1.1 Propositional Logic Examples

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Do your homework.

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NO! - It's a command. A proposition is a declarative sentence that's either true or false.

Pigs can fly.

Pigs can fly.

YES! The proposition is false.

x is greater than y.

x is greater than y.

NO! - Until we know the values of *x* and *y*, we can't say whether the statement is true or false.

What's the negation?

p: It is sunny outside.

¬р:

What's the negation?

p: It is sunny outside.

¬p: It isn't sunny outside.

What's the implication?

p: It is raining.

q: I carry my umbrella.

$$p \rightarrow q$$
:

What's the implication?

p: It is raining.

q: I carry my umbrella.

 $p \rightarrow q$: If it's raining, then I carry my umbrella.

What's the inverse?

p → q: If it's raining, then I carry my umbrella.

What's the inverse?

p → q: If it's raining, then I carry my umbrella.

¬p → ¬q: If it's not raining, then I don't carry my umbrella.

What's the converse?

p → q: If it's raining, then I carry my umbrella.

What's the converse?

p → q: If it's raining, then I carry my umbrella.

q → p: If I carry my umbrella, then it's raining.

What's the contrapositive?

p → q: If it's raining, then I carry my umbrella.

What's the contrapositive?

p → q: If it's raining, then I carry my umbrella.

 $\neg q \rightarrow \neg p$: If I don't carry my umbrella, then it's not raining.

What's the conjuction?

p: It is raining.

q: I carry my umbrella.

What's the conjuction?

p: It is raining.

q: I carry my umbrella.

p ∧ q: It's raining and I carry my umbrella.

What's the disjunction?

p: It is raining.

q: I carry my umbrella.

What's the disjunction?

p: It is raining.

q: I carry my umbrella.

p v q: It's raining or I carry my umbrella.

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

3 propositions (p, q, r) \rightarrow 2³ rows = 8 rows

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r

 $((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$

р	q	r
		F
		Т
		F
		Т
		F
		Т
		F
		Т

 $((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$

р	q	r
	F	F
	Ш	Т
	\vdash	F
	Τ	Т
	Ш	F
	H	Т
	Т	F
	Т	T

 $((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$

р	q	r
П	H	F
F	F	Т
F	T	F
F	Т	Т
Т	F	F
Τ	F	Т
Т	Т	F
Т	T	Т

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q
F	H	П	
F	F	Т	
F	Т	F	
F	Т	Т	
Т	F	F	
Т	F	Т	
Т	Т	F	
Т	Т	Т	

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q
F	Т	П	
F	F	Т	
F	Т	F	
F	Т	Т	
Т	F	F	
Т	F	Т	
Т	Т	F	
Τ	Т	Т	

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q
F	H	П	Τ
F	F	Т	Т
F	Т	F	F
F	Т	Т	F
Т	F	F	Т
Т	F	Т	Т
Т	Т	F	F
Τ	Т	T	F

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q
F	H	П	Т
F	F	Т	Т
F	Т	F	F
F	Т	Т	F
Т	F	F	Т
Т	F	T	Т
Τ	Т	F	F
T	Т	Т	F

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q	p∧¬q
F	F	F	Τ	
F	F	Т	Т	
F	Т	F	F	
F	Т	Т	F	
T	F	F	Т	
T	F	Т	Т	
T	Т	F	F	
T	Т	T	F	

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q	p∧¬q
F	F	F	_	
F	F	Т	Т	
F	Т	F	F	
F	Т	Т	F	
Т	F	F	Т	
Т	F	Т	Т	
Т	Т	F	F	
T	Т	Τ	H	

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q	p∧¬q
F	F	F	_	F
F	F	Т	Т	F
F	Т	F	F	F
F	Т	Т	F	F
Т	F	F	Т	Т
Т	F	Т	Т	Т
Т	Т	F	F	F
Т	Т	Т	H	F

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q	p∧¬q	p∨q
F	F	F	Т	F	
F	F	Т	Т	F	
F	Т	F	F	F	
F	T	T	F	F	
T	F	F	Т	Т	
Т	F	Т	Т	Т	
T	T	F	F	F	
T	Т	Т	F	F	

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q	p∧¬q	p∨q
F	F	F	Т	F	
F	F	Т	Т	F	
F	Т	F	F	F	
F	Т	Т	F	F	
Т	F	F	Т	Т	
Т	H	Τ	Т	Т	
T	Τ	IL	L	F	
T	Τ	Т	F	F	

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q	p∧¬q	p∨q
F	F	П	Т	F	F
F	F	Т	Т	F	F
F	Т	F	F	F	Т
F	Т	Т	F	F	Т
Т	F	F	Т	Т	Т
Т	F	Т	Т	Т	Т
Т	Т	F	F	F	Т
Т	Т	Т	F	F	Т

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q	p∧¬q	p∨q
F	F	F	Т	F	F
F	L	Τ	Т	F	F
F	Т	F	F	F	Т
F	T	Т	F	F	Т
Т	F	F	Т	Т	Т
Т	F	Т	Т	Т	Т
T	Τ	IL	L	F	Т
T	Т	Т	F	F	Т

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q	p∧¬q	p∨q	(p∧¬q)↔(p∨q)
F	П	T	Т	F	F	
F	F	Т	Т	F	F	
F	Т	F	F	F	Т	
F	Т	Т	F	F	Т	
Т	F	F	Т	Т	Т	
Т	F	Т	Т	Т	Т	
Т	Т	F	F	F	Т	
Т	Т	Т	F	F	Т	

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q	p∧¬q	p∨q	(p∧¬q)↔(p∨q)
F	F	F	Т	F	F	
F	F	Τ	Т	F	F	
F	Т	F	F	F	Т	
F	Т	Τ	F	F	Т	
T	F	F	Т	Т	Т	
T	F	Τ	Т	Т	Т	
T	Т	F	F	F	T	
T	Т	T	F	F	T	

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q	p∧¬q	p∨q	(p∧¬q)↔(p∨q)
F	F	T	Т	F	F	Т
F	F	Т	Т	F	F	Т
F	Т	F	F	F	Т	F
F	Т	Т	F	F	Т	F
Т	F	F	Т	Т	Т	Т
Т	F	Т	Т	Т	Т	Т
Т	T	F	F	F	Т	F
Т	T	Т	F	F	Т	F

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q	p∧¬q	p∨q	(p∧¬q)↔(p∨q)
F	F	T	Т	F	F	Т
F	F	Т	Т	F	F	Т
F	Т	F	F	F	Т	F
F	Т	Т	F	F	Т	F
T	F	F	Т	Т	Т	Т
T	F	Т	Т	Т	Т	Т
T	Т	F	F	F	Т	F
Τ	Т	Т	F	F	Т	F

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q	p∧¬q	p∨q	(p∧¬q)↔(p∨q)	$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$
F	F	F	Т	F	П	T	
F	F	Т	Т	F	F	Т	
F	Т	F	F	F	Т	F	
F	Т	Т	F	F	Т	F	
T	F	F	Т	Т	Т	Т	
T	F	Т	Т	Т	Т	Т	
T	Т	F	F	F	Т	F	
T	Т	Т	F	F	Т	F	

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q	p∧¬q	pvq	(p∧¬q)↔(p∨q)	$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$
F	H	F	Т	F	F	Т	
F	F	Т	Т	F	F	Т	
F	Т	F	F	F	Т	F	
F	Т	Т	F	F	Т	F	
T	F	F	Т	Т	Т	Т	
T	F	Т	Т	Т	Т	Т	
T	Τ	F	F	F	Т	F	
T	Τ	Т	F	F	Т	F	

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q	p∧¬q	pvq	(p∧¬q)↔(p∨q)	$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$
F	П	F	Т	F	F	Т	Т
F	П	Т	Т	F	F	Т	F
F	Τ	F	F	F	Т	F	F
F	Τ	Т	F	F	Т	F	Т
T	F	F	Т	Т	Т	Т	Т
T	F	Т	Т	Т	Т	Т	F
Т	Τ	F	F	F	Т	F	F
Т	T	Т	F	F	Т	F	T

$$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$$

р	q	r	¬q	p∧¬q	pvq	(p∧¬q)↔(p∨q)	$((p \land \neg q) \leftrightarrow (p \lor q) \oplus r)$
F	F	F	Τ	H	F	Т	Т
F	F	Т	Τ	H	F	Т	F
F	Т	F	F	F	Т	F	F
F	Т	Т	F	F	Т	F	Т
T	F	F	Т	Т	Т	Т	Т
T	F	Т	Т	Т	Т	Т	F
T	Т	F	F	F	Т	F	F
T	Т	Т	F	F	Т	F	Т