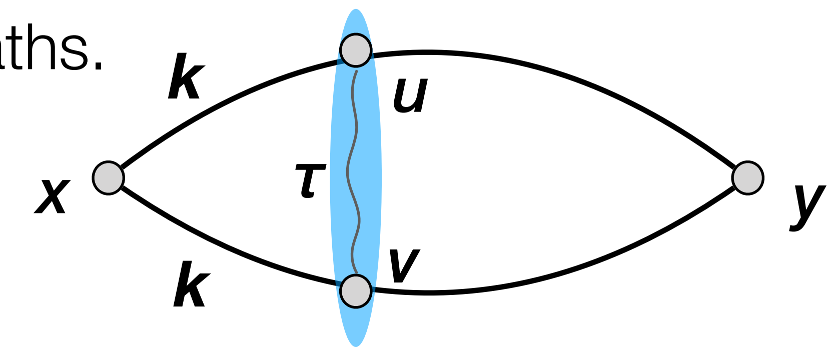
(**Q1**) Interval thinness governs hyperbolicity in Helly graphs

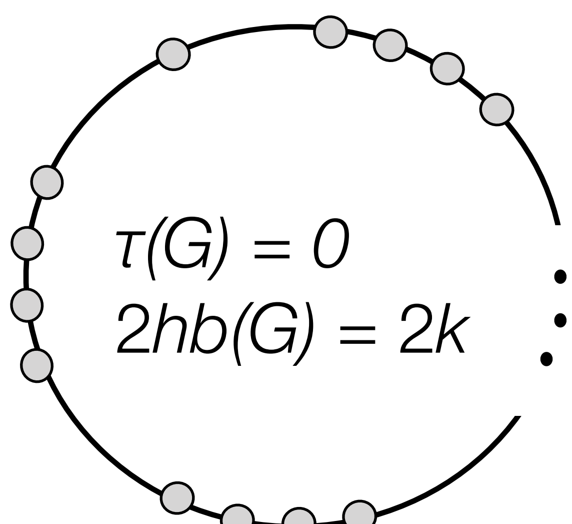
- An interval $I(x,y)$ is the set of all vertices from shortest (x,y)-paths.
- A slice of an interval at distance k is defined as:

$$S_k(x,y) = \{z \in I(x,y) : d(z,x) = k\}$$



- An interval is τ-thin if for any natural number k and any two u,v vertices of $S_k(x,y)$ are at most τ apart.
- A graph is τ-thin if all of its intervals are at most τ-thin.

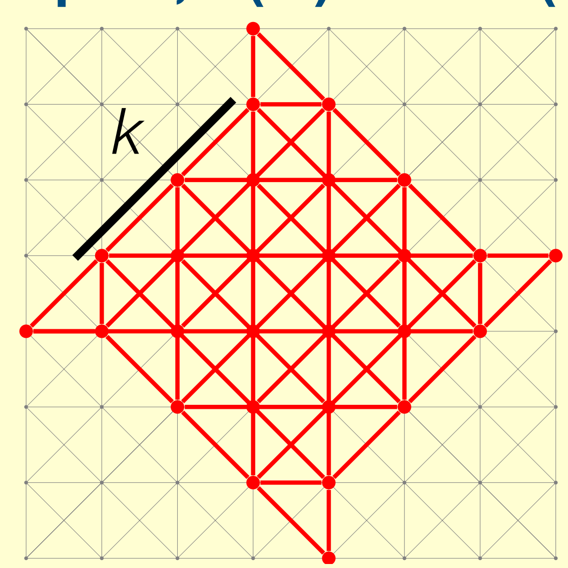
For general graphs τ(*G*) ≤ 2*hb*(*G*), but τ(*G*) and *hb*(*G*) can be far apart.



example: odd cycle with 4k+1 vertices

Our Contribution

Theorem [4]:
For Helly graphs, τ(G) ≤ 2hb(G) ≤ τ(G) + 1.



Example when $2hb(G) = \tau(G) + 1$

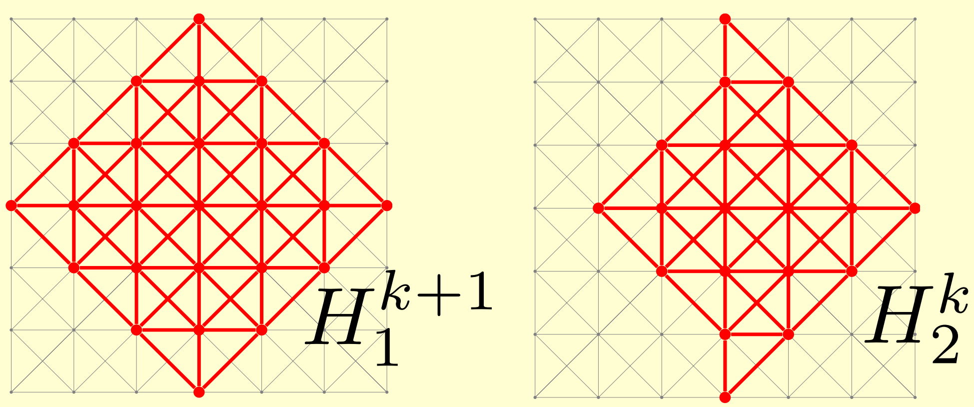
[4] F. Dragan and H. Guarnera, *Obstructions to a small hyperbolicity in Helly graphs*, under review.

($\mathbf{Q1}$) Special subgraphs of a chess grid govern hyperbolicity in Helly graphs

Our Contribution

Theorem [4]: We show that for Helly graphs and any integer *k*,

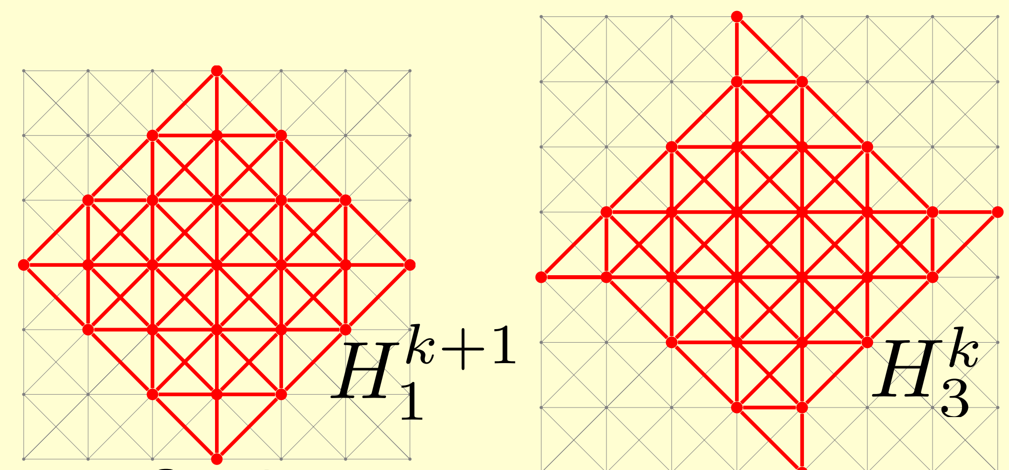
- ***hb*(*G*) ≤ *k* if and only if *G* has neither isometric H_1^{k+1} nor H_2^k**



δ = *k* δ = *k* + 1/2

hb(*G*) is an integer

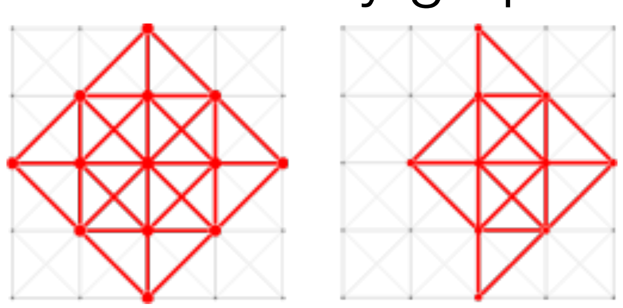
- ***hb*(*G*) ≤ *k*+1/2 if and only if *G* has neither isometric H_1^{k+1} nor H_3^k**



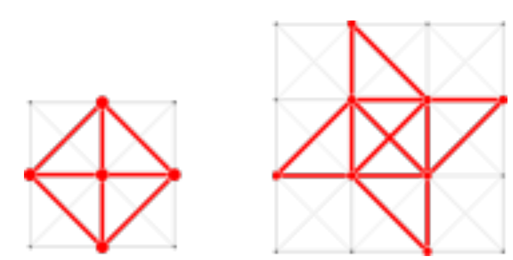
δ = *k* δ = *k* + 1

hb(*G*) is a half-integer

Example: forbidden isometric subgraphs for **1-hyperbolic** Helly graphs.



Example: forbidden isometric subgraphs for **1/2-hyperbolic** Helly graphs.



[4] F. Dragan and H. Guarnera, *Obstructions to a small hyperbolicity in Helly graphs*, under review.