1.5 Examples

Translate $\exists x \forall y (xy = y)$ into English, where the domain for each variable consists of all real numbers.

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$\exists x$ - There exists a real number x

$\forall y$ - For every real number y

"There exists a real number x such that for every real number y, $xy = y$."

This asserts the existence of a multiplicative identity for the real numbers. It's true, for example x =1.

Translate $\forall x \forall y(((x \geq 0) \wedge (y < 0)) \rightarrow (x - y > 0))$ into English, where the domain for each variable consists of all real numbers.

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For every real number x and real number y, if x *i*s nonnegative and y is negative, then the difference x - y is positive*.*

A nonnegative number minus a negative number is positive (true!).

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$\exists z$ - There exists a real number z

For every real number x and real number y, there exists a real number z such that x = y + z.

True, because in each case we can make z = x - y.

Let *Q(x, y)* be the statement "student *x* has been a contestant on quiz show *y*." Express the following in terms of *Q(x,y)*, quantifiers, and logical connectives, where the domain for *x* consists of all students at your school and for *y* consists of all quiz shows on television.

1. There is a student at your school who has been a contestant on a television quiz show.

2. No student at your school has ever been a contestant on a television quiz show.

3. There is a student at your school who has been a contestant on Jeopardy and on Wheel of Fortune.

4. Every television quiz show has had a student from your school as a contestant.

5. At least two students from your school have been contestants on Jeopardy.

a. $\exists x(Q(x, \text{Jeopardy}) \wedge Q(x, \text{Wheel of Fortune}))$

b. $\neg\exists x\exists y Q(x,y)$

c. $\forall y\exists x Q(x, y)$

d. $\exists x\exists y Q(x,y)$

e. $\forall x\forall y\neg Q(x,y)$

f. $\exists x1\exists x2(Q(x1, \text{Jeopardy}) \wedge Q(x2, \text{Jeopardy}) \wedge x1\neq x2)$

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Let $Q(x,y)$ be the statement "$x + y = xy$". If the domain for both variables consists of all integers, what are the truth values?

A. $Q(0,0)$
B. $Q(1,0)$
C. $Q(0, 1)$
D. $Q(1,1)$
E. $\forall y Q(0,y)$
F. $\exists x Q(x, 0)$

Let $Q(x,y)$ be the statement "$x + y = xy$". If the domain for both variables consists of all integers, what are the truth values?

A. $Q(0,0)$ True. $0 + 0 = 0$

B. $Q(1,0)$ False. $1 + 0 = 0$

C. $Q(0, 1)$ False. $0 + 1 = 0$

D. $Q(1,1)$ False. $1 + 1 = 1$

E. $\forall y Q(0,y)$ False. C is a counterexample

F. $\exists x Q(x, 0)$ True. A is an example.

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