

This is given:

matrix	A_0	A_1	A_2	A_3	A_4	A_5
dimension	30×35	35×15	15×5	5×10	10×20	20×25

N	0	1	2	3	4	5
0	0					
1		0				
2			0			
3				0		
4					0	
5						0

d_0	30
d_1	35
d_2	15
d_3	5
d_4	10
d_5	20
d_6	25

Step 1. If there's only one matrix, it costs nothing to multiply since it is already provided.

$$\begin{aligned} A_0 A_1 &= 30 \cdot 35 \cdot 15 = 15750 \\ A_1 A_2 &= 35 \cdot 15 \cdot 5 = 2625 \\ A_2 A_3 &= 15 \cdot 5 \cdot 10 = 750 \\ A_3 A_4 &= 5 \cdot 10 \cdot 20 = 1000 \\ A_4 A_5 &= 10 \cdot 20 \cdot 25 = 5000 \end{aligned}$$

Step 2. Multiplying two matrices together

N	0	1	2	3	4	5
0	0	15750				
1		0	2625			
2			0	750		
3				0	1000	
4					0	5000
5						0

K	0	1	2	3	4	5
0	0	0				
1		1				
2			2			
3				3		
4					4	
5						

$$A_0 A_1 A_2 = \min \left\{ \begin{array}{l} [f(A_0 A_1) A_2] \\ [f(A_0 A_2) A_1] \end{array} \right\}$$

$$\begin{aligned} 0 + 2625 + 30 \cdot 35 \cdot 15 &= 7875 \quad k=0 \\ 15750 + 0 + 30 \cdot 15 \cdot 5 &= 18000 \quad k=1 \end{aligned}$$

Step 3. Multiplying three matrices together

N	0	1	2	3	4	5
0	0	15750	7875			
1		0	2625			
2			0	750		
3				0	1000	
4					0	5000
5						0

K	0	1	2	3	4	5
0	0	0	0			
1		1				
2			2			
3				3		
4					4	
5						

$$A_1 A_2 A_3 = \min \left\{ \begin{array}{l} A_1 (A_2 A_3) \\ (A_1 A_2) A_3 \end{array} \right\}$$

$$\begin{aligned} 0 + 750 + 35 \cdot 15 \cdot 10 &= 6000 \quad k=1 \\ 2625 + 0 + 35 \cdot 5 \cdot 10 &= 4375 \quad k=2 \end{aligned}$$

N	0	1	2	3	4	5
0	0	15750	7875			
1		0	2625	4375		
2			0	750		
3				0	1000	
4					0	5000
5						0

K	0	1	2	3	4	5
0	0	0	0	0		
1		1	2			
2			2			
3				3		
4					4	
5						

Note we record $k[0][2]=0$ because (highlighted in green above), the minimum number that we recorded in $N[0][2]$ came when we did the last multiplication at matrix A_0 .

$k[1][2]=2$ because (highlighted in green above), the minimum number that we recorded in $N[1][3]$ came when we did the last multiplication at matrix A_0

$$i=2 \quad A_2 A_3 A_4 \quad \min \left\{ \begin{array}{l} (A_2)(A_3 A_4) \\ (A_2 A_3) \cdot (A_4) \end{array} \right\} \quad \begin{array}{l} 0 + 1000 + 15 \cdot 5 \cdot 20 = 2500 \\ 750 + 0 + 15 \cdot 10 \cdot 20 = 3750 \end{array} \quad k=2 \quad k=3$$

$$i=3 \quad A_3 A_4 A_5 \quad \min \left\{ \begin{array}{l} (A_3)(A_4 A_5) \\ (A_3 A_4) \cdot (A_5) \end{array} \right\} \quad \begin{array}{l} 0 + 5000 + 5 \cdot 10 \cdot 25 = 6250 \\ 1000 + 0 + 5 \cdot 20 \cdot 25 = 3500 \end{array} \quad k=3 \quad k=4$$

N	0	1	2	3	4	5
0	0	15750	7875			
1	0	2625	4375			
2		0	750	2500		
3			0	1000	3500	
4				0	5000	
5					0	

K	0	1	2	3	4	5
0	0	0	0			
1			1	2		
2				2	2	
3					3	4
4						4
5						

$$i=0 \quad A_0 A_1 A_2 A_3 \quad \min \left\{ \begin{array}{l} (A_0)(A_1 A_2 A_3) \\ (A_0 A_1)(A_2 A_3) \\ (A_0 A_1 A_2)(A_3) \end{array} \right\} \quad \begin{array}{l} 0 + 4375 + 30 \cdot 35 \cdot 10 = 14875 \quad k=0 \\ 15750 + 750 + 30 \cdot 15 \cdot 10 = 21000 \quad k=1 \\ 7875 + 0 + 30 \cdot 5 \cdot 10 = 9375 \quad k=2 \end{array}$$

Step 4. Multiplying four matrices together

$$j=4 \quad A_1 A_2 A_3 A_4 \quad \min \left\{ \begin{array}{l} (A_1)(A_2 A_3 A_4) \\ (A_1 A_2)(A_3 A_4) \\ (A_1 A_2 A_3)(A_4) \end{array} \right\} \quad \begin{array}{l} 0 + 2500 + 35 \cdot 15 \cdot 20 = 13000 \quad k=1 \\ 2625 + 1000 + 35 \cdot 5 \cdot 20 = 7125 \quad k=2 \\ 4375 + 0 + 35 \cdot 10 \cdot 20 = 11375 \quad k=3 \end{array}$$

$$j=5 \quad A_2 A_3 A_4 A_5 \quad \min \left\{ \begin{array}{l} A_2 A_3 A_4 A_5 \\ A_2 A_3 A_4 A_5 \\ A_2 A_3 A_4 A_5 \end{array} \right\} \quad \begin{array}{l} 0 + 3500 + 15 \cdot 5 \cdot 25 = 5375 \quad k=2 \\ 750 + 5000 + 15 \cdot 10 \cdot 25 = 9500 \quad k=3 \\ 2500 + 0 + 15 \cdot 20 \cdot 25 = 10000 \quad k=4 \end{array}$$

N	0	1	2	3	4	5
0	0	15750	7875	9375		
1	0	2625	4375	7125		
2		0	750	2500	5375	
3			0	1000	3500	
4				0	5000	
5					0	

K	0	1	2	3	4	5
0	0	0	0	2		
1			1	2	2	
2				2	2	2
3					3	4
4						4
5						

$$i=0 \quad A_0 A_1 A_2 A_3 A_4 \quad \min \left\{ \begin{array}{l} A_0(A_1 A_2 A_3 A_4) \\ A_0 A_1(A_2 A_3 A_4) \\ A_0 A_1 A_2(A_3 A_4) \\ A_0 A_1 A_2 A_3(A_4) \end{array} \right\} \quad \begin{array}{l} 0 + 7125 + 30 \cdot 35 \cdot 20 = 28125 \quad k=0 \\ 15750 + 2500 + 30 \cdot 15 \cdot 20 = 27250 \quad k=1 \\ 7875 + 1000 + 30 \cdot 5 \cdot 20 = 11875 \quad k=2 \\ 9375 + 0 + 30 \cdot 10 \cdot 20 = 15375 \quad k=3 \end{array}$$

Step 5. Multiplying five matrices together

$$j=5 \quad A_1 A_2 A_3 A_4 A_5 \quad \min \left\{ \begin{array}{l} A_1 A_2 A_3 A_4 A_5 \\ A_1 A_2 A_3 A_4 A_5 \\ A_1 A_2 A_3 A_4 A_5 \\ A_1 A_2 A_3 A_4 A_5 \end{array} \right\} \quad \begin{array}{l} 0 + 5375 + 35 \cdot 15 \cdot 25 = 18500 \quad k=1 \\ 2625 + 3500 + 35 \cdot 5 \cdot 25 = 10500 \quad k=2 \\ 4375 + 5000 + 35 \cdot 10 \cdot 25 = 18125 \quad k=3 \\ 7125 + 0 + 35 \cdot 20 \cdot 25 = 24625 \quad k=4 \end{array}$$

N	0	1	2	3	4	5
0	0	15750	7875	9375	11875	
1		0	2625	4375	7125	10500
2			0	750	2500	5375
3				0	1000	3500
4					0	5000
5						0

K	0	1	2	3	4	5
0	0	0	0	2	2	
1			1	2	2	2
2				2	2	2
3					3	4
4						4
5						

$\min_{\substack{i=0 \\ j=5}} \left\{ \begin{array}{l} A_0 | A_1 A_2 A_3 A_4 A_5 \\ A_0 A_1 | A_2 A_3 A_4 A_5 \\ A_0 A_1 A_2 | A_3 A_4 A_5 \\ A_0 A_1 A_2 A_3 | A_4 A_5 \\ A_0 A_1 A_2 A_3 A_4 | A_5 \\ A_0 A_1 A_2 A_3 A_4 A_5 \end{array} \right. \quad \begin{array}{llll} 0 + 10500 & + 30 \cdot 35 \cdot 25 = 36750 & k=0 \\ 15750 + 5375 & + 30 \cdot 15 \cdot 25 = 32375 & k=1 \\ 7875 + 3500 & + 30 \cdot 5 \cdot 25 = 15125 & k=2 \\ 9375 + 5000 & + 30 \cdot 10 \cdot 25 = 21875 & k=3 \\ 11875 + 0 & + 30 \cdot 20 \cdot 25 = 26875 & k=4 \end{array} \quad \text{Step 6. Multiplying six matrices together}$

N	0	1	2	3	4	5
0	0	15750	7875	9375	11875	15125
1		0	2625	4375	7125	10500
2			0	750	2500	5375
3				0	1000	3500
4					0	5000
5						0

K	0	1	2	3	4	5
0	0	0	0	2	2	2
1			1	2	2	2
2				2	2	2
3					3	4
4						4
5						

Finished building N table (left). It says that the minimum number of scalar operations it will take to multiply these 6 matrices together is 15125. The k table (right) explains how the chain of matrices was parenthesized which minimized the number of scalar operations. Interpreting the k-table gives the solution

$$((A_0) \times (A_1 \times A_2)) \times ((A_3 \times A_4) \times (A_5))$$

$k[0][5]$ determines the last multiplication, effectively splitting the chain of matrices into large matrices from $A_0 \dots A_2$ on the left and $A_3 \dots A_5$ on the right.

$k[0][2]$ describes how to multiply the left chain (from $A_0 \dots A_2$)
 $k[3][5]$ describes how to multiply the right chain (from $A_3 \dots A_5$)