1.1 Propositional Logic Examples

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Is it a proposition?

Do your homework.

Is it a proposition?

Do your homework.

NO! - It's a command. A proposition is a declarative sentence that's either true or false.

Is it a proposition?

Pigs can fly.

Is it a proposition?

Pigs can fly.

YES! The proposition is false.

Is it a proposition?

x is greater than y.

Is it a proposition?

x is greater than y.

NO! - Until we know the values of x and y, we can't say whether the statement is true or false.

What's the negation?

p: It is sunny outside.

¬p:

What's the negation?

p: It is sunny outside.

¬p: It isn't sunny outside.

What's the implication?

p: It is raining.

q: I carry my umbrella.

p → *q:*

What's the implication?

p: It is raining.

q: I carry my umbrella.

p → *q:* If it's raining, then I carry my umbrella.

What's the inverse?

p → q: If it's raining, then I carry my umbrella.

What's the inverse?

p → q: If it's raining, then I carry my umbrella.

¬p → ¬q: If it's not raining, then I don't carry my umbrella.

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What's the converse?

p → q: If it's raining, then I carry my umbrella.

What's the converse?

p → q: If it's raining, then I carry my umbrella.

q → p: If I carry my umbrella, then it's raining.

What's the contrapositive?

p → q: If it's raining, then I carry my umbrella.

What's the contrapositive?

p → q: If it's raining, then I carry my umbrella.

¬*q* → ¬*p:* If I don't carry my umbrella, then it's not raining.

What's the conjuction?

p: It is raining.

q: I carry my umbrella.

What's the conjuction?

p: It is raining.

q: I carry my umbrella.

p ∧ q: It's raining **and** I carry my umbrella.

What's the disjunction?

p: It is raining.

q: I carry my umbrella.

What's the disjunction?

p: It is raining.

q: I carry my umbrella.

p ∨ *q:* It's raining **or** I carry my umbrella.

Constructing a truth table

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

Constructing a truth table

$$((p \wedge \neg q) \leftrightarrow (p \vee q) \oplus r)$$

3 propositions (p, q, r) -> 2^3 rows = 8 rows

Constructing a truth table

$$((p \wedge \neg q) \leftrightarrow (p \vee q) \oplus r)$$

p	q	r

Constructing a truth table

$$((p \wedge \neg q) \leftrightarrow (p \vee q) \oplus r)$$

p	q	r
		F
		T
		F
		T
		F
		T
		F
		T

Constructing a truth table

$$((p \wedge \neg q) \leftrightarrow (p \vee q) \oplus r)$$

p	q	r
	F	F
	F	T
	T	F
	T	T
	F	F
	F	T
	T	F
	T	T

Constructing a truth table

$$((p \wedge \neg q) \leftrightarrow (p \vee q) \oplus r)$$

p	q	r
F	F	F
F	F	T
F	T	F
F	T	T
T	F	F
T	F	T
T	T	F
T	T	T

Constructing a truth table

$$((p \wedge \neg q) \leftrightarrow (p \vee q) \oplus r)$$

p	q	r	¬q
F	F	F	
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	

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F	F	T	
F	T	F	
F	T	T	
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p	q	r	¬q
F	F	F	T
F	F	T	T
F	T	F	F
F	T	T	F
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	F

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$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

p	q	r	¬q
F	F	F	T
F	F	T	T
F	T	F	F
F	T	T	F
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	F

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$$((p \wedge \neg q) \leftrightarrow (p \vee q) \oplus r)$$

p	q	r	¬q	p∧¬q
F	F	F	T	
F	F	T	T	
F	T	F	F	
F	T	T	F	
T	F	F	T	
T	F	T	T	
T	T	F	F	
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F	F	T	T	F
F	T	F	F	F
F	T	T	F	F
T	F	F	T	T
T	F	T	T	T
T	T	F	F	F
T	T	T	F	F

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F	F	T	T	F	
F	T	F	F	F	
F	T	T	F	F	
T	F	F	T	T	
T	F	T	T	T	
T	T	F	F	F	
T	T	T	F	F	

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T	F	T	T	T	
T	T	F	F	F	
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F	F	T	T	F	F
F	T	F	F	F	T
F	T	T	F	F	T
T	F	F	T	T	T
T	F	T	T	T	T
T	T	F	F	F	T
T	T	T	F	F	T

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F	F	T	T	F	F
F	T	F	F	F	T
F	T	T	F	F	T
T	F	F	T	T	T
T	F	T	T	T	T
T	T	F	F	F	T
T	T	T	F	F	T

Constructing a truth table

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

p	q	r	¬q	p∧¬q	p∨q	(p∧¬q)↔(p∨q)
F	F	F	T	F	F	
F	F	T	T	F	F	
F	T	F	F	F	T	
F	T	T	F	F	T	
T	F	F	T	T	T	
T	F	T	T	T	T	
T	T	F	F	F	T	
T	T	T	F	F	T	

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F	F	T	T	F	F	
F	T	F	F	F	T	
F	T	T	F	F	T	
T	F	F	T	T	T	
T	F	T	T	T	T	
T	T	F	F	F	T	
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F	F	T	T	F	F	T
F	T	F	F	F	T	F
F	T	T	F	F	T	F
T	F	F	T	T	T	T
T	F	T	T	T	T	T
T	T	F	F	F	T	F
T	T	T	F	F	T	F

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F	T	F	F	F	T	F
F	T	T	F	F	T	F
T	F	F	T	T	T	T
T	F	T	T	T	T	T
T	T	F	F	F	T	F
T	T	T	F	F	T	F

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F	F	F	T	F	F	T	
F	F	T	T	F	F	T	
F	T	F	F	F	T	F	
F	T	T	F	F	T	F	
T	F	F	T	T	T	T	
T	F	T	T	T	T	T	
T	T	F	F	F	T	F	
T	T	T	F	F	T	F	

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F	F	F	T	F	F	T	
F	F	T	T	F	F	T	
F	T	F	F	F	T	F	
F	T	T	F	F	T	F	
T	F	F	T	T	T	T	
T	F	T	T	T	T	T	
T	T	F	F	F	T	F	
T	T	T	F	F	T	F	

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F	F	F	T	F	F	T	T
F	F	T	T	F	F	T	F
F	T	F	F	F	T	F	F
F	T	T	F	F	T	F	T
T	F	F	T	T	T	T	T
T	F	T	T	T	T	T	F
T	T	F	F	F	T	F	F
T	T	T	F	F	T	F	T

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F	F	F	T	F	F	T	T
F	F	T	T	F	F	T	F
F	T	F	F	F	T	F	F
F	T	T	F	F	T	F	T
T	F	F	T	T	T	T	T
T	F	T	T	T	T	T	F
T	T	F	F	F	T	F	F
T	T	T	F	F	T	F	T