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A New Planar Circle-based Approach for Camera Self-calibration

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Abstract

According to the projective property of curve of the second order, a new camera self-calibration approach based on planar circle is proposed. Using the projective relations between lines that link the points with others of a circle and the projective invariance of the cross-ratio, image coordinates of circular points are solved. And constrains of the camera intrinsic parameters are established according to the property of circular points. This approach only requires the camera to take a few (at least three) photos of the planar pattern from different unknown orientations. This planar pattern includes a circle and its any three diameters. Then all the intrinsic parameters are solved linearly. The results of simulated data and real data verify the better accuracy and the better robustness of the algorithm.

Keywords: Computer Vision; Camera Calibration; Conic Curve; Circular Point; Absolute Conic

1. Introduction

Three-dimensional (3D) reconstruction is a process of obtaining information of 3D spatial points from two-dimensional (2D) image points. The relations between the coordinates of 2D image points and the coordinates of corresponding 3D measured points are determined by geometry models of the camera imaging. The parameters of these geometry models are camera parameters. Solving the parameters of these geometry models is called camera calibration[1].

Related work. Generally speaking, there are two camera calibration approaches, namely tradition calibration and self calibration. Where self-calibration approaches have better flexibility require only images information without the need for points matching and physical measurement of the calibration objects. Essentially, a variety of self-calibration approaches are solving Kruppa equations based on the nature of absolute conic and polar line transformation, stratified calibration approaches and calibration approaches based on absolute quadric and so on[2]. For example in the literatures [3][4][5][6][7], however many of which are require to solve nonlinear equations and have complex calculation. Since 2000, circular points are first introduced into camera calibration by Meng and others [8], we have seen the emergence of many camera self-calibration approaches based on circular points, such as literatures [8][9][10]. Most of them are based on the Laguerre theorem in projective geometry, solving image coordinates of circular points through the harmonic relation between the vanishing points which are orthogonal and the circular

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points. However the calibration process requires more stringent about the orthogonality of vanishing points, and sometimes it is hard to implement.

This paper proposes a more flexible approach solving camera intrinsic parameters linearly based on planar circle. Using the projective relations between the lines that link the points with others of a circle and the projective invariance of the cross-ratio, the image coordinates of circular points are solved. And then the five intrinsic parameters are solved linearly [11]. The calibration pattern is a circle which includes any three diameters. The process of calibration requires only the image coordinates of intersection points of the diameters and the circle, and it does not need to know coordinates of the center of the circle and physical measurement of the calibration objects and the directions of camera motion, so the flexibility of calibration is increased.

2. Calibration Principles and Methods

2.1. Pinhole Imaging Model

Shown in Fig.1, any point P in the world coordinate system (WCS), which homogeneous coordinates is $(x_w, y_w, z_w, 1)^T$, and p is its corresponding image point in the imaging plane, which homogeneous coordinates is $(u, v, 1)^T$. In the camera pinhole imaging model, the projection formula of the spatial point and its image point is:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_u & s & u_0 \\ 0 & f_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} [R, T] \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} = K[R, T] \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} \tag{1}$$

where λ is a scale factor. K is camera intrinsic parameters matrix, (u_0, v_0) are the coordinates of the principal point and (f_u, f_v) are the effective focal length and s is the skewness of the two axis of coordinates. $[R, T]$ are camera extrinsic parameters matrix, which are the rotation and translation from the WCS to the camera coordinate system.

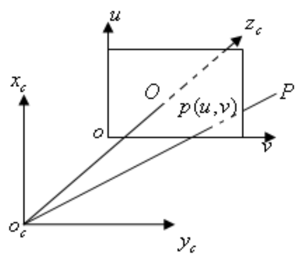

Fig.1 Pinhole Imaging Model

2.2. Circular Points and Its Nature

Without loss of generality, assume that pattern plane Π locates in the $x-y$ plane of WCS, thus the equation of Π is $z=0$. In the homogeneous coordinates system (HCS), $(x,y,0,w)^T$ denotes any appoint in plane Π. According to projective geometry theory, circular points are the intersection of the line at infinity and any circle in the same plane. Let $I=(1 \quad i \quad 0 \quad 0)^T$, $J=(1 \quad -i \quad 0 \quad 0)^T$ be the circular points in plane Π. It is easy to show that the circular points are conjugate points on the absolute conic $(X^T X=0)$ in the plane at infinity.

Let m be the image of point X on the absolute conic, according to (1), one gets the following equation:

$$m^T K^{-T} K^{-1} m = 0 \tag{2}$$

So the images of points of absolute conic are really a conic, which relates only to camera intrinsic parameters. If $m_i=(x_r+x_i i, y_r+y_i i, 1)^T$, $m_j=(x_r-x_i i,\ y_r-y_i i, 1)^T$ are the images of I,J respectively, then m_i, m_j are on the images of the absolute conic, one has the following equation:

$$\begin{cases} m_i^T \omega m_i = 0 \\ m_j^T \omega m_j = 0 \end{cases} \tag{3}$$

where $\omega = K^{-T}K^{-1}$. Because m_i, m_j are a pair of conjugate points under projective transformation, so only two constraint equations can be gotten form foregoing equations, that are:

$$\begin{cases} \mathrm{Re}(m_i^T \omega m_i) = 0 \\ \mathrm{Im}\left(m_i^T \omega m_i\right) = 0 \end{cases} \tag{4}$$

2.3. Determining the Image Coordinates of Circular Points

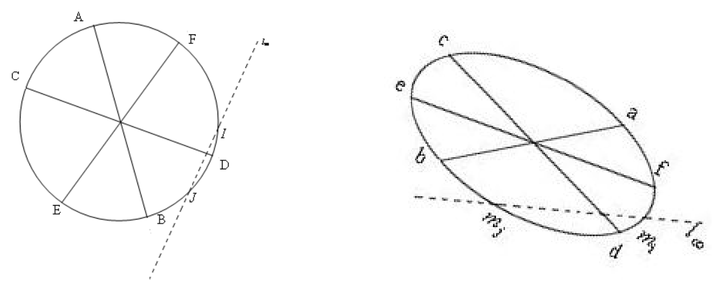

Fig.2 Planar Circle and Its Image

Any circle O in the plane Π is shown as Fig.2, there are any three diameters AB, CD, EF. l_∞ is the line at infinity of the plane Π. The intersections of l_∞ and circle O are I, J. The corresponding imaging points of points A,B,C,D,E,F,I,J are $a(u_a,v_a)^T$, $b(u_b,v_b)^T$, $c(u_c,v_c)^T$, $d(u_d,v_d)^T$, $e(u_e,v_e)^T$, $f(u_f,v_f)^T$,

$m_i\left(x_r+x_i i, y_r+y_i i\right)^T$, $m_j\left(x_r-x_i i, y_r-y_i i\right)^T$. Let $p_1\left(u_{p1}, v_{p1}\right)^T$, $p_2\left(u_{p2}, v_{p2}\right)^T$, $p_3\left(u_{p3}, v_{p3}\right)^T$, $p_4\left(u_{p4}, v_{p4}\right)^T$ be the vanishing points of ad, ac, ed, ec. According to elementary geometry and projective geometry, one has the following equations:

$$\begin{cases} p_1 = l_{ad} \times l_{bc} \\ p_2 = l_{ac} \times l_{bd} \\ p_3 = l_{ed} \times l_{cf} \\ p_4 = l_{ec} \times l_{df} \end{cases} \tag{5}$$

According to the projective property of curve of the second order in projective geometry, if I, J, C, D are four fixed points of circle O, one has the equations:

$$\left(l_{AI} l_{AJ}, l_{AC} l_{AD}\right) = \left(l_{BI} l_{BJ}, l_{BC} l_{BD}\right) = \left(l_{EI} l_{EJ}, l_{EC} l_{ED}\right) = \left(l_{FI} l_{FJ}, l_{FC} l_{FD}\right) \tag{6}$$

Thereinafter the equation can be gotten based on the projective of cross-ratio as follows:

$$\left(m_i m_j, p_1 p_2\right) = \left(m_i m_j, p_2 p_1\right) = \left(m_i m_j, p_3 p_4\right) = \left(m_i m_j, p_4 p_3\right) \tag{7}$$

So, there are:

$$\begin{cases} \left(u_{p1}+u_{p4}-u_{p2}-u_{p3}\right)\left(x_r^{\,2}+x_i^2\right)+2\left(u_{p2}u_{p3}-u_{p1}u_{p4}\right)x_r = u_{p1}u_{p2}\left(u_{p3}-u_{p4}\right)+u_{p3}u_{p4}\left(u_{p2}-u_{p1}\right) \\ x_r^{\,2}+x_i^2-\left(u_{p1}+u_{p2}\right)x_r = -u_{p1}u_{p2} \\ x_r^{\,2}+x_i^2-\left(u_{p3}+u_{p4}\right)x_r = -u_{p3}u_{p4} \end{cases} \tag{8}$$

then m_i, m_j can be obtained by solving (8).

2.4. Solving Camera Intrinsic Parameters

Suppose $\omega = K^{-T}K^{-1} = \begin{bmatrix} c_1 & c_2 & c_3 \\ c_2 & c_4 & c_5 \\ c_3 & c_5 & c_6 \end{bmatrix}$, it is a symmetric matrix. Let C be a 6 dimensional column vector, and $C = \left(c_1, c_2, c_3, c_4, c_5, c_6\right)^T$. According to (4), one has the equation as follows:

$$\begin{bmatrix} x_r^{\,2}-x_i^2 & 2\left(x_r y_r - x_i y_i\right) & 2x_r & y_r^{\,2}-y_i^2 & 2y_r & 1 \\ x_r x_i & x_r y_i + x_i y_r & x_i & y_r y_i & y_i & 0 \end{bmatrix} C = 0 \tag{9}$$

We take n clear-cut photos of the planar pattern from n $\left(R^{(i)}, T^{(i)}\right)$ $\left(i = 1, 2\cdots\cdots, n\right)$ different orientations. Then the image coordinates of n pairs circular points can be obtained, which are denoted by $m_i^{(i)}, m_j^{(i)}$ $\left(i = 1, 2\cdots\cdots, n\right)$, and n equations as (9) can be gotten. Simultaneous these n equations, one gets the equations:

$$AC = 0 \tag{10}$$

Where A is a $2n\times 6$ matrix. If $n\geq 3$, the unique solution C is solved from (10), so ω can be gotten. Then the camera intrinsic parameters matrix is obtained by taking ω Choleskey factorization and normalizing the last element of the invertible matrix K^{-1}.

The Algorithm of camera calibration is as follows:

Step1. Print the planar pattern including a circle and its any three diameters and attach it on the adamant plane.

Step2. Take at least three images of the plane circle from different orientations by moving either camera or pattern.

Step3. Extract coordinates of intersection points of the diameters and ellipse in images according to Harris corner detector [12].

Step4. Solve coordinates of vanishing points according to (5).

Step5. Solve image coordinates of circular points in images according to (8).

Step6. Solve the image of absolute conic ω according to (10) and solve the camera intrinsic parameters by taking ω Choleskey factorization and normalizing the last element of the invertible matrix K^{-1}.

3. Experiments

3.1. Simulation Experiment

The simulated camera has the following property: $f_u=1500$, $f_v=1500$, $s=0.2$, $u_0=640$, $v_0=512$, the image resolution is 1280×1024. The relative coordinator locations of calibration circle in plane Π are that the coordinates of the center of circle O is $(50,50)^T$ and the radius of circle O is 30, and the included angles between diameters AB, CD, EF and positive direction of X axis are respectively 131.81°, 70.53°, 0°. The spatial coordinates of points are estimated, the imaging coordinates of these points are computed according to the camera parameters foregoing sets. Then the camera intrinsic parameters are solved through the foregoing algorithm. Gaussian random noises with 0 mean and σ (0~7.2) standard deviation are added to the projected image points in order to verify the robustness of the algorithm. For each noise level, the estimate value of each parameter is the average value of fifty reciprocal independent experiments. The results of simulation experiments with varying noise levels are shown in Table 1. Fig. 3 is the curve where the standard deviation of f_u, f_v, s, u_0 and v_0 change with the varying noise levels, which indicates the algorithm this paper proposed has the better accuracy and the better robustness. Although the noise levels are sizeable, the standard deviations of each parameter are still small.

In order to verify the impact of the algorithm with the number of the images increasing, simulation experiment is given different number of the images from 3 to 20. For each image, the intrinsic parameters are fixed and the extrinsic parameters are random. For each number, Gaussian random noise with 0 mean and 0.3 standard deviations is added to image points, and the results are shown in Fig.4, which indicates the algorithm has the better stability. The absolute errors of parameters are becoming smaller and smaller as the number of images increasing.

Table 1 Calibration Results of the Computer Simulations with Varying Noise Levels

σ	f_u	f_v	s	u_0	v_0
0	1500.0000	1500.0000	0.2000	640.0000	512.0000
0.6	1499.9943	1499.9742	0.1789	640.3149	512.4071
1.2	1500.1139	1500.0687	0.1880	640.1378	512.1832
1.8	1499.7115	1499.4825	0.1878	640.4540	512.5170
2.4	1499.4072	1498.9988	0.1830	640.6404	512.7380
3.0	1499.5653	1499.0642	0.1628	640.7246	512.9014
3.6	1499.4736	1498.8754	0.1557	640.8690	513.0796
4.2	1499.3804	1498.6861	0.1488	641.0133	513.2571
4.8	1498.9941	1497.9674	0.1292	641.6809	514.0239
5.4	1498.7234	1497.3382	0.0999	641.6643	514.1407
6.0	1499.0911	1498.1151	0.1290	641.4453	513.7853
6.6	1499.1236	1498.0922	0.1198	641.5536	513.9314
7.2	1498.8903	1497.7317	0.1164	641.7324	514.1340

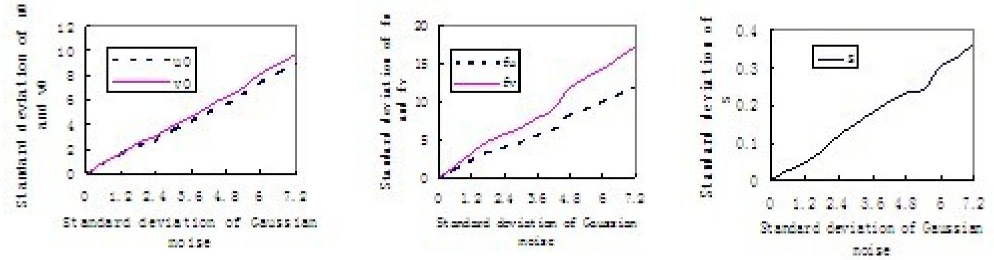

Fig.3 Standard Deviations of Camera Parameters with Varying Noise Levels

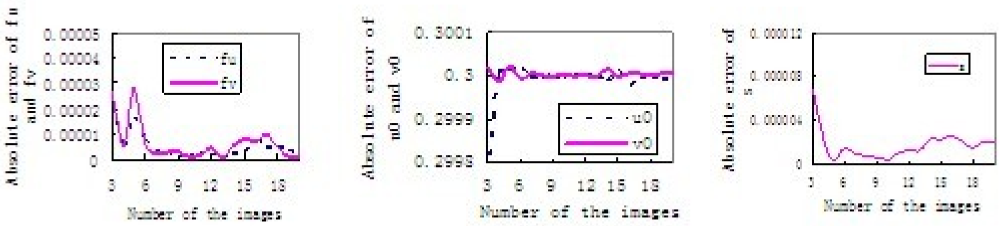

Fig.4 Absolute Errors of Camera Parameters with Varying Number of Images

3.2. Real Experiment

The real experiment uses a CCD digital camera, the image resolution is 640×480. We print the planar pattern including a circle and its any three diameters and attach it on the adamant plane. The diameter of the circle is 6cm. We take five photos from different orientations, shown as Fig.5. We extract coordinates of intersection points of the diameters and ellipse using Harris corner operator, five intrinsic parameters solved by the approach which this paper proposes and the approach which the literature [13] proposed. Comparisons of the results of the two approaches are shown in Table 2.

Fig.5 Real Images of Planar Circle

Table 2 Comparisons of the Results of the Two Approaches

Parameters	f_u	f_v	s	u_0	v_0
Our approach	870.9526	841.6010	1.0070	331.7986	223.8942
Literature[13]	847.1182	845.0571	3.3282	308.7238	267.5337

4. Conclusion

Based on the camera self-calibration approaches using circular points[14], this paper proposes a new a new camera self calibration approach based on planar circle pattern, which includes any three diameters. The process only require the camera to take a few (at least three) photos of the pattern from different unknown orientations and does not to know physical measurement of the pattern. The intrinsic parameters are solved by complete linearization techniques. This approach which solves the coordinates only based on fundamental geometry properties of the pattern is very flexible, and it applies any curve of the second order as soon as the coordinates of the vanishing points can be obtained.

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