Design and Analysis of Algorithms: Homework 4

1. (10 points) Let $S = \{a, b, c, d, e, f, g\}$ be a collection of items with weight-benefit values as follows: $a(3, 12)$, $b(6, 12)$, $c(6, 9)$, $d(1, 5)$, $e(2, 5)$, $f(10, 10)$, $g(3, 9)$. For example, item a weighs 3 lbs and is worth a total of \$12. What is an optimal solution to the **fractional** knapsack problem for S assuming we have a knapsack that can hold a total of 11 lbs? Show your work.

2. (10 points) Suppose we are given a set of tasks specified by pairs of start times and finish times as $T = \{(1, 2), (1, 3), (1, 4), (2, 5), (3, 7), (4, 9), (5, 6), (6, 8), (7, 9), (7, 10)\}$. Solve the task scheduling problem for these tasks.

3. (10 points) For each of the following recurrence equations which describe the running time $T(n)$ of a recursive algorithm, use the master method to express the asymptotic complexity (assuming that $T(n) = c$ for $n < d$, for constants $c > 0$ and $d \geq 1$).

   (a) $T(n) = 2T(n/2) + \log n$

   (b) $T(n) = 8T(n/2) + n^2$

   (c) $T(n) = 7T(n/3) + n$

   (d) $T(n) = 4T(n/2) + n^2$

   (e) $T(n) = 3T(n/2) + n^2$

4. (10 points) What is the best way to multiply a chain of matrices with dimensions that are 10×5, 5×2, 2×20, 20×12, 12×4, and 4×60? Show your work (including two tables - one indicating the index k which gives the final multiply index for each subproblem, and the second table which indicates the optimal number of multiplications for each subproblem).

5. Consider the single machine scheduling problem where we are given a set T of tasks specified by their start times and finish times, as in the task scheduling problem, except now we have only one machine and we wish to maximize the number of tasks that this single machine performs.

   (a) (7 points) Design a greedy algorithm for this single machine scheduling problem. What is the running time of your algorithm?

   (b) (3 points) Prove the correctness of your algorithm.

6. (10 points) A company named RT&T has a network of n switching stations connected by m high-speed communication links. Each customer's phone is directly connected to one station in his or her area. The engineers of RT&T have developed a prototype video-phone system that allows two customers to see eachother during a phone call. In order to have acceptable image quality, however, the number of links used to transmit video signals between the two parties cannot exceed 4. Suppose that RT&T network is represented by a graph. Design and give the **pseudo-code** for an efficient algorithm that computes, for each station, the set of stations it can reach using no more than 4 links. Analyze its running time.

7. (5 points) An Eulerian cycle of a directed graph G with n vertices and m edges is a cycle that traverses each edge of G exactly once according to its direction. Such a cycle always exists if the in-degree is equal to the out-degree for each vertex in G. Describe a $O(n+m)$ time algorithm for finding an Euler cycle of such a graph G. Analyze its running time.