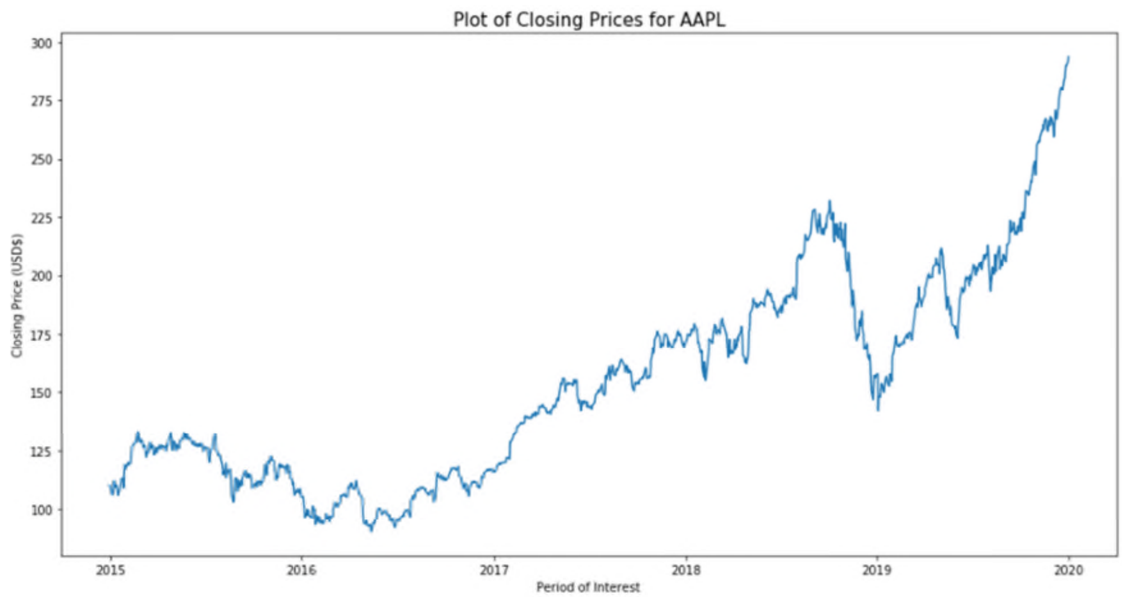
ECA5315 Financial Econometrics Assignment

Apple (AAPL) is the chosen listed company.

Part (i): Descriptive Statistics

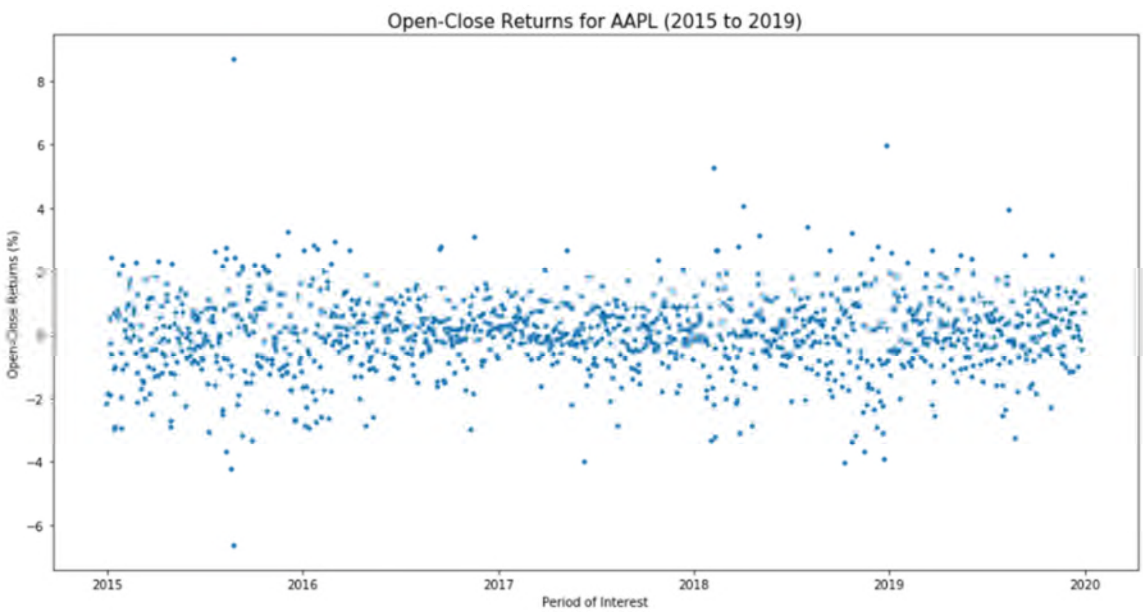
a) Plot of (Adjusted) Closing Share Prices Across Time



b) Plotting of Different Returns Across Time

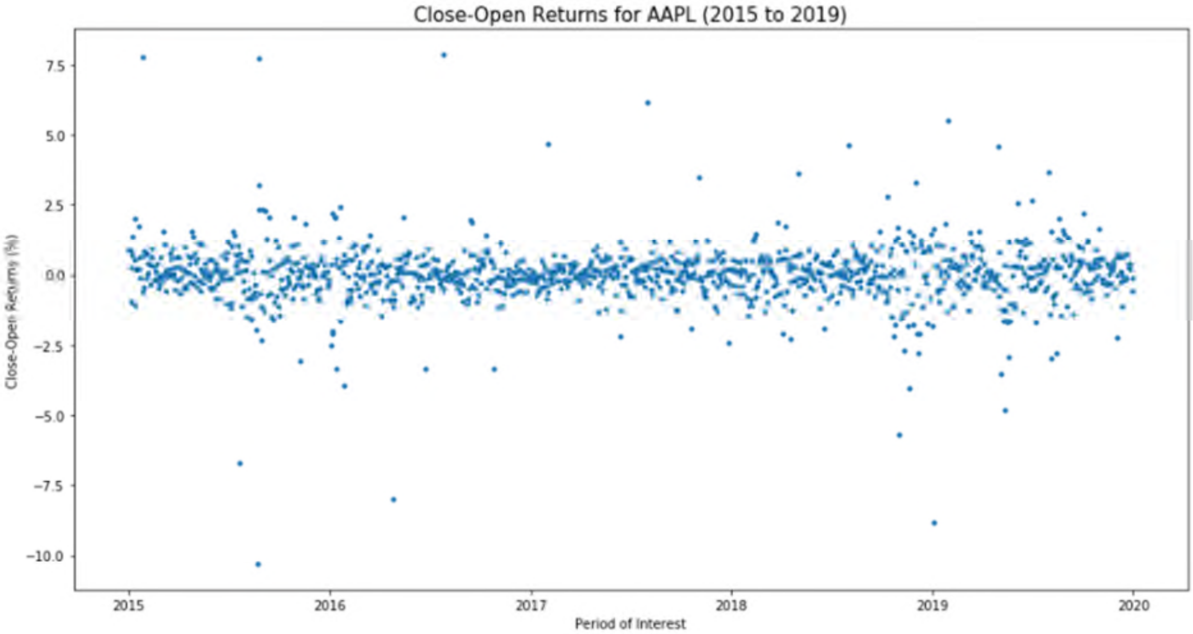
Open-Close

$$\text{Open} - \text{Close Returns } (\%) = \frac{Close - \ Open}{Open} \times 100\%$$



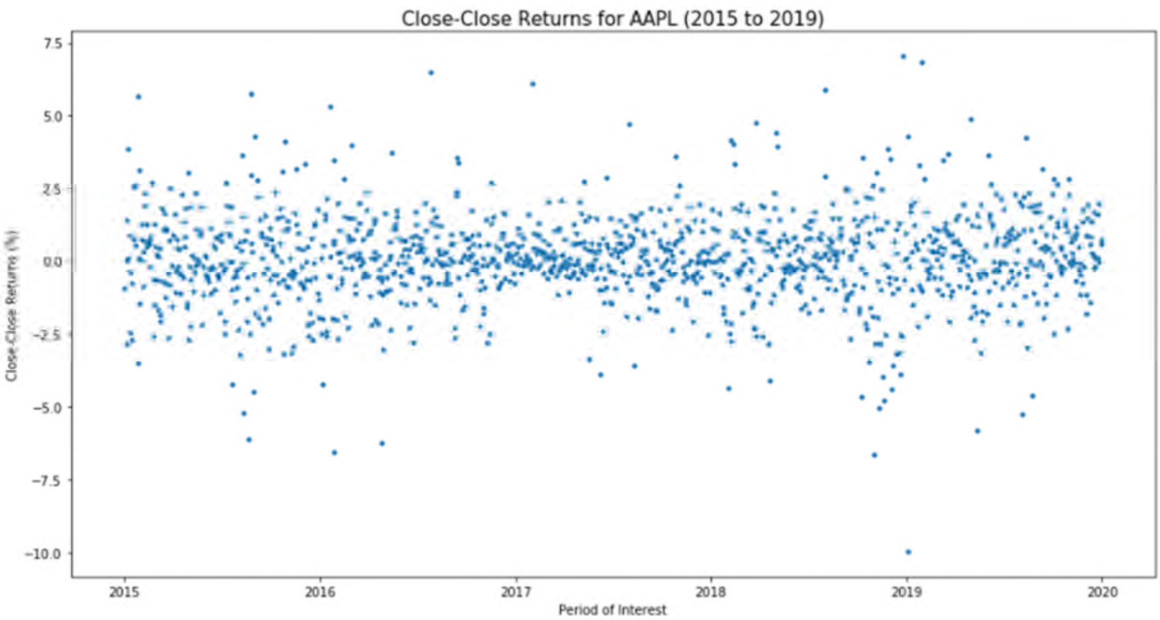
Close-Open

$$\text{Close} - \text{Open Returns}\ (\%) = \frac{Close - \ Previous\ Open}{Previous\ Open} \times 100\%$$



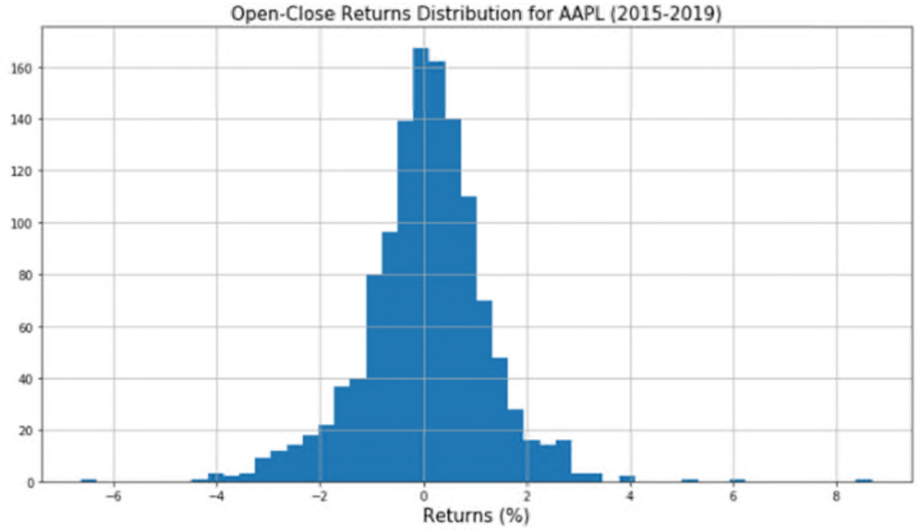
Close-Close

$$\text{Close} - \text{Close Returns}\ (\%) = \frac{Close - \ Previous\ Close}{Previous\ Close} \times 100\%$$

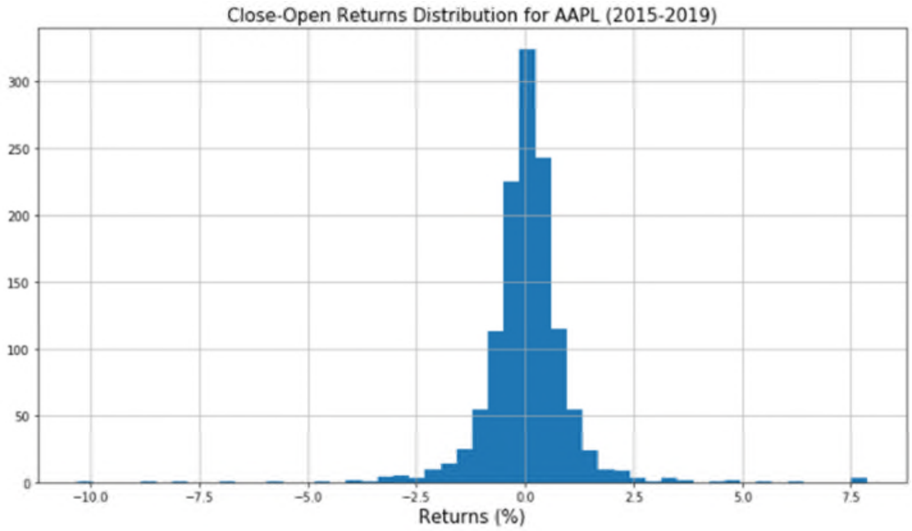


c) Histogram of Empirical Distribution of Different Returns

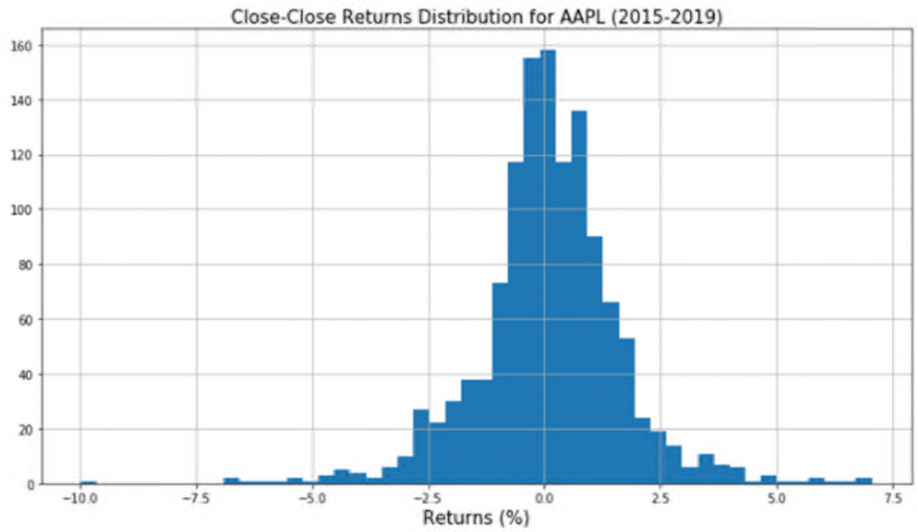
Open-Close



Close-Open



Close-Close



d) Summary Tables and Comparison

	Open-Close	Close-Open	Close-Close
Standard Deviation (%)	1.204	1.079	1.566
Skewness	0.050	-0.682	-0.229
Kurtosis	3.934	22.63	3.805

It is found that the **Open-Close** and **Close-Close** returns summary statistics are broadly similar except for the **Close-Open** returns. The latter is likely to be different due to out-of-market-hours information being priced into the AAPL opening share price, resulting in price shocks and higher trading volatilities. The gapping up/down of AAPL opening share prices from previous close causes the higher kurtosis (heavy-tailed relative to normal distribution) and absolute skew values of **22.63** and **-0.682** respectively. While containing a little bit of skew and kurtosis, the **Open-Close** and **Close-Close** returns broadly follows a normal distribution, making them useful for econometric analysis, since many tests makes the standard assumption of normality in distributions.

Part (ii): Efficient Market Hypothesis (EMH)

Three tests will be conducted here to verify whether AAPL share distribution follows random walk, namely Augmented Dicky-Fuller (ADF) and Variance Ratio Test:

$$X_t = X_{t-1} + u_t \qquad \text{or} \qquad \Delta X_t = u_t$$

a) Augmented Dicky-Fuller (ADF) Test

Given random walk formula above, ADF Test (lags = 1 and no constant terms) would be used here to test for the presence of unit root at first difference.

```
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                -26.217
P-value                         0.000
Lags                                1
-------------------------------------

Trend: No Trend
Critical Values: -2.57 (1%), -1.94 (5%), -1.62 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
```

```
                                 OLS Regression Results
=======================================================================================
Dep. Variable:                      y   R-squared (uncentered):                   0.497
Model:                            OLS   Adj. R-squared (uncentered):              0.496
Method:                 Least Squares   F-statistic:                              618.8
Date:                Sat, 07 Mar 2020   Prob (F-statistic):                   1.11e-187
Time:                        22:11:09   Log-Likelihood:                          3439.5
No. Observations:                1256   AIC:                                     -6875.
Df Residuals:                    1254   BIC:                                     -6865.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Level.L1      -1.0387      0.040    -26.217      0.000      -1.116      -0.961
Diff.L1        0.0495      0.028      1.757      0.079      -0.006       0.105
==============================================================================
Omnibus:                      128.614   Durbin-Watson:                   2.002
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              754.736
Skew:                          -0.256   Prob(JB):                    1.29e-164
Kurtosis:                       6.763   Cond. No.                         2.59
==============================================================================
```

The results above would lead to **rejection** of **null hypothesis** that the **process contains a unit root** at first difference, even at **two-tailed 1% significance level**, since the p-value is nearly zero. This likely indicates that AAPL share price follows a **random walk**.

b) Variance Ratio Test

There are **1258** data points, of which they would be segregated into 6 different intervals to perform the Variance Ratio test (de-biased to account for 2 degrees of freedom). The results have been tabulated below:

Lags	Test-Statistics	P-value
1256	-0.651	0.515
627	-0.260	0.795
312	-0.550	0.582
155	-0.700	0.484
76	-0.667	0.505
37	-0.233	0.816

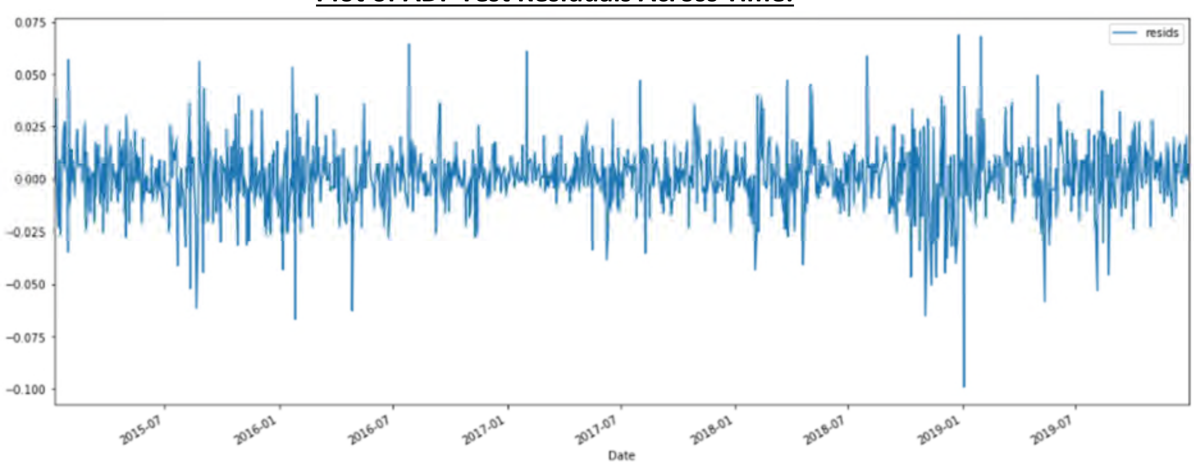
From the above results, the null hypothesis that the series is a **pure random walk cannot be rejected** for each of the 6 intervals for the Variance Ratio Test at 5% significance level, confirming the earlier ADF tests that the AAPL's Close-Close returns likely follows a random walk model.

c) Discussion

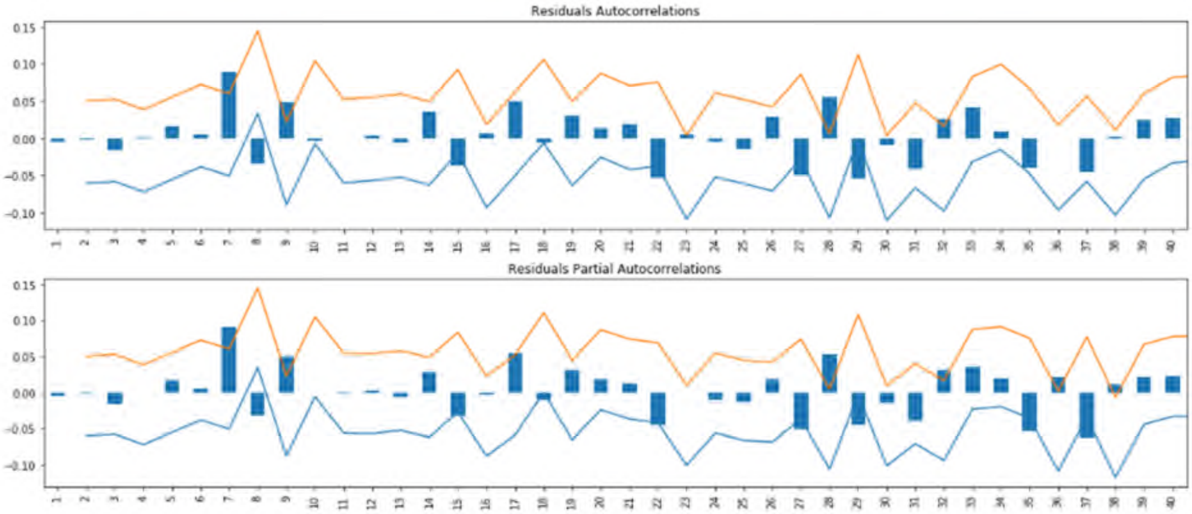
Random Walk:
The above two tests provide evidence of the random walk nature of AAPL's Close-Close Returns.

EMH:
Though random walk is consistent with EMH, to confirm its existence, randomness and non-serially correlated Close-Close returns residual would needs to be established. The residuals obtained from earlier ADF results for a random walk model can be exploited here to check for no serial autocorrelations.

Plot of ADF Test Residuals Across Time:



The next steps would include plotting **PACF** and **ACF** correlograms of the residuals to verify no autocorrelations between them:

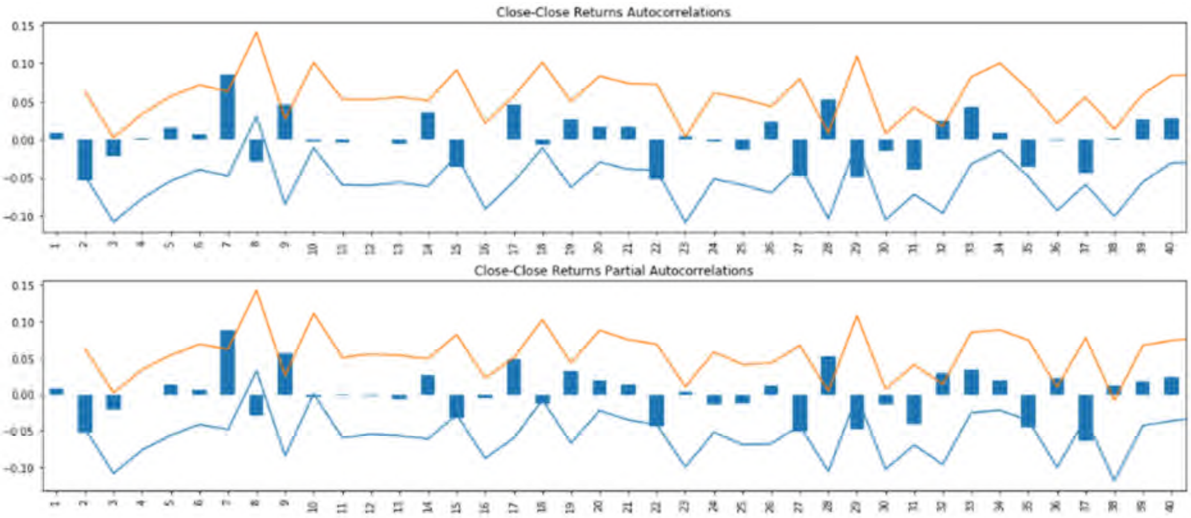


Given the residuals' autocorrelations falling within the 5% confidence intervals in the plots, this indicates **EMH is likely to be valid**, given the established randomness in the residuals from the ADF tests' results upon first differencing of the returns.

<u>Part (iii): Building your own model</u>

a) ACF and PACF

The Close-Close returns' ACF and PACF correlograms are plotted below:



b) ARMA Model

To judge the proper ARMA model, the AIC and BIC criterions would be used to judge as parameters are fitted against the AAPL's Close-Close returns

Model	Constants	AIC	BIC
ARMA(1,0)	Yes	-6883.365	-6867.953
ARMA(1,0)	No	-6881.283	-6871.008
ARMA(2,0)	Yes	-6884.906	-6864.357

ARMA(2,0)	No	-6882.372	-6866.960
ARMA(3,0)	Yes	-6883.469	-6857.782
ARMA(3,0)	No	-6880.752	-6860.203
ARMA(2,1)	Yes	-6883.345	-6857.658
ARMA(2,1)	No	-6880.645	-6860.096
ARMA(3,2)	Yes	-6884.505	-6848.544
ARMA(3,2)	No	-6882.498	-6851.674

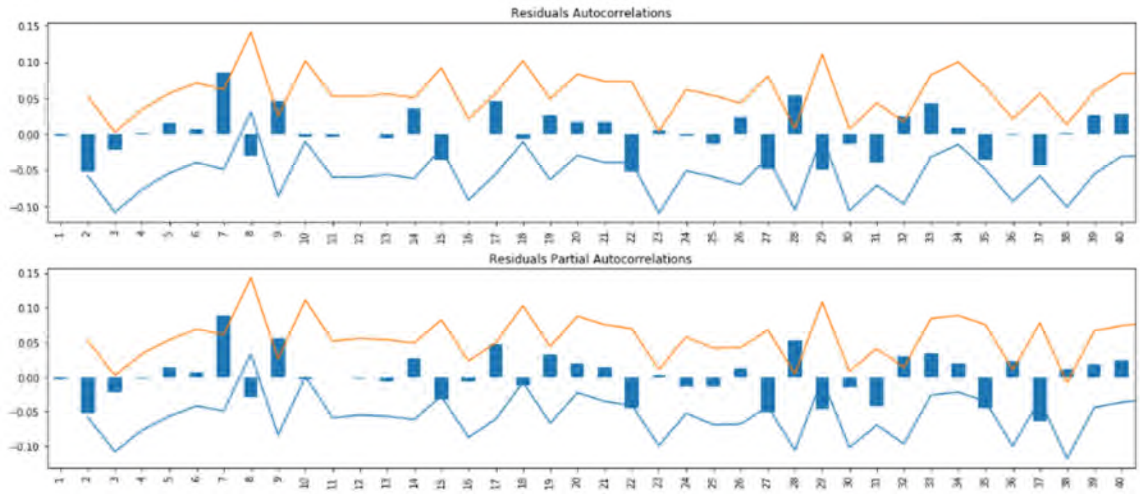
Given above AIC and BIC figures, **ARMA(1,0) without constants** would be selected as the model of choice, mostly because of earlier sections indicating it follows a random walk model, i.e. AR(1) process, and that it has the **lowest BIC values** out of all the models ran. Most of the models above had comparable **AIC values**.

The resulting model residuals' summary statistics are:

Durbin Watson Values	1.998
Heteroskedasticity ARCH F-Statistics	4.067
Heteroskedasticity ARCH F-Test P-value	5.069e-10

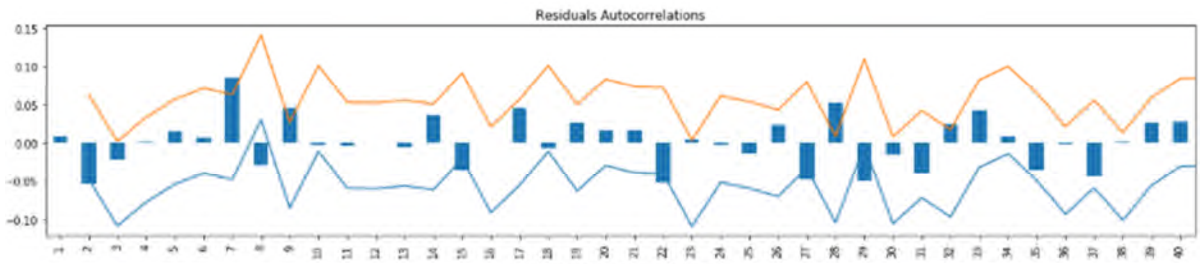
With a Durbin Watson value close to **2** (slightly less), it is found that there is **little autocorrelation between the model residuals**. However, with a heteroskedasticity arch F-test P-value of nearly zero (after correcting for 1 ddof for the ARMA(1,0) estimation), the **null hypothesis** can be rejected even at **1% significance level**, implying the **model residuals' variances are heteroskedastic**.

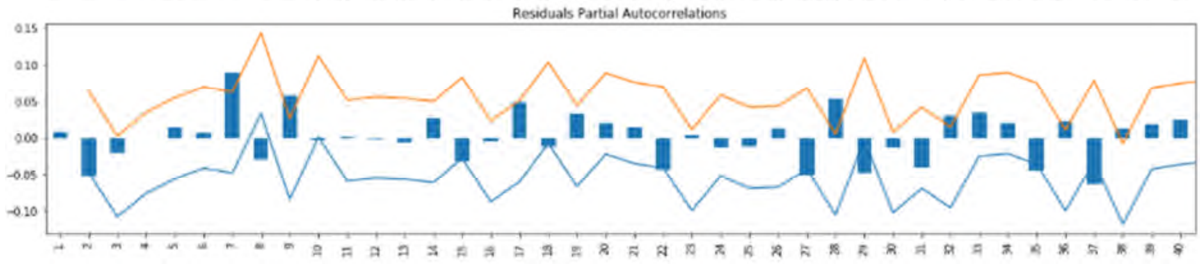
The resulting ACF and PACF correlograms are plotted below:



To confirm the validity of the model, the Q stats have been generated below. It can be seen that up to 40 lags, the p-values are all above 0.025 by some margins, indicating the **non-rejection of the null hypothesis at two tailed 5% significance level**, and hence **absence of autocorrelations between the model residuals**. Besides, with most values within the residuals' ACF and PACF correlograms falling within the 5% confidence intervals in the above plots, it can also confirm the absence of any serial correlation between residuals generated from the **ARMA(1,0)** model.

lag	AC	Q	Prob(>Q)
1.0	-0.002690	0.009127	0.923888
2.0	-0.052770	3.523355	0.171757
3.0	-0.021302	4.096499	0.251231
4.0	0.001471	4.099234	0.392743
5.0	0.015865	4.417637	0.490978
6.0	0.006565	4.472198	0.613051
7.0	0.085942	13.830654	0.054277
8.0	-0.030464	15.007499	0.059000
9.0	0.045658	17.653068	0.039419
10.0	-0.003604	17.669561	0.060800
11.0	-0.003721	17.687159	0.089129
12.0	-0.000059	17.687164	0.125525
13.0	-0.005582	17.726827	0.168170
14.0	0.035892	19.368309	0.151348
15.0	-0.035257	20.953431	0.138324
16.0	0.000785	20.954218	0.180277
17.0	0.045254	23.569937	0.131599
18.0	-0.007021	23.632950	0.167424
19.0	0.026852	24.555349	0.175705
20.0	0.016861	24.919367	0.204533
21.0	0.016452	25.266219	0.235818
22.0	-0.052748	28.834329	0.149652
23.0	0.005503	28.873197	0.184551
24.0	-0.002741	28.882850	0.224571
25.0	-0.013673	29.123170	0.258800
26.0	0.023980	29.862999	0.273326
27.0	-0.048501	32.891938	0.200660
28.0	0.054112	36.665394	0.126425
29.0	-0.049237	39.792056	0.087395
30.0	-0.013759	40.036405	0.104161
31.0	-0.039672	42.069606	0.088657
32.0	0.025198	42.890491	0.094577
33.0	0.042955	45.277955	0.075468
34.0	0.008253	45.366160	0.092079
35.0	-0.035976	47.043592	0.083956
36.0	-0.000870	47.044574	0.102929
37.0	-0.043646	49.517557	0.081814
38.0	0.002013	49.522820	0.099827
39.0	0.026594	50.442422	0.103683
40.0	0.027314	51.413327	0.106643





c) GARCH models

Various GARCH models will be tried here to establish if any is suitable, with the tables populating their respective AIC and BIC criterions:

Model	Constants	AIC	BIC	Log Likelihood
GARCH(1,1)	Yes	4572.56	4593.11	-2282.28
GARCH(1,1)	No	4588.01	4603.42	-2291.00
TARCH(1,1,1)	Yes	4379.05	4409.88	-2183.53
TARCH(1,1,1)	No	4387.62	4413.31	-2188.81
EGARCH(1,1)	Yes	4589.68	4605.10	-2291.84
EGARCH(1,1)	No	4573.33	4593.88	-2282.66

Based on the above **AIC/BIC Criterion**, it would be determined which GARCH models would be the most appropriate. Log-likelihood is provided above for reference, though larger models tend to fare better in this respect, so AIC/BIC would penalise them with a penalty term. Based on the above criterions, it seems **TARCH(1,1,1) with constants** is likely to be the appropriate model to model the dynamic variances across time. To verify its validity, a QQ plot of the residuals is made to confirm that little autocorrelation exists post-implementation of the model. A look below at the QQ plots shows the non-rejection of the AC up to 40 lags at 5% significance level, indicating the absence of any serial correlations.

However, it must be noted that, despite the fitting of **TARCH(1,1,1) on AAPL Close-Close returns**, it is likely to fare as well on **ARMA(1,0)** given the previous section showing there's little autocorrelation anyway and significant evidence of a random walk model. Also, the ARMA(1,0) has more negative AIC and BIC values, indicating least information loss vs a true model.

lag	AC	Q	Prob(>Q)
1.0	0.007854	0.077784	0.780324
2.0	-0.052918	3.611745	0.164331
3.0	-0.021872	4.215966	0.239068
4.0	0.001399	4.218442	0.377249
5.0	0.015963	4.540809	0.474442
6.0	0.007693	4.615729	0.593954
7.0	0.085690	13.919404	0.052634
8.0	-0.029000	14.985873	0.059421
9.0	0.045294	17.589406	0.040247
10.0	-0.003141	17.601932	0.062061
11.0	-0.003761	17.619917	0.090829
12.0	-0.000162	17.619950	0.127727
13.0	-0.005189	17.654230	0.171095
14.0	0.035438	19.254454	0.155452
15.0	-0.034847	20.802958	0.143244
16.0	0.000905	20.804003	0.186168
17.0	0.045181	23.411268	0.136334
18.0	-0.006221	23.460734	0.173500
19.0	0.026969	24.391237	0.181570
20.0	0.017341	24.776275	0.210125
21.0	0.016061	25.106816	0.242567
22.0	-0.052505	28.642134	0.155364
23.0	0.004083	28.672735	0.191406
24.0	-0.002834	28.683052	0.232288
25.0	-0.013446	28.915488	0.267464
26.0	0.023295	29.613679	0.283924
27.0	-0.047642	32.536307	0.212804
28.0	0.053032	36.160536	0.138555
29.0	-0.048001	39.232105	0.097297
30.0	-0.014741	39.512593	0.114655
31.0	-0.039551	41.533408	0.097964
32.0	0.025237	42.356863	0.104204
33.0	0.043319	44.784953	0.082743
34.0	0.008334	44.874891	0.100501
35.0	-0.035898	46.549046	0.091791
36.0	-0.001755	46.549041	0.111994
37.0	-0.043640	49.021314	0.089268
38.0	0.001827	49.025650	0.108498
39.0	0.026916	49.967674	0.112107
40.0	0.027504	50.952132	0.114910

Example of fitted TGARCH Model:

Zero Mean - TARCH/ZARCH Model Results

Dep. Variable:	Close	R-squared:	0.000
Mean Model:	Zero Mean	Adj. R-squared:	0.001
Vol Model:	TARCH/ZARCH	Log-Likelihood:	-2188.81
Distribution:	Standardized Student's t	AIC:	4387.62
Method:	Maximum Likelihood	BIC:	4413.31
		No. Observations:	1258
Date:	Sun, Mar 15 2020	Df Residuals:	1253
Time:	12:23:54	Df Model:	5

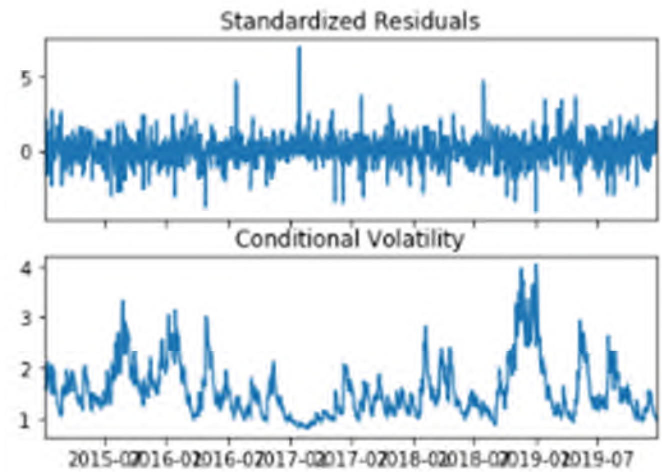
Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.0788	2.471e-02	3.188	1.434e-03	[3.034e-02, 0.127]
alpha[1]	0.0102	2.116e-02	0.482	0.630	[-3.127e-02,5.168e-02]
gamma[1]	0.1724	3.263e-02	5.284	1.263e-07	[0.108, 0.236]
beta[1]	0.8846	2.641e-02	33.492	6.283e-246	[0.833, 0.936]

Distribution

	coef	std err	t	P>|t|	95.0% Conf. Int.
nu	4.4309	0.551	8.045	8.609e-16	[3.351, 5.510]

Covariance estimator: robust



d) Discuss if model is able to provide good forecasts of future share price

Since it has been discussed that **ARMA(1,0)** likelier produce a better model than the **TARCH(1,1,1) with constants**, the discussion whether the model is able to provide good forecast of future share price would be centered on **ARMA(1,0) with constant** model.

ARMA(1,0) Model

Based on the above model, the mean absolute forecast error is calculated against the original Close-Close Returns on the period of **1st Jan 2015** to **31st Dec 2019**, giving an average of **1.103%**. Given a mean absolute returns of **1.104%**, this mean absolute forecast error is almost the same magnitude as the mean absolute returns, indicating the lack of predictive power of the model.

To test the efficacy of the model on unseen data, the forecast error is then computed against **1st Jan 2020** to **31st Jan 2020**, a period outside the modelled period. The forecast errors were approximately **1.381%**, slightly more than modelled period of **1.103%**. As the period stretch further in the future, it likely means the predictive power of the time series model made will tend to wane, i.e. in line with what one would expect.

Sample of 1st January to 31st January AAPL Data:

Date	High	Low	Open	Close	Volume	Adj Close
2019-12-31	293.679993	289.519989	289.929993	293.649994	25201400	292.954712
2020-01-02	300.600006	295.190002	296.239990	300.350006	33870100	299.638885
2020-01-03	300.579987	296.500000	297.149994	297.429993	36580700	296.725769
2020-01-06	299.959991	292.750000	293.790009	299.799988	29596800	299.090149
2020-01-07	300.899994	297.480011	299.839996	298.390015	27218000	297.683533

Computed Returns:

```
Date
2020-01-02     0.022816
2020-01-03    -0.009722
2020-01-06     0.007968
2020-01-07    -0.004703
2020-01-08     0.016086
2020-01-09     0.021241
2020-01-10     0.002261
2020-01-13     0.021364
2020-01-14    -0.013503
2020-01-15    -0.004286
2020-01-16     0.012526
2020-01-17     0.011071
2020-01-21    -0.006777
2020-01-22     0.003570
2020-01-23     0.004816
2020-01-24    -0.002882
2020-01-27    -0.029405
2020-01-28     0.028289
2020-01-29     0.020932
2020-01-30    -0.001449
2020-01-31    -0.044339
Name: Close, dtype: float64
```