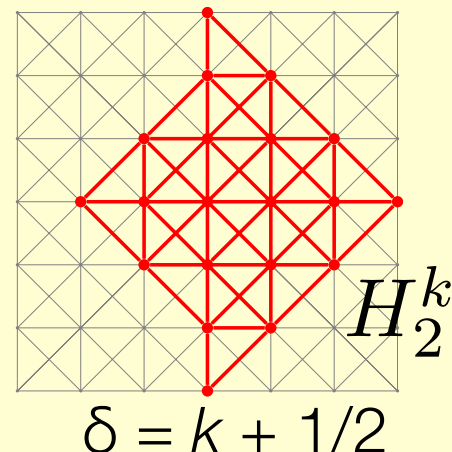
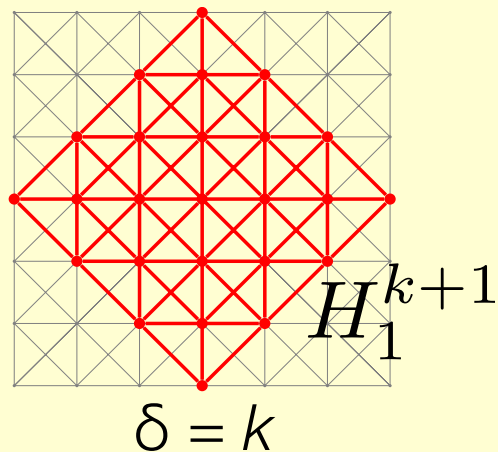


# (Q1) Special subgraphs of a chess grid govern hyperbolicity in Helly graphs

## Our Contribution

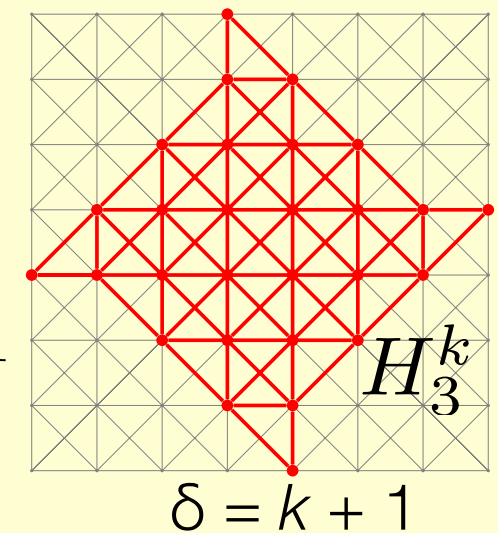
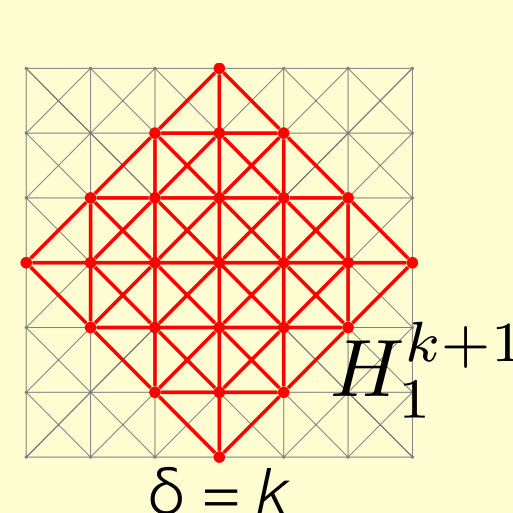
**Theorem [4]:** We show that for Helly graphs and any integer  $k$ ,

- $hb(G) \leq k$  if and only if  $G$  has neither isometric  $H_1^{k+1}$  nor  $H_2^k$



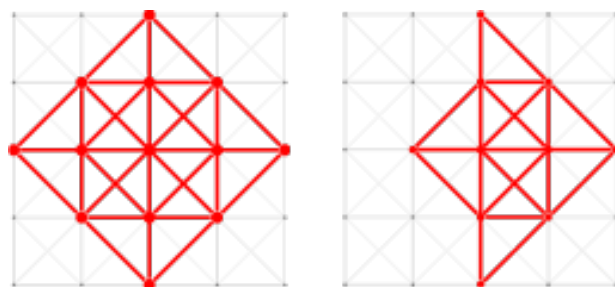
$hb(G)$  is an integer

- $hb(G) \leq k + 1/2$  if and only if  $G$  has neither isometric  $H_1^{k+1}$  nor  $H_3^k$

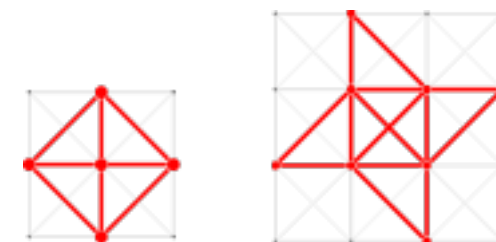


$hb(G)$  is a half-integer

Example: forbidden isometric subgraphs for 1-hyperbolic Helly graphs.

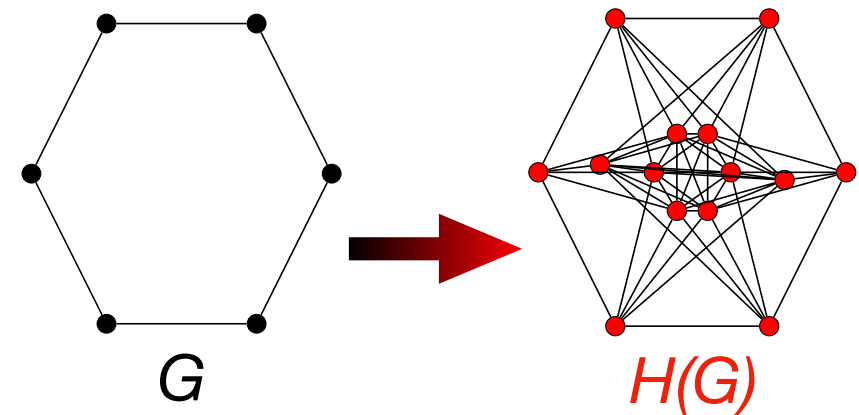


Example: forbidden isometric subgraphs for 1/2-hyperbolic Helly graphs.



# (Q2) How big is $H(G)$ with respect to $G$ ?

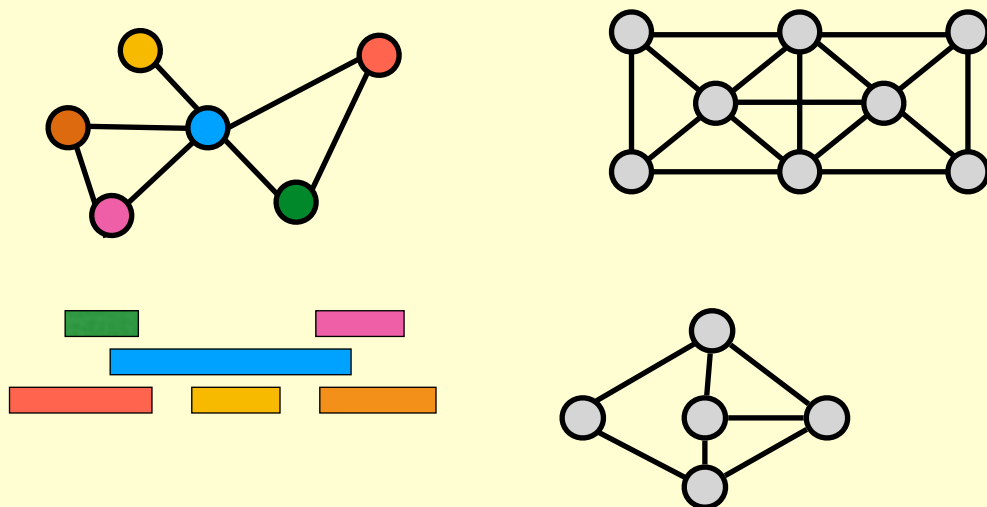
Every graph  $G$  can be isometrically embedded into the smallest **Helly** graph  $H(G)$ , called the **injective hull** of  $G$ .



## Our Contribution

**Theorem [5]:** There are some graph classes for which  $H(G)$  ... contains at most  $2n$  vertices.

- Interval graphs
- Helly graphs
- Distance hereditary graphs



... can contain at least  $2^n$  vertices.

- Chordal bipartite graphs
- Chordal graphs
- Cocomparability graphs

