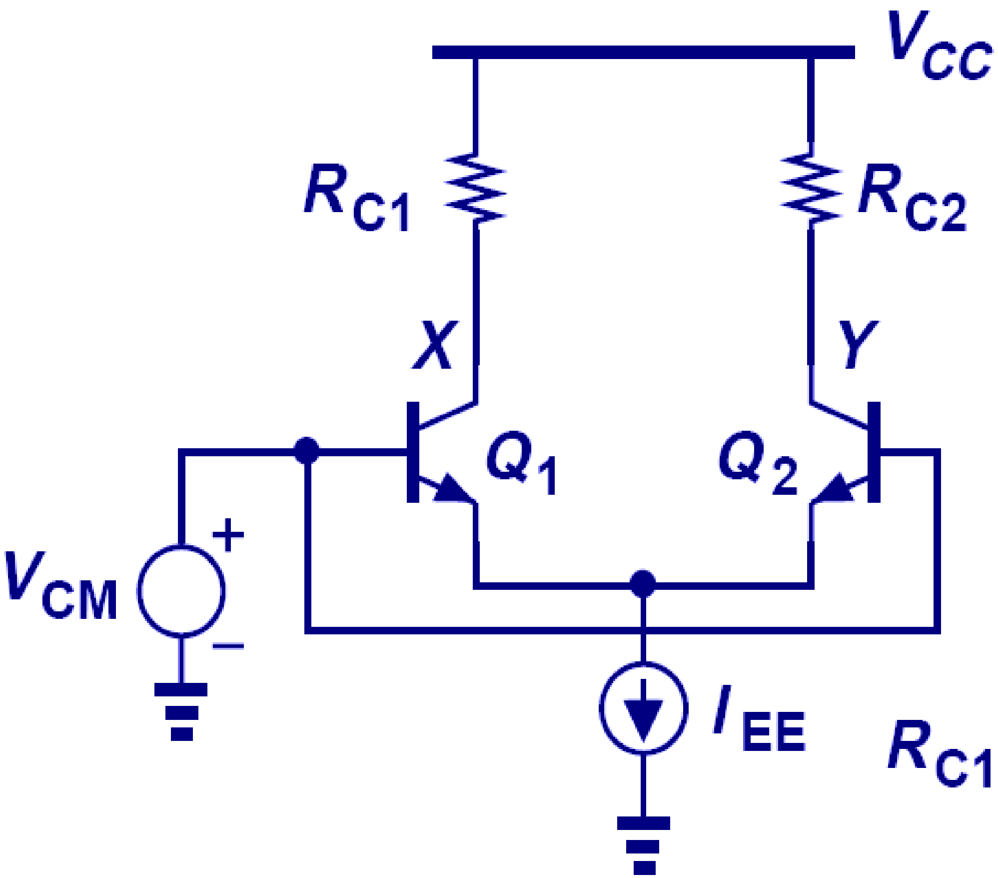
(b)

Differential Pair



- With the addition of a tail current, the circuits above operate as an elegant, yet robust differential pair.

Common-Mode Response



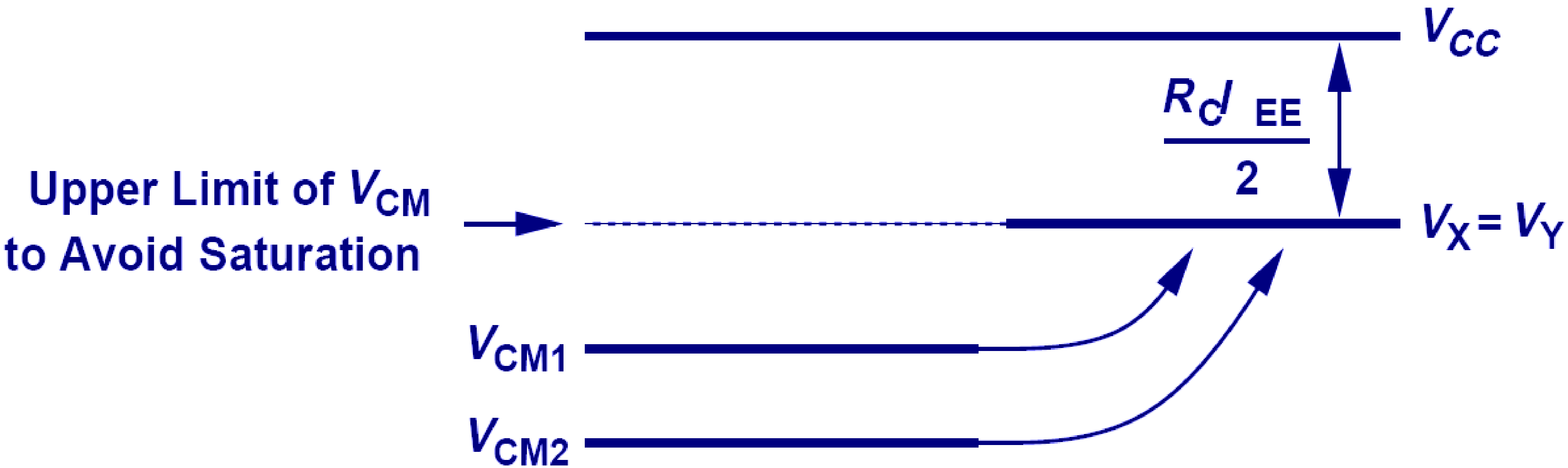
$$V_{BE1} = V_{BE2}$$

$$I_{C1} = I_{C2} = \frac{I_{EE}}{2}$$

$$V_X = V_Y = V_{CC} - R_C \frac{I_{EE}}{2}$$

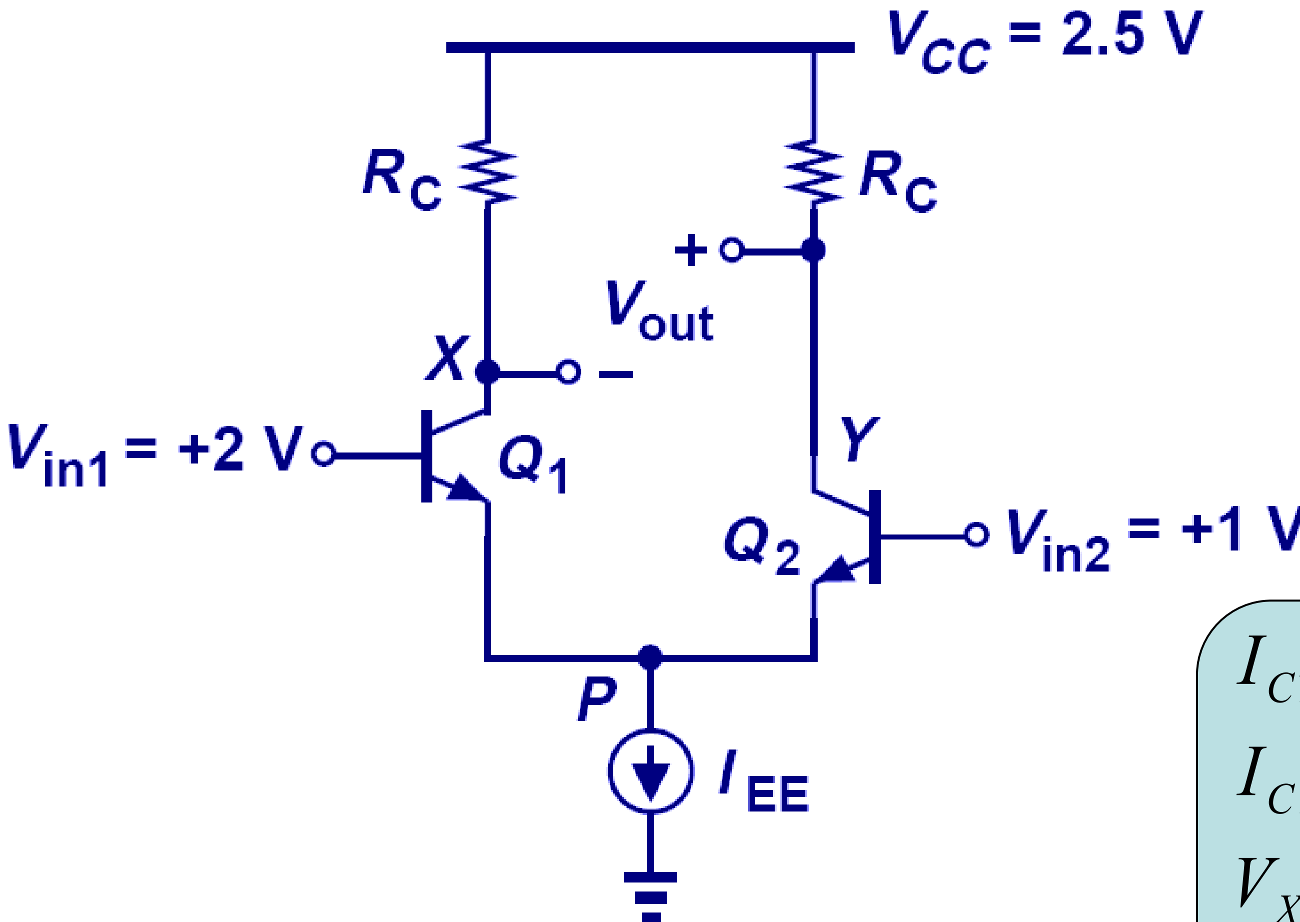
$R_{C1} = R_{C2} = R_C$

Common-Mode Rejection



- Due to the fixed tail current source, the input common-mode value can vary without changing the output common-mode value.

Differential Response I



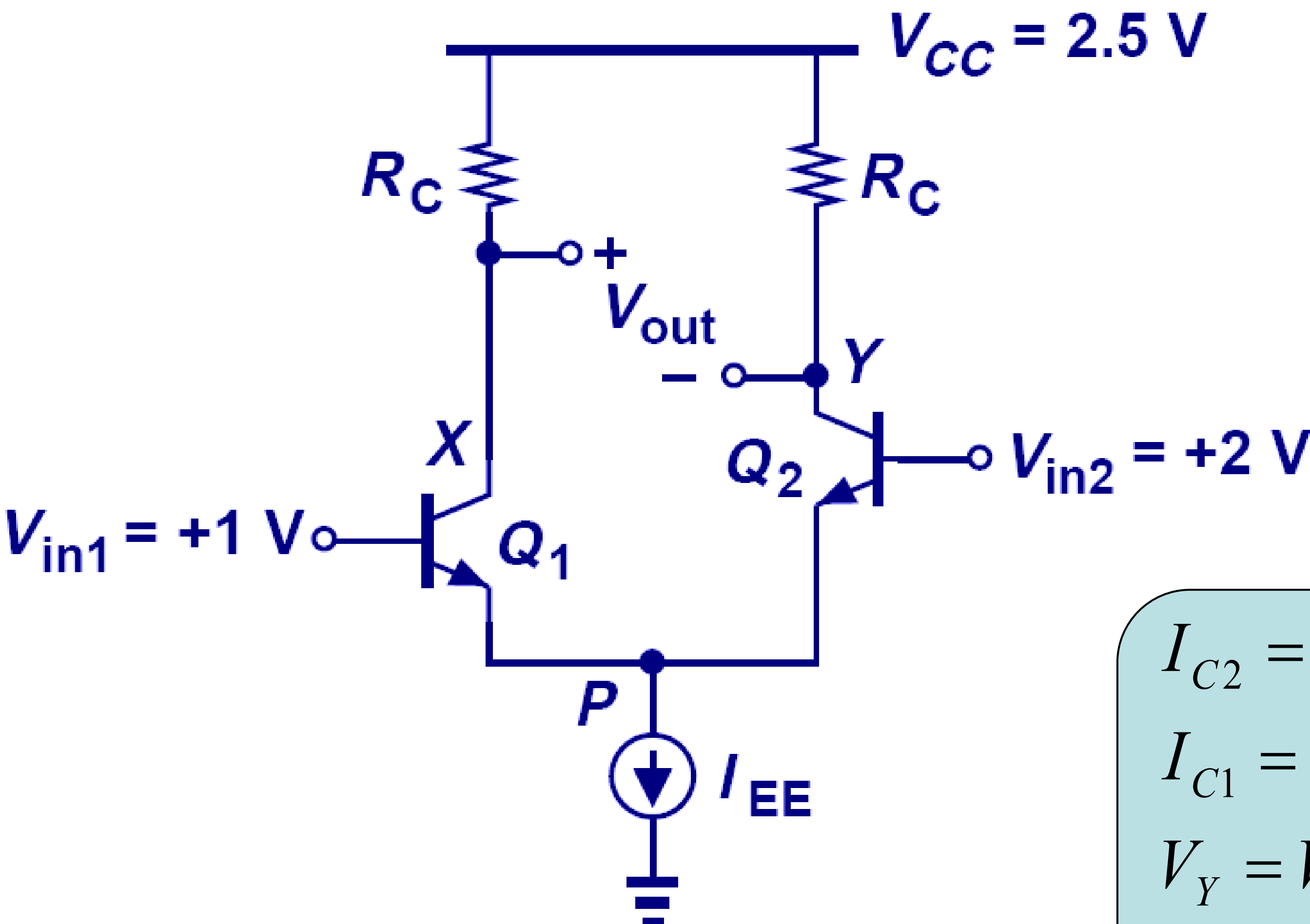
$$I_{C1} = I_{EE}$$

$$I_{C2} = 0$$

$$V_X = V_{CC} - R_C I_{EE}$$

$$V_Y = V_{CC}$$

Differential Response II



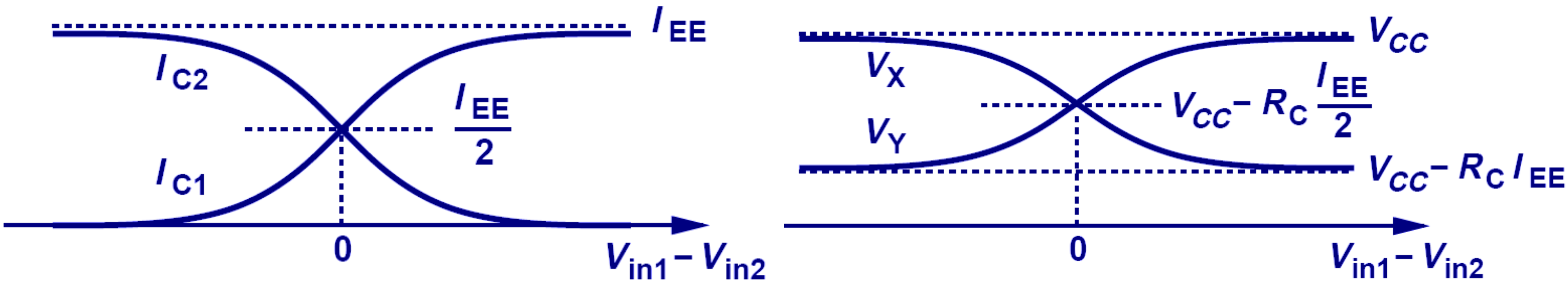
$$I_{C2} = I_{EE}$$

$$I_{C1} = 0$$

$$V_Y = V_{CC} - R_C I_{EE}$$

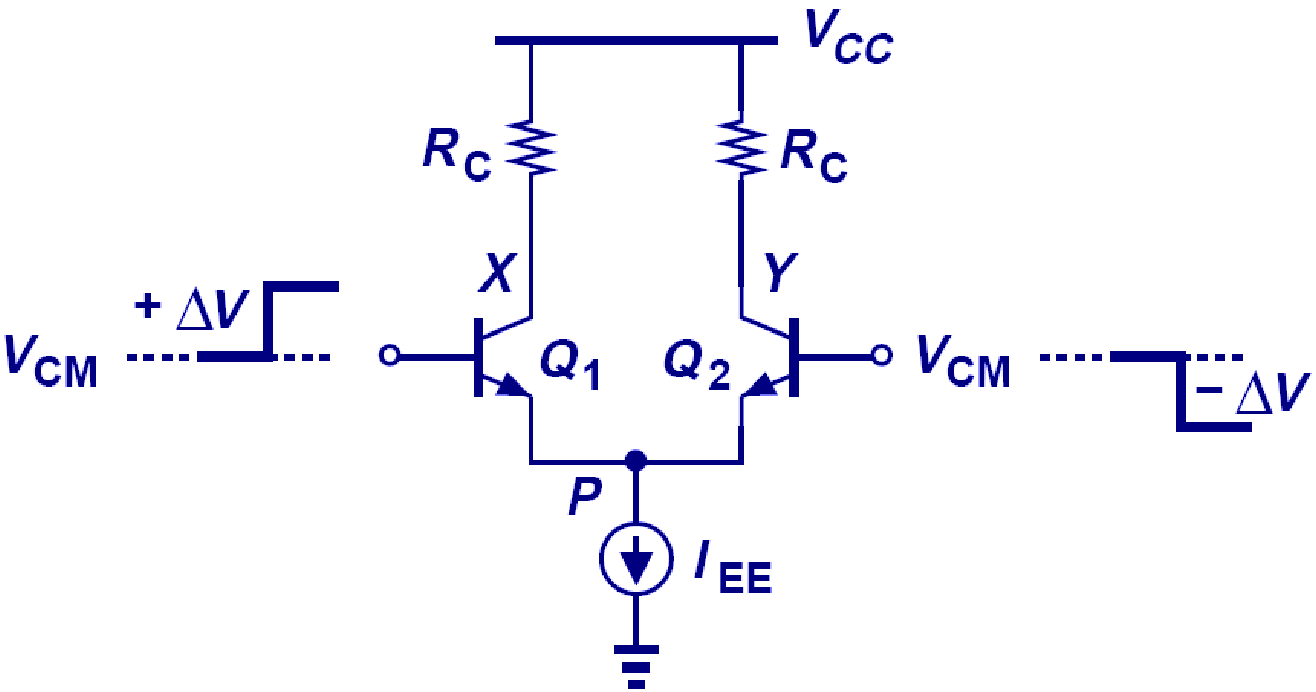
$$V_X = V_{CC}$$

Differential Pair Characteristics



- **None-zero differential input produces variations in output currents and voltages, whereas common-mode input produces no variations.**

Small-Signal Analysis

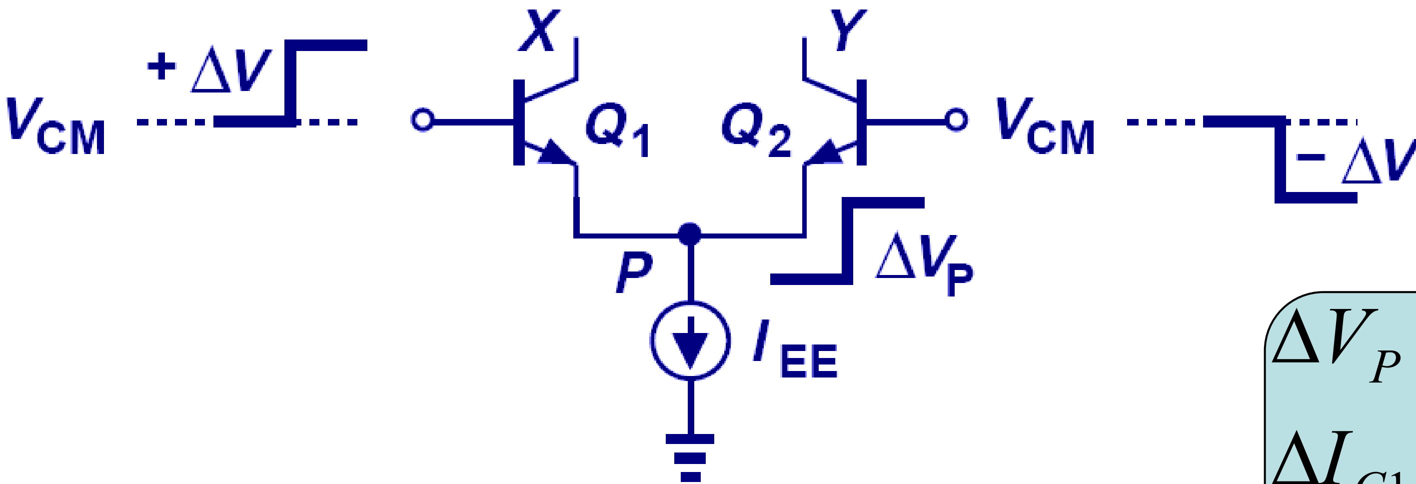


$$I_{C1} = \frac{I_{EE}}{2} + \Delta I$$

$$I_{C2} = \frac{I_{EE}}{2} - \Delta I$$

- Since the input to Q_1 and Q_2 rises and falls by the same amount, and their bases are tied together, the rise in I_{C1} has the same magnitude as the fall in I_{C2}.

Virtual Ground



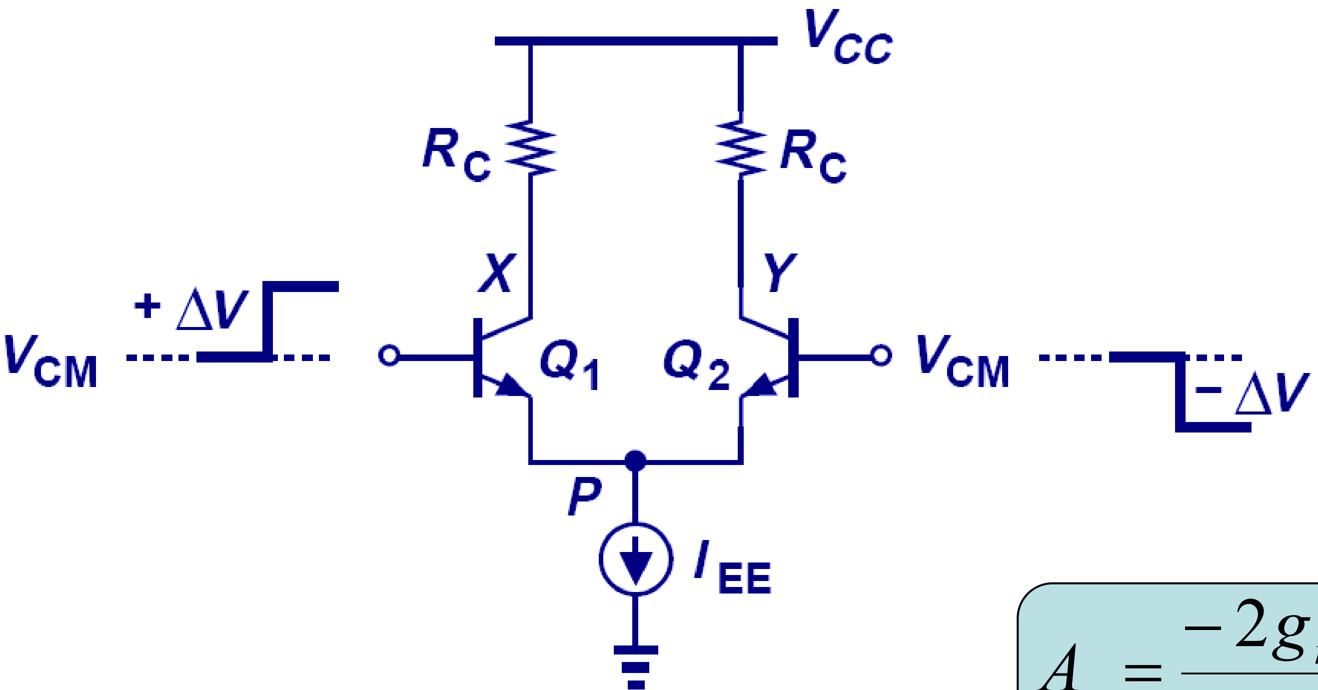
$$\Delta V_P = 0$$

$$\Delta I_{C1} = g_m \Delta V$$

$$\Delta I_{C2} = -g_m \Delta V$$

- **For small changes at inputs, the g_m's are the same, and the respective increase and decrease of I_{C1} and I_{C2} are the same, node P must stay constant to accommodate these changes. Therefore, node P can be viewed as AC ground.**

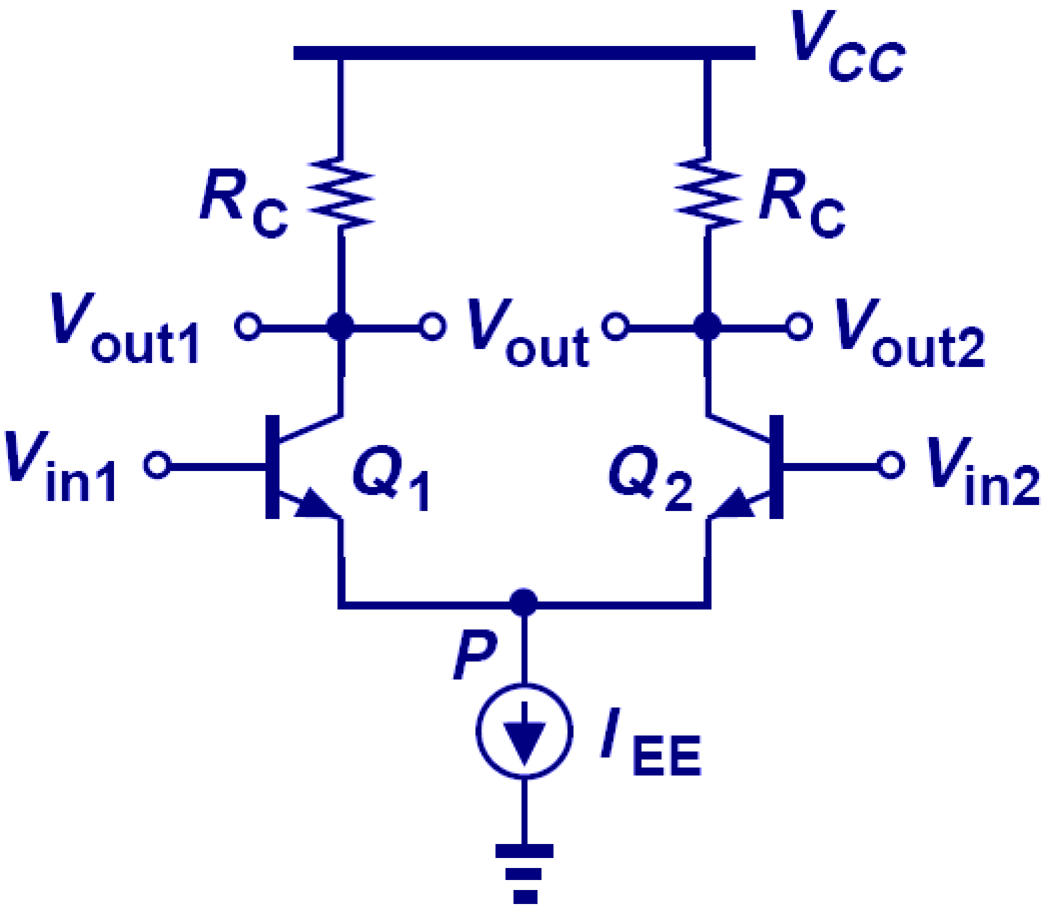
Small-Signal Differential Gain



$$A_v = \frac{-2g_m \Delta V R_C}{2\Delta V} = -g_m R_C$$

- **Since the output changes by $-2g_m\Delta VR_C$ and input by $2\Delta V$, the small signal gain is $-g_mR_C$, similar to that of the CE stage. However, to obtain same gain as the CE stage, power dissipation is doubled.**

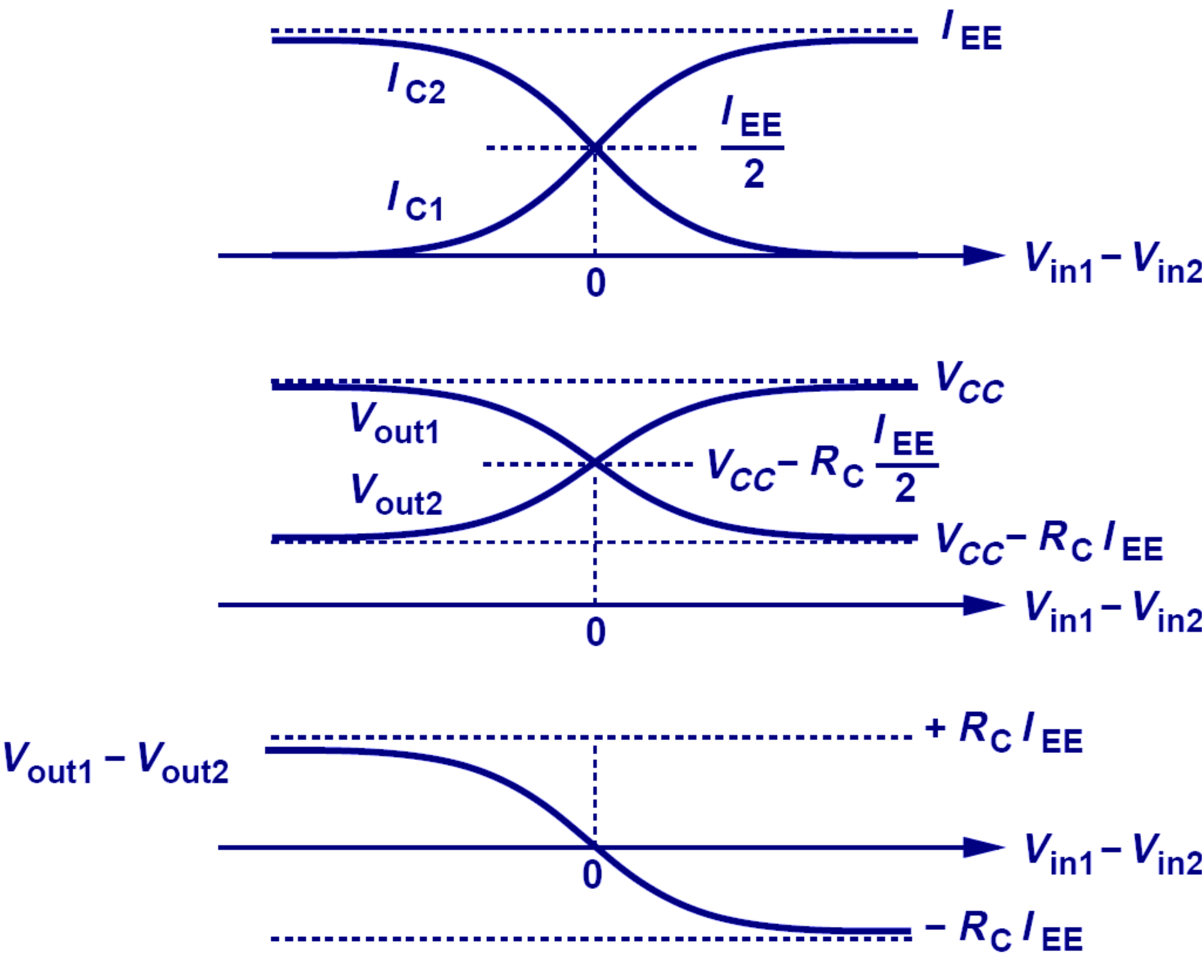
Large Signal Analysis



$$I_{C1} = \frac{I_{EE}\exp\dfrac{V_{in1}-V_{in2}}{V_T}}{1+\exp\dfrac{V_{in1}-V_{in2}}{V_T}}$$

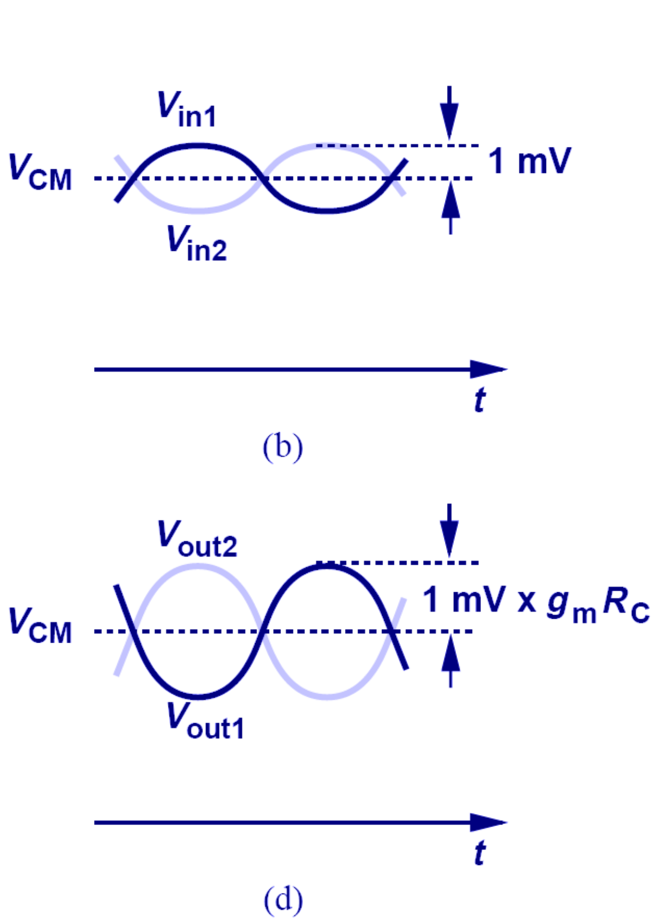
$$I_{C2} = \frac{I_{EE}}{1+\exp\dfrac{V_{in1}-V_{in2}}{V_T}}$$

Input/Output Characteristics



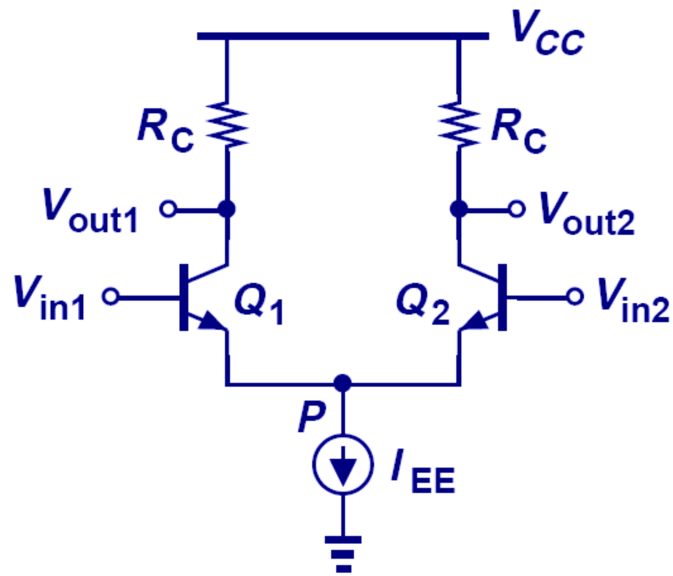
$$V_{out1} - V_{out2} = -R_C I_{EE} \tanh \frac{V_{in1} - V_{in2}}{2V_T}$$

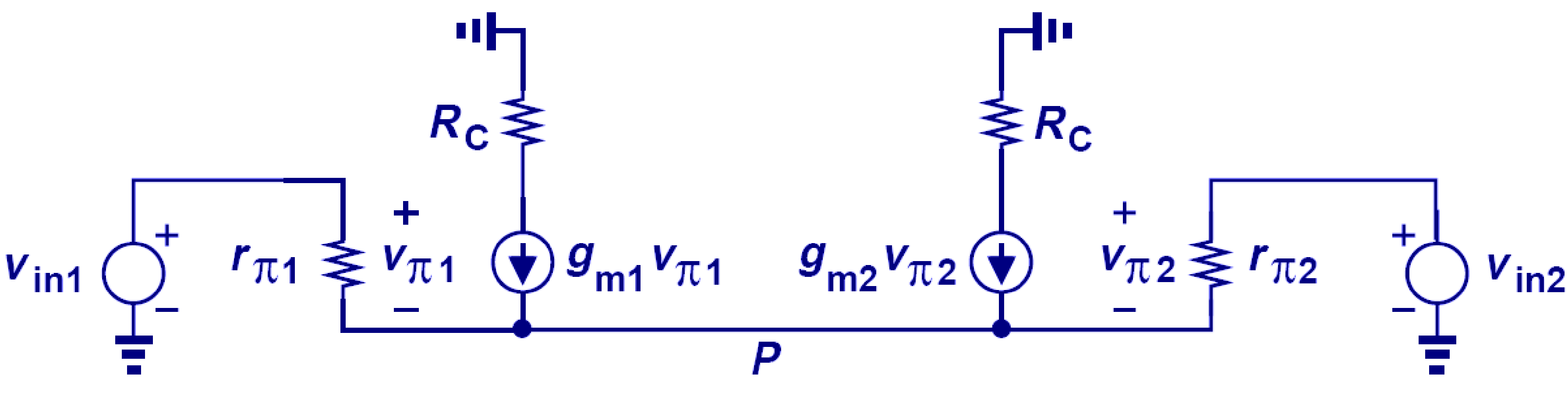
Linear/Nonlinear Regions



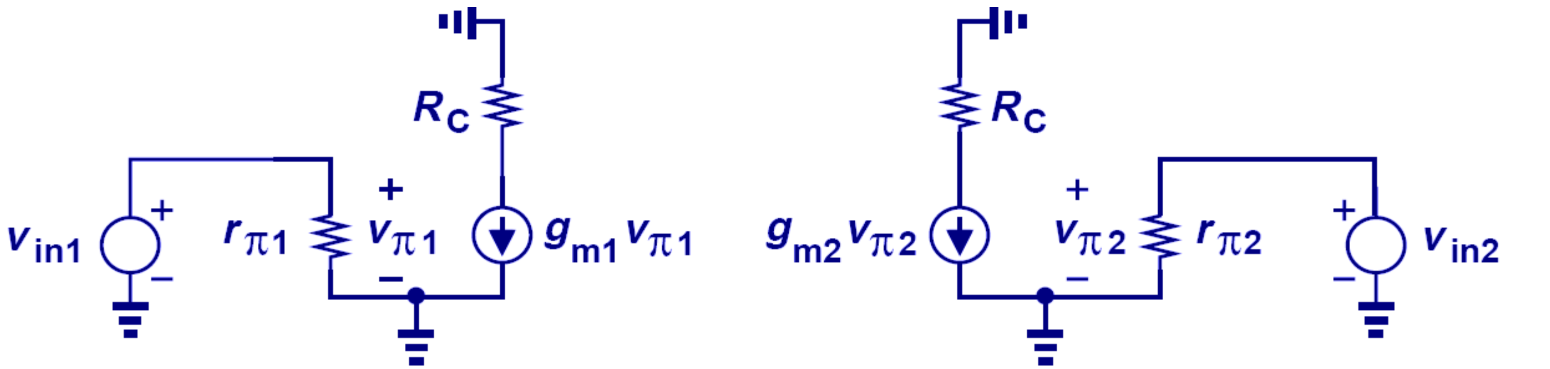
- **The left column operates in linear region, whereas the right column operates in nonlinear region.**

Small-Signal Model

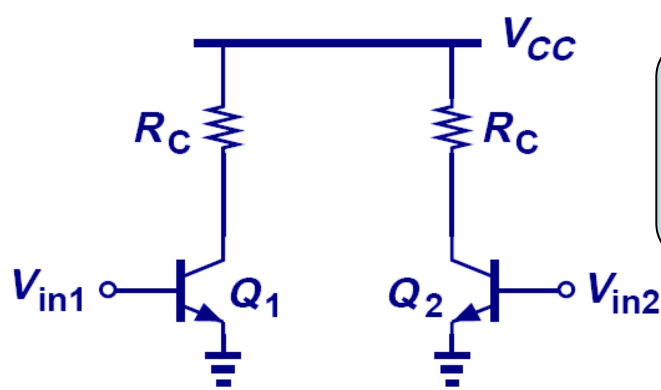




Half Circuits



(b)

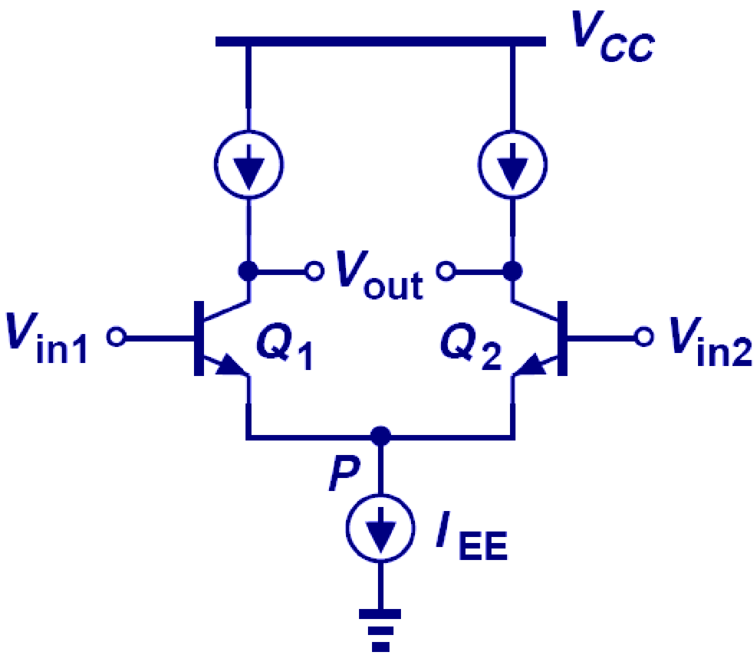


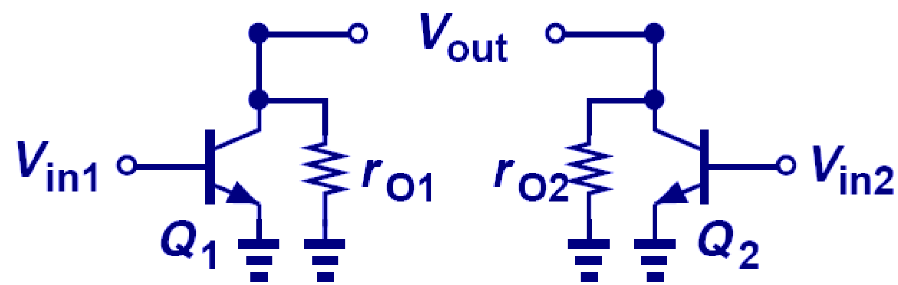
(c)

$$\frac{v_{out1} - v_{out2}}{v_{in1} - v_{in2}} = -g_m R_C$$

- Since V_P is grounded, we can treat the differential pair as two CE "half circuits", with its gain equal to one half circuit's single-ended gain.

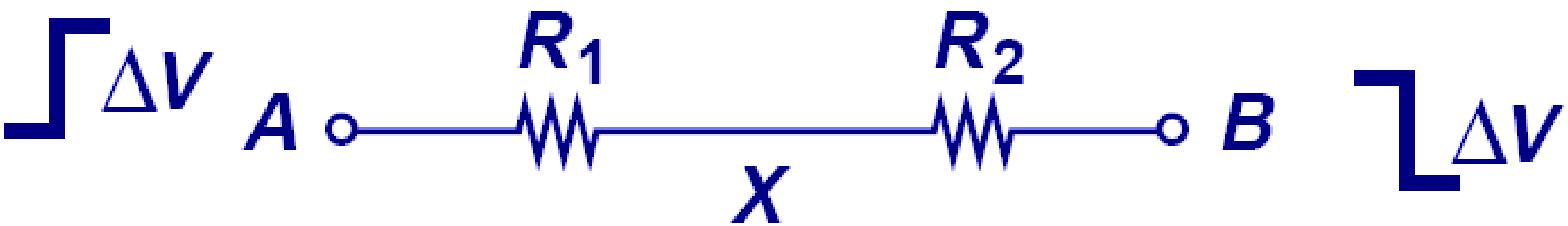
Example: Differential Gain





$$\frac{v_{out1} - v_{out2}}{v_{in1} - v_{in2}} = -g_m r_O$$

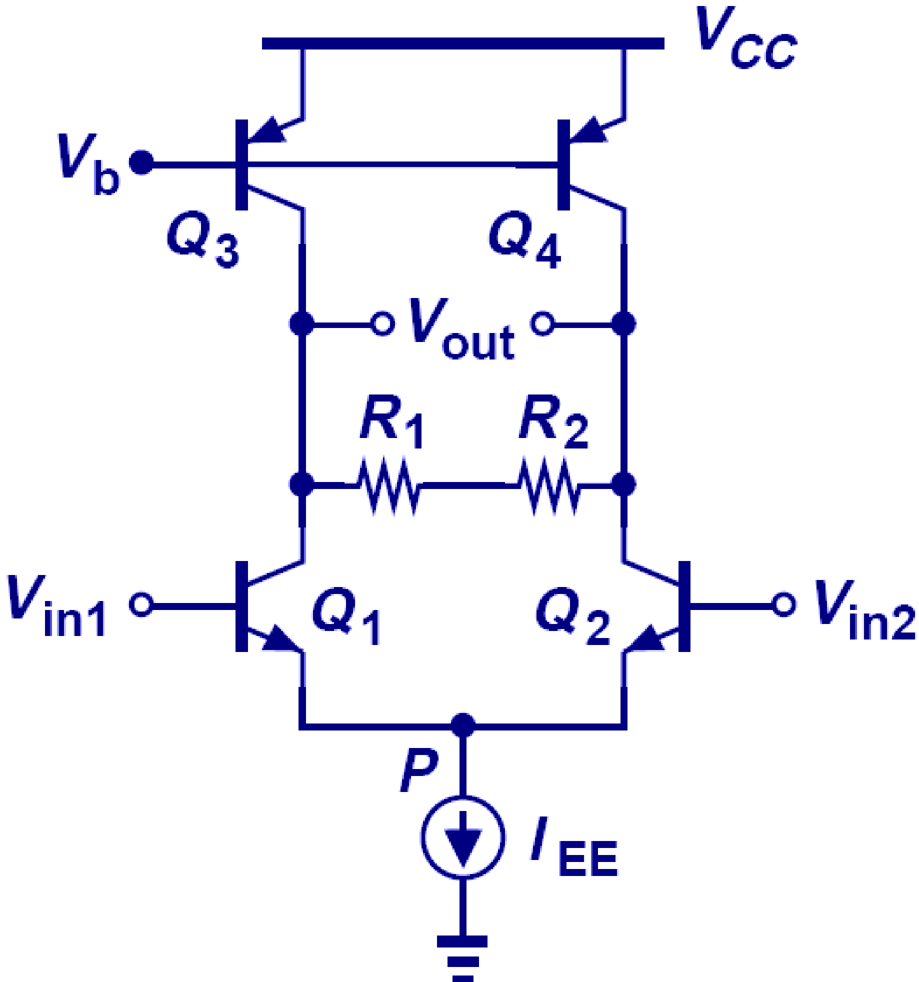
Extension of Virtual Ground

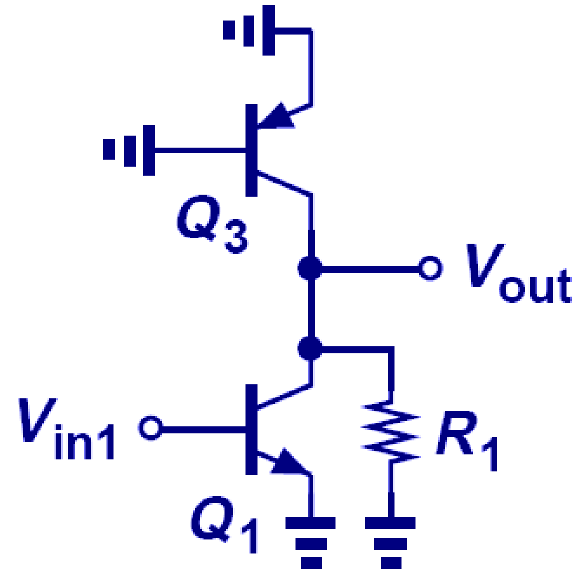


$$V_X = 0$$

- It can be shown that if $R_1 = R_2$, and points A and B go up and down by the same amount respectively, V_X does not move.

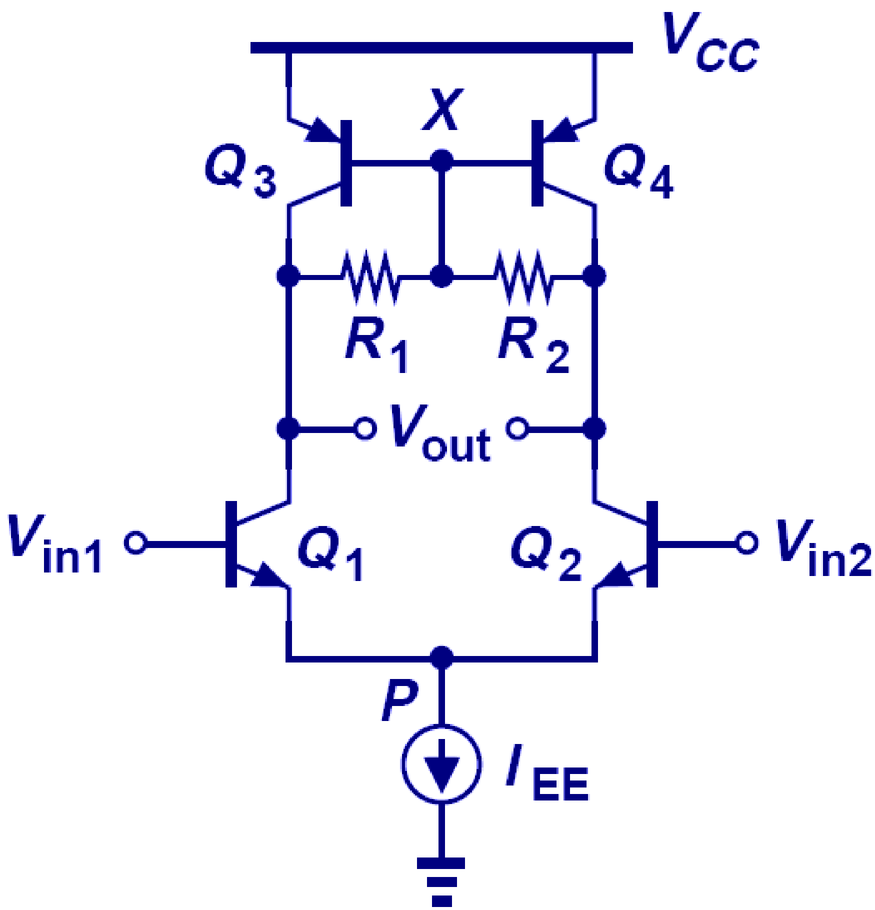
Half Circuit Example I

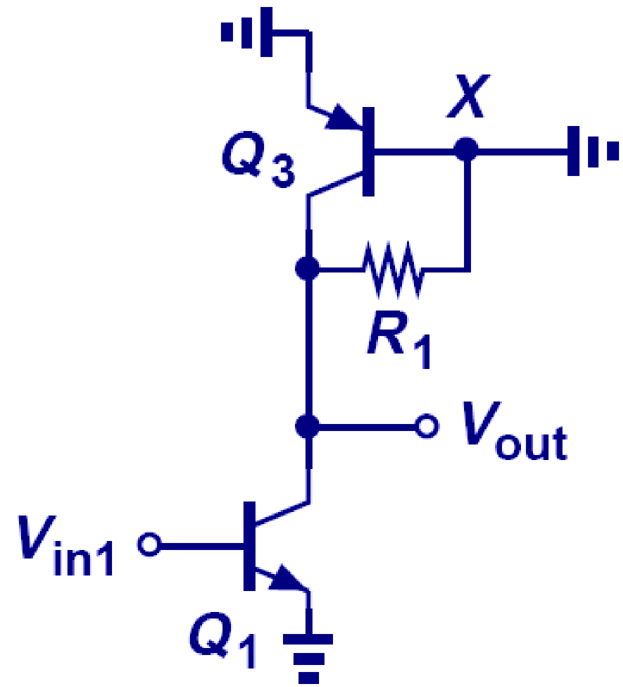




$$A_v = -g_{m1}\left(r_{O1} \parallel r_{O3} \parallel R_1\right)$$

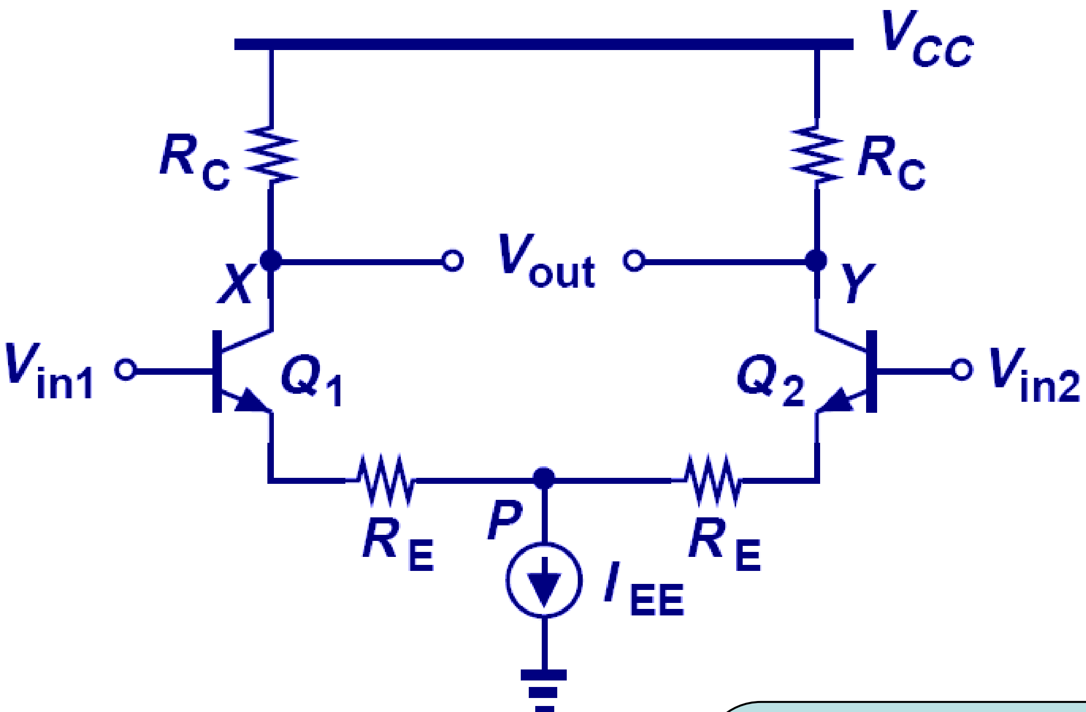
Half Circuit Example II

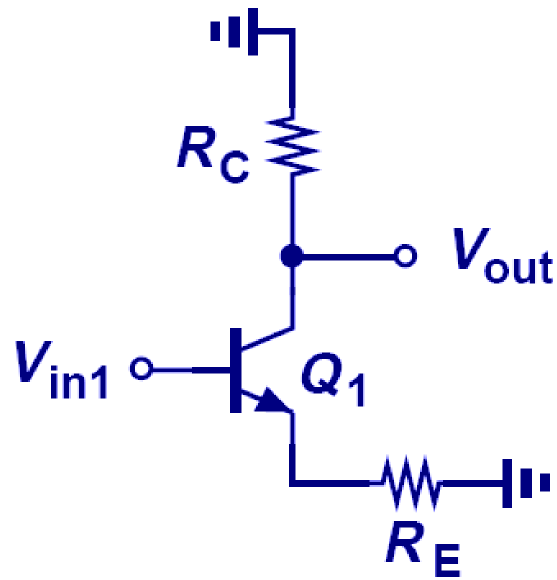




$$A_v = -g_{m1}\left(r_{O1} \| r_{O3} \| R_1\right)$$

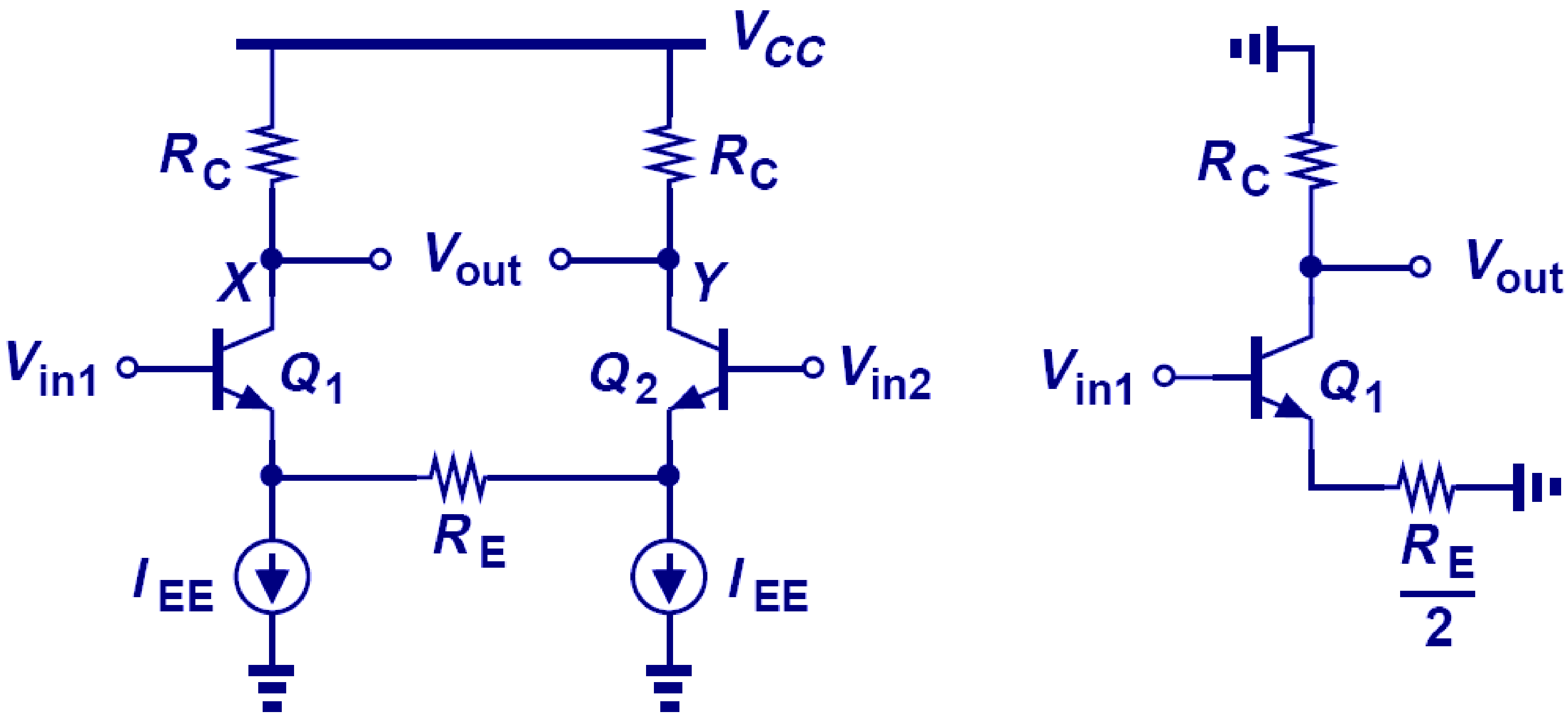
Half Circuit Example III





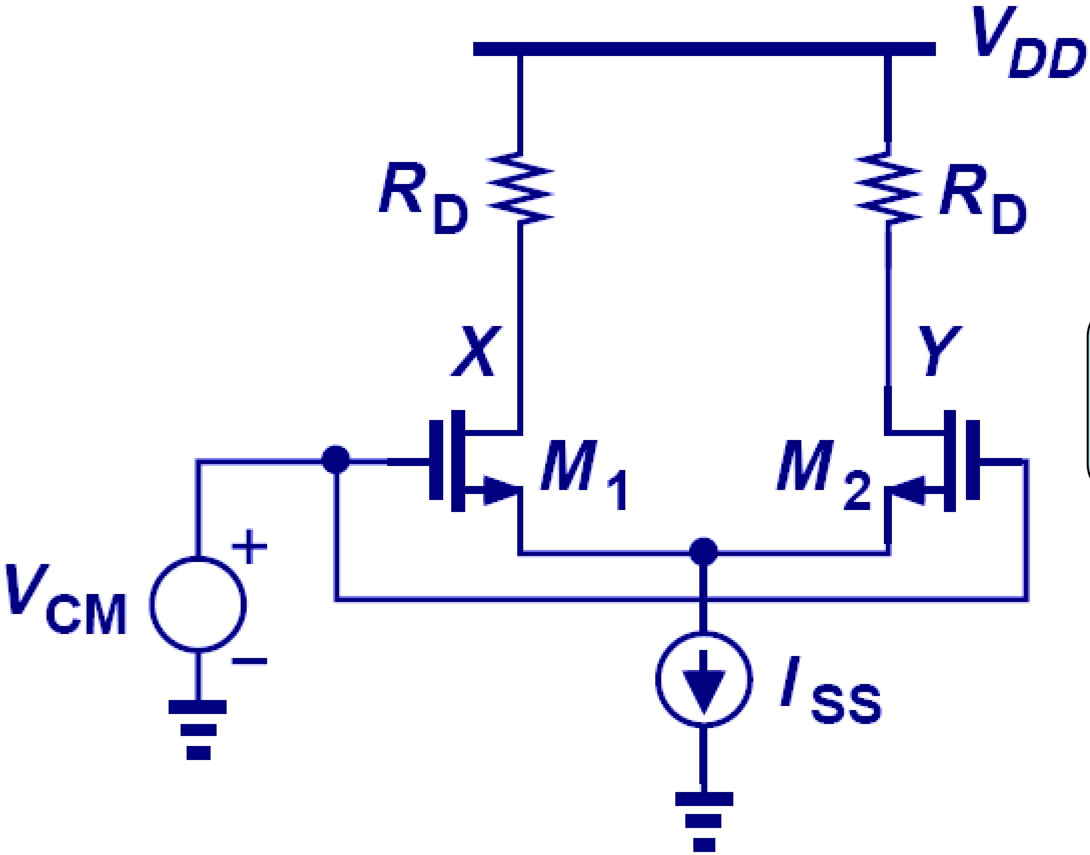
$$A_v = -\frac{R_C}{R_E + \frac{1}{g_m}}$$

Half Circuit Example IV



$$A_v = -\frac{R_C}{\frac{R_E}{2} + \frac{1}{g_m}}$$

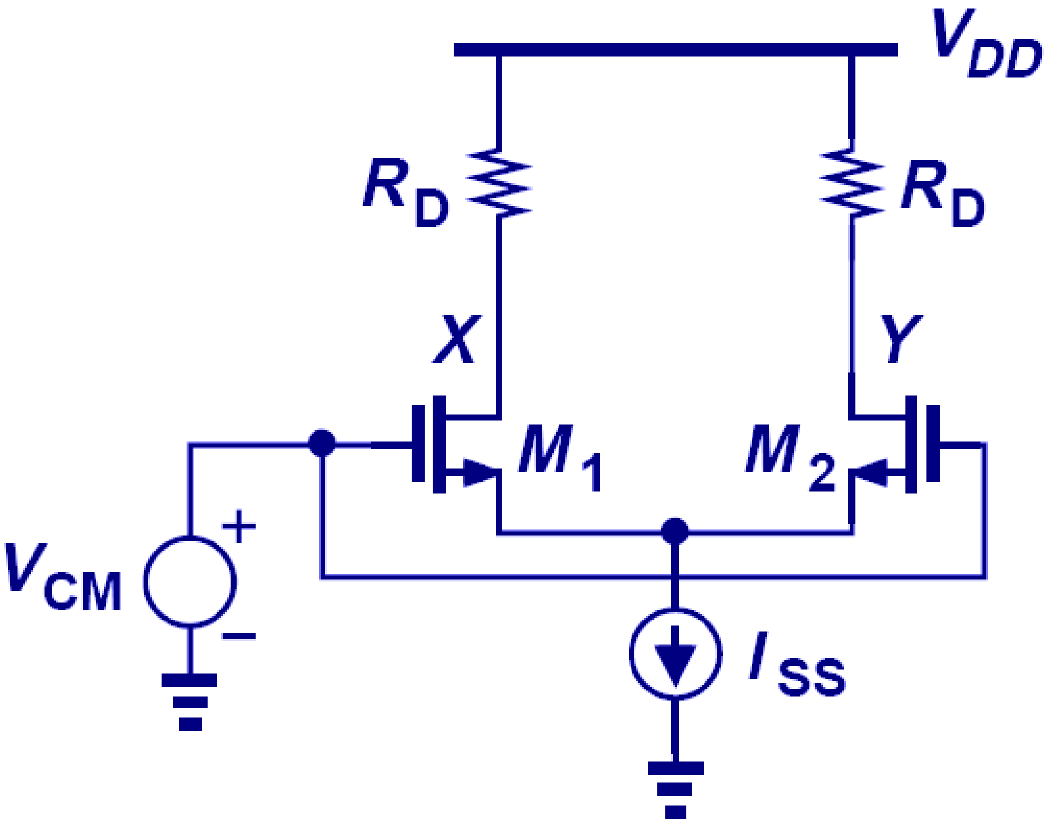
MOS Differential Pair's Common-Mode Response



$$V_X = V_Y = V_{DD} - R_D \frac{I_{SS}}{2}$$

- Similar to its bipolar counterpart, MOS differential pair produces zero differential output as V_{CM} changes.

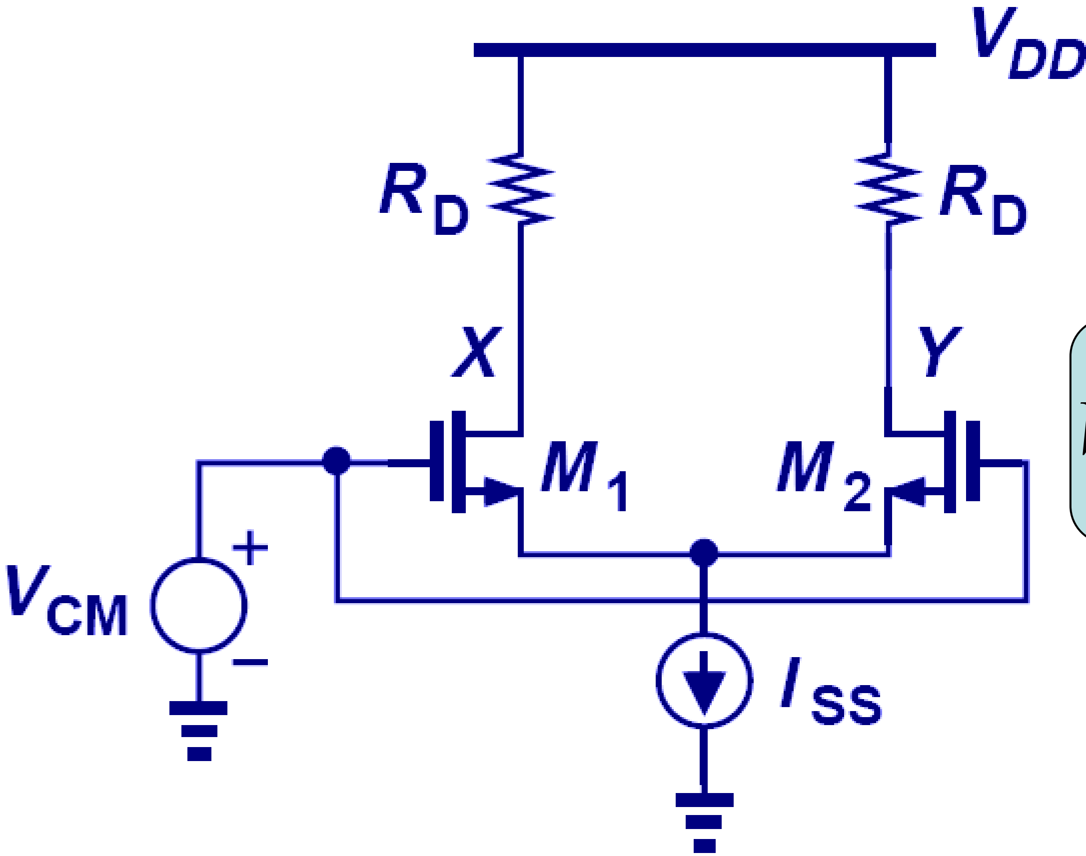
Equilibrium Overdrive Voltage



$$\left(V_{GS}-V_{TH}\right)_{equil}=\sqrt{\frac{I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$$

- The equilibrium overdrive voltage is defined as the overdrive voltage seen by M_1 and M_2 when both of them carry a current of $I_{SS}/2$.

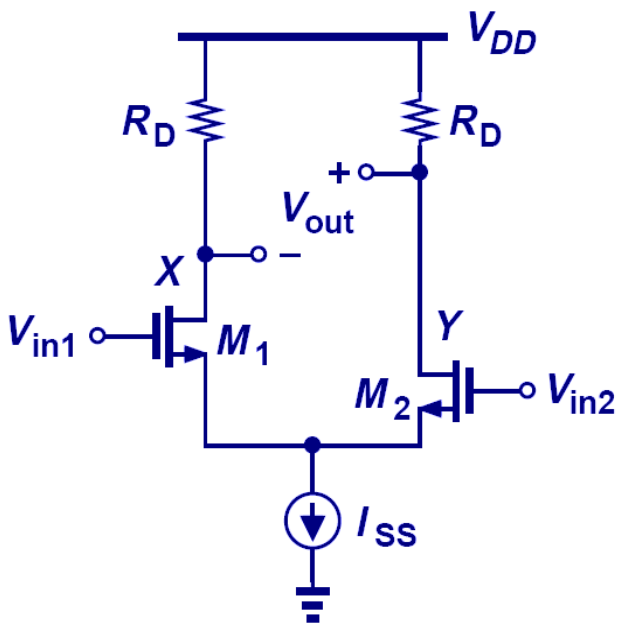
Minimum Common-mode Output Voltage



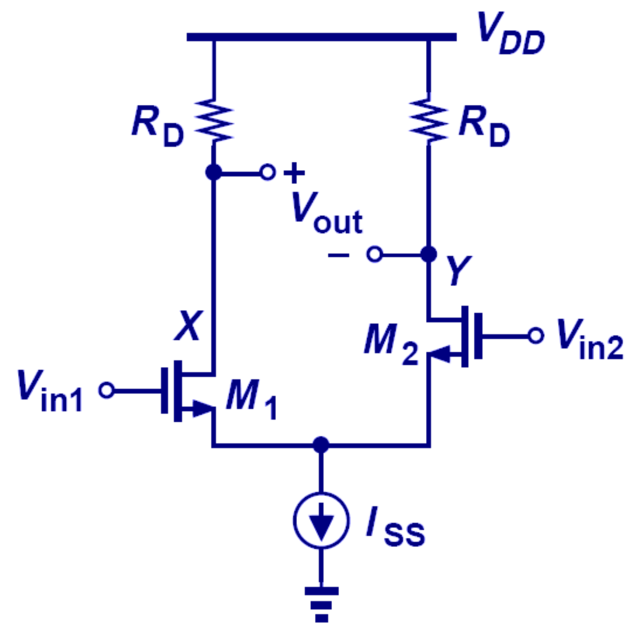
$$V_{DD} - R_D \frac{I_{SS}}{2} > V_{CM} - V_{TH}$$

- In order to maintain M_1 and M_2 in saturation, the common-mode output voltage cannot fall below the value above.
- This value usually limits voltage gain.

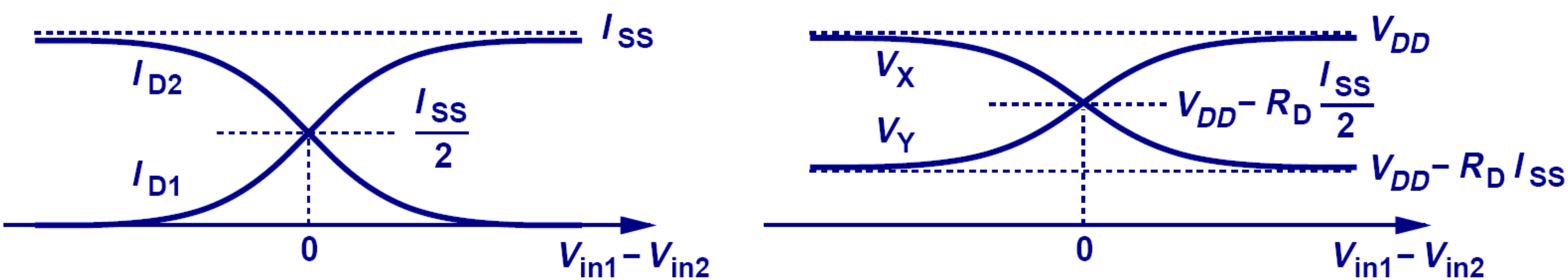
Differential Response



(a)

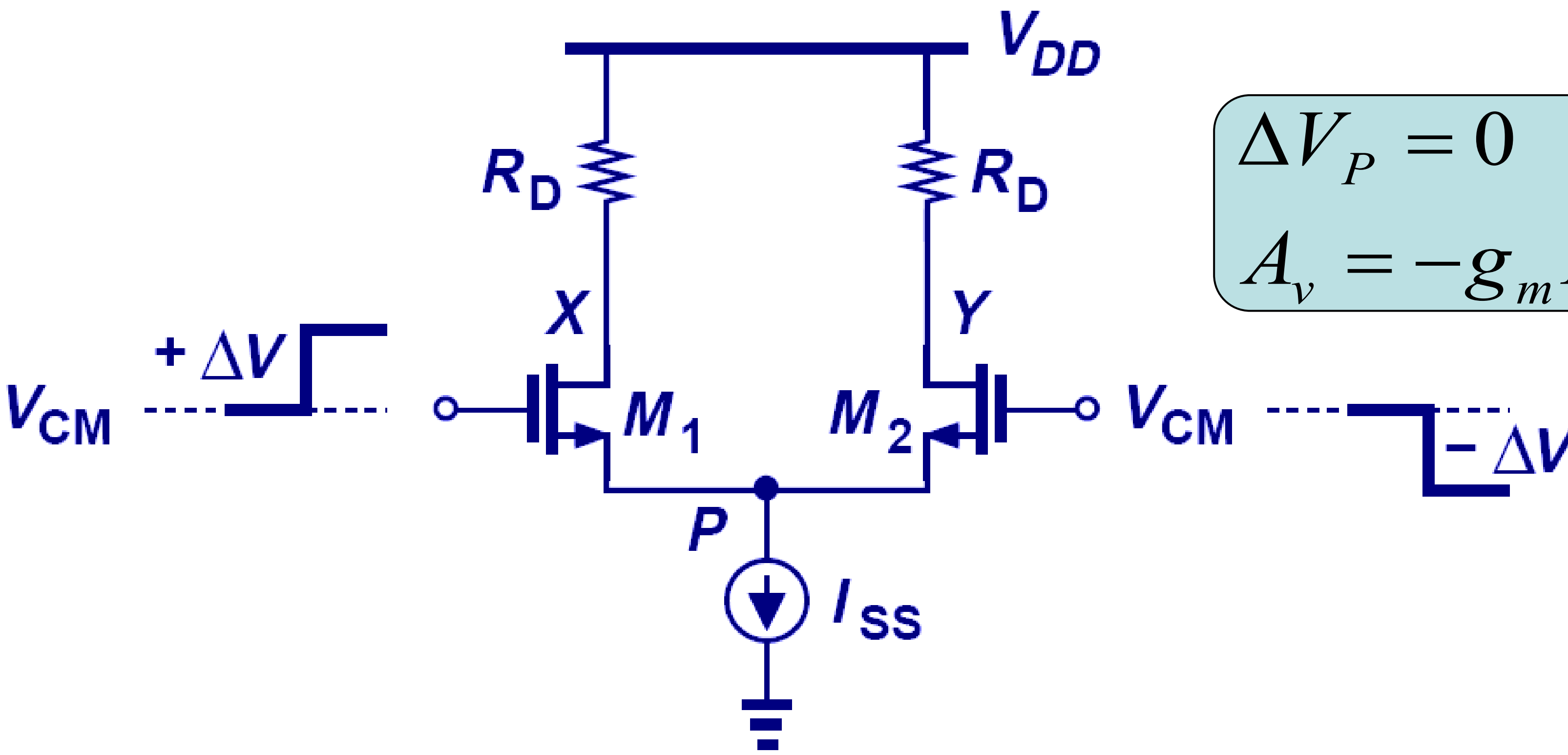


(b)



(c)

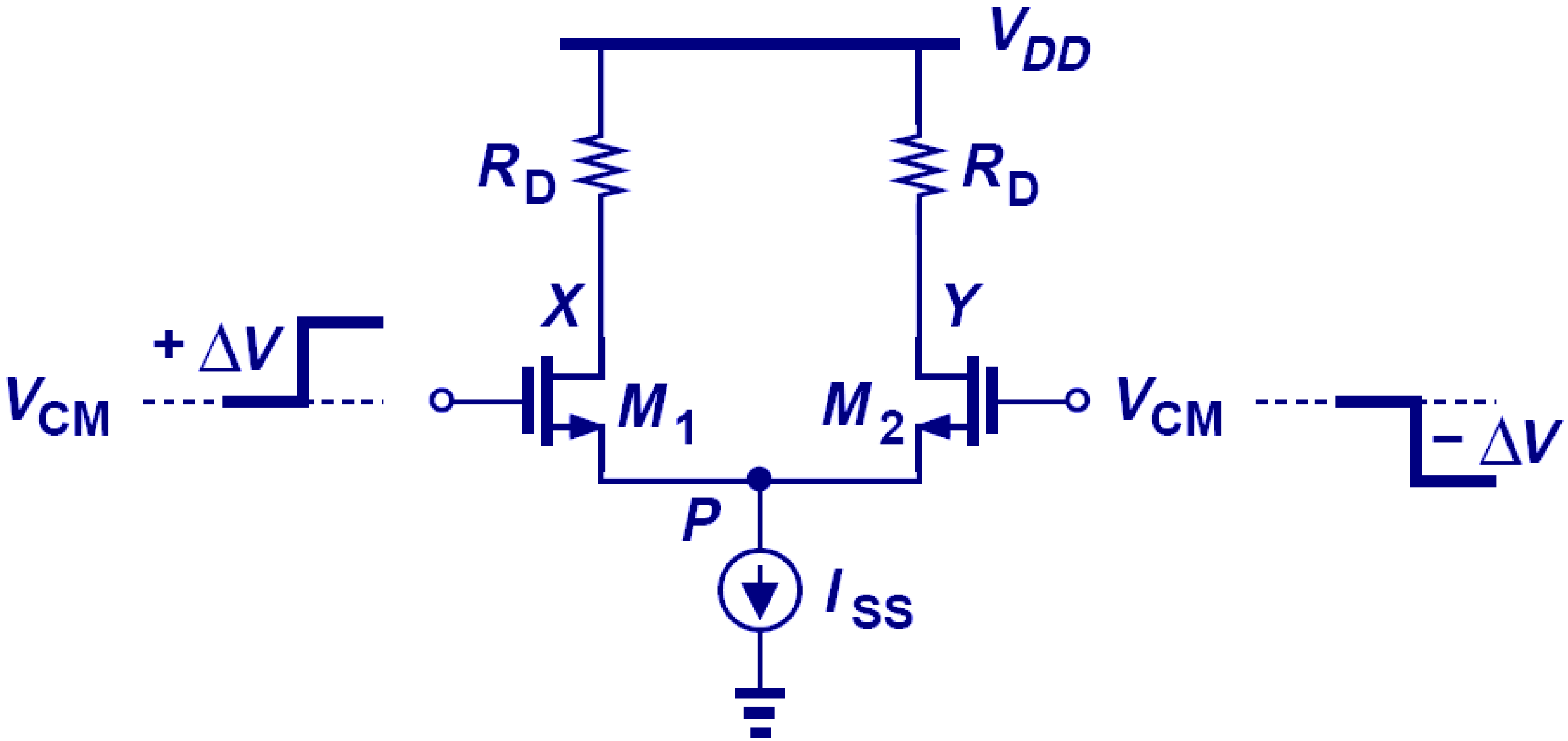
Small-Signal Response



$$\Delta V_P = 0$$
$$A_v = -g_m R_D$$

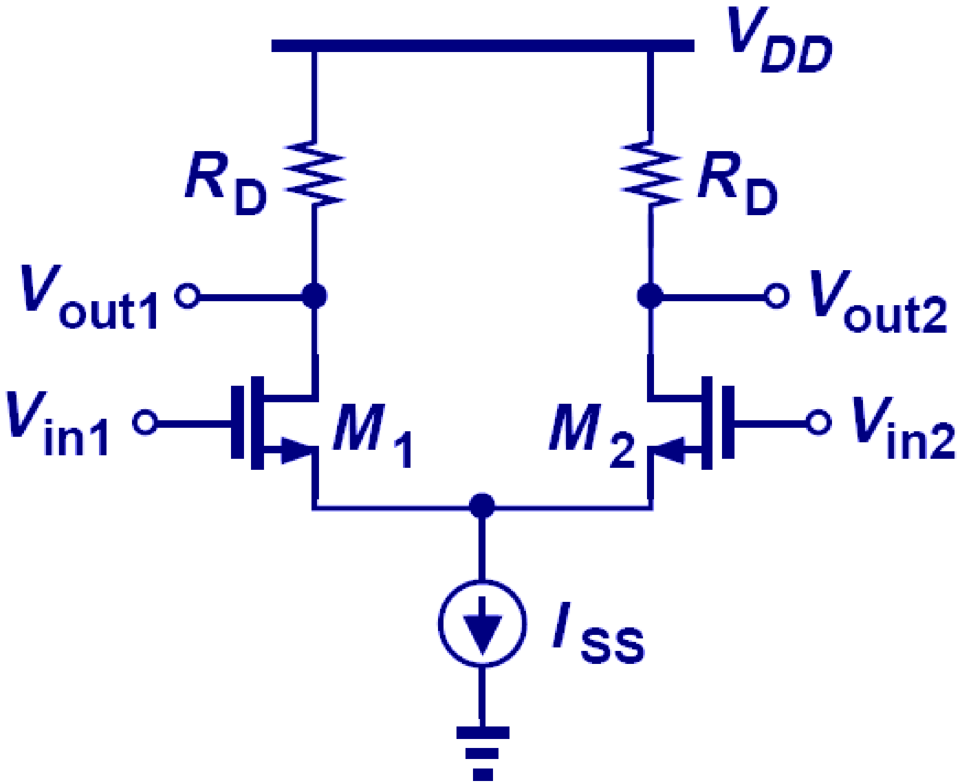
- Similar to its bipolar counterpart, the MOS differential pair exhibits the same virtual ground node and small signal gain.

Power and Gain Tradeoff



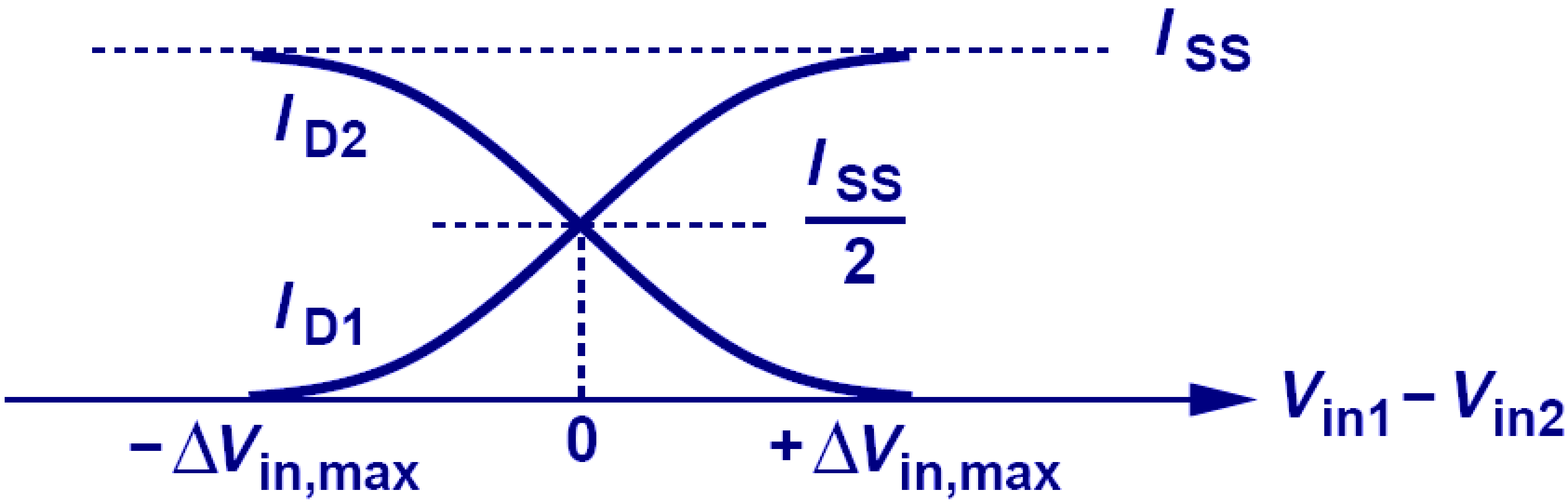
- In order to obtain the source gain as a CS stage, a MOS differential pair must dissipate twice the amount of current. This power and gain tradeoff is also echoed in its bipolar counterpart.

MOS Differential Pair's Large-Signal Response



$$I_{D1} - I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{in1} - V_{in2}\right)\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - \left(V_{in1} - V_{in2}\right)^2}$$

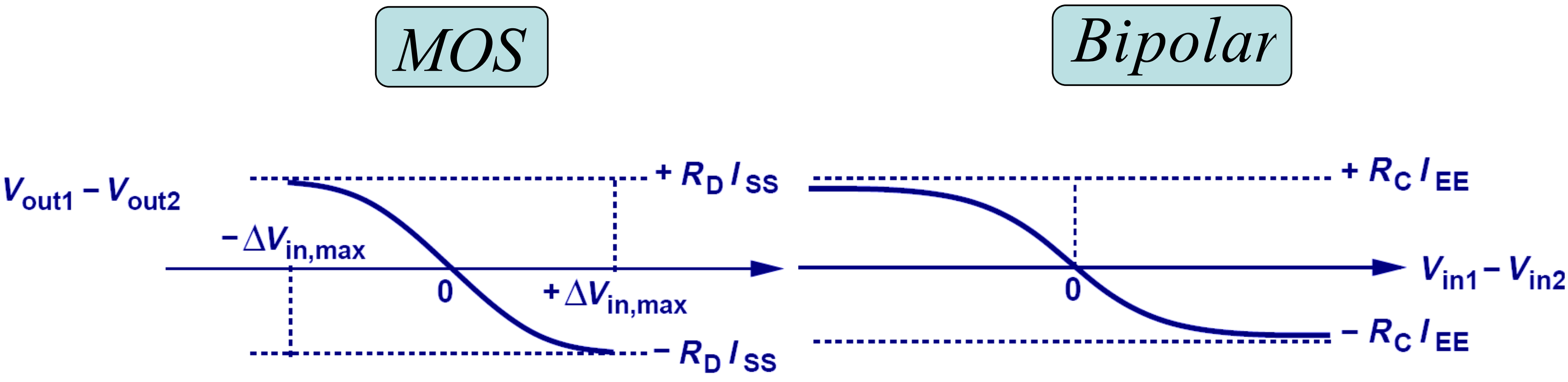
Maximum Differential Input Voltage



$$\left|V_{in1} - V_{in2}\right|_{\max} = \sqrt{2}\left(V_{GS} - V_{TH}\right)_{equil}$$

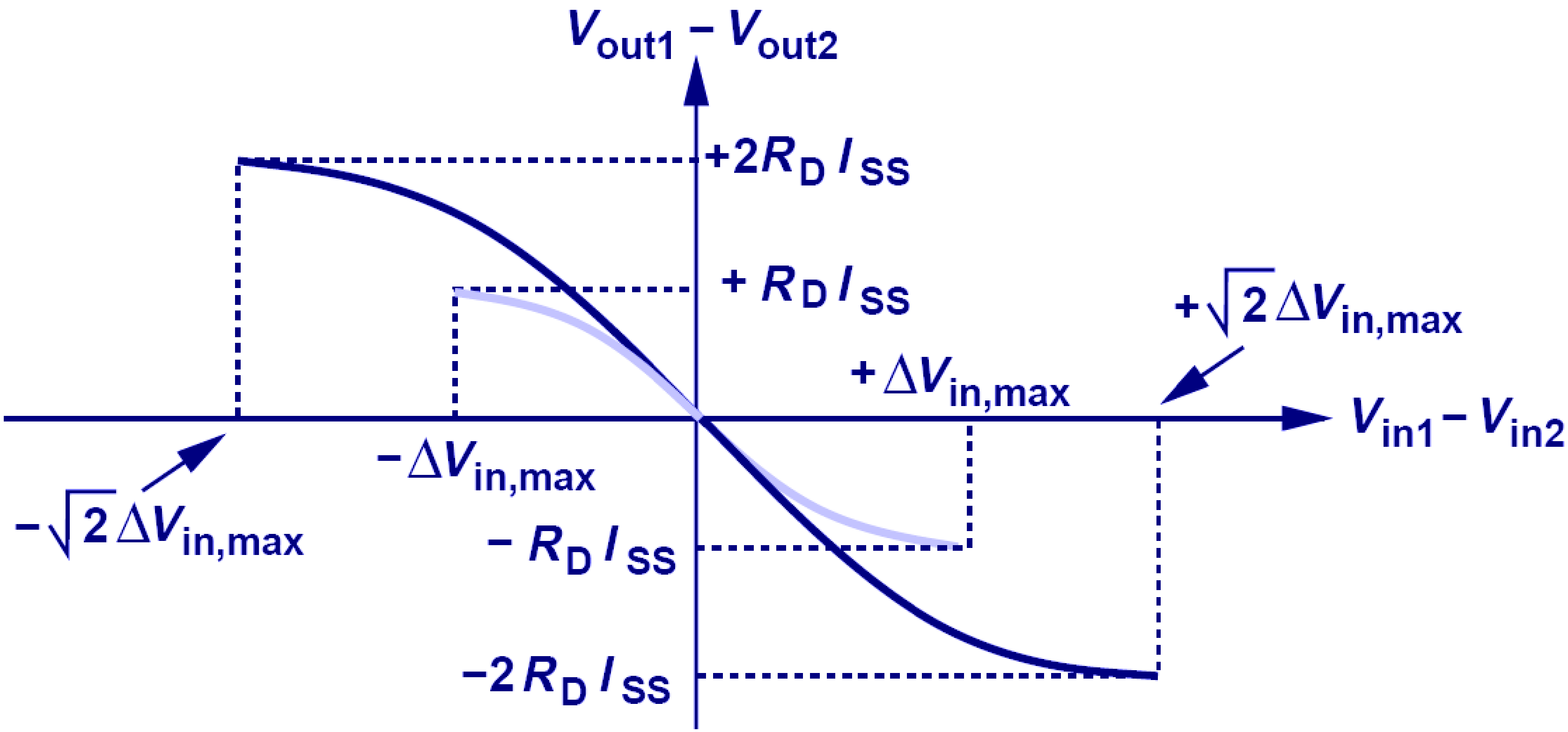
- **There exists a finite differential input voltage that completely steers the tail current from one transistor to the other. This value is known as the maximum differential input voltage.**

Contrast Between MOS and Bipolar Differential Pairs



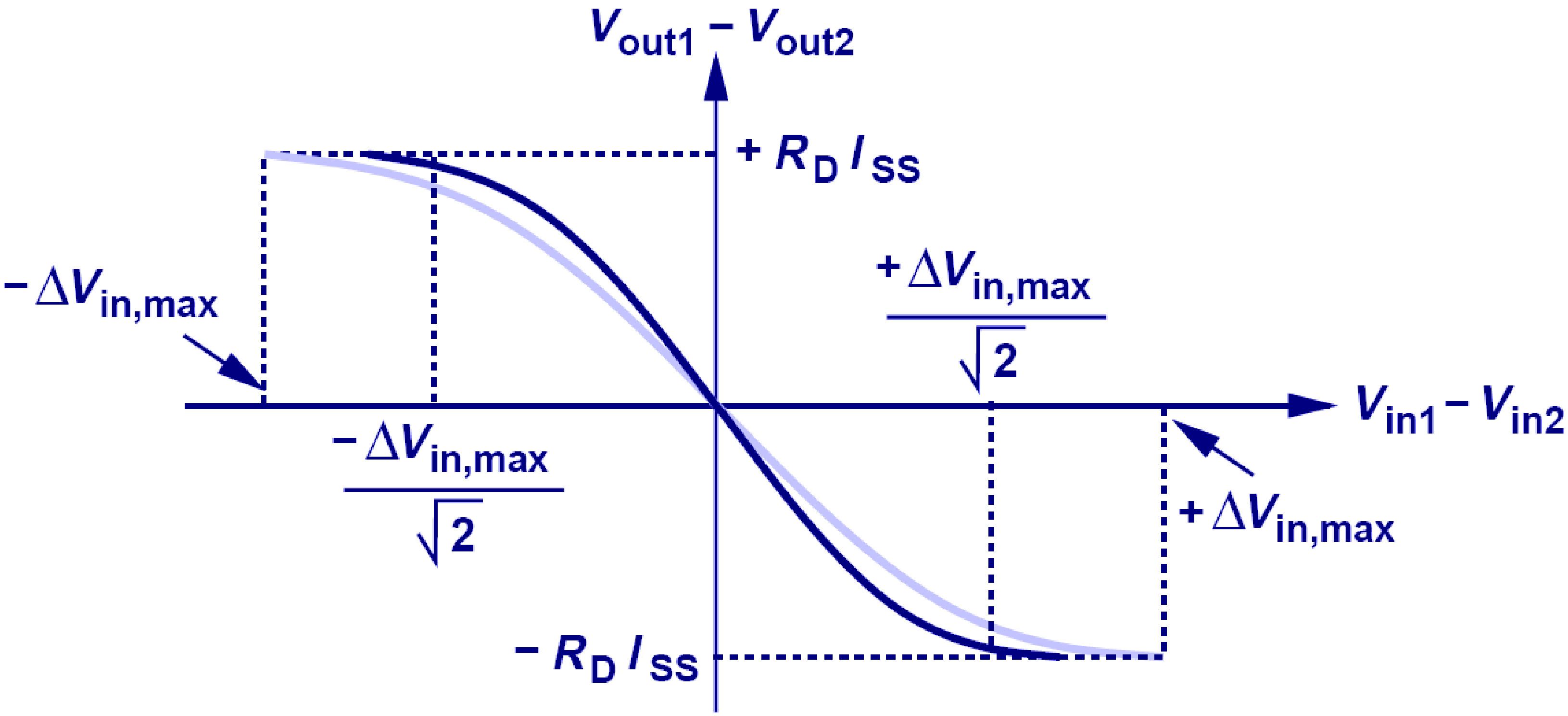
- In a MOS differential pair, there exists a finite differential input voltage to completely switch the current from one transistor to the other, whereas, in a bipolar pair that voltage is infinite.

The effects of Doubling the Tail Current



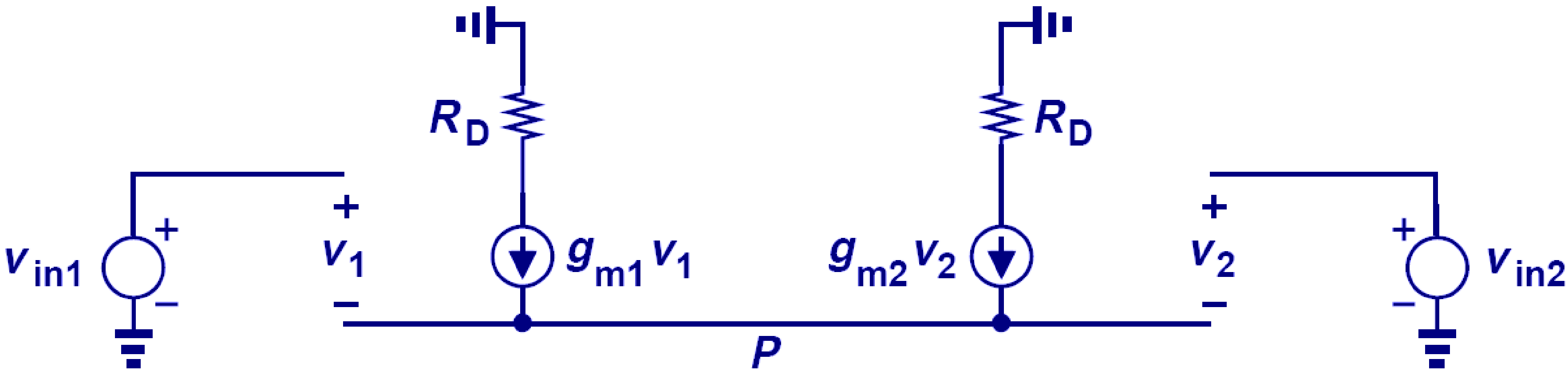
- Since I_{SS} is doubled and W/L is unchanged, the equilibrium overdrive voltage for each transistor must increase by $\sqrt{2}$ to accommodate this change, thus $\Delta V_{in,max}$ increases by $\sqrt{2}$ as well. Moreover, since I_{SS} is doubled, the differential output swing will double.

The effects of Doubling W/L



- **Since W/L is doubled and the tail current remains unchanged, the equilibrium overdrive voltage will be lowered by $\sqrt{2}$ to accommodate this change, thus $\Delta V_{in,max}$ will be lowered by $\sqrt{2}$ as well. Moreover, the differential output swing will remain unchanged since neither I_{SS} nor R_D has changed**

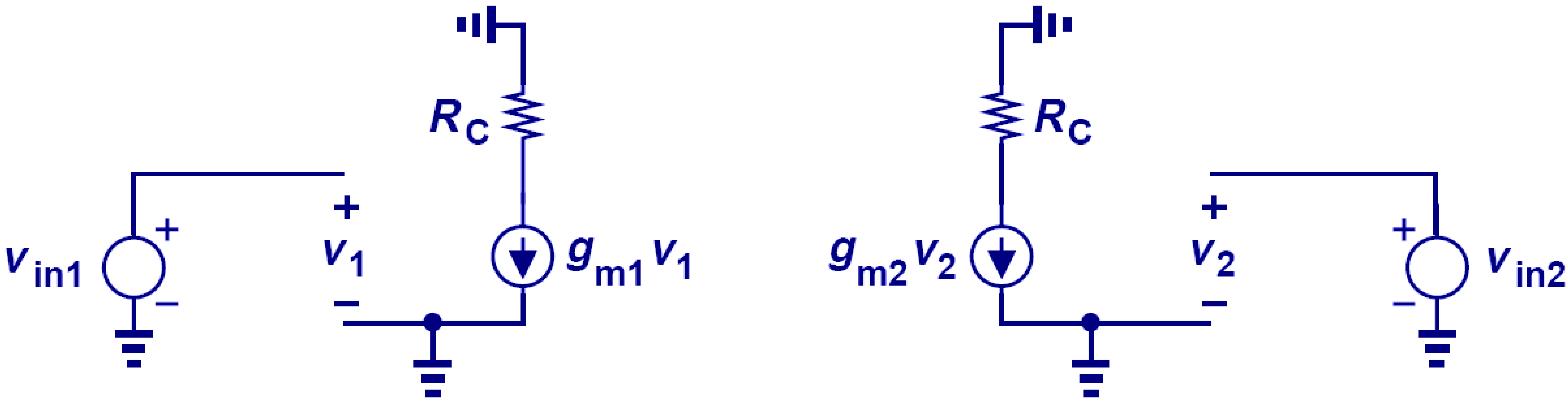
Small-Signal Analysis of MOS Differential Pair



$$I_{D1} - I_{D2} \approx \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{in2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} = \sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}(V_{in1} - V_{in2})$$

- **When the input differential signal is small compared to $4I_{SS}/\mu_n C_{ox}(W/L)$, the output differential current is linearly proportional to it, and small-signal model can be applied.**

Virtual Ground and Half Circuit

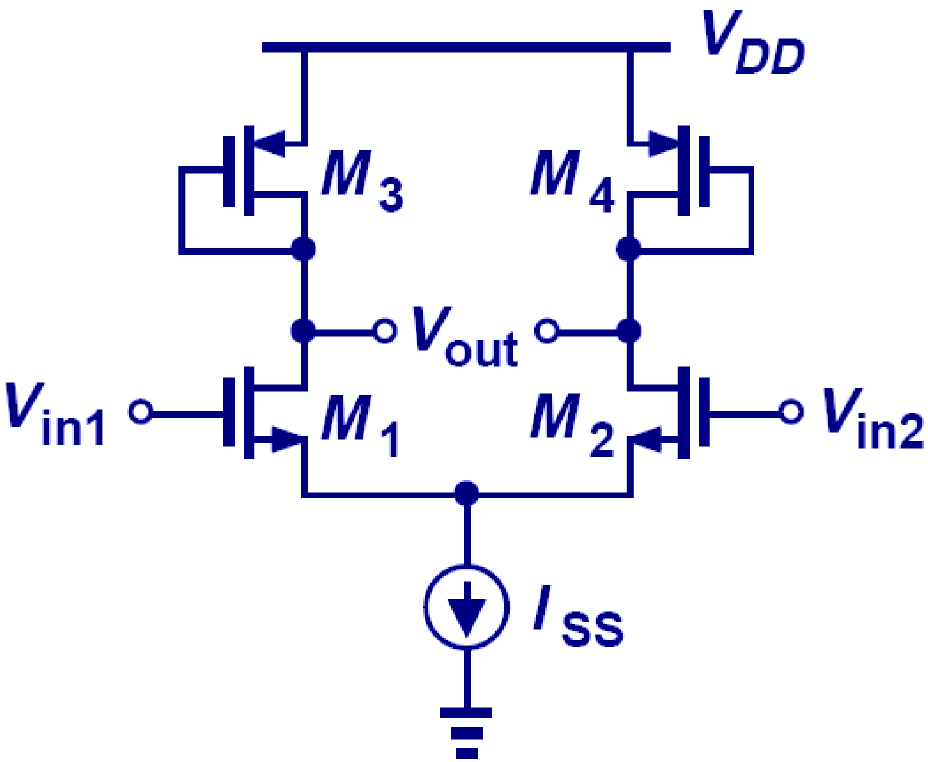


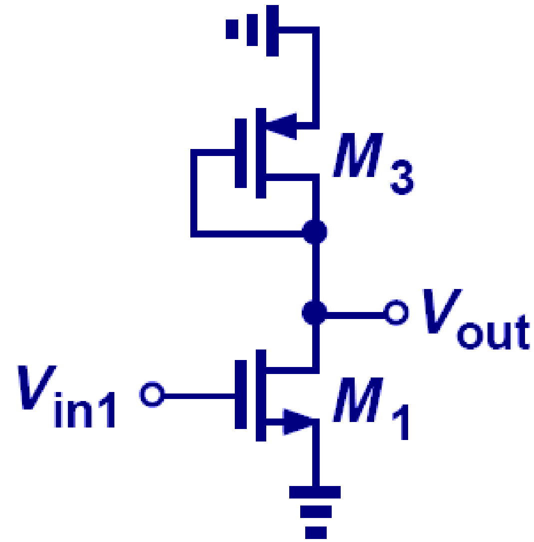
$$\Delta V_P = 0$$

$$A_v = -g_m R_C$$

- Applying the same analysis as the bipolar case, we will arrive at the same conclusion that node P will not move for small input signals and the concept of half circuit can be used to calculate the gain.

MOS Differential Pair Half Circuit Example I

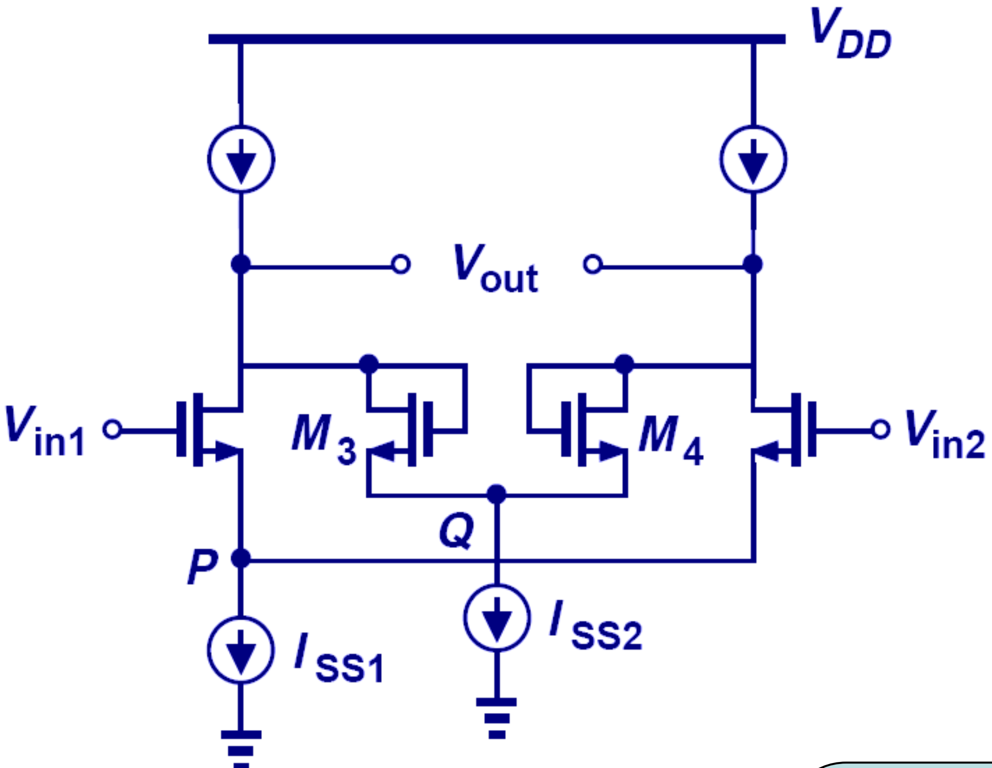


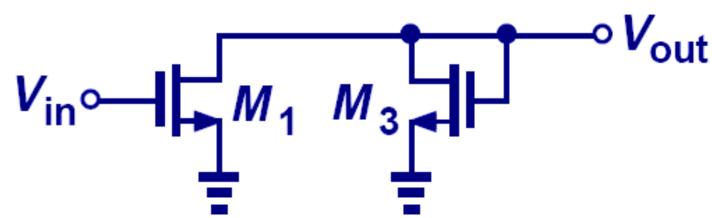


$$\lambda \neq 0$$

$$A_v = -g_{m1}\left(\frac{1}{g_{m3}} \| r_{O3} \| r_{O1}\right)$$

MOS Differential Pair Half Circuit Example II

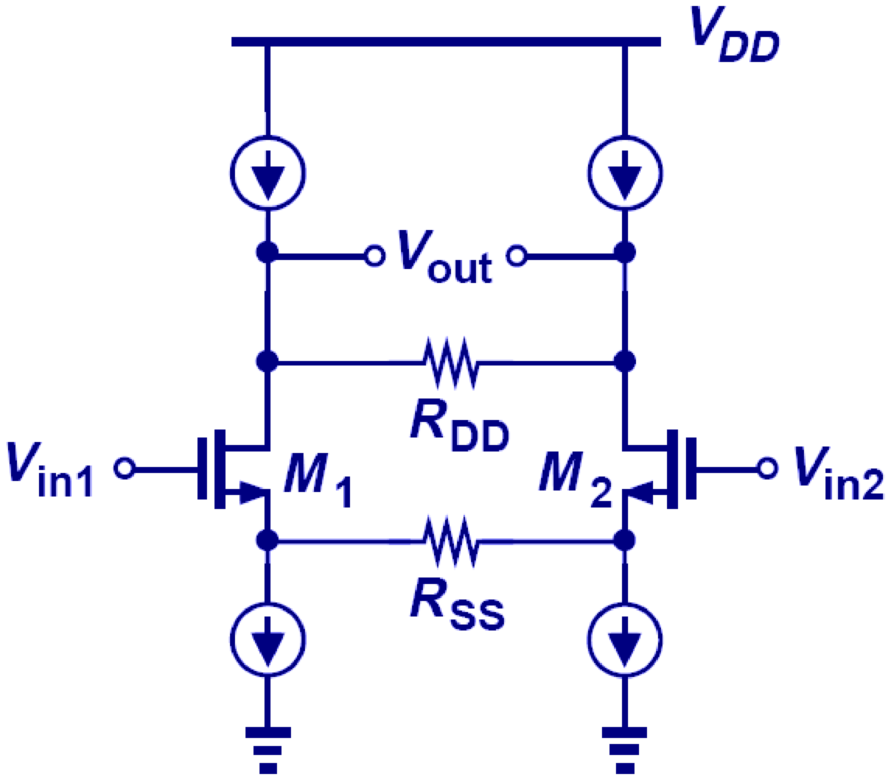


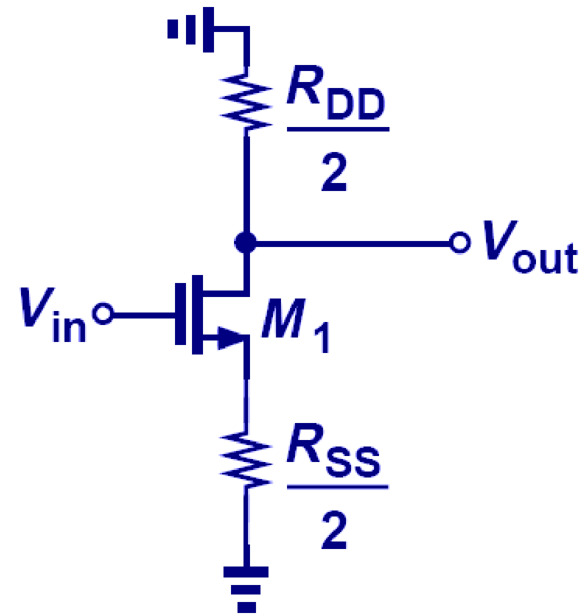


$$\lambda = 0$$

$$A_v = -\frac{g_{m1}}{g_{m3}}$$

MOS Differential Pair Half Circuit Example III

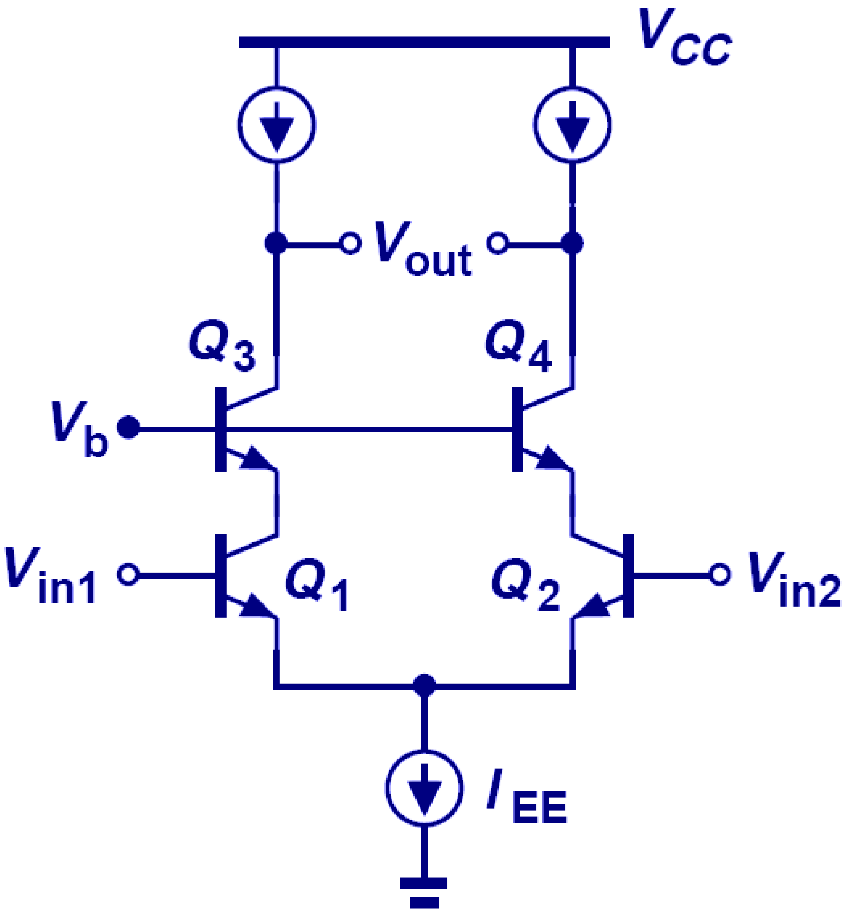


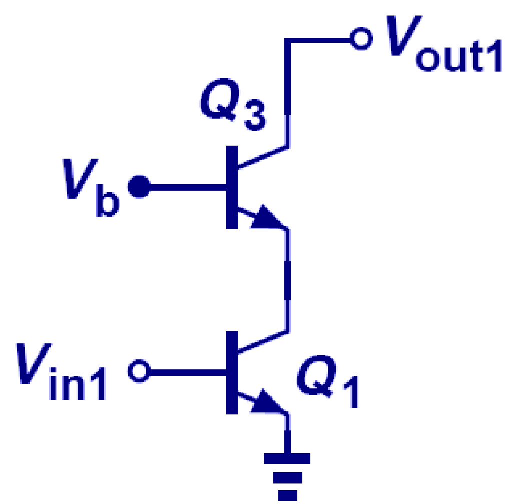


$$\lambda = 0$$

$$A_v = -\frac{R_{DD}/2}{R_{SS}/2 + 1/g_m}$$

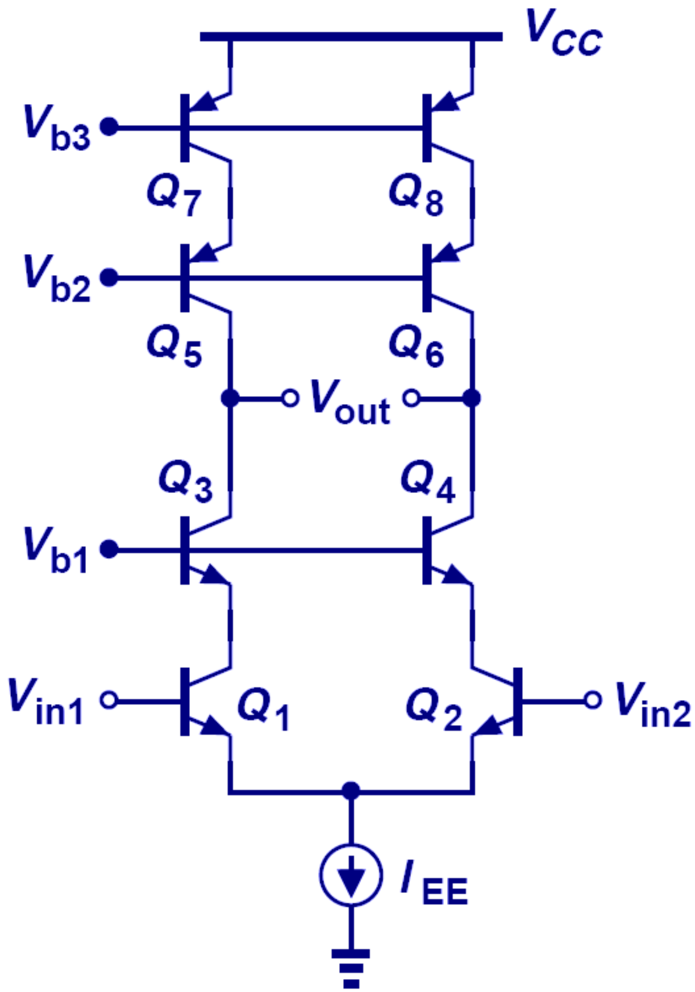
Bipolar Cascode Differential Pair

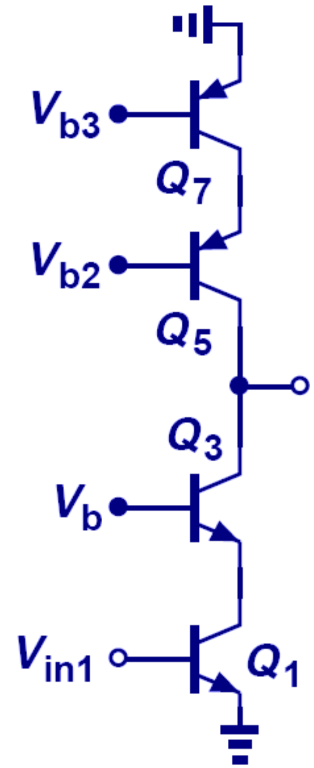




$$A_v = -g_{m1}\left[g_{m3}\left(r_{O1} \parallel r_{\pi 3}\right)r_{O3} + r_{O1}\right]$$

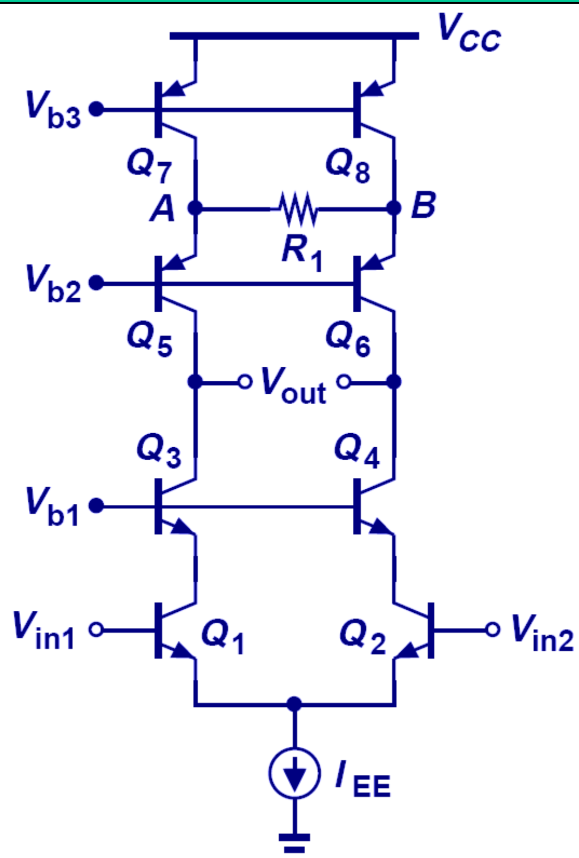
Bipolar Telescopic Cascode

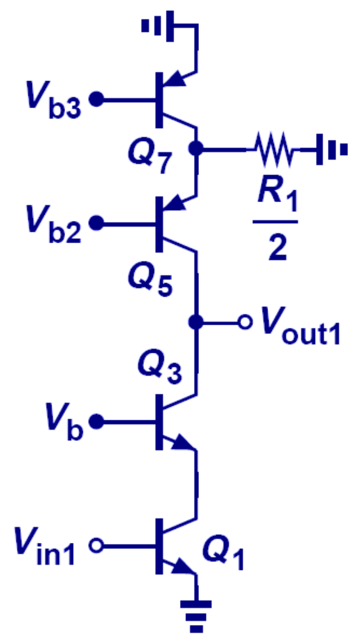




$$A_v \approx -g_{m1}\left[g_{m3}r_{O3}(r_{O1} \parallel r_{\pi3})\right] \parallel \left[g_{m5}r_{O5}(r_{O7} \parallel r_{\pi5})\right]$$

Example: Bipolar Telescopic Parasitic Resistance

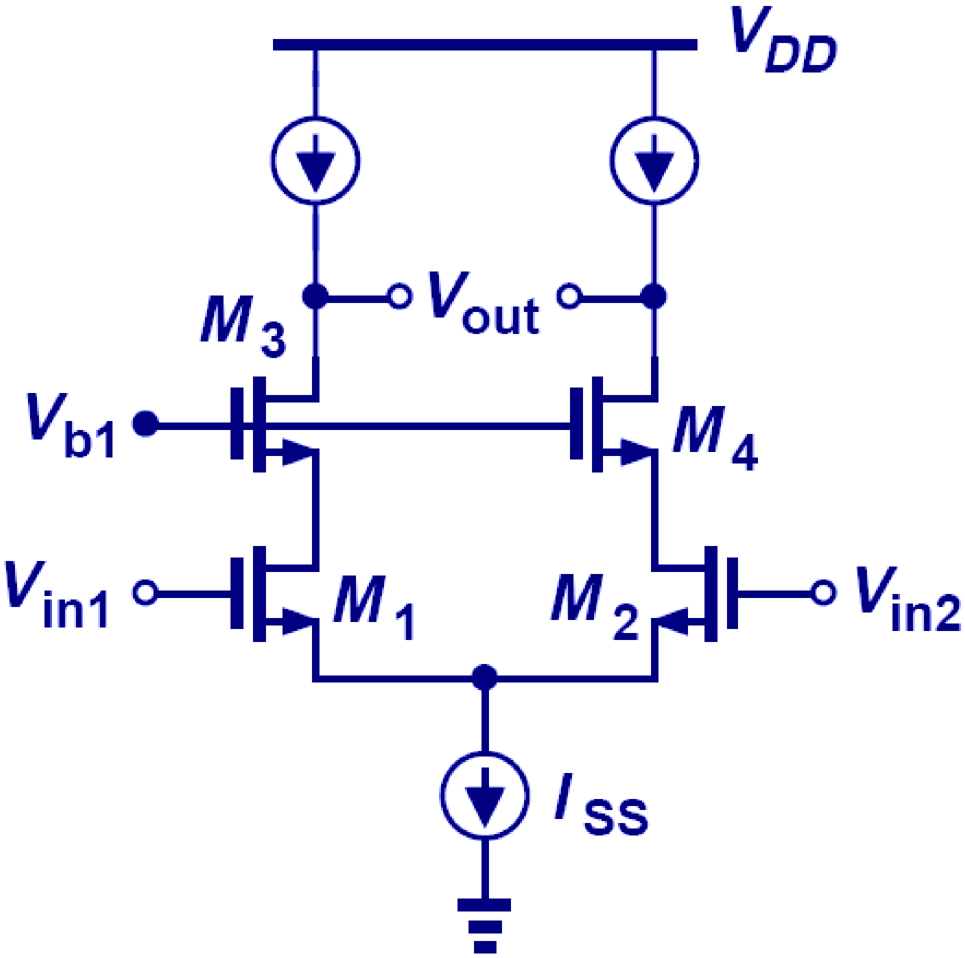


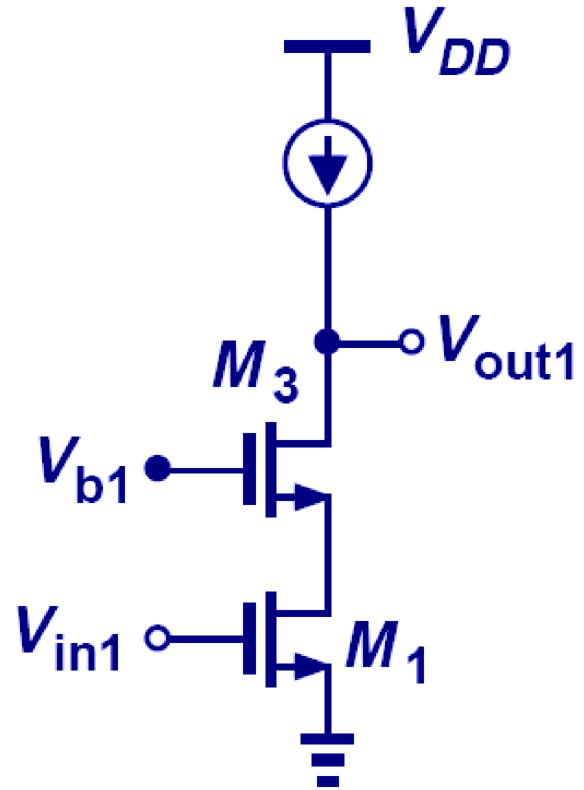


$$R_{op} = r_{O5}\left[1 + g_{m5}\left(r_{O7} \parallel r_{\pi 5} \parallel \frac{R_1}{2}\right)\right] + r_{O7} \parallel r_{\pi 5} \parallel \frac{R_1}{2}$$

$$A_v = -g_{m1}\left[g_{m3} r_{O3}(r_{O1} \parallel r_{\pi 3})\right] \parallel R_{op}$$

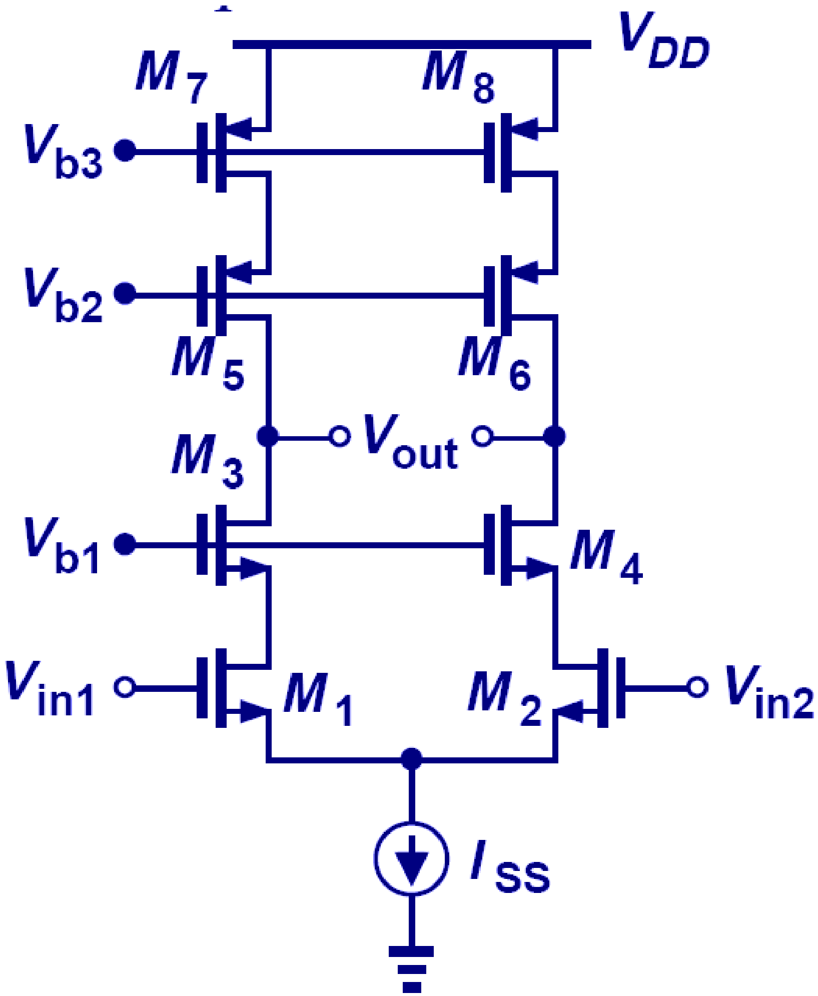
MOS Cascode Differential Pair

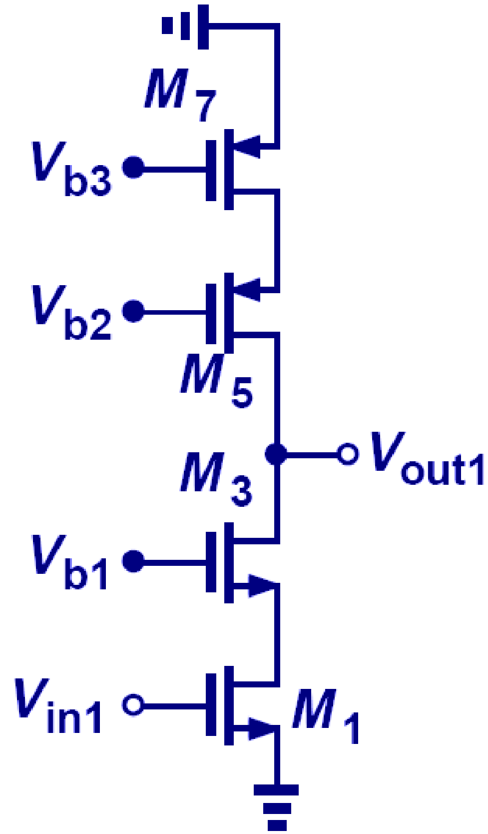




$$A_v \approx -g_{m1} r_{O3} g_{m3} r_{O1}$$

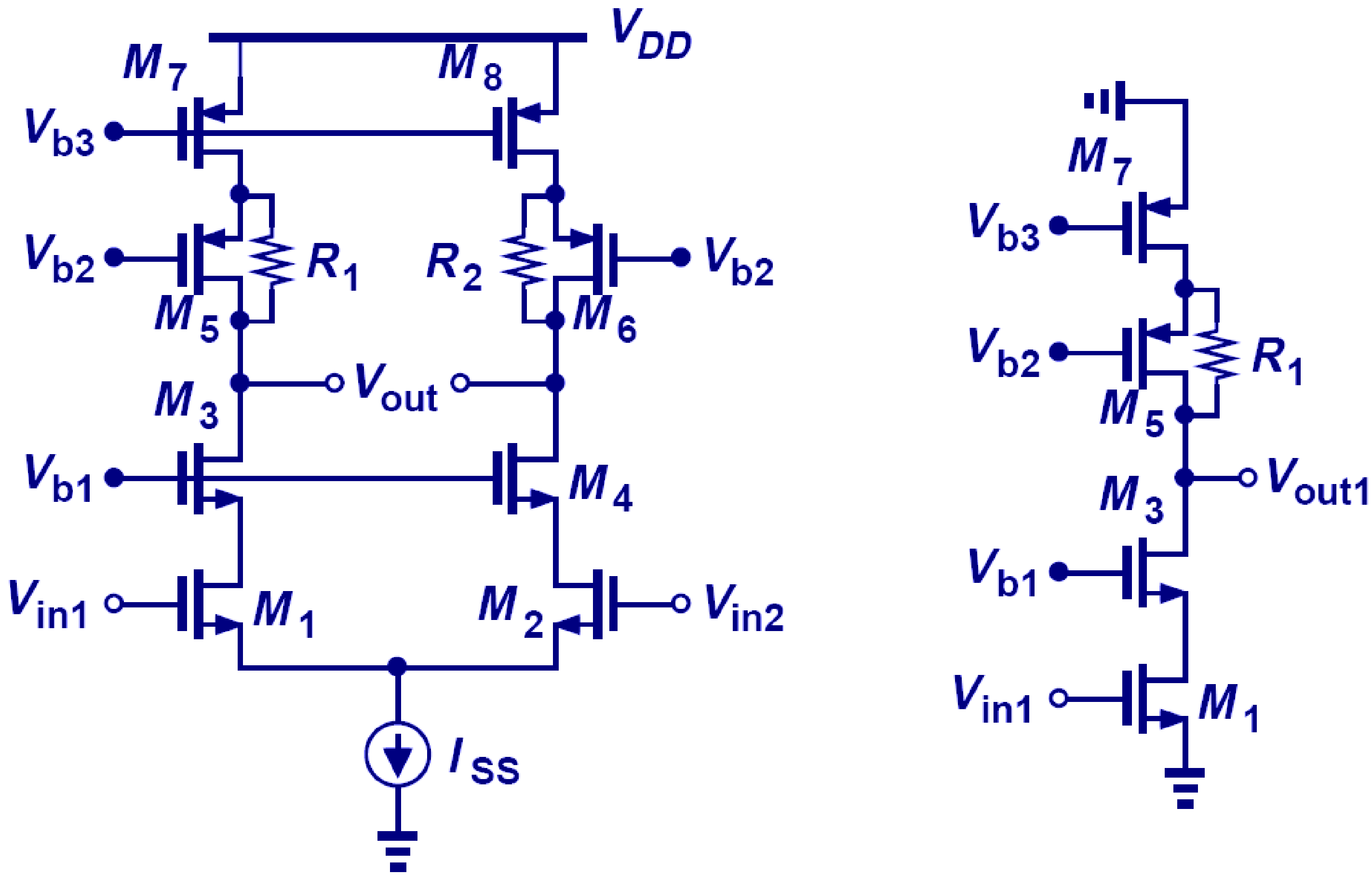
MOS Telescopic Cascode





$$A_v \approx -g_{m1}\left[\left(g_{m3}r_{O3}r_{O1}\right) \| \left(g_{m5}r_{O5}r_{O7}\right)\right]$$

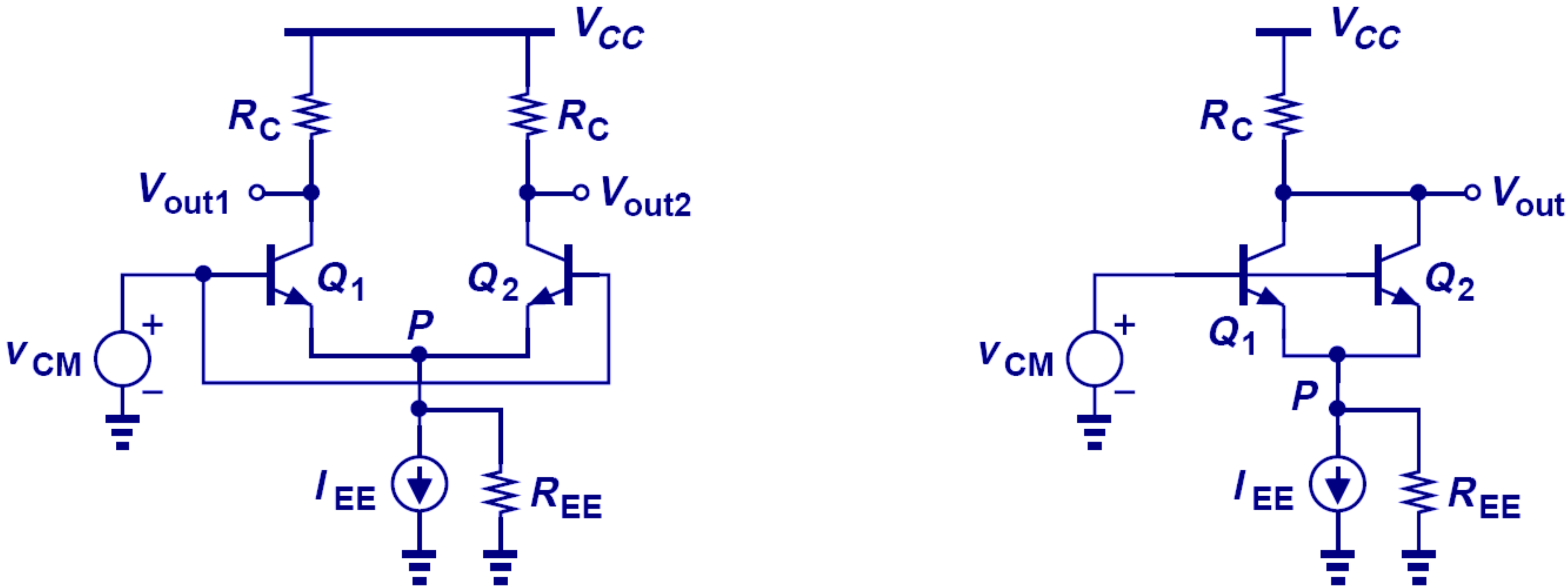
Example: MOS Telescopic Parasitic Resistance



$$R_{op} = r_{O5} \parallel [R_1(1+g_{m5}r_{O7})+r_{O7}]$$

$$A_v \approx -g_{m1}(R_{op} \parallel r_{O3}g_{m3}r_{O1})$$

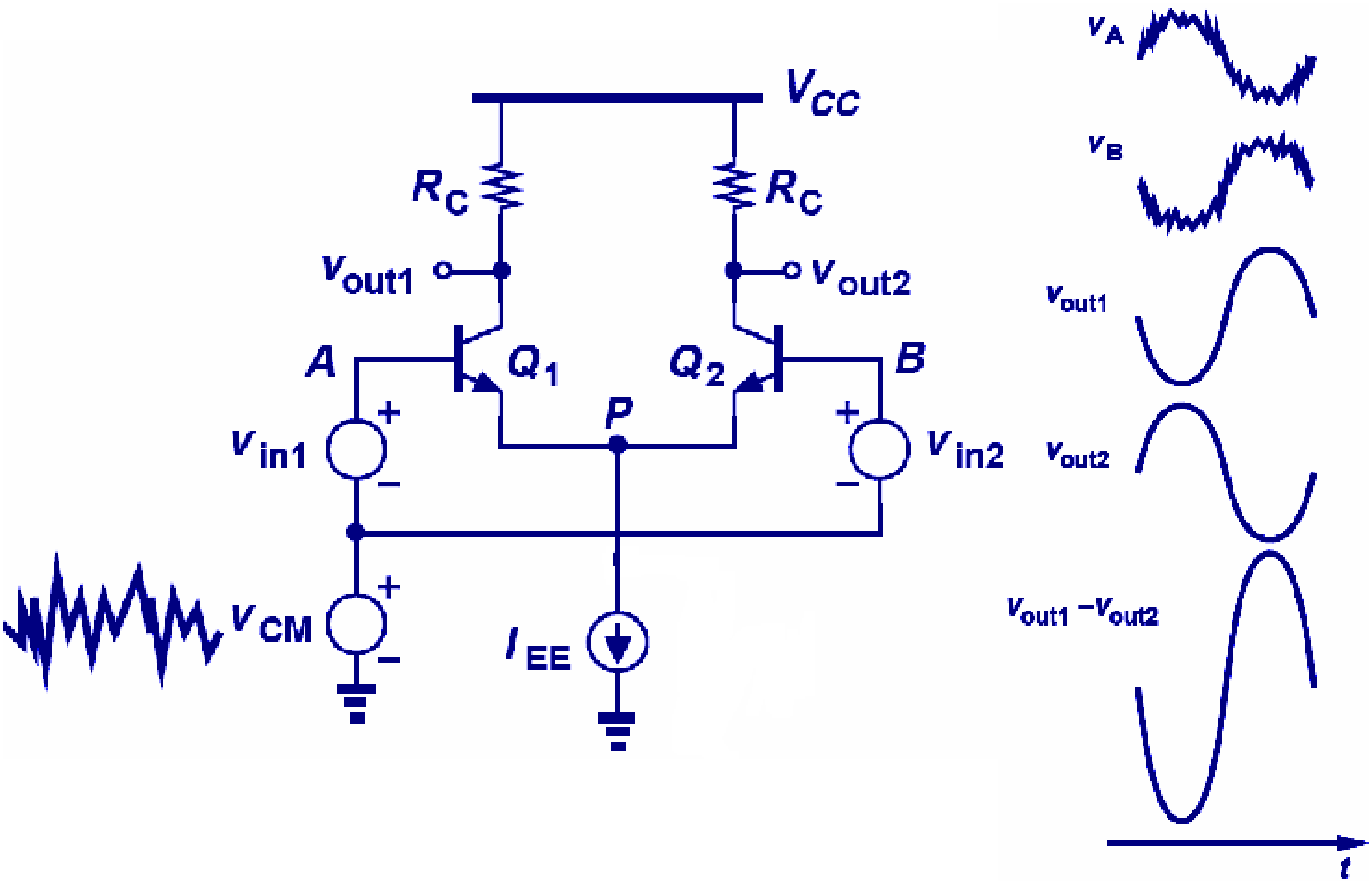
Effect of Finite Tail Impedance



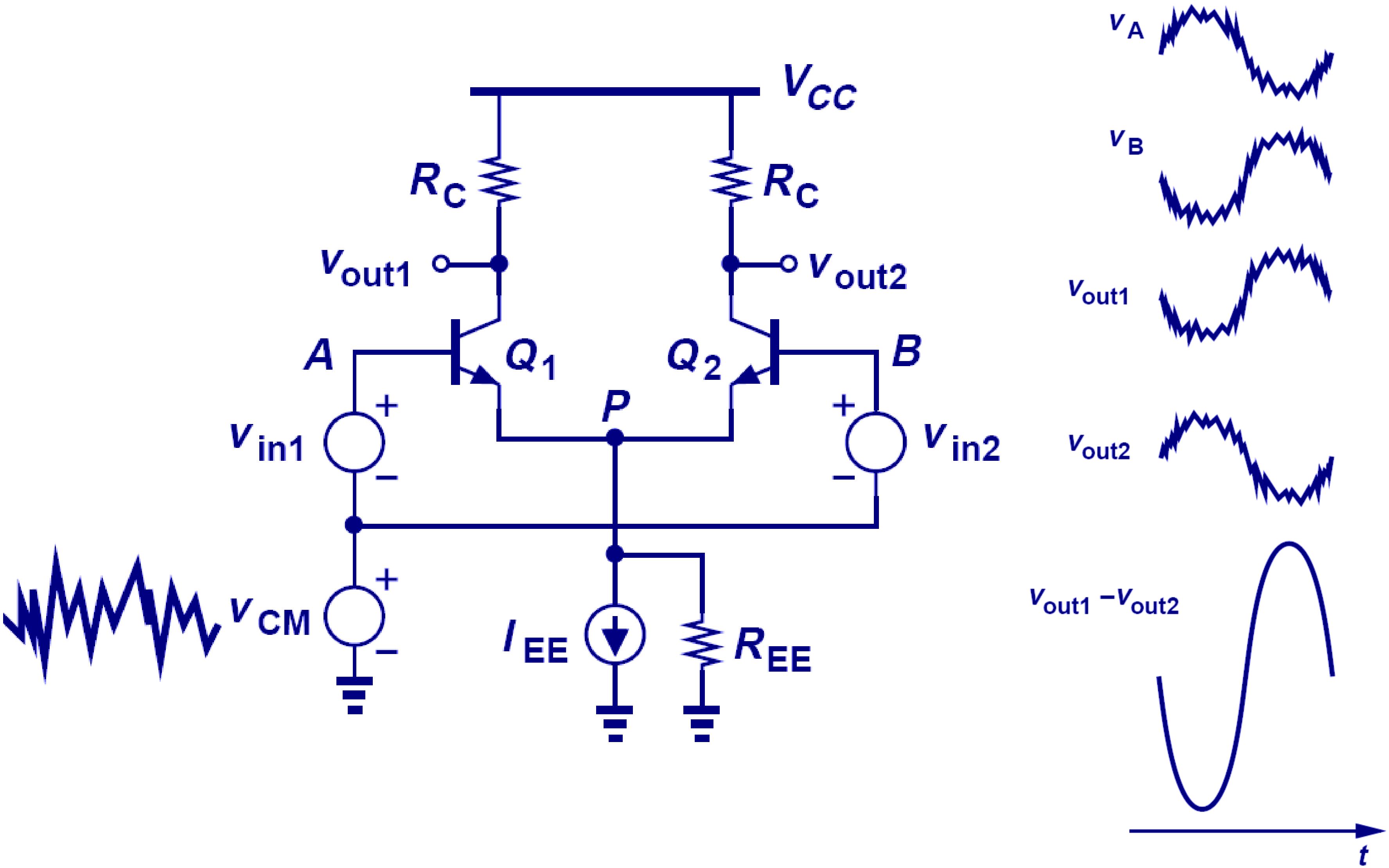
$$\frac{\Delta V_{out,CM}}{\Delta V_{in,CM}} = -\frac{R_C/2}{R_{EE}+1/2g_m}$$

- **If the tail current source is not ideal, then when a input CM voltage is applied, the currents in Q_1 and Q_2 and hence output CM voltage will change.**

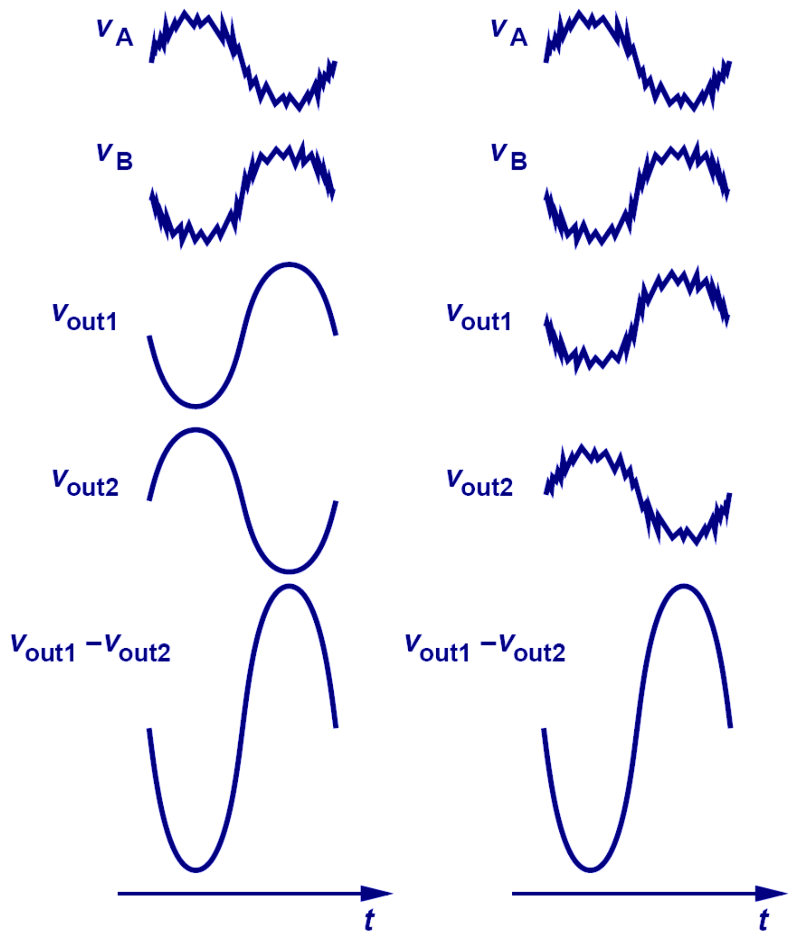
Input CM Noise with Ideal Tail Current



Input CM Noise with Non-ideal Tail Current

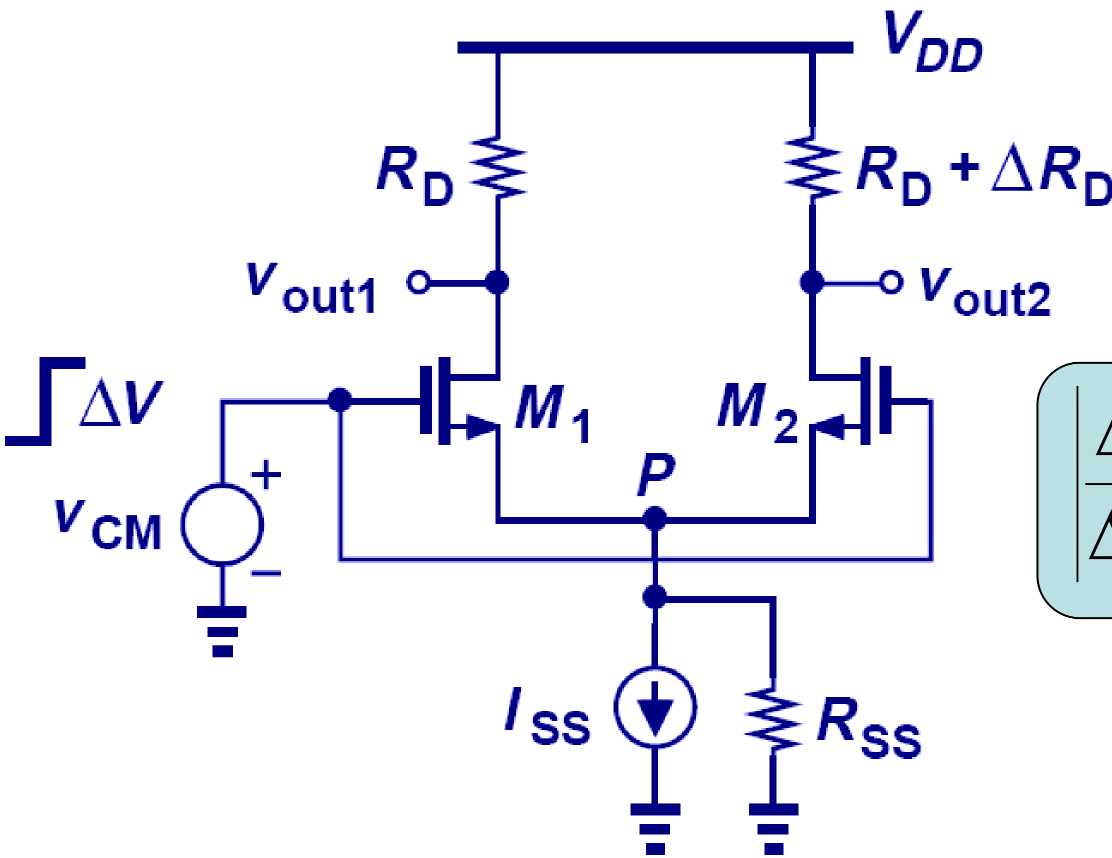


Comparison



- **As it can be seen, the differential output voltages for both cases are the same. So for small input CM noise, the differential pair is not affected.**

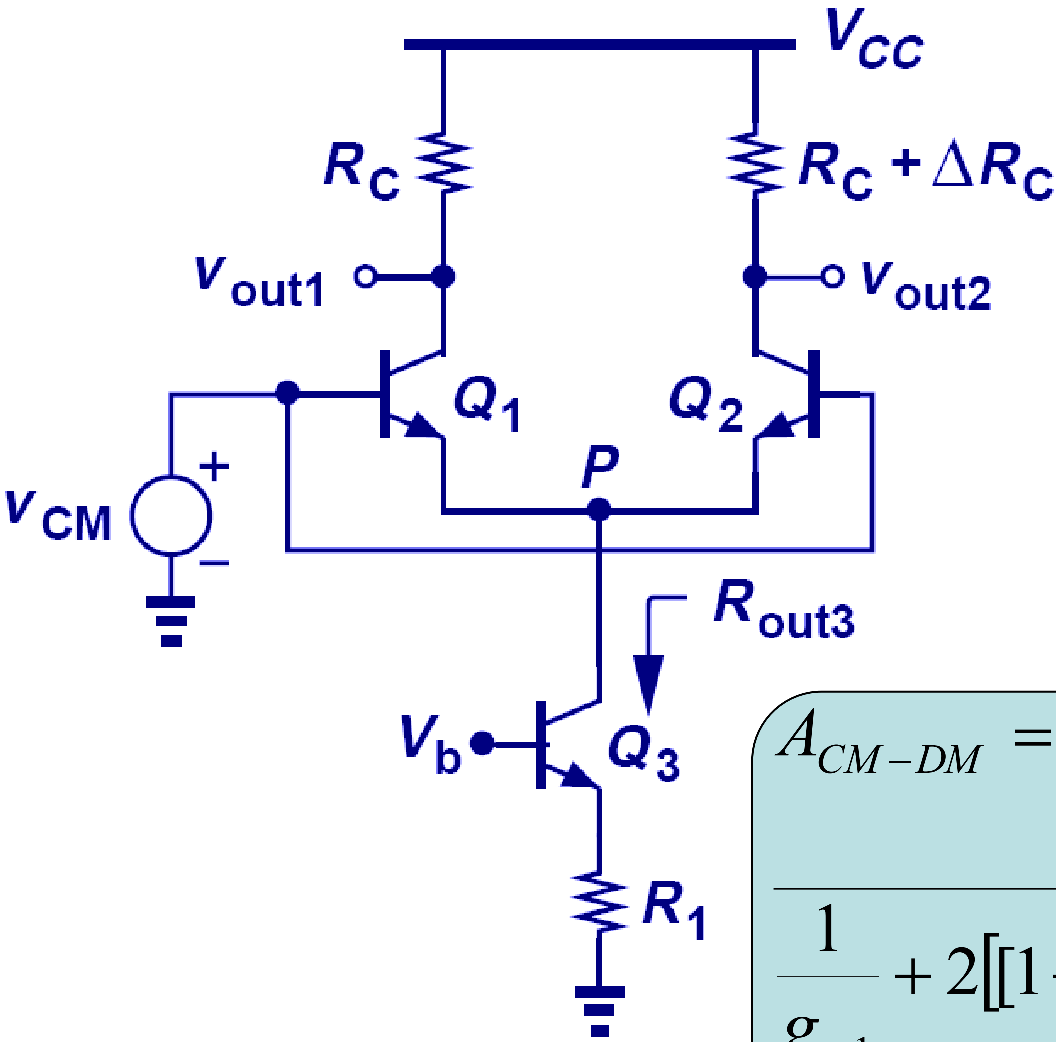
CM to DM Conversion, $A_{CM\text{-}DM}$



$$\left|\frac{\Delta V_{out}}{\Delta V_{CM}}\right| = \frac{\Delta R_D}{1/g_m + 2R_{EE}}$$

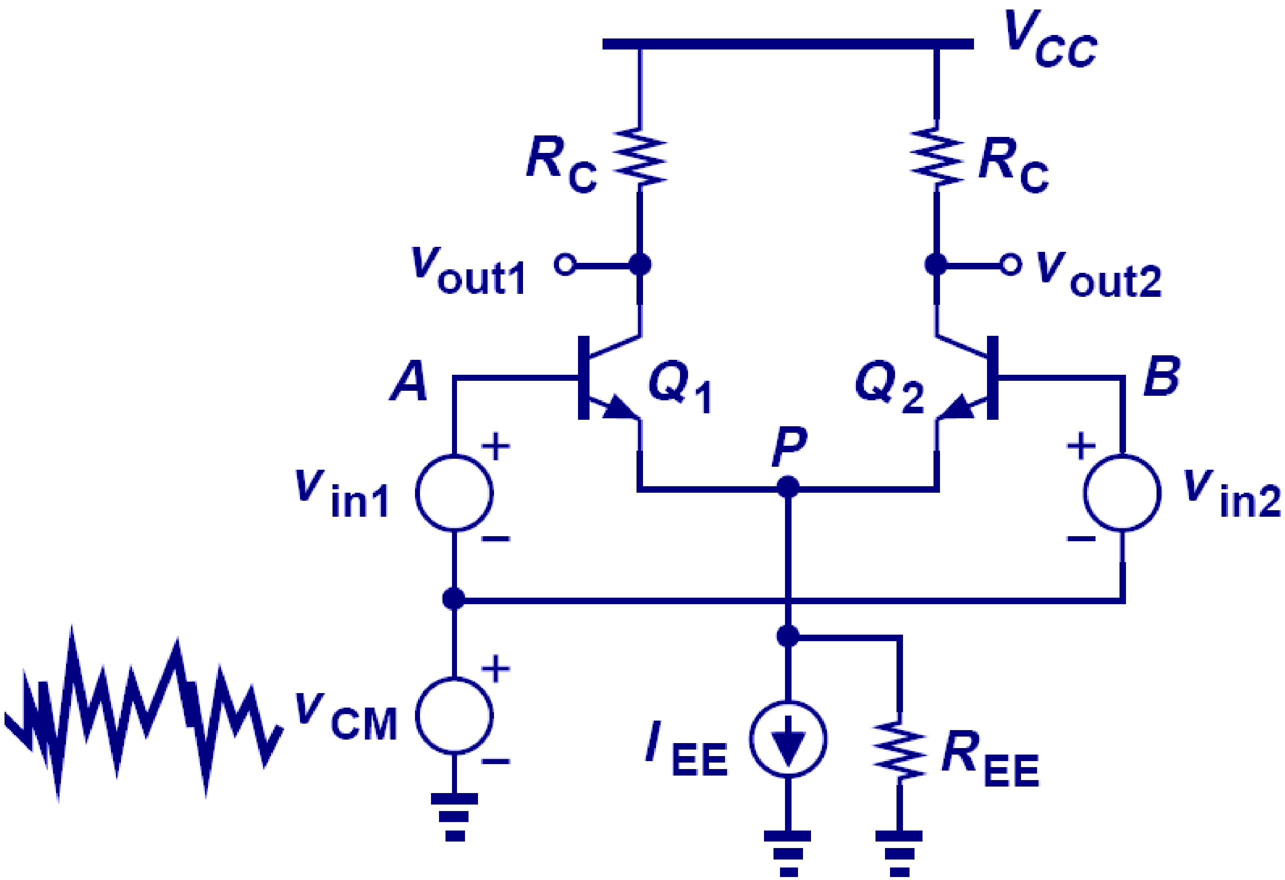
- **If finite tail impedance and asymmetry are both present, then the differential output signal will contain a portion of input common-mode signal.**

Example: A_{CM-DM}



$$A_{CM-DM} = \frac{\Delta R_C}{\frac{1}{g_{m1}} + 2[[1 + g_{m3}(R_1 \parallel r_{\pi 3})]r_{O3} + R_1 \parallel r_{\pi 3}]}$$

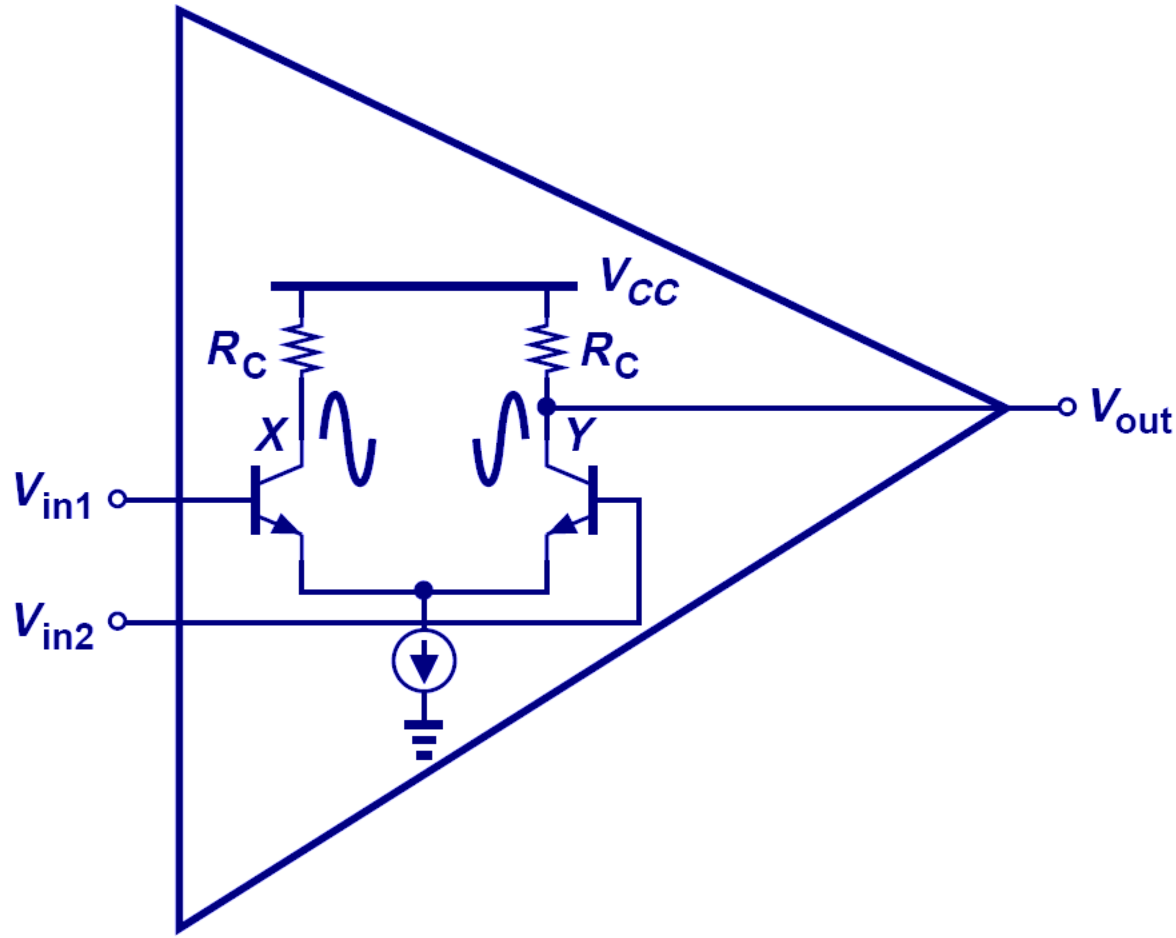
CMRR



$$CMRR = \frac{A_{DM}}{A_{CM-DM}}$$

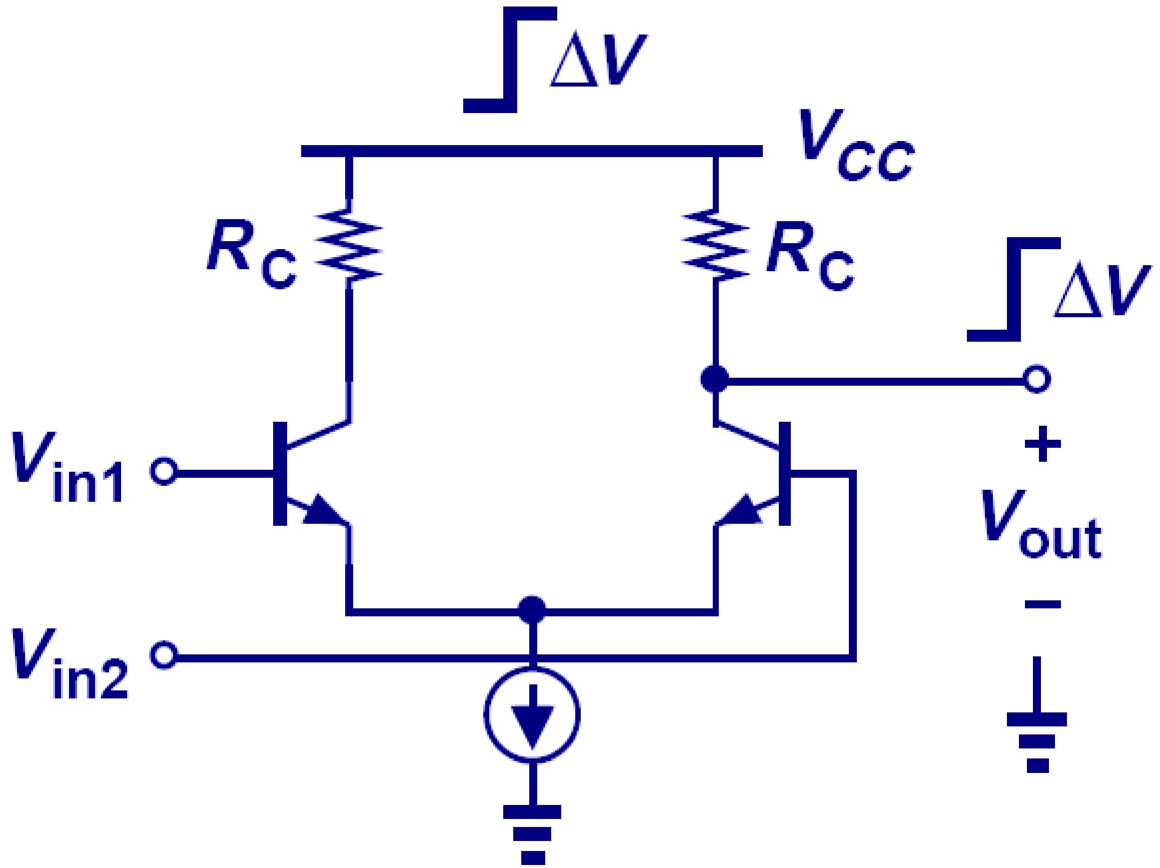
- **CMRR defines the ratio of wanted amplified differential input signal to unwanted converted input common-mode noise that appears at the output.**

Differential to Single-ended Conversion



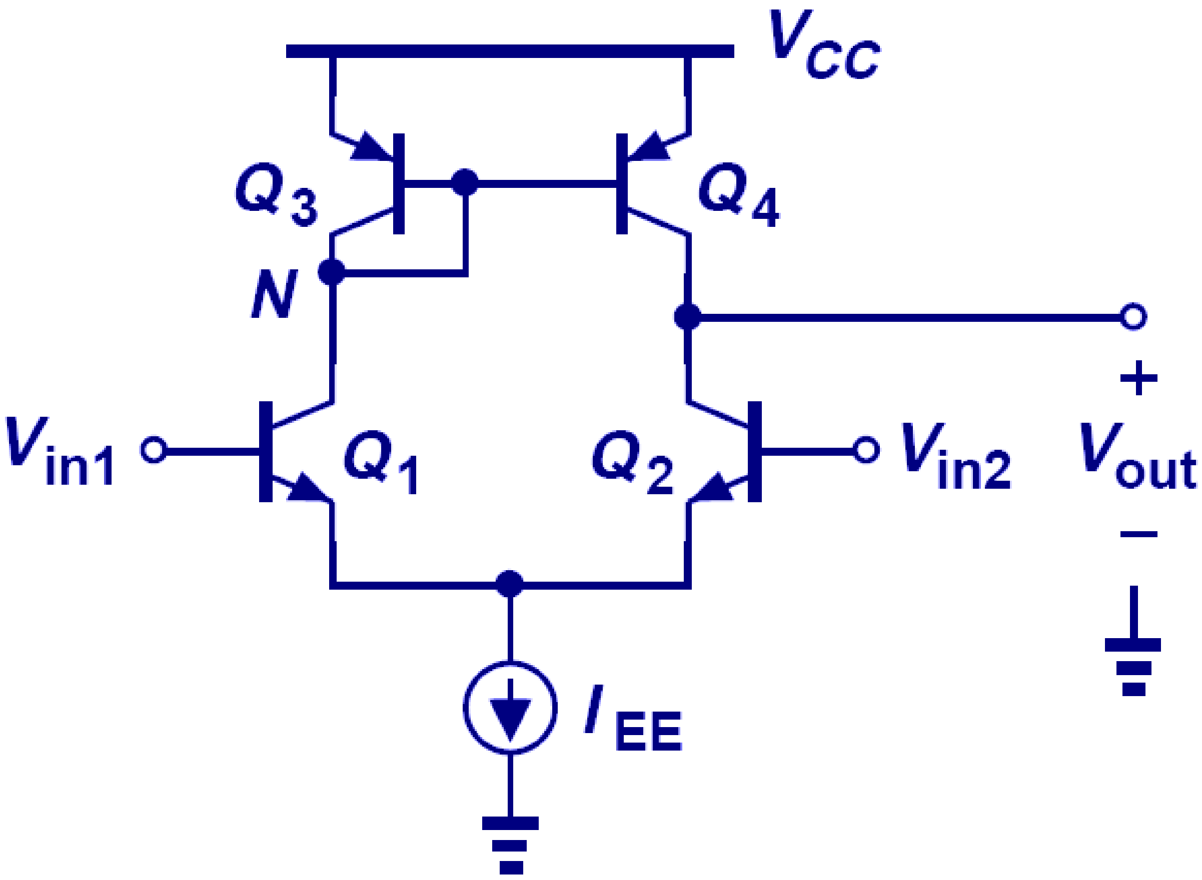
- Many circuits require a differential to single-ended conversion, however, the above topology is not very good.

Supply Noise Corruption



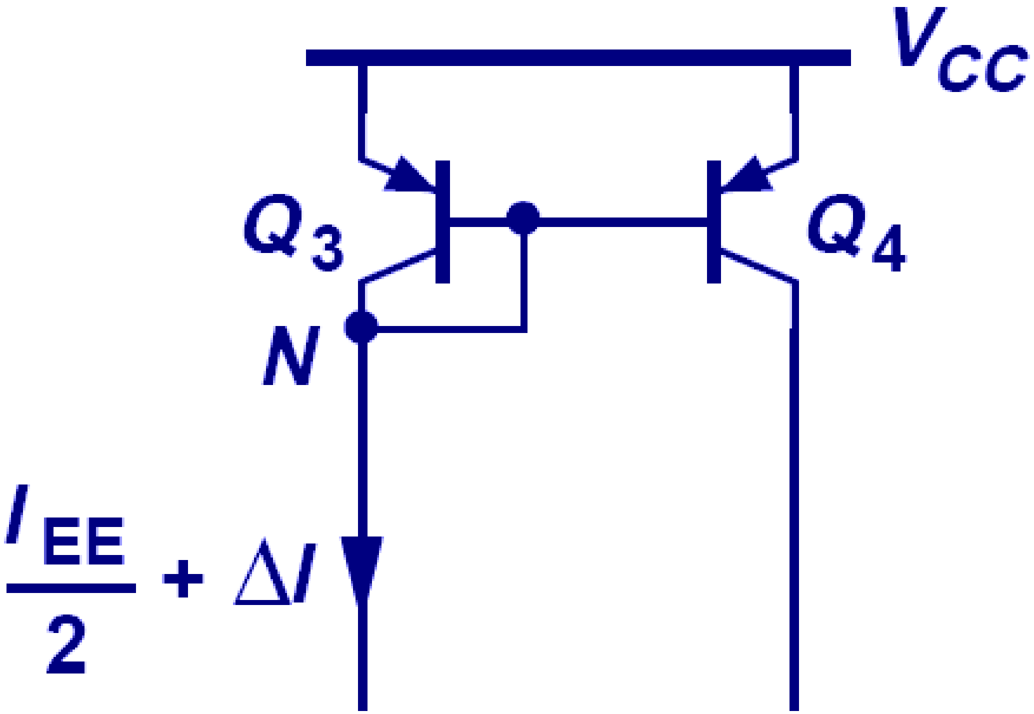
- The most critical drawback of this topology is supply noise corruption, since no common-mode cancellation mechanism exists. Also, we lose half of the signal.

Better Alternative



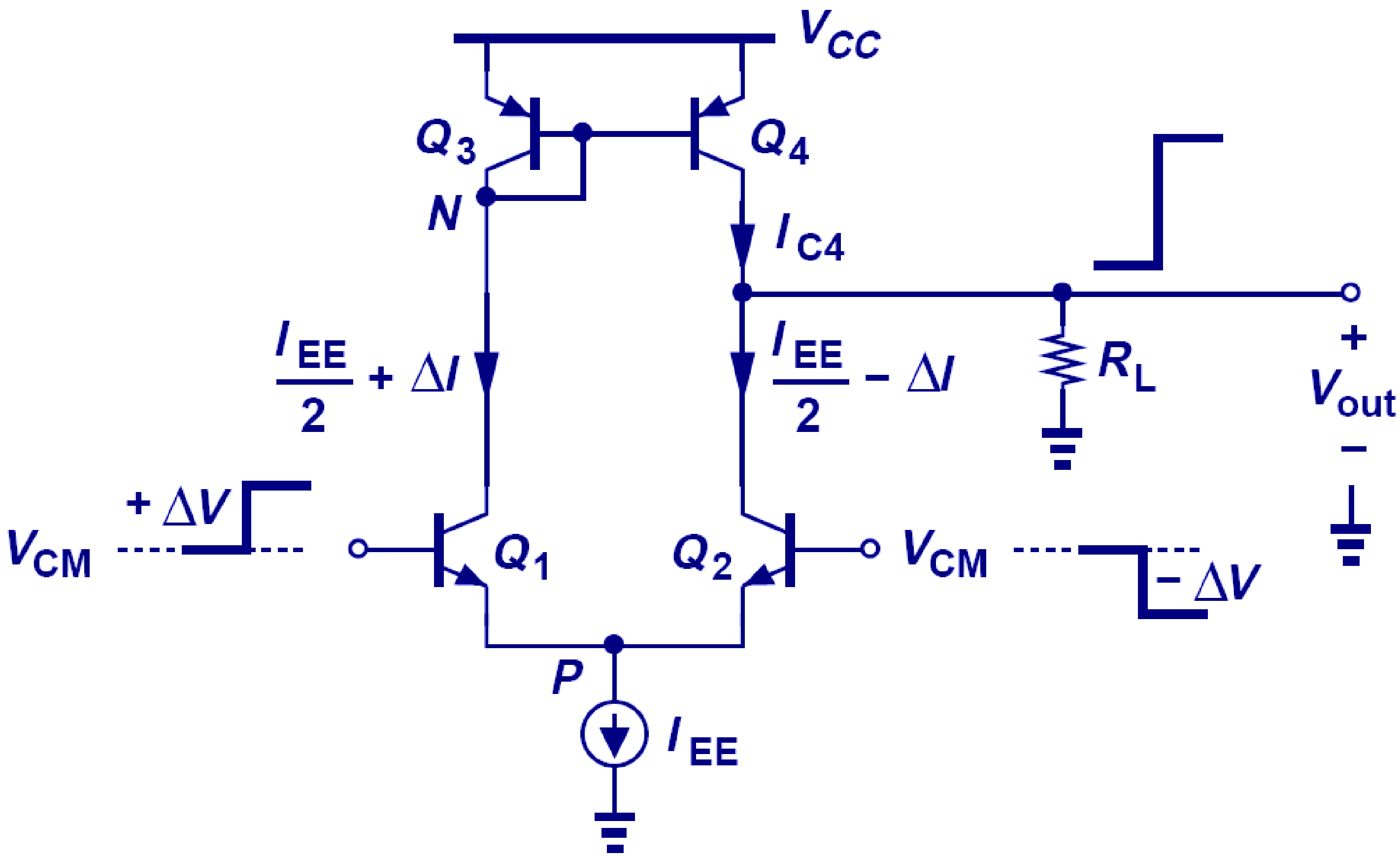
- **This circuit topology performs differential to single-ended conversion with no loss of gain.**

Active Load



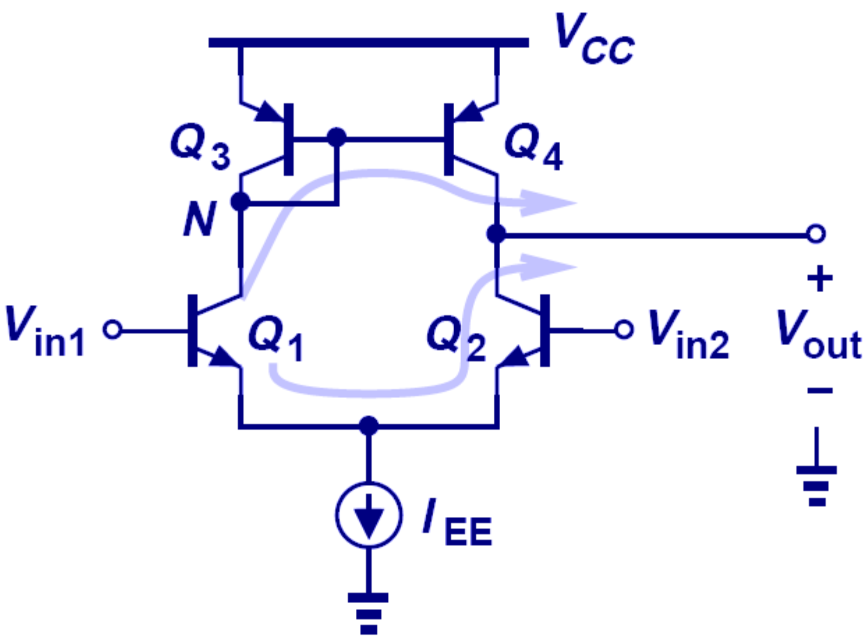
- With current mirror used as the load, the signal current produced by the Q_1 can be replicated onto Q_4.
- This type of load is different from the conventional “static load” and is known as an “active load”.

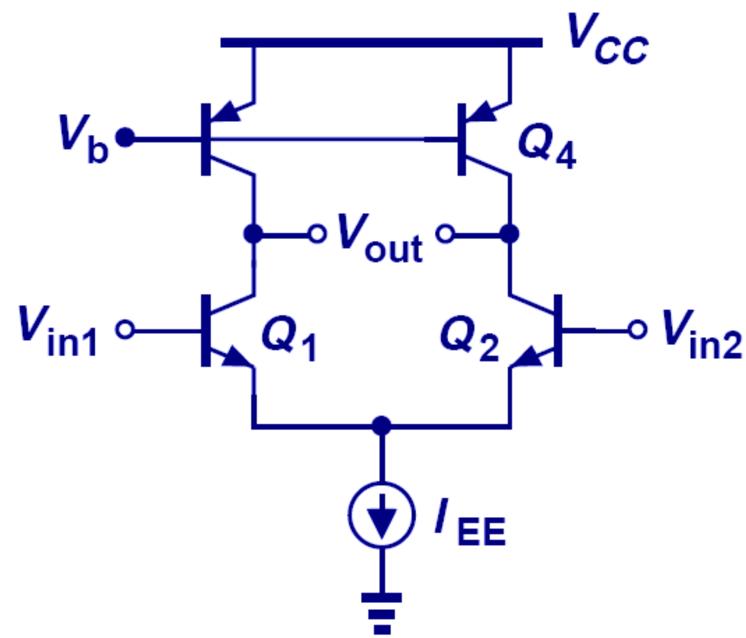
Differential Pair with Active Load



- The input differential pair decreases the current drawn from R_L by ΔI and the active load pushes an extra ΔI into R_L by current mirror action; these effects enhance each other.

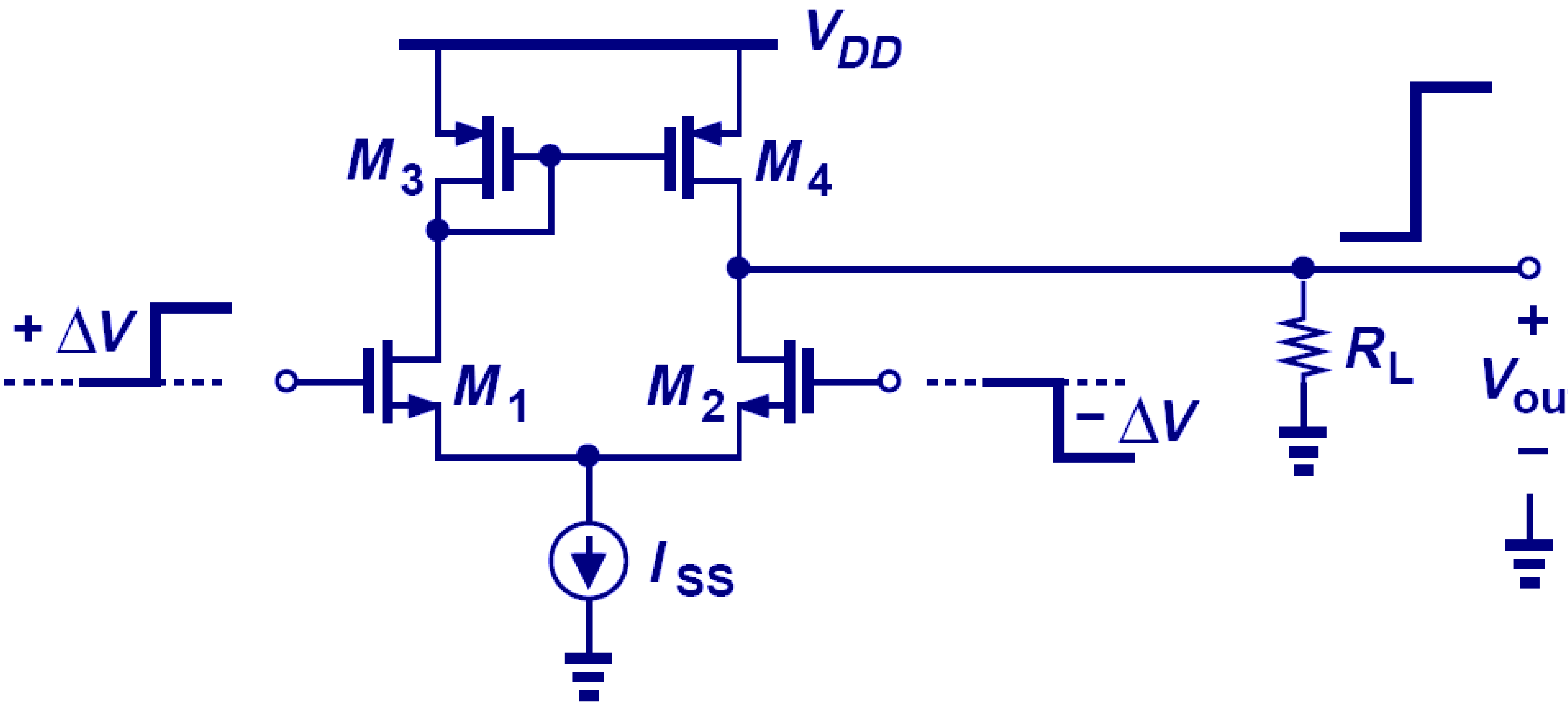
Active Load vs. Static Load





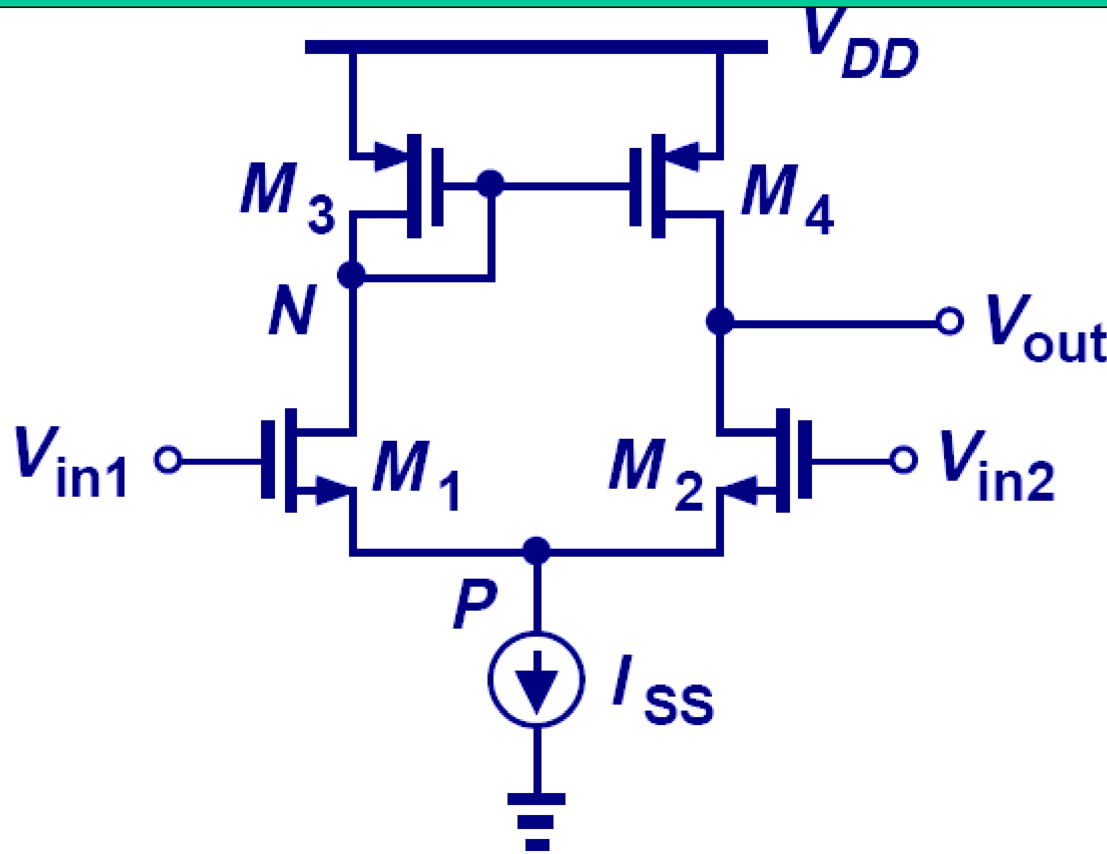
- The load on the left responds to the input signal and enhances the single-ended output, whereas the load on the right does not.

MOS Differential Pair with Active Load



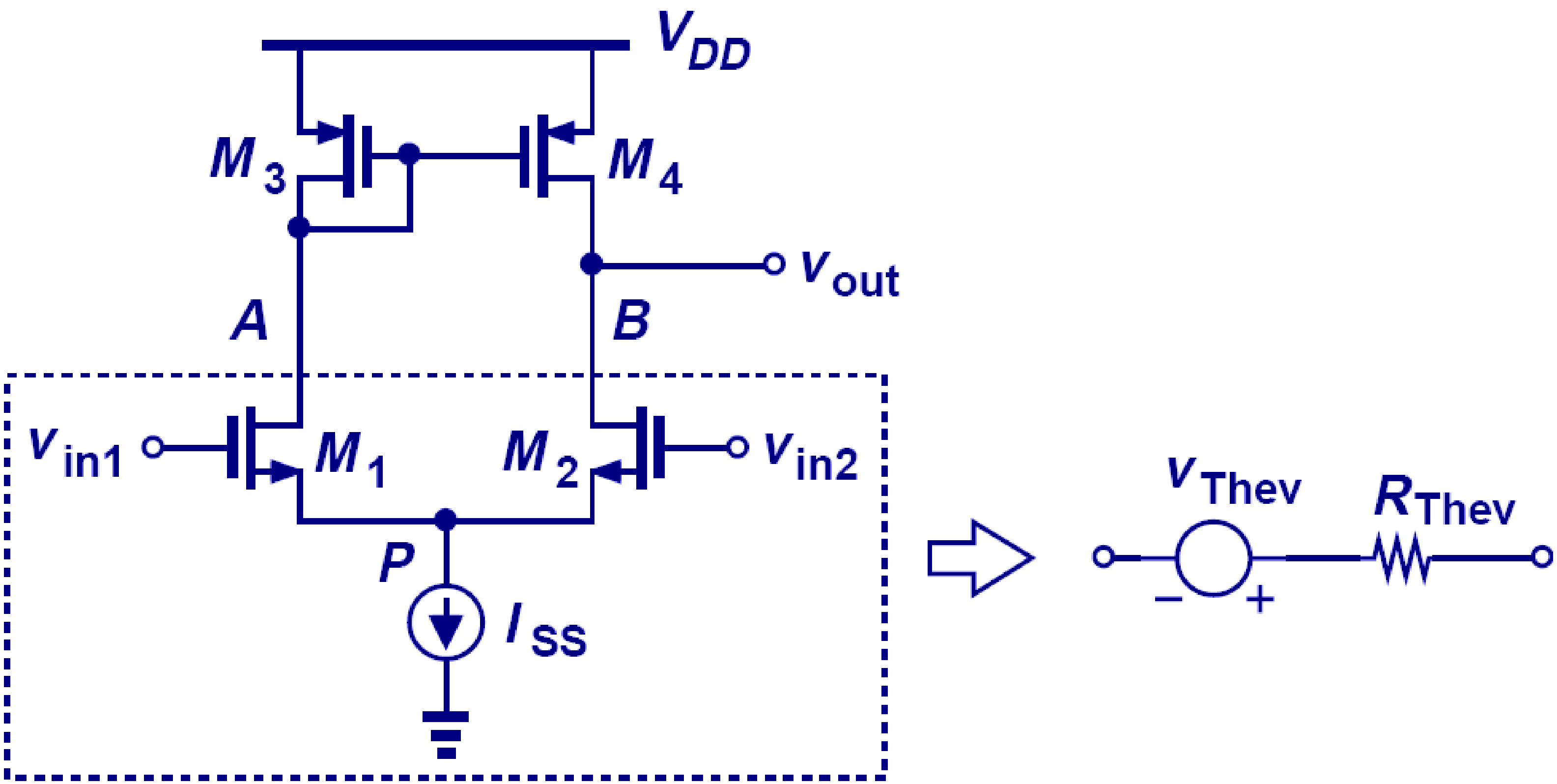
- Similar to its bipolar counterpart, MOS differential pair can also use active load to enhance its single-ended output.

Asymmetric Differential Pair



- **Because of the vastly different resistance magnitude at the drains of M_1 and M_2, the voltage swings at these two nodes are different and therefore node P cannot be viewed as a virtual ground.**

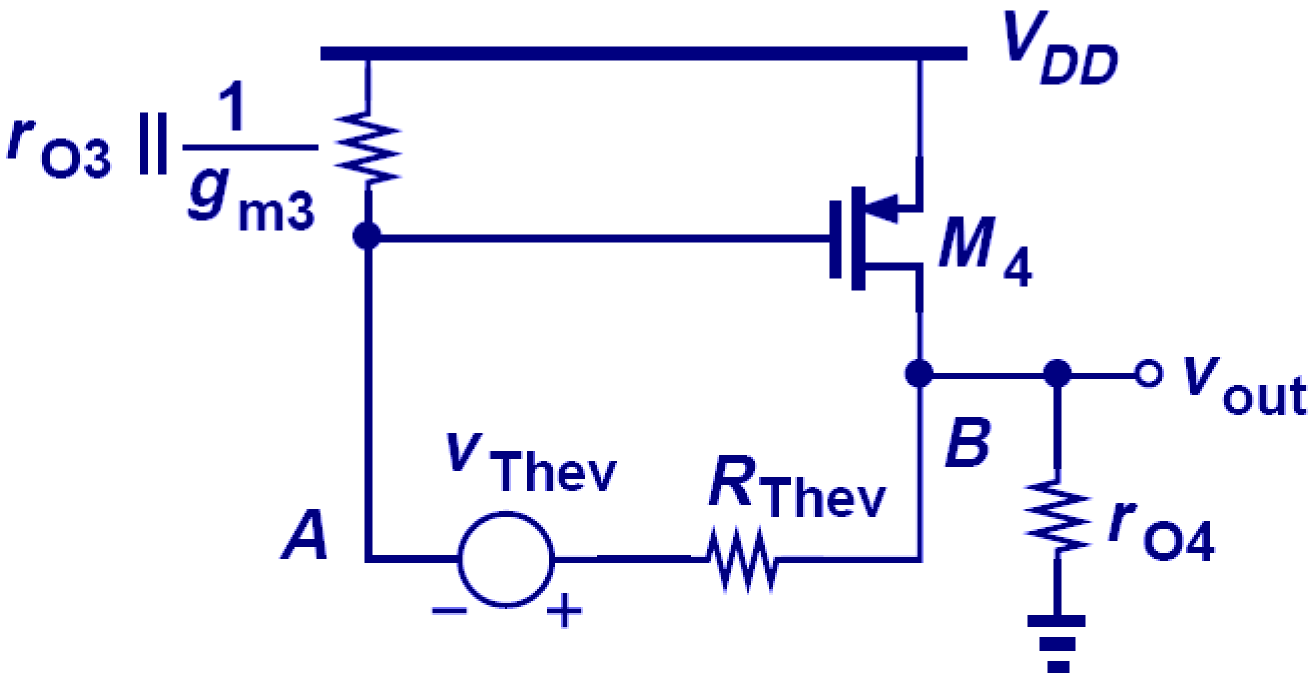
Thevenin Equivalent of the Input Pair



$$v_{Thev} = -g_{mN} r_{oN} (v_{in1} - v_{in2})$$

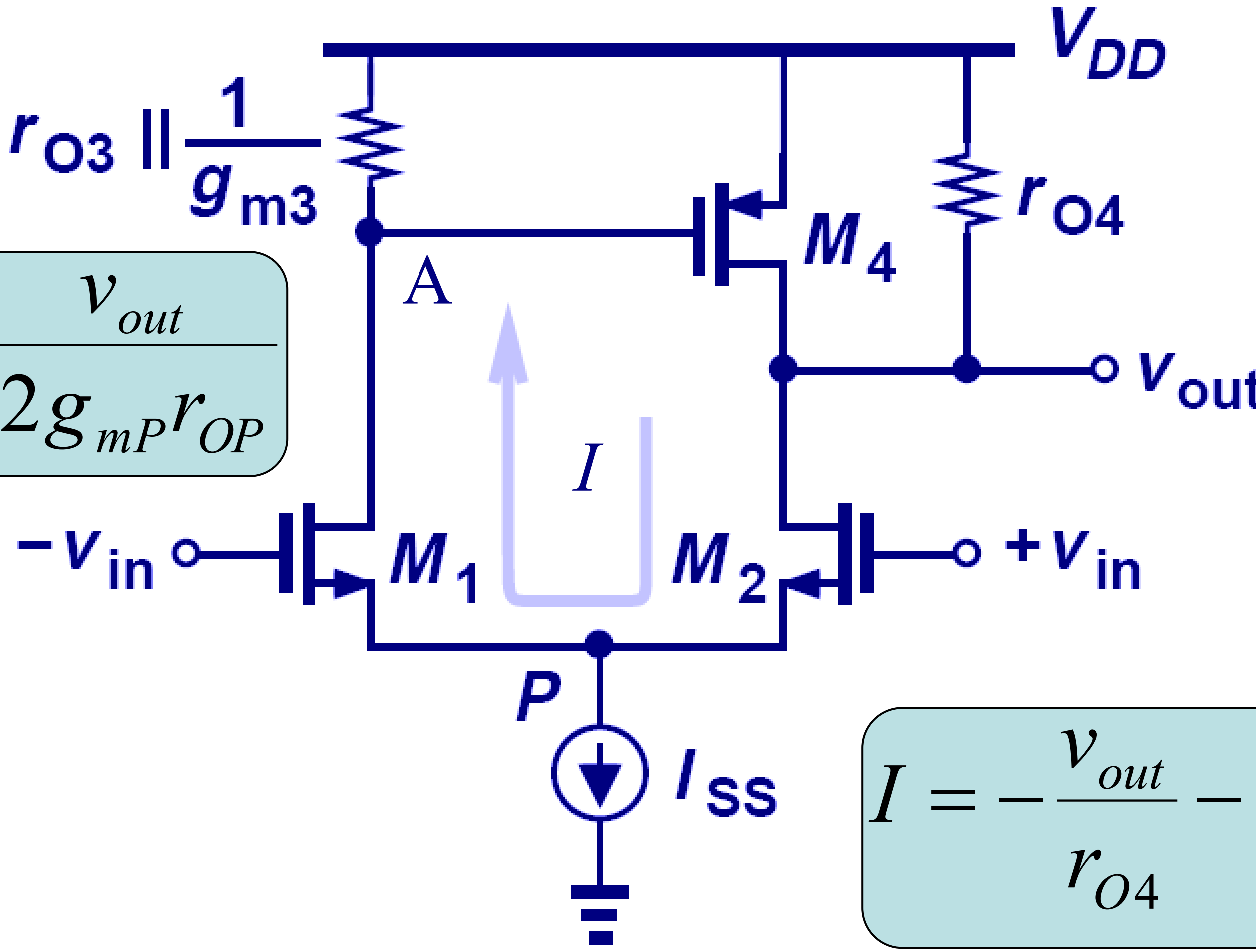
$$R_{Thev} = 2 r_{oN}$$

Simplified Differential Pair with Active Load



$$\frac{v_{out}}{v_{in1} - v_{in2}} = g_{mN}(r_{ON} \,||\, r_{OP})$$

Proof of $V_A \ll V_{out}$



$$v_A \approx -\frac{v_{out}}{2g_{mP}r_{OP}}$$

$$I = -\frac{v_{out}}{r_{O4}} - g_{m4}v_A$$

Chapter 11 Frequency Response

Chapter Outline

Fundamental Concepts

- Bode's Rules
- Association of Poles with Nodes
- Miller's Theorem

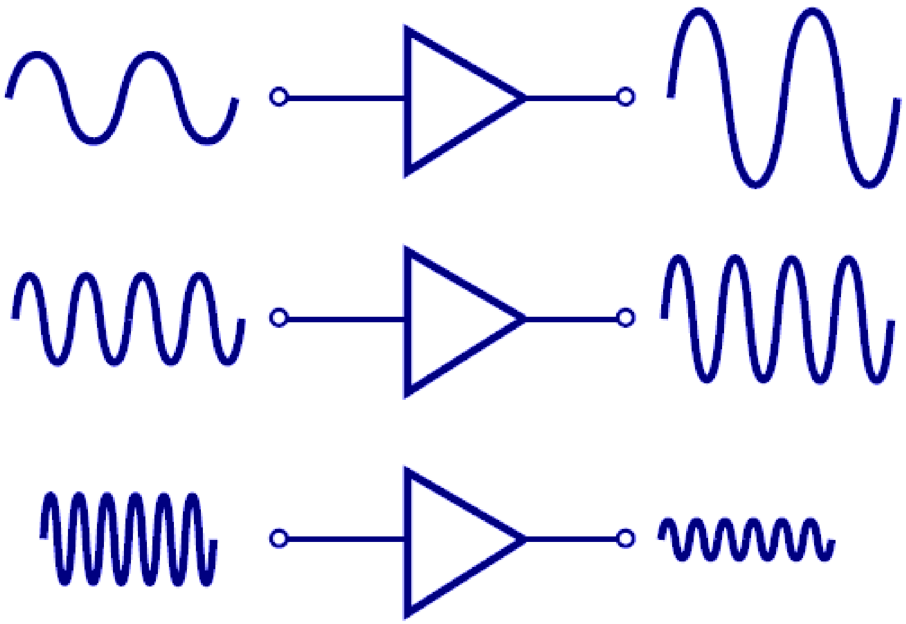
High-Frequency Models of Transistors

- Bipolar Model
- MOS Model
- Transit Frequency

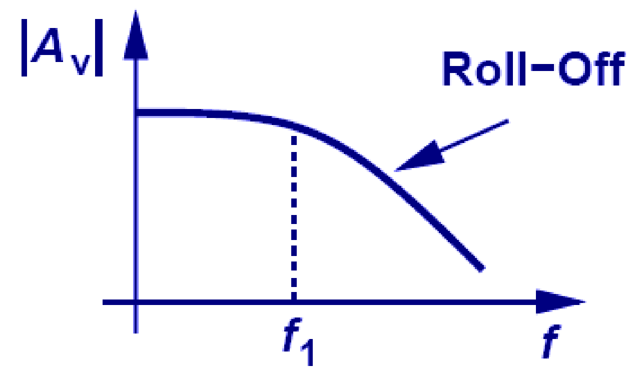
Frequency Response of Circuits

- CE/CS Stages
- CB/CG Stages
- Followers
- Cascode Stage
- Differential Pair

High Frequency Roll-off of Amplifier



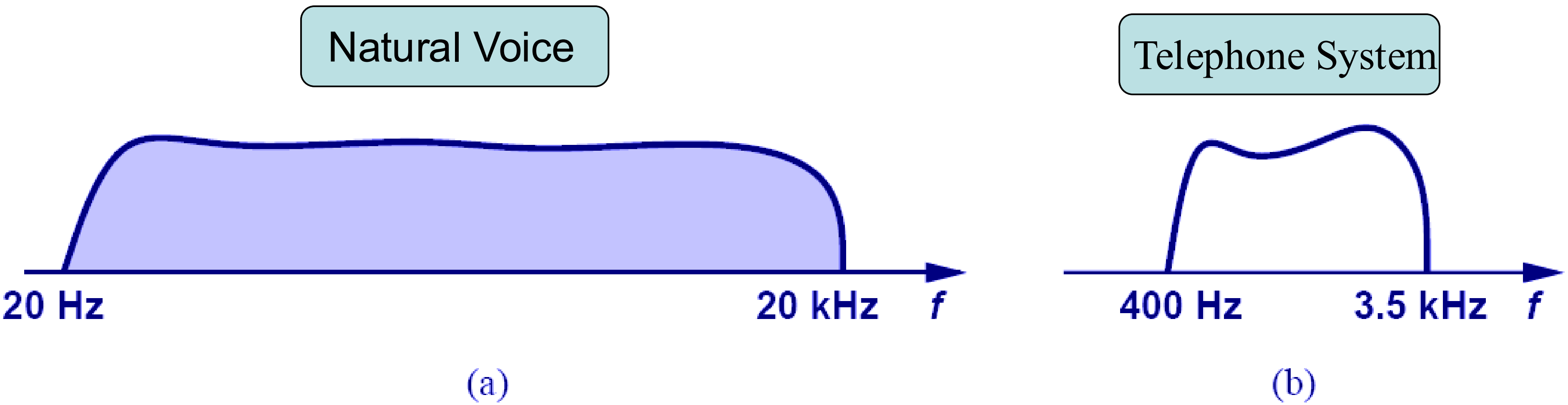
(a)



(b)

- As frequency of operation increases, the gain of amplifier decreases. This chapter analyzes this problem.

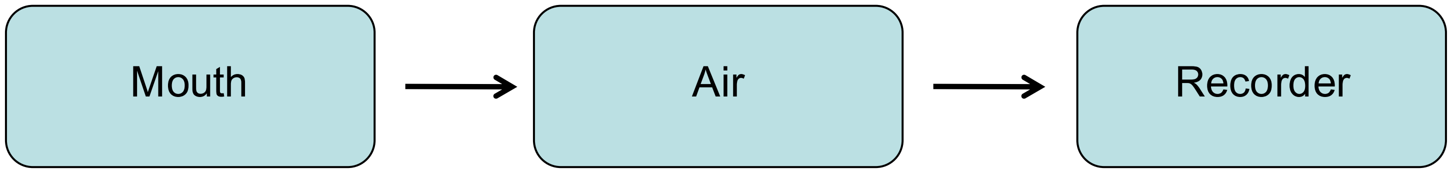
Example: Human Voice I



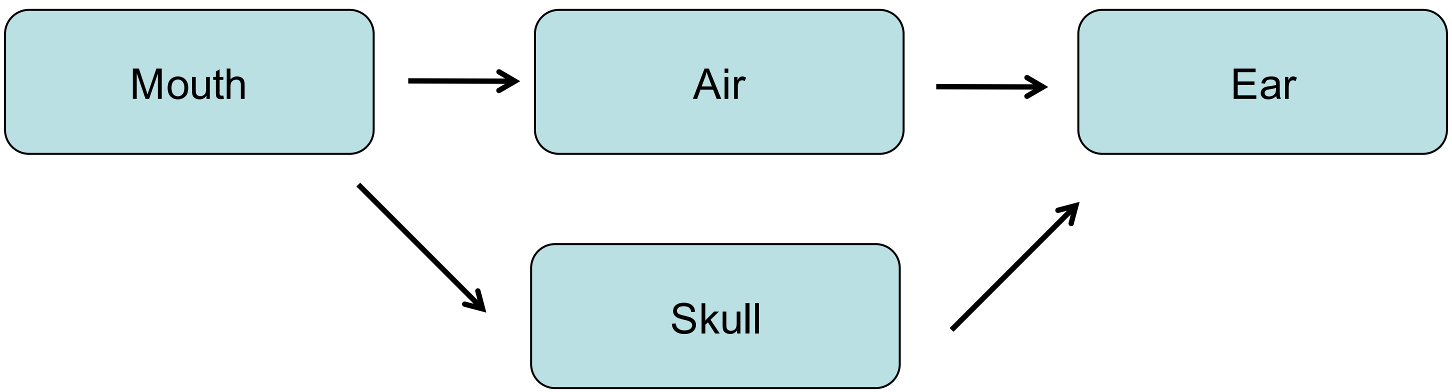
- Natural human voice spans a frequency range from 20Hz to 20KHz, however conventional telephone system passes frequencies from 400Hz to 3.5KHz. Therefore phone conversation differs from face-to-face conversation.

Example: Human Voice II

Path traveled by the human voice to the voice recorder

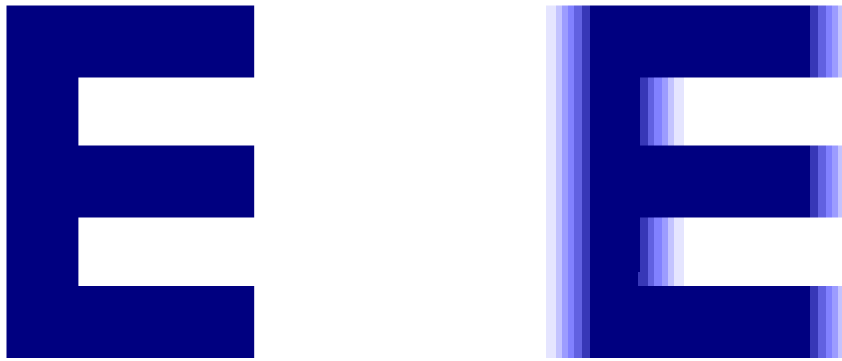


Path traveled by the human voice to the human ear



- **Since the paths are different, the results will also be different.**

Example: Video Signal



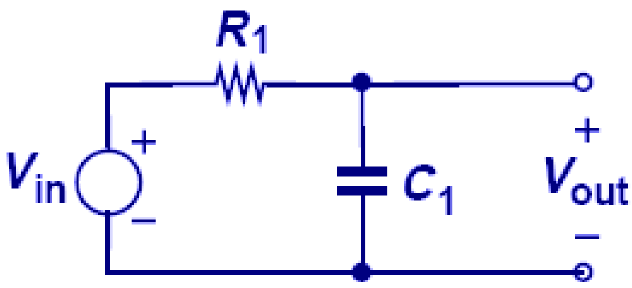
(a) (b)

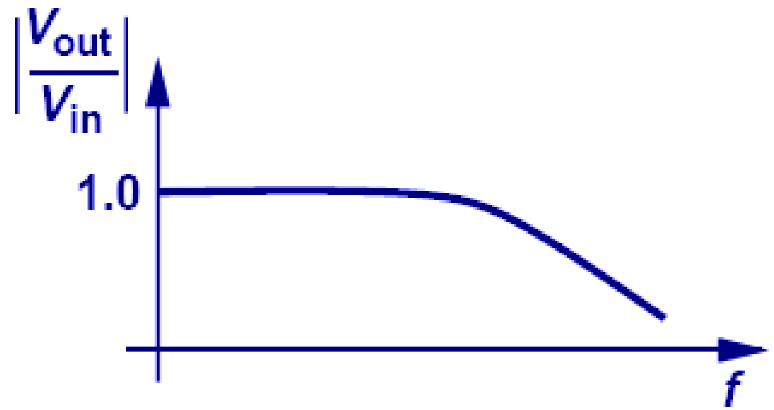
High Bandwidth

Low Bandwidth

- Video signals without sufficient bandwidth become fuzzy as they fail to abruptly change the contrast of pictures from complete white into complete black.

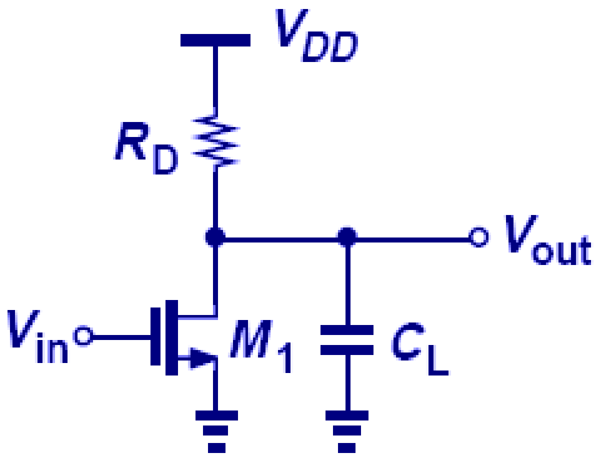
Gain Roll-off: Simple Low-pass Filter



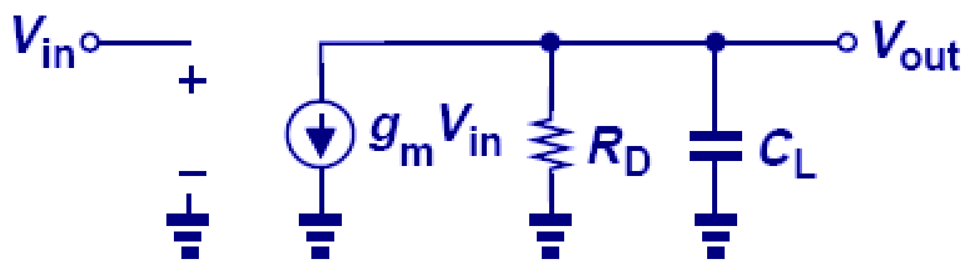


- In this simple example, as frequency increases the impedance of C_1 decreases and the voltage divider consists of C_1 and R_1 attenuates V_{in} to a greater extent at the output.

Gain Roll-off: Common Source



(a)

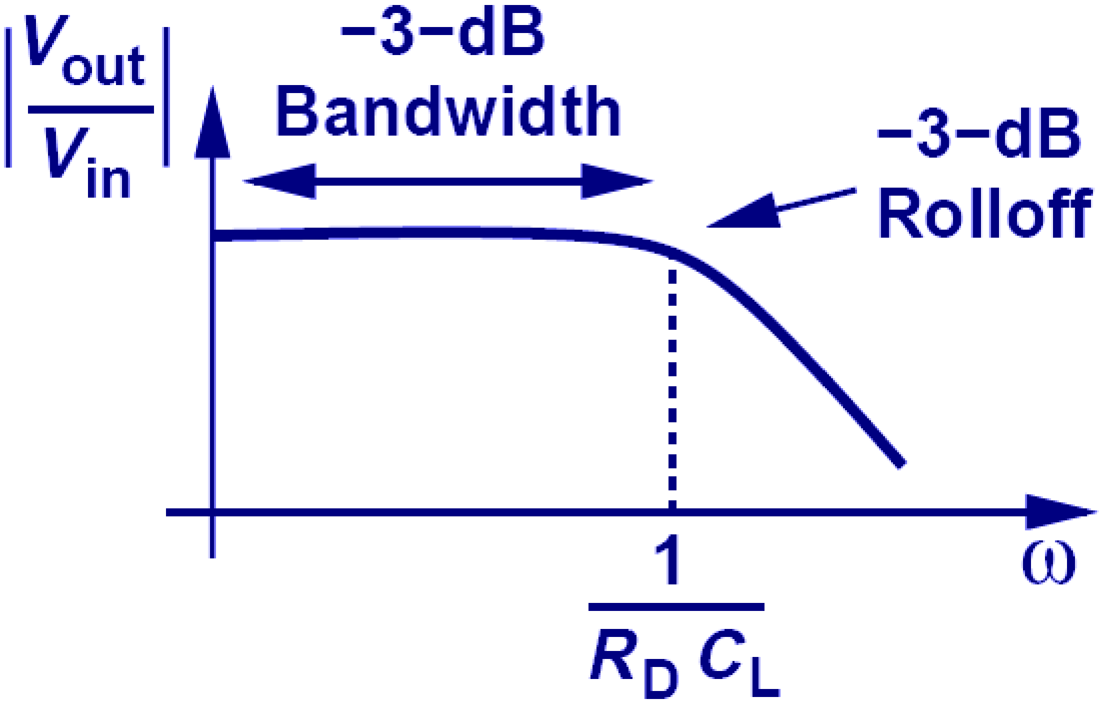


(b)

$$V_{out} = -g_m V_{in} \left(R_D \,||\, \frac{1}{C_L s} \right)$$

- The capacitive load, C_L, is the culprit for gain roll-off since at high frequency, it will "steal" away some signal current and shunt it to ground.

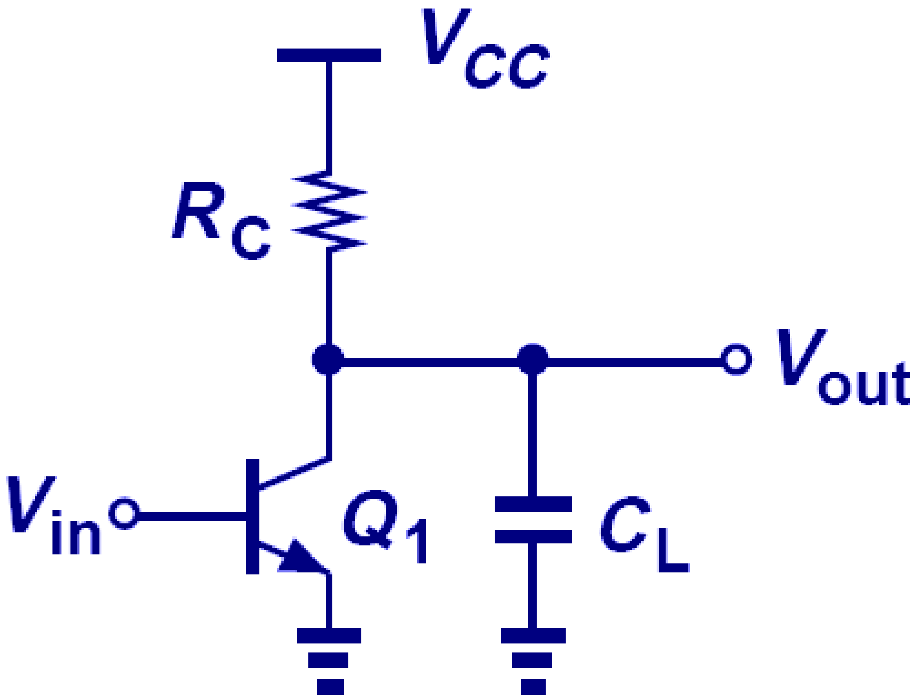
Frequency Response of the CS Stage



$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{R_D^2 C_L^2 \omega^2 + 1}}$$

- At low frequency, the capacitor is effectively open and the gain is flat. As frequency increases, the capacitor tends to a short and the gain starts to decrease. A special frequency is $\omega=1/(R_D C_L)$, where the gain drops by 3dB.

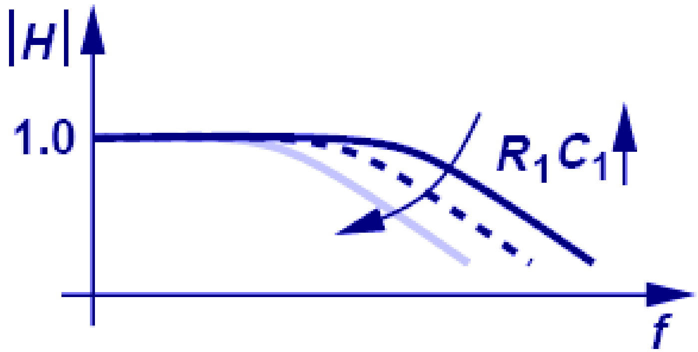
Example: Figure of Merit



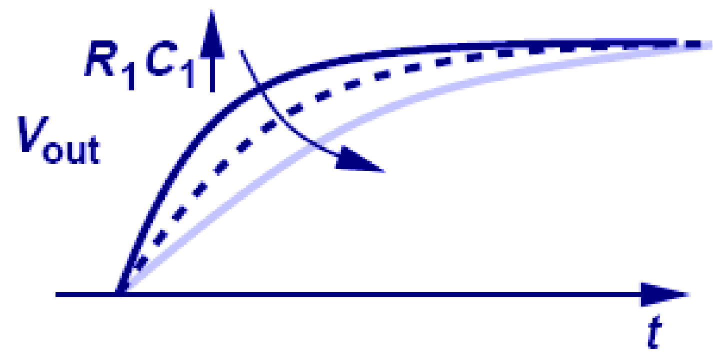
$$F.O.M. = \frac{1}{V_T V_{CC} C_L}$$

- **This metric quantifies a circuit's gain, bandwidth, and power dissipation. In the bipolar case, low temperature, supply, and load capacitance mark a superior figure of merit.**

Example: Relationship between Frequency Response and Step Response



(a)



(b)

$$\left|H(s=j\omega)\right| = \frac{1}{\sqrt{R_1^2C_1^2\omega^2+1}}$$

$$V_{out}(t) = V_0\left(1-\exp\frac{-t}{R_1C_1}\right)u(t)$$

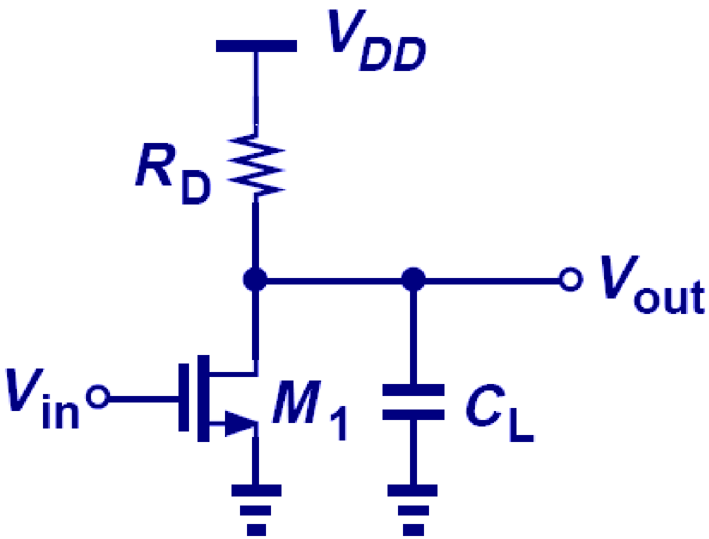
- **The relationship is such that as R_1C_1 increases, the bandwidth *drops* and the step response becomes *slower.***

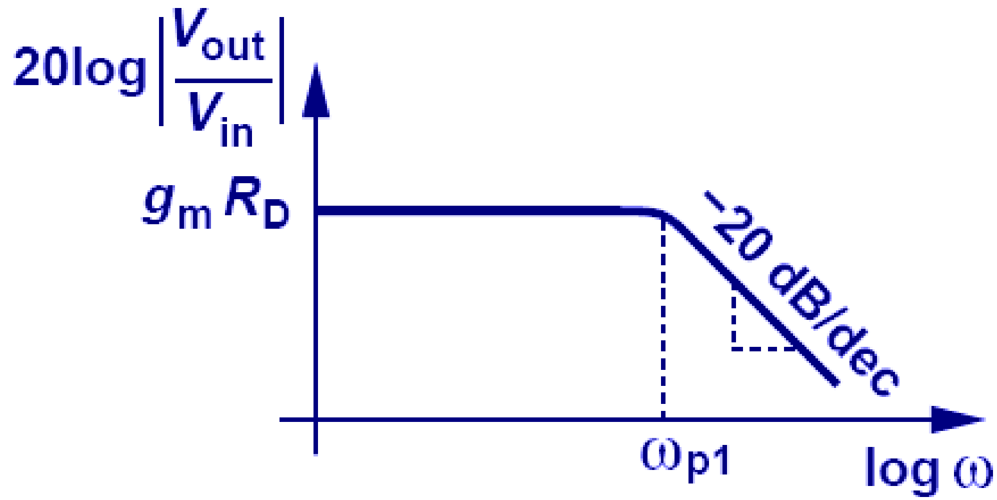
Bode Plot

$$H(s) = A_0 \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\cdots}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\cdots}$$

- When we hit a zero, ω_{zj}, the Bode magnitude rises with a slope of +20dB/dec.
- When we hit a pole, ω_{pj}, the Bode magnitude falls with a slope of -20dB/dec

Example: Bode Plot

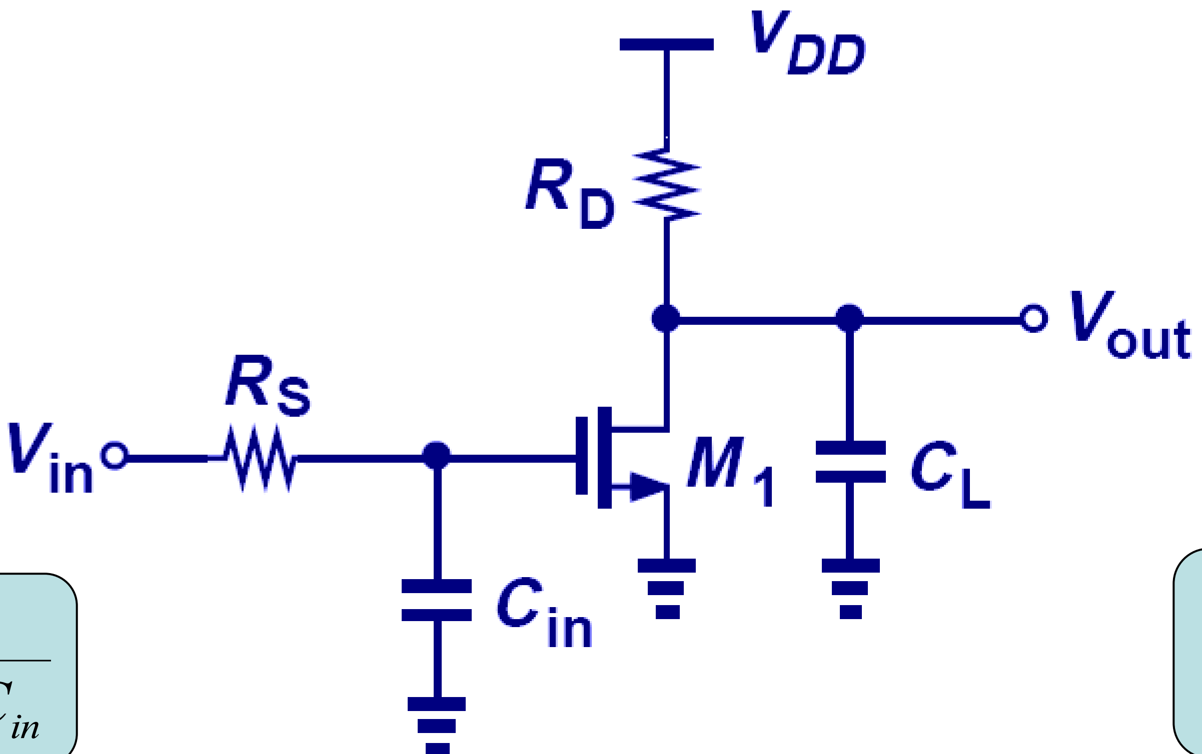




$$\left|\omega_{p1}\right| = \frac{1}{R_D C_L}$$

- The circuit only has one pole (no zero) at $1/(R_D C_L)$, so the slope drops from 0 to -20dB/dec as we pass ω_{p1}.

Pole Identification Example I

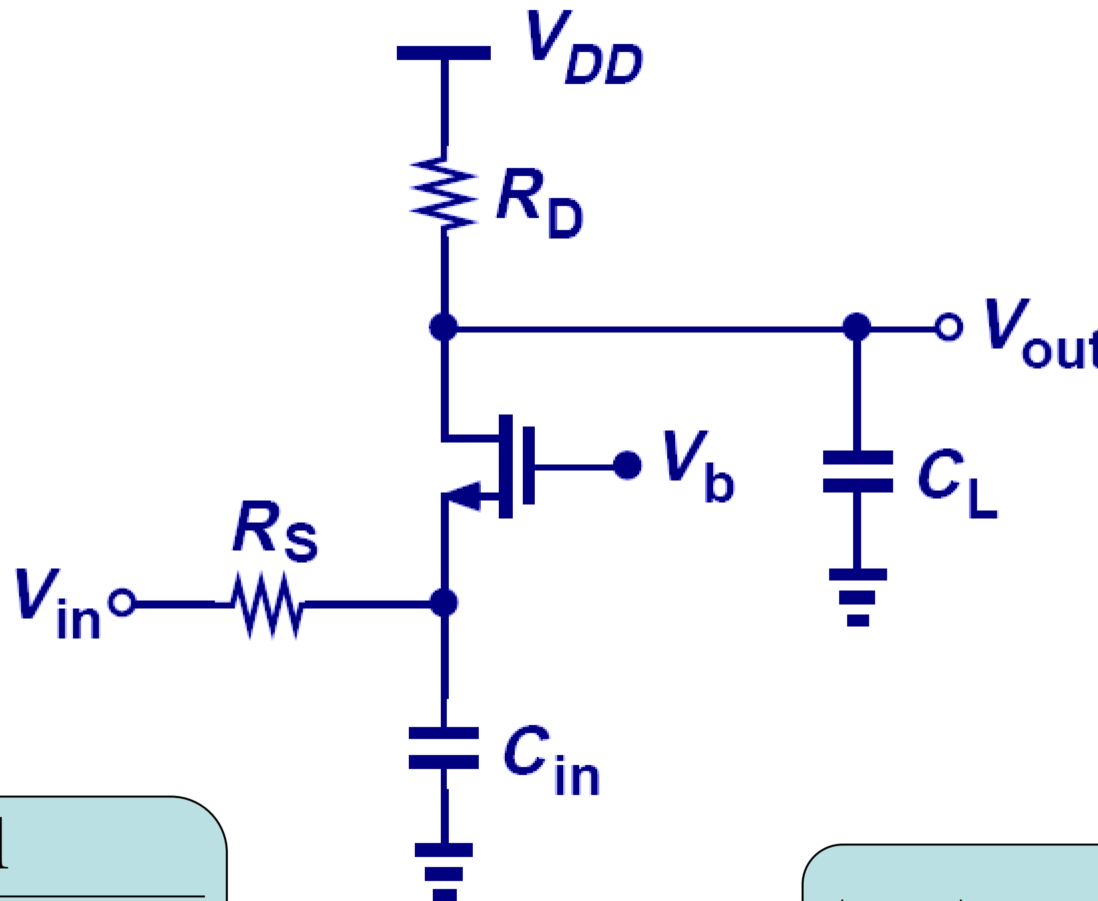


$$|\omega_{p1}| = \frac{1}{R_S C_{in}}$$

$$|\omega_{p2}| = \frac{1}{R_D C_L}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{\left(1+\omega^2/\omega_{p1}^2\right)\left(1+\omega^2/\omega_{p2}^2\right)}}$$

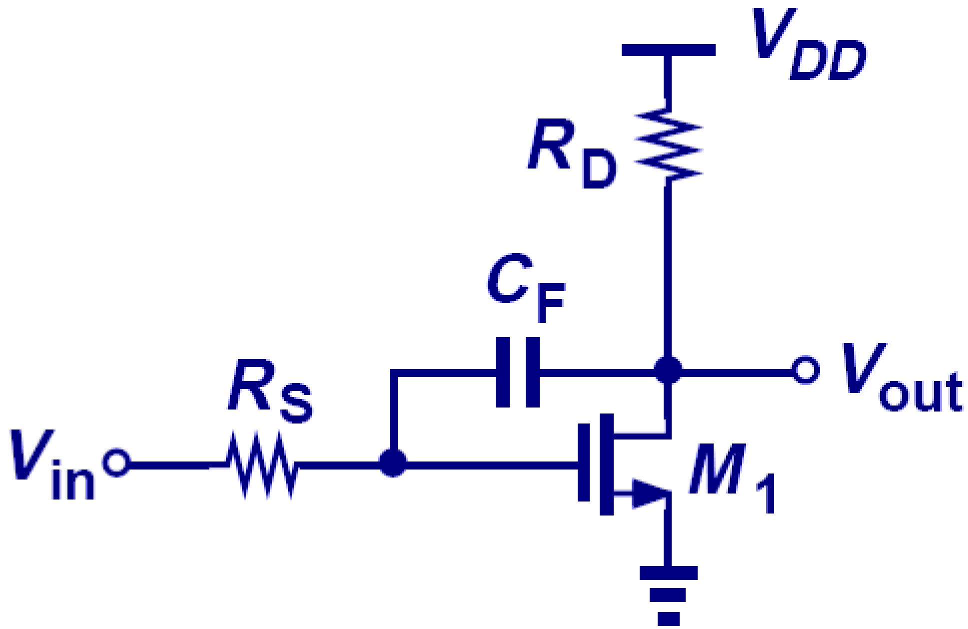
Pole Identification Example II



$$|\omega_{p1}| = \frac{1}{\left(R_S \parallel \frac{1}{g_m}\right) C_{in}}$$

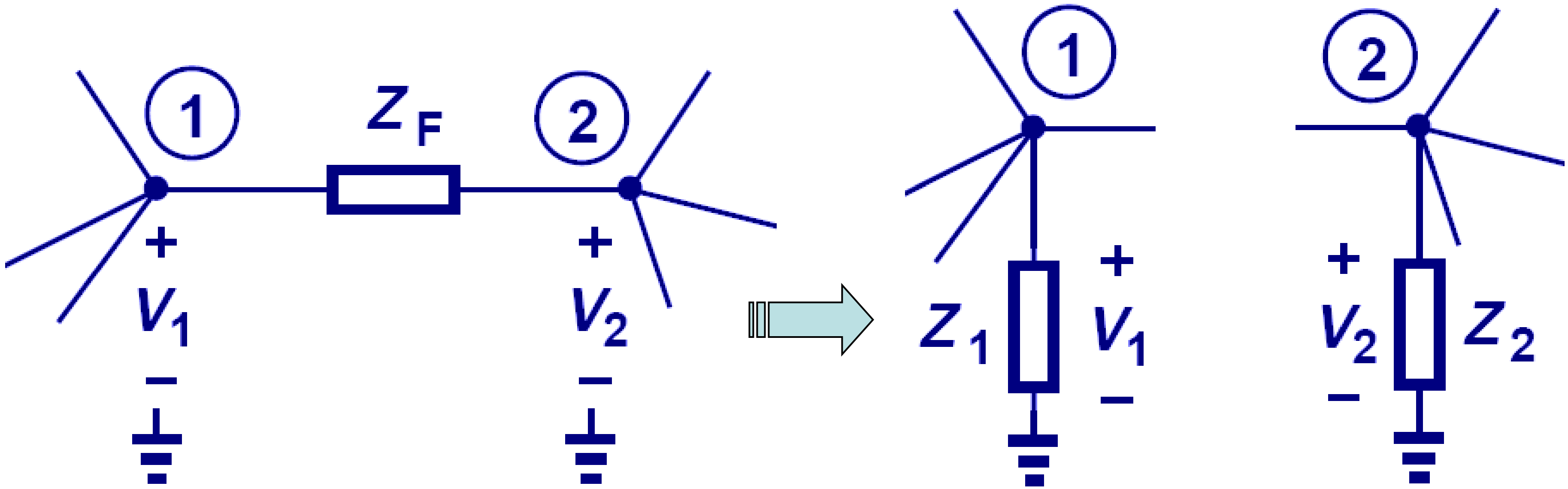
$$|\omega_{p2}| = \frac{1}{R_D C_L}$$

Circuit with Floating Capacitor



- The pole of a circuit is computed by finding the effective resistance and capacitance from a node to GROUND.
- The circuit above creates a problem since neither terminal of C_F is grounded.

Miller's Theorem

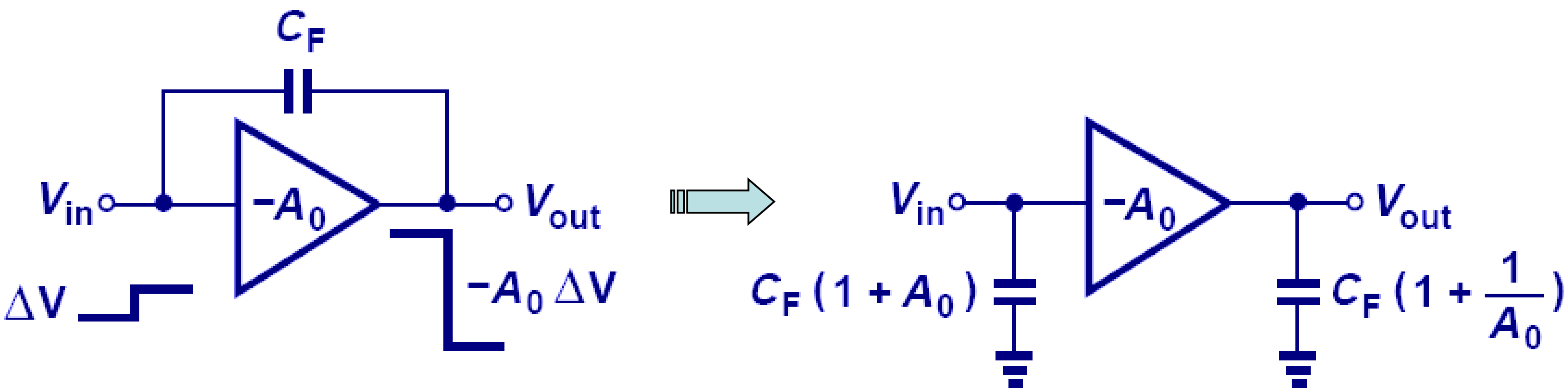


$$Z_1 = \frac{Z_F}{1 - A_v}$$

$$Z_2 = \frac{Z_F}{1 - 1/A_v}$$

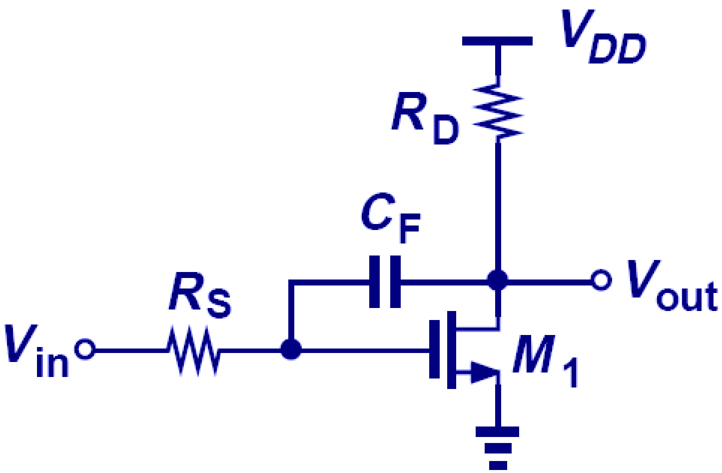
➢ **If A_v is the gain from node 1 to 2, then a floating impedance Z_F can be converted to two grounded impedances Z_1 and Z_2.**

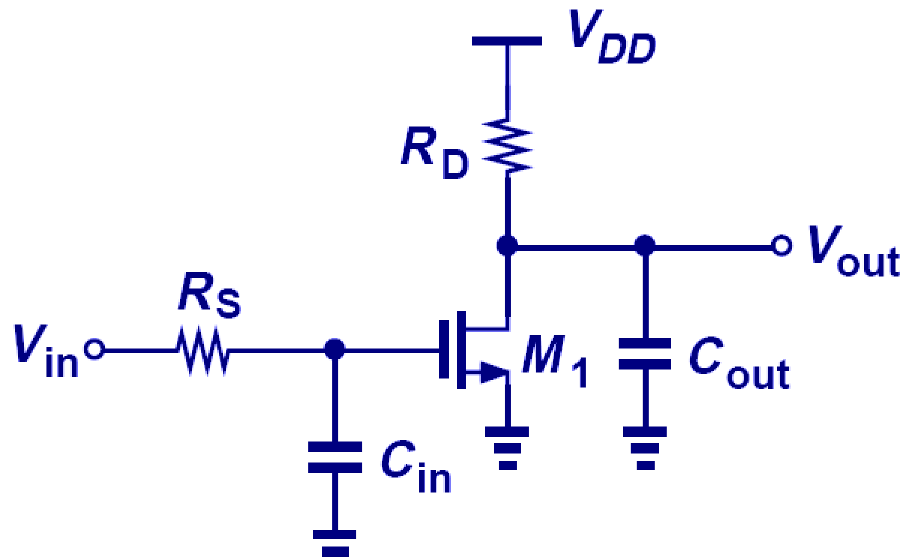
Miller Multiplication



- With Miller's theorem, we can separate the floating capacitor. However, the input capacitor is larger than the original floating capacitor. We call this Miller multiplication.

Example: Miller Theorem

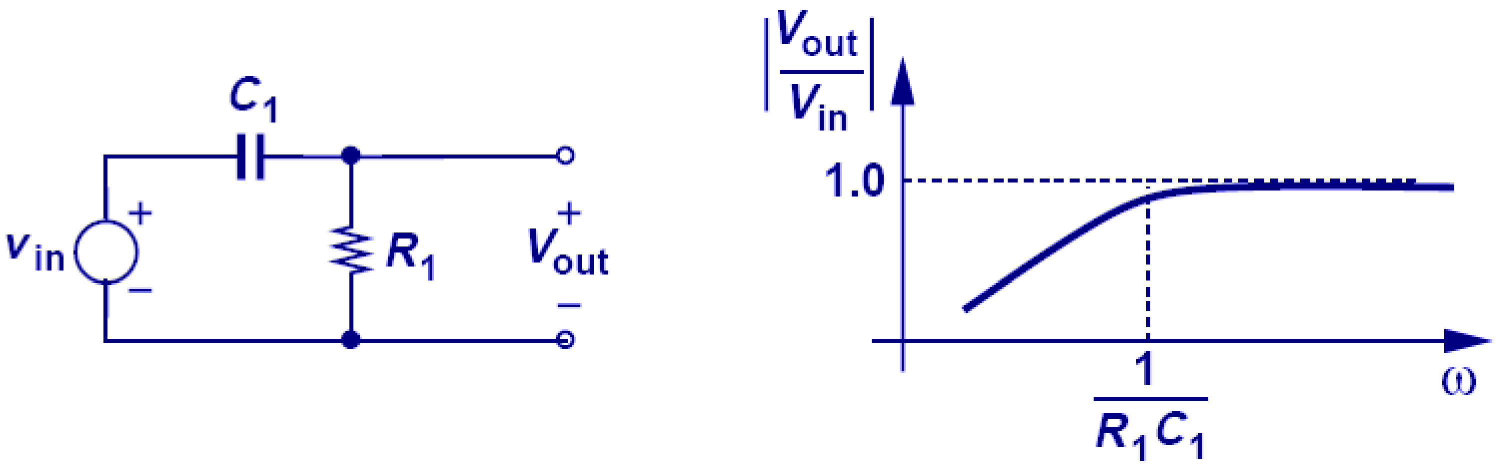




$$\omega_{in} = \frac{1}{R_S(1+g_m R_D)C_F}$$

$$\omega_{out} = \frac{1}{R_D\left(1+\frac{1}{g_m R_D}\right)C_F}$$

High-Pass Filter Response

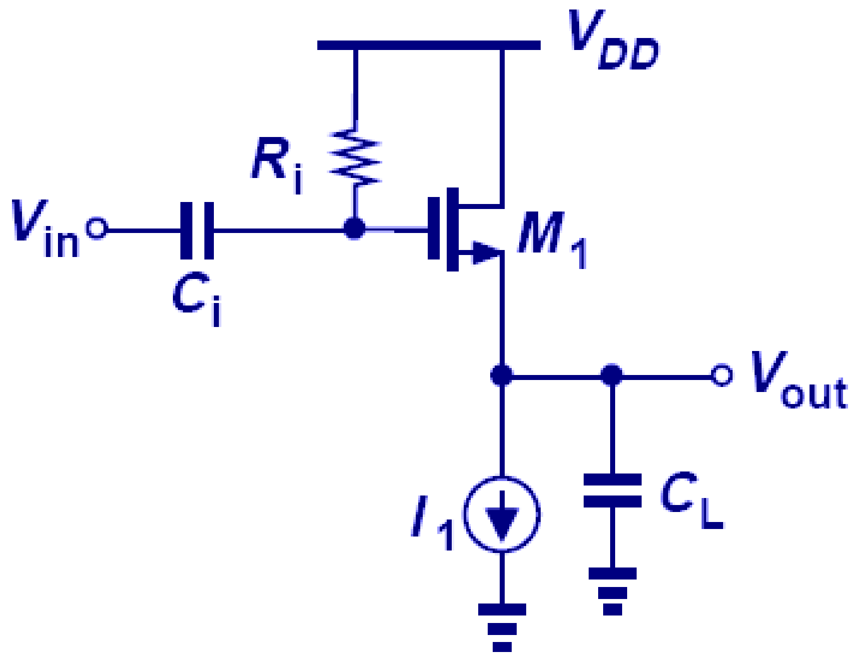


(a) (b)

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{R_1C_1\omega}{\sqrt{R_1^2C_1^2\omega_1^2+1}}$$

- The voltage division between a resistor and a capacitor can be configured such that the gain at low frequency is reduced.

Example: Audio Amplifier



$$C_i = 79.6nF$$

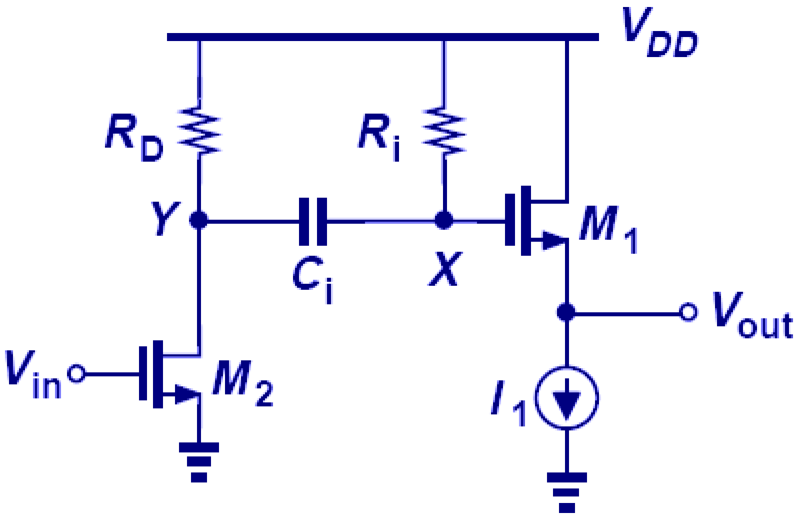
$$C_L = 39.8nF$$

$$R_i = 100K\Omega$$

$$g_m = 1/200\Omega$$

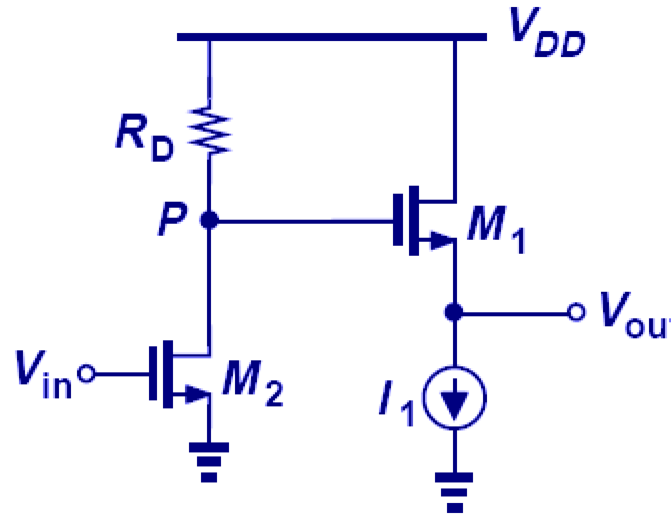
- In order to successfully pass audio band frequencies (20 Hz-20 KHz), large input and output capacitances are needed.

Capacitive Coupling vs. Direct Coupling



(a)

Capacitive Coupling

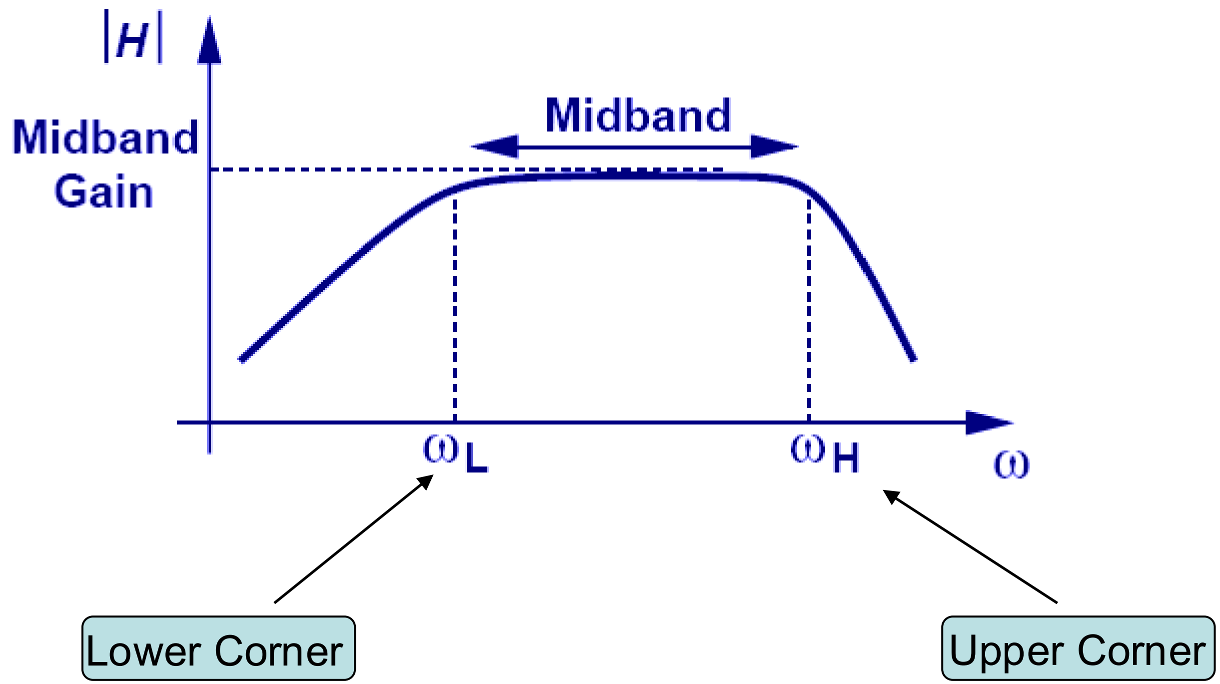


(b)

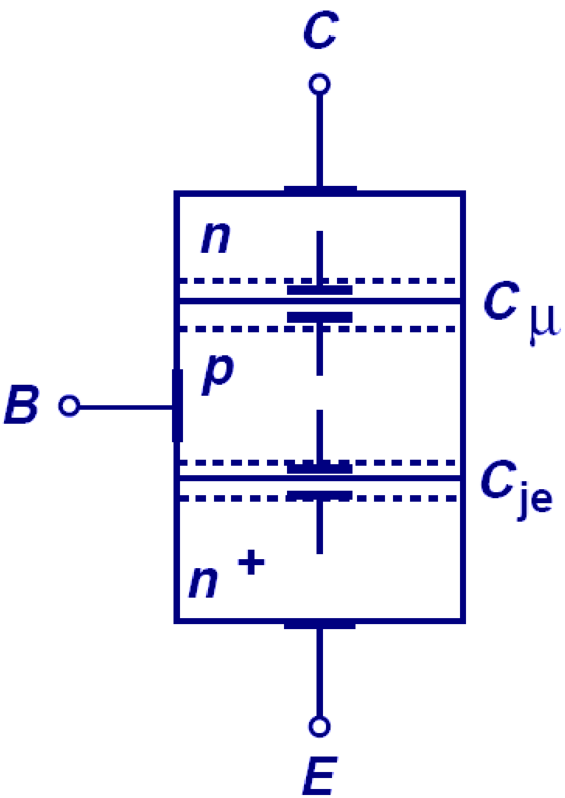
Direct Coupling

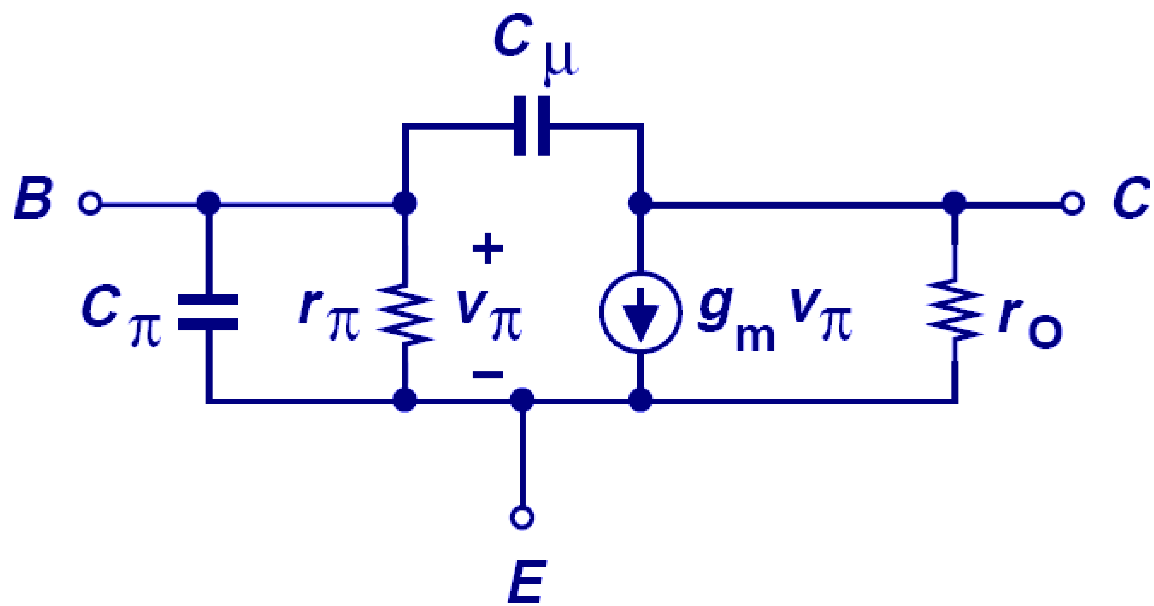
- **Capacitive coupling, also known as AC coupling, passes AC signals from Y to X while blocking DC contents.**
- **This technique allows independent bias conditions between stages. Direct coupling does not.**

Typical Frequency Response



High-Frequency Bipolar Model

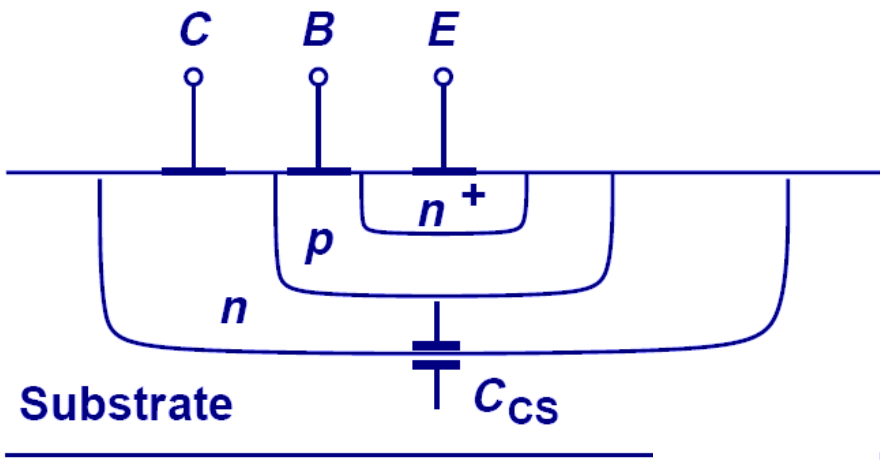


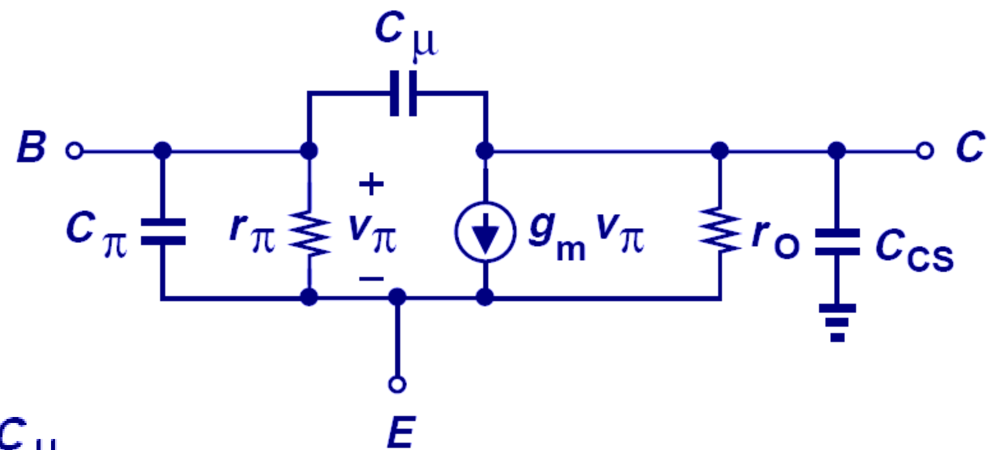


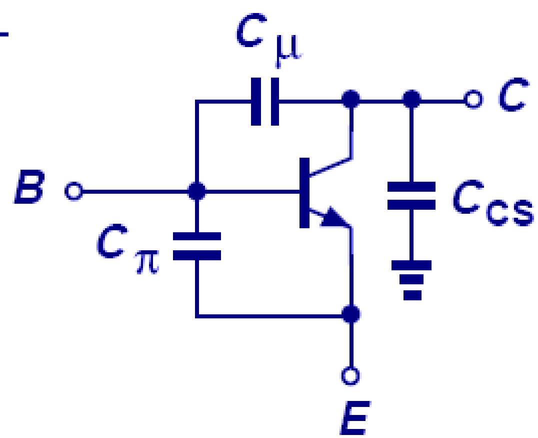
$$C_{\pi} = C_b + C_{je}$$

- At high frequency, capacitive effects come into play. C_b represents the base charge, whereas C_{μ} and C_{je} are the junction capacitances.

High-Frequency Model of Integrated Bipolar Transistor

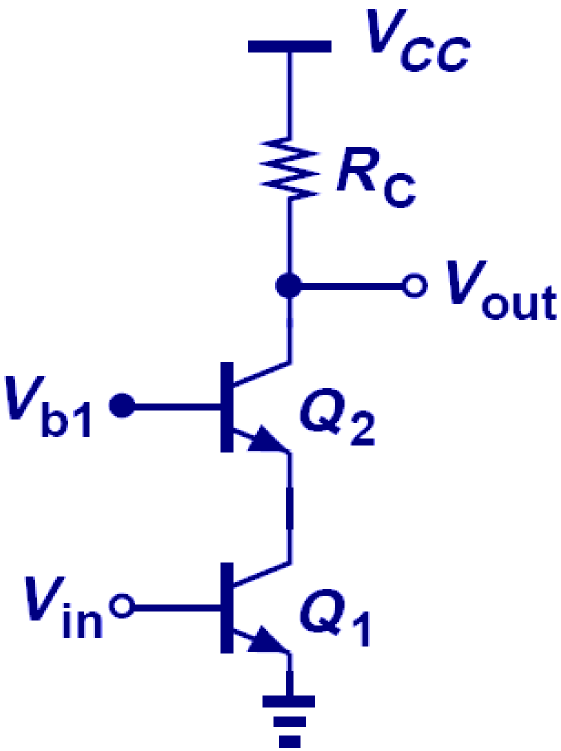


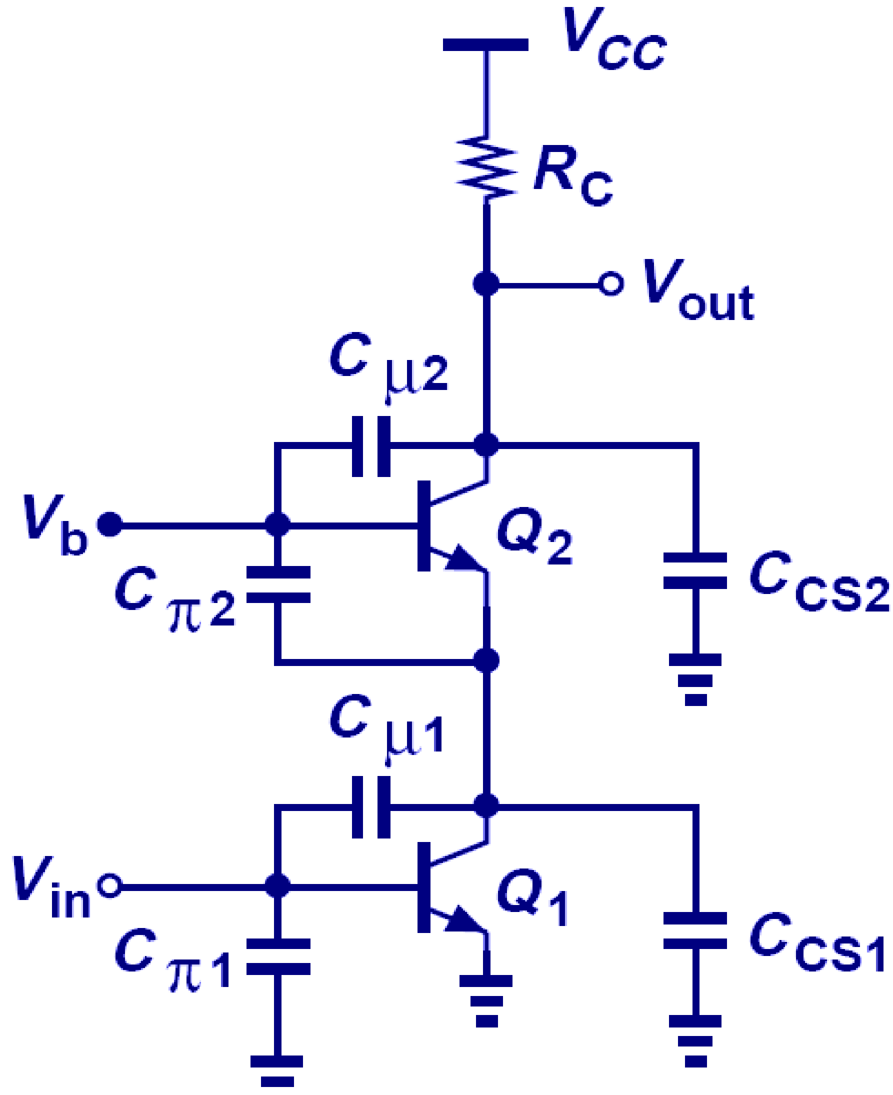




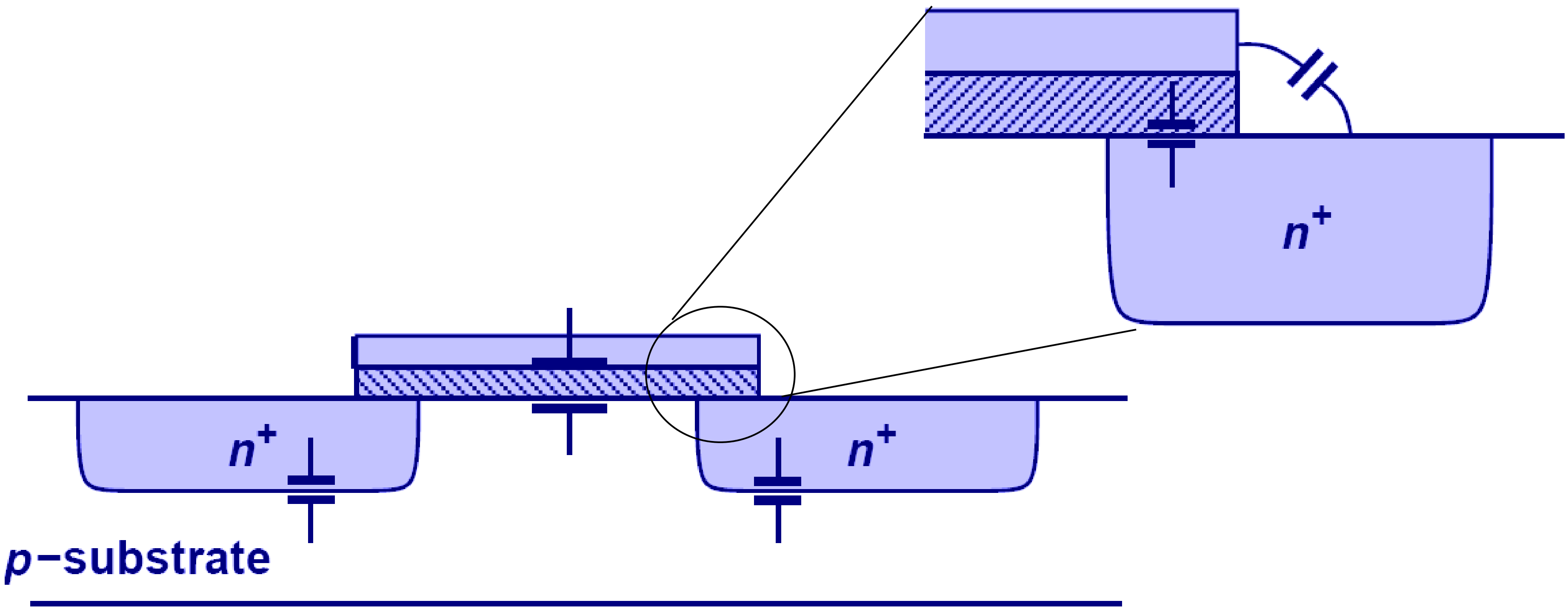
- Since an integrated bipolar circuit is fabricated on top of a substrate, another junction capacitance exists between the collector and substrate, namely C_{CS}.

Example: Capacitance Identification



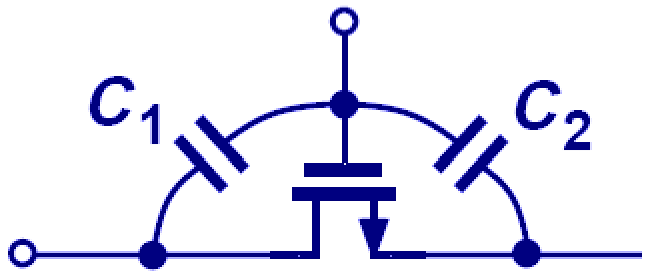


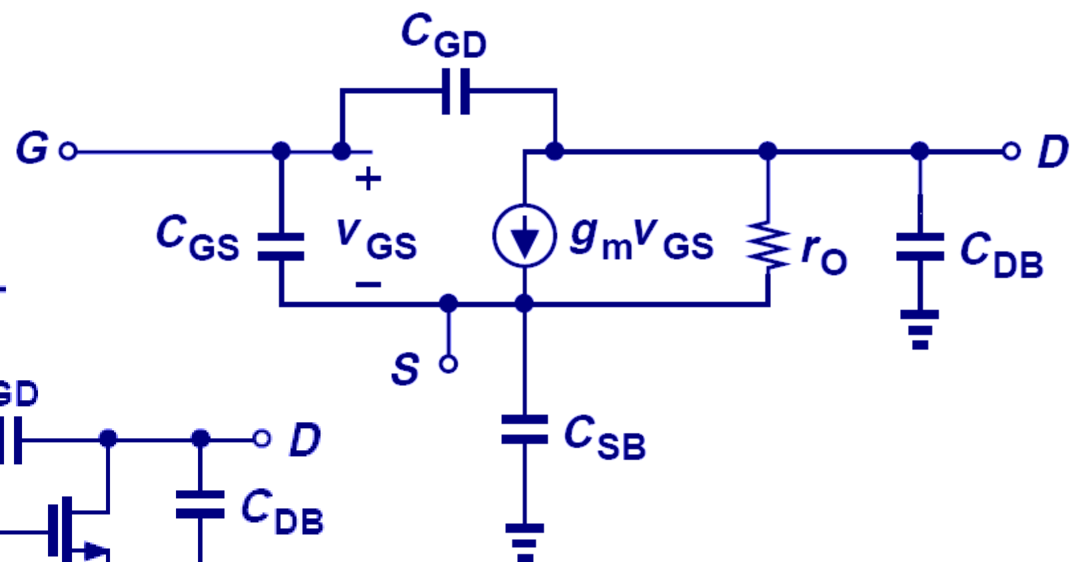
MOS Intrinsic Capacitances

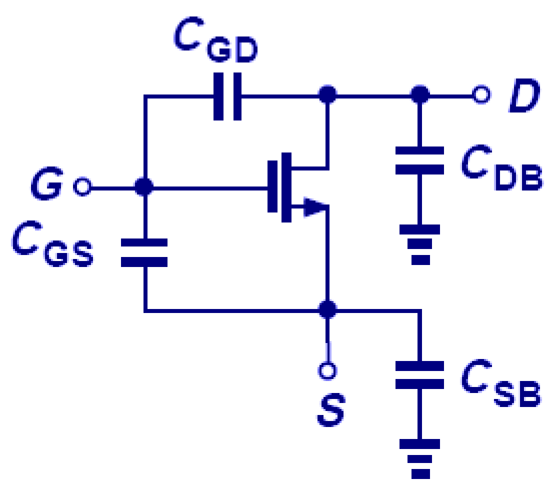


- For a MOS, there exist oxide capacitance from gate to channel, junction capacitances from source/drain to substrate, and overlap capacitance from gate to source/drain.

Gate Oxide Capacitance Partition and Full Model

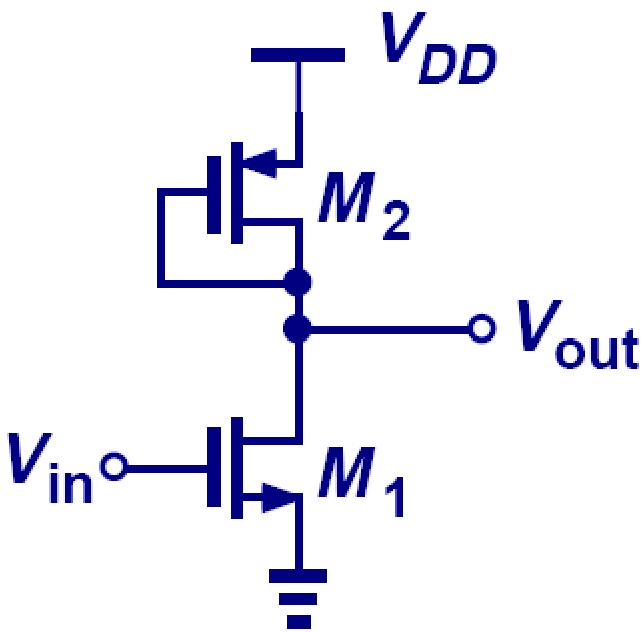


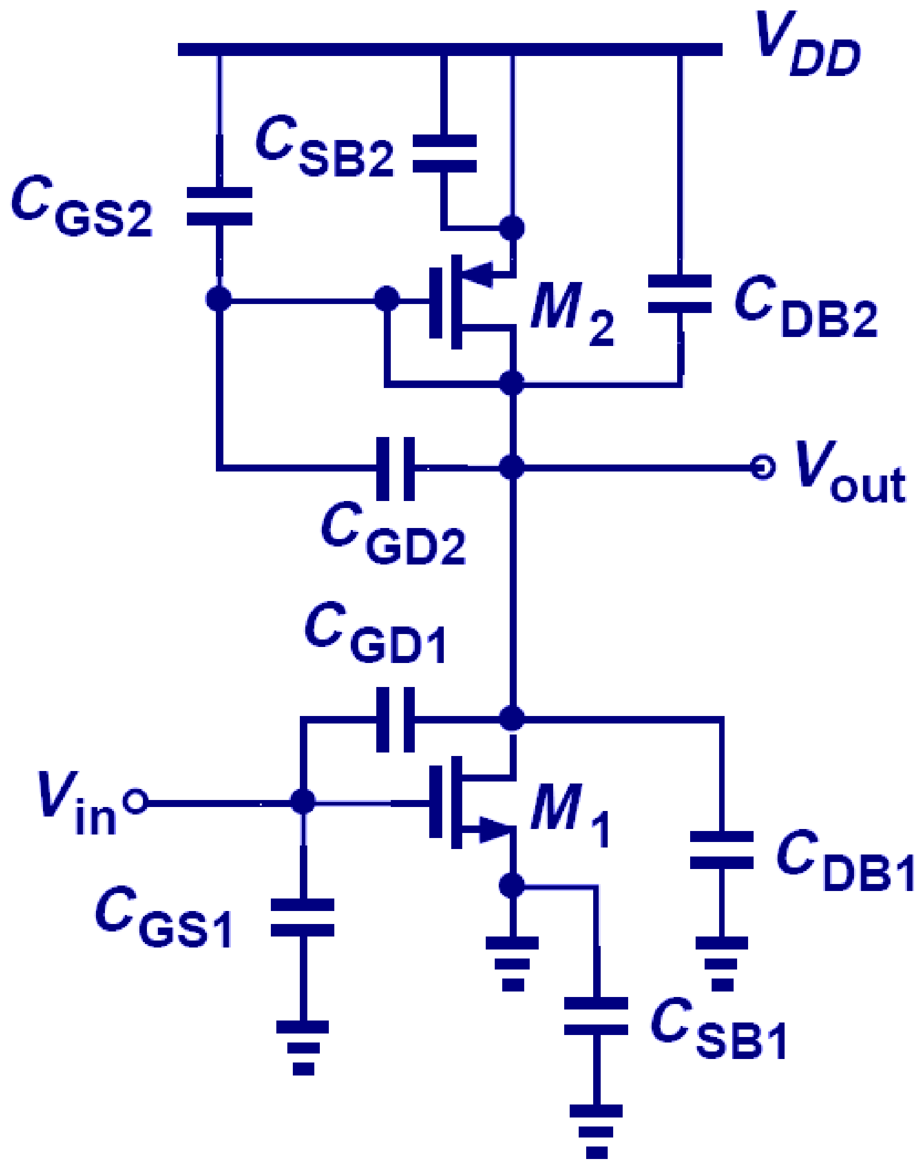




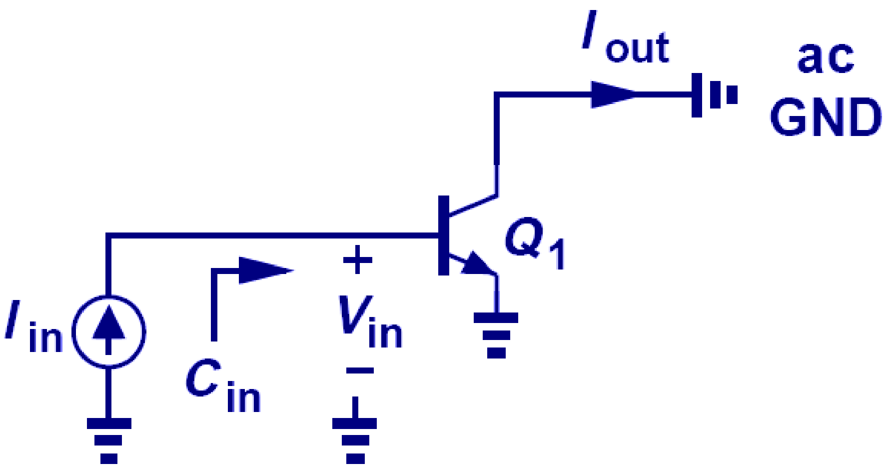
- The gate oxide capacitance is often partitioned between source and drain. In saturation, $C_2 \sim C_{gate}$, and $C_1 \sim 0$. They are in parallel with the overlap capacitance to form C_{GS} and C_{GD}.

Example: Capacitance Identification

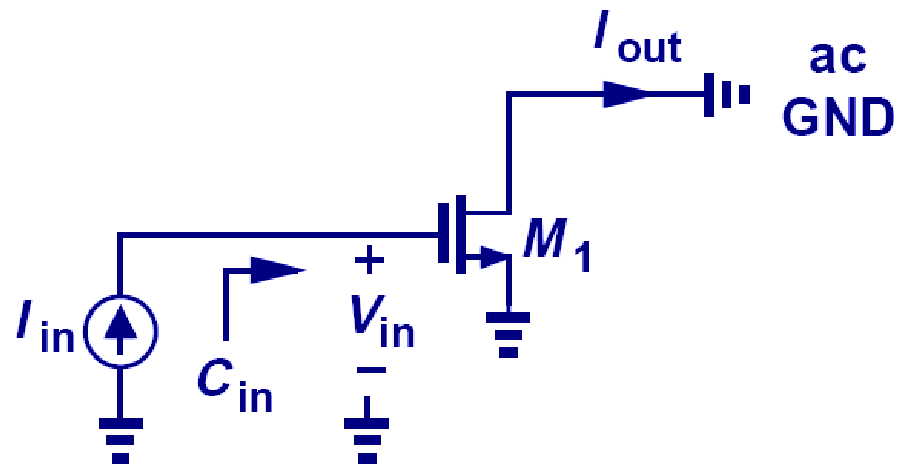




Transit Frequency



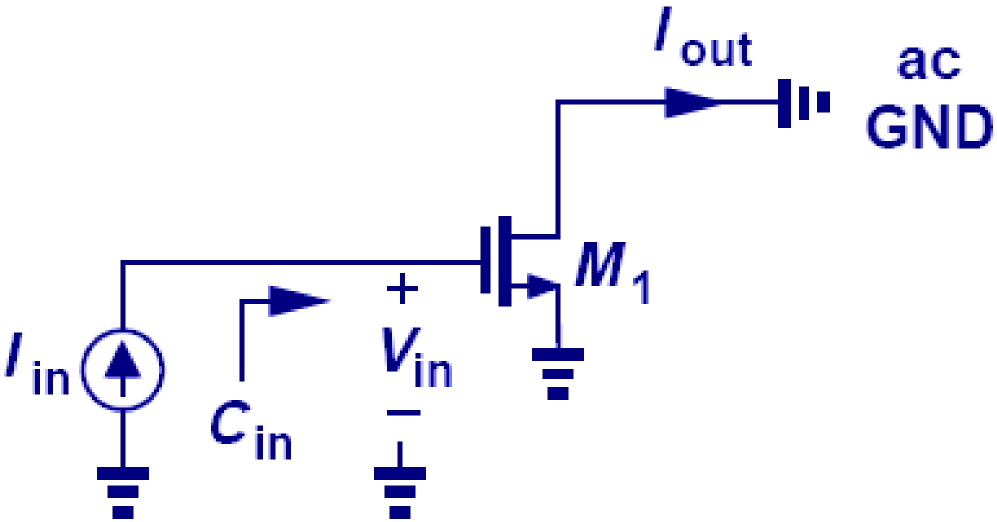
$$2\pi f_T = \frac{g_m}{C_{GS}}$$



$$2\pi f_T = \frac{g_m}{C_\pi}$$

- Transit frequency, f_T, is defined as the frequency where the current gain from input to output drops to 1.

Example: Transit Frequency Calculation



$$2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}\left(V_{GS} - V_{TH}\right)$$

$$L = 65nm$$
$$V_{GS} - V_{TH} = 100mV$$
$$\mu_n = 400cm^2/(V.s)$$
$$f_T = 226GHz$$

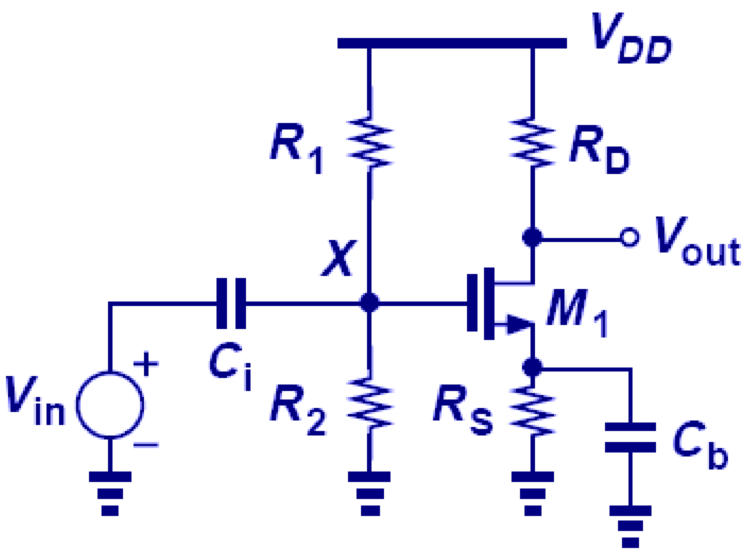
Analysis Summary

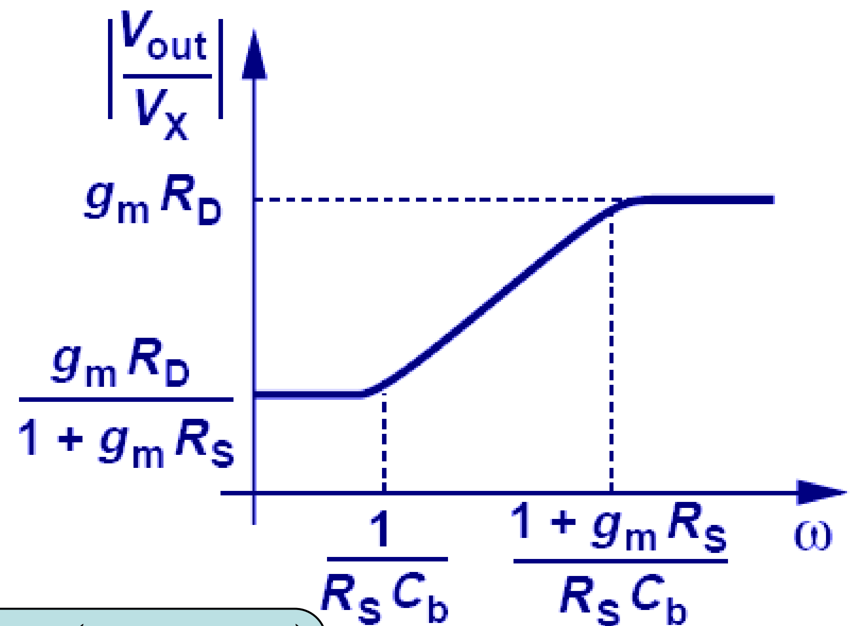
- The frequency response refers to the magnitude of the transfer function.
- Bode's approximation simplifies the plotting of the frequency response if poles and zeros are known.
- In general, it is possible to associate a pole with each node in the signal path.
- Miller's theorem helps to decompose floating capacitors into grounded elements.
- Bipolar and MOS devices exhibit various capacitances that limit the speed of circuits.

High Frequency Circuit Analysis Procedure

- Determine which capacitor impact the low-frequency region of the response and calculate the low-frequency pole (neglect transistor capacitance).
- Calculate the midband gain by replacing the capacitors with short circuits (neglect transistor capacitance).
- Include transistor capacitances.
- Merge capacitors connected to AC grounds and omit those that play no role in the circuit.
- Determine the high-frequency poles and zeros.
- Plot the frequency response using Bode's rules or exact analysis.

Frequency Response of CS Stage with Bypassed Degeneration

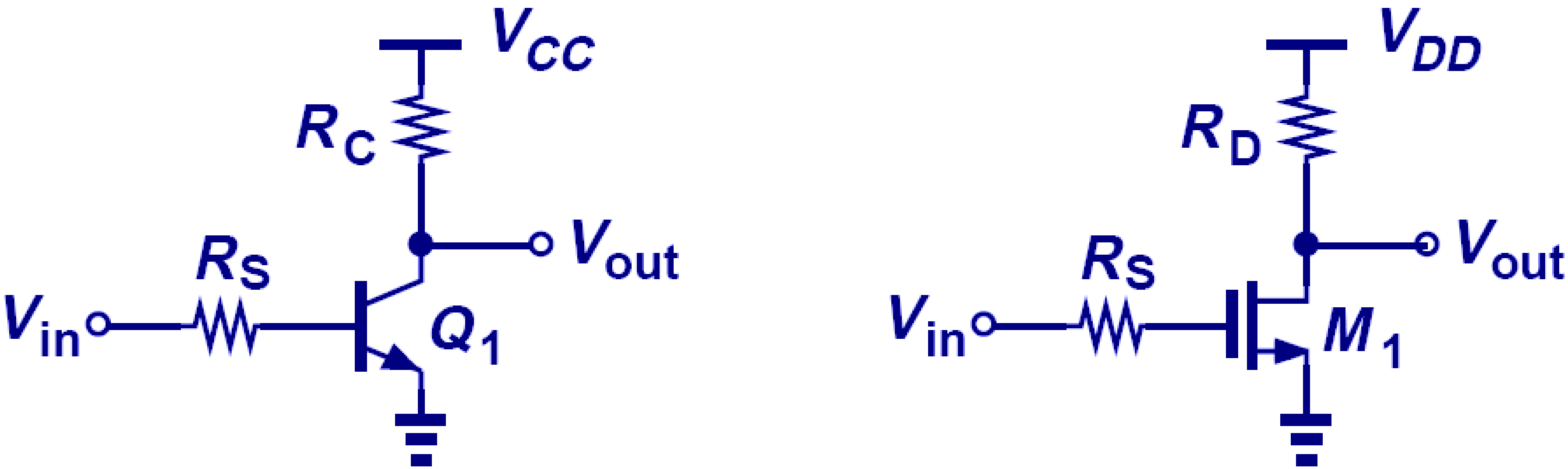


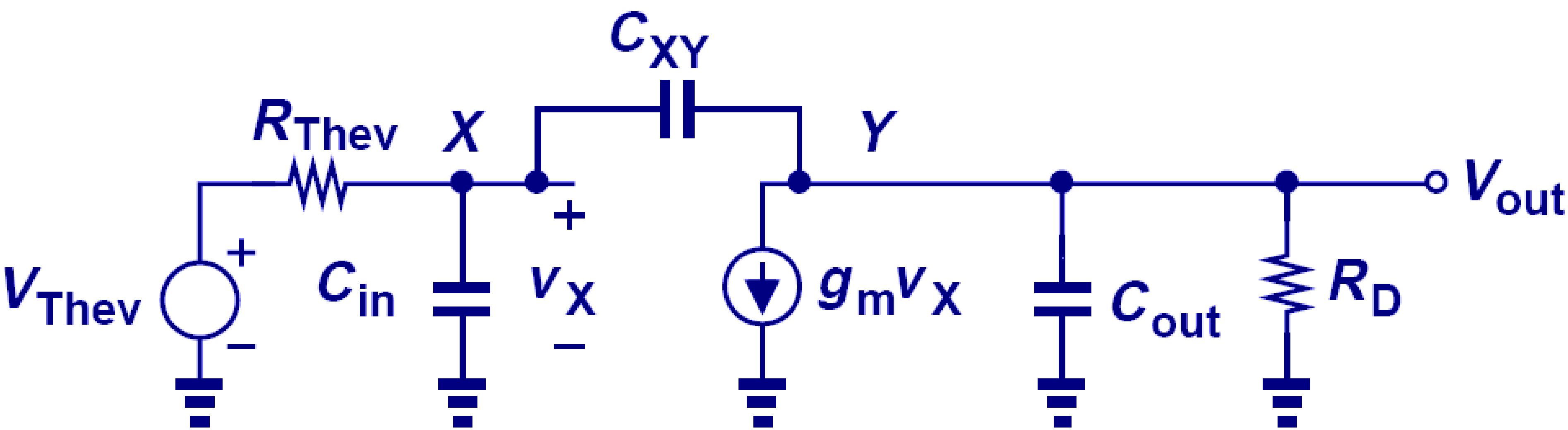


$$\left|\frac{V_{out}}{V_X}(s)\right| = \frac{-g_m R_D (R_S C_b s + 1)}{R_S C_b s + g_m R_S + 1}$$

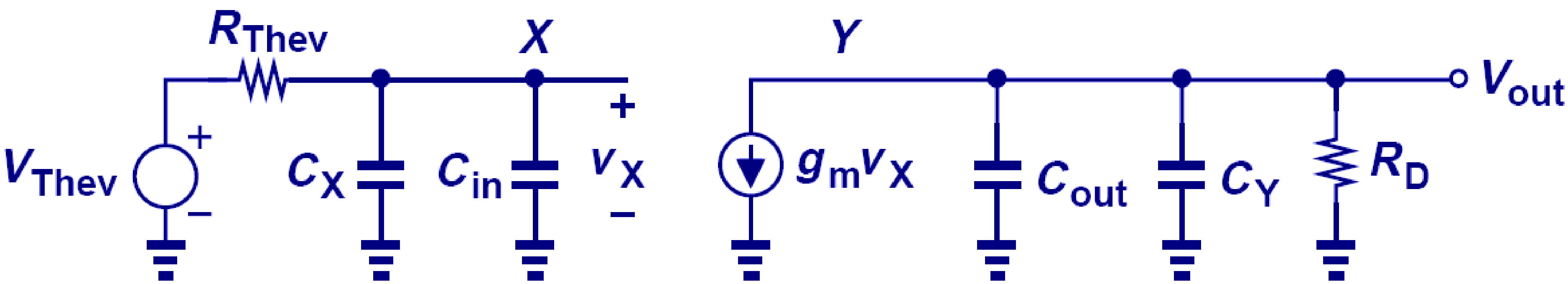
- In order to increase the midband gain, a capacitor C_b is placed in parallel with R_s.
- The pole frequency must be well below the lowest signal frequency to avoid the effect of degeneration.

Unified Model for CE and CS Stages





Unified Model Using Miller's Theorem



CE Stage

$$V_{Thev} = V_{in}\frac{r_\pi}{r_\pi + R_S}$$

$$R_{Thev} = R_S$$

$$C_X = C_\mu(1 + g_m R_D)$$

$$C_Y = C_\mu(1 + \frac{1}{g_m R_D})$$

CS Stage

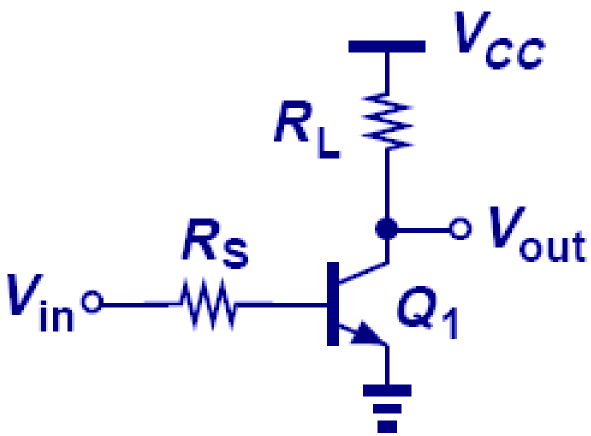
$$V_{Thev} = V_{in}$$

$$R_{Thev} = R_S$$

$$C_X = C_{GD}(1 + g_m R_D)$$

$$C_Y = C_{GD}(1 + \frac{1}{g_m R_D})$$

Example: CE Stage

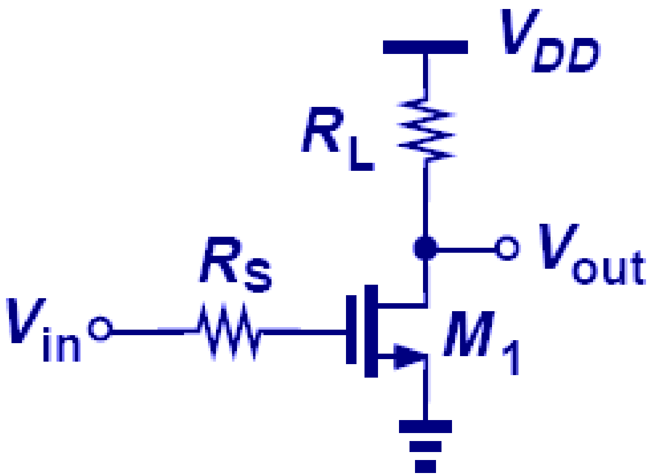


$$R_S = 200\Omega$$
$$I_C = 1mA$$
$$\beta = 100$$
$$C_\pi = 100fF$$
$$C_\mu = 20fF$$
$$C_{CS} = 30fF$$

$$\left|\omega_{p,in}\right| = 2\pi \times (516MHz)$$
$$\left|\omega_{p,out}\right| = 2\pi \times (1.59GHz)$$

- The input pole is the bottleneck for speed.

Example: Half Width CS Stage



$$W \downarrow 2X$$

$$\left|\omega_{p,in}\right| = \frac{1}{R_S\left[\frac{C_{in}}{2} + \left(1 + \frac{g_m R_L}{2}\right)\frac{C_{XY}}{2}\right]}$$

$$\left|\omega_{p,out}\right| = \frac{1}{R_L\left[\frac{C_{out}}{2} + \left(1 + \frac{2}{g_m R_L}\right)\frac{C_{XY}}{2}\right]}$$

Direct Analysis of CE and CS Stages

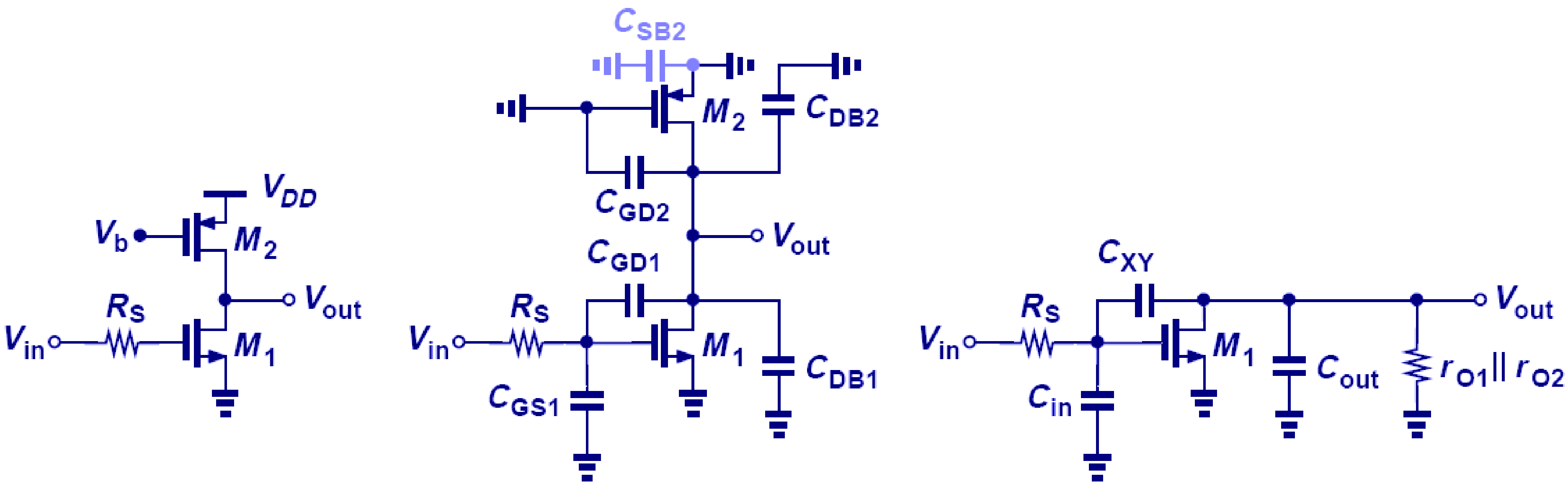
$$|\omega_z| = \frac{g_m}{C_{XY}}$$

$$|\omega_{p1}| = \frac{1}{(1+g_m R_L)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_L(C_{XY}+C_{out})}$$

$$|\omega_{p2}| = \frac{(1+g_m R_L)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_L(C_{XY}+C_{out})}{R_{Thev}R_L(C_{in}C_{XY}+C_{out}C_{XY}+C_{in}C_{out})}$$

- **Direct analysis yields different pole locations and an extra zero.**

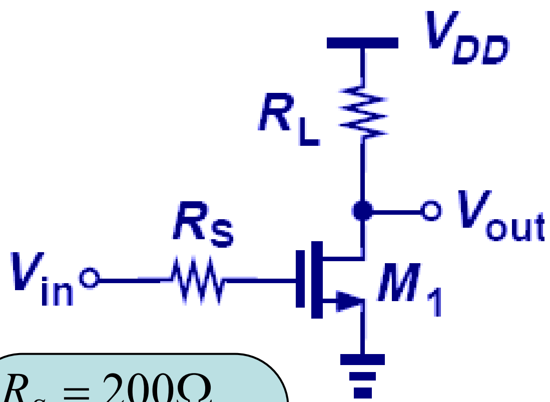
Example: CE and CS Direct Analysis



$$\omega_{p1} \approx \frac{1}{[1+g_{m1}(r_{O1} \| r_{O2})]C_{XY}R_S + R_S C_{in} + (r_{O1} \| r_{O2})(C_{XY} + C_{out})}$$

$$\omega_{p2} \approx \frac{[1+g_{m1}(r_{O1} \| r_{O2})]C_{XY}R_S + R_S C_{in} + (r_{O1} \| r_{O2})(C_{XY} + C_{out})}{R_S(r_{O1} \| r_{O2})(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})}$$

Example: Comparison Between Different Methods



$R_S = 200\Omega$

$C_{GS} = 250\,fF$

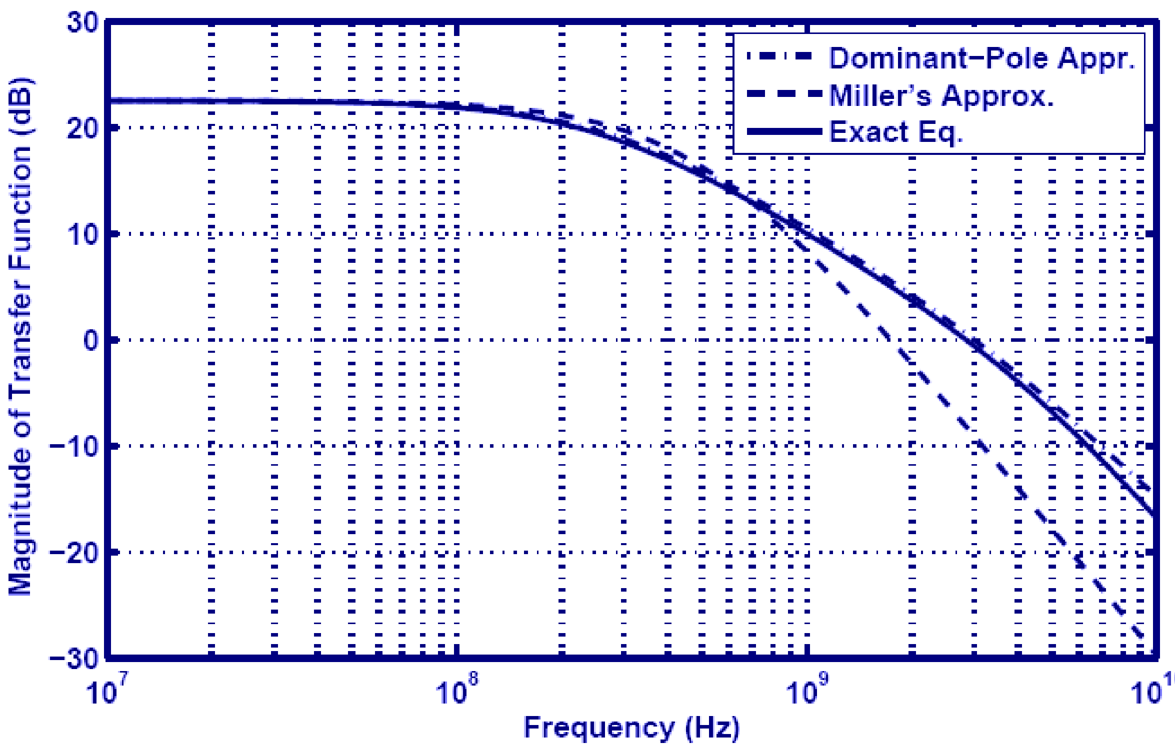
$C_{GD} = 80\,fF$

$C_{DB} = 100\,fF$

$g_m = (150\Omega)^{-1}$

$\lambda = 0$

$R_L = 2K\Omega$



Miller's

$$|\omega_{p,in}| = 2\pi \times (571MHz)$$
$$|\omega_{p,out}| = 2\pi \times (428MHz)$$

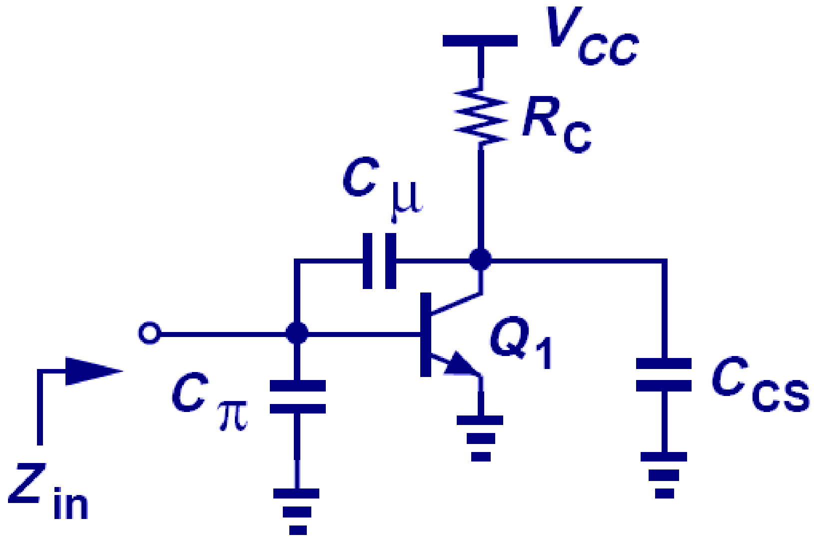
Exact

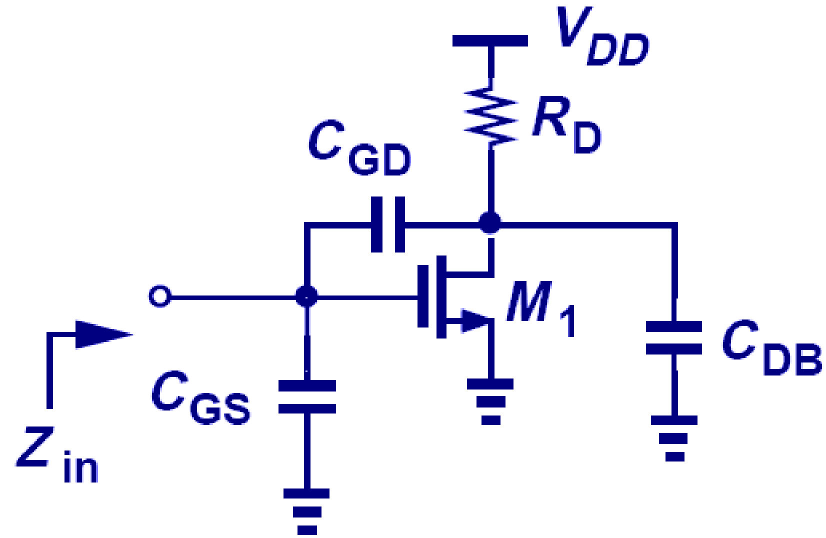
$$|\omega_{p,in}| = 2\pi \times (264MHz)$$
$$|\omega_{p,out}| = 2\pi \times (4.53GHz)$$

Dominant Pole

$$|\omega_{p,in}| = 2\pi \times (249MHz)$$
$$|\omega_{p,out}| = 2\pi \times (4.79GHz)$$

Input Impedance of CE and CS Stages

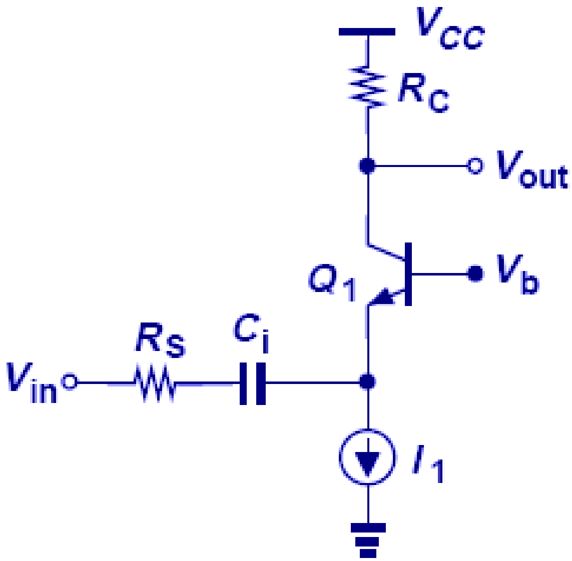




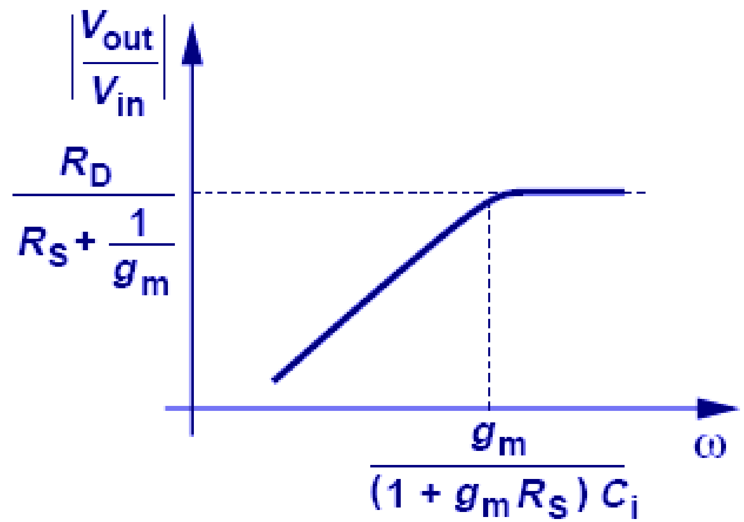
$$Z_{in} \approx \frac{1}{\left[C_\pi + (1 + g_m R_C)C_\mu\right]s} \parallel r_\pi$$

$$Z_{in} \approx \frac{1}{\left[C_{GS} + (1 + g_m R_D)C_{GD}\right]s}$$

Low Frequency Response of CB and CG Stages



(a)

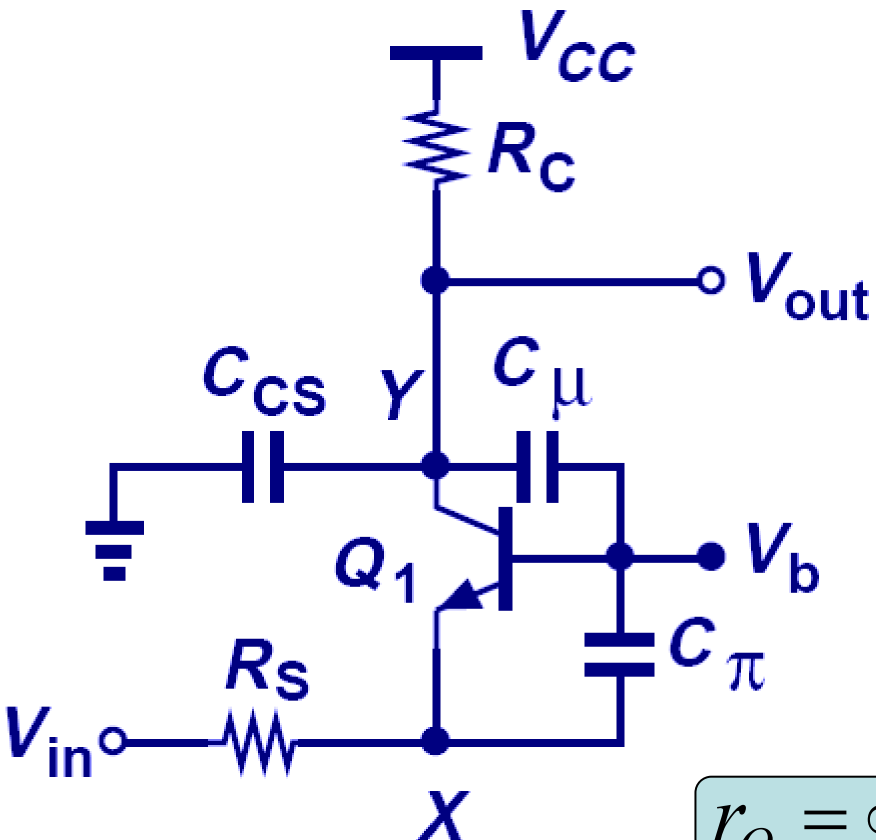


(b)

$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m R_C C_i s}{(1 + g_m R_S) C_i s + g_m}$$

- As with CE and CS stages, the use of capacitive coupling leads to low-frequency roll-off in CB and CG stages (although a CB stage is shown above, a CG stage is similar).

Frequency Response of CB Stage



$r_O = \infty$

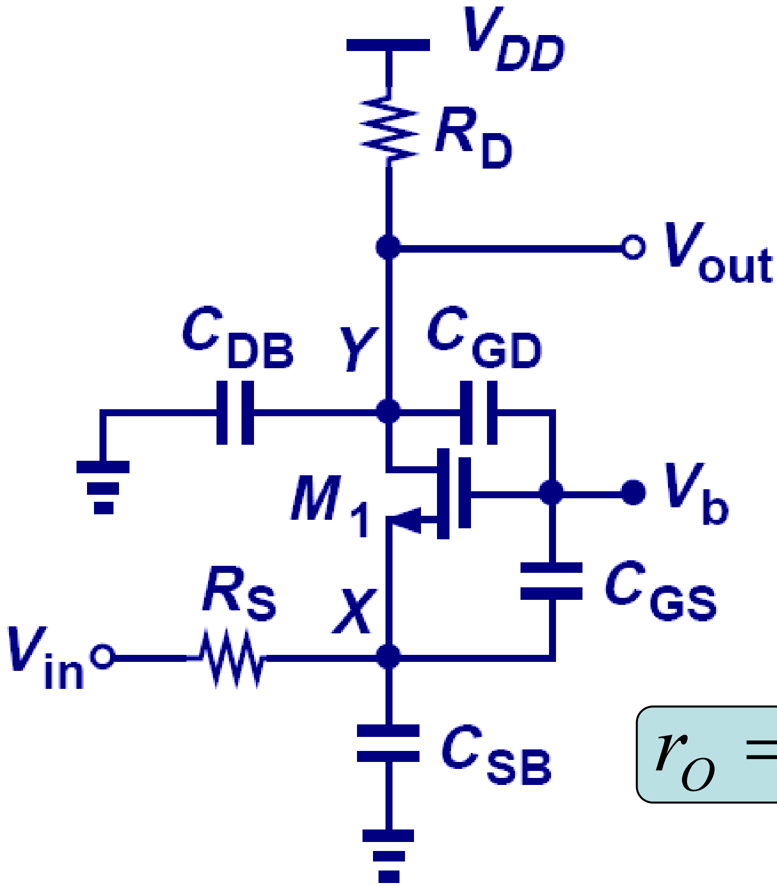
$$\omega_{p,X} = \frac{1}{\left(R_S \parallel \frac{1}{g_m}\right) C_X}$$

$$C_X = C_\pi$$

$$\omega_{p,Y} = \frac{1}{R_L C_Y}$$

$$C_Y = C_\mu + C_{CS}$$

Frequency Response of CG Stage



$$r_O = \infty$$

$$\omega_{p,X} = \frac{1}{\left(R_S \parallel \frac{1}{g_m}\right) C_X}$$

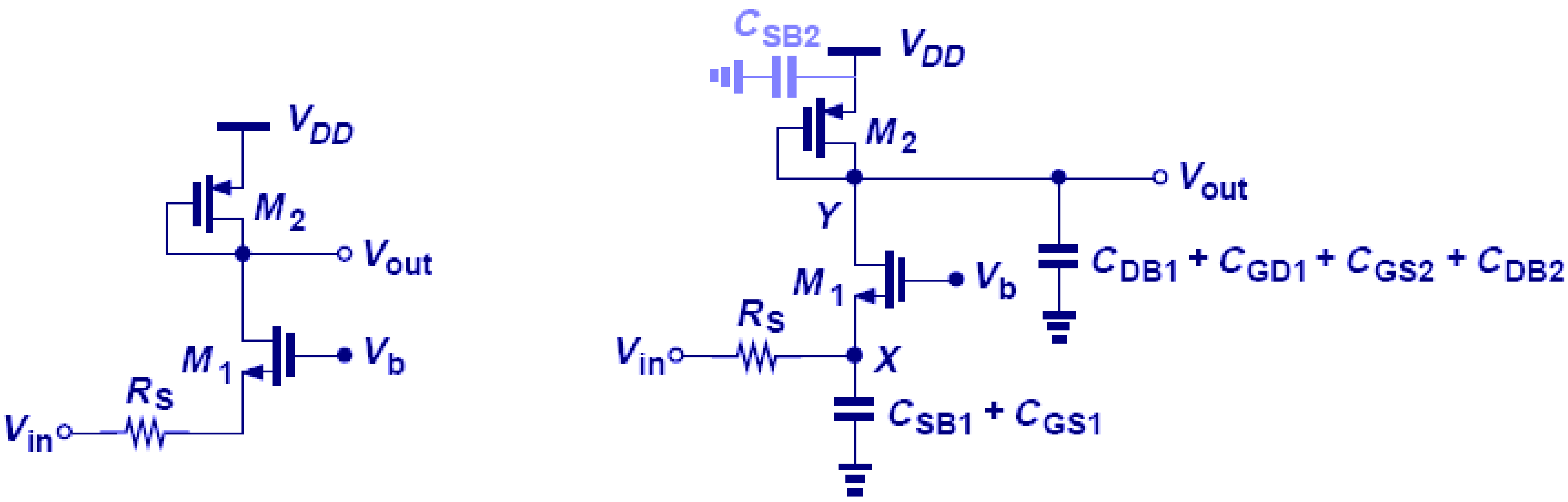
$$C_X = C_{GS} + C_{SB}$$

$$\omega_{p,Y} = \frac{1}{R_L C_Y}$$

$$C_Y = C_{GD} + C_{DB}$$

- Similar to a CB stage, the input pole is on the order of f_T, so rarely a speed bottleneck.

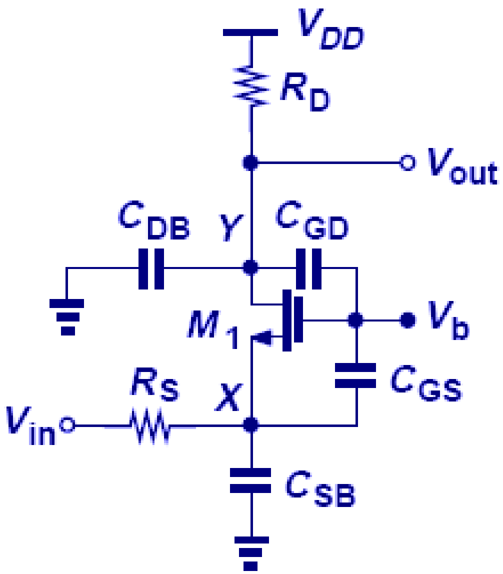
Example: CG Stage Pole Identification



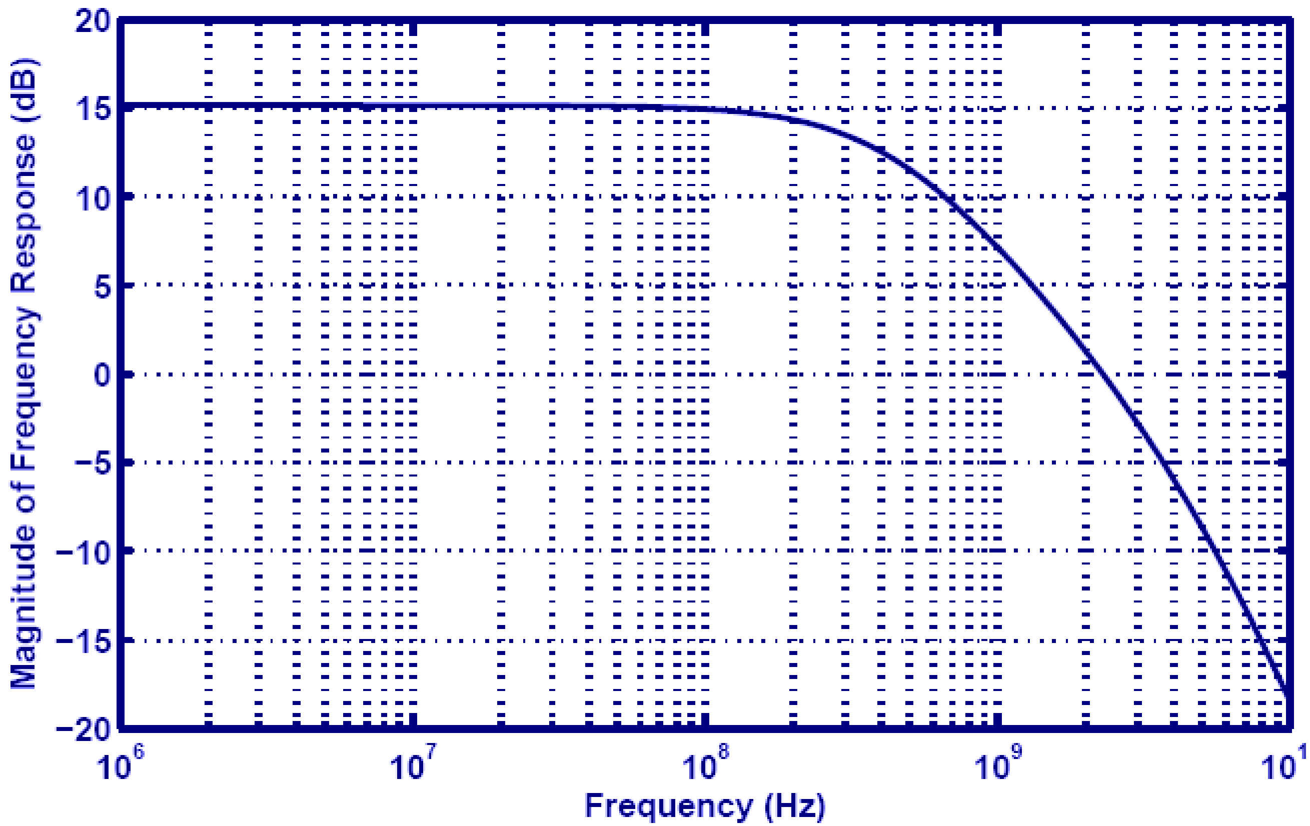
(a) (b)

$$\omega_{p,X} = \frac{1}{\left(R_S \parallel \frac{1}{g_{m1}}\right)\left(C_{SB1} + C_{GD1}\right)} \qquad \omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}\left(C_{DB1} + C_{GD1} + C_{GS2} + C_{DB2}\right)}$$

Example: Frequency Response of CG Stage



$$R_S = 200\Omega$$
$$C_{GS} = 250\,fF$$
$$C_{GD} = 80\,fF$$
$$C_{DB} = 100\,fF$$
$$g_m = (150\Omega)^{-1}$$
$$\lambda = 0$$
$$R_d = 2K\Omega$$



$$|\omega_{p,X}| = 2\pi \times (5.31GHz)$$
$$|\omega_{p,Y}| = 2\pi \times (442MHz)$$

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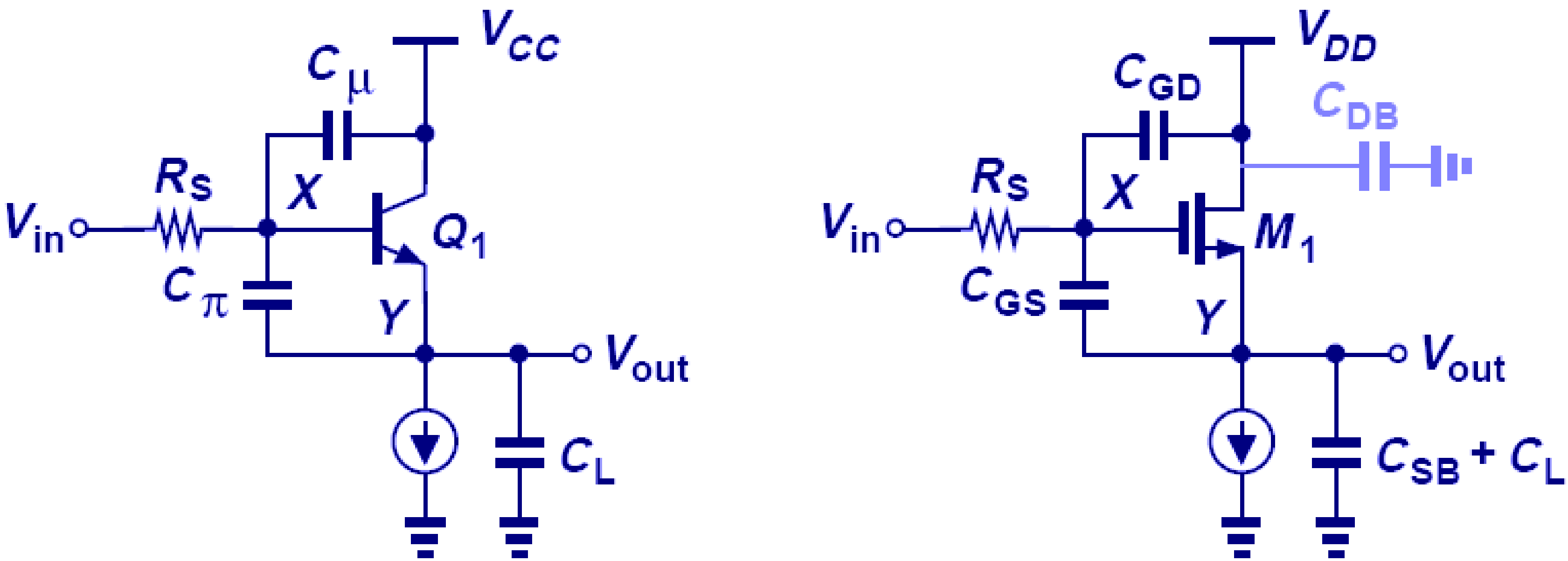
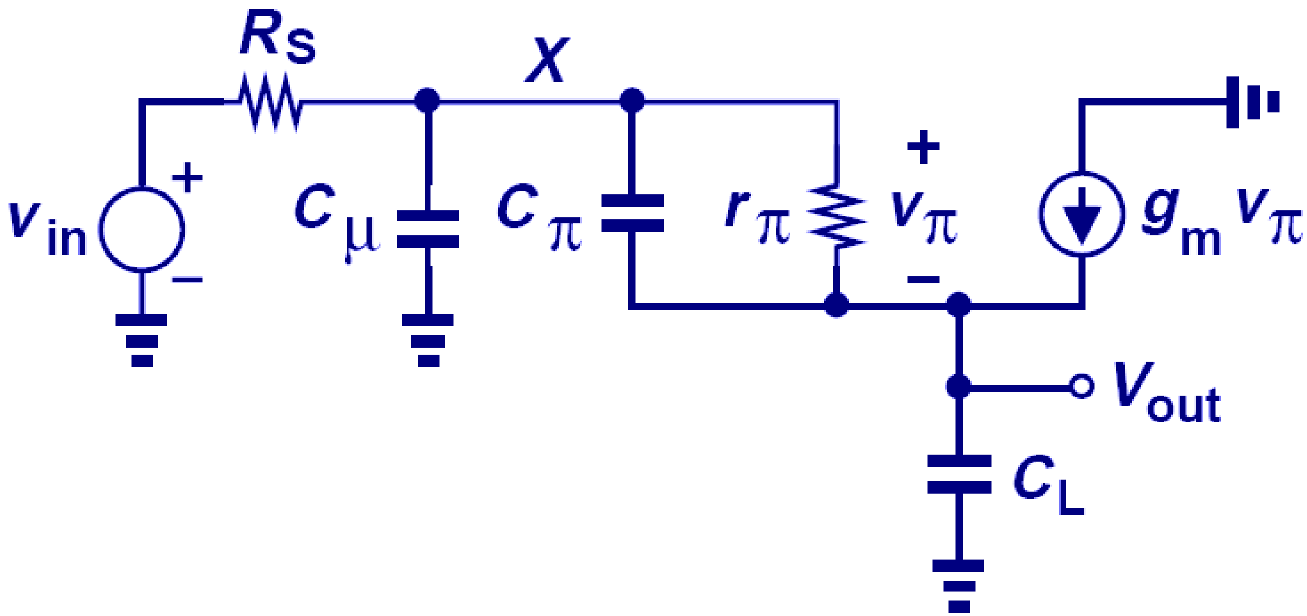
Emitter and Source Followers



(a) (b)

- The following will discuss the frequency response of emitter and source followers using direct analysis.
- Emitter follower is treated first and source follower is derived easily by allowing r_π to go to infinity.

Direct Analysis of Emitter Follower

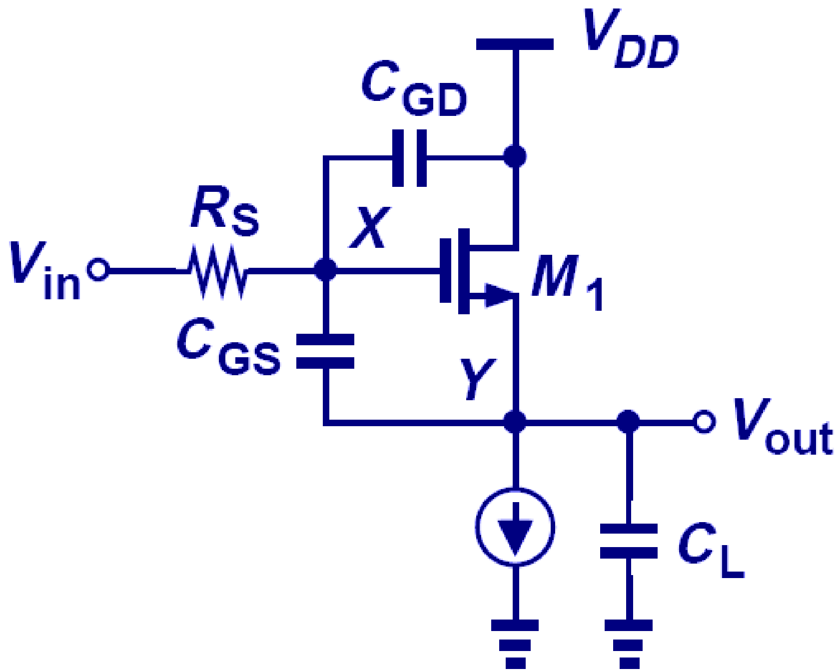


$$\frac{V_{out}}{V_{in}} = \frac{1 + \dfrac{C_\pi}{g_m} s}{a s^2 + b s + 1}$$

$$a = \frac{R_S}{g_m}\left(C_\mu C_\pi + C_\mu C_L + C_\pi C_L\right)$$

$$b = R_S C_\mu + \frac{C_\pi}{g_m} + \left(1 + \frac{R_S}{r_\pi}\right)\frac{C_L}{g_m}$$

Direct Analysis of Source Follower Stage

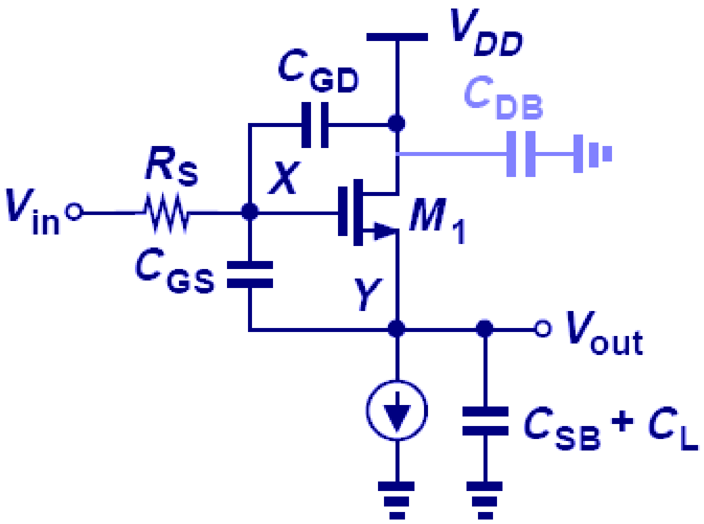


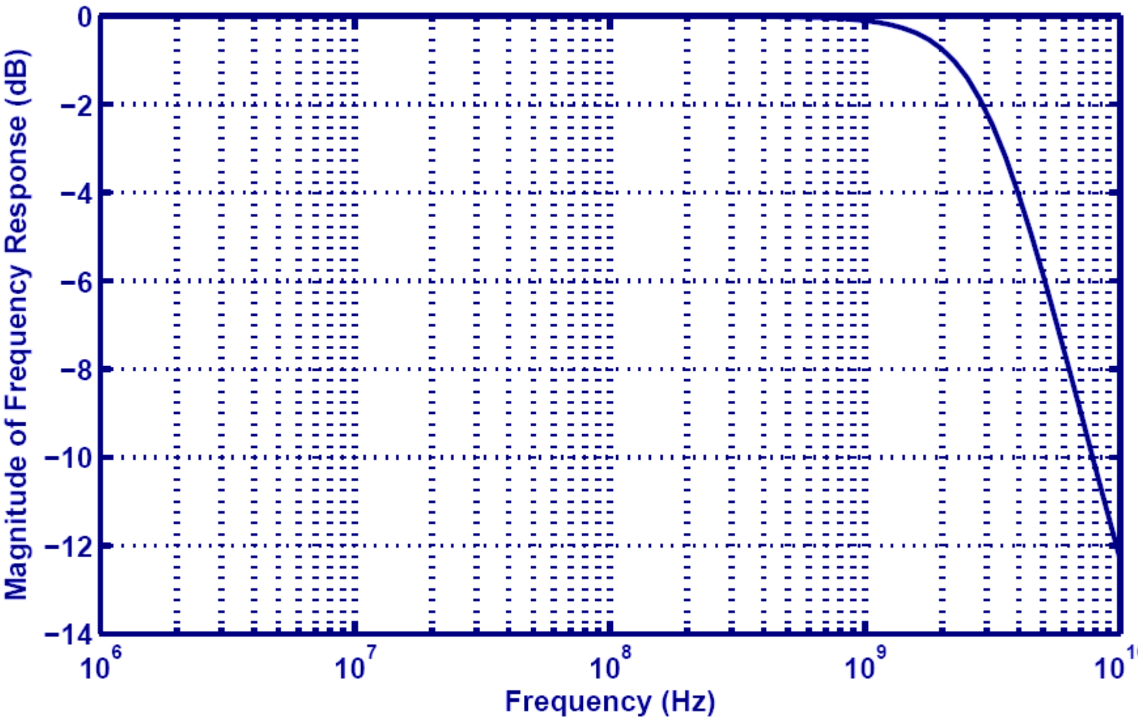
$$\frac{V_{out}}{V_{in}} = \frac{1 + \dfrac{C_{GS}}{g_m} s}{a s^2 + b s + 1}$$

$$a = \frac{R_S}{g_m}\left(C_{GD}C_{GS} + C_{GD}C_{SB} + C_{GS}C_{SB}\right)$$

$$b = R_S C_{GD} + \frac{C_{GD} + C_{SB}}{g_m}$$

Example: Frequency Response of Source Follower





$$R_S = 200\Omega$$
$$C_L = 100\,fF$$
$$C_{GS} = 250\,fF$$
$$C_{GD} = 80\,fF$$
$$C_{DB} = 100\,fF$$
$$g_m = (150\Omega)^{-1}$$
$$\lambda = 0$$

$$\omega_{p1} = 2\pi[-1.79GHz + j(2.57GHz)]$$
$$\omega_{p2} = 2\pi[-1.79GHz - j(2.57GHz)]$$

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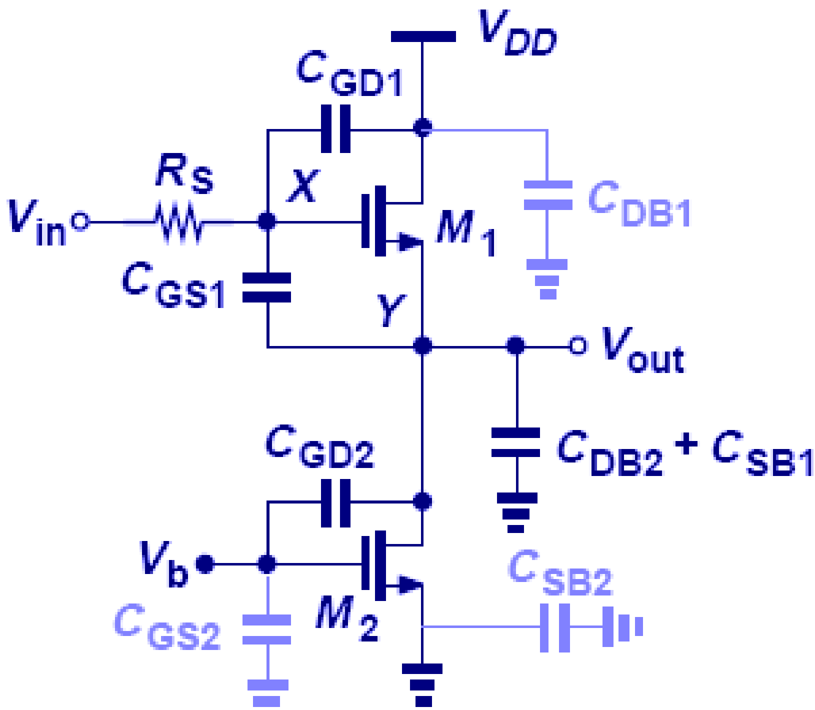
Example: Source Follower

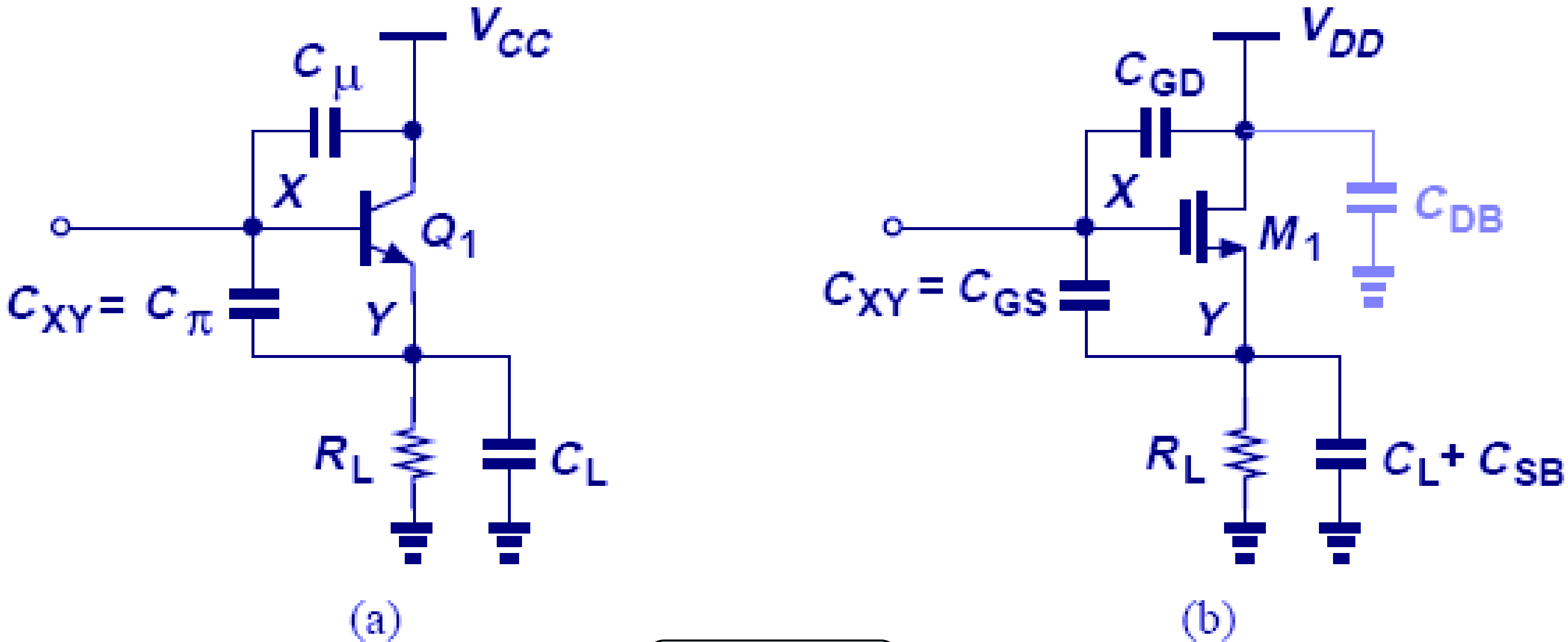


$$\frac{V_{out}}{V_{in}}=\frac{1+\dfrac{C_{GS}}{g_m}s}{as^2+bs+1}$$

$$a=\frac{R_S}{g_{m1}}\left[C_{GD1}C_{GS1}+(C_{GD1}+C_{GS1})(C_{SB1}+C_{GD2}+C_{DB2})\right]$$

$$b=R_S C_{GD1}+\frac{C_{GD1}+C_{SB1}+C_{GD2}+C_{DB2}}{g_{m1}}$$

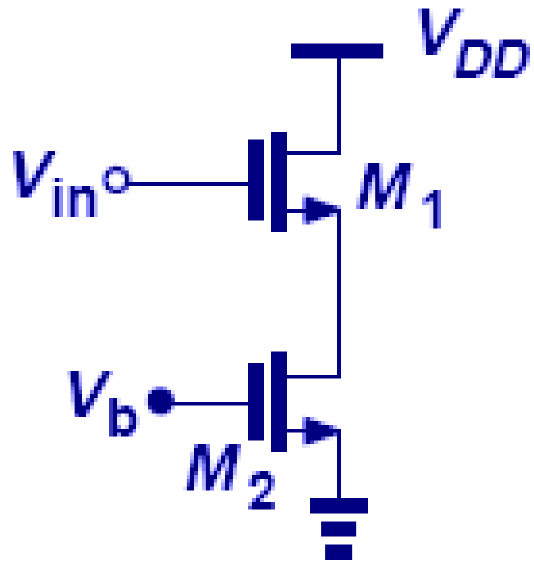
Input Capacitance of Emitter/Source Follower



$$r_O = \infty$$

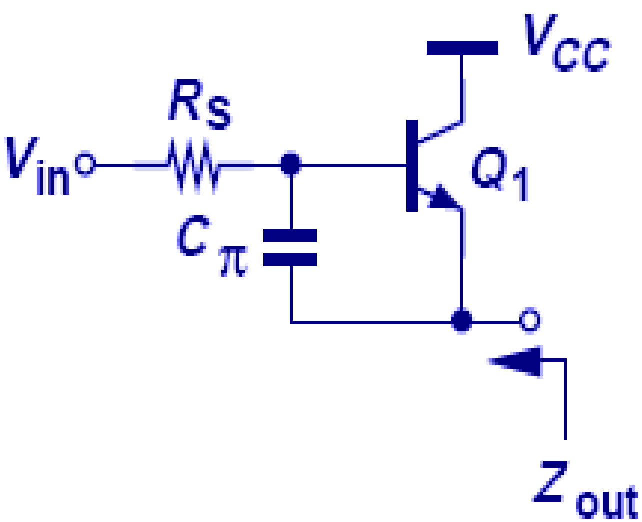
$$C_{in} = C_\mu / C_{GD} + \frac{C_\pi / C_{GS}}{1 + g_m R_L}$$

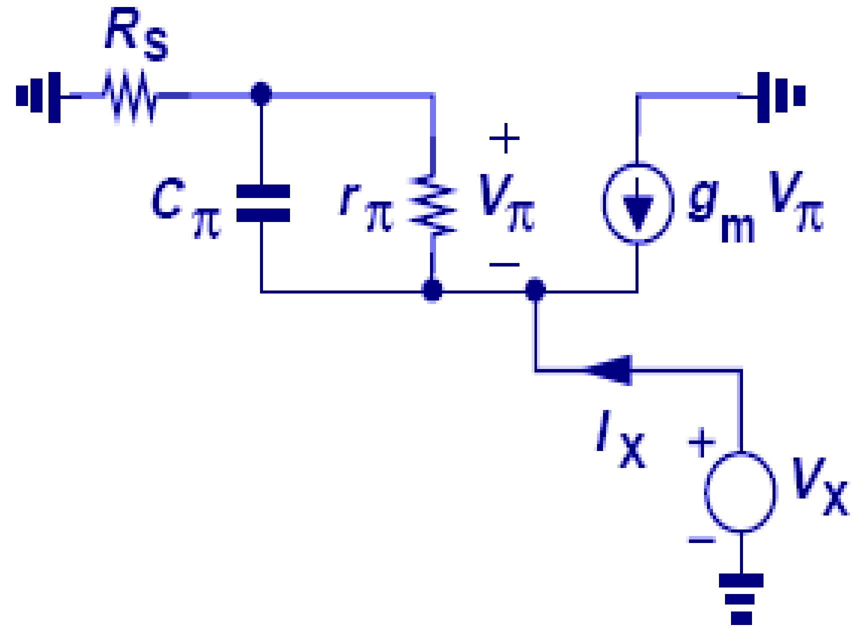
Example: Source Follower Input Capacitance



$$C_{in} = C_{GD1} + \frac{1}{1 + g_{m1}\left(r_{O1} \parallel r_{O2}\right)} C_{GS1}$$

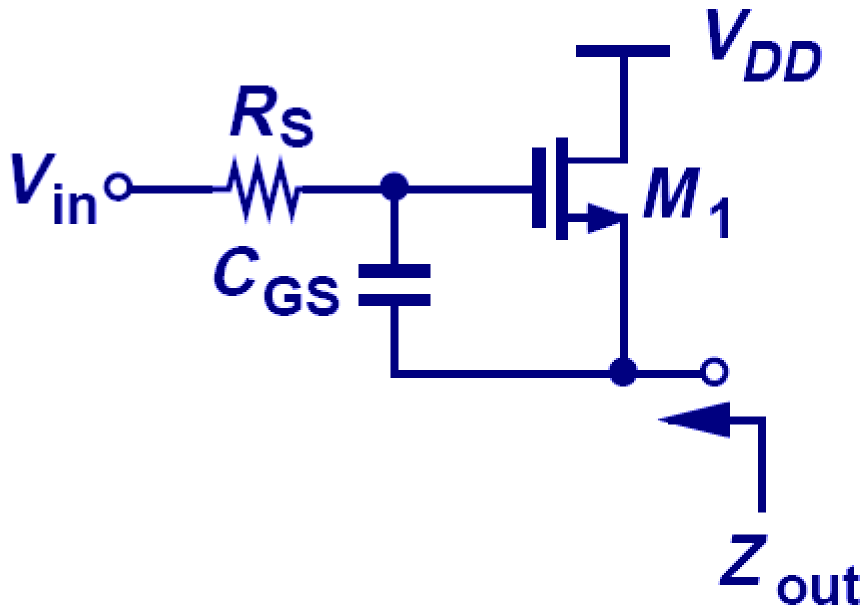
Output Impedance of Emitter Follower





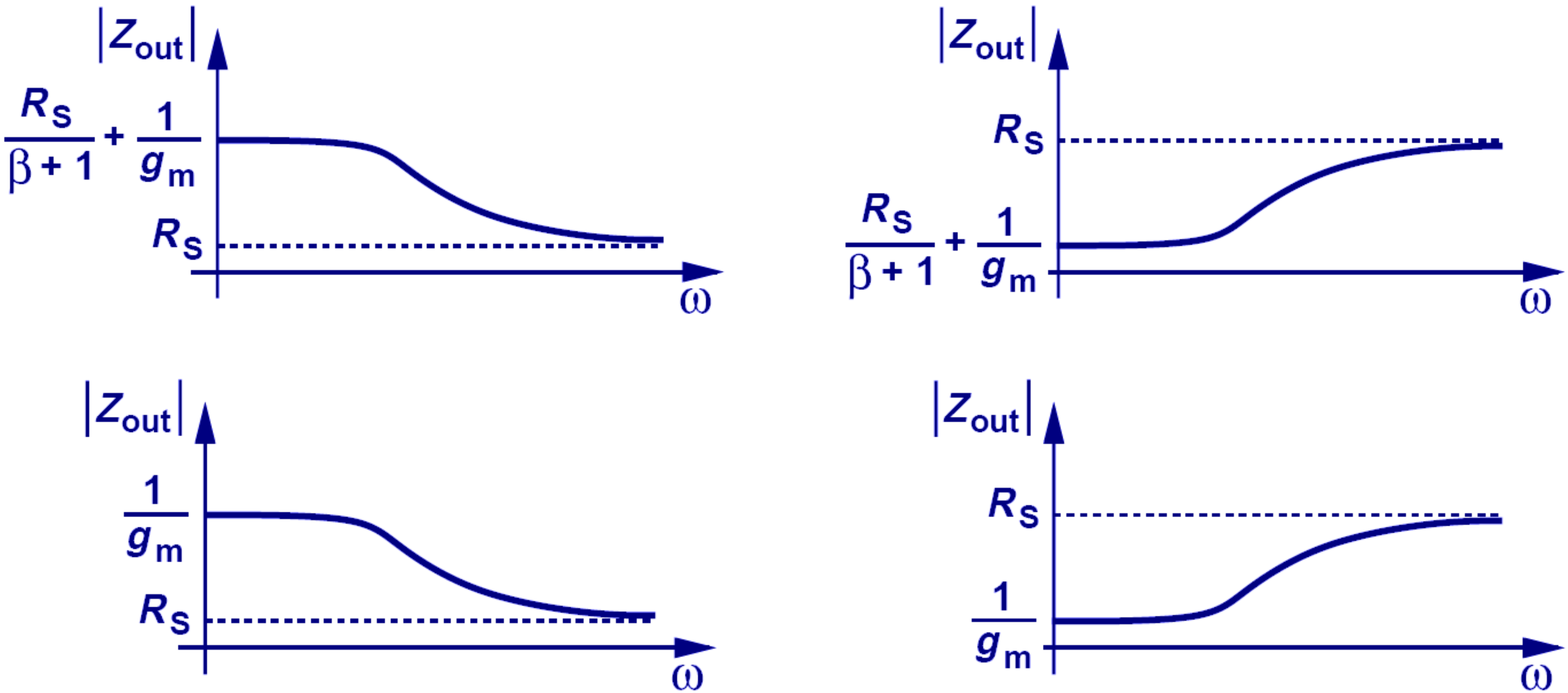
$$\frac{V_X}{I_X} = \frac{R_S r_\pi C_\pi s + r_\pi + R_S}{r_\pi C_\pi s + \beta + 1}$$

Output Impedance of Source Follower



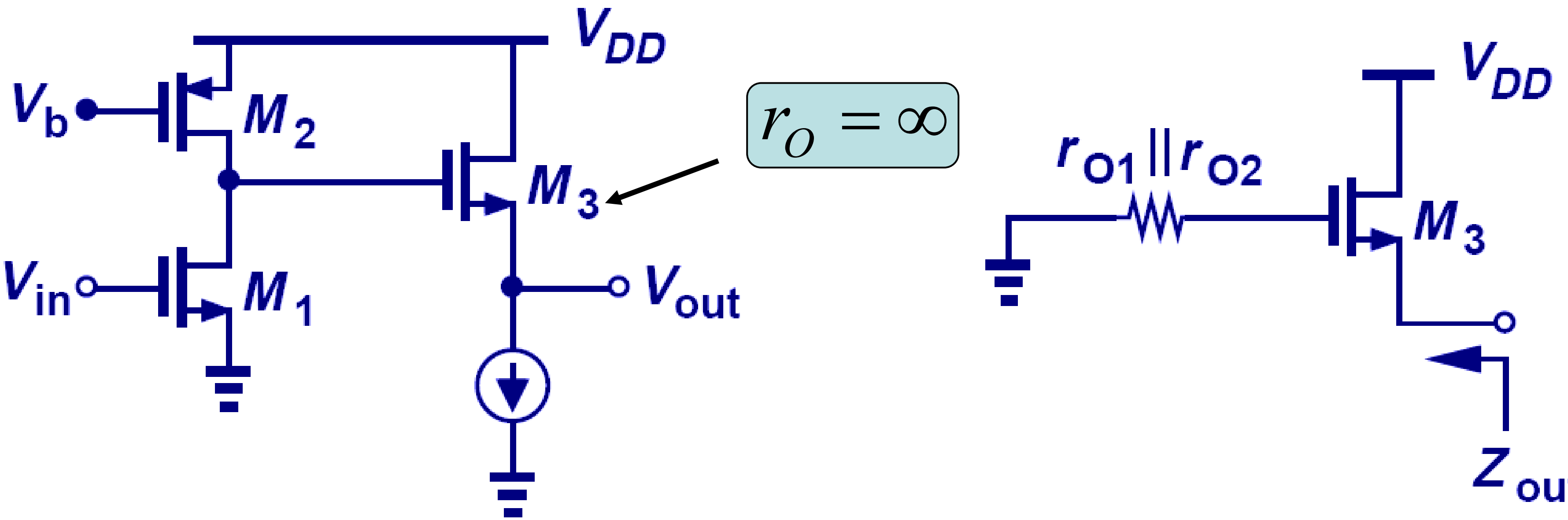
$$\frac{V_X}{I_X} = \frac{R_S C_{GS} s + 1}{C_{GS} s + g_m}$$

Active Inductor



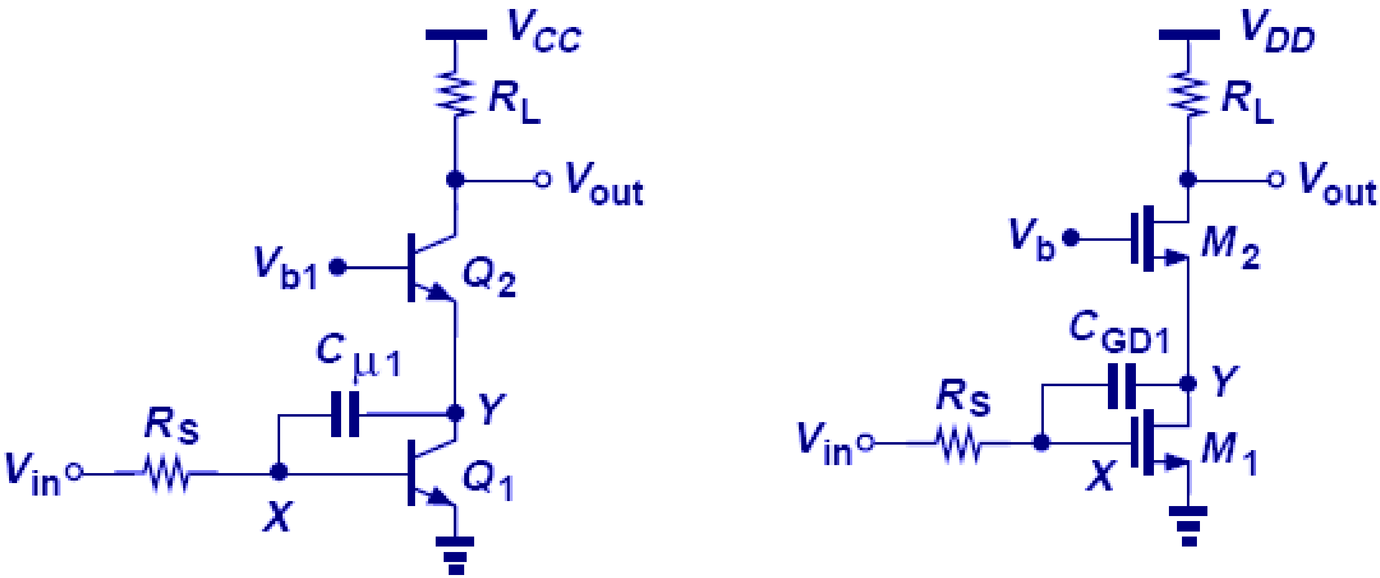
- **The plot above shows the output impedance of emitter and source followers. Since a follower's primary duty is to lower the driving impedance ($R_S>1/g_m$), the "active inductor" characteristic on the right is usually observed.**

Example: Output Impedance



$$\frac{V_X}{I_X} = \frac{(r_{O1} \| r_{O2})C_{GS3}s + 1}{C_{GS3}s + g_{m3}}$$

Frequency Response of Cascode Stage



(a) (b)

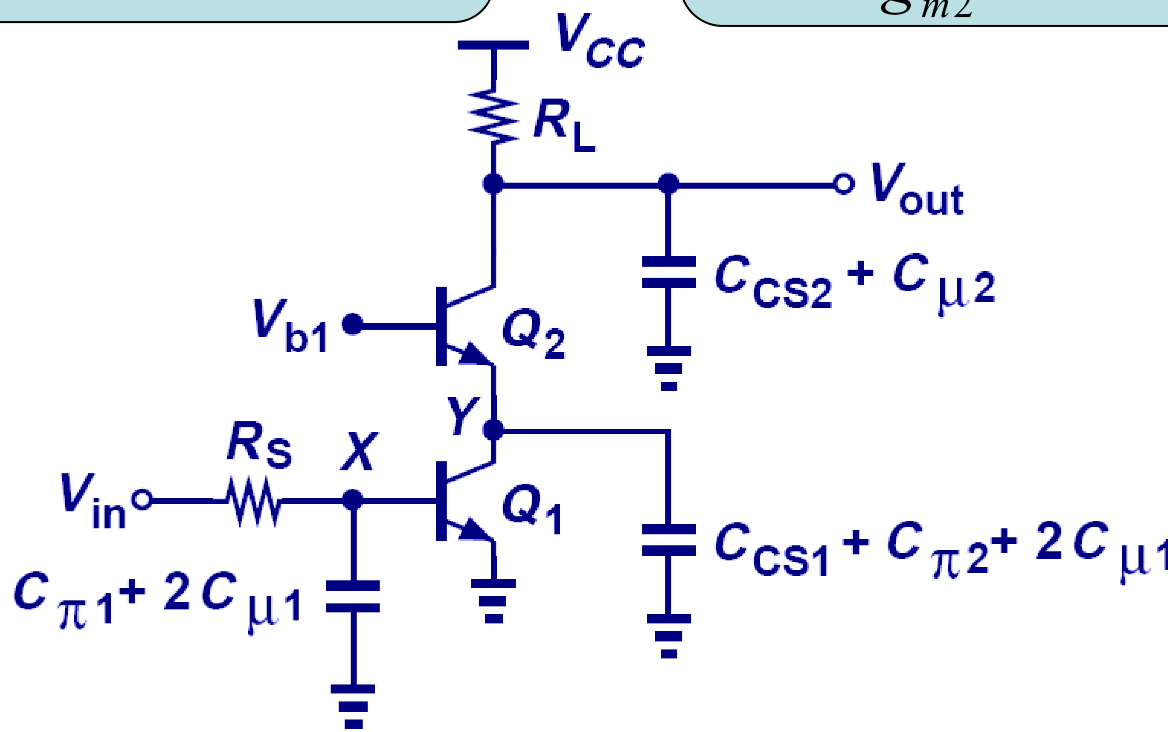
$$A_{v,XY} = \frac{-g_{m1}}{g_{m2}} \approx -1 \qquad C_x \approx 2C_{XY}$$

- **For cascode stages, there are three poles and Miller multiplication is smaller than in the CE/CS stage.**

Poles of Bipolar Cascode

$$\omega_{p,X} = \frac{1}{\left(R_S \parallel r_{\pi 1}\right)\left(C_{\pi 1} + 2C_{\mu 1}\right)}$$

$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}\left(C_{CS1} + C_{\pi 2} + 2C_{\mu 1}\right)}$$

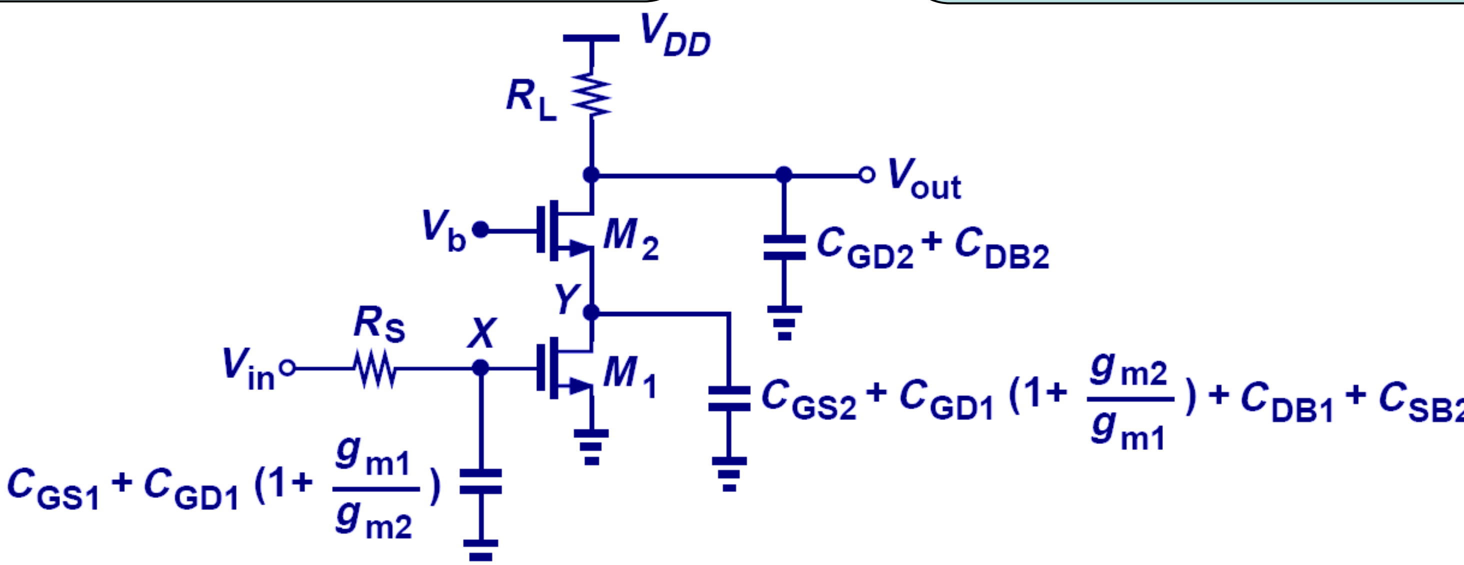


$$\omega_{p,out} = \frac{1}{R_L\left(C_{CS2} + C_{\mu 2}\right)}$$

Poles of MOS Cascode

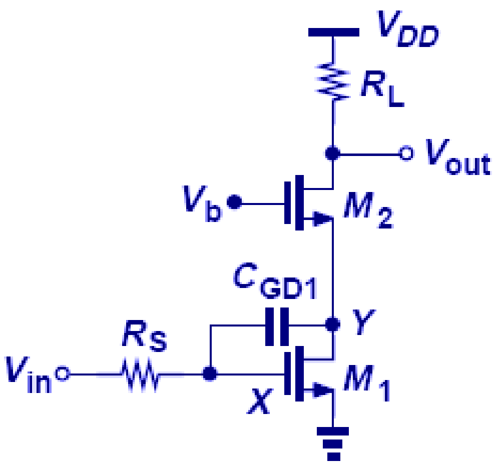
$$\omega_{p,X} = \frac{1}{R_S\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right)C_{GD1}\right]}$$

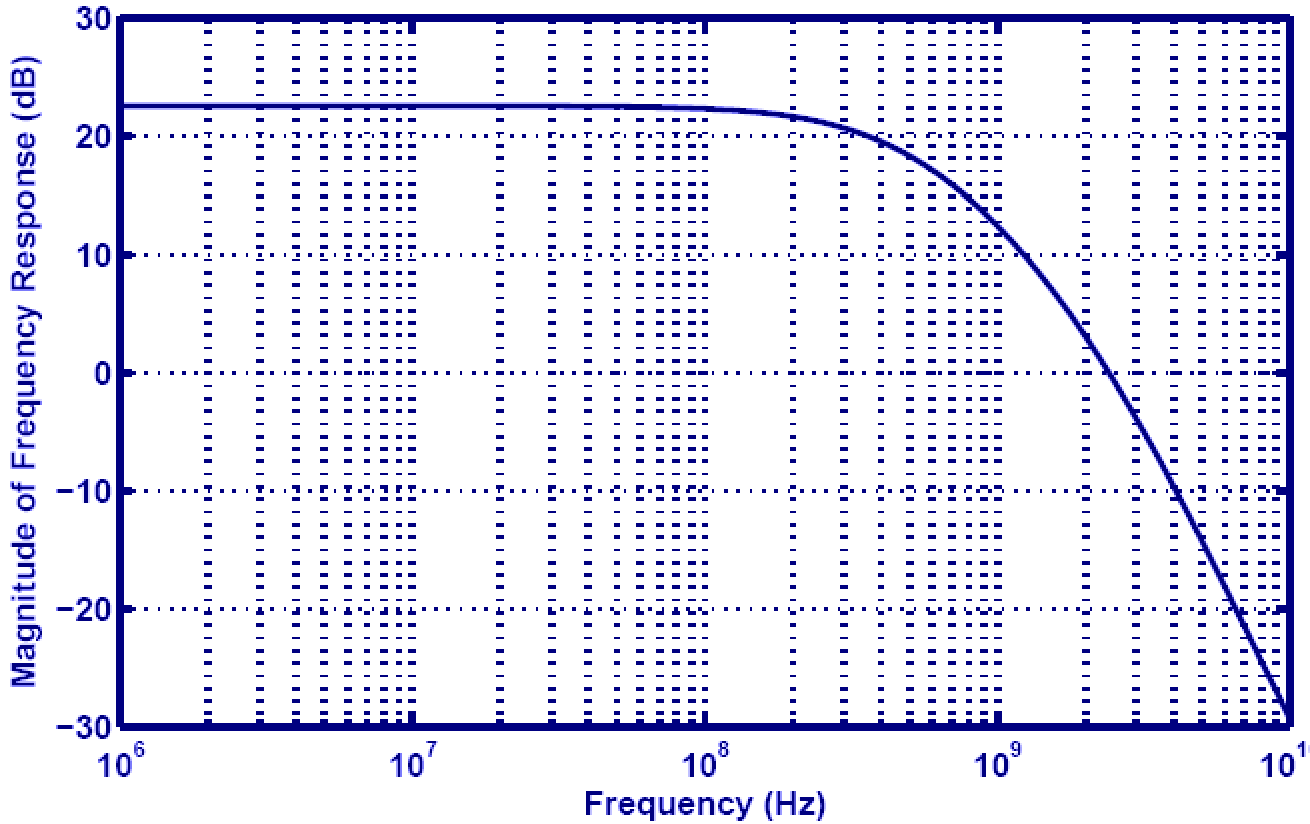
$$\omega_{p,out} = \frac{1}{R_L\left(C_{DB2} + C_{GD2}\right)}$$



$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}\left[C_{DB1} + C_{GS2} + \left(1 + \frac{g_{m2}}{g_{m1}}\right)C_{GD1}\right]}$$

Example: Frequency Response of Cascode





$$R_S = 200\Omega$$
$$C_{GS} = 250\,fF$$
$$C_{GD} = 80\,fF$$
$$C_{DB} = 100\,fF$$
$$g_m = (150\Omega)^{-1}$$
$$\lambda = 0$$
$$R_L = 2K\Omega$$

$$|\omega_{p,X}| = 2\pi \times (1.95GHz)$$
$$|\omega_{p,Y}| = 2\pi \times (1.73GHz)$$
$$|\omega_{p,out}| = 2\pi \times (442MHz)$$

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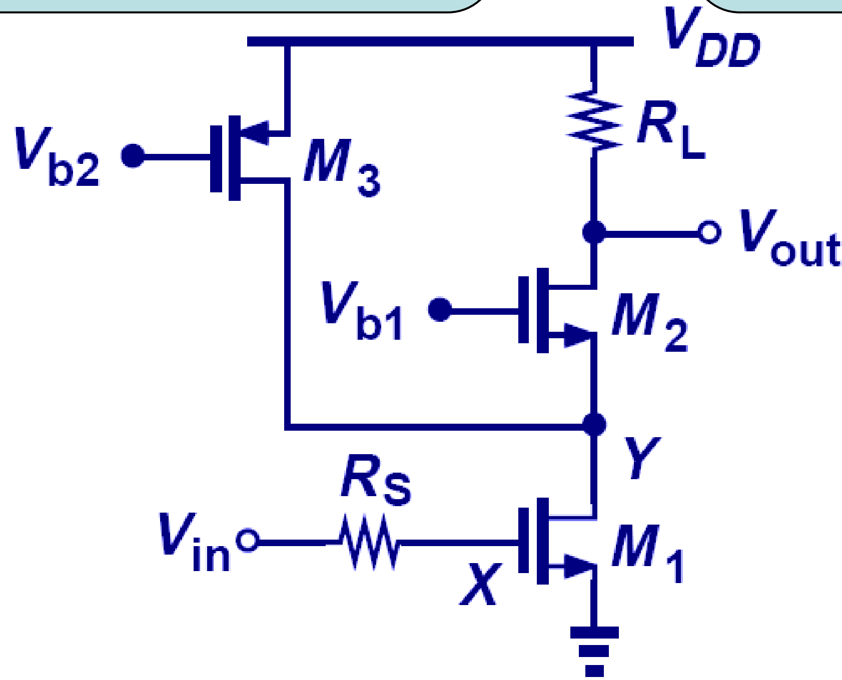
MOS Cascode Example

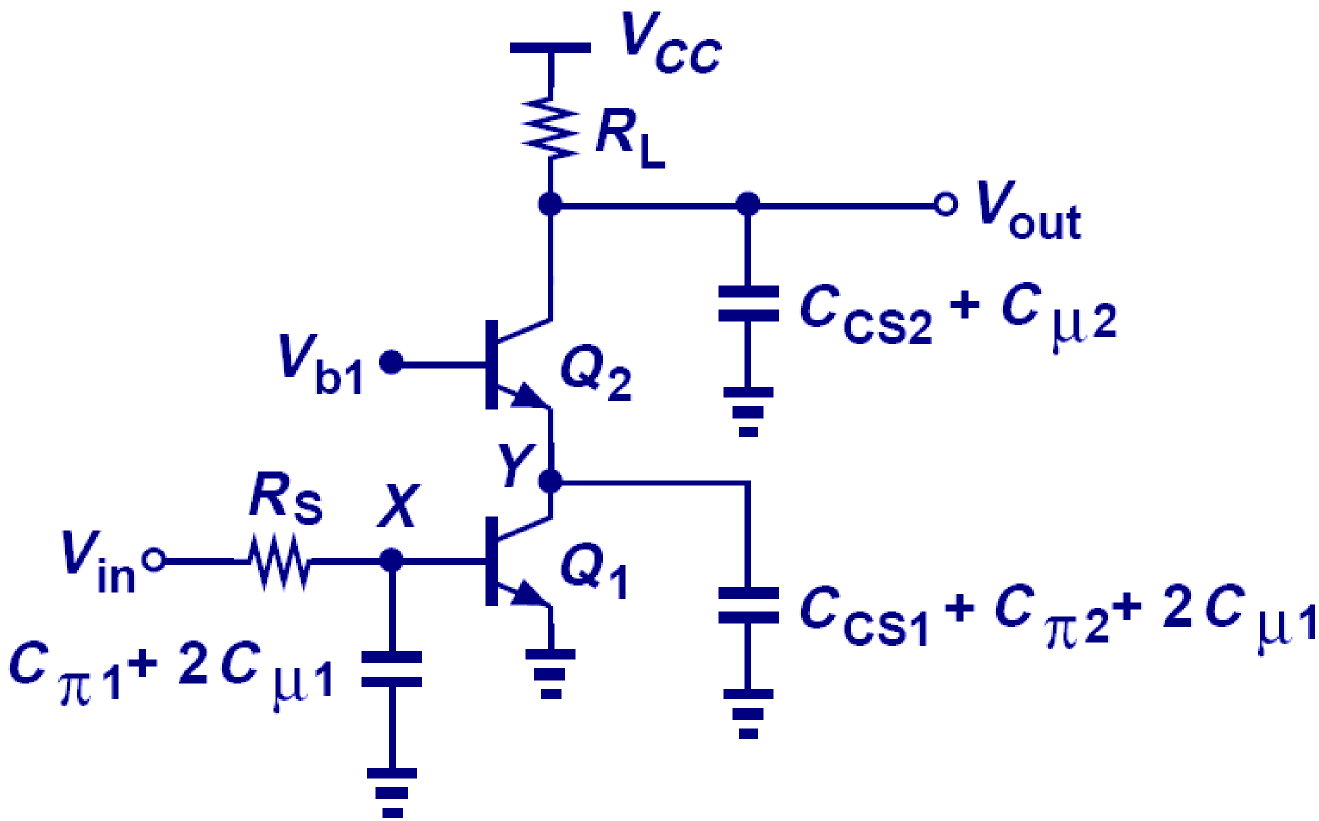
$$\omega_{p,X} = \frac{1}{R_S\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right)C_{GD1}\right]}$$

$$\omega_{p,out} = \frac{1}{R_L\left(C_{DB2} + C_{GD2}\right)}$$



$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}\left[C_{DB1} + C_{GS2} + \left(1 + \frac{g_{m2}}{g_{m1}}\right)C_{GD1} + C_{GD3} + C_{DB3}\right]}$$

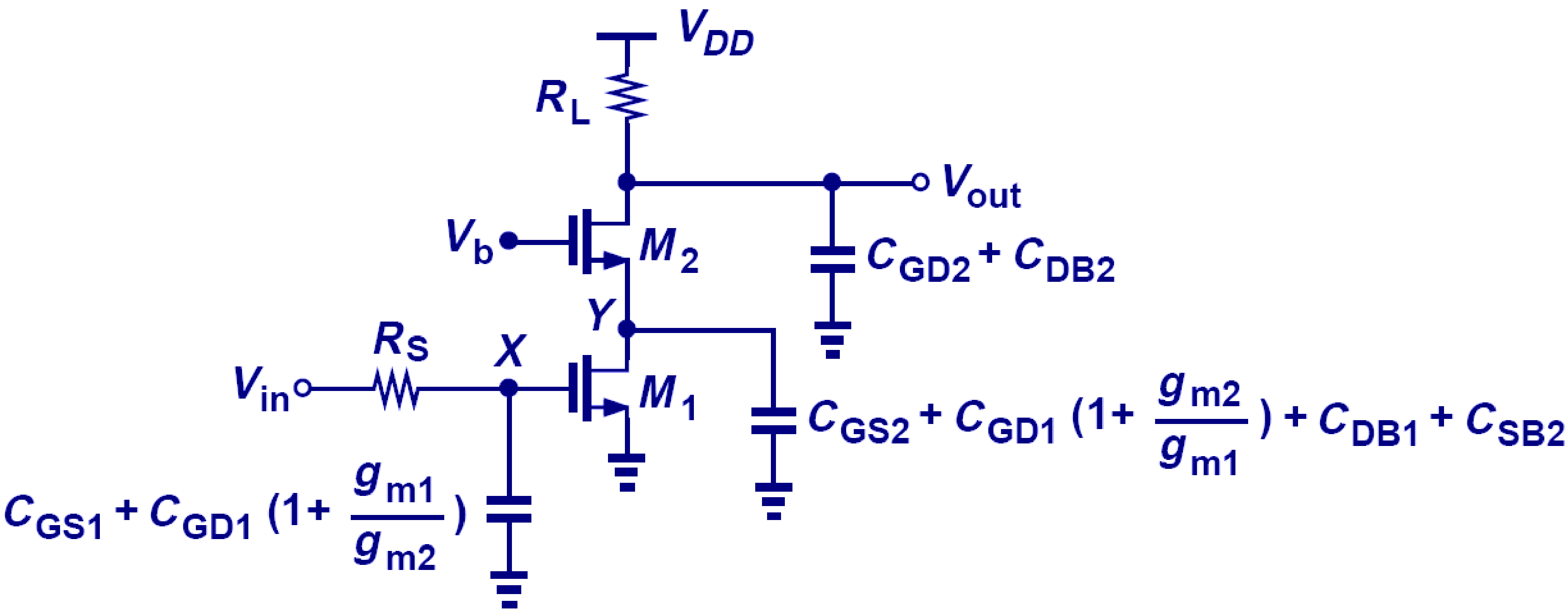
I/O Impedance of Bipolar Cascode



$$Z_{in} = r_{\pi 1} \parallel \frac{1}{(C_{\pi 1} + 2C_{\mu 1})s}$$

$$Z_{out} = R_L \parallel \frac{1}{(C_{\mu 2} + C_{CS2})s}$$

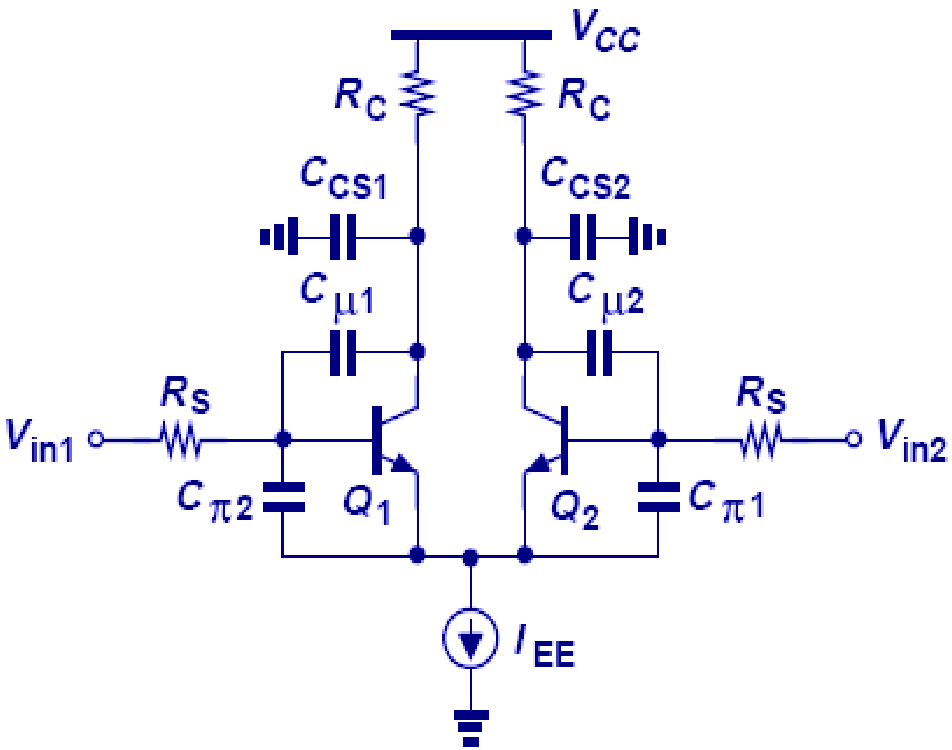
I/O Impedance of MOS Cascode

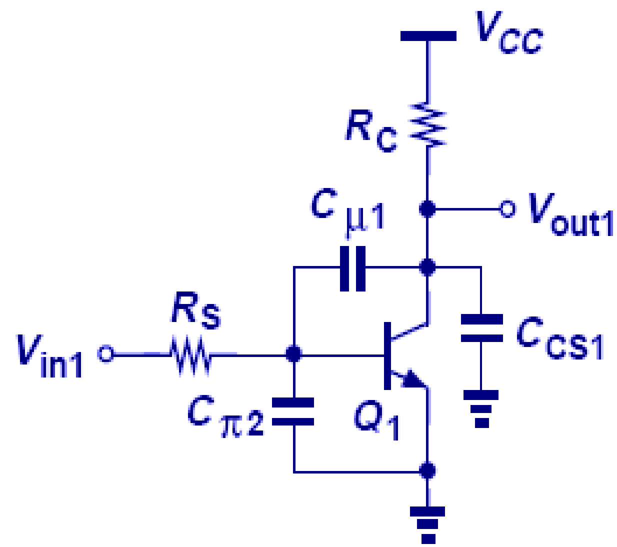


$$Z_{in} = \frac{1}{\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right)C_{GD1}\right]s}$$

$$Z_{out} = R_L \parallel \frac{1}{(C_{GD2} + C_{DB2})s}$$

Bipolar Differential Pair Frequency Response

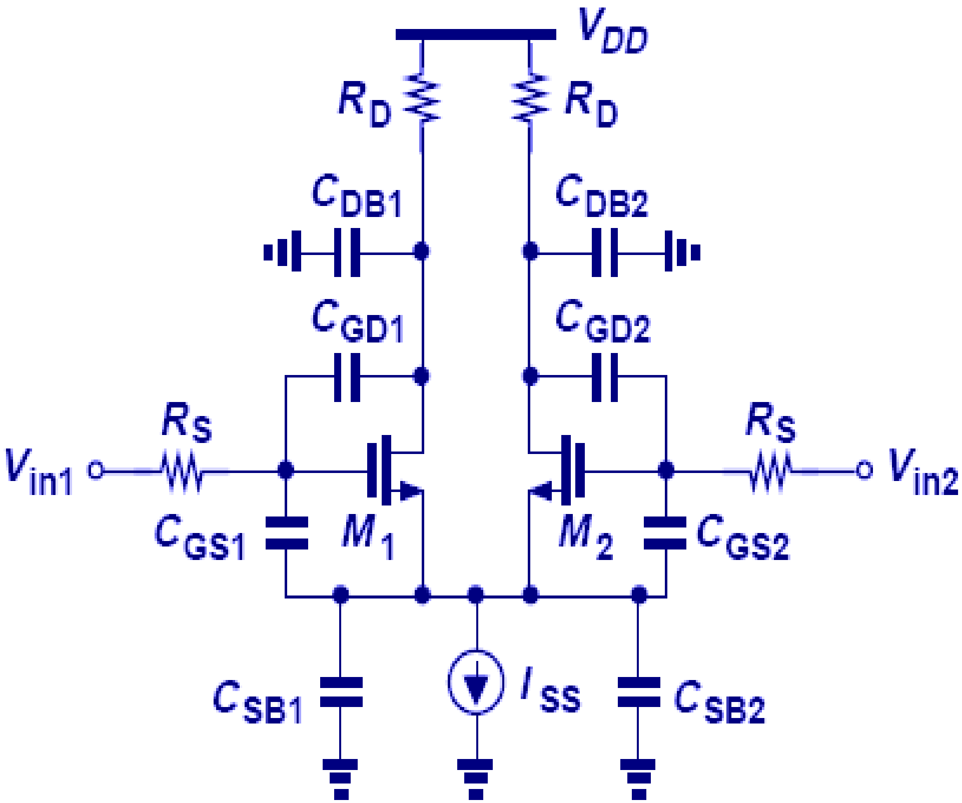


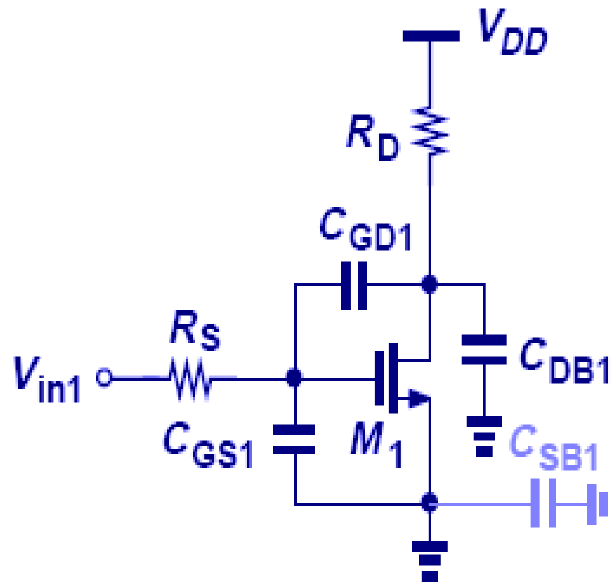


Half Circuit

- Since bipolar differential pair can be analyzed using half-circuit, its transfer function, I/O impedances, locations of poles/zeros are the same as that of the half circuit's.

MOS Differential Pair Frequency Response

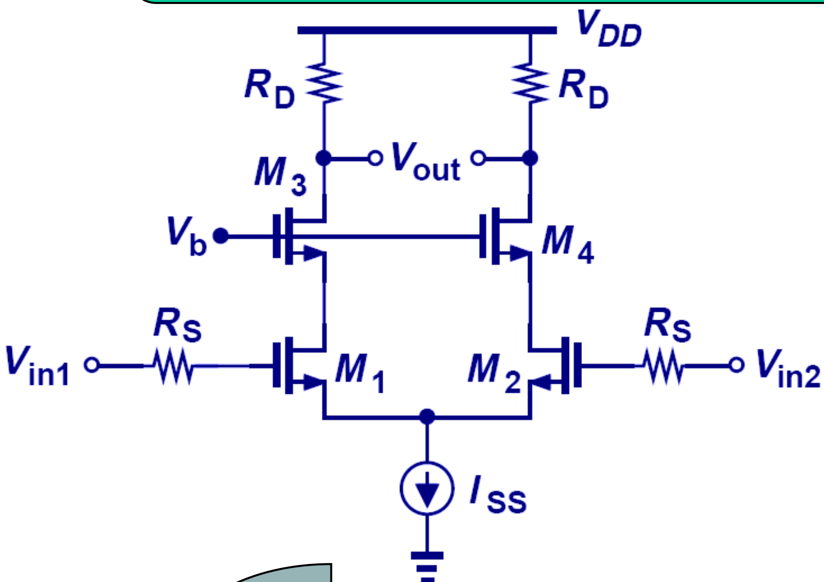




Half Circuit

- Since MOS differential pair can be analyzed using half-circuit, its transfer function, I/O impedances, locations of poles/zeros are the same as that of the half circuit's.

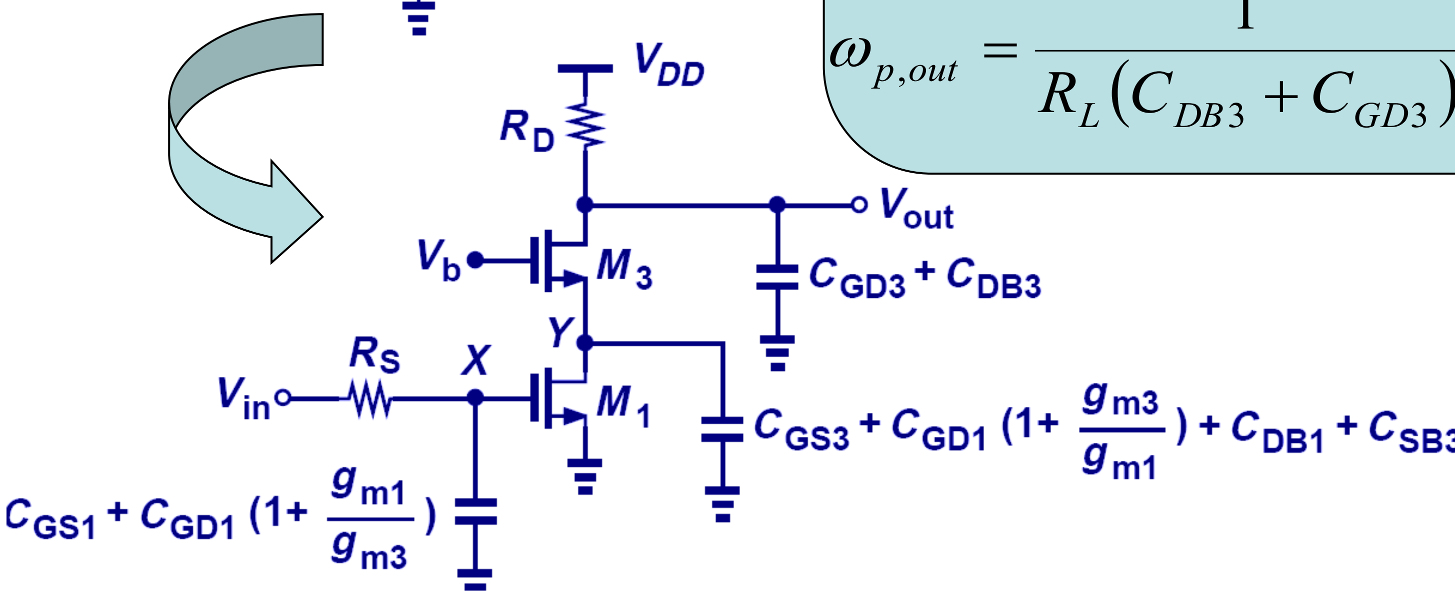
Example: MOS Differential Pair



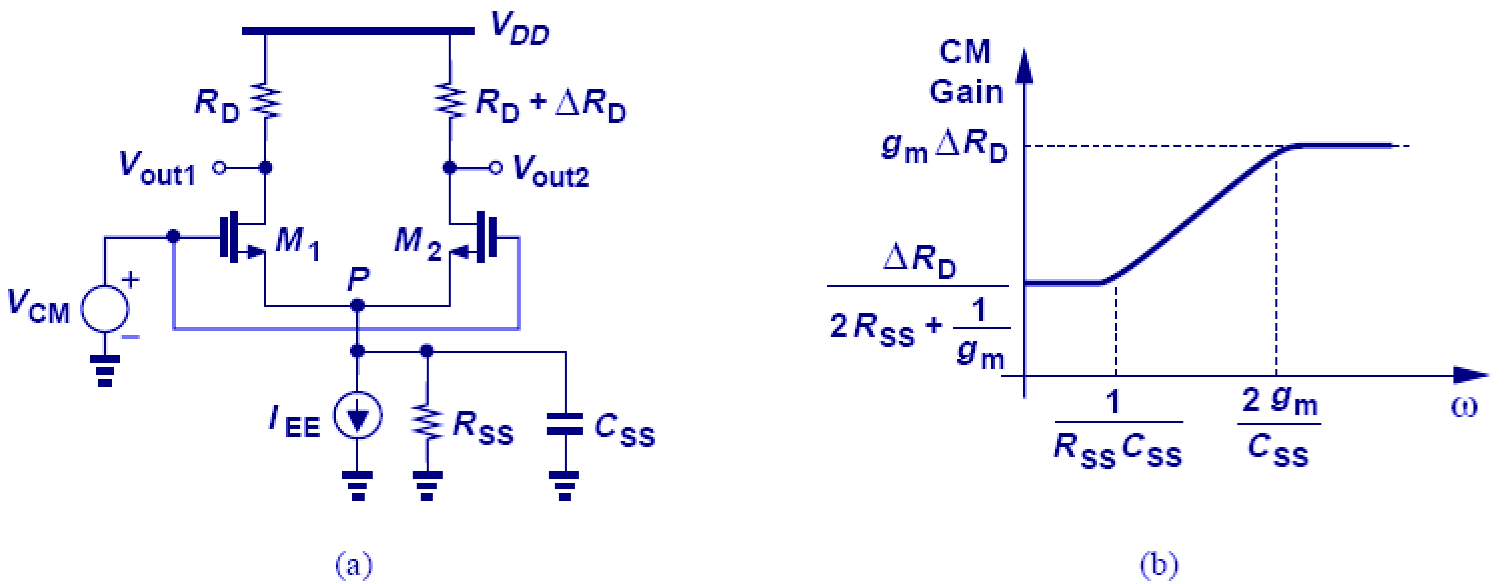
$$\omega_{p,X} = \frac{1}{R_S[C_{GS1} + (1 + g_{m1}/g_{m3})C_{GD1}]}$$

$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m3}}\left[C_{DB1} + C_{GS3} + \left(1 + \frac{g_{m3}}{g_{m1}}\right)C_{GD1}\right]}$$

$$\omega_{p,out} = \frac{1}{R_L(C_{DB3} + C_{GD3})}$$



Common Mode Frequency Response



$$\left|\frac{\Delta V_{out}}{\Delta V_{CM}}\right| = \frac{g_m \Delta R_D (R_{SS}C_{SS} + 1)}{R_{SS}C_{SS}s + 2g_m R_{SS} + 1}$$

- **C_{ss} will lower the total impedance between point P to ground at high frequency, leading to higher CM gain which degrades the CM rejection ratio.**

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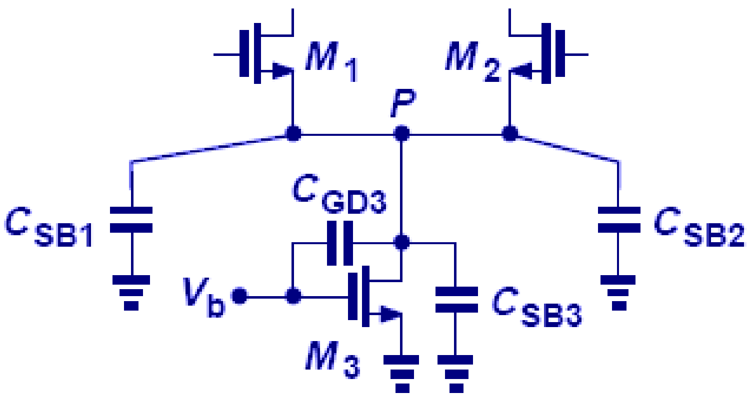
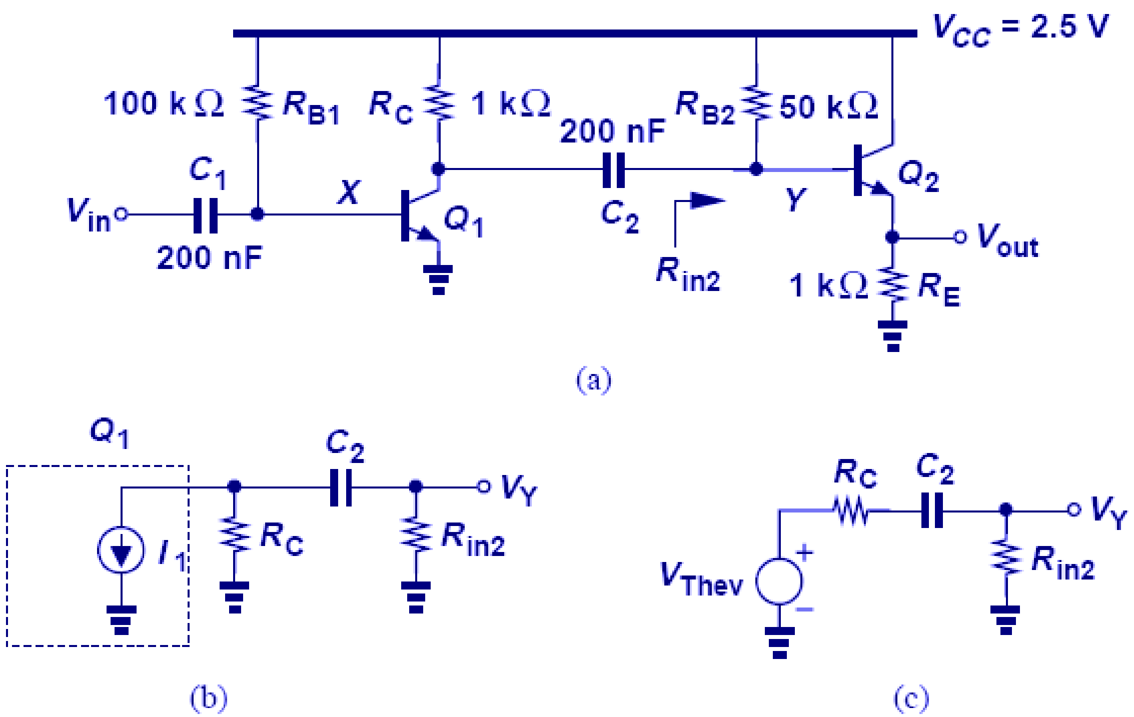
Tail Node Capacitance Contribution



- **Source-Body Capacitance of M_1, M_2 and M_3**
- **Gate-Drain Capacitance of M_3**

Example: Capacitive Coupling

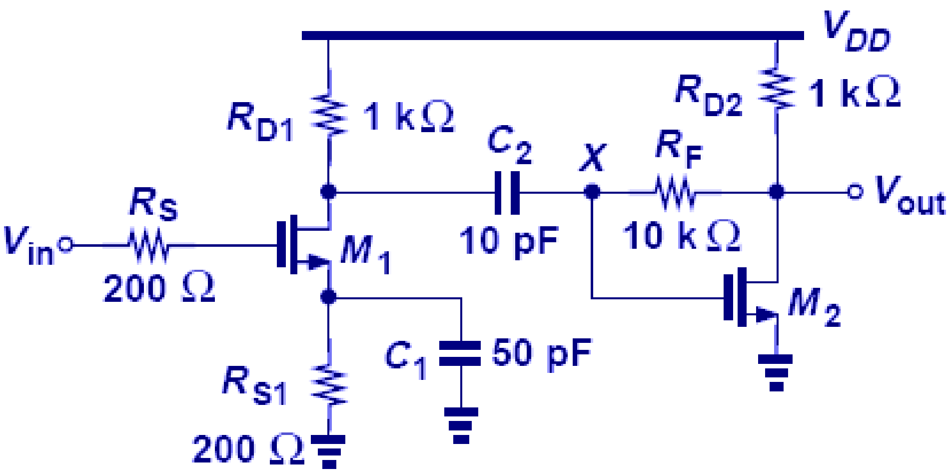


$$R_{in2} = R_{B2} \,||\, \left[r_{\pi 2} + (\beta + 1)R_E\right]$$

$$\omega_{L1} = \frac{1}{(r_{\pi 1} \,||\, R_{B1})C_1} = 2\pi \times (542Hz)$$

$$\omega_{L2} = \frac{1}{(R_C + R_{in2})C_2} = \pi \times (22.9Hz)$$

Example: IC Amplifier – Low Frequency Design

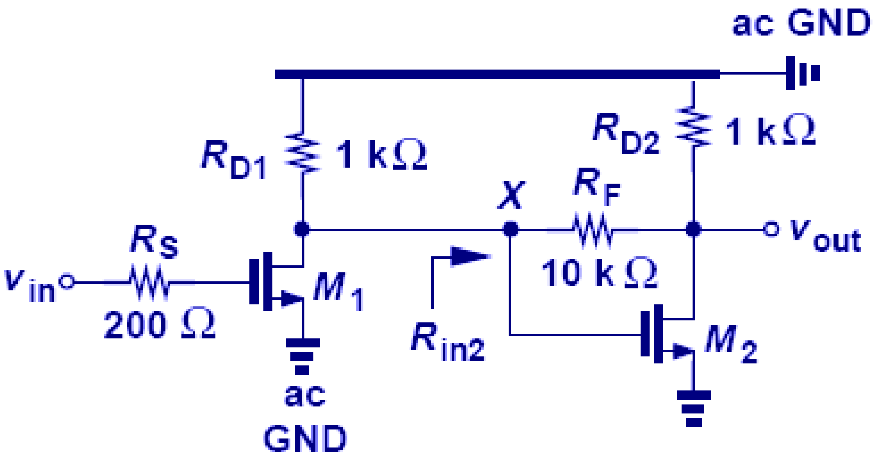


$$R_{in2} = \frac{R_F}{1 - A_{v2}}$$

$$\omega_{L1} = \frac{g_{m1}R_{S1} + 1}{R_{S1}C_1} = 2\pi \times (42.4MHz)$$

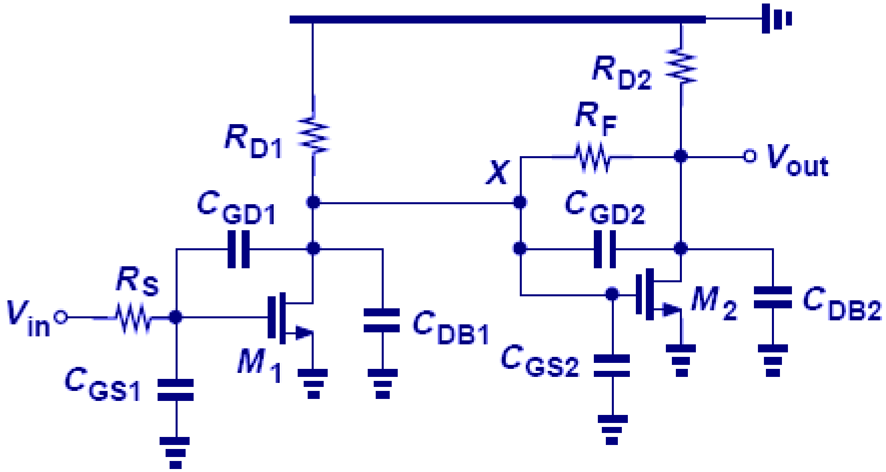
$$\omega_{L2} = \frac{1}{(R_{D1} + R_{in2})C_2} = 2\pi \times (6.92MHz)$$

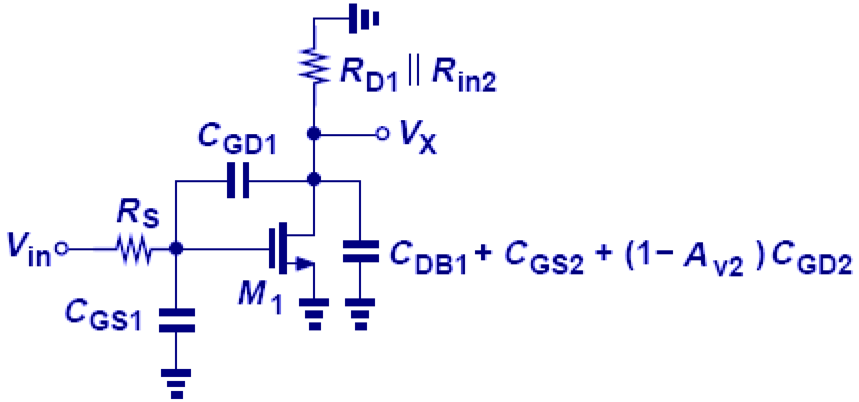
Example: IC Amplifier – Midband Design

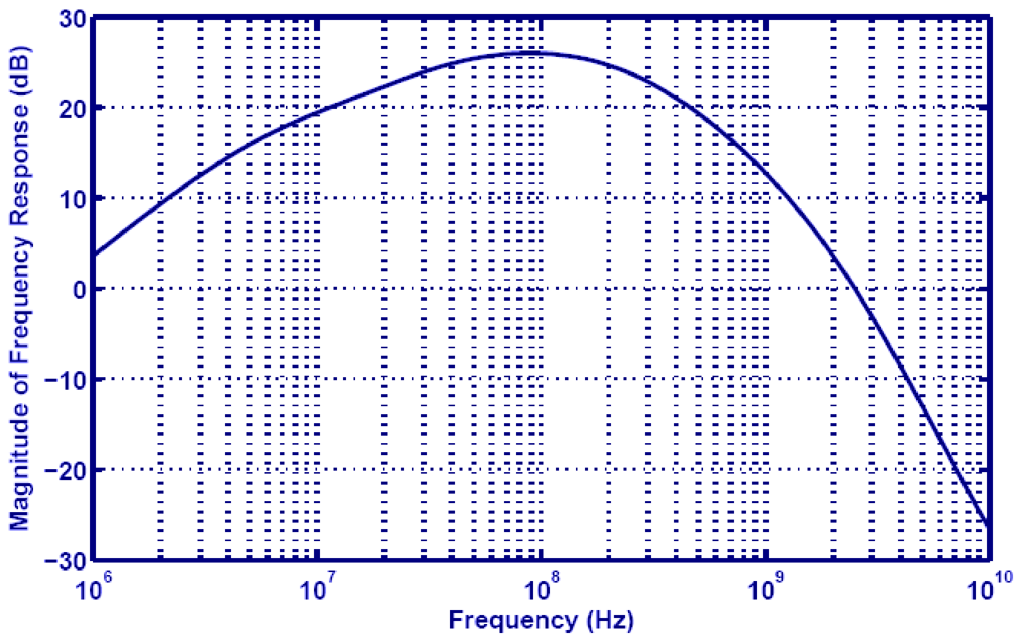


$$\frac{v_X}{v_{in}} = -g_{m1}\left(R_{D1} \parallel R_{in2}\right) = -3.77$$

Example: IC Amplifier – High Frequency Design







$$|\omega_{p1}| = 2\pi \times (308\ MHz)$$

$$|\omega_{p2}| = 2\pi \times (2.15\ GHz)$$

$$|\omega_{p3}| = \frac{1}{R_{L2}(1.15C_{GD2} + C_{DB2})} = 2\pi \times (1.21\ GHz)$$

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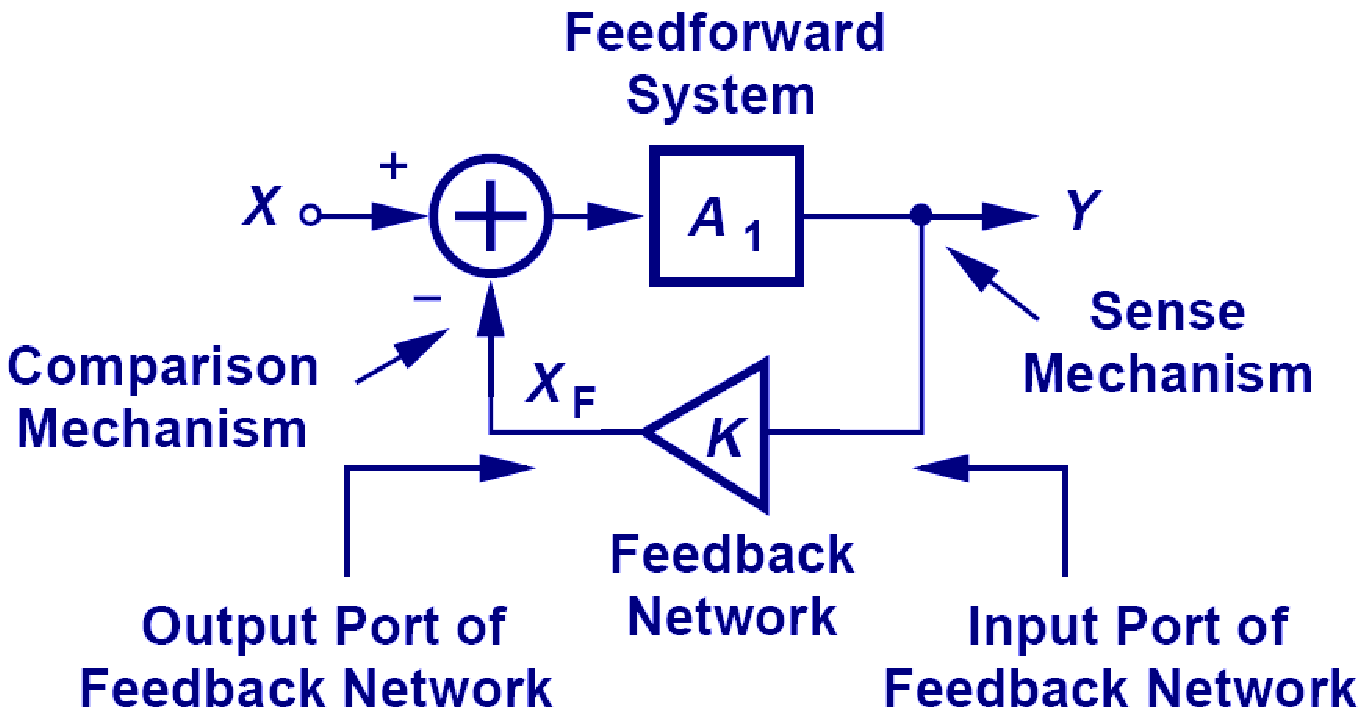
Chapter 12 Feedback

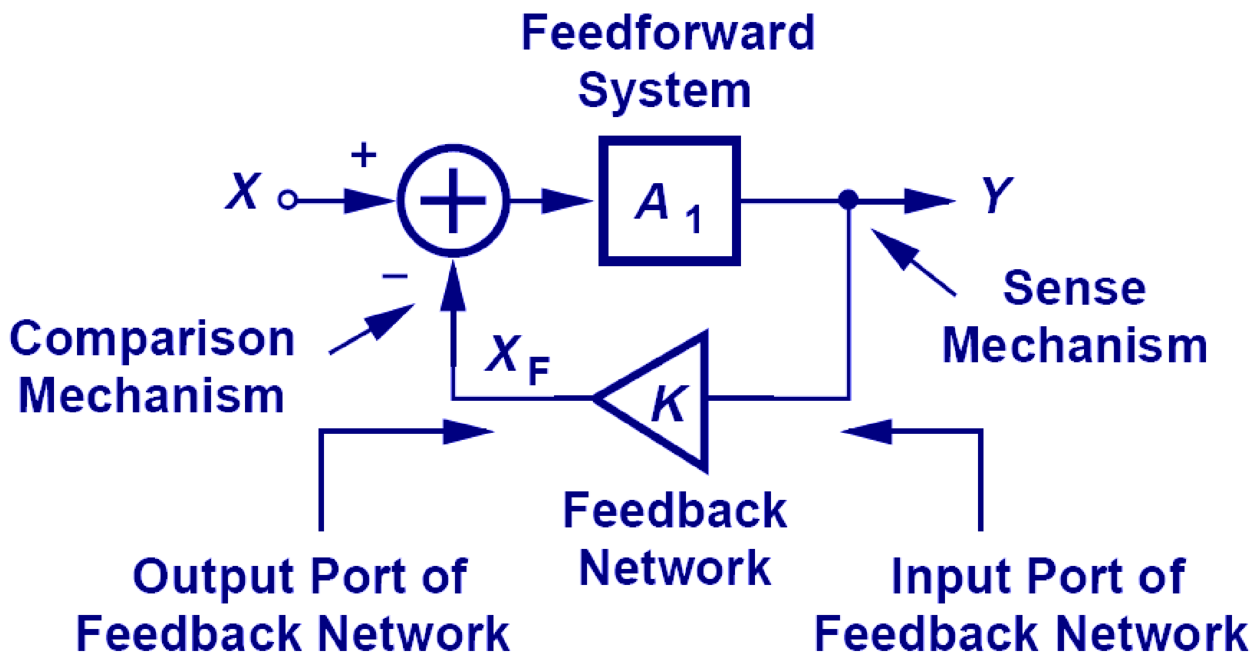
- 12.1 General Considerations
- 12.2 Types of Amplifiers
- 12.3 Sense and Return Techniques
- 12.4 Polarity of Feedback
- 12.5 Feedback Topologies
- 12.6 Effect of Finite I/O Impedances
- 12.7 Stability in Feedback Systems

Negative Feedback System



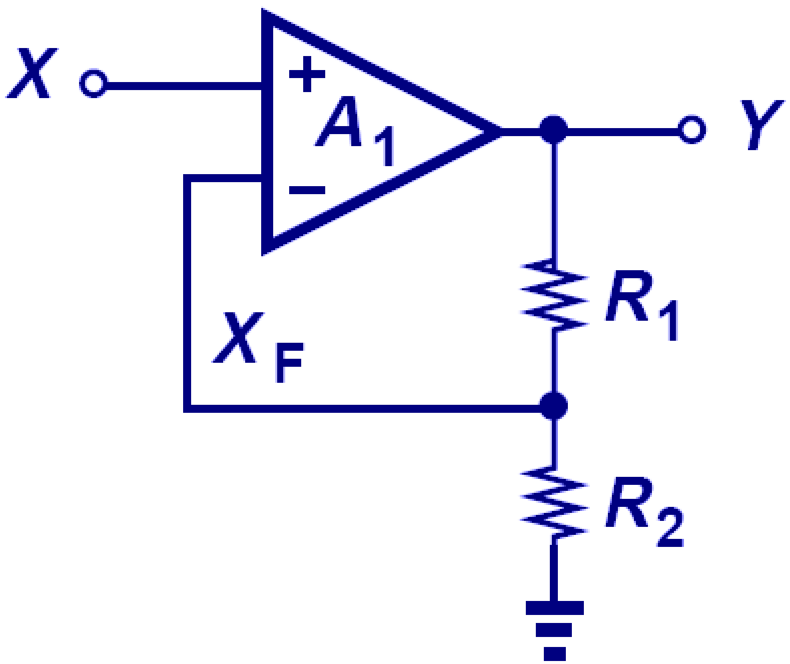
- A negative feedback system consists of four components: 1) feedforward system, 2) sense mechanism, 3) feedback network, and 4) comparison mechanism.

Close-loop Transfer Function



$$\frac{Y}{X} = \frac{A_1}{1 + KA_1}$$

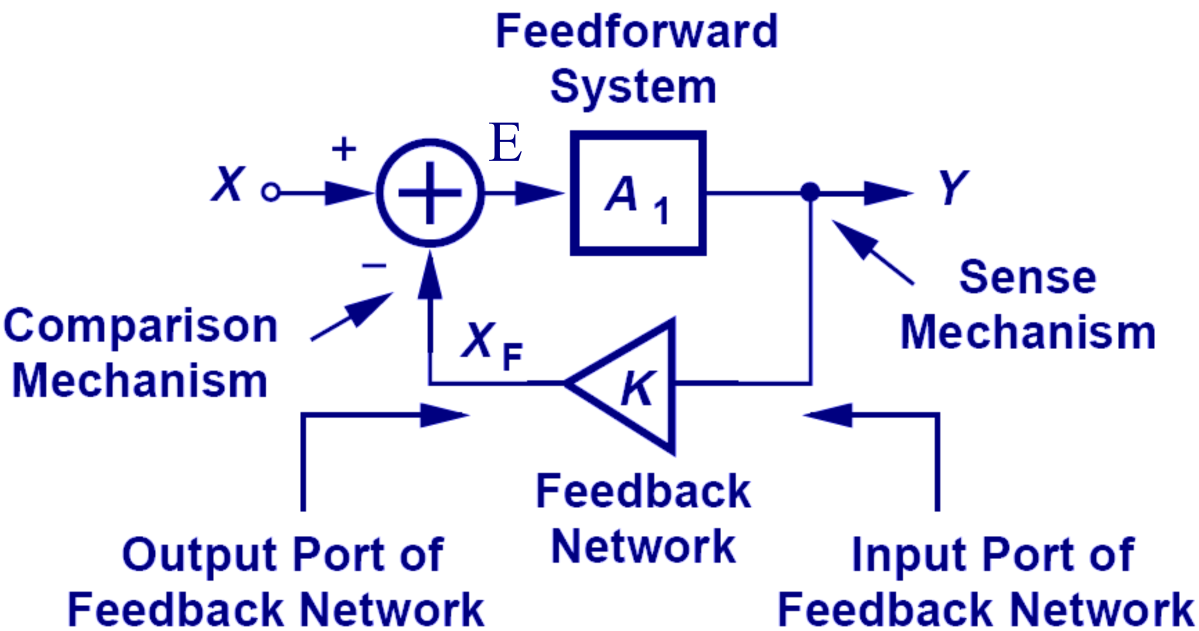
Feedback Example



$$\frac{Y}{X} = \frac{A_1}{1 + \dfrac{R_2}{R_1 + R_2} A_1}$$

- A_1 is the feedforward network, R_1 and R_2 provide the sensing and feedback capabilities, and comparison is provided by differential input of A_1.

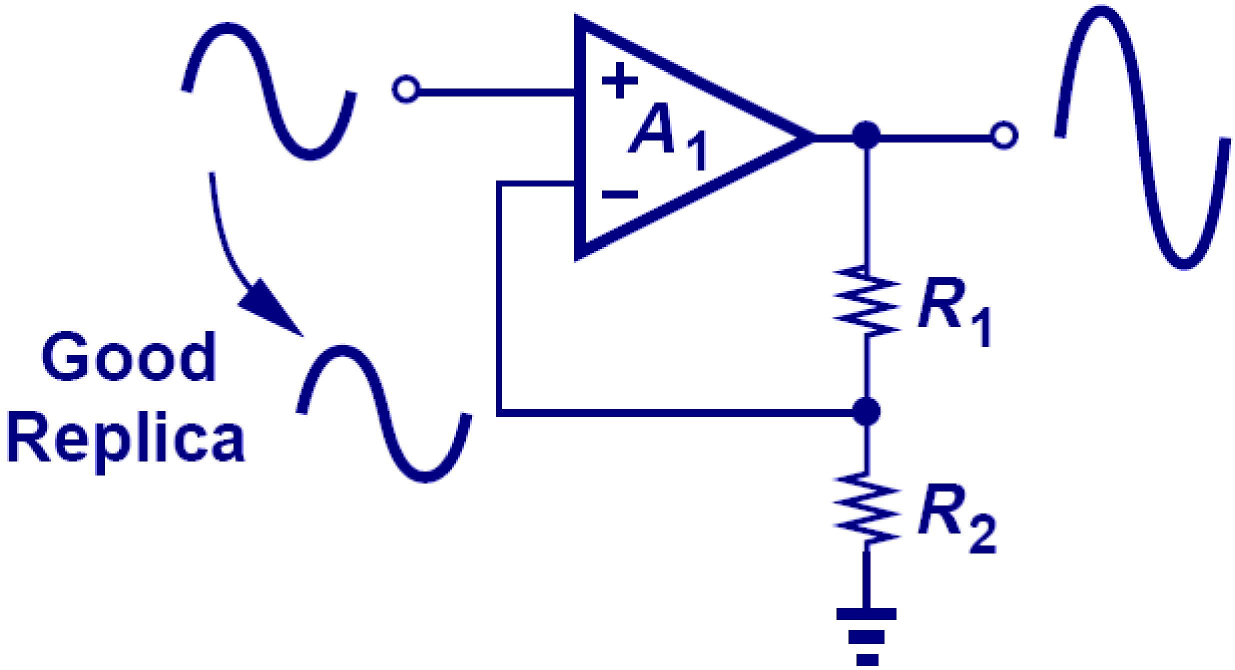
Comparison Error



$$E = \frac{X}{1 + A_1 K}$$

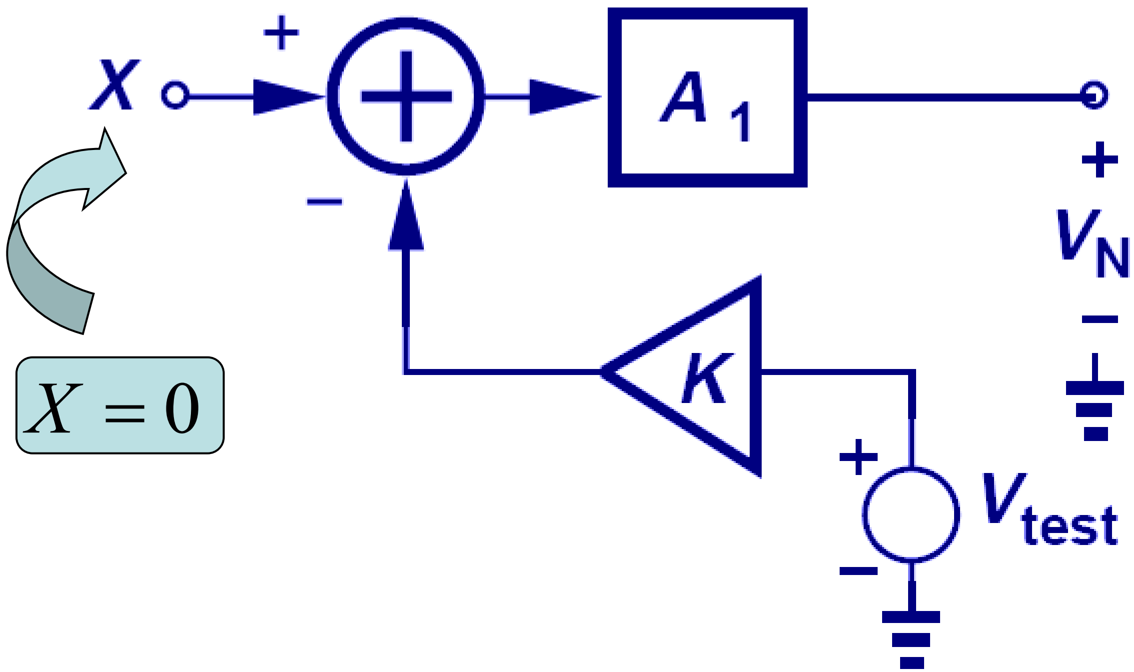
- As A_1K increases, the error between the input and fed back signal decreases. Or the fed back signal approaches a good replica of the input.

Comparison Error



$$\frac{Y}{X} \approx 1 + \frac{R_1}{R_2}$$

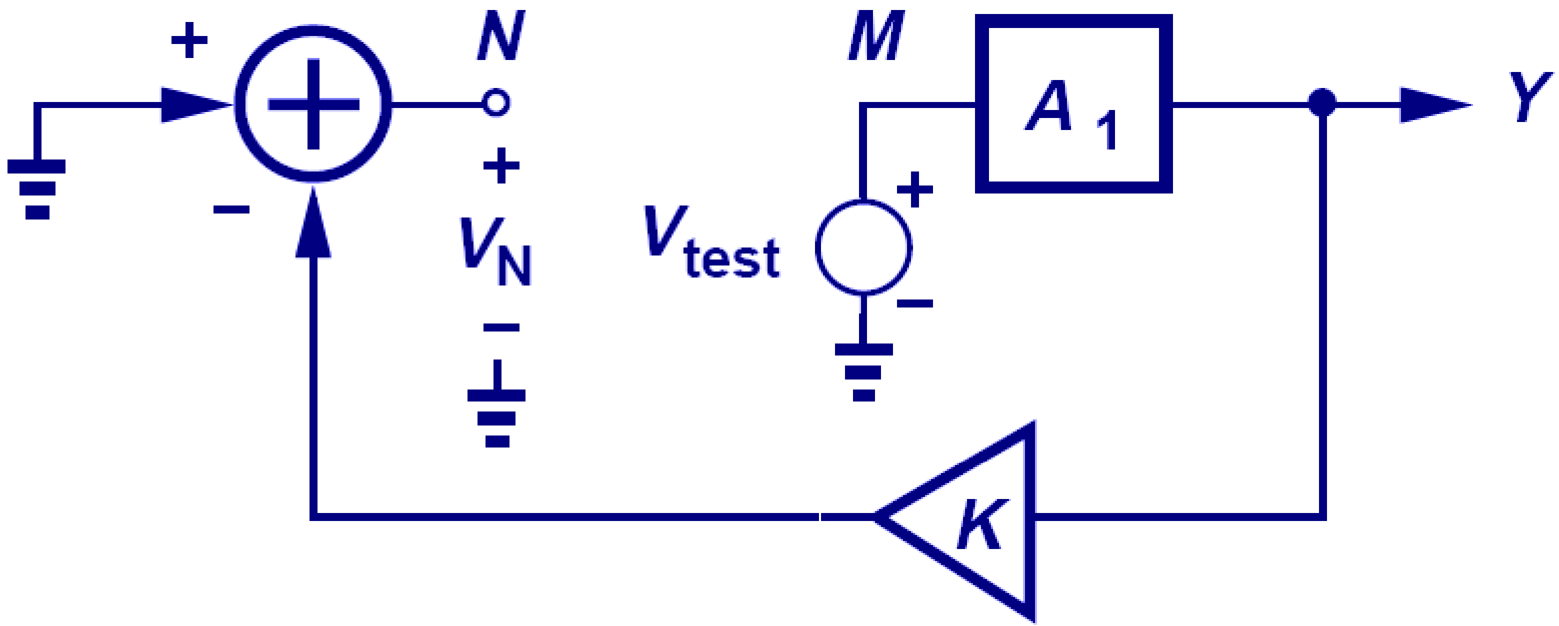
Loop Gain



$$KA_1 = -\frac{V_N}{V_{test}}$$

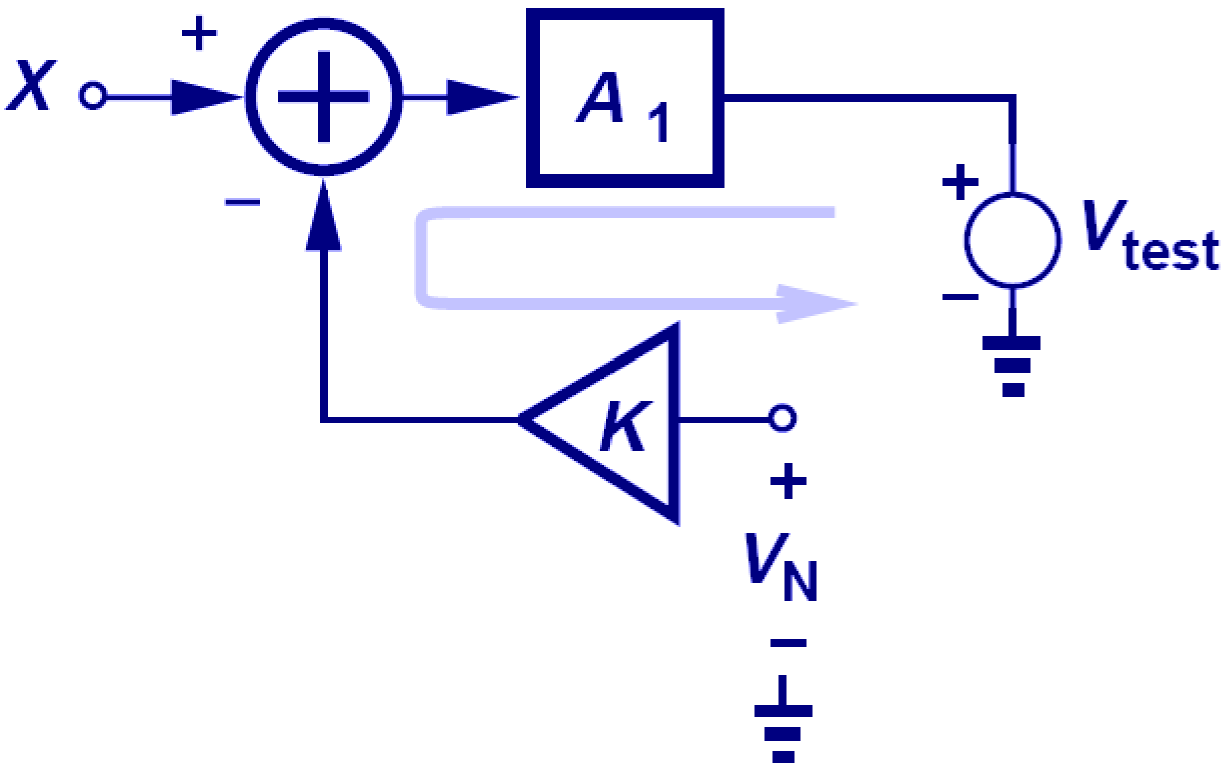
- When the input is grounded, and the loop is broken at an arbitrary location, the loop gain is measured to be KA_1.

Example: Alternative Loop Gain Measurement



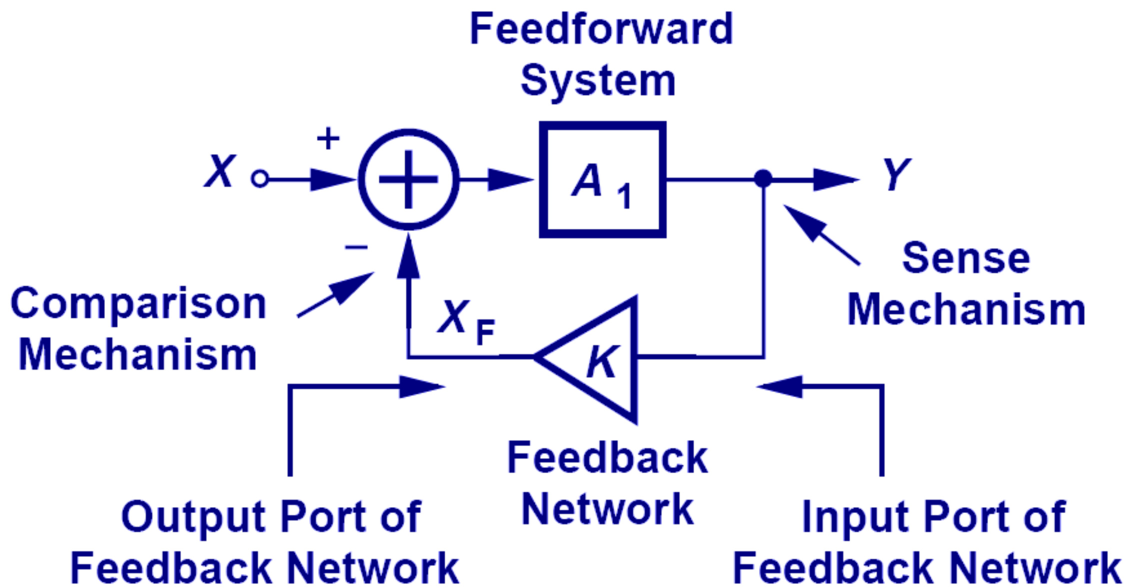
$$V_N = -KA_1V_{test}$$

Incorrect Calculation of Loop Gain



- Signal naturally flows from the input to the output of a feedforward/feedback system. If we apply the input the other way around, the "output" signal we get is not a result of the loop gain, but due to poor isolation.

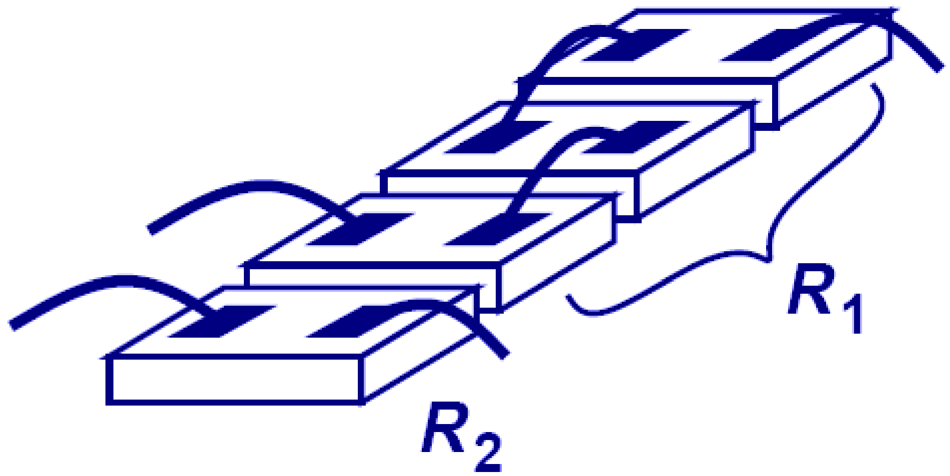
Gain Desensitization



$$A_1K >> 1 \Rightarrow \frac{Y}{X} \approx \frac{1}{K}$$

- **A large loop gain is needed to create a precise gain, one that does not depend on A_1, which can vary by ±20%.**

Ratio of Resistors



➢ **When two resistors are composed of the same unit resistor, their ratio is very accurate. Since when they vary, they will vary together and maintain a constant ratio.**

Merits of Negative Feedback

- 1) Bandwidth enhancement
- 2) Modification of I/O Impedances
- 3) Linearization

Bandwidth Enhancement

Open Loop

$$A(s) = \frac{A_0}{1 + \frac{s}{\omega_0}}$$

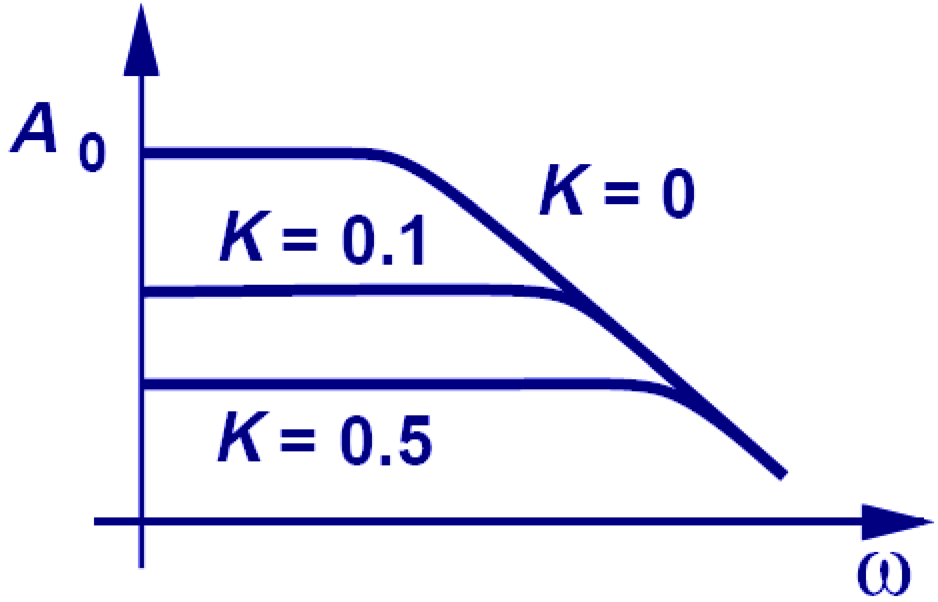
Negative Feedback

Closed Loop

$$\frac{Y}{X}(s) = \frac{\frac{A_0}{1 + KA_0}}{1 + \frac{s}{(1 + KA_0)\omega_0}}$$

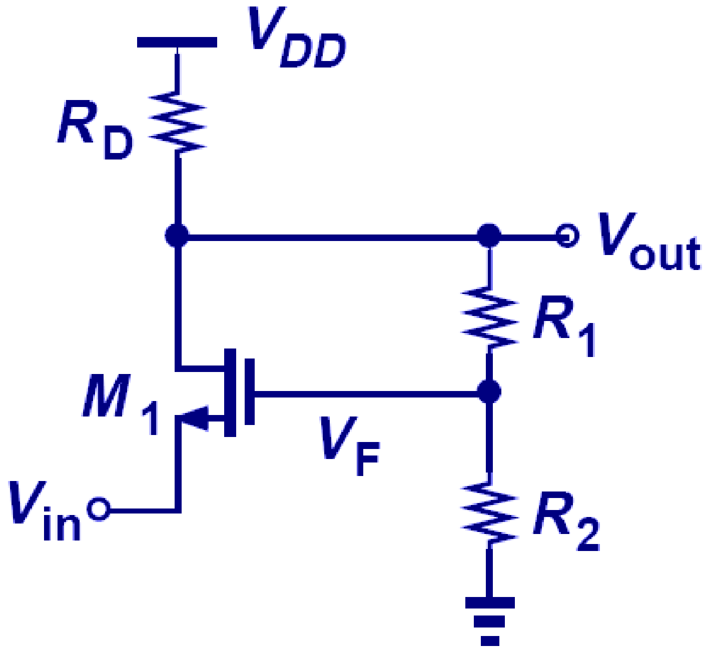
- **Although negative feedback lowers the gain by $(1+KA_0)$, it also extends the bandwidth by the same amount.**

Bandwidth Extension Example



- **As the loop gain increases, we can see the decrease of the overall gain and the extension of the bandwidth.**

Example: Open Loop Parameters

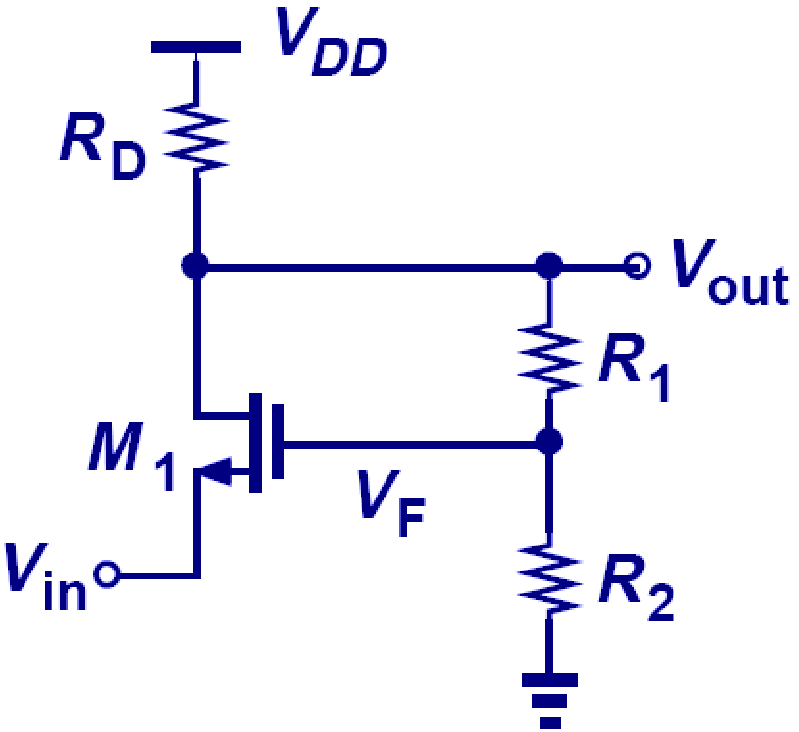


$$A_0 \approx g_m R_D$$

$$R_{in} = \frac{1}{g_m}$$

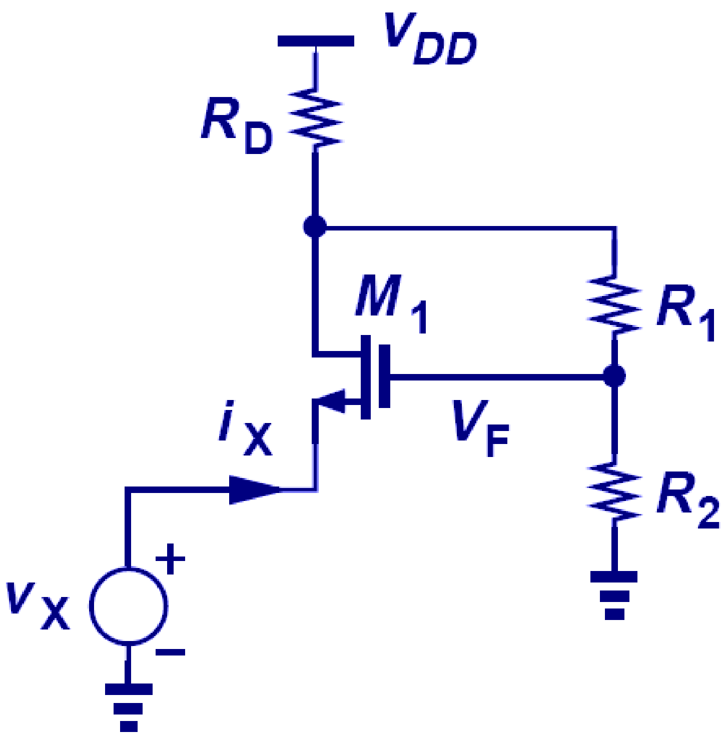
$$R_{out} = R_D$$

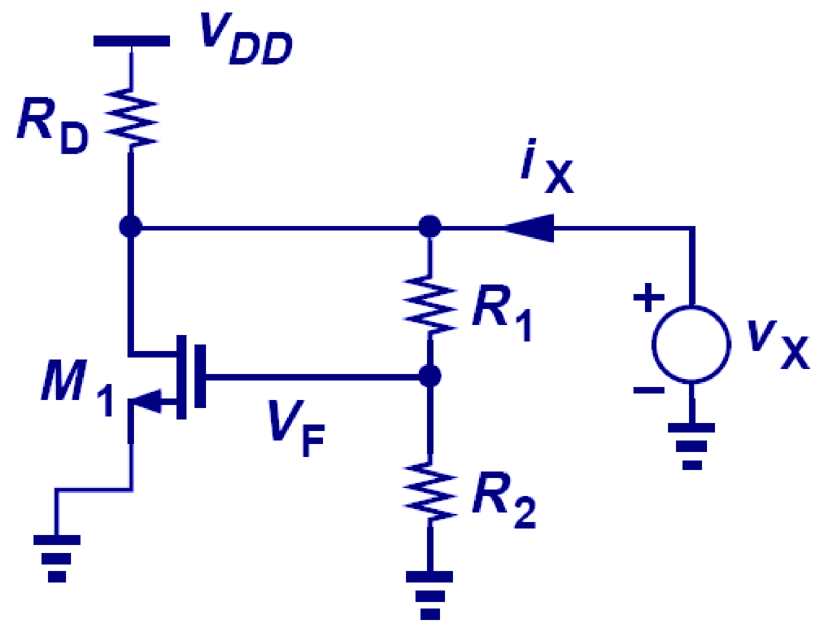
Example: Closed Loop Voltage Gain



$$\frac{v_{out}}{v_{in}} = \frac{g_m R_D}{1 + \dfrac{R_2}{R_1 + R_2} g_m R_D}$$

Example: Closed Loop I/O Impedance

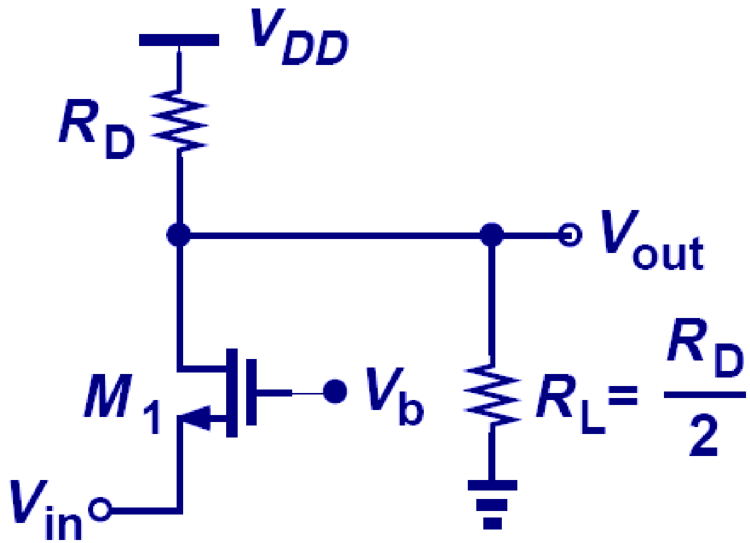




$$R_{in} = \frac{1}{g_m}\left(1 + \frac{R_2}{R_1 + R_2} g_m R_D\right)$$

$$R_{out} = \frac{R_D}{1 + \frac{R_2}{R_1 + R_2} g_m R_D}$$

Example: Load Desensitization



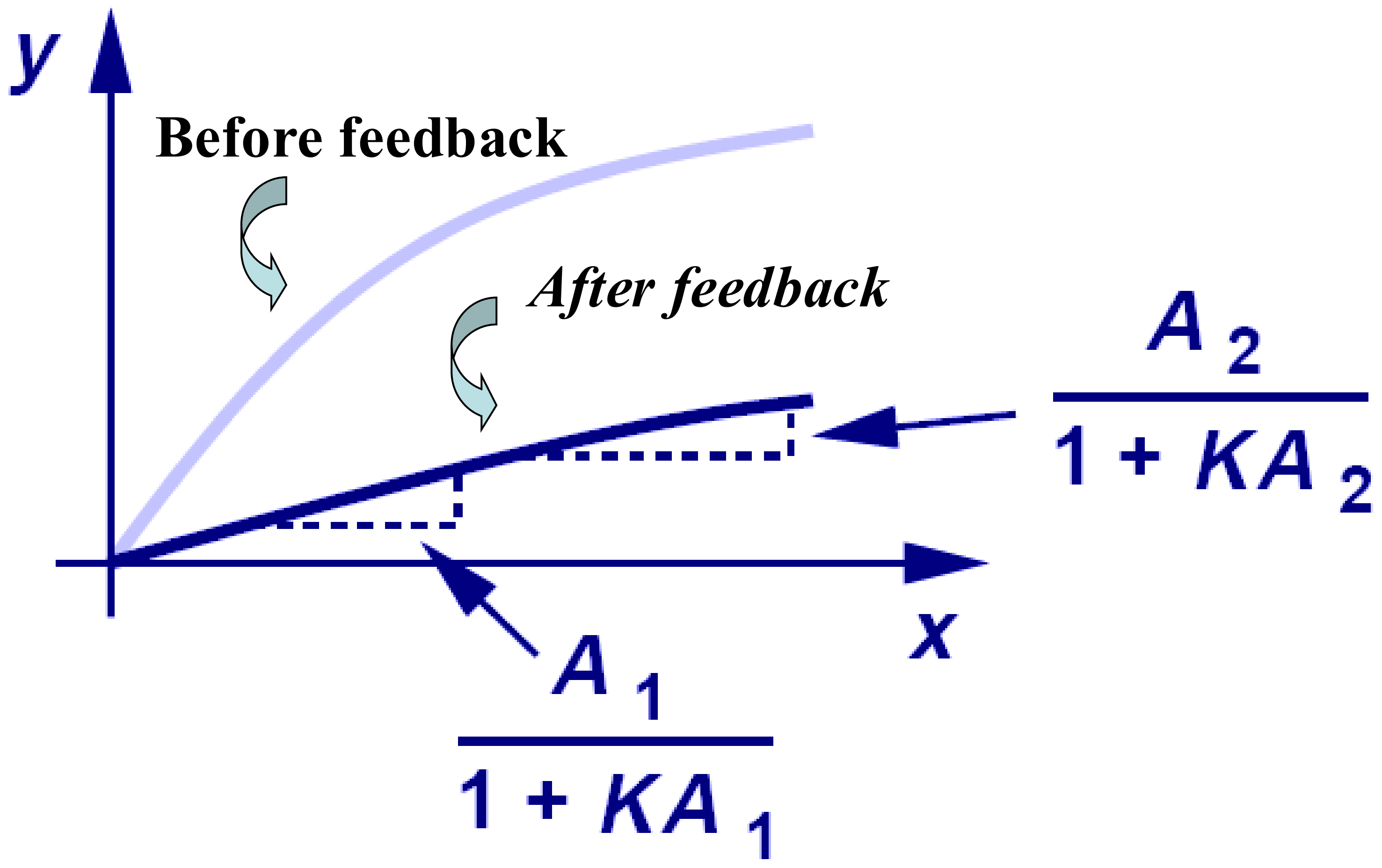
W/O Feedback
Large Difference

$$g_m R_D \rightarrow g_m R_D / 3$$

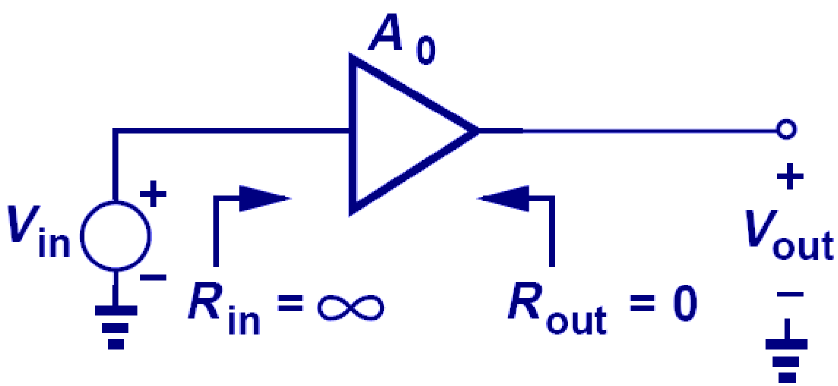
With Feedback
Small Difference

$$\frac{g_m R_D}{1 + \frac{R_2}{R_1 + R_2} g_m R_D} \rightarrow \frac{g_m R_D}{3 + \frac{R_2}{R_1 + R_2} g_m R_D}$$

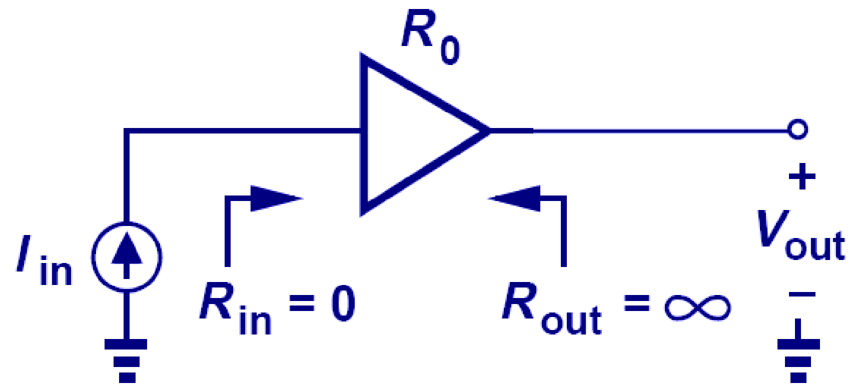
Linearization



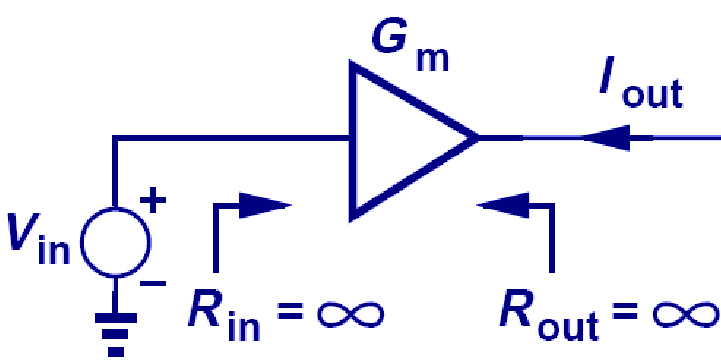
Four Types of Amplifiers

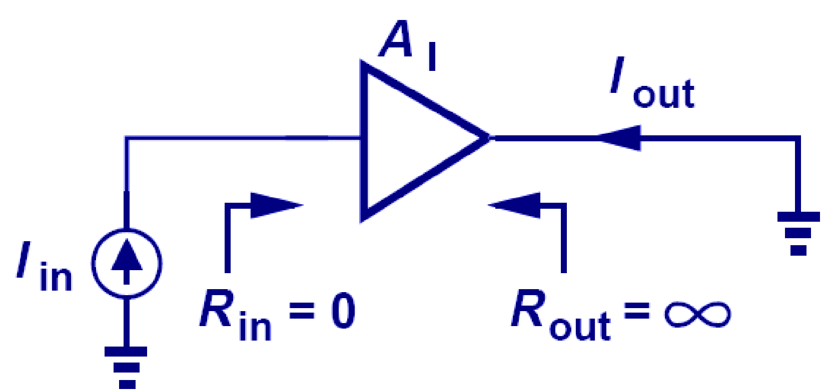


(a)

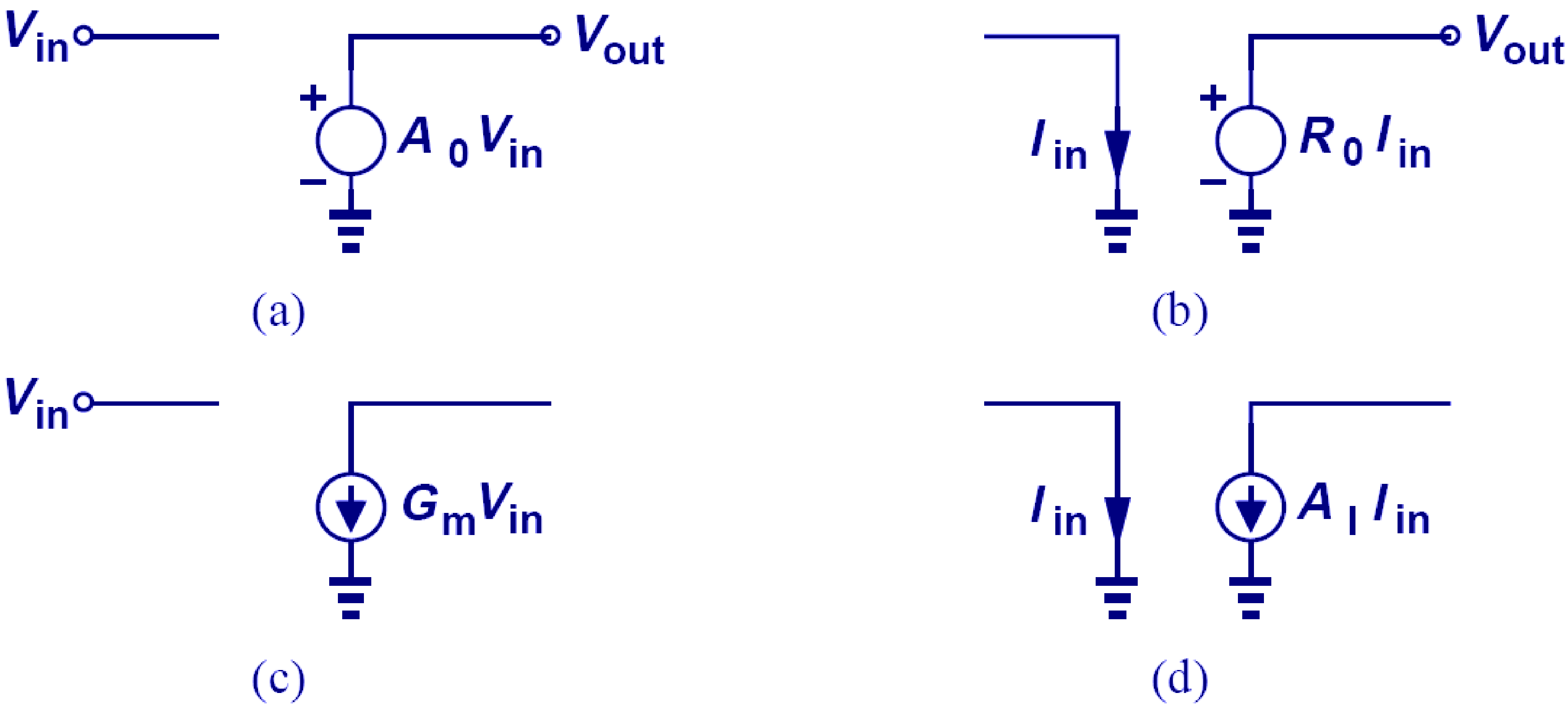


(b)

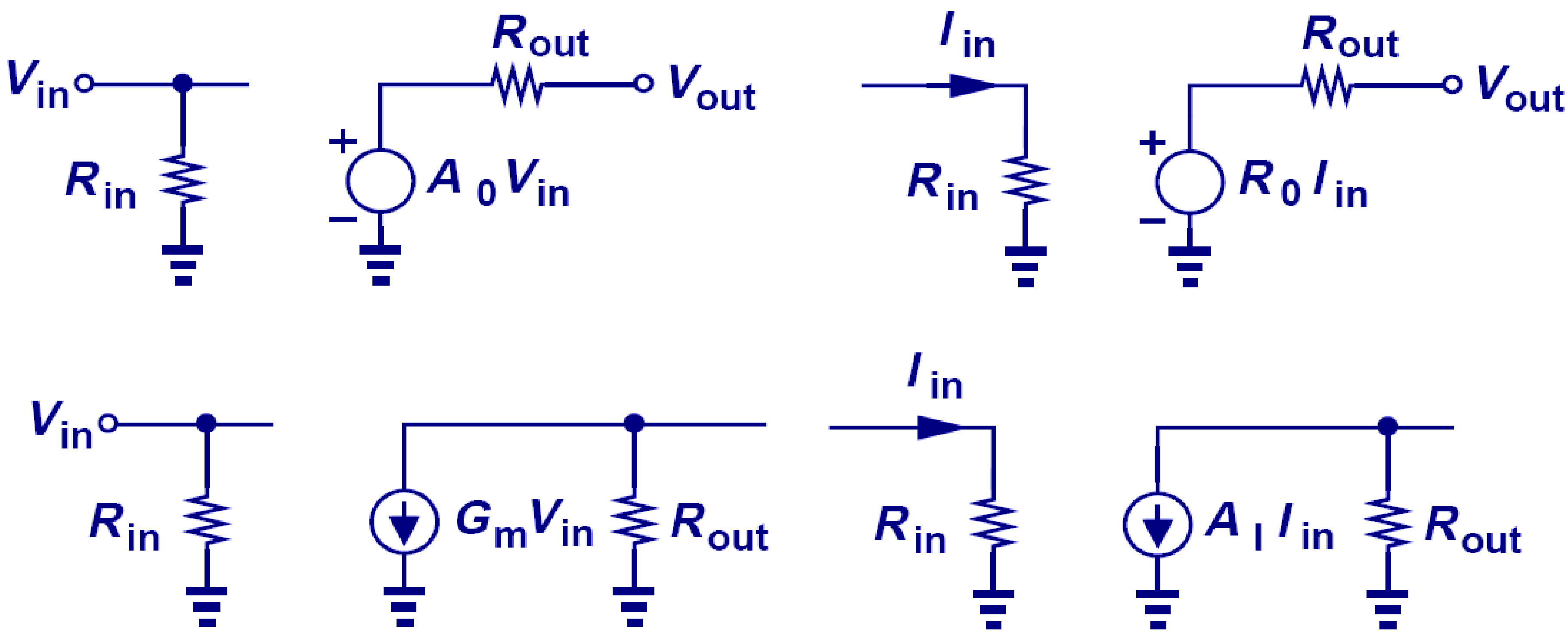




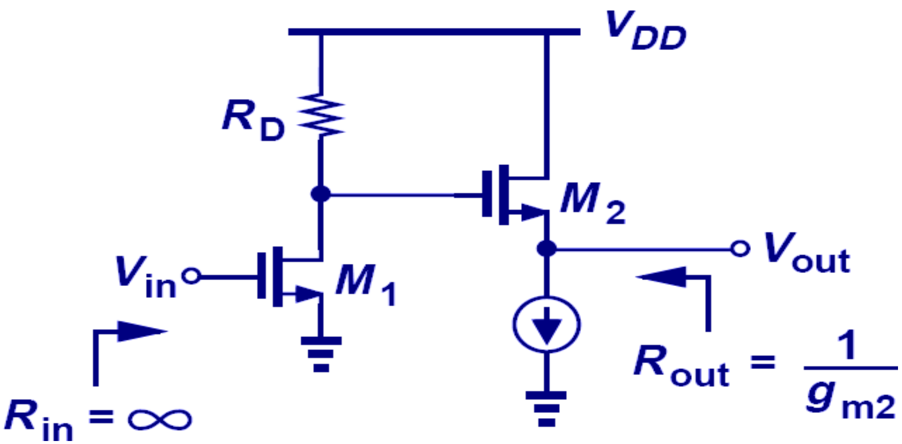
Ideal Models of the Four Amplifier Types

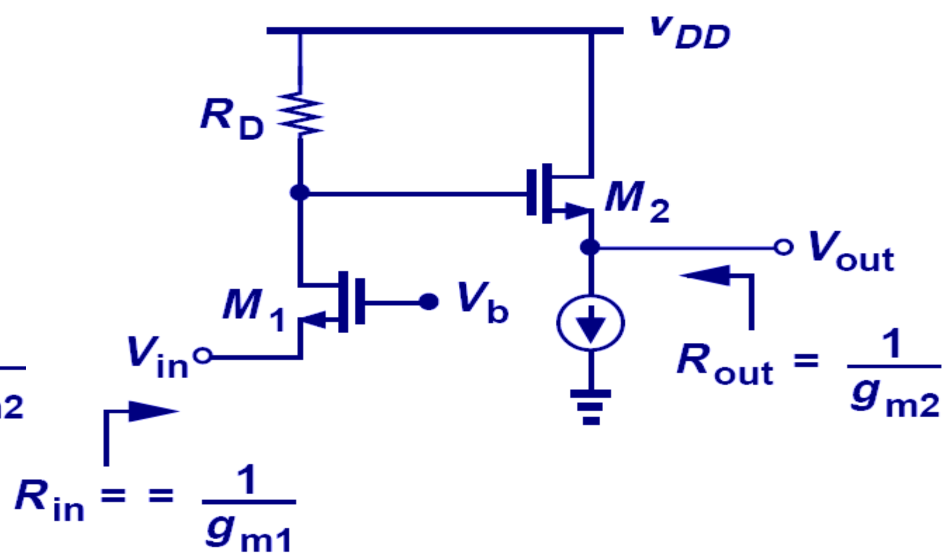


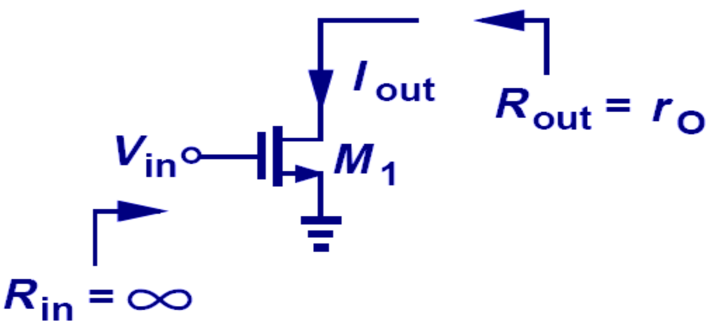
Realistic Models of the Four Amplifier Types

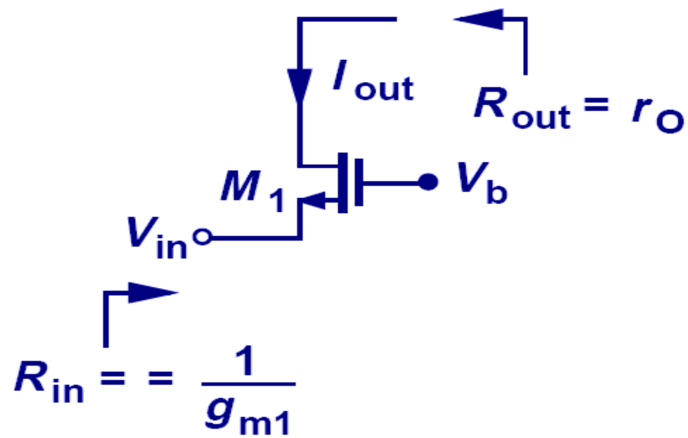


Examples of the Four Amplifier Types

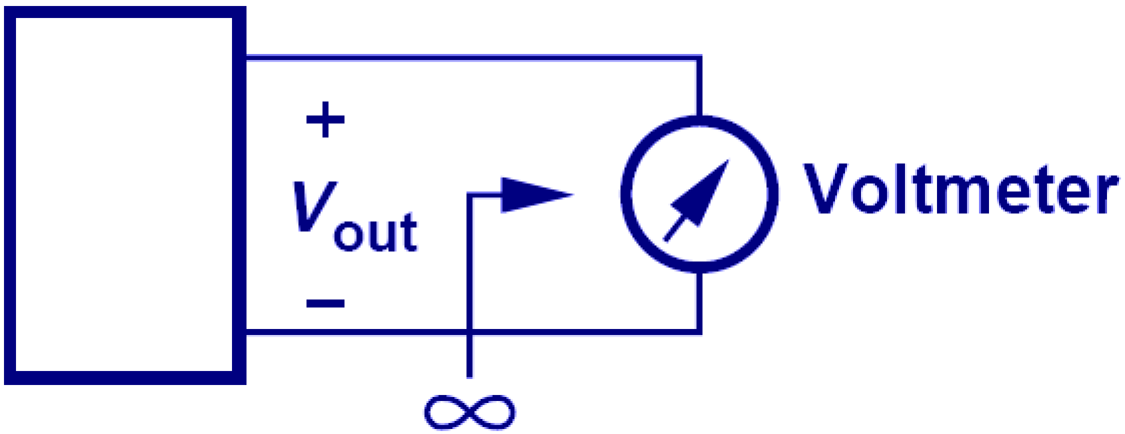






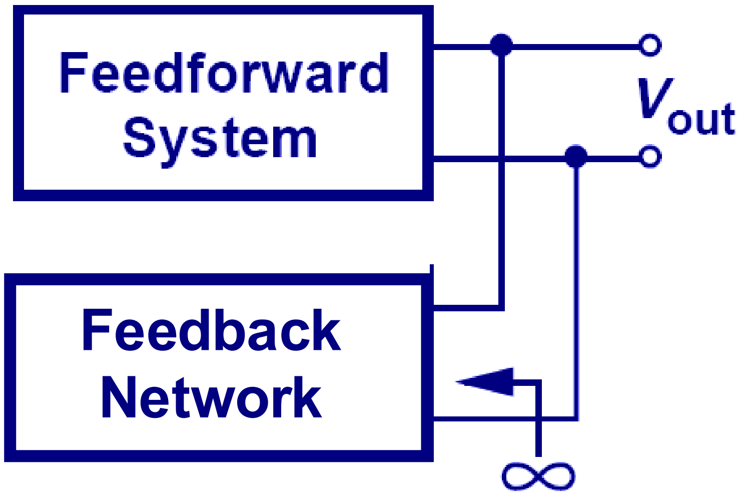


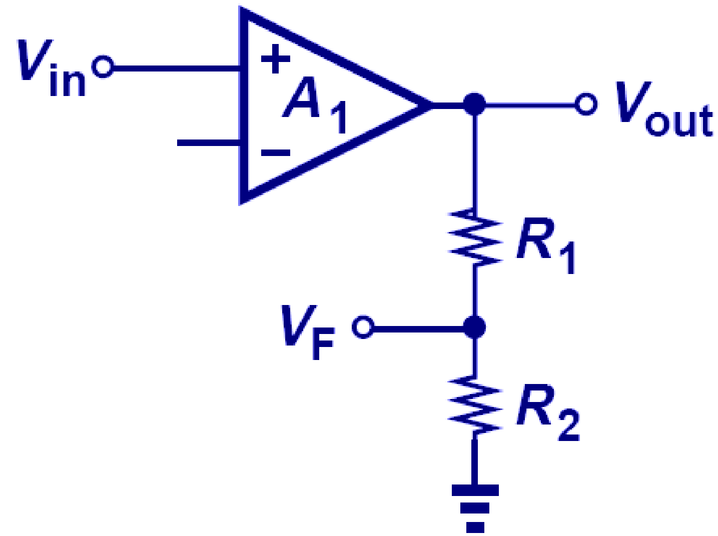
Sensing a Voltage



- In order to sense a voltage across two terminals, a voltmeter with ideally infinite impedance is used.

Sensing and Returning a Voltage

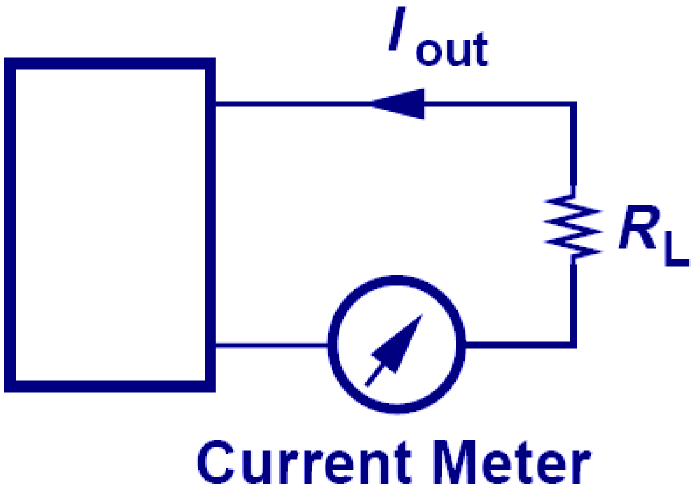


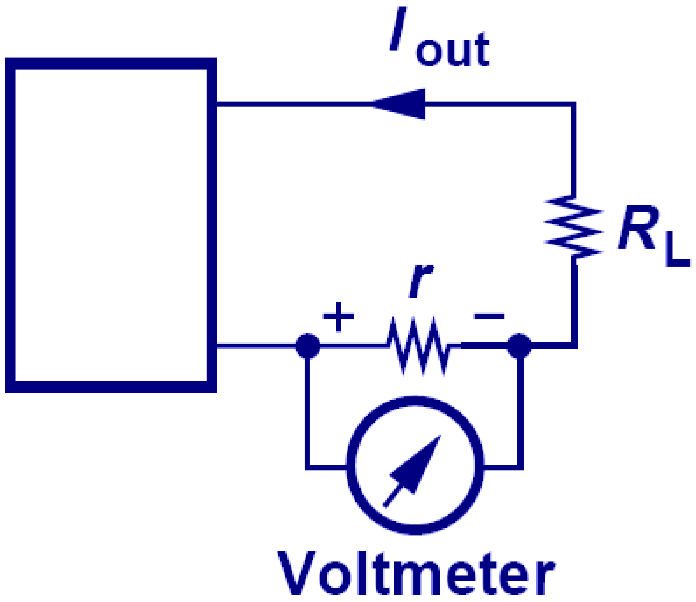


$$R_1 + R_2 \approx \infty$$

- Similarly, for a feedback network to correctly sense the output voltage, its input impedance needs to be large.
- R_1 and R_2 also provide a mean to return the voltage.

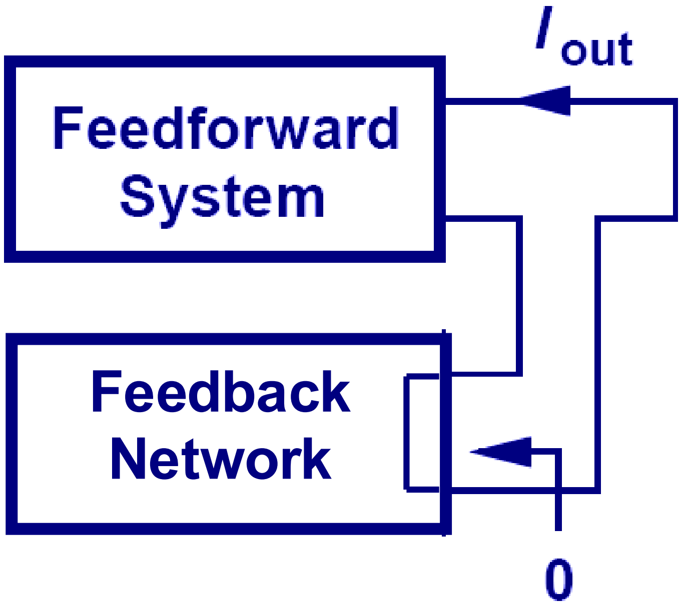
Sensing a Current

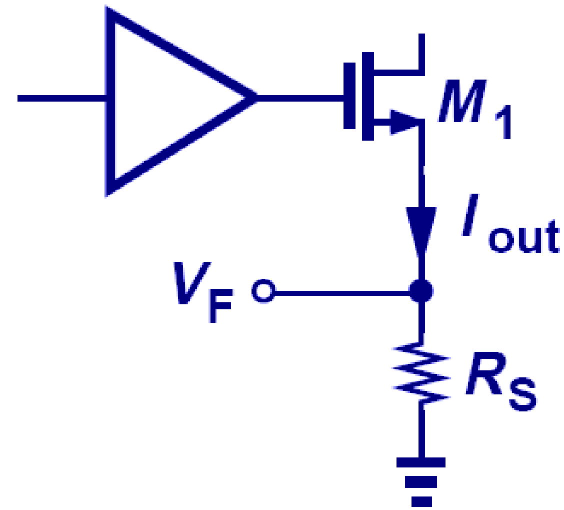




- A current is measured by inserting a current meter with ideally zero impedance in series with the conduction path.
- The current meter is composed of a small resistance r in parallel with a voltmeter.

Sensing and Returning a Current

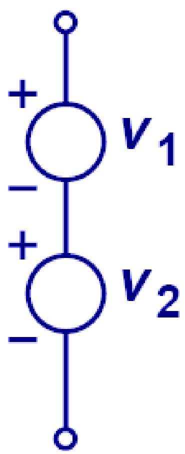




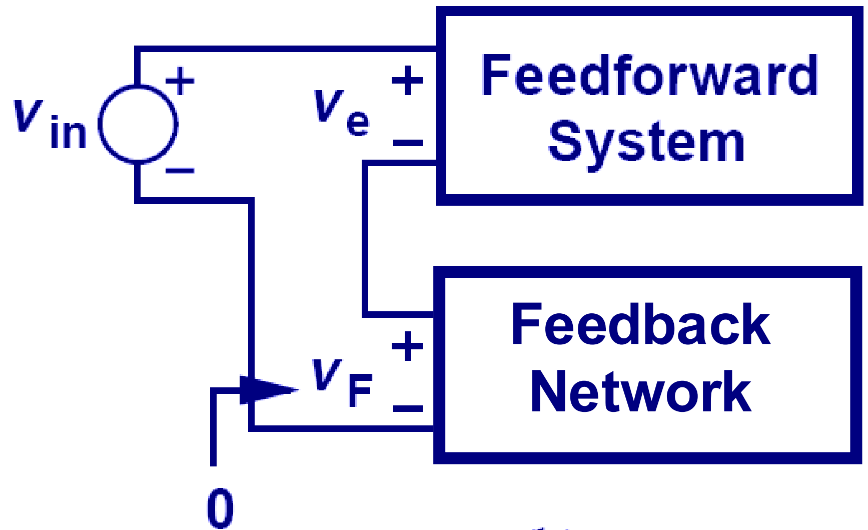
$$R_S \approx 0$$

- Similarly for a feedback network to correctly sense the current, its input impedance has to be small.
- R_S has to be small so that its voltage drop will not change I_{out}.

Addition of Two Voltage Sources



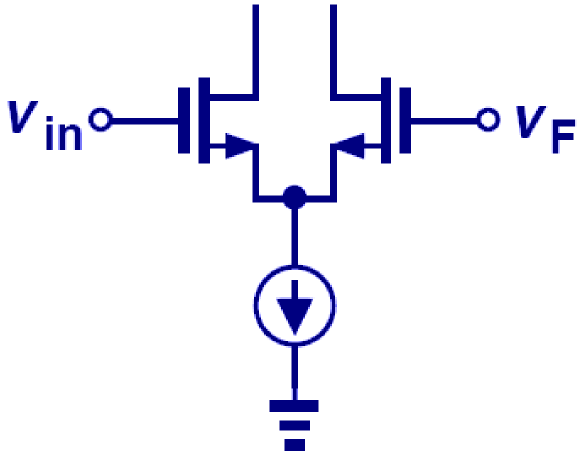
(a)



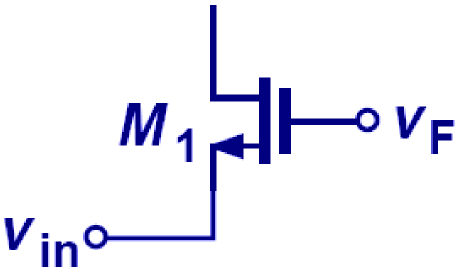
(b)

- In order to add or substrate two voltage sources, we place them in series. So the feedback network is placed in series with the input source.

Practical Circuits to Subtract Two Voltage Sources



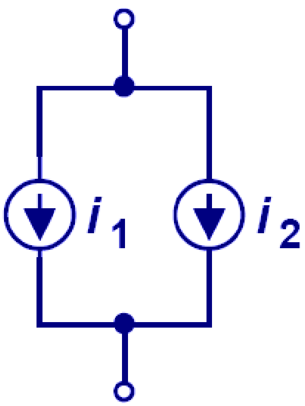
(c)



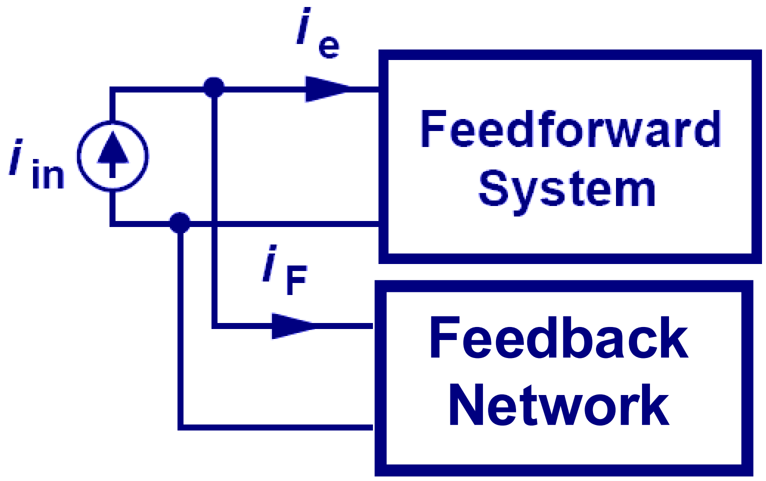
(d)

- Although not directly in series, V_{in} and V_F are being subtracted since the resultant currents, differential and single-ended, are proportional to the difference of V_{in} and V_F.

Addition of Two Current Sources



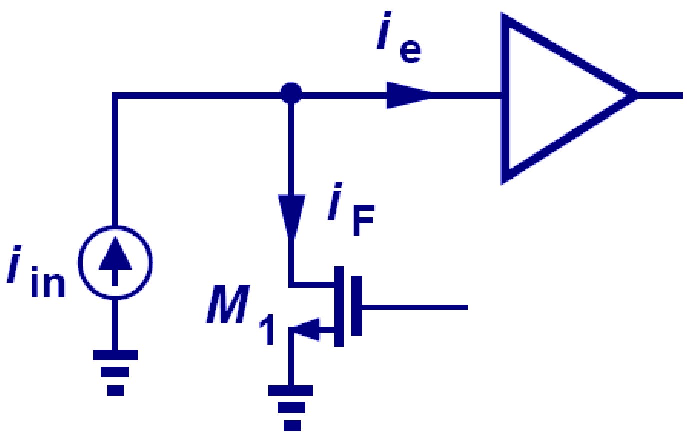
(a)



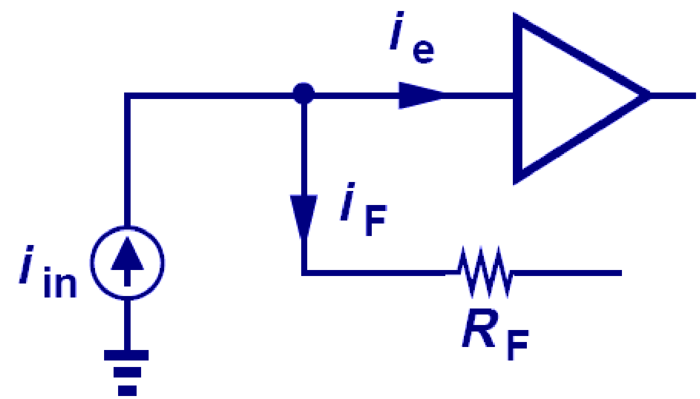
(b)

- In order to add two current sources, we place them in parallel. So the feedback network is placed in parallel with the input signal.

Practical Circuits to Subtract Two Current Sources



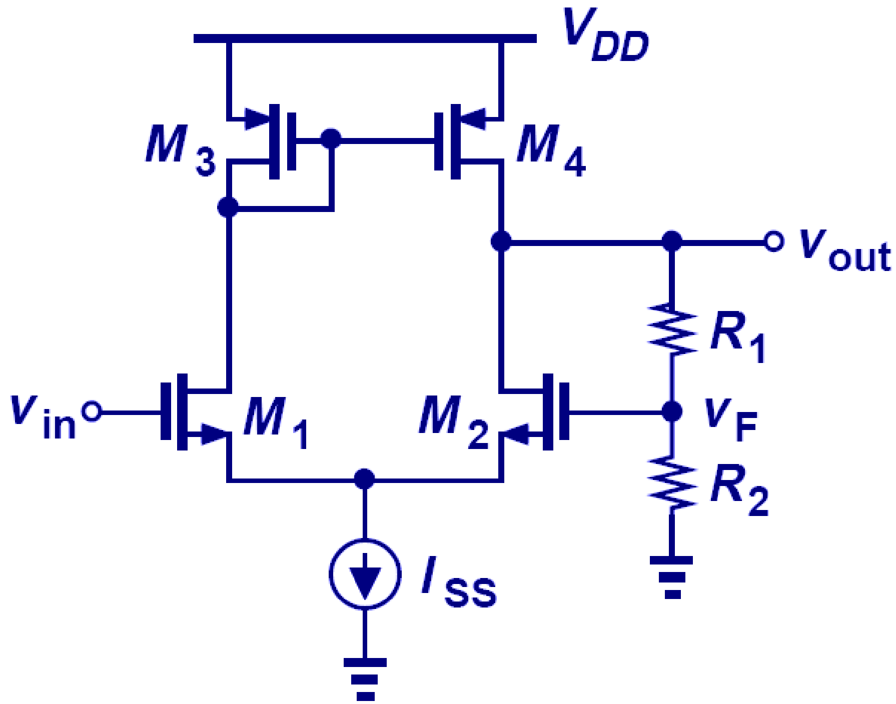
(c)



(d)

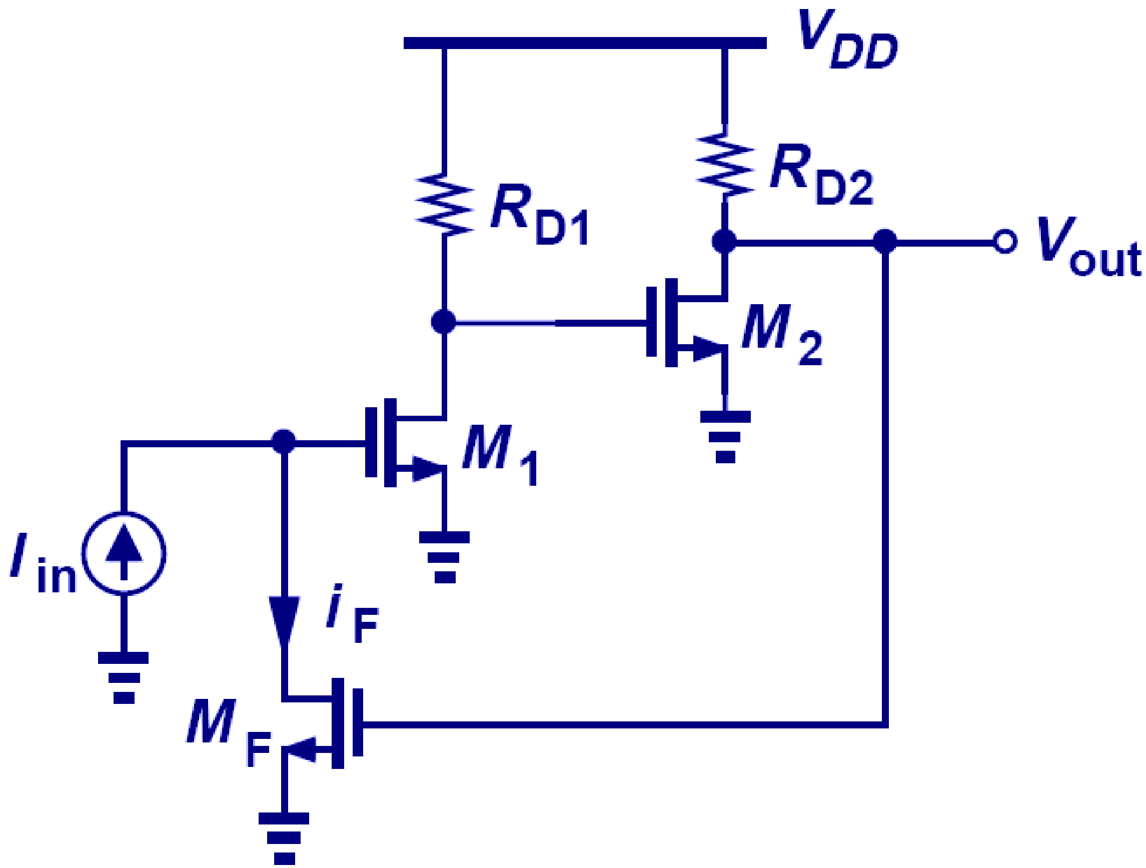
- Since M_1 and R_F are in parallel with the input current source, their respective currents are being subtracted. Note, R_F has to be large enough to approximate a current source.

Example: Sense and Return



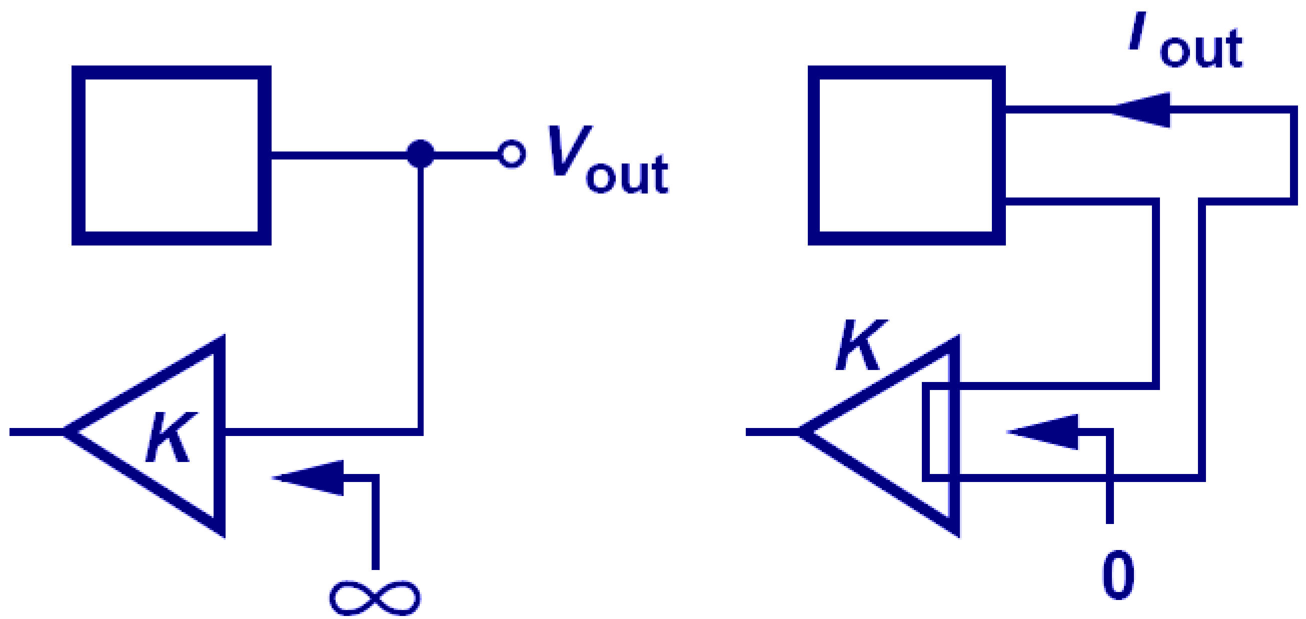
- R_1 and R_2 sense and return the output voltage to feedforward network consisting of M_1- M_4.
- M_1 and M_2 also act as a voltage subtractor.

Example: Feedback Factor



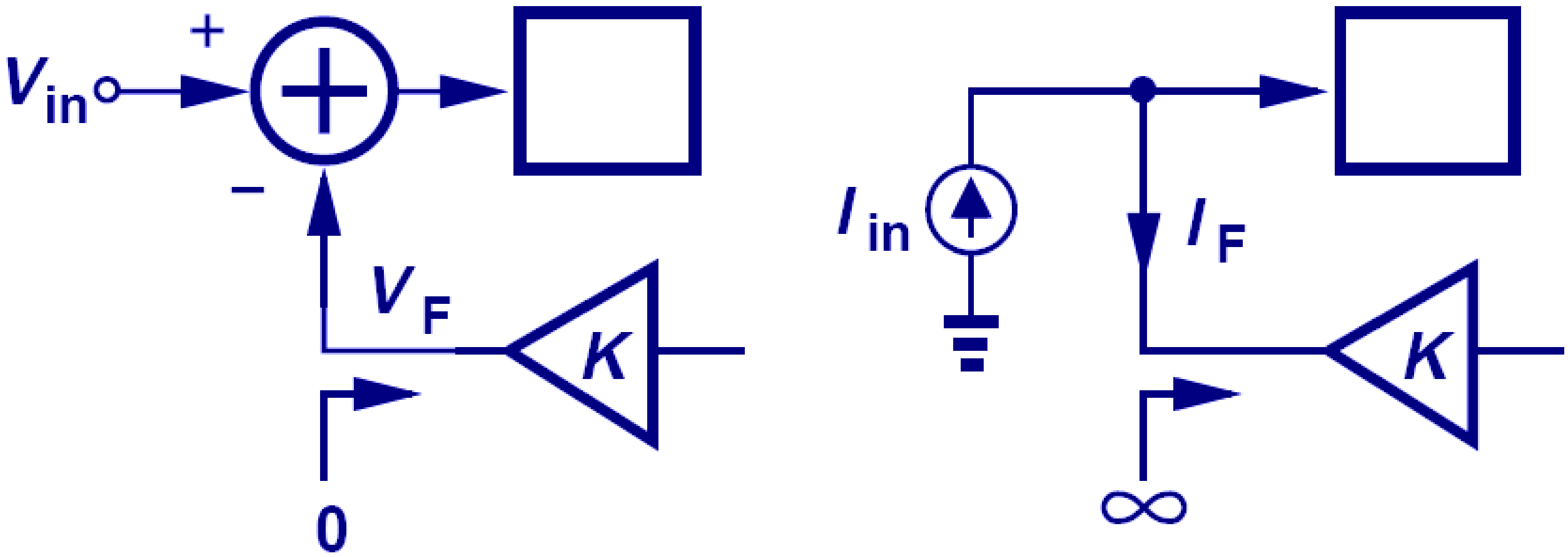
$$K = \frac{i_F}{v_{out}} = g_{mF}$$

Input Impedance of an Ideal Feedback Network



- To sense a voltage, the input impedance of an ideal feedback network must be infinite.
- To sense a current, the input impedance of an ideal feedback network must be zero.

Output Impedance of an Ideal Feedback Network

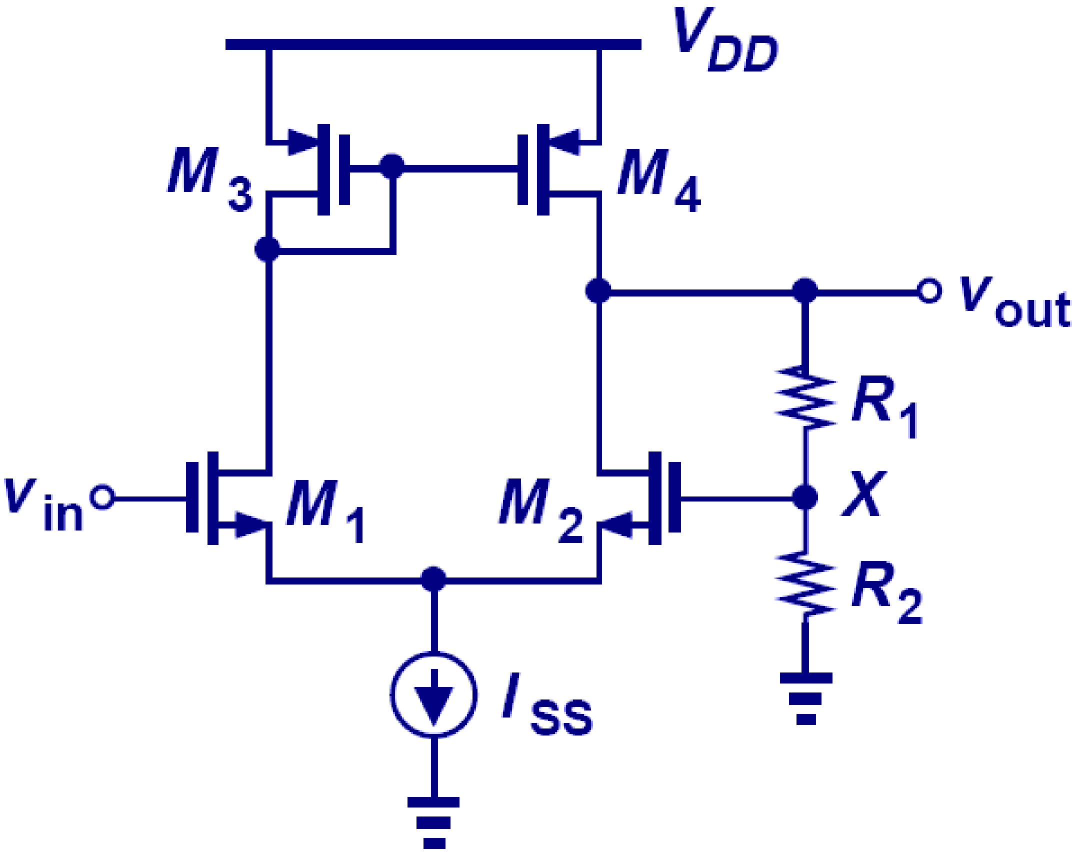


- To return a voltage, the output impedance of an ideal feedback network must be zero.
- To return a current, the output impedance of an ideal feedback network must be infinite.

Determining the Polarity of Feedback

- 1) Assume the input goes either up or down.
- 2) Follow the signal through the loop.
- 3) Determine whether the returned quantity enhances or opposes the original change.

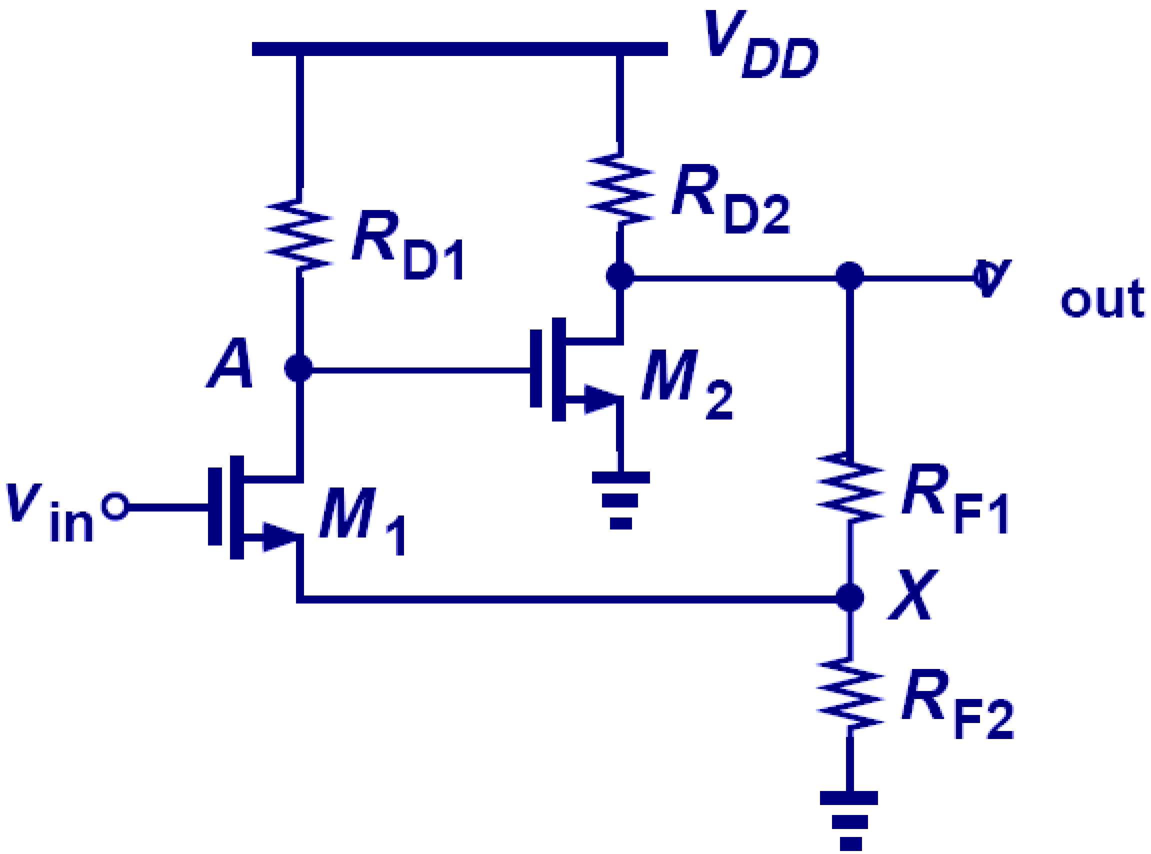
Polarity of Feedback Example I



$$V_{in} \uparrow \Rightarrow I_{D1} \uparrow, I_{D2} \downarrow \Rightarrow V_{out} \uparrow, V_x \uparrow \Rightarrow I_{D2} \uparrow, I_{D1} \downarrow$$

Negative Feedback

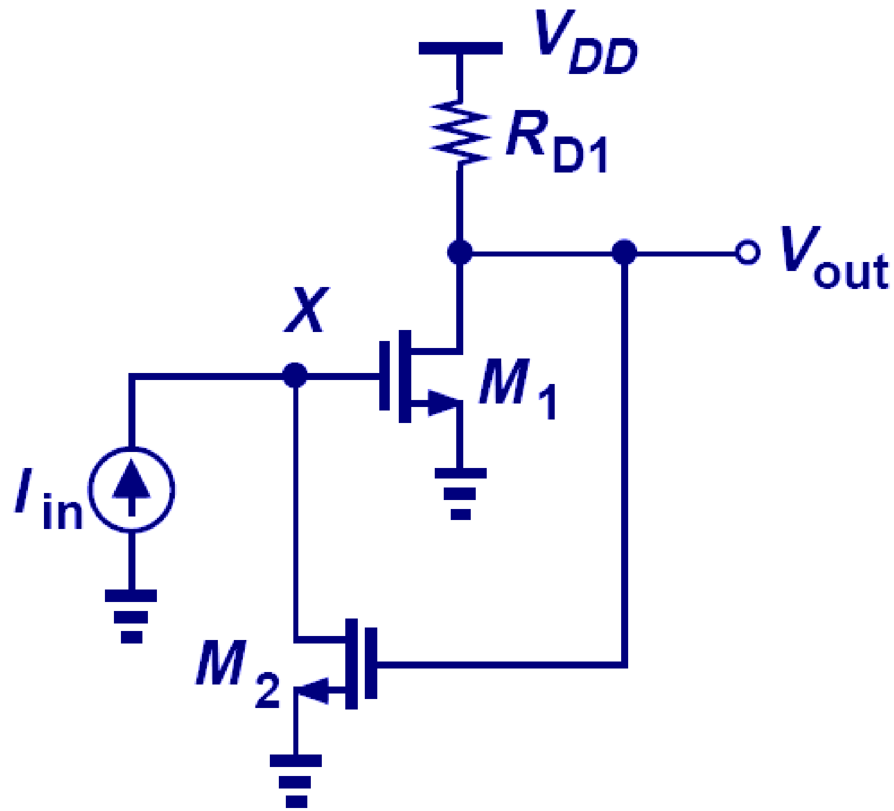
Polarity of Feedback Example II



$$V_{in}\uparrow \Rightarrow I_{D1}\uparrow, V_A\downarrow \Rightarrow V_{out}\uparrow, V_x\uparrow \Rightarrow I_{D1}\downarrow, V_A\uparrow$$

Negative Feedback

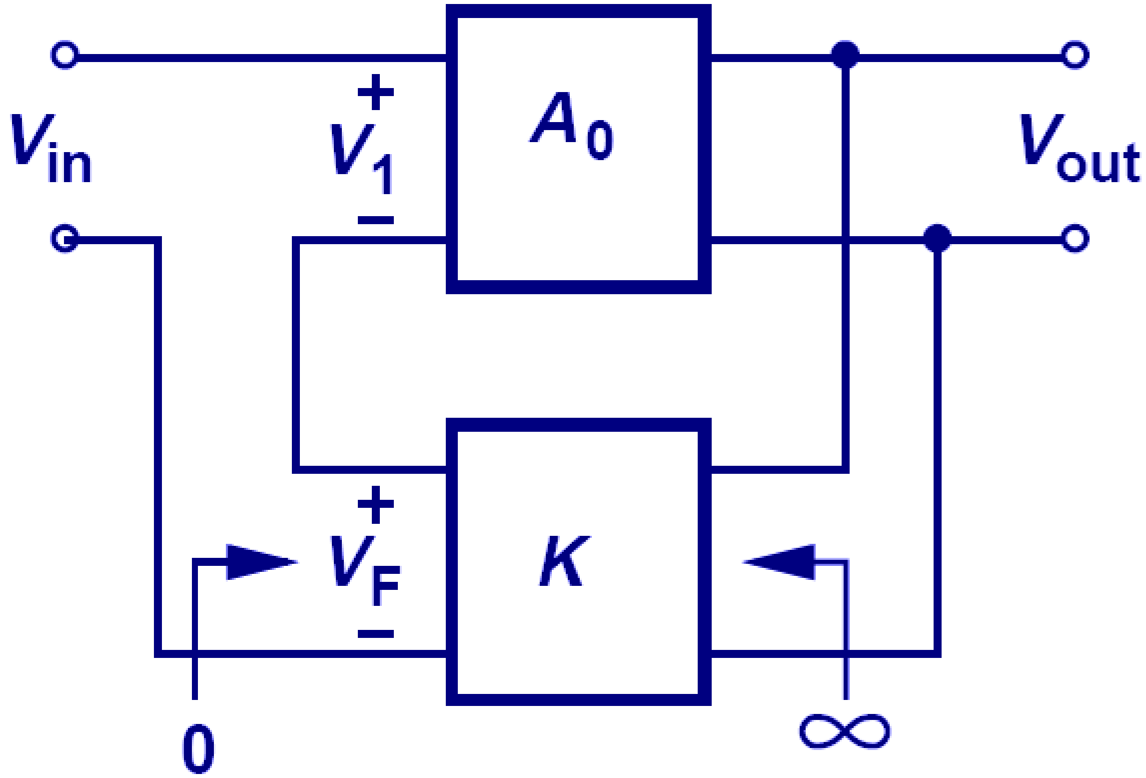
Polarity of Feedback Example III



$$I_{in}\uparrow \;\Rightarrow\; I_{D1}\uparrow, V_X\uparrow \;\Rightarrow\; V_{out}\downarrow, I_{D2}\downarrow \;\Rightarrow\; I_{D1}\uparrow, V_X\uparrow$$

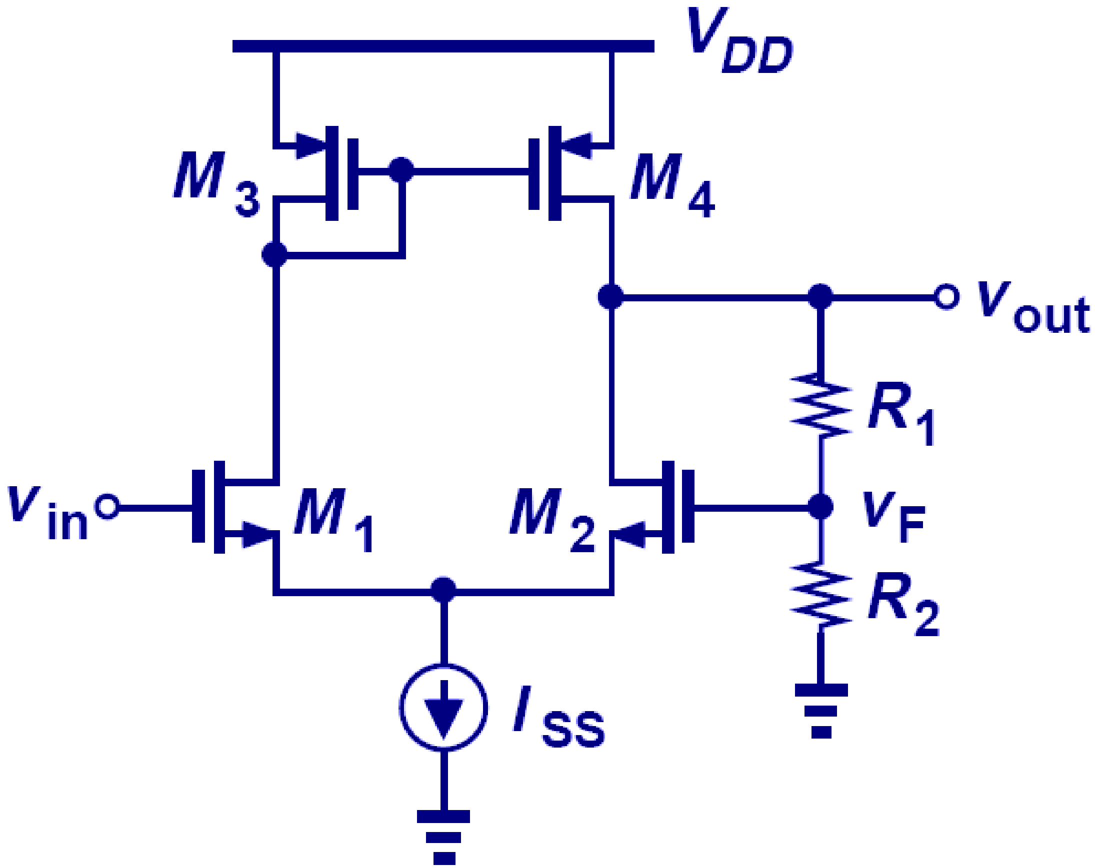
Positive Feedback

Voltage-Voltage Feedback



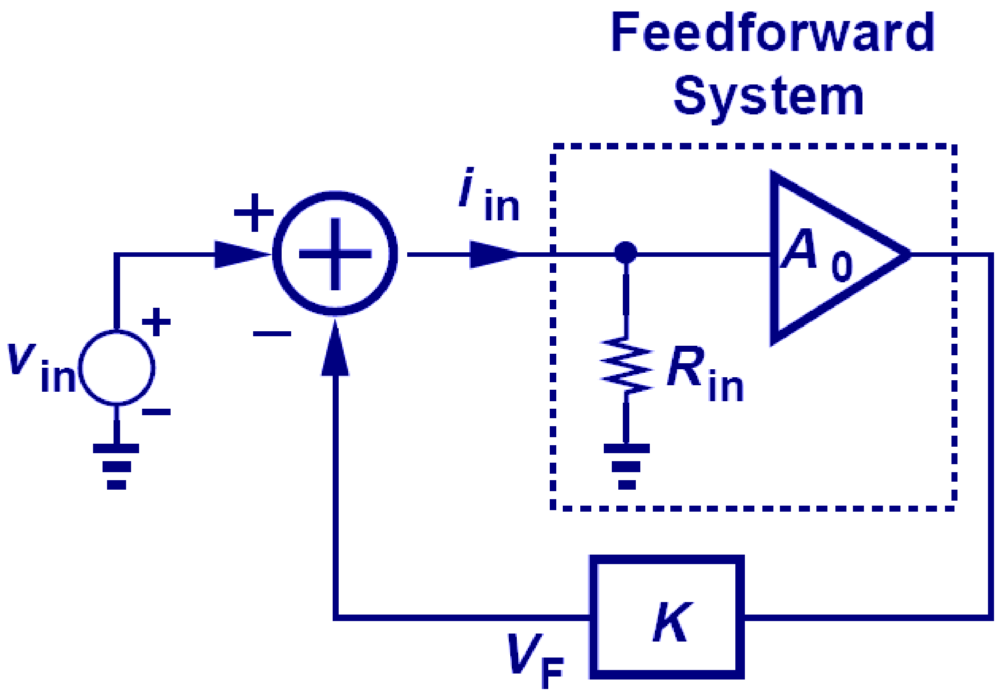
$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1 + KA_0}$$

Example: Voltage-Voltage Feedback



$$\frac{V_{out}}{V_{in}} = \frac{g_{mN}(r_{ON} \| r_{OP})}{1 + \frac{R_2}{R_1 + R_2} g_{mN}(r_{ON} \| r_{OP})}$$

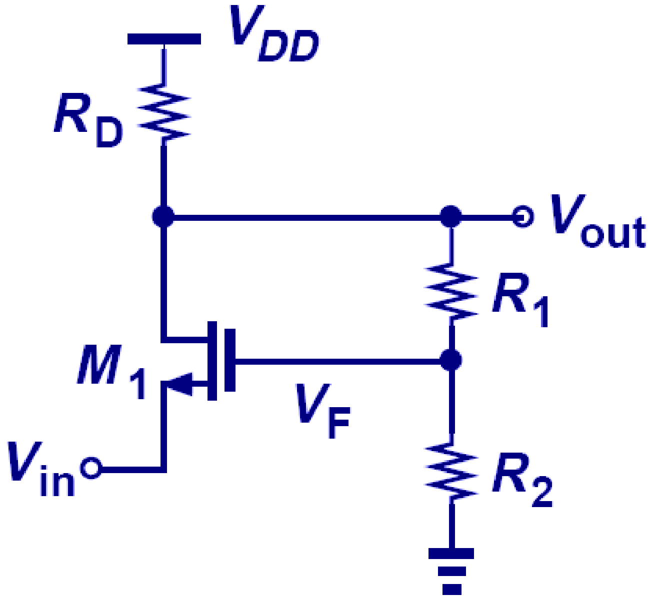
Input Impedance of a V-V Feedback

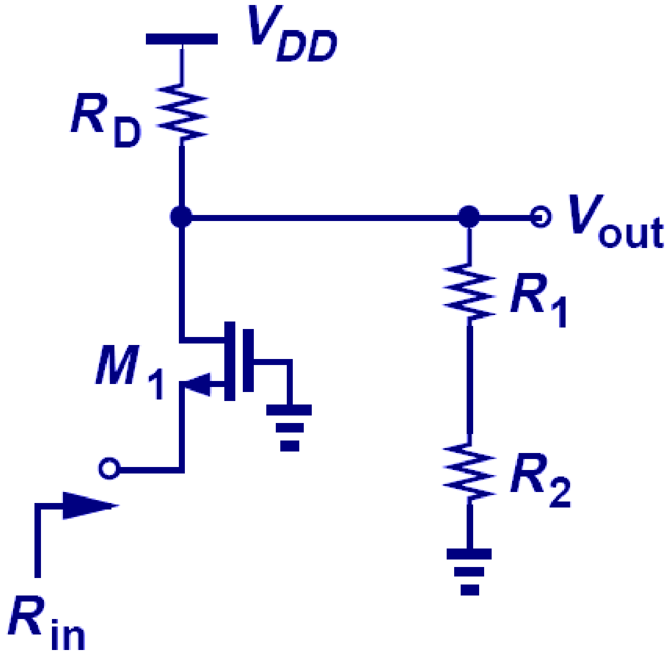


$$\frac{V_{in}}{I_{in}} = R_{in}(1 + A_0 K)$$

- A better voltage sensor

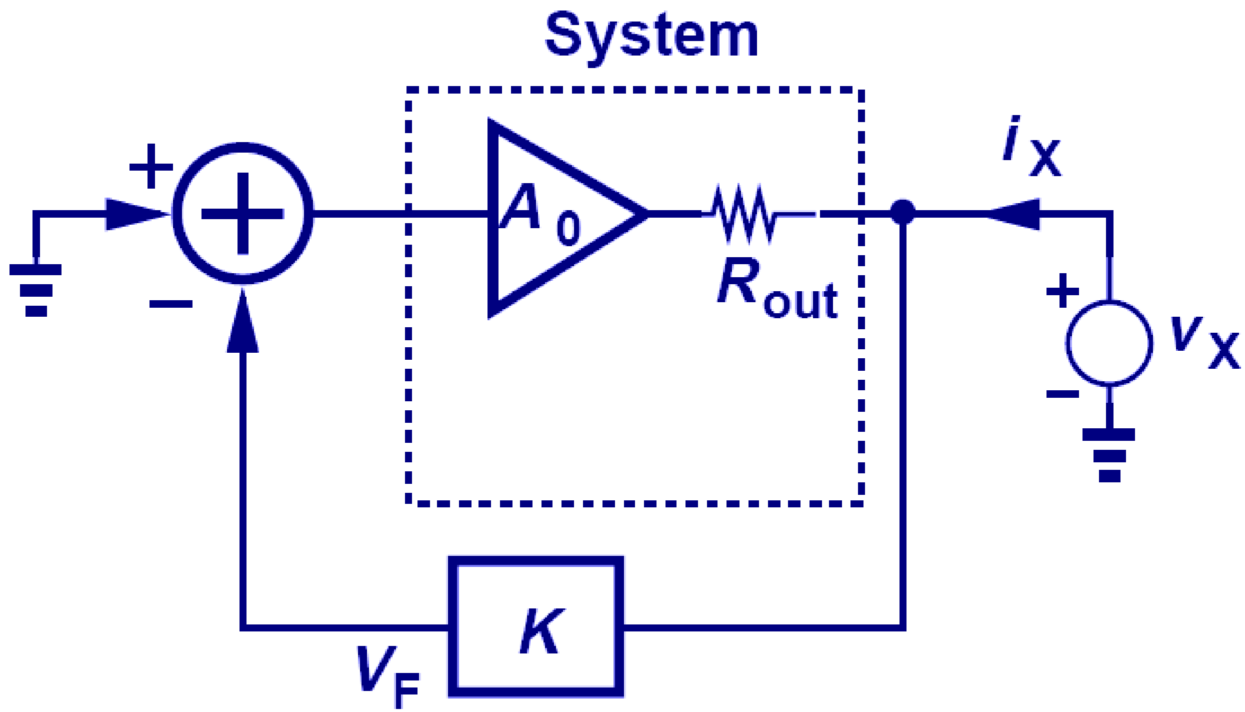
Example: V-V Feedback Input Impedance





$$\frac{V_{in}}{I_{in}} = \frac{1}{g_m}\left(1 + \frac{R_2}{R_1 + R_2} g_m R_D\right)$$

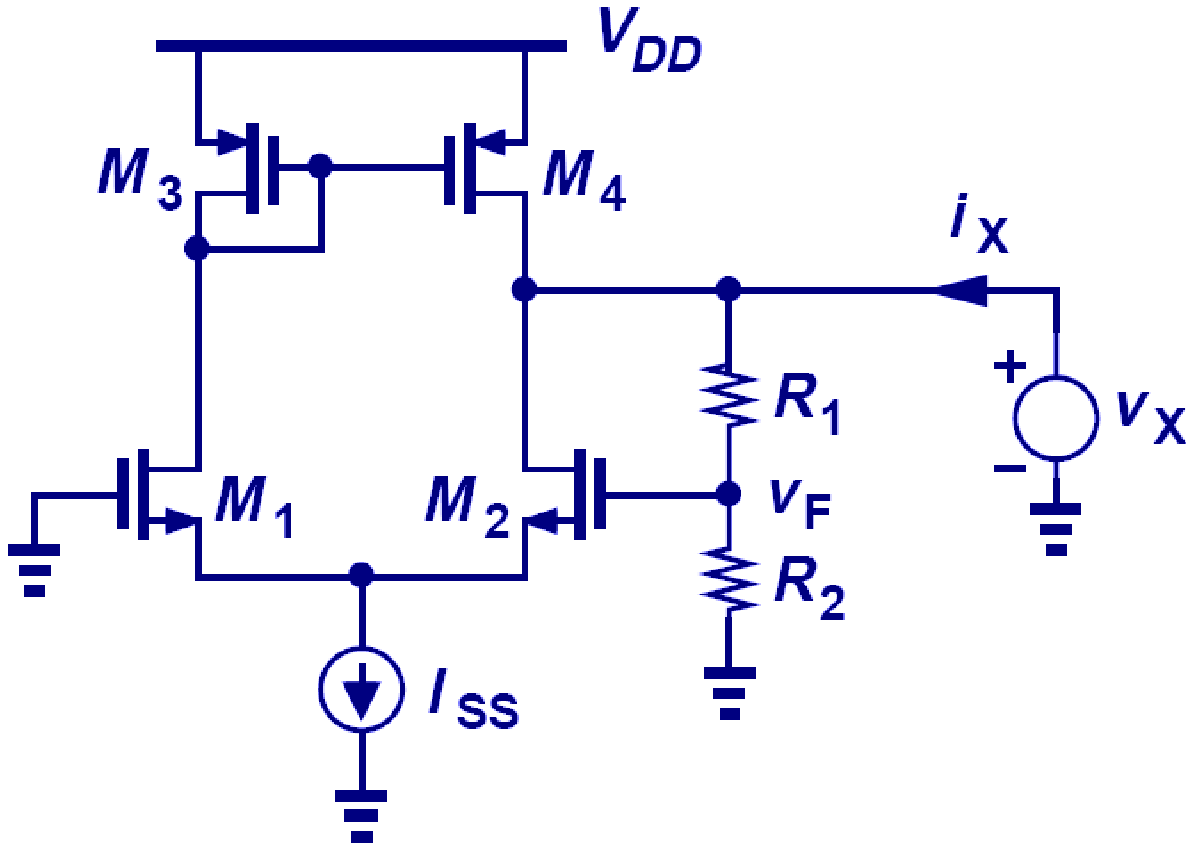
Output Impedance of a V-V Feedback



$$\frac{V_X}{I_X} = \frac{R_{out}}{(1 + KA_0)}$$

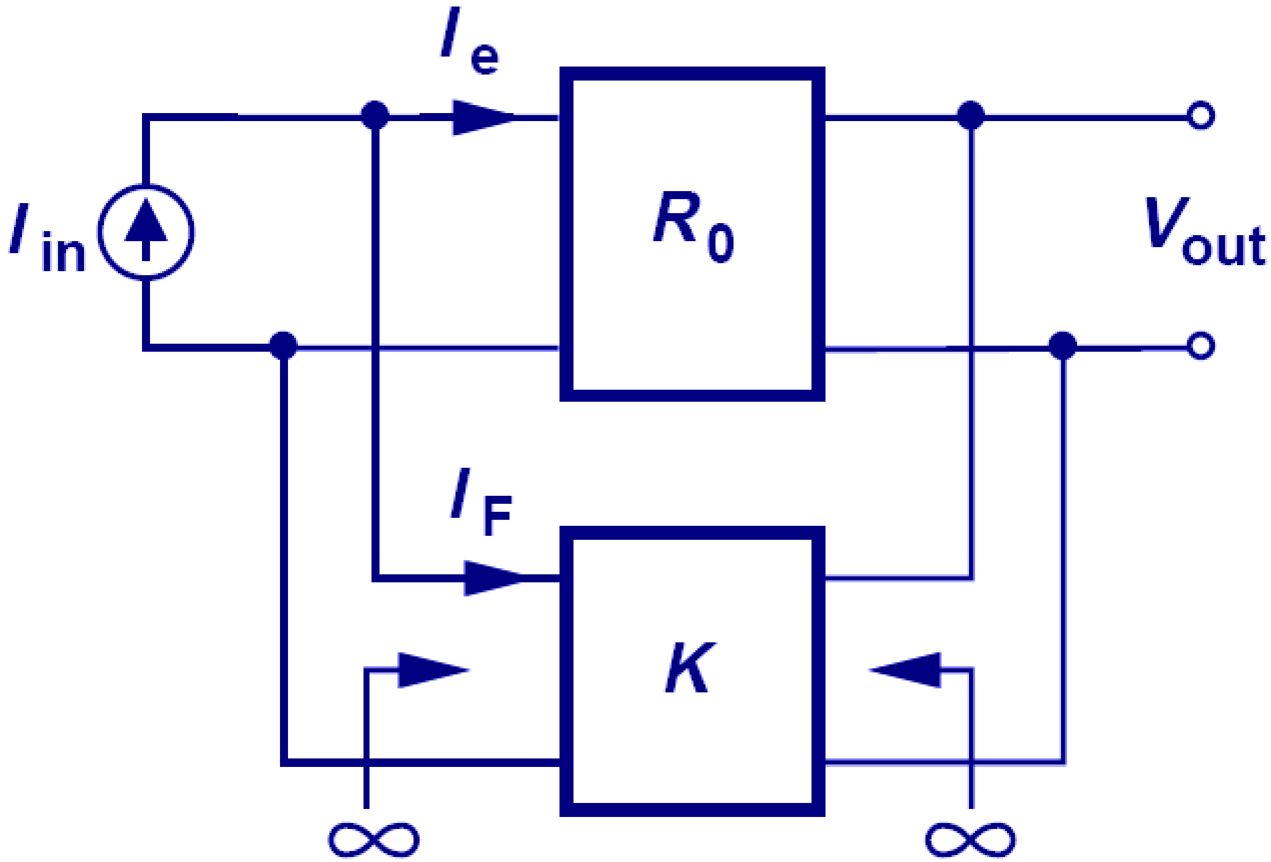
- A better voltage source

Example: V-V Feedback Output Impedance



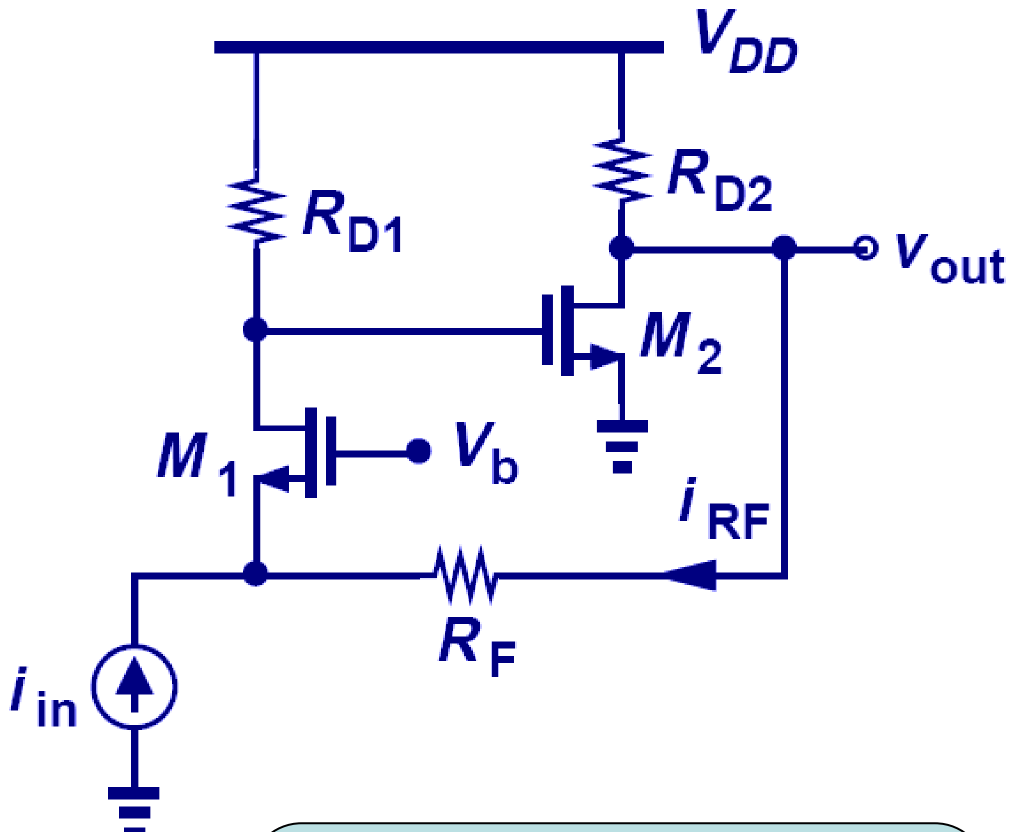
$$R_{out,closed} \approx \left(1+\frac{R_1}{R_2}\right)\frac{1}{g_{mN}}$$

Voltage-Current Feedback



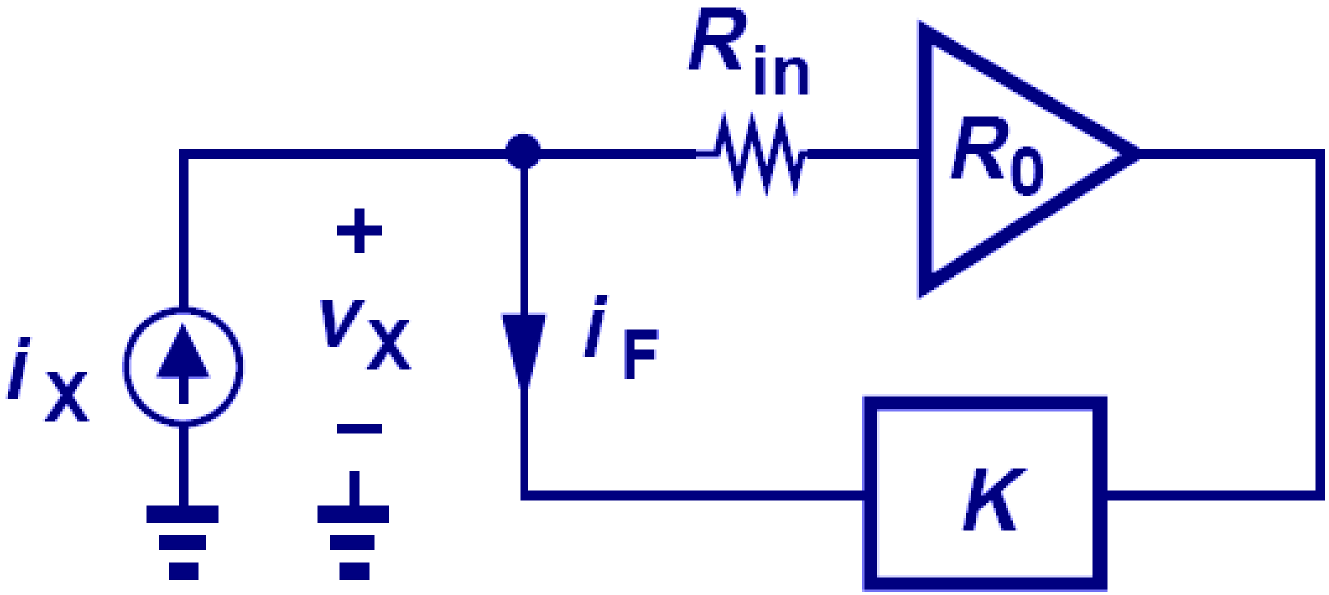
$$\frac{V_{out}}{I_{in}} = \frac{R_O}{1 + KR_O}$$

Example: Voltage-Current Feedback



$$\frac{V_{out}}{I_{in}} = \frac{-g_{m2}R_{D1}R_{D2}}{1+\dfrac{g_{m2}R_{D1}R_{D2}}{R_F}}$$

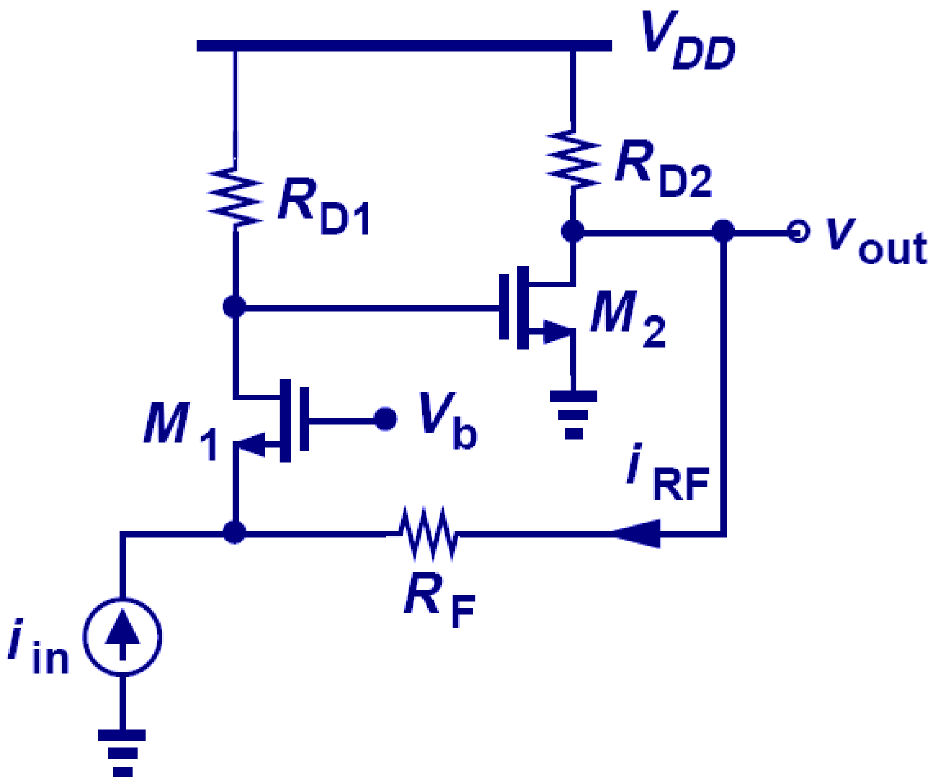
Input Impedance of a V-C Feedback



$$\frac{V_X}{I_X} = \frac{R_{in}}{1 + R_0 K}$$

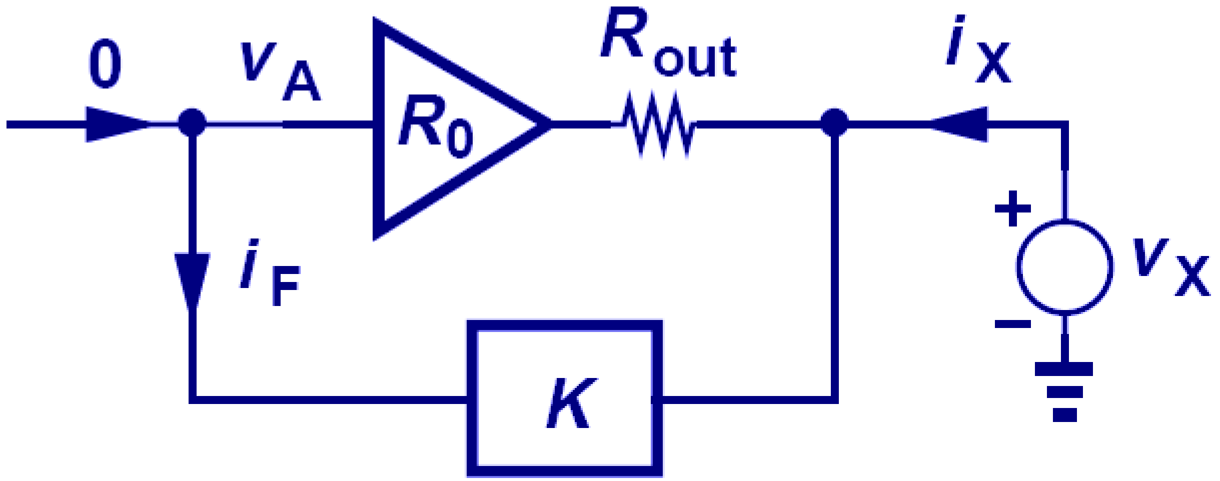
- A better current sensor.

Example: V-C Feedback Input Impedance



$$R_{in,closed} = \frac{1}{g_{m1}} \cdot \frac{1}{1 + \dfrac{g_{m2} R_{D1} R_{D2}}{R_F}}$$

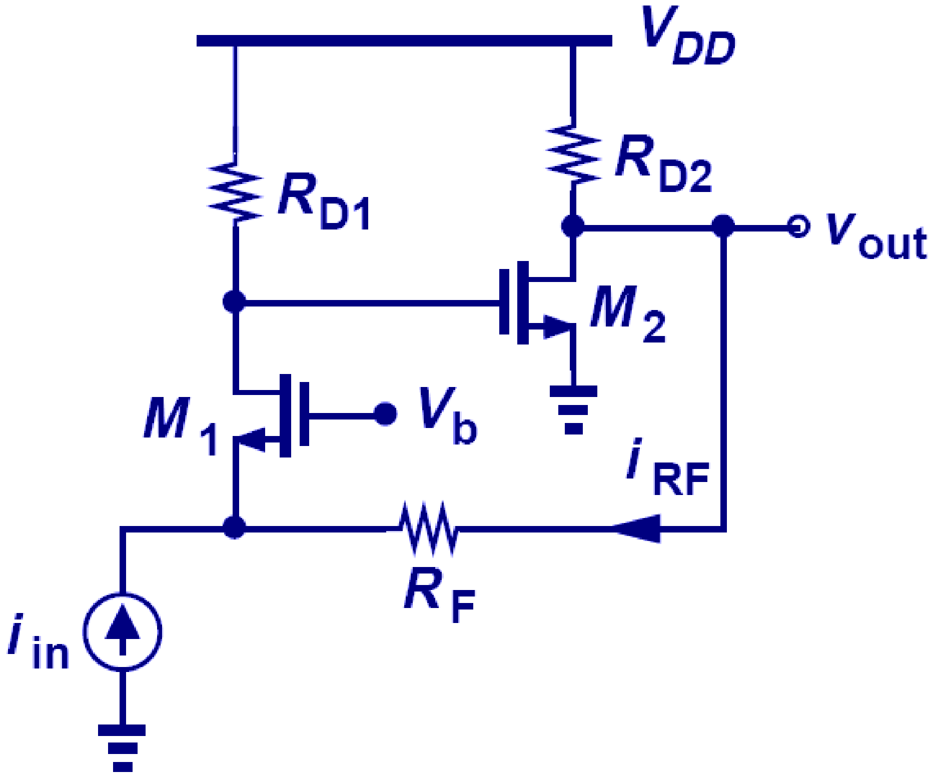
Output Impedance of a V-C Feedback



$$\frac{V_X}{I_X} = \frac{R_{out}}{1 + R_0 K}$$

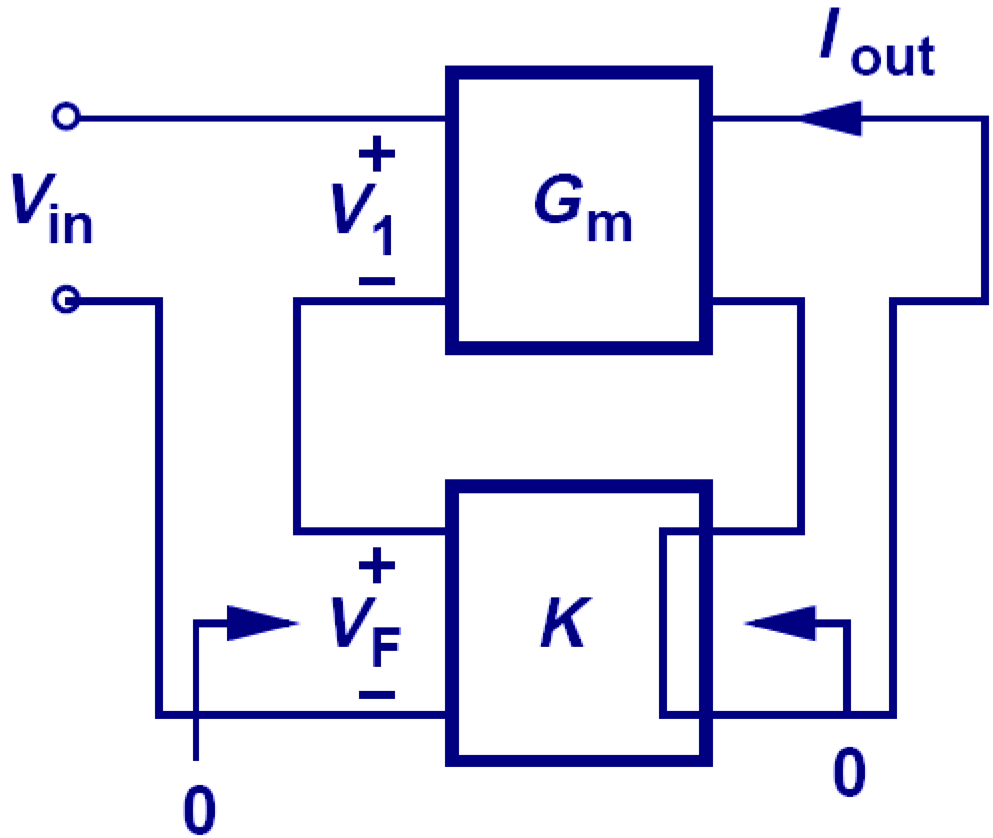
- A better voltage source.

Example: V-C Feedback Output Impedance



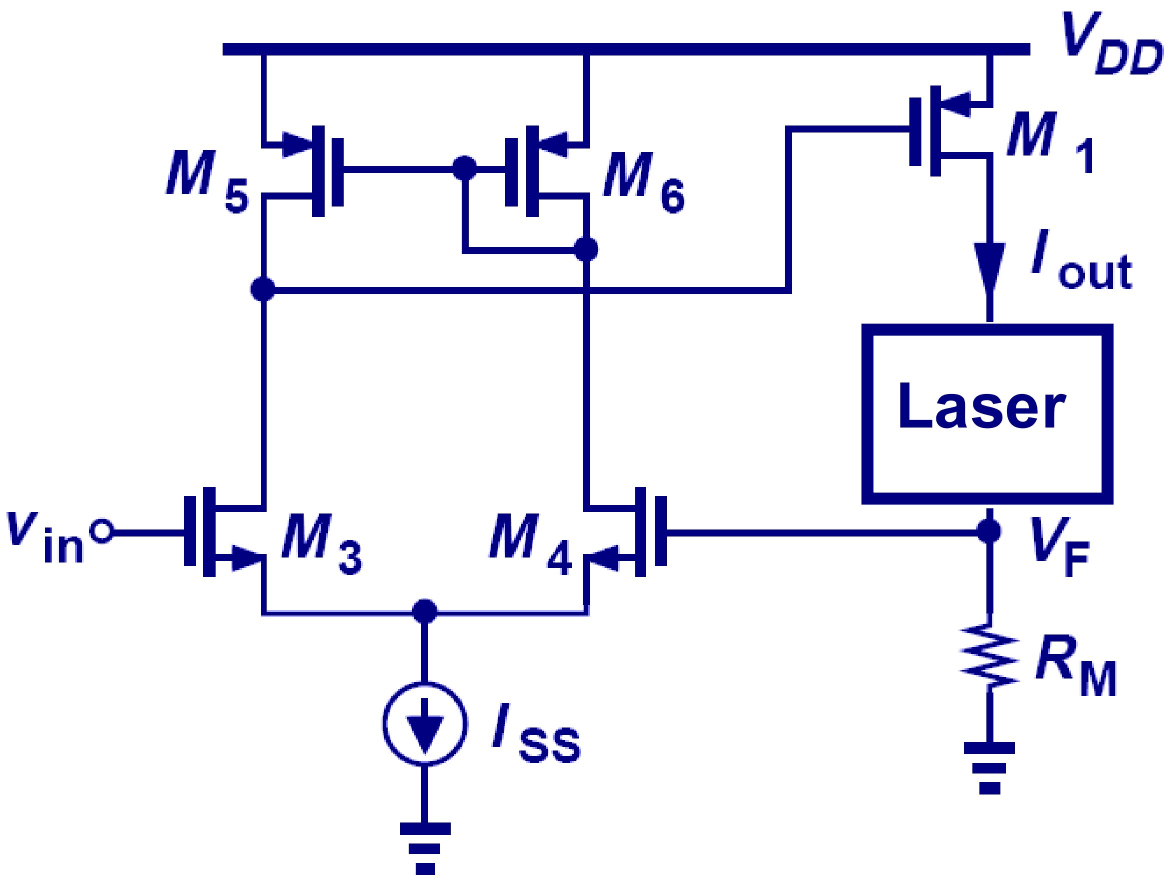
$$R_{out,closed} = \frac{R_{D2}}{1 + \dfrac{g_{m2} R_{D1} R_{D2}}{R_F}}$$

Current-Voltage Feedback



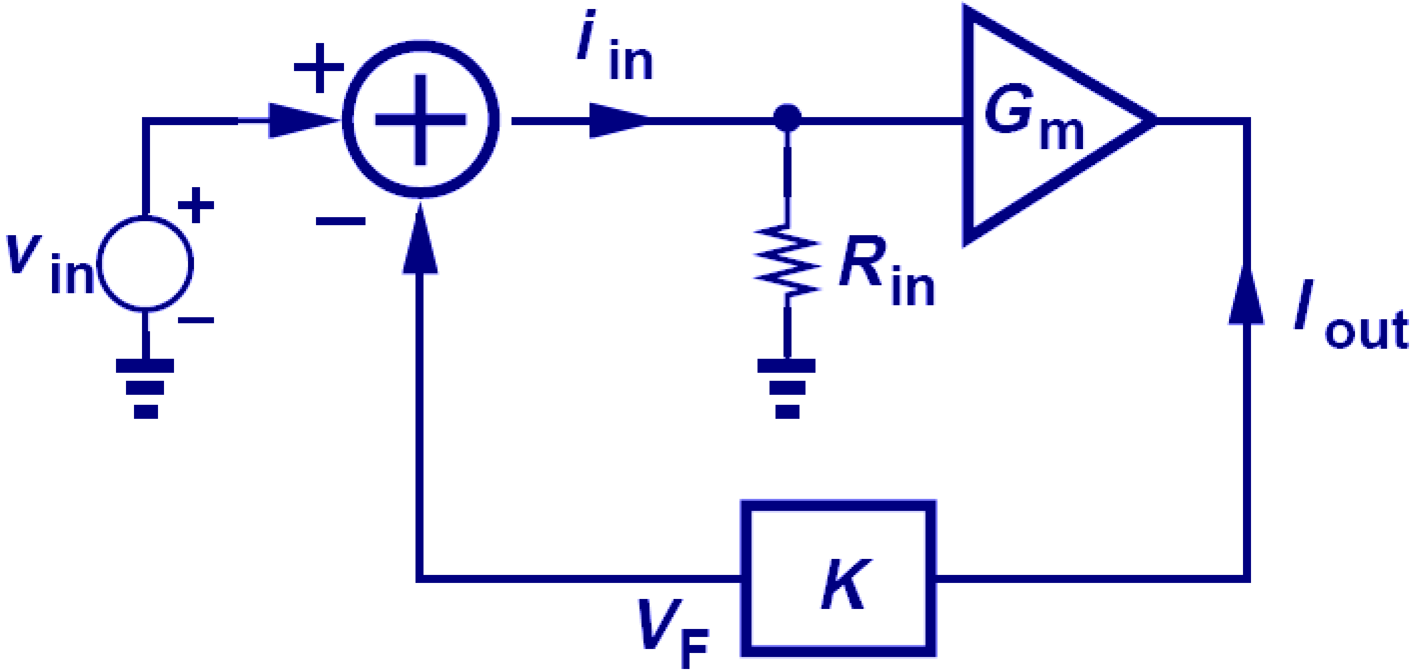
$$\frac{I_{out}}{V_{in}} = \frac{G_m}{1+KG_m}$$

Example: Current-Voltage Feedback



$$\frac{I_{out}}{V_{in}}\Big|_{closed} = \frac{g_{m1}g_{m3}\left(r_{O3} \parallel r_{O5}\right)}{1+g_{m1}g_{m3}\left(r_{O3} \parallel r_{O5}\right)R_M}$$

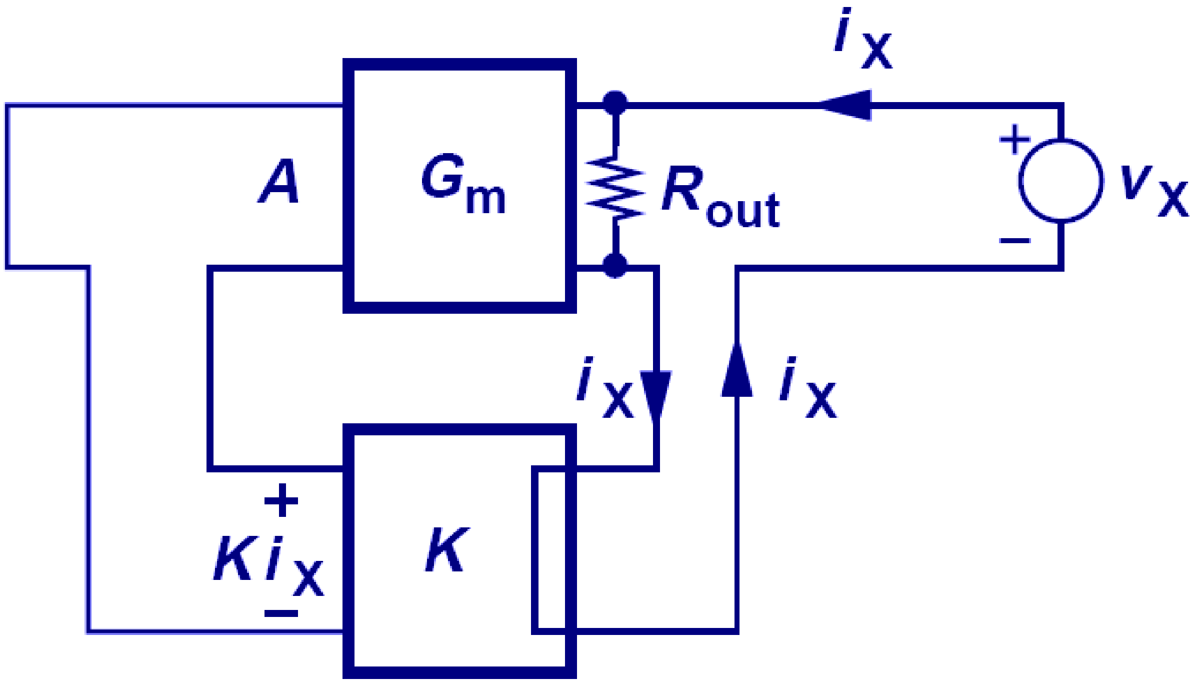
Input Impedance of a C-V Feedback



$$\frac{V_{in}}{I_{in}} = R_{in}(1 + KG_m)$$

- A better voltage sensor.

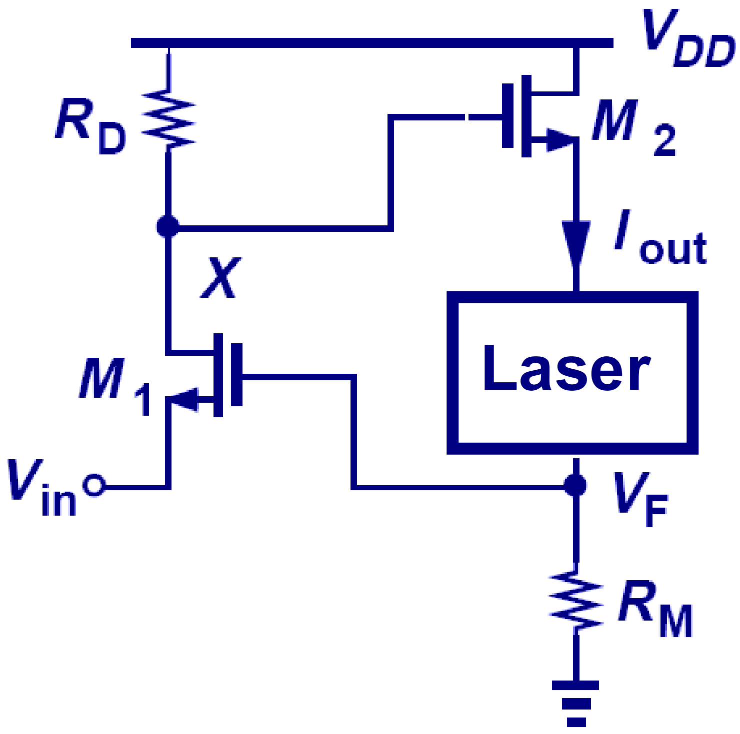
Output Impedance of a C-V Feedback



$$\frac{V_X}{I_X} = R_{out}(1 + KG_m)$$

- A better current source.

Example: Current-Voltage Feedback

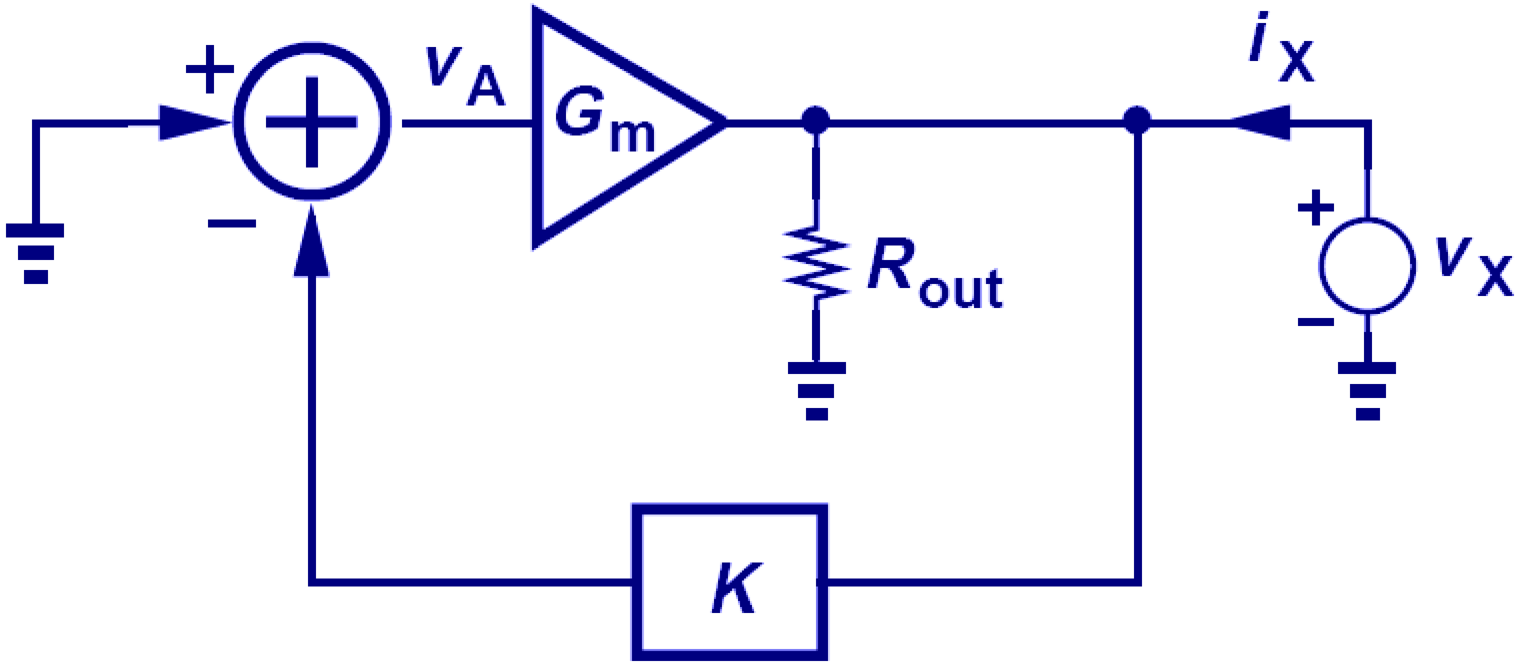


$$\frac{I_{out}}{V_{in}}\bigg|_{closed} = \frac{g_{m1}g_{m2}R_D}{1+g_{m1}g_{m2}R_DR_M}$$

$$R_{in}\big|_{closed} = \frac{1}{g_{m1}}(1+g_{m1}g_{m2}R_DR_M)$$

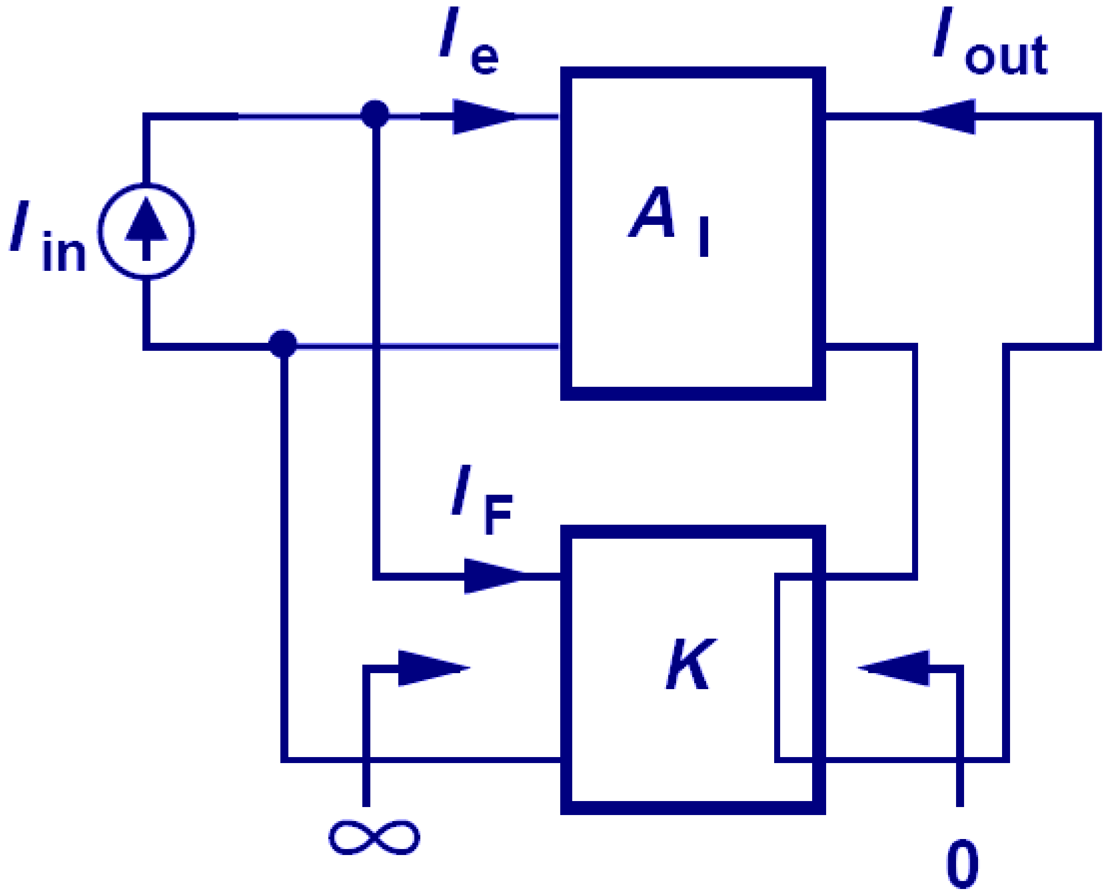
$$R_{out}\big|_{closed} = \frac{1}{g_{m2}}(1+g_{m1}g_{m2}R_DR_M)$$

Wrong Technique for Measuring Output Impedance



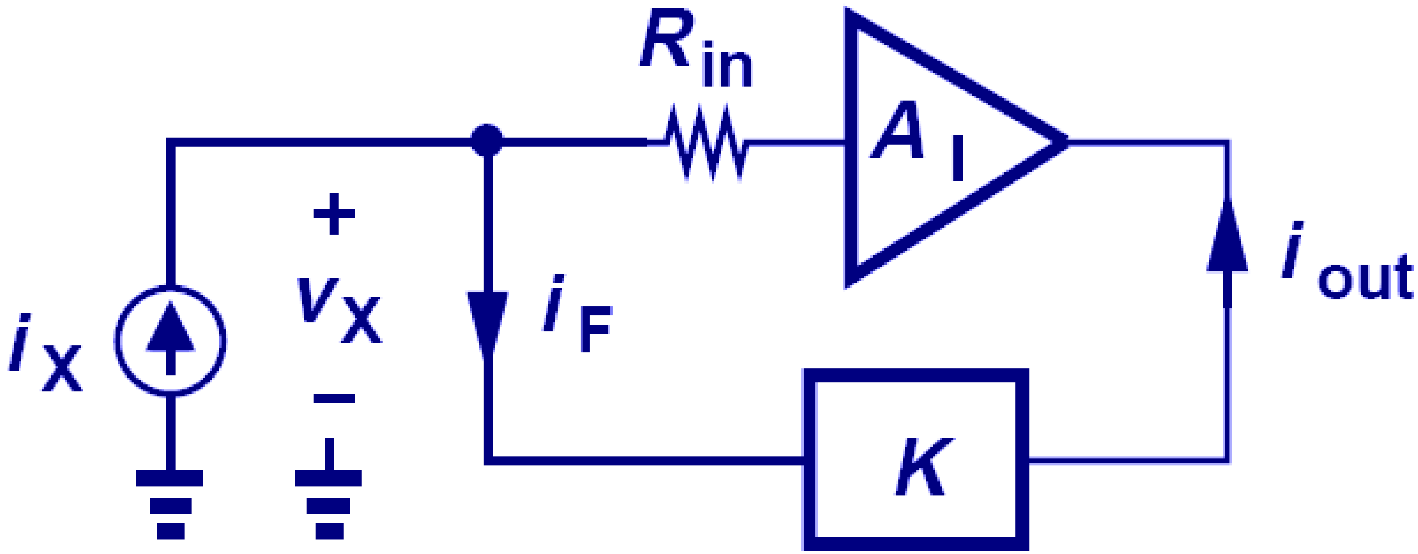
- **If we want to measure the output impedance of a C-V closed-loop feedback topology directly, we have to place V_X in series with K and R_{out}. Otherwise, the feedback will be disturbed.**

Current-Current Feedback



$$\frac{I_{out}}{I_{in}} = \frac{A_I}{1+KA_I}$$

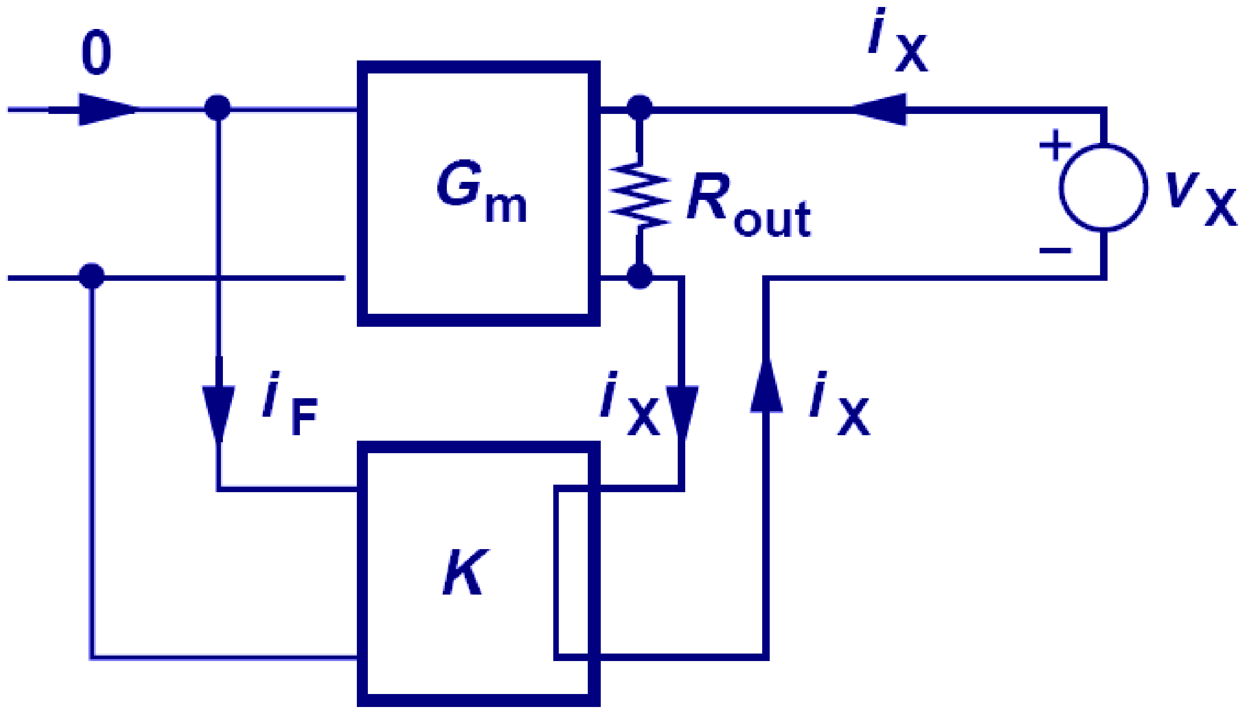
Input Impedance of C-C Feedback



$$\frac{V_X}{I_X} = \frac{R_{in}}{1 + KA_I}$$

- A better current sensor.

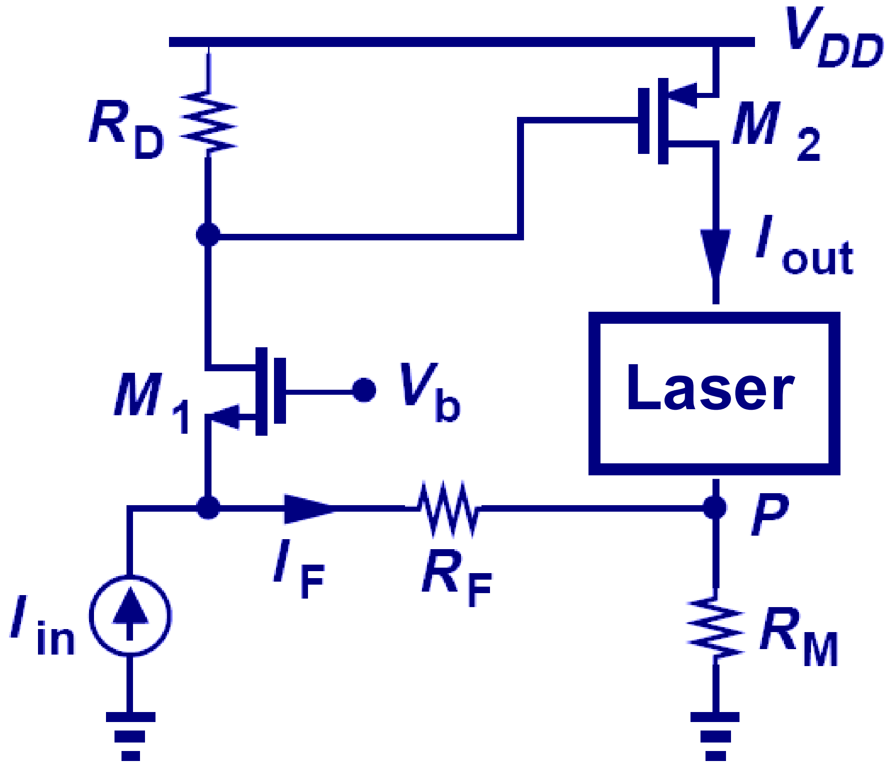
Output Impedance of C-C Feedback



$$\frac{V_X}{I_X} = R_{out}(1 + KA_I)$$

- A better current source.

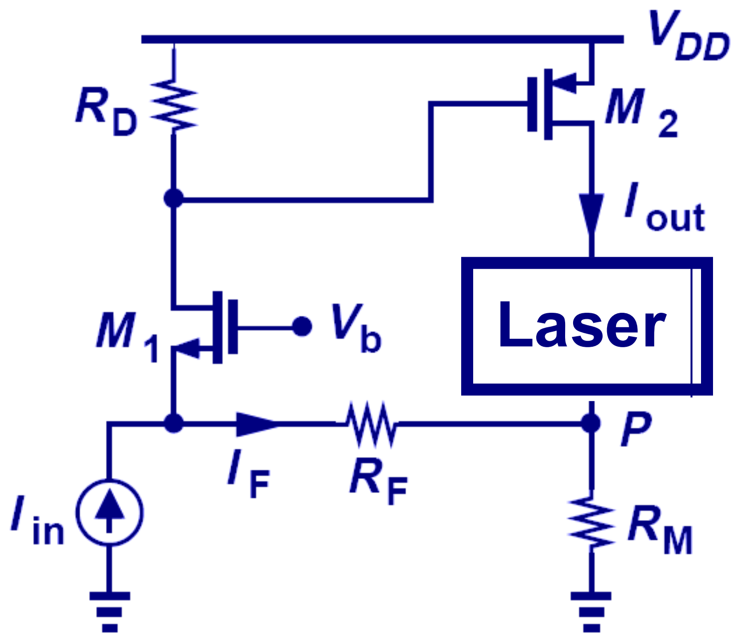
Example: Test of Negative Feedback



$$I_{in}\uparrow \Rightarrow V_{D1}\uparrow, I_{out}\downarrow \Rightarrow V_P\downarrow, I_F\uparrow \Rightarrow V_{D1}\downarrow, I_{out}\uparrow$$

Negative Feedback

Example: C-C Negative Feedback

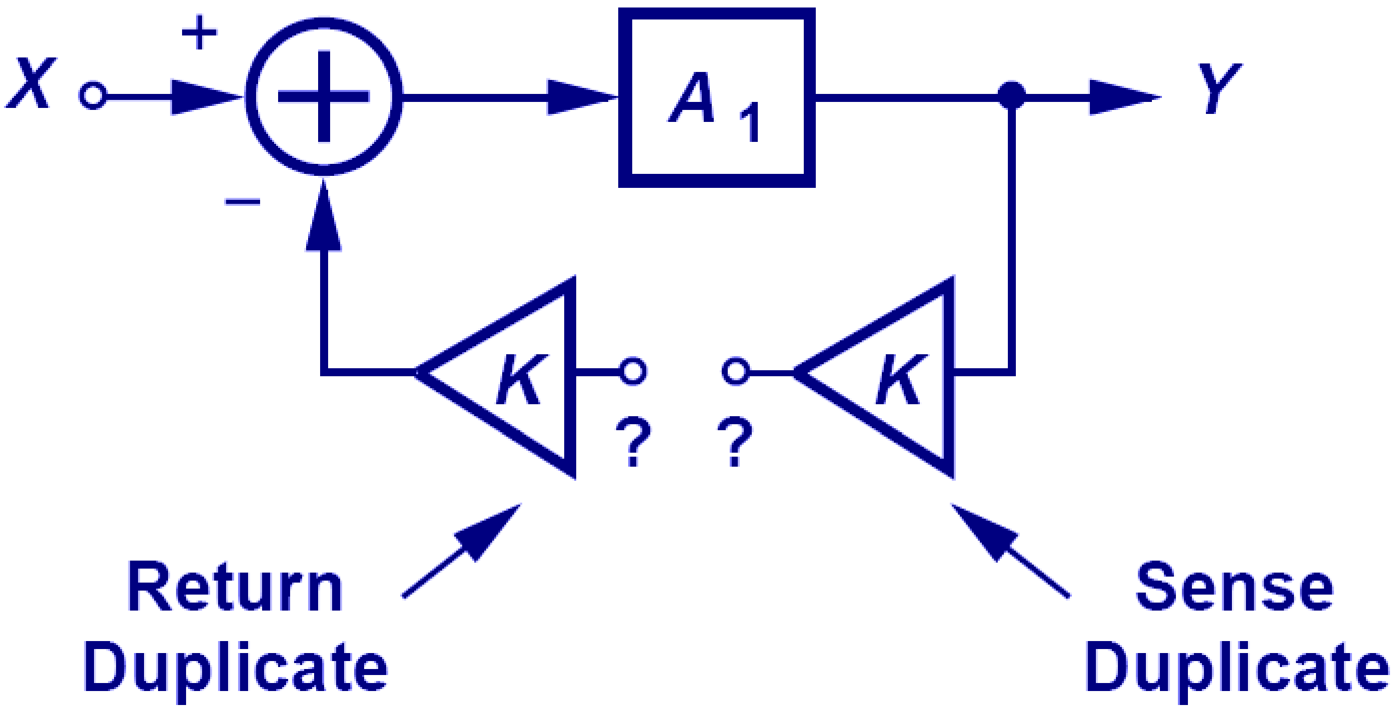


$$A_I|_{closed} = \frac{-g_{m2}R_D}{1+g_{m2}R_D(R_M/R_F)}$$

$$R_{in}|_{closed} = \frac{1}{g_{m1}} \cdot \frac{1}{1+g_{m2}R_D(R_M/R_F)}$$

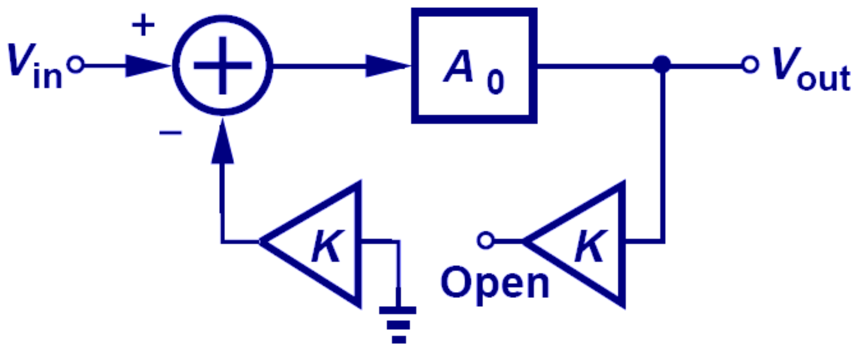
$$R_{out}|_{closed} = r_{O2}[1+g_{m2}R_D(R_M/R_F)]$$

How to Break a Loop

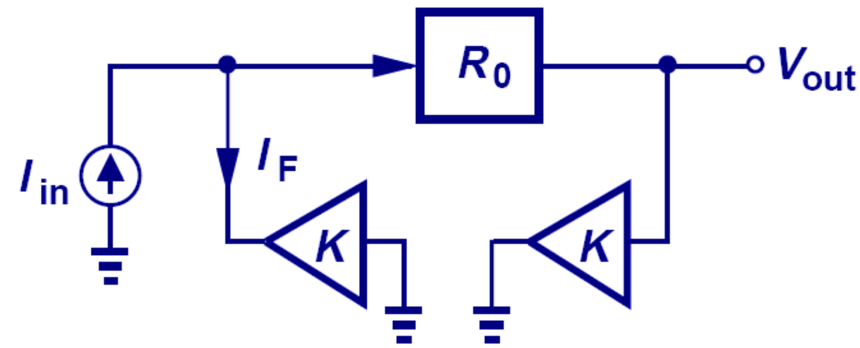


- The correct way of breaking a loop is such that the loop does not know it has been broken. Therefore, we need to present the feedback network to both the input and the output of the feedforward amplifier.

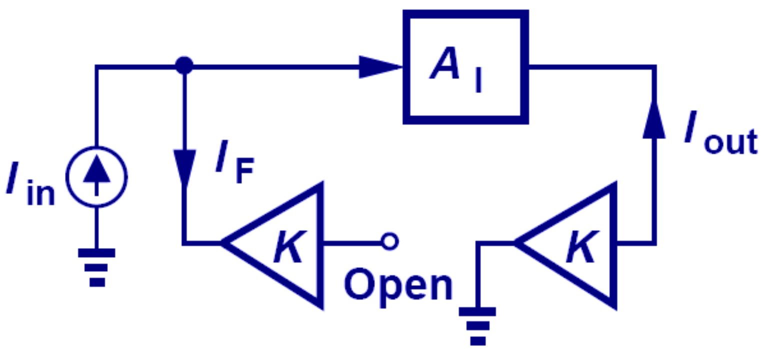
Rules for Breaking the Loop of Amplifier Types



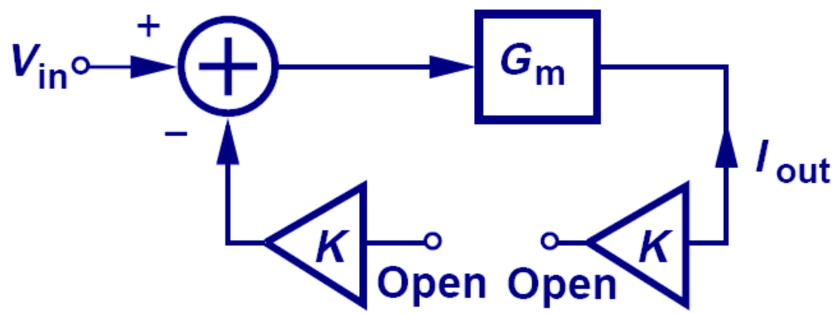
(a)



(b)



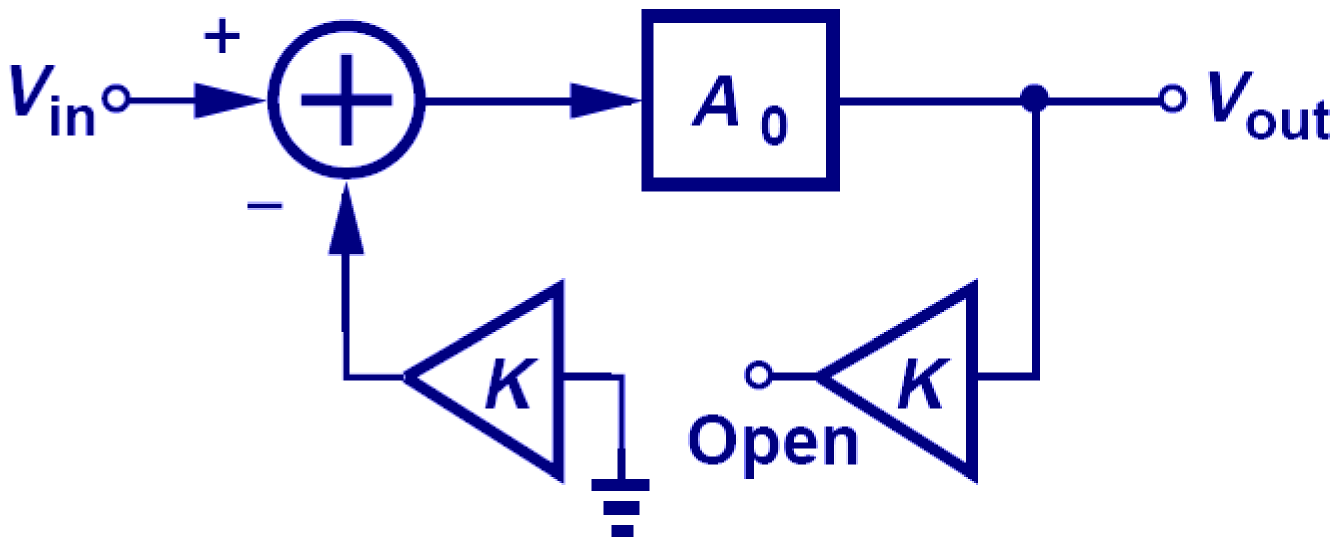
(c)



(d)

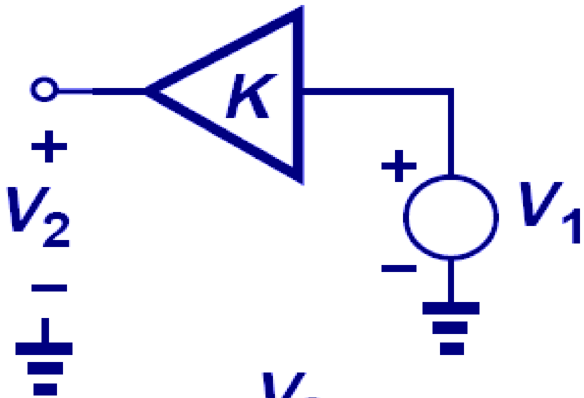
Intuitive Understanding of these Rules

Voltage-Voltage Feedback

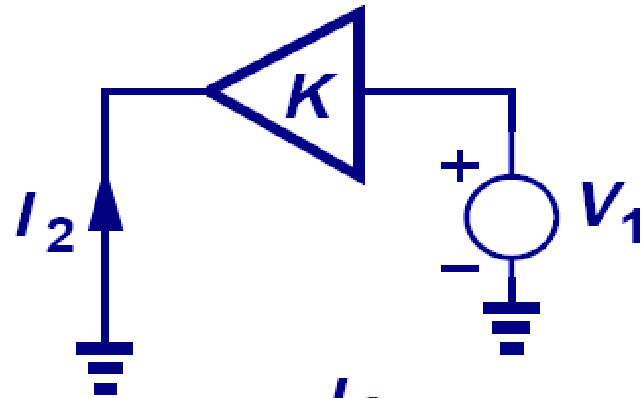


- Since ideally, the input of the feedback network sees zero impedance (Z_{out} of an ideal voltage source), the return replicate needs to be grounded. Similarly, the output of the feedback network sees an infinite impedance (Z_{in} of an ideal voltage sensor), the sense replicate needs to be open.
- Similar ideas apply to the other types.

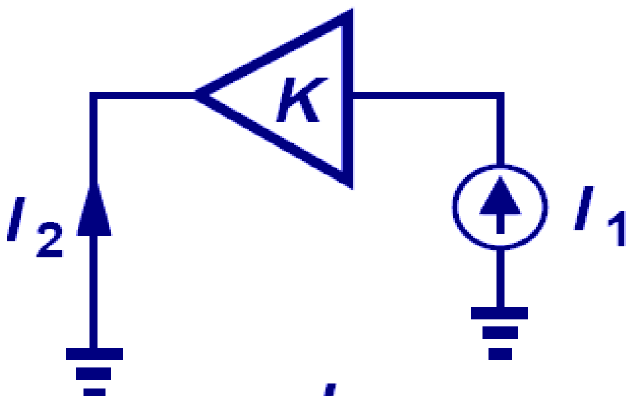
Rules for Calculating Feedback Factor



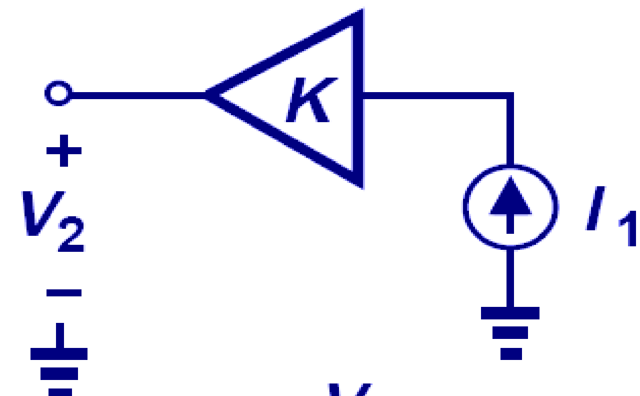
$$K = \frac{V_2}{V_1}$$



$$K = \frac{I_2}{V_1}$$



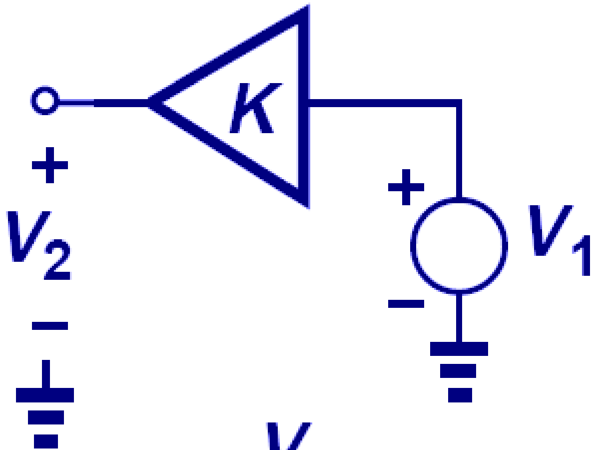
$$K = \frac{I_2}{I_1}$$



$$K = \frac{V_2}{I_1}$$

Intuitive Understanding of these Rules

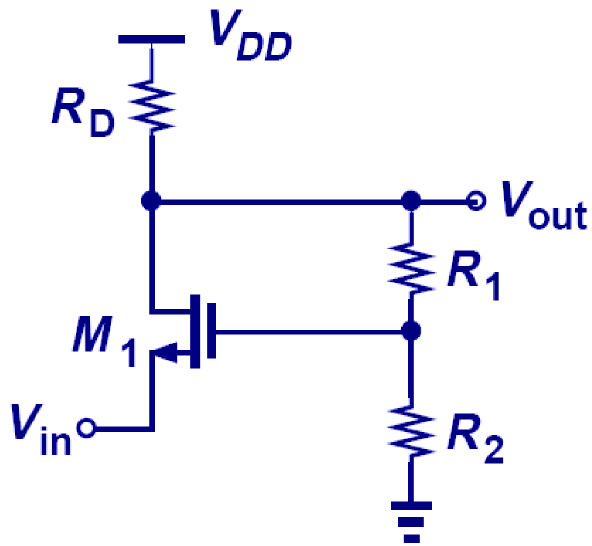
Voltage-Voltage Feedback

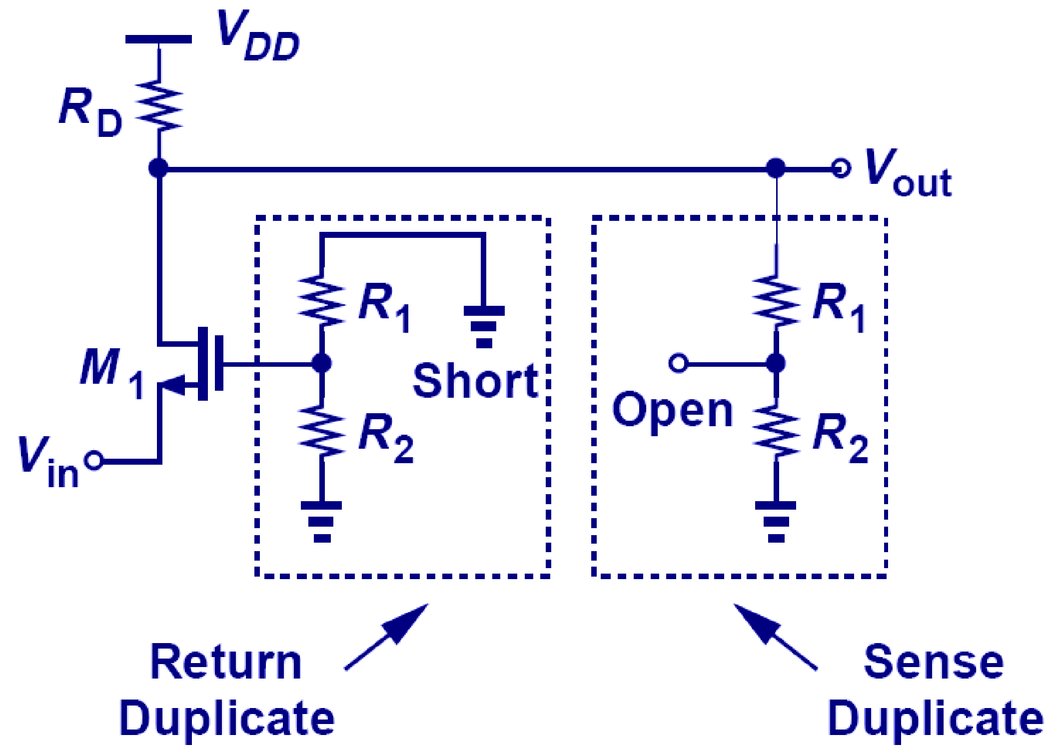


$$K = \frac{V_2}{V_1}$$

- Since the feedback senses voltage, the input of the feedback is a voltage source. Moreover, since the return quantity is also voltage, the output of the feedback is left open (a short means the output is always zero).
- Similar ideas apply to the other types.

Breaking the Loop Example I



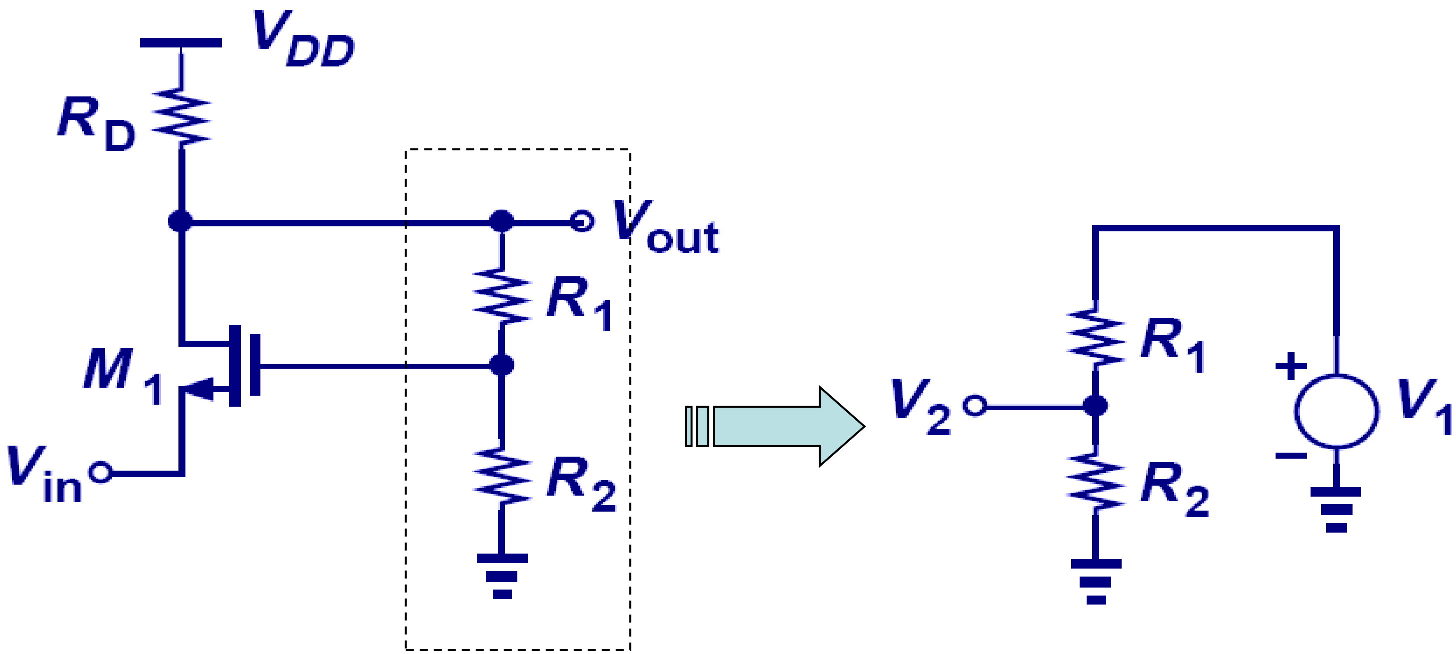


$$A_{v,open} = g_{m1}\left[R_D \,\|\, \left(R_1 + R_2\right)\right]$$

$$R_{in,open} = 1/g_{m1}$$

$$R_{out,open} = R_D \,\|\, \left(R_1 + R_2\right)$$

Feedback Factor Example I



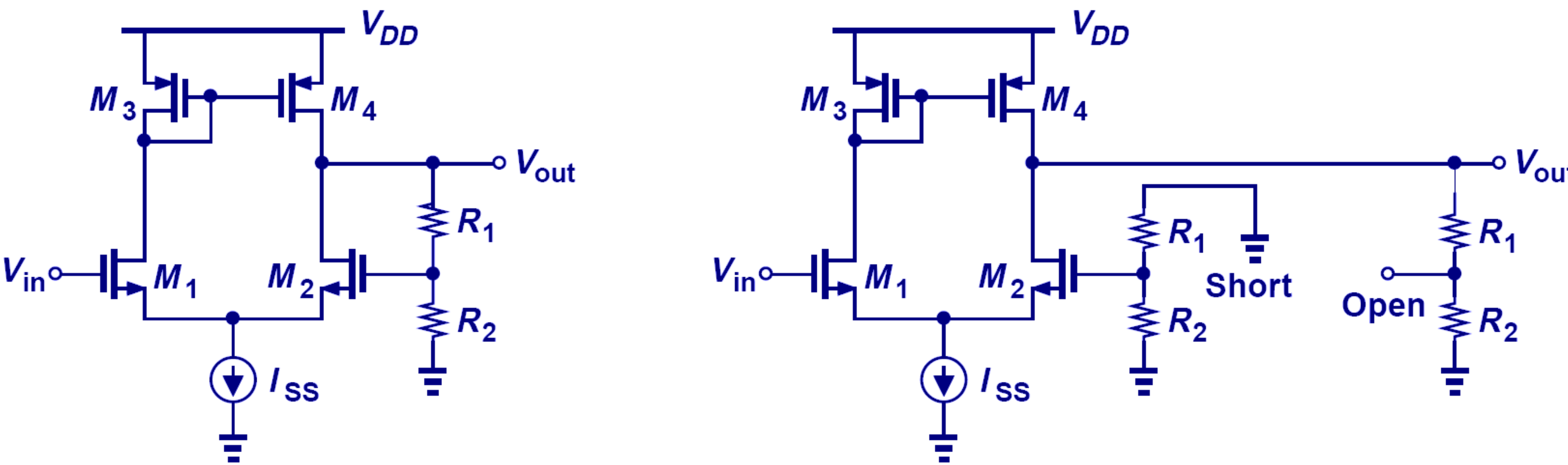
$$K = R_2 / (R_1 + R_2)$$

$$A_{v,closed} = A_{v,open} / (1 + KA_{v,open})$$

$$R_{in,closed} = R_{in,open} (1 + KA_{v,open})$$

$$R_{out,closed} = R_{out,closed} / (1 + KA_{v,open})$$

Breaking the Loop Example II

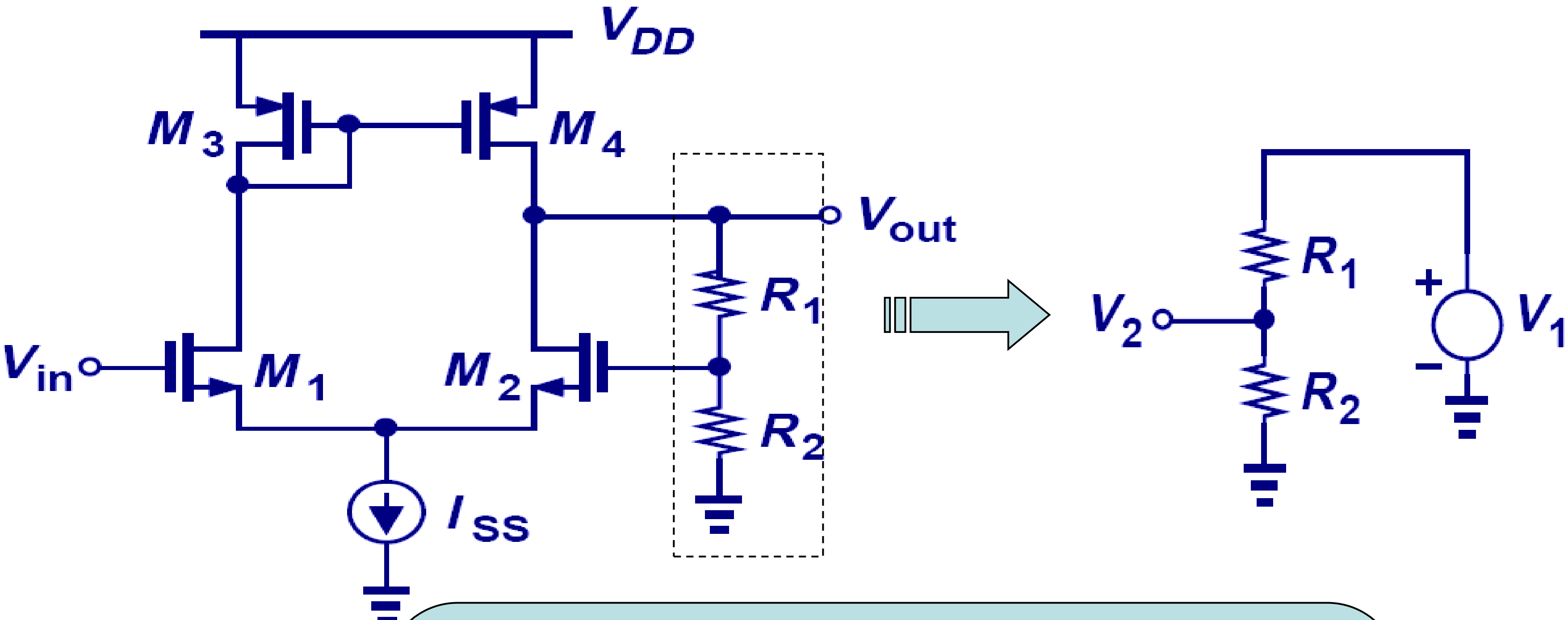


$$A_{v,open} = g_{mN}\left[r_{ON} \,\|\, r_{OP} \,\|\, (R_1 + R_2)\right]$$

$$R_{in,open} = \infty$$

$$R_{out,open} = r_{ON} \,\|\, r_{OP} \,\|\, (R_1 + R_2)$$

Feedback Factor Example II



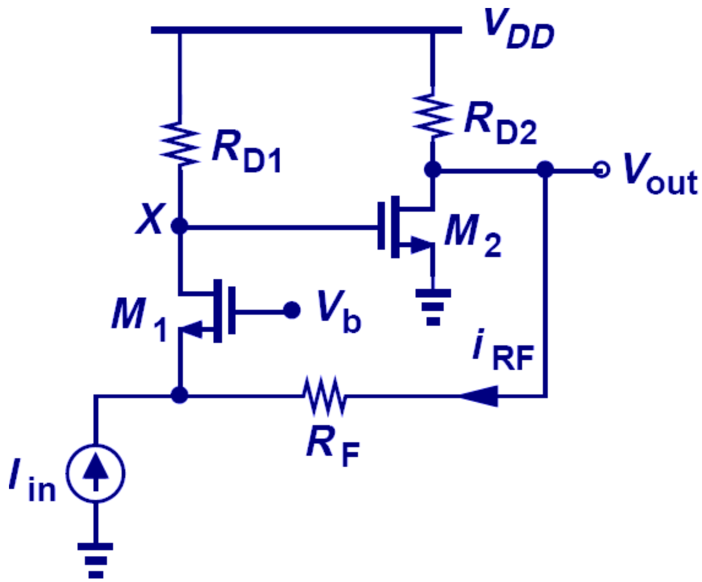
$$K = R_2/(R_1 + R_2)$$

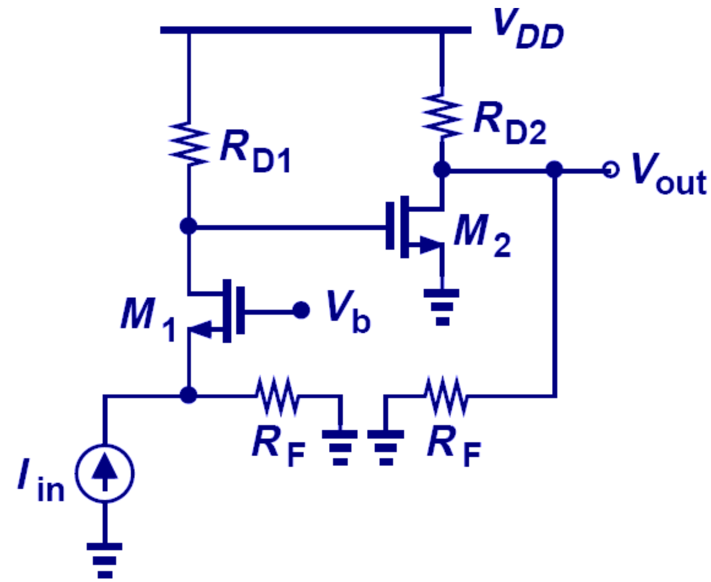
$$A_{v,closed} = A_{v,open}/(1 + KA_{v,open})$$

$$R_{in,closed} = \infty$$

$$R_{out,closed} = R_{out,open}/(1 + KA_{v,open})$$

Breaking the Loop Example IV



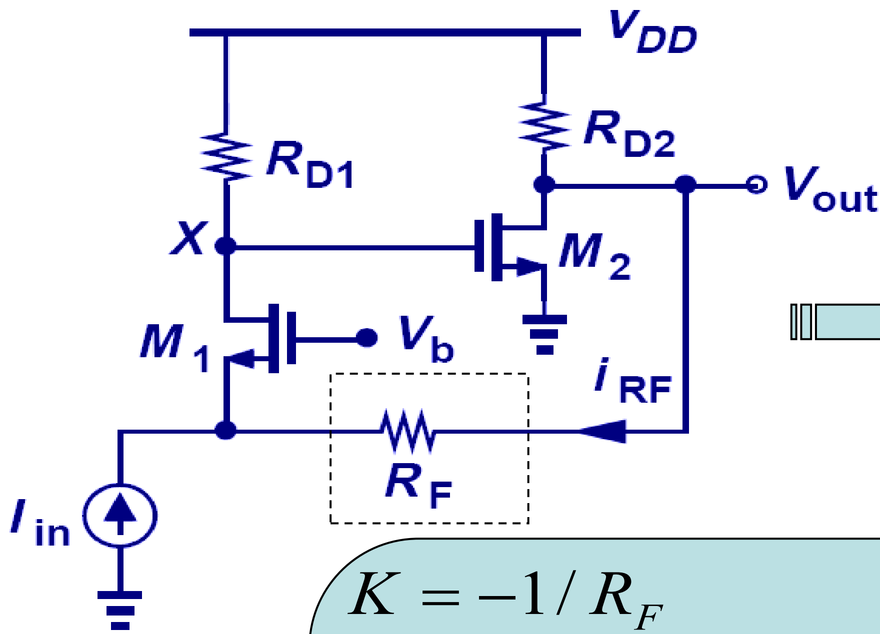


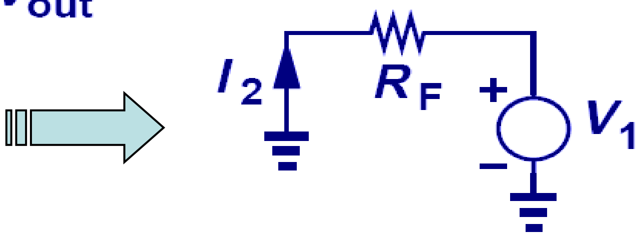
$$\frac{V_{out}}{I_{in}}|_{open} = \frac{R_F R_{D1}}{R_F + \frac{1}{g_{m1}}} . [-g_{m2}(R_{D2} \parallel R_F)]$$

$$R_{in,open} = \frac{1}{g_{m1}} \parallel R_F$$

$$R_{out,open} = R_{D2} \parallel R_F$$

Feedback Factor Example IV





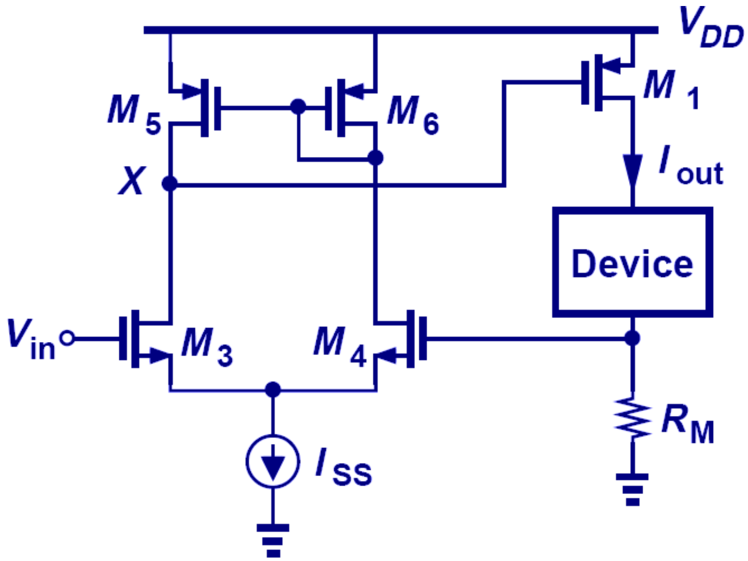
$$K = -1/R_F$$

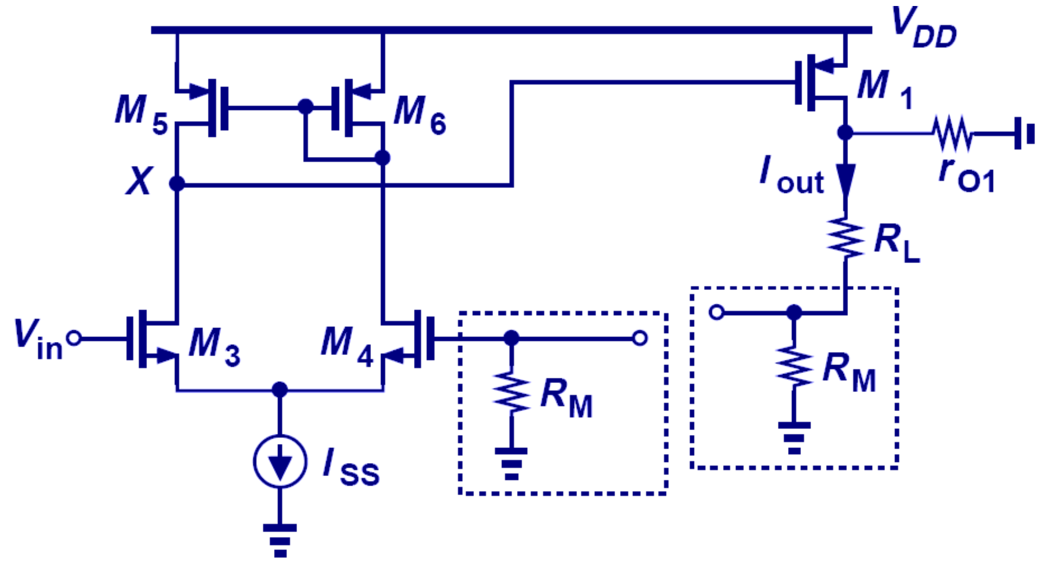
$$\frac{V_{out}}{I_{in}}\bigg|_{closed} = \frac{V_{out}}{I_{in}}\bigg|_{open} \Big/ \left(1 + K\frac{V_{out}}{I_{in}}\bigg|_{open}\right)$$

$$R_{in,closed} = R_{in,open} \Big/ \left(1 + K\frac{V_{out}}{I_{in}}\bigg|_{open}\right)$$

$$R_{out,closed} = R_{out,open} \Big/ \left(1 + K\frac{V_{out}}{I_{in}}\bigg|_{open}\right)$$

Breaking the Loop Example V



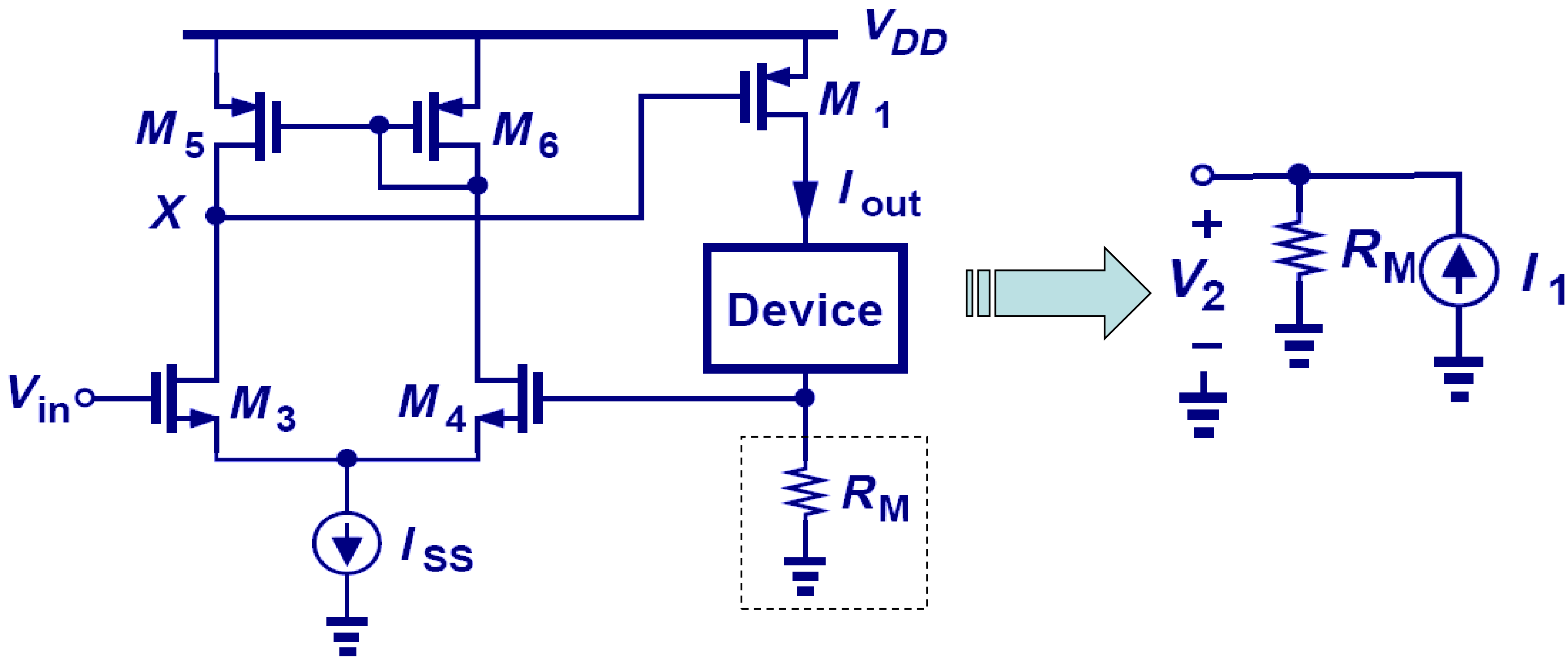


$$\frac{I_{out}}{V_{in}}\bigg|_{open} = \frac{g_{m3}(r_{O3} \parallel r_{O5})g_{m1}r_{O1}}{r_{O1} + R_L + R_M}$$

$$R_{in,open} = \infty$$

$$R_{out,open} = r_{O1} + R_M$$

Feedback Factor Example V



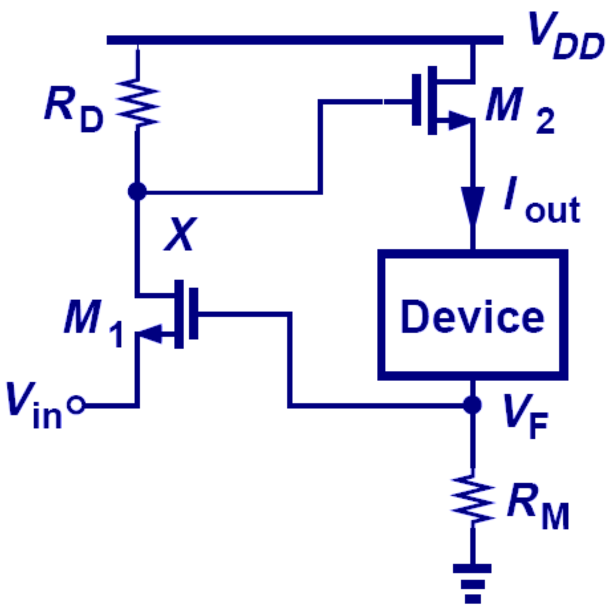
$$K = R_M$$

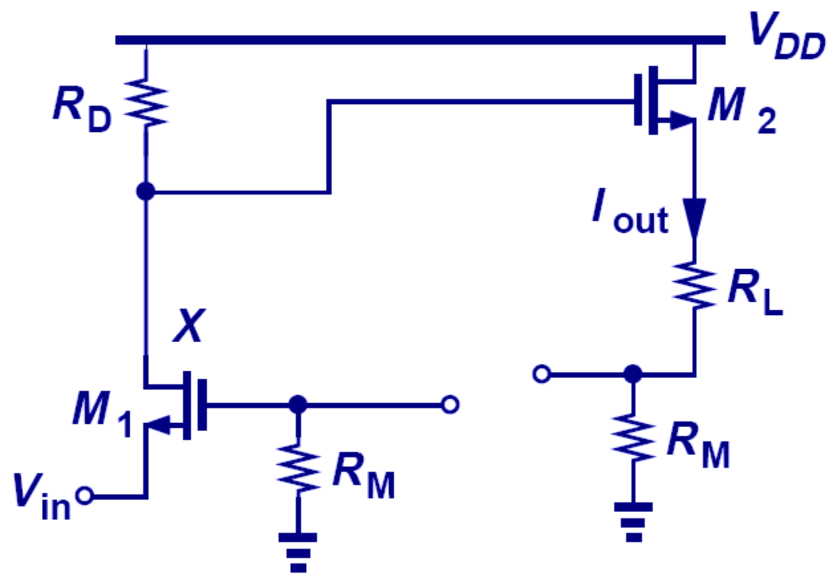
$$(I_{out} / V_{in} |_{closed}) = (I_{out} / V_{in} |_{open}) / [1 + K(I_{out} / V_{in}) |_{open}]$$

$$R_{in,closed} = \infty$$

$$R_{out,closed} = R_{out,open} [1 + K(I_{out} / V_{in}) |_{open}]$$

Breaking the Loop Example VI



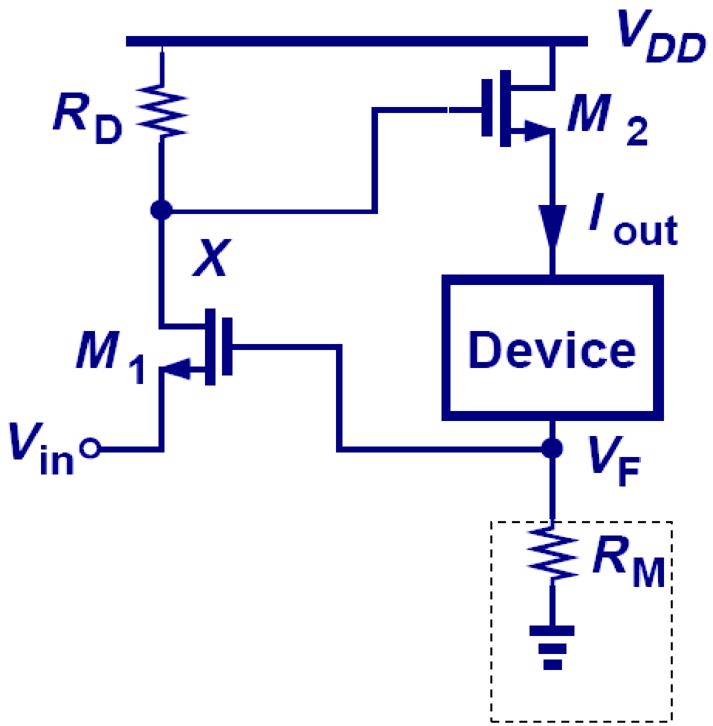


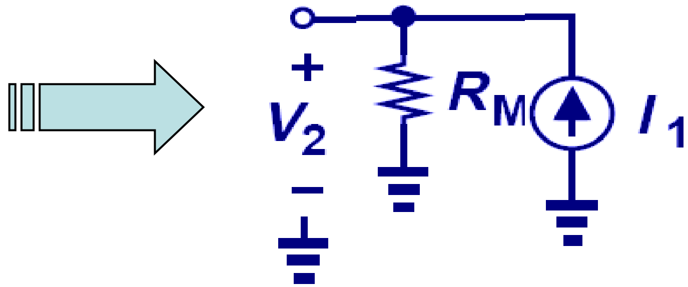
$$\frac{I_{out}}{V_{in}}\bigg|_{open} = \frac{g_{m1}R_D}{R_L + R_M + 1/g_{m2}}$$

$$R_{in,open} = 1/g_{m1}$$

$$R_{out,open} = (1/g_{m2}) + R_M$$

Feedback Factor Example VI





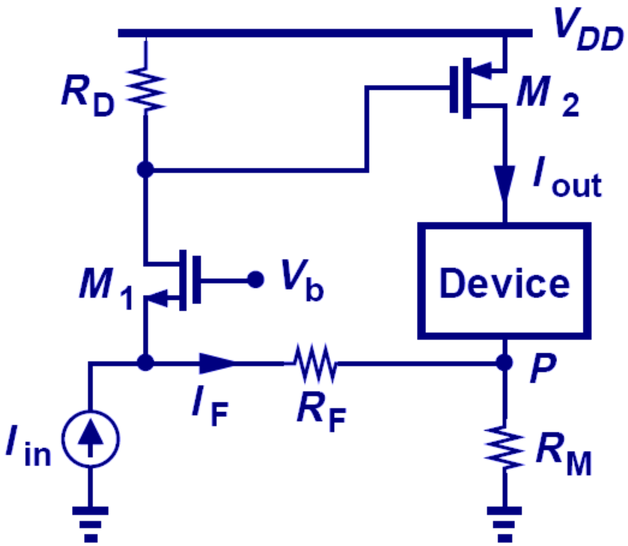
$$K = R_M$$

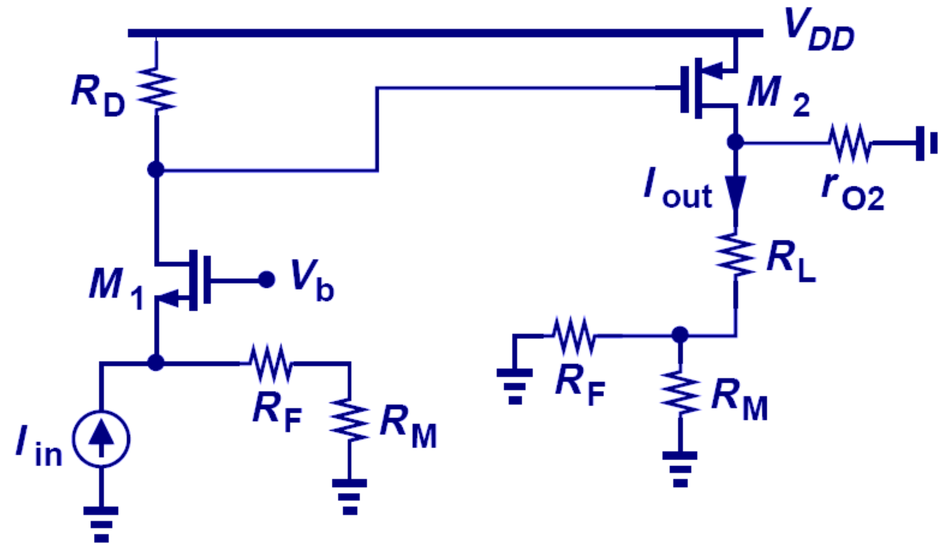
$$(I_{out} / V_{in}|_{closed}) = (I_{out} / V_{in}|_{open}) / [1 + K(I_{out} / V_{in})|_{open}]$$

$$R_{in,closed} = R_{in,open}[1 + K(I_{out} / V_{in})|_{open}]$$

$$R_{out,closed} = R_{out,open}[1 + K(I_{out} / V_{in})|_{open}]$$

Breaking the Loop Example VII



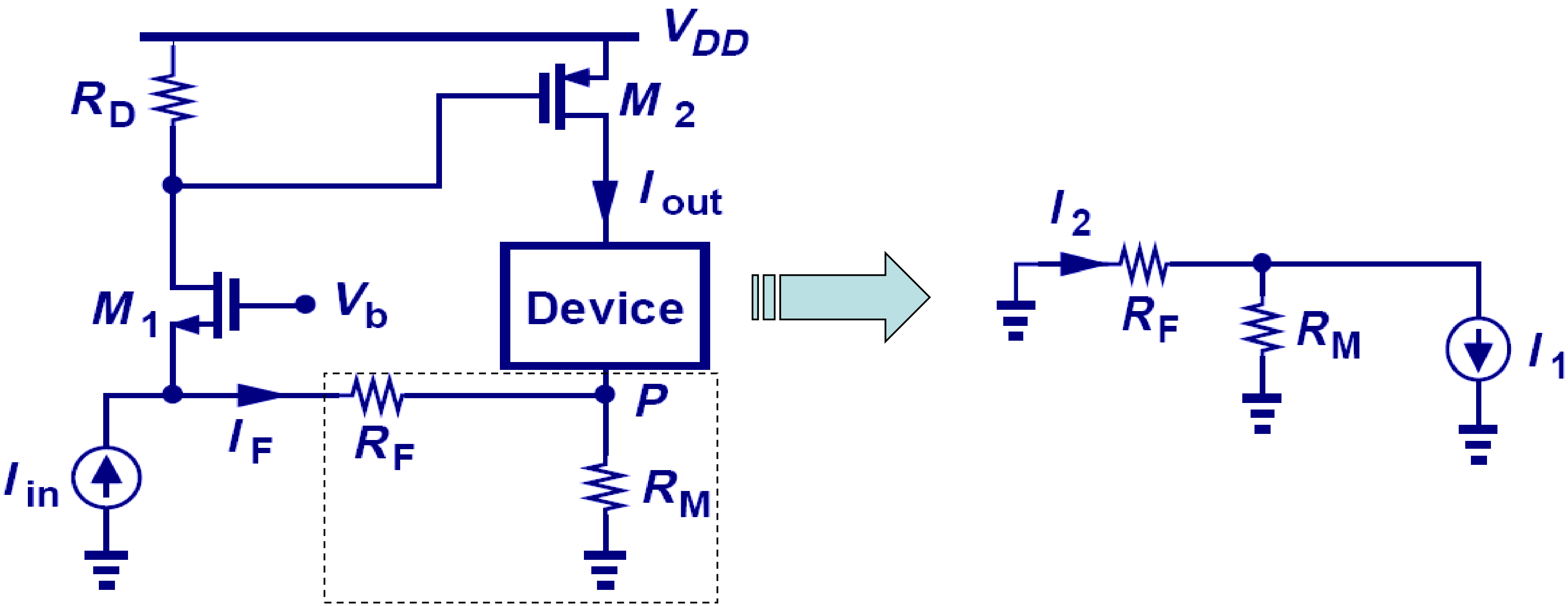


$$A_{I,open} = \frac{(R_F + R_M)R_D}{R_F + R_M + \dfrac{1}{g_{m1}}} \cdot \frac{-g_{m2} r_{O2}}{r_{O2} + R_L + R_M \parallel R_F}$$

$$R_{in,open} = \frac{1}{g_{m1}} \parallel (R_F + R_M)$$

$$R_{out,open} = r_{O2} + R_F \parallel R_M$$

Feedback Factor Example VII



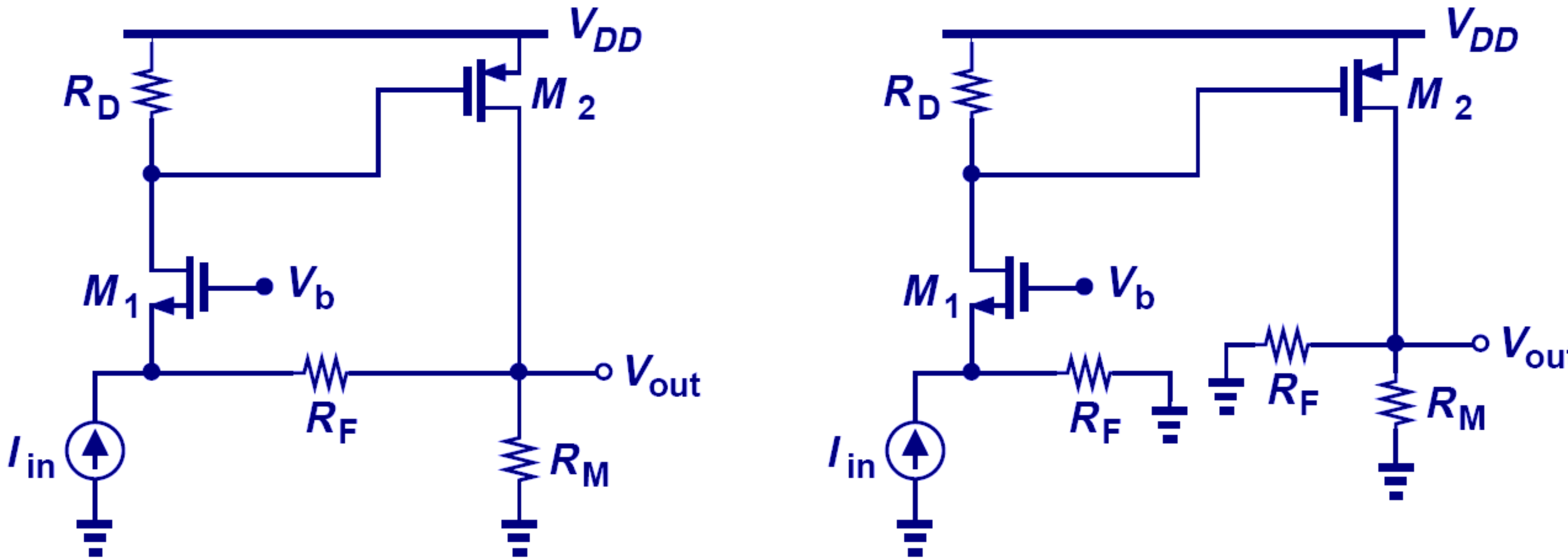
$$K = -R_M / (R_F + R_M)$$

$$A_{I,closed} = A_{I,open} / (1 + KA_{I,open})$$

$$R_{in,closed} = R_{in,open} / (1 + KA_{I,open})$$

$$R_{out,closed} = R_{out,open} (1 + KA_{I,open})$$

Breaking the Loop Example VIII

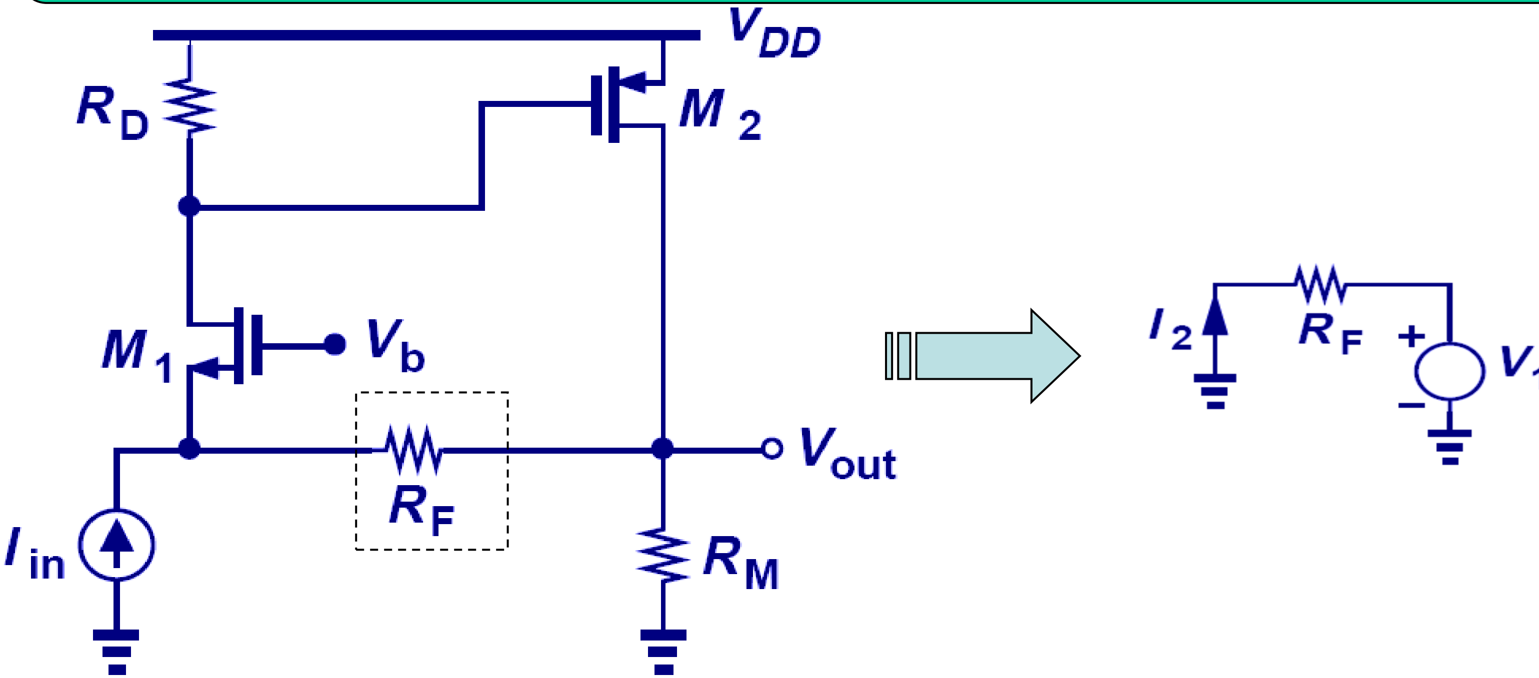


$$\frac{V_{out}}{I_{in}}\Big|_{open} = \frac{R_F R_D}{R_F + 1/g_{m1}}[-g_{m2}(R_F \parallel R_M)]$$

$$R_{in,open} = \frac{1}{g_{m1}} \parallel R_F$$

$$R_{out,open} = R_F \parallel R_M$$

Feedback Factor Example VIII



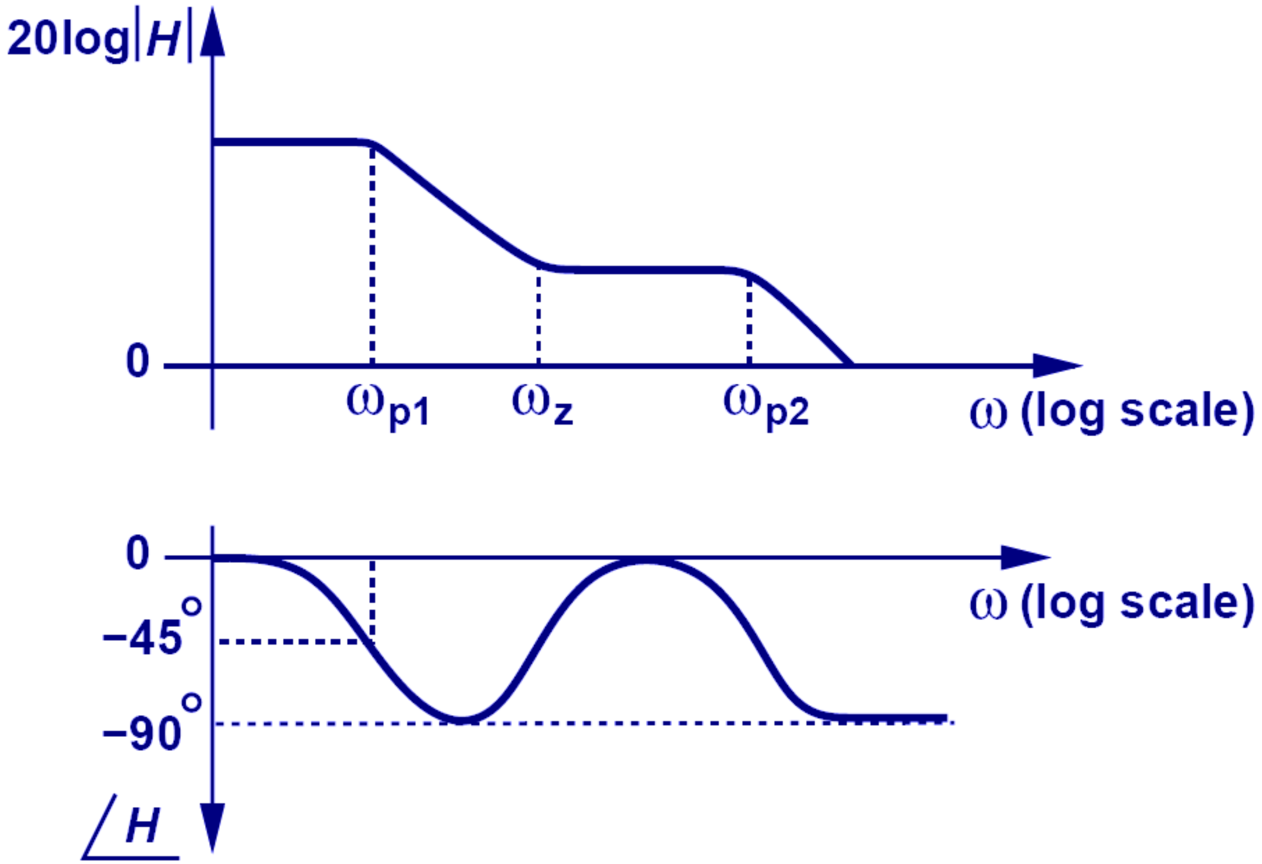
$$K = -1/R_F$$

$$(V_{out}/I_{in})|_{closed} = (V_{out}/I_{in})|_{open} / [1 + K(V_{out}/I_{in})|_{open}]$$

$$R_{in,closed} = R_{in,open} / [1 + K(V_{out}/I_{in})|_{open}]$$

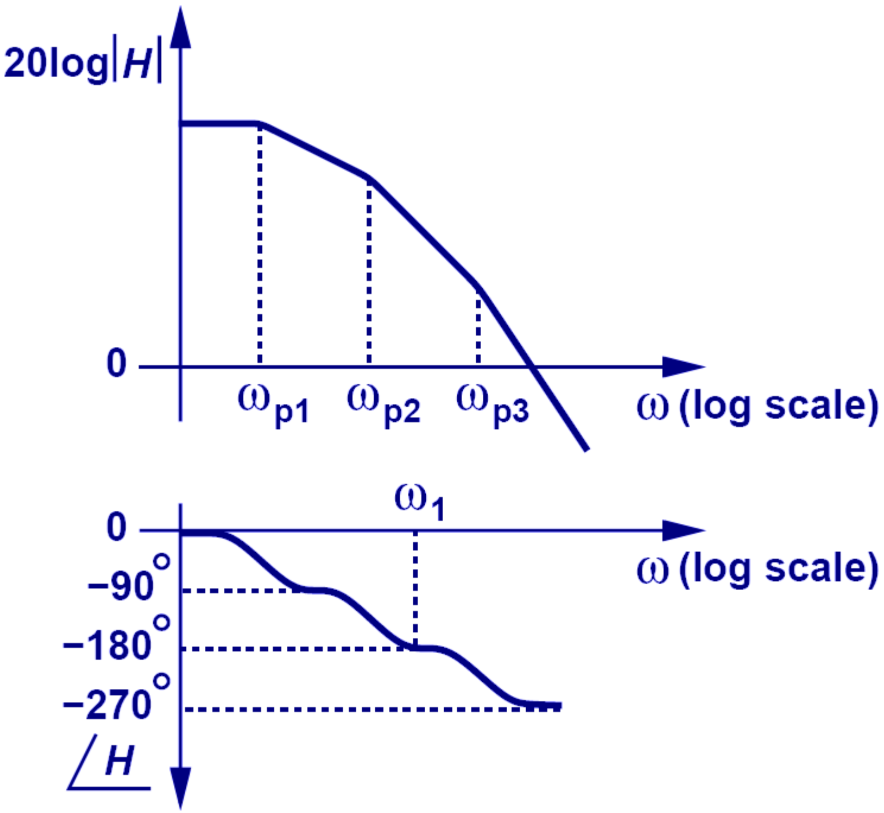
$$R_{out,closed} = R_{out,open} / [1 + K(V_{out}/I_{in})|_{open}]$$

Example: Phase Response



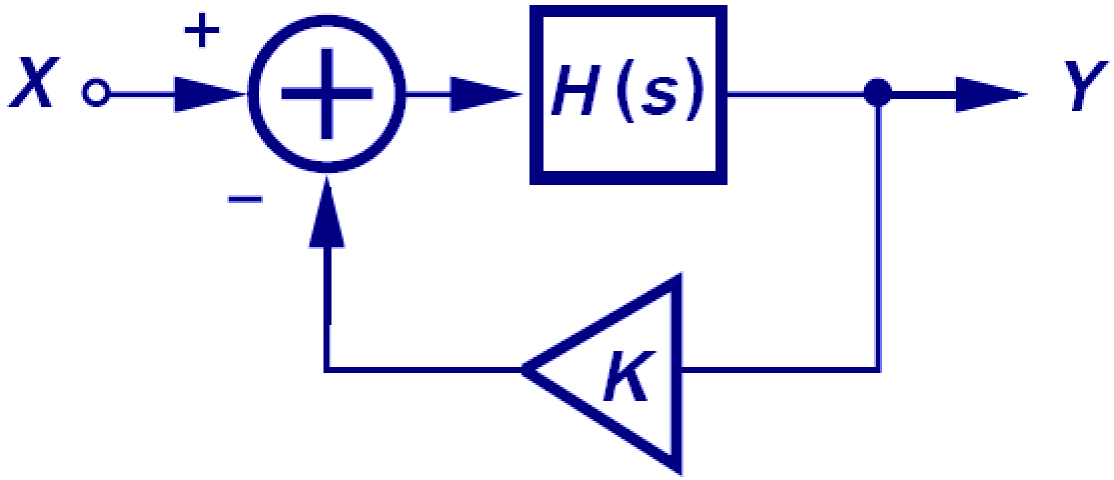
- As it can be seen, the phase of H(jω) starts to drop at 1/10 of the pole, hits -45° at the pole, and approaches -90° at 10 times the pole.

Example: Three-Pole System



- For a three-pole system, a finite frequency produces a phase of -180°, which means an input signal that operates at this frequency will have its output inverted.

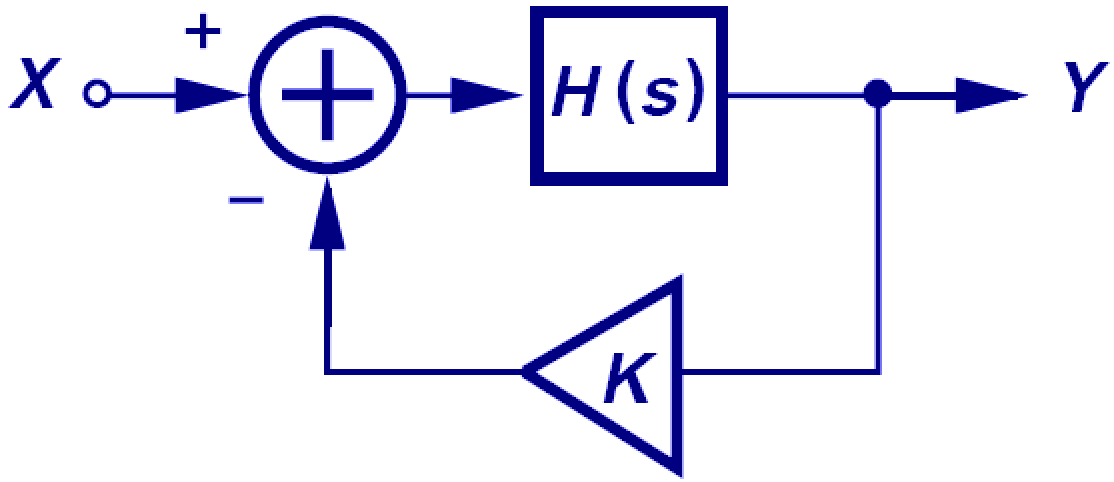
Instability of a Negative Feedback Loop



$$\frac{Y}{X}(s) = \frac{H(s)}{1+KH(s)}$$

- Substitute $j\omega$ for s. If for a certain ω_1, $KH(j\omega_1)$ reaches -1, the closed loop gain becomes infinite. This implies for a very small input signal at ω_1, the output can be very large. Thus the system becomes unstable.

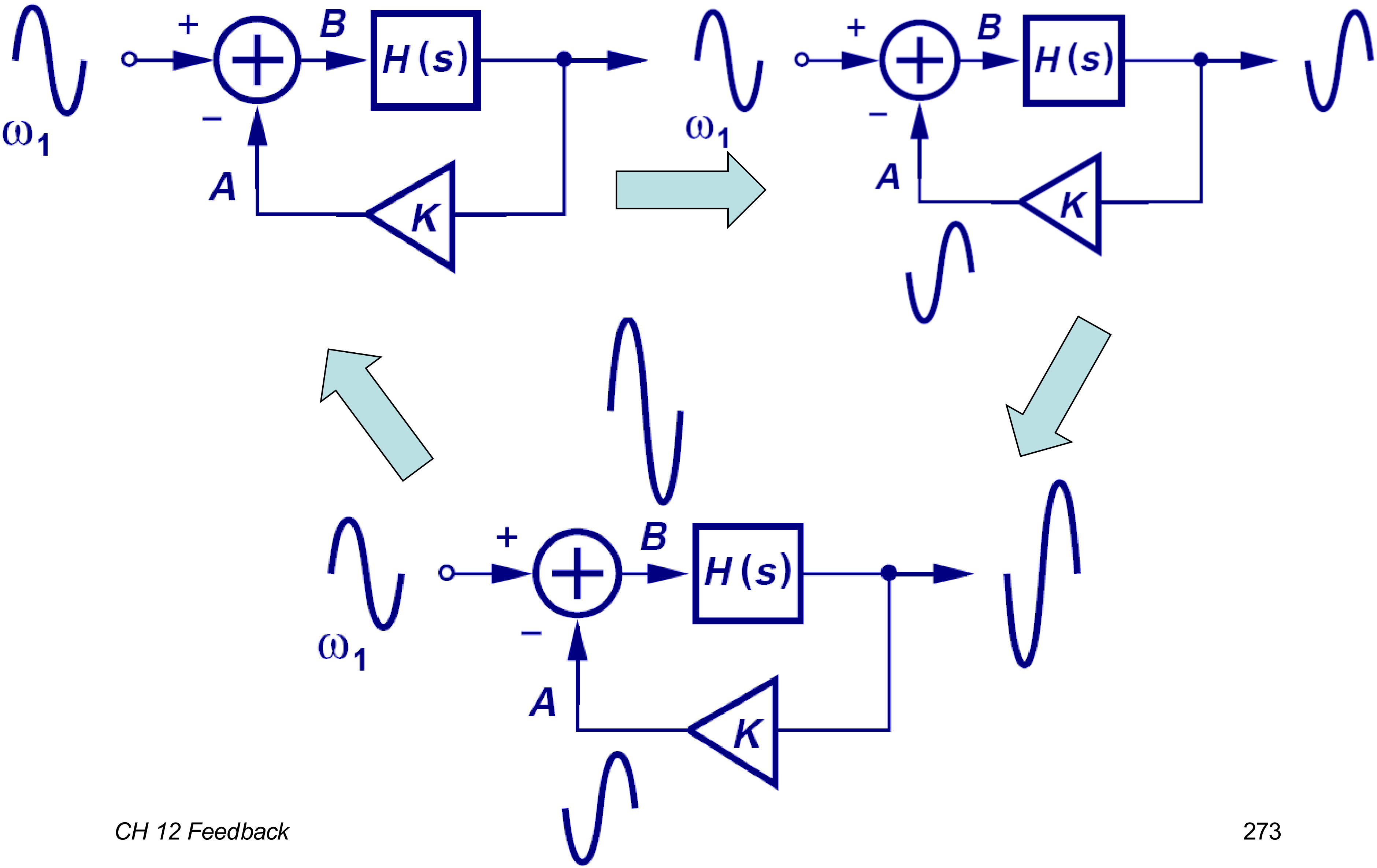
"Barkhausen's Criteria" for Oscillation



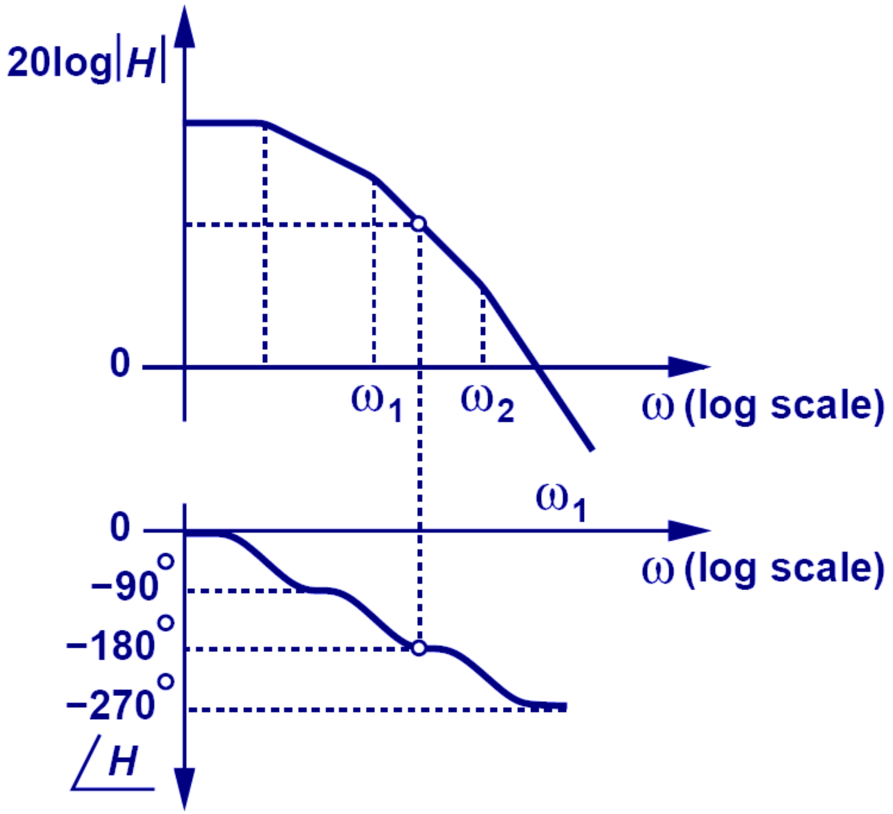
$$|KH(j\omega_1)| = 1$$

$$\angle KH(j\omega_1) = -180$$

Time Evolution of Instability



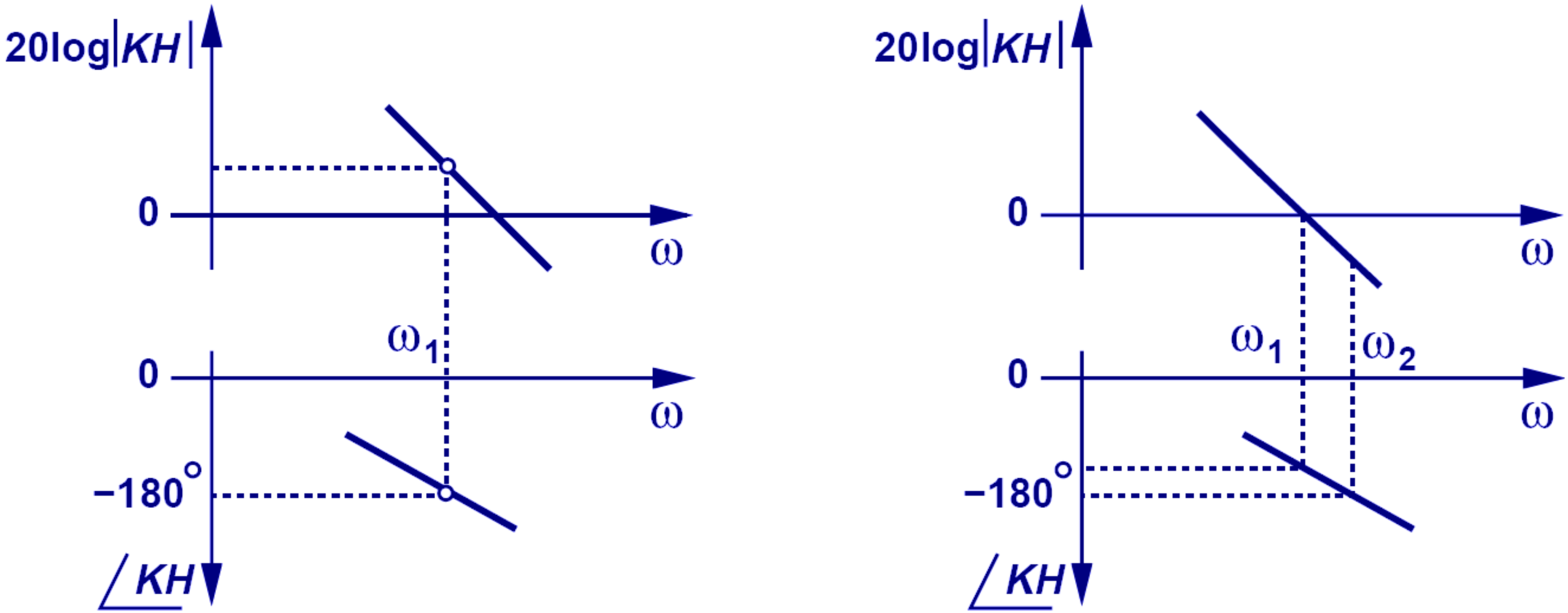
Oscillation Example



(c)

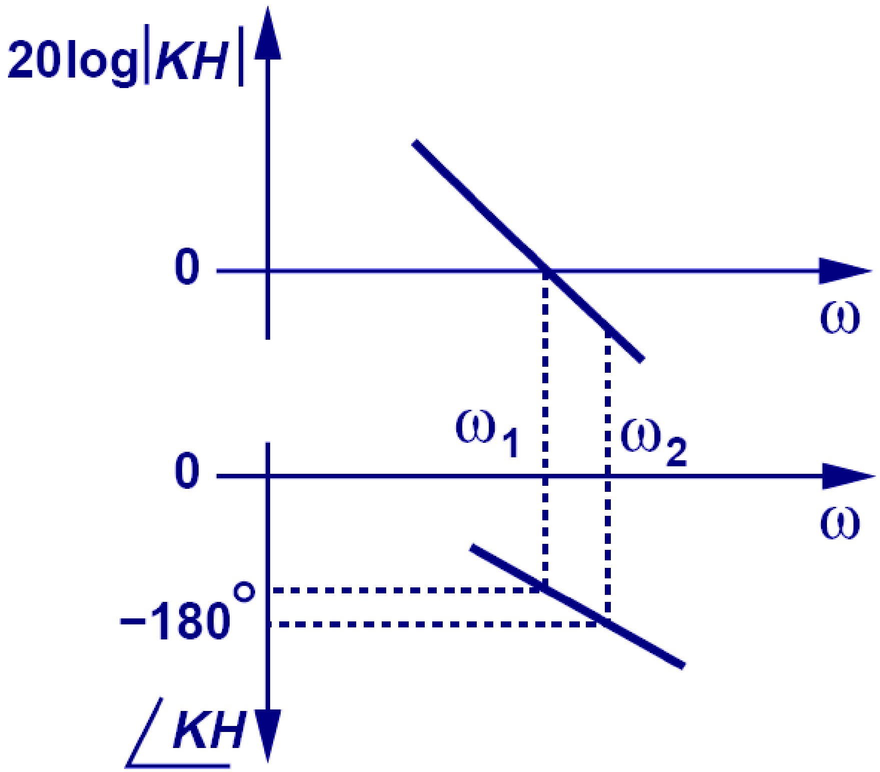
- This system oscillates, since there's a finite frequency at which the phase is -180° and the gain is greater than unity. In fact, this system exceeds the minimum oscillation requirement.

Condition for Oscillation



- Although for both systems above, the frequencies at which |KH|=1 and ∠KH=-180° are different, the system on the left is still unstable because at ∠KH=-180°, |KH|>1. Whereas the system on the right is stable because at ∠KH=-180°, |KH|<1.

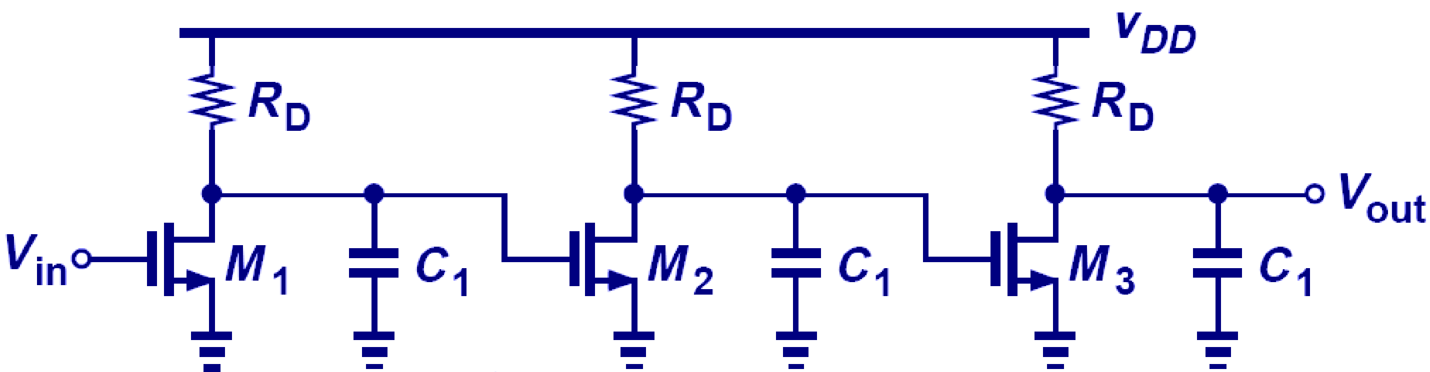
Condition for Stability

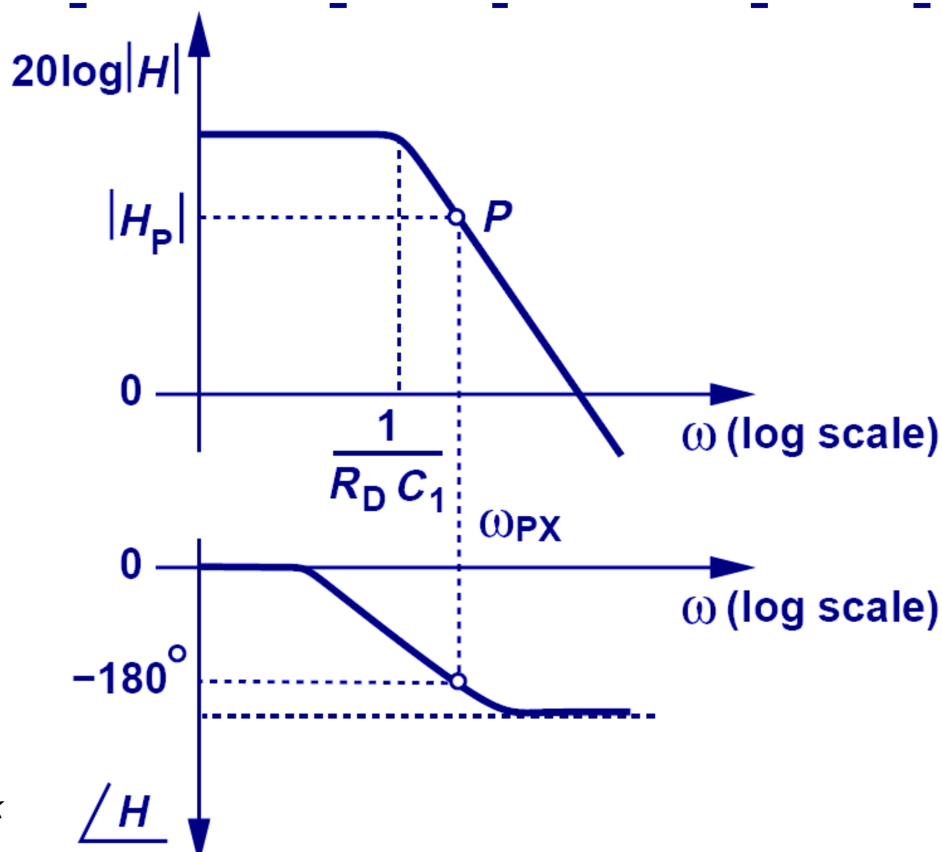


$$\omega_{GX} < \omega_{PX}$$

- **ω_{PX}, (“phase crossover”), is the frequency at which $\angle KH = -180^o$.**
- **ω_{GX}, (“gain crossover”), is the frequency at which $|KH|=1$.**

Stability Example I

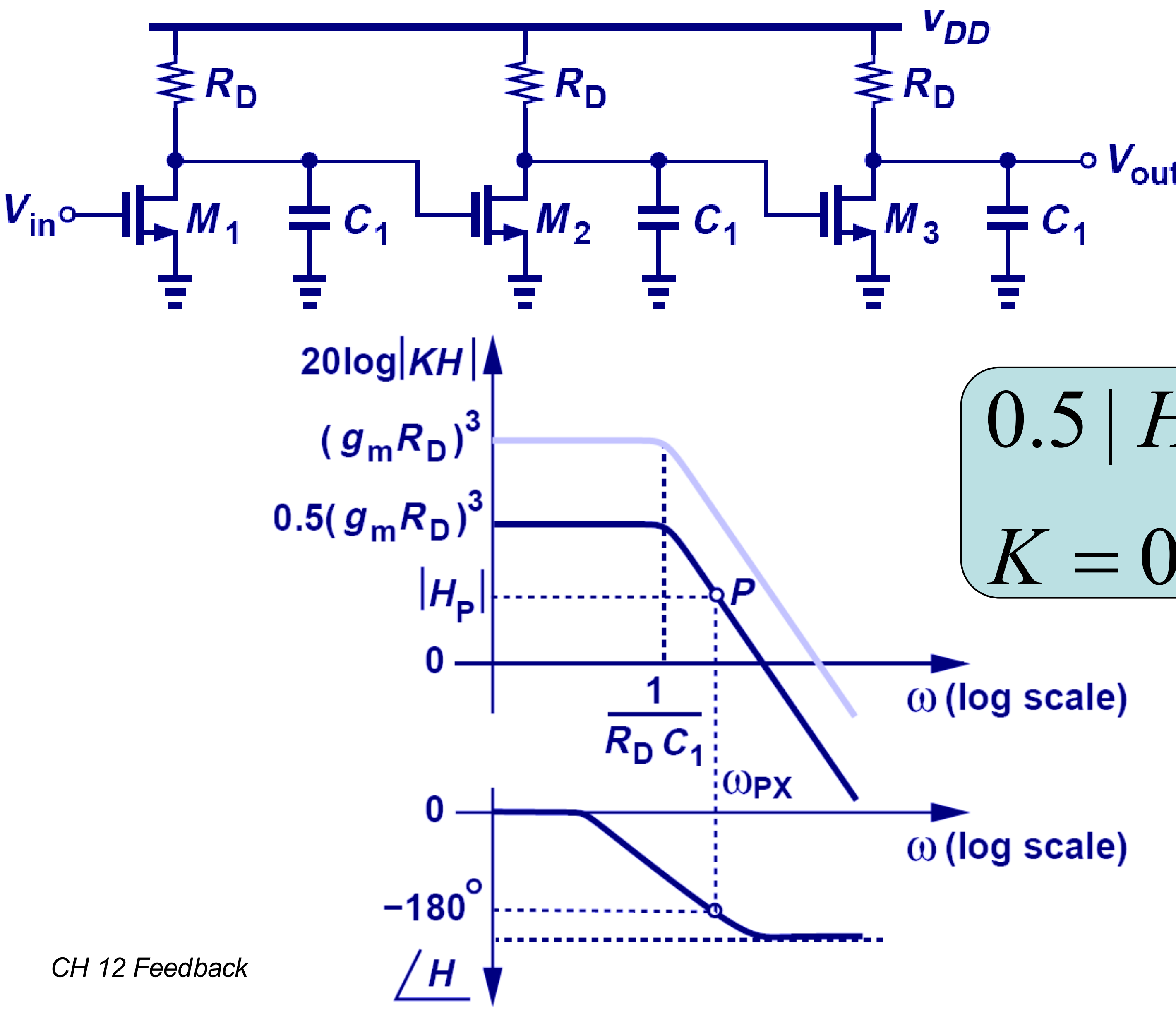




$$|H_p| < 1$$

$$K = 1$$

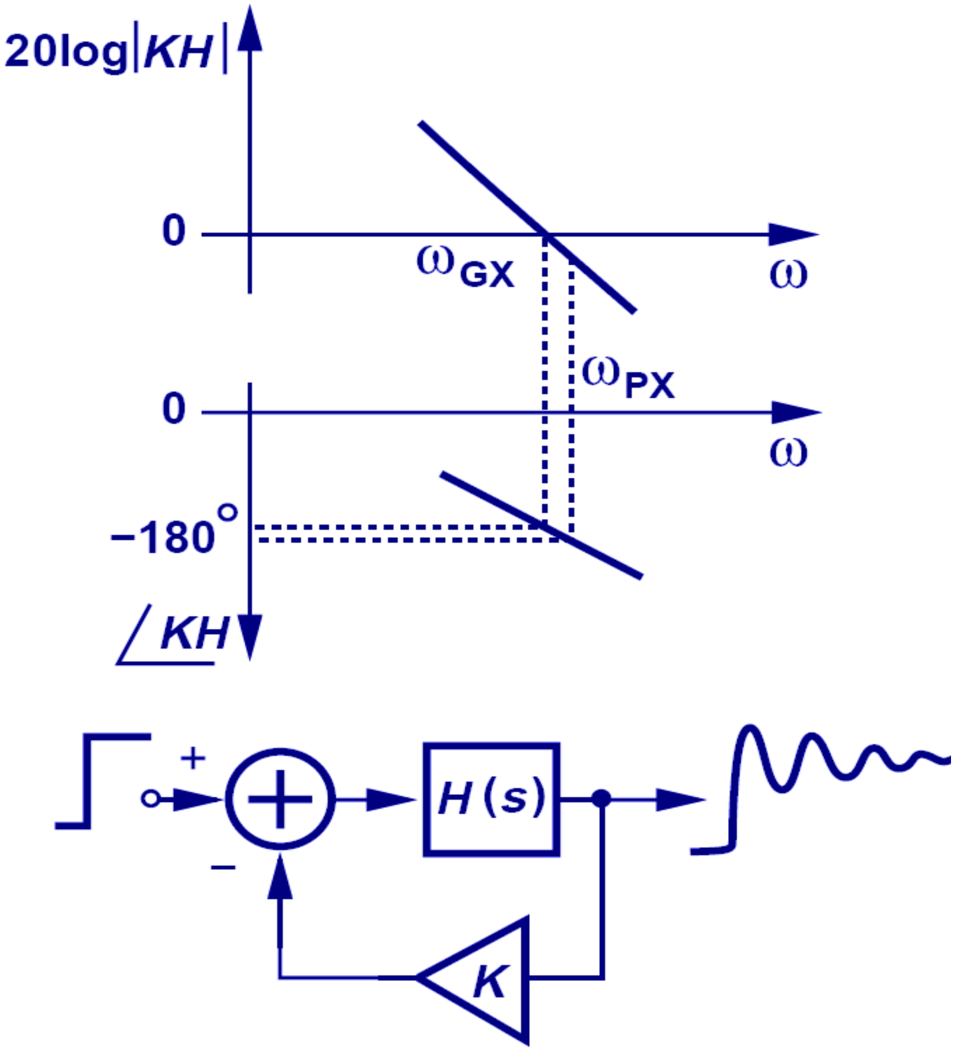
Stability Example II



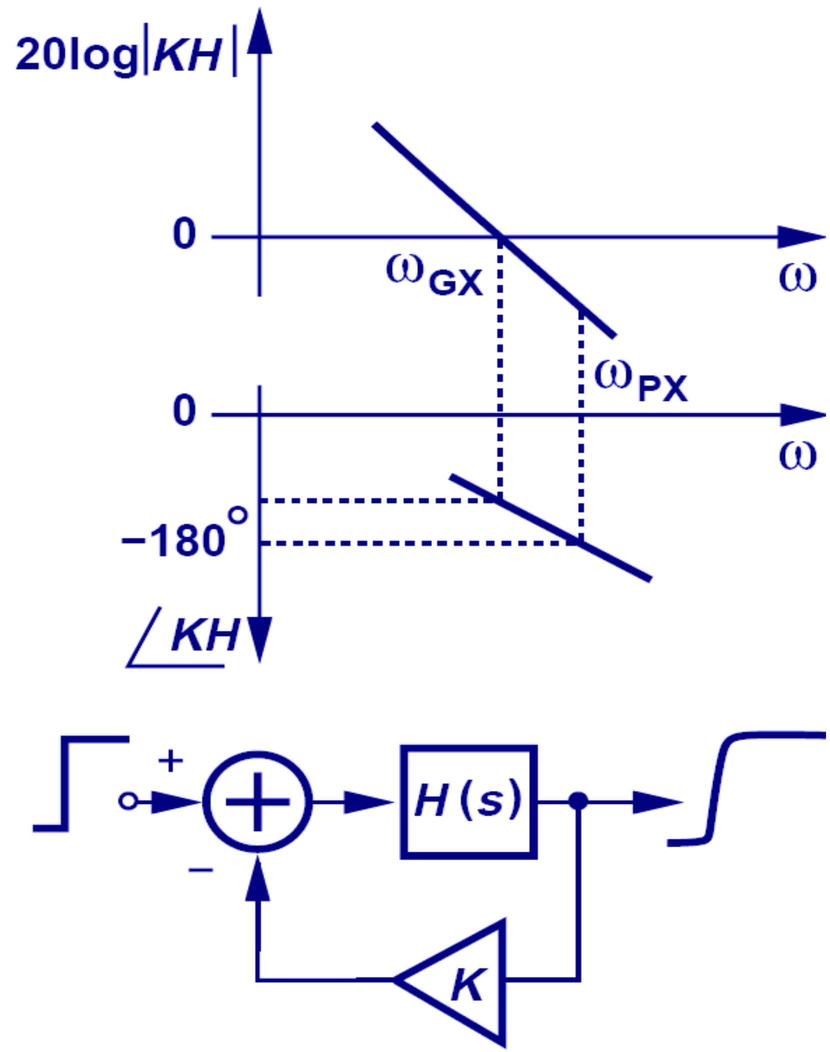
$$0.5|H_p| < 1$$

$$K = 0.5$$

Marginally Stable vs. Stable



Marginally Stable



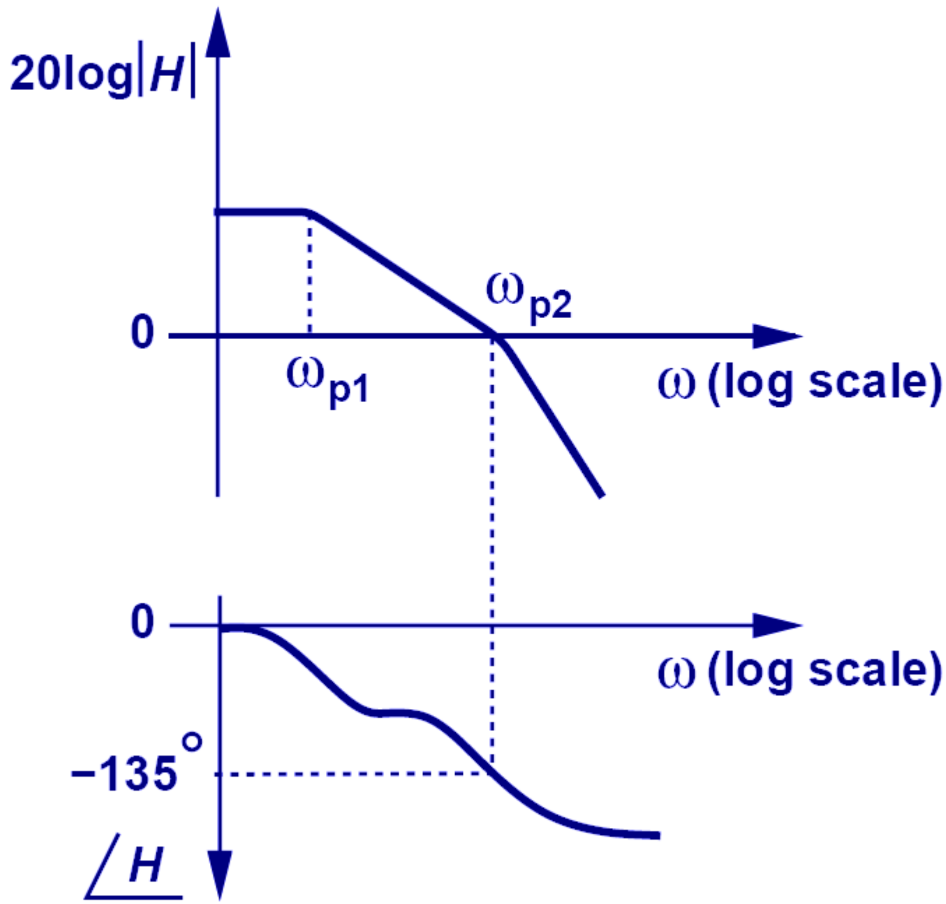
Stable

Phase Margin

- **Phase Margin = $\angle H(\omega_{GX})+180$**

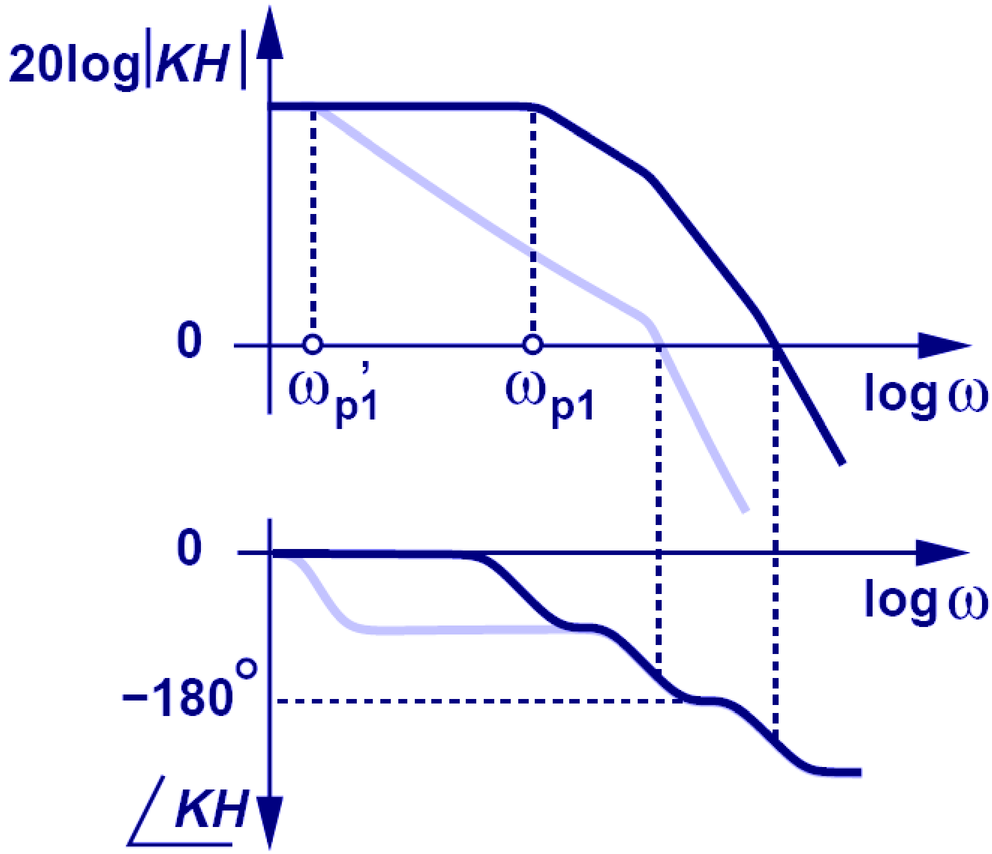
- **The larger the phase margin, the more stable the negative feedback becomes**

Phase Margin Example



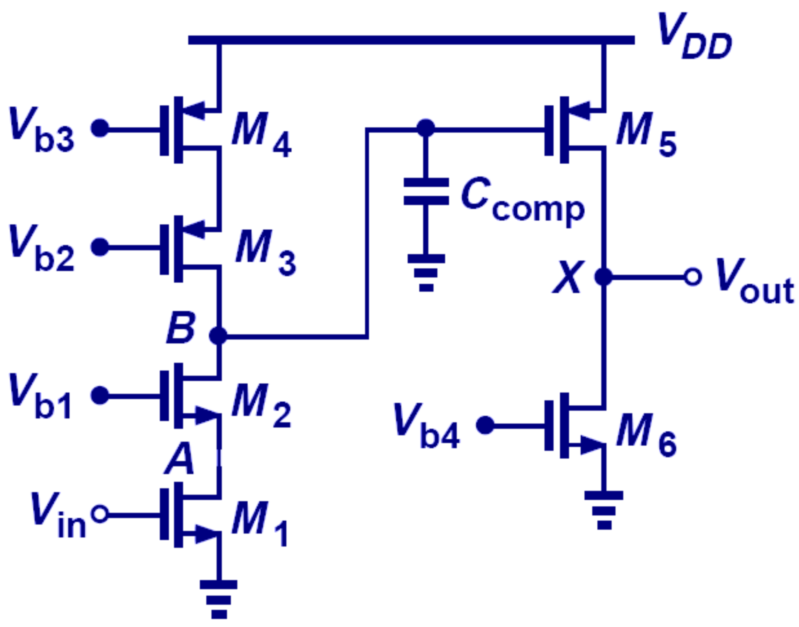
$$PM = 45^\circ$$

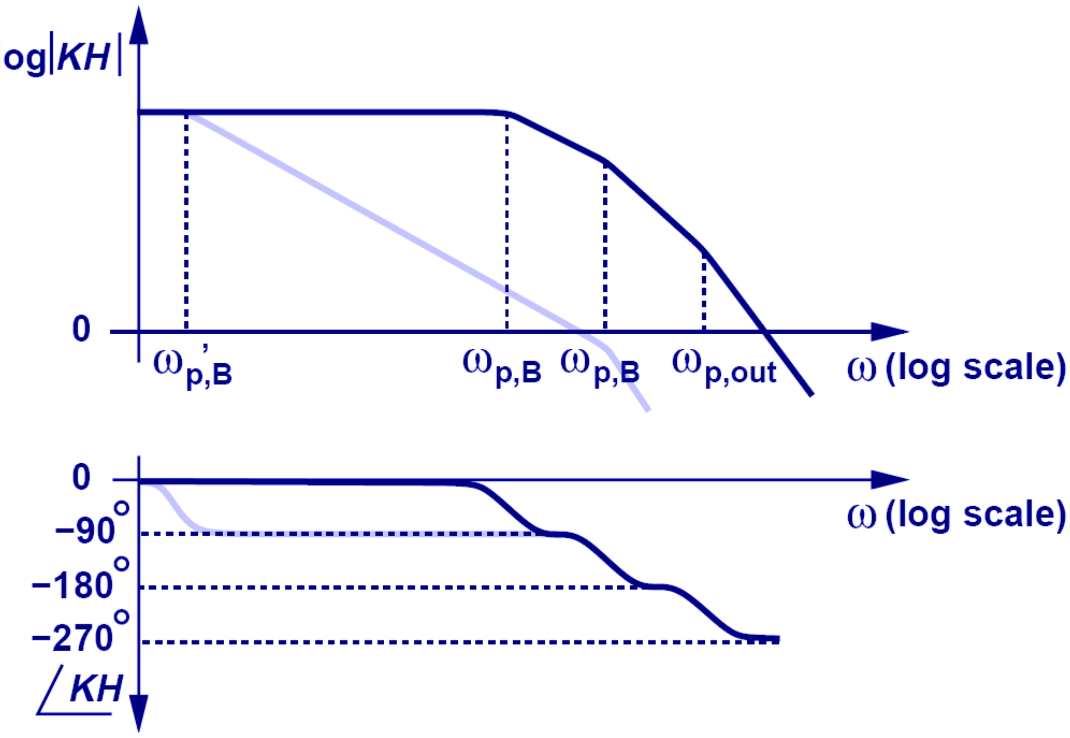
Frequency Compensation



- Phase margin can be improved by moving ω_{GX} closer to origin while maintaining ω_{PX} unchanged.

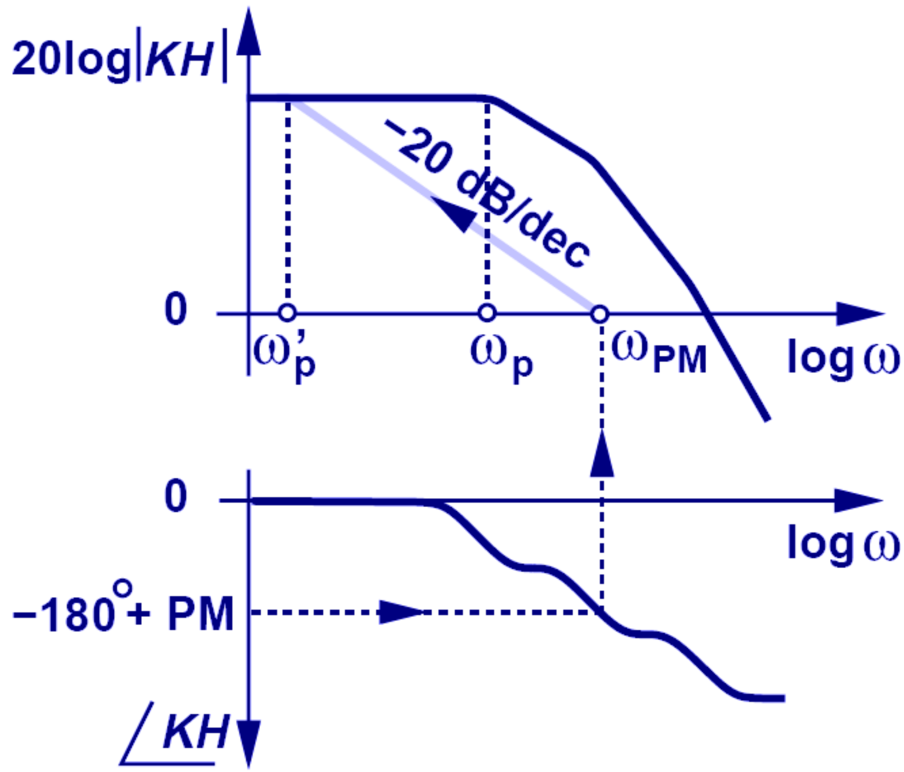
Frequency Compensation Example





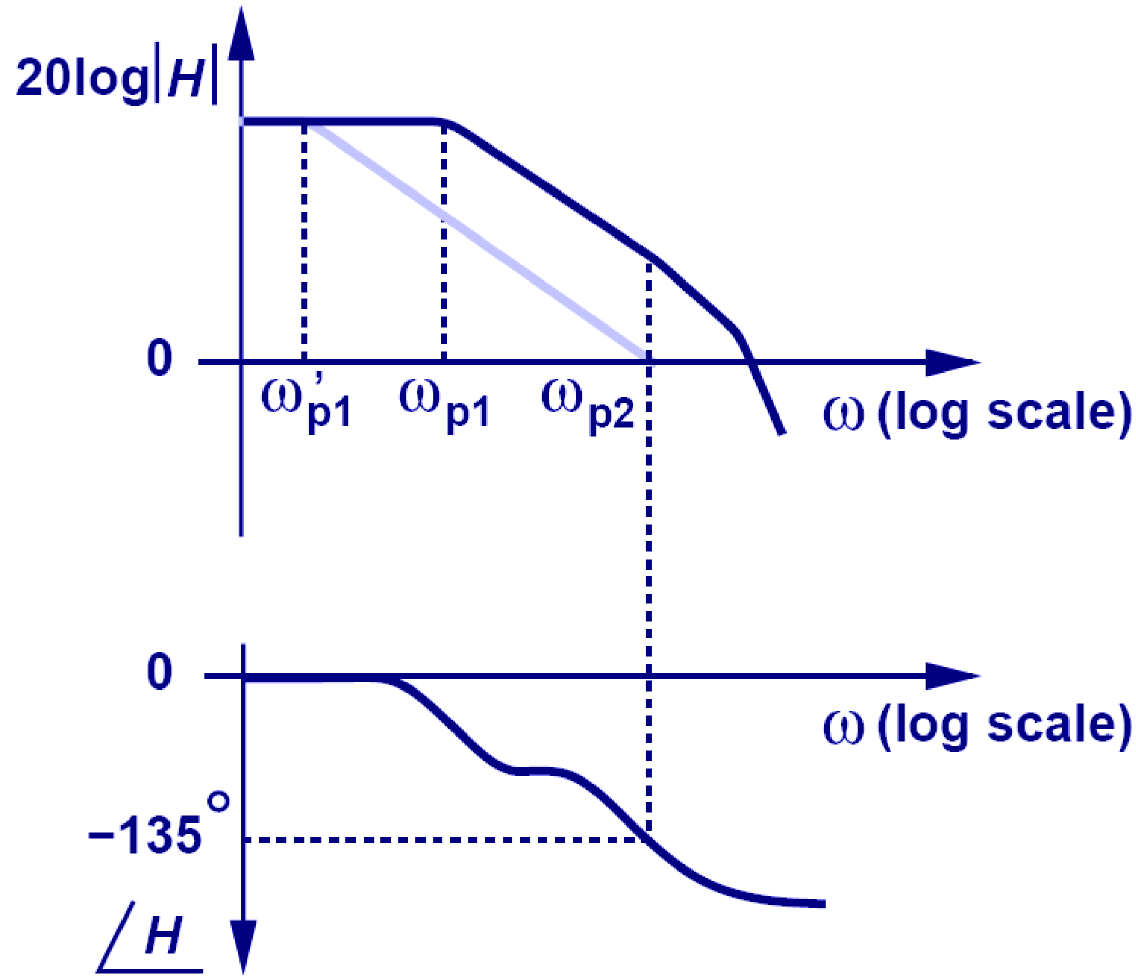
- C_{comp} is added to lower the dominant pole so that ω_{GX} occurs at a lower frequency than before, which means phase margin increases.

Frequency Compensation Procedure



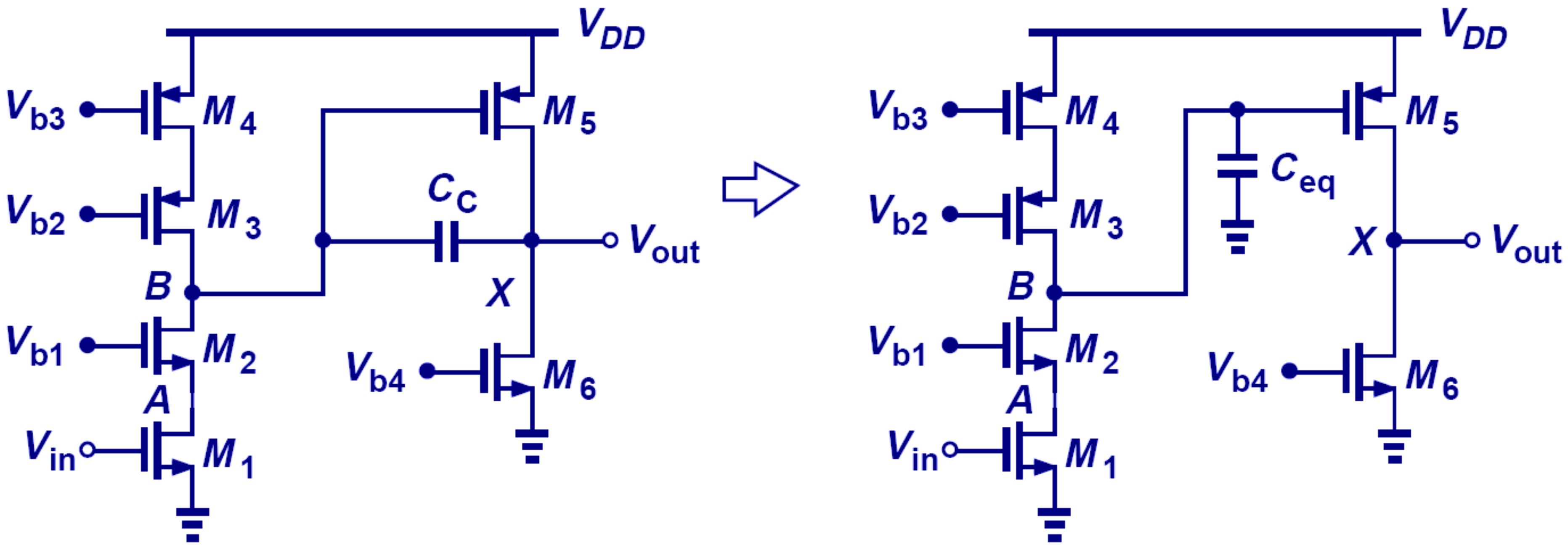
- 1) We identify a PM, then -180°+PM gives us the new ω_{GX}, or ω_{PM}.
- 2) On the magnitude plot at ω_{PM}, we extrapolate up with a slope of +20dB/dec until we hit the low frequency gain then we look "down" and the frequency we see is our new dominant pole, ω_P'.

Example: 45° Phase Margin Compensation



$$\omega_{PM} = \omega_{p2}$$

Miller Compensation



$$C_{eq} = [1 + g_{m5}(r_{O5} \parallel r_{O6})]C_c$$

- **To save chip area, Miller multiplication of a smaller capacitance creates an equivalent effect.**