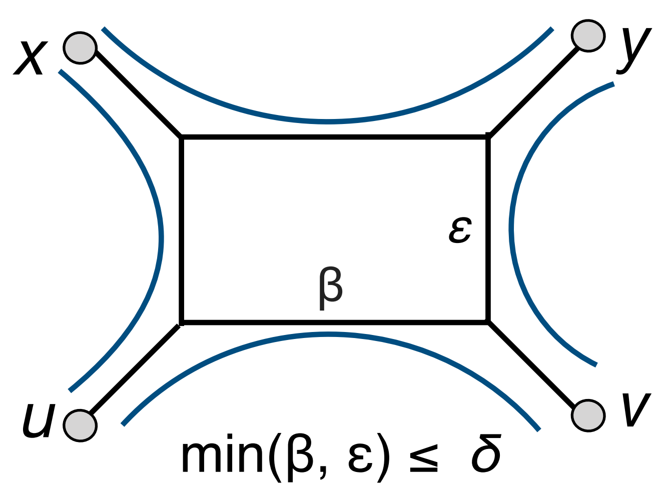
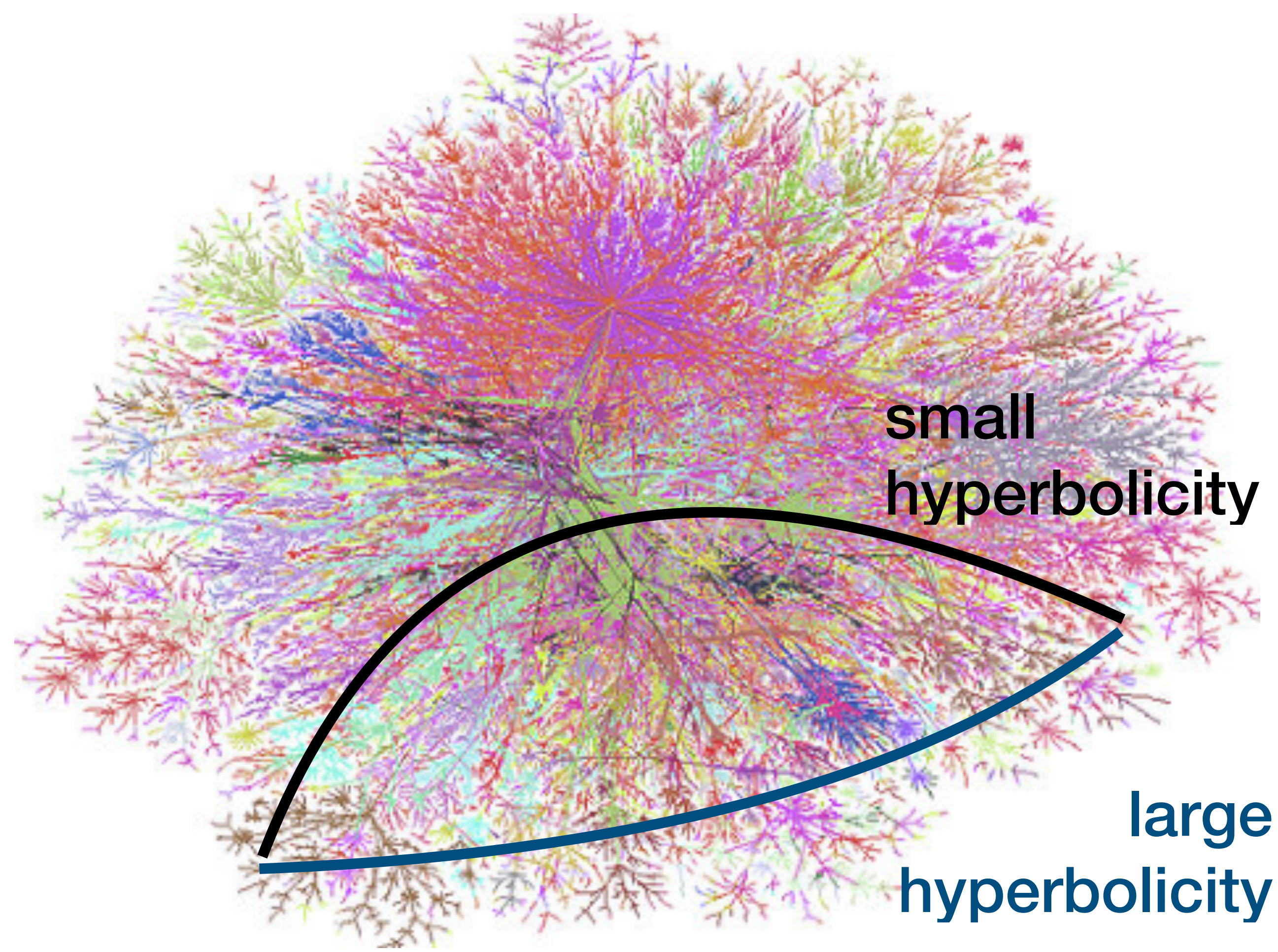
Applications of Hyperbolicity

- Many real world networks have small hyperbolicity (biological, social, collaboration, communication, etc.)
- Smaller value means the network
  - is metrically closer to a tree
  - has negative curvature



A graph is δ-hyperbolic provided for any vertices x, y, u, v in it, the two larger of the three sums
d(u,v) + d(x,y),
d(u,x) + d(v,y), and
d(u,y) + d(v,x) differ by at most 2δ.



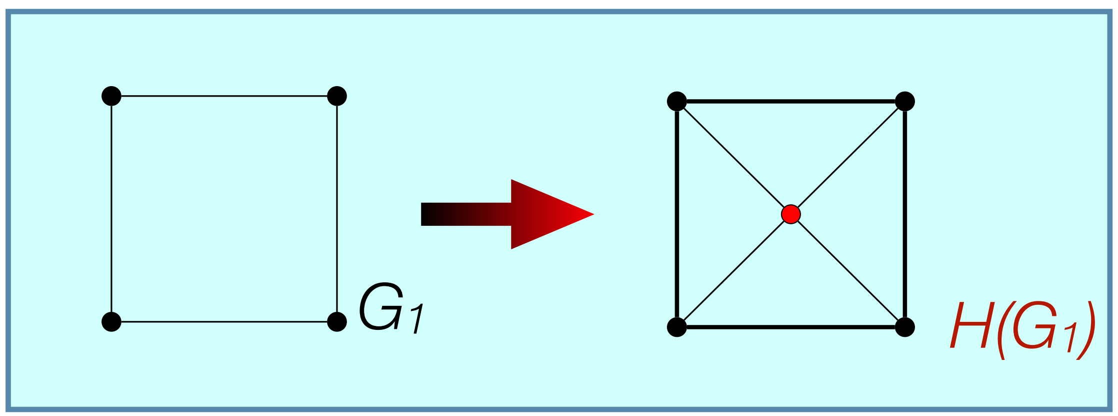
Small hyperbolicity implies that the shortest path between two points curves inward towards the core of the network.

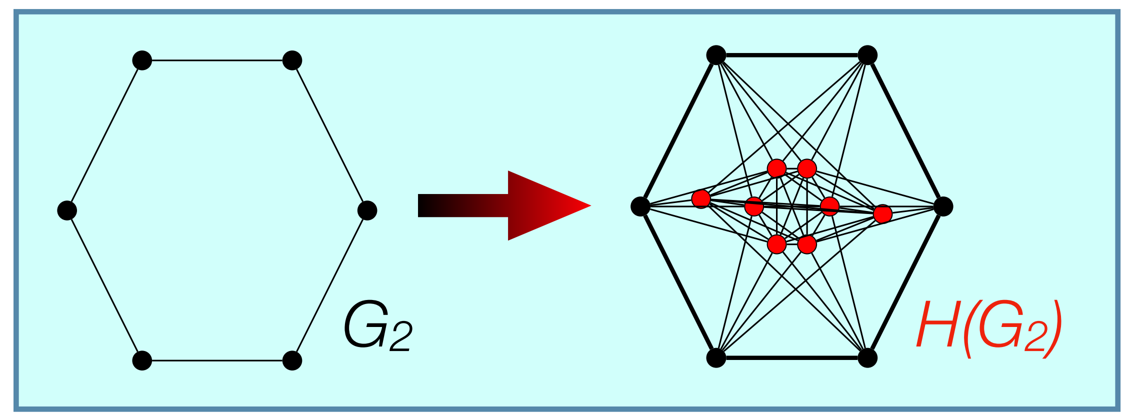
How hyperbolicity relates to Injective Hulls

Every graph G can be isometrically embedded into the smallest Helly graph $H(G)$ [1,2]

- $H(G)$ is called the injective hull of G
- $H(G)$ preserves hyperbolicity
- If G is δ-hyperbolic, any vertex in $H(G)$ is within 2δ to a vertex in G [3]

- A set S of sets S_i has the Helly property if for every subset T of S the following hold: if the elements of T pairwise intersect, then the intersection of all elements of T is also non-empty.
- A graph is called Helly if its family of disks satisfies the Helly property.





We want to understand:

- **(Q1) what governs hyperbolicity in Helly graphs in order to understand what governs hyperbolicity in regular graphs, and**
- **(Q2) how does the injective hull grow for various graph classes?**

[1] J. Isbell. *Six theorems about injective metric spaces*, Comment. Math. Helv (1964).
[2] A. Dress. *Trees, tight extensions of metric spaces, and the cohomological dimension of certain groups*, Adv. in Math (1984).
[3] U. Lang, *Injective hulls of certain discrete metric spaces and groups*, J. Topol. Anal. (2013)