1.6 Examples

Determine whether the argument is correct or incorrect

Everyone majoring in computer science has Linux installed.

George doesn’t have Linux installed.

Therefore, George isn’t majoring in computer science.

Determine whether the argument is correct or incorrect

Everyone majoring in computer science has Linux installed. $\boxed{\forall x(C(x) \rightarrow L(x))}$

George doesn't have Linux installed. $\boxed{\neg L(g)}$

Therefore, George isn't majoring in computer science. $\boxed{\neg C(g)}$

Determine whether the argument is correct or incorrect

Everyone majoring in computer science has Linux installed. $\boxed{\forall x(C(x) \rightarrow L(x))}$

George doesn't have Linux installed. $\boxed{\neg L(g)}$

Therefore, George isn't majoring in computer science. $\boxed{\neg C(g)}$

$$\begin{array}{l} \forall x(C(x) \rightarrow L(x)) \\ \neg L(g) \\ \hline \therefore \neg C(g) \end{array}$$

Determine whether the argument is correct or incorrect

Everyone majoring in computer science has Linux installed. $\boxed{\forall x(C(x) \rightarrow L(x))}$

George doesn't have Linux installed. $\boxed{\neg L(g)}$

Therefore, George isn't majoring in computer science. $\boxed{\neg C(g)}$

Correct! Universal Modus Tollens

$$\begin{array}{l} \forall x(C(x) \rightarrow L(x)) \\ \neg L(g) \\ \hline \therefore\ \neg C(g) \end{array}$$

Determine whether the argument is correct or incorrect

A Dvorak keyboard is efficient to use.

Jake’s keyboard is not a Dvorak keyboard.

Therefore, Jake’s keyboard is not efficient.

Determine whether the argument is correct or incorrect

A Dvorak keyboard is efficient to use. $\boxed{\forall x(D(x) \rightarrow E(x))}$

Jake's keyboard is not a Dvorak keyboard. $\boxed{\neg D(j)}$

Therefore, Jake's keyboard is not efficient. $\boxed{\neg E(j)}$

Determine whether the argument is correct or incorrect

A Dvorak keyboard is efficient to use. $\boxed{\forall x(D(x) \rightarrow E(x))}$

Jake's keyboard is not a Dvorak keyboard. $\boxed{\neg D(j)}$

Therefore, Jake's keyboard is not efficient. $\boxed{\neg E(j)}$

$$\begin{array}{l} \forall x(D(x) \rightarrow E(x)) \\ \neg D(j) \\ \hline \therefore \neg E(j) \end{array}$$

Determine whether the argument is correct or incorrect

A Dvorak keyboard is efficient to use. $\boxed{\forall x(D(x) \to E(x))}$

Jake's keyboard is not a Dvorak keyboard. $\boxed{\neg D(j)}$

Therefore, Jake's keyboard is not efficient. $\boxed{\neg E(j)}$

Incorrect! We can't conclude ¬ E(j) with this information

$$\begin{array}{l} \forall x(D(x) \to E(x)) \\ \neg D(j) \\ \hline \therefore \neg E(j) \end{array}$$

Determine whether the argument is correct or incorrect

A Dvorak keyboard is efficient to use. $\boxed{\forall x(D(x) \rightarrow E(x))}$

Jake's keyboard is not a Dvorak keyboard. $\boxed{\neg D(j)}$

Therefore, Jake's keyboard is not efficient. $\boxed{\neg E(j)}$

$$\begin{array}{l} \forall x(D(x) \rightarrow E(x)) \\ \neg E(j) \\ \hline \therefore \neg D(j) \end{array}$$

$$\cancel{\begin{array}{l} \forall x(D(x) \rightarrow E(x)) \\ \neg D(j) \\ \hline \therefore \neg E(j) \end{array}}$$

Determine whether the argument is correct or incorrect

A Dvorak keyboard is efficient to use.

Jake’s keyboard is not efficient to use.

Therefore, Jake’s keyboard is not a Dvorak.

Determine whether the argument is correct or incorrect

A Dvorak keyboard is efficient to use. $\boxed{\forall x(D(x) \rightarrow E(x))}$

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Correct! Universal Modus Tollens

$$\begin{array}{l} \forall x(D(x) \rightarrow E(x)) \\ \neg E(j) \\ \hline \therefore \neg D(j) \end{array}$$

#6 from the book

Show that the following hypothesis:

- *“If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on,”*
- *“If the sailing race is held, then the trophy will be awarded,”*
- “*The trophy was not awarded.*”

imply the conclusion “*It rained.*”

Show that the following hypothesis:

- *"If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on,"*
- *"If the sailing race is held, then the trophy will be awarded,"*
- "*The trophy was not awarded.*"

imply the conclusion "*It rained.*"

f="It's foggy." s="The sailing race is held."

r="It rains." t="The trophy is awarded."

l="The life saving demonstrations will go on."

Show that the following hypothesis:

- *"If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on,"* $\boxed{(\neg r \vee \neg f) \rightarrow (s \wedge l)}$
- *"If the sailing race is held, then the trophy will be awarded,"* $\boxed{s \rightarrow t}$
- "*The trophy was not awarded.*" $\boxed{\neg t}$

imply the conclusion "*It rained.*" $\boxed{r}$

f="It's foggy."

s="The sailing race is held."

r="It rains."

t="The trophy is awarded."

l="The life saving demonstrations will go on."

$$(\neg r \vee \neg f) \rightarrow (s \wedge l)$$

$$s \rightarrow t$$

$$\neg t$$

$$\therefore r$$

$(\neg r \vee \neg f) \rightarrow (s \wedge l)$

$s \rightarrow t$

$\neg t$

$\therefore\ r$

Step	Reason
1. $\neg t$	Premise

$$(\neg r \vee \neg f) \rightarrow (s \wedge l)$$
$$s \rightarrow t$$
$$\underline{\neg t \qquad\qquad\qquad\qquad}$$
$$\therefore r$$

Step	Reason
1. $\neg t$	Premise
2. $s \rightarrow t$	Premise

$$(\neg r \vee \neg f) \rightarrow (s \wedge l)$$
$$s \rightarrow t$$
$$\neg t$$

$$\therefore r$$

Step	Reason
1. $\neg t$	Premise
2. $s \rightarrow t$	Premise
3. $\neg s$	Modus tollens using (1) and (2)

$$(\neg r \vee \neg f) \rightarrow (s \wedge l)$$
$$s \rightarrow t$$
$$\underline{\neg t \qquad\qquad\qquad\qquad}$$
$$\therefore r$$

Step	Reason
1. $\neg t$	Premise
2. $s \rightarrow t$	Premise
3. $\neg s$	Modus tollens using (1) and (2)
4. $(\neg r \vee \neg f) \rightarrow (s \wedge l)$	Premise

$$\begin{array}{l} (\neg r \lor \neg f) \to (s \land l) \\ s \to t \\ \underline{\neg t \qquad\qquad\qquad\qquad} \\ \therefore\ r \end{array}$$

Step	Reason
1. $\neg t$	Premise
2. $s \to t$	Premise
3. $\neg s$	Modus tollens using (1) and (2)
4. $(\neg r \lor \neg f) \to (s \land l)$	Premise
5. $(\neg(s \land l)) \to \neg(\neg r \lor \neg f)$	Contrapositive of (4)

$$(\neg r \vee \neg f) \rightarrow (s \wedge l)$$
$$s \rightarrow t$$
$$\underline{\neg t \qquad\qquad\qquad\qquad}$$
$$\therefore r$$

Step	Reason
1. $\neg t$	Premise
2. $s \rightarrow t$	Premise
3. $\neg s$	Modus tollens using (1) and (2)
4. $(\neg r \vee \neg f) \rightarrow (s \wedge l)$	Premise
5. $(\neg(s \wedge l)) \rightarrow \neg(\neg r \vee \neg f)$	Contrapositive of (4)
6. $(\neg s \vee \neg l) \rightarrow (r \wedge f)$	De Morgan's law and double negative

$$(\neg r \vee \neg f) \rightarrow (s \wedge l)$$
$$s \rightarrow t$$
$$\neg t$$

$$\therefore r$$

Step	Reason
1. $\neg t$	Premise
2. $s \rightarrow t$	Premise
3. $\neg s$	Modus tollens using (1) and (2)
4. $(\neg r \vee \neg f) \rightarrow (s \wedge l)$	Premise
5. $(\neg(s \wedge l)) \rightarrow \neg(\neg r \vee \neg f)$	Contrapositive of (4)
6. $(\neg s \vee \neg l) \rightarrow (r \wedge f)$	De Morgan's law and double negative
7. $\neg s \vee \neg l$	Addition using (3)

$$(\neg r \vee \neg f) \to (s \wedge l)$$
$$s \to t$$
$$\neg t$$

$$\therefore r$$

Step	Reason
1. $\neg t$	Premise
2. $s \to t$	Premise
3. $\neg s$	Modus tollens using (1) and (2)
4. $(\neg r \vee \neg f) \to (s \wedge l)$	Premise
5. $(\neg(s \wedge l)) \to \neg(\neg r \vee \neg f)$	Contrapositive of (4)
6. $(\neg s \vee \neg l) \to (r \wedge f)$	De Morgan's law and double negative
7. $\neg s \vee \neg l$	Addition using (3)
8. $r \wedge f$	Modus ponens using (6) and (7)

$$(\neg r \lor \neg f) \to (s \land l)$$
$$s \to t$$
$$\neg t$$
$$\therefore r$$

Step	Reason
1. $\neg t$	Premise
2. $s \to t$	Premise
3. $\neg s$	Modus tollens using (1) and (2)
4. $(\neg r \lor \neg f) \to (s \land l)$	Premise
5. $(\neg(s \land l)) \to \neg(\neg r \lor \neg f)$	Contrapositive of (4)
6. $(\neg s \lor \neg l) \to (r \land f)$	De Morgan's law and double negative
7. $\neg s \lor \neg l$	Addition using (3)
8. $r \land f$	Modus ponens using (6) and (7)
9. r	Simplification using (8)

$$(\neg r \vee \neg f) \rightarrow (s \wedge l)$$
$$s \rightarrow t$$
$$\neg t$$
$$\therefore r$$

Step	Reason
1. $\neg t$	Premise
2. $s \rightarrow t$	Premise
3. $\neg s$	Modus tollens using (1) and (2)
4. $(\neg r \vee \neg f) \rightarrow (s \wedge l)$	Premise
5. $(\neg(s \wedge l)) \rightarrow \neg(\neg r \vee \neg f)$	Contrapositive of (4)
6. $(\neg s \vee \neg l) \rightarrow (r \wedge f)$	De Morgan's law and double negative
7. $\neg s \vee \neg l$	Addition using (3)
8. $r \wedge f$	Modus ponens using (6) and (7)
9. r	Simplification using (8)

More than one way to do this…

$$(\neg r \vee \neg f) \rightarrow (s \wedge l)$$
$$s \rightarrow t$$
$$\underline{\neg t \qquad\qquad\qquad\qquad}$$
$$\therefore\ r$$

Step	Reason
1. $\neg t$	Premise
2. $s \rightarrow t$	Premise
3. $\neg s$	Modus tollens using (1) and (2)
4. $(\neg r \vee \neg f) \rightarrow (s \wedge l)$	Premise

$$(\neg r \vee \neg f) \rightarrow (s \wedge l)$$
$$s \rightarrow t$$
$$\neg t$$
$$\therefore r$$

Step	Reason
1. $\neg t$	Premise
2. $s \rightarrow t$	Premise
3. $\neg s$	Modus tollens using (1) and (2)
4. $(\neg r \vee \neg f) \rightarrow (s \wedge l)$	Premise
5. $\neg s \vee \neg l$	Addition using (3)

$$
\begin{array}{l}
(\neg r \lor \neg f) \rightarrow (s \land l) \\
s \rightarrow t \\
\underline{\neg t \qquad\qquad\qquad\qquad} \\
\therefore\ r
\end{array}
$$

Step	Reason
1. $\neg t$	Premise
2. $s \rightarrow t$	Premise
3. $\neg s$	Modus tollens using (1) and (2)
4. $(\neg r \lor \neg f) \rightarrow (s \land l)$	Premise
5. $\neg s \lor \neg l$	Addition using (3)
6. $\neg(s \land l)$	De Morgan's law using (5)

$$(\neg r \vee \neg f) \rightarrow (s \wedge l)$$
$$s \rightarrow t$$
$$\neg t$$
$$\therefore r$$

Step	Reason
1. $\neg t$	Premise
2. $s \rightarrow t$	Premise
3. $\neg s$	Modus tollens using (1) and (2)
4. $(\neg r \vee \neg f) \rightarrow (s \wedge l)$	Premise
5. $\neg s \vee \neg l$	Addition using (3)
6. $\neg(s \wedge l)$	De Morgan's law using (5)
7. $\neg(\neg r \vee \neg f)$	Modus tollens using (4) and (6)

$$\begin{array}{l} (\neg r \vee \neg f) \rightarrow (s \wedge l) \\ s \rightarrow t \\ \underline{\neg t \qquad\qquad\qquad\qquad} \\ \therefore r \end{array}$$

Step	Reason
1. $\neg t$	Premise
2. $s \rightarrow t$	Premise
3. $\neg s$	Modus tollens using (1) and (2)
4. $(\neg r \vee \neg f) \rightarrow (s \wedge l)$	Premise
5. $\neg s \vee \neg l$	Addition using (3)
6. $\neg(s \wedge l)$	De Morgan's law using (5)
7. $\neg(\neg r \vee \neg f)$	Modus tollens using (4) and (6)
8. $r \wedge f$	De Morgan's law and double negation (using (7)

$$(\neg r \vee \neg f) \rightarrow (s \wedge l)$$
$$s \rightarrow t$$
$$\underline{\neg t \qquad\qquad\qquad\qquad}$$
$$\therefore r$$

Step	Reason
1. $\neg t$	Premise
2. $s \rightarrow t$	Premise
3. $\neg s$	Modus tollens using (1) and (2)
4. $(\neg r \vee \neg f) \rightarrow (s \wedge l)$	Premise
5. $\neg s \vee \neg l$	Addition using (3)
6. $\neg(s \wedge l)$	De Morgan's law using (5)
7. $\neg(\neg r \vee \neg f)$	Modus tollens using (4) and (6)
8. $r \wedge f$	De Morgan's law and double negation (using (7)
9. r	Simplification using (8)