Pin-hole Modelled Camera Calibration from a Single Image

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Camera calibration from a single image is of importance in computer vision. It requires a priori scene information. In recent years, some methods have been studied to solve this problem. One approach is to adopt the idea of using vanishing points and vanishing lines. Circles and balls are also introduced for the calibration target. Besides, symmetry and coplanar points and lines can be applied as the constraints for calibration. This survey is concerned with research on single image calibration and contains the annotations for milestone publications.

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1. INTRODUCTION

Classical camera calibration methods require more than one image: they depend or partly depend on the relationship between images. For example: [Hartley 1994], [Hartley and Zisserman 2003], [Zhang 2000]. Since camera calibration is essential for 3D reconstruction, classical methods lack the power to deal with single images from unknown sources such as the Internet or magazines.

Another direction is camera calibration from only one image. Unlike camera calibration from multiple images, previously known geometrical information in a single image is always required. Several kinds of information have already been studied.

Camera calibration using vanishing points and vanishing lines is the earliest to be proposed. Caprile and Torre [1990] were the first to propose this method. Caprile and Torre [1990] used three vanishing points. In 2000, Guillou et al. [2000] proposed a method of using two vanishing points and a known line segment. Wilczkowiak et al. [2001] and Wilczkowiak et al. [2005] replace the cube, which is used in Caprile and Torre [1990] to generate three vanishing points, with parallelepipeds. Avinash and Murali [2008] used a rectangular prism instead.

Circles and balls are also used to compute the intrinsic parameters. Fremont and Chellali [2002] introduced the use of two concentric circles. Teramoto and Xu [2002] proposed the use of three balls. Chen et al. [2004] and Chen et al. [2008] proposed the use of two coplanar circles. Colombo et al. [2006] used two coaxial circles. Sometimes in this approach vanishing points and vanishing lines are also used to perform additional constraints.

Other approaches have also been proposed: Hong et al. [2004] proposed the usage of symmetry, Shang et al. [2004] proposed the application of four coplanar control lines and Park [2007] proposed the usage of three coplanar points.

Note that in this survey, camera calibration means the computation of intrinsic parameters. Besides, it is widely accepted that the modern CCD camera has square pixels, zero skew factor and the fixed principal point which lies at the centre of the image. Therefore, if the method is only about the computation of focal length, it is also considered as a calibration method.

2. OVERVIEW OF THE RESEARCH

This chapter surveys the research on camera calibration methods from a single image. These methods use different approaches: calibration based on vanishing points and vanishing lines, calibration based on circles, calibration based on the collaboration of vanishing points/lines and circles, calibration based on symmetry, calibration based on four coplanar control lines and calibration based on three known coplanar points.

2.1 Calibration Based on Vanishing Points and Vanishing Lines

The first idea of adopting vanishing points to calibrate the camera was proposed by Caprile and Torre [1990]. It inspired by the usage of vanishing points for other targets, such as locating objects in space or recovering the extrinsic parameters of a pair of images ([Odone et al. 1986], [Nagy and Falsafi 1988] and [Jiang and Chen 1987]).

The method proposed by Caprile and Torre [1990] is based on the view of a cube, whose orientation is set at 45 degree in order to achieve optimized accuracy. Three vanishing points can be retrieved from the image of the cube and the intrinsic parameters can be calculated from the property of the vanishing points.

It has been many years since the work of Caprile and Torre [1990]. Then in the year of 2000, Guillou et al. [2000] extended the work of Caprile and Torre [1990] in the field of camera calibration from a single image. This camera calibration technique uses one image without any calibration pattern. The assumptions for the proposed method, claimed by Guillou et al. [2000], are: a single real image which contains two vanishing points and a known line segment, the principal point of which lies in the centre of the image and the user-fixed aspect ratio.

In the same year, the work of Svedberg and Carlsson [2000], which is derived from the work of Caprile and Torre [1990], was also proposed. That method is to use the seemingly orthogonal wedge, which is defined as two rectangular planes intersecting at right angles, as a reference structure. Such a wedge is assumed to behave the way straight-angled corner do. Vanishing points are extracted from the projections of the wedge. The determination of vanishing points brings three quadratic constraint equations for the unknown camera parameters. Thus it is possible to imagine new views. The way of choosing a reference structure to imagine new views is equivalent to camera calibration.

In the next year, the work of Wilczkowiak et al. [2001] was proposed as the expansion of the work of Caprile and Torre [1990]. It uses parallelepipeds rather than cubes, which are a subclass of parallelepipeds, to calibrate the camera. The use of parallelepipeds is claimed to be an improvement since computing vanishing points is numerically unstable, and dealing with individual vanishing points does not allow fully exploiting the redundancy contained in all of the vanishing points which stem from the projection of a single parallelepiped. Wilczkowiak et al. [2001] states that the proposed method does not compute explicitly vanishing points but first estimates the best projection matrix such that the defined image points correspond to a parallelepiped in space. From this projection matrix and a priori knowledge of the parallelepiped, such as angles, length ratios and so on, the constraints on the intrinsic parameters can be derived. The authors claim that most of these constraints are linear. These constraints are then used to compute intrinsic parameters.

In the year of 2002, the work of Deutscher et al. [2002] was proposed. It assumes the image satisfies a Manhattan world, in which edges lie predominantly in three orthogonal directions. Deutscher et al. [2002] claim that this method exploits the orthogonality constraint explicitly to compute a joint estimate of the three vanishing points rather than trying to enforce orthogonality and estimate the intrinsic parameters after the vanishing points have been found.

Another calibration method, which registers two planes in the scene and uses vanishing points as assistance, was also proposed by Kushal et al. [2002]. Assuming that in the image, the two world planes are orthogonal and their common axis of intersection is visible, two homography relationships can be obtained. Thus the intrinsic parameters can be calculated by the properties of homography. If an axis is invisible, then by introducing vanishing points, the effect of that axis can be ignored and the camera intrinsic parameters can also be calculated.

In the year of 2005, the work of Wilczkowiak et al. [2005] was proposed as a follow up to that of Wilczkowiak et al. [2001]. Wilczkowiak et al. [2005] proposes the camera calibration using geometric constraints through parallelepipeds. By adding the ideas of normalization and factorization, the intrinsic parameters can be computed and the errors can be suppressed. Wilczkowiak et al. [2001] also discusses the singularities brought by the factorization: it lists every possible solution under different environments and then determines and presents the certain and unique solution to every situation.

The calibration method of using three vanishing points of orthogonal directions was proposed by Grammatikopoulos et al. [2007] who state that camera calibration is achieved iteratively through a simultaneous least-squares adjustment of all image points belonging to lines, which intersect in the three dominant vanishing points of the scene, on the assumption that these correspond to orthogonal space directions.

In the same year, Wu et al. [2007] proposed the calibration method by adopting RANSAC algorithm, vanishing points and vanishing lines. The vanishing points, the intersection of two lines, can perform an equation set which contains the intrinsic parameters. Besides, the vanishing lines, the intersection of two planes, can use another equation set which also contains the intrinsic parameters. This ensures the unique solution of the intrinsic parameters. The RANSAC algorithm is also adopted to eliminate the outliers and to cull the input points to improve the accurateness of estimation. Besides, foot-to-head homology is used to converge the symmetric transfer errors.

The latest work on this approach is the work of Avinash and Murali [2008] who propose a method of camera calibration from a single image containing a rectangular prism. The rectangular prism is used to generate two vanishing points for camera calibration. Then fixing up the picture plane and fixing up the station point are carried out. Thus the intrinsic parameters can be calculated from the formed equation.

Year	Authors	Title	Contribution
1990	Caprile and Torre	Using Vanishing Points for Camera Calibration	Propose the idea of adopting three vanishing points for camera calibration from a single image.
2000	Guillou et al.	Using Vanishing Points for Camera Calibration and Coarse 3D Reconstruction from a Single Image	Propose the idea of using a single image containing two vanishing points and a known line segment.
2000	Svedberg and Carlsson	Calibration, Pose and Novel Views from Single Images of Constrained Scenes	Introducing the orthogonal wedge for the camera calibration.
2001	Wilczkowiak et al.	Camera Calibration and 3D Reconstruction from Single Images Using Parallelepipeds	Introducing the parallelepipeds. It is the extension of Caprile and Torre [1990].
2002	Deutscher et al.	Automatic Camera Calibration from a Single Manhattan Image	Restrict the orthogonality by adopting Manhattan images.
2002	Kushal et al.	A Simple Method for Interactive 3D Reconstruction and Camera Calibration from a Single View	Propose the calibration method by registering two planes in the scene and using vanishing points as assistance.
2005	Wilczkowiak et al.	Using Geometric Constraints Through Parallelepipeds for Calibration and 3D Modeling	Propose a more thorough study of parallelepipeds.
2007	Grammatikopoulos et al.	An Automatic Approach for Camera Calibration from Vanishing Points	Calibrate the camera through the points in lines intersecting at vanishing points.
2007	Wu et al.	Robust Self-calibration from Single Image Using RANSAC	Adopt RANSAC algorithm, vanishing points and vanishing lines for camera calibration.
2008	Avinash and Murali	Perspective Geometry Based Single Image Camera Calibration	Introduce a rectangular prism as calibration pattern.

Table I. Major Contributions in Camera Calibration based on Vanishing Points/Lines

2.2 Calibration Based on Circles and Balls

The first work which uses circles in a single image to calibrate the camera is the work of Fremont and Chellali [2002], who proposes the method of calibrating the camera using one image of two concentric circles of known radii. The assumptions of zero skew, unit aspect ratio, no distortions and square pixels are made. First, the perspective projections of concentric circles, which are ellipses in a classic configuration, are detected. Then the coefficients of the conic (the ellipse) are estimated. Such coefficients are functions of intrinsic and extrinsic parameters as well as the projected circle centre [Kanatani and Liu 1993]. Finally, from the n image points of the estimated ellipse, the system can be solved.

A similar attempt is proposed by Teramoto and Xu [2002]: camera calibration from a single view which contains three balls. The authors state that the camera intrinsic parameters can be represented as an image of the absolute conic. The implementation consists of following steps: (1) extract boundary for each ball and sample 40-60 points for each boundary, (2) fit each point set by a conic, (3) choose

Year	Authors	Title	Contribution
2002	Fremont and Chellali	Direct Camera Calibration Using Two Concentric Circles from a Single Image	Calibrate the camera with two concentric circles of known radii.
2002	Teramoto and Xu	Camera Calibration by a Single Image of Balls: from Conics to the Absolute Conic	Calibrate the camera with three balls.
2004	Chen et al.	Camera Calibration with Two Arbitrary Coplanar Circles	Calibrate the camera with two coplanar circles.
2005	Jiang and Quan	Detection of Concentric Circles for Camera Calibration	Propose a way of detecting concentric circles and form a correspondent calibration method.
2006	Colombo et al.	Camera Calibration with Two Arbitrary Coaxial Circles	Calibrate the camera with two coaxial circles.

Table II. Major Contributions in Camera Calibration based on Circles and Balls

the ball that is the forest from the image centre and estimate the focal length, (4) normalize the image (points) with the focal length determined in (3), (5) estimate the relative size and position of each ball, (6) optimally estimate all of the parameters in the normalized image.

Another method was proposed by Chen et al. [2004]: camera calibration with two arbitrary coplanar circles, even if the centres of the circles and/or part of the circles are occluded. After viewing the scene containing two coplanar circles, an image with two ellipses can be obtained. Then two oblique elliptical cones can be formed from the detected ellipses if the focal length is correct. So that the focal length can be determined by making sure each of the formed cones is stable and is similar to the surfaces of the real cones.

In the year of 2005, the work of Jiang and Quan [2005] was proposed as the improvement of Fremont and Chellali [2002]: camera calibration by using planar pattern of pairs of concentric circles. The authors introduce a geometric method that constructs a sequence of points strictly convergent to the image of the circle centre from an arbitrary point. The method automatically detects the points of the pattern features by the construction method, and identify them by invariants. It then takes advantage of homological constraints to consistently and optimally estimate the features in the image. Then the complete system of practical and automatic calibration method can be constructed.

In the next year, the work of Colombo et al. [2006] was proposed: camera calibration with two arbitrary coaxial circles. This method is based on the automatic localization of a surface of revolution in an single image. Colombo et al. [2006] states that fixed entities of the imaged surface of revolution are strictly related to the matrix of intrinsic parameters. Since cross sections are parallel circles in 3D, they intersect at the circular points of the families of planes orthogonal to the surface of revolution symmetry axis. Besides, their projection in the image are also related to the image of the absolute conic. So that four linear constraints can be used to compute the equation set which contains four unknowns. The assumptions of zero skew factor and unit aspect ratio are made.

Year	Authors	Title	Contribution
2005	Wang et al.	Camera Calibration and 3D Reconstruction from a Single View Based on Scene Constraints	Camera calibration from two line segments with equal length or known length ratio, circle and a vanishing point.
2008	Chen et al.	Full Camera Calibration from a Single View of Planar Scene	Calculate all intrinsic parameters from two coplanar circles and vanishing points/lines.

Table III. Major Contributions in Camera Calibration based on the Collaboration of Vanishing Points/Lines and Circles

2.3 Calibration Based on the Collaboration of Vanishing Points/Lines and Circles

Wang et al. [2005] proposed the following method: camera calibration from a single image which contains a man-made environment. The man-made environment is defined as one containing three orthogonal principal directions. Wang et al. [2005] state that the proposed method is inspired by Caprile and Torre [1990]. The authors propose that two line segments with equal length or known length ratio on an object can provide an additional independent constraint of the image of the absolute conic. They use this constraint to compute the image of circular points. Four independent linear constraints on the image of absolute conic can be obtained from the image of circular points and a vertical vanishing point, which is the image perpendicular to the plane of the image of absolute conic. Then an equation set, in which camera calibration coefficients can be retrieved, can be acquired.

Recently, the work of Chen et al. [2008] proposes a method of camera calibration from a single image containing two coplanar circles. The authors claim that it is an improvement on the work of Chen et al. [2004]: it calculates not only the focal length but also the principal point coordinates and the aspect ratio. The calibration steps are listed: first, calibration-free planar rectification [Ip and Chen 2005] is performed and then extended to recover the vanishing line, the centres of circles and orthogonal vanishing point pairs; then the distribution of the focal length are computed from all orthogonal vanishing point pairs; next, a statistical optimization routine is performed to estimate the focal length, the principal point and the aspect ratio simultaneously.

2.4 Calibration Based on Symmetry

The use of symmetry in a single image to calibrate the camera was proposed by Hong et al. [2004]. There are three steps in this method. The first is the calibration from translational symmetry. The second is the calibration from reflective symmetry. Hong et al. [2004] states that if the reflective symmetry is along three mutually orthogonal directions, this method is similar to the first subsection. The last step is the calibration from rotational symmetry.

In detail, Hong et al. [2004] focus on the fundamental matrix and construct an equation under the epipolar constraints between pairs of hidden images. Then as described above, they analyze it in three steps. In the first step, rotation is avoided; and since three mutually orthogonal translations are given, three linear equations can be determined to solve intrinsic parameters. Hong et al. [2004] claim that such three orthogonal translations correspond precisely to the notion of "vanishing point"

Year	Authors	Title	Contribution
2004	Hong et al.	On Symmetry and Multiple-View Geometry: Structure, Pose, and Calibration from a Single Image	Introduce the symmetry in the image for calibration.

Table IV. Major Contributions in Camera Calibration based on Symmetry

Year	Authors	Title	Contribution
2004	Shang et al.	Analytical Method for Camera Calibration from a Single Image with Four Coplanar Control Lines	Adopt four coplanar lines in the image for calibration.

Table V. Major Contributions in Camera Calibration based on Four Coplanar Control Lines

Year	Authors	Title	Contribution
2007	Park	Quaternion-Based Camera Calibration and 3D Scene Reconstruction	Adopt three known coplanar points in the image for calibration.

Table VI. Major Contributions in Camera Calibration based on Three Known Coplanar Points

or "vanishing line" as defined in Criminisi et al. [2000]. The second step is similar to the first one. In the last step, the calibration matrix satisfies the normalized Kruppa's equation. If only the focal length is unknown, it can be retrieved from the equation derived from the rotational symmetry.

2.5 Calibration Based on Four Coplanar Control Lines

A method of camera calibration from a single view with four coplanar control lines was proposed by Shang et al. [2004], who use control lines rather than control points to perform constraints. These constraints can be used to compute intrinsic parameters. The authors claim that using control lines also brings the benefit of suppressing the image noise so that the calibrated result is better than the analogous method by using of control points.

Since the four coplanar control lines are obtained, they can be transformed to four constraints with four linear equations: because a planar line is defined by two parameters. Then combined with the homography equation sets, the unique solution can be acquired finally.

2.6 Calibration Based on Three Known Coplanar Points

A method of camera calibration from a single image which contains three known coplanar points was proposed by Park [2007], who defines the Heptanion as the extension of the quaternion by adding an additional part of translation vector. Therefore the Heptanion can represent 6 DOFs (Degrees Of Freedom). By projection model, the equation can be derived to describe the camera. The co-plane relationship can provide the mapping between ground and camera image, and combining with three known points the equation can be solved and the intrinsic parameters can be retrieved.

3. DETAILED DISCUSSION OF THE RESEARCH

3.1 Calibration Based on Vanishing Points and Vanishing Lines

3.1.1 *Caprile [1990]: Using Vanishing Points for Camera Calibration.* The authors appear to be the first to identify the problem that: using vanishing points to calculate camera's intrinsic parameters. They state that such method allows the calculation of camera geometry by using only a single view of a calibrating pattern. They claim that a preliminary report [Odone et al. 1986] of this work has already been presented.

The authors refer to the work of Tsai [1987], Tsai and Lenz [1989] and Faugeras and Toscani [1987] to introduce the method of camera calibration by viewing a suitable calibrating pattern without using direct measurements. The authors also refer to the work of Magee and Aggarwal [1984], Pacioli [1956] and Francesca [1942] to state properties of vanishing points they use for camera calibration. Besides, the authors state that the idea of using vanishing points to locate objects in space or to recover the extrinsic parameters of a pair of images has already been suggested by Odone et al. [1986], Nagy and Falsafi [1988] and Jiang and Chen [1987].

The authors propose that the orthocenter of the triangle with vertexes in the three vanishing points is the intersection of the optical axis and the image plane as the requisite to estimate the intrinsic parameters. Then at first the authors propose the way of determine the ratio between the values of the focal length of the device expressed in Y-pixel and in X-pixel units by counting pixels on the screen; next, based on the ratio obtained before and equation sets derived from properties of vanishing points, the authors claim that the rest intrinsic parameters can be calculated; at last, the authors propose that if the cube is the main object in the scene, the accuracy in the localization of the three vanishing points in the image of the calibrating cube increases if the cube is positioned so that its visible faces form with the image plane an angle as close to 45 degrees as possible.

The authors perform three experiments. The first experiment is to compare the value of rotation and translation as obtained by proposed calibration procedure with values of rotation and translation measured by direct techniques, the authors compare different data sets since the rotation angle and translation length both can be set to a fixed number within certain absolute error range. The second experiment is to control viability for stereo matching of the epipolar constraint based on calibration data, which is designed to establish whether the calibration parameters are precise enough to provide a good exploitation of the epipolar constraint. The authors reproduce a pair of images of objects on a table, then determine when the two images are re-projected on a plane through the optical centres of the two cameras, whether conjugate points on the two images lie on parallel lines which are the i rectified conjugate epipolar lines. The last experiment is to estimate the length of a known segment by a stereo algorithm which uses the obtained calibration data. It aims to check the accuracy of the metrology that can be obtained with a stereo system using the proposed technique for calibration. The measurement method is based on simple triangulation principles assuming the segment is almost orthogonal to the optical axis of the cameras. The authors also state that the proposed algorithm for camera calibration complements has already been presented in Faugeras and Toscani [1987], Lenz and Tsai [1987] and Tsai [1987].

The authors claim that the method they proposed has been tested and is extensively used in several national laboratories and provides calibration data with the accuracy adequate for metrology and stereoscopic vision. They also claim that this technique, which does not require the solution of a nonlinear system of equations, can be performed almost in real time.

The authors claim that they are the first to introduce this method and they do not perform any comparison with other methods.

3.1.2 *Guillou [2000]: Using Vanishing Points for Camera Calibration and Coarse 3D Reconstruction from a Single Image.* The author claim that for most IBM (Image-based Modeling Techniques), camera calibration is needed. Then they propose a camera calibration technique which uses only one image without any calibration pattern. The assumptions for the proposed method, claimed by the authors, are: a single real image which contains two vanishing points, a known line segment in the image, the principal point which lies in the centre of the image and the user-fixed aspect ratio.

The authors refer to the work of Haralick [1999] and indicate that Haralick [1999] does not explicitly make use of vanishing points but use a similar idea. Then the authors refer to the work of Caprile and Torre [1990] to introduce the idea of calibrating a camera through vanishing points. Next, the authors refer to the work of Shigang et al. [1990] which is under certain assumptions. Chen and Jiang [1991] is also introduced for its calibrating group idea. Wang and Tsai [1991] employs a hexahedron as a calibration target to generate a vanishing line of the ground plane. At last, the authors introduce the work of Liebowitz et al. [1999] for its usage of three vanishing to calibrate the camera.

The authors propose that in order to determine two vanishing points, two sets of parallel lines, by which two basic vectors within world coordinate system are defined, are selected manually. If these two vectors are perpendicular, the third vector can be computed as the cross product of anterior vectors. Therefore, a vanishing plane and another plane, which is assumed not parallel to the image, can be formed. Since the principal point is fixed and a segment of certain length is known, the exact value of focal length can be computed from simple Euclidean geometry and transformation of trigonometric functions. The authors denote that the resolution of used single image must be high in order to achieve precision since the vanishing points are chosen manually. Authors also claim that vanishing points can be extracted automatically ([Brillault-O'Mahony 1991] and [Tai et al. 1993]).

The authors first use a scanned image and manually select two sets of parallel lines, so that the two vanishing points can be determined. Then the authors perform the calibration routine to calibrate the camera. Through recovered camera intrinsic parameters, 3D reconstruction of the objects depicted are performed. Next, the authors apply an image shot by a classical camera with already accurately known intrinsic parameters. Then two vanishing points are selected and calibration routine is executed. Once the recovered intrinsic parameters are acquired, the authors compare the recovered parameters with previously and precisely known parameters.

The author claim that regarding to the 3D reconstruction from the first scanned image, the quality of the reconstruction is acceptable. To the experiment related to the second image, the author claim that the manual selection of parallel lines is sufficiently precise. The authors also claim that an error of 50 pixels, which appears in the experiment, is not noticeable since it entails an error of 0.05cm on the calculated focal length. What's more, they state that their method is robust.

The authors claim that they are the first to introduce this method. In compared to Shum et al. [1999], Parodi and Piccioli [1996] and Liebowitz et al. [1999], the authors claim that their work needs less image and less vanishing points and is pattern-free so that their work is more practical. The authors suggest to limiting computation time by applying C/C++ programming language.

3.1.3 *Svedberg [2000]: Calibration, Pose and Novel Views from Single Images of Constrained Scenes.* The authors state that a central problem in image-based rendering, explicit 3D reconstruction and so on is of camera calibration. The authors also state that generally the 3D structure of a scene is constrained. If these constraints can be identified, they can be exploited in the calibration and reconstruction process. Thus, the authors propose the problem: camera calibration from a single image with a right-angle corner constraint.

The authors refer to the work of Liedtke et al. [1995] and Faugeras et al. [1998] to demonstrate that the identified constraints in a 3D scene can be exploited in the calibration and reconstruction process. The authors also refer to the work of Sugihara [1986] and Rothwell et al. [1993], and based on them, the authors claim that if the scene is constrained, a single image can provide a substantial amount of information for camera calibration and 3D reconstruction. Besides, the authors refer to the work of Seitz and Dyer [1996] and Vetter and Poggio [1997] to introduce the method of novel view synthesis by using bilateral symmetry constraint in single images. At last, the authors refer to the work of Caprile and Torre [1990] to introduce the idea related to the vanishing points and claim that their method is derived from the use of vanishing points.

The authors state that exploiting the constraint of a right-angle corner in the scene can perform camera calibration from a single perspective image of a building without assuming any metric 3D information about the corner. The method relies on the use of vanishing points for calibration and pose estimation as originally presented in Caprile and Torre [1990] and is in conjunction with a specific constrained and perspective camera model. The two image coordinate axes of a camera are orthogonal and have the same scale factor. The method is to use the seemingly orthogonal wedge, which is defined as two rectangular planes intersecting at right angles, as a reference structure. Such wedge is assumed to behave the way straight-angle corners do. Thus it is possible to imagine new views. The way of choosing a reference structure to imagine new views is equivalent to camera calibration. The authors assume that the camera is orthogonal.

The authors propose that the constraint of a right-angle corner brings two identified orthogonal planes. These two planes can set a left-handed system with coordinate directions along the edges of the wedge. In consequence, the intrinsic parameters can be obtained by using vanishing points. These vanishing points are extracted from the projections of the intersections, which are from the parallel edges of the rectangular 3D planes. The determination of vanishing points brings three quadratic constraint equations for the unknown camera parameters. Then, pair-wise subtraction is applied so that two linear homogeneous equations for two known principal point coordinates. At last, SVD (Singular Value Decomposition) is applied to determine the scale factor.

The authors perform experiments with two real images to test the method. These images are taken from a digital camera. The right-angle corners are extracted from each image to perform the calibration process. After the calibration, the 3D reconstruction and texture mapping are applied as the result of experiments.

The authors claim that the calibration result can be used to perform correct 3D reconstruction. The authors also state that an exact camera calibration is not necessary to produce realistic-looking novel views.

The authors claim that they are the first to introduce this method. They do not compare their work with the others.

3.1.4 *Wilczkowiak [2001]: Camera Calibration and 3D Reconstruction from Single Images Using Parallelepipeds.* The authors propose the calibration method by using the single image which contains parallelepipeds. They claim that a subclass of parallelepipeds, the cuboids, has been used over the past to partially calibrate the camera. They also claim that they exploit the features of parallelepipeds to invent the proposed method.

The authors refer to the work of Caprile and Torre [1990] to introduce the calibration method by using cuboids, which are parallelepipeds with right angles, to estimate some camera parameters. However, the authors state that computing vanishing points and lines in the image is numerically unstable. Moreover, dealing with individual vanishing points does not allow to fully exploit the redundancy contained in all the vanishing points stem from the projection of a single parallelepiped. Besides, the authors refer to the work of Malis and Cipolla [2000], Sturm and Maybank [1999], Triggs [1998] and Zhang [2000] to introduce the calibration method by using planar patterns and homography. The authors state that their method is a generalization of plane-based methods with metric information to 3D parallelepiped patterns. What's more, the work of Chen et al. [1999] is referred to introduce the 3D reconstruction by using cuboids. The work of Chen et al. [1999] is also referred to introduce the camera calibration in augmented reality applications by using parallelepipeds.

The authors state that their approach is based on the duality between the intrinsic parameters of a camera and those of a parallelepiped. Given an image of a parallelepiped, each known intrinsic parameter of either camera or parallelepiped gives a constraint on the parameter sets of both entities. Based on such constraint, intrinsic parameters can be calculated through formed equation set.

The authors state that the proposed method does not compute explicitly vanishing points but first estimates the best projection matrix such that the defined image points correspond to a parallelepiped in space. The error between re-projected and measured image points is minimized, so that the intersection of lines is avoided, and all information is used simultaneously. From this projection matrix and a priori knowledge on the parallelepiped, such as angles, length ratios and so on, the constraints on the intrinsic parameters can be derived. The authors claim that most of these constraints are linear. These constraints are then used to compute intrinsic parameters.

The authors conduct experiments with synthetic and real data. In simulation, the authors evaluate the sensitivity of the calibration method in the presence of noise. The information of right angles is applied as the priori information. In the experiment with real images, the reconstruction, which based on the calibration, is performed to test the method.

The authors claim that their method is effective and accurate.

The authors claim that they are the first to propose this method. They do not compare their work with others.

3.1.5 *Deutscher [2002]: Automatic Camera Calibration from a Single Manhattan Image.* The authors refer to the work of Coughlan and Yuille [1999], and based on it, the authors propose the problem: camera calibration from a single image which is assumed to satisfy a Manhattan world. The Manhattan world assumption is introduced by Coughlan and Yuille [1999] to model scenes in which edges lie predominantly in three orthogonal directions. The authors claim that this assumption is often satisfied by outdoor or indoor views of man-made structures and environments.

The authors refer to the work of Faugeras [1993], Tsai [1987] and Zhang [2000] to introduce the calibration strategy of applying calibration patterns. The authors also refer to the work of Hartley and Zisserman [2003] to introduce the calibration method of inferring a camera calibration in parallel with scene geometry from a sequence of images of rigid structures. Besides, the authors refer to the work of Coughlan and Yuille [1999] to introduce the idea of the Manhattan world assumption and the Manhattan framework. What's more, the authors refer to the work of McLachlan and Basford [1988] to introduce the stochastic search algorithm which performs mixture density estimation in an annealing framework. At last, the authors refer to the work of Caprile and Torre [1990], Kanatani [1992], Cipolla et al. [1999] and Antone and Teller [2000] to introduce the idea of using vanishing points or vanishing edges.

The authors propose the calibration method with a single image fitting the Manhattan world assumption. The authors make use of the Manhattan framework and combine it with a stochastic research algorithm to estimate the focal length and alignment of an unknown camera given a single image of a Manhattan scene. The authors claim that by using a likelihood model for the image together with Bayesian inference, all of the image data without a separate edge-detection and clustering stage can be incorporated. Besides, the authors claim that this method exploits the orthogonality constraint explicitly to compute a joint estimate of the three vanishing points rather than trying to enforce orthogonality and estimate the intrinsic parameters after the vanishing points have been found.

The authors first construct a parameter vector which contains the rotation coefficients and focal length in pixels. Because estimation of the principal point of a camera is very sensitive to noise ([Hartley and Zisserman 2003]), the authors assume that the principal point is at the centre of the image. Besides, the authors assume no radial distortion. Then the authors apply the Manhattan Likelihood to make it be decomposed into a product of independent likelihoods. These likelihoods represent the probability if points or edges in the scene can be mapped to the image. Since the overall sum value for those likelihoods is 1, the stochastic search algorithm can be adopted iteratively to estimate the density. At last the optimized estimation of intrinsic parameters can be retrieved from such search algorithm.

The authors perform experiments by using three different digital cameras, and choosing two focal-length settings for each camera giving six distinct internal calibrations in all. The authors first calibrate the camera by the code provided by Bouguet [2001] as the reference. Then the authors perform their proposed method to calibrate the camera from a number of images in an office building. Some of the images can be termed as two types of poorly-conditioned images. The first type is the one in which the vanishing points cannot be easily inferred due to the lack of oriented edges or textures in the image. The second type is the one in which two of the three vanishing points corresponding to orthogonal directions are at infinity. At last the two sets of calibration data from two methods are compared.

The authors claim that experiments with some images conduct poor calibration results. The authors then claim that these poor results are because of the poorly-conditioned images. The authors also claim that the calibration results obtained through proposed method are less accurate than those from standard methods employing a calibration pattern or multiple images. However, the authors state that the outputs with proposed method are certainly good enough for common vision tasks such as tracking.

The authors claim that they are the first to address this problem. The authors also claim that their method is of less accuracy compared with standard methods such as the methods applying a calibration pattern or multiple images. The authors state that this method can be applied automatically and without use of a calibration pattern, which can be considered as the advantages of the proposed method.

3.1.6 *Kushal [2002]: A Simple Method for Interactive 3D Reconstruction and Camera Calibration from a Single View.* The authors claim that 3D reconstruction from a single image must necessarily be through an interactive process, where the user provides information and constraints about the scene structure. Then the authors propose the method of 3D reconstruction from a single image by registering two planes in the scene. Meanwhile, the authors propose the problem: camera calibration from a single image through the registration of two planes.

The authors refer to the work of Horry et al. [1997], Criminisi et al. [2000], Liebowitz et al. [1999] and Cipolla and Robertson [1999] to introduce the method of embedding vanishing points or lines to the single image. The authors also refer to the work of Sturm and Maybank [1999] to introduce the method of adding planarity to the single image. Besides, Debevec et al. [1996] and Grossmann et al. [2002] are referred for the addition of spatial inter-relationship of features. What's more, Wilczkowiak et al. [2001] is been referred by the authors to introduce the camera constraints.

The authors propose the method for 3D reconstruction and camera calibration from a single image based on the registration of two planes. This method is based on computation of homography from two world planes to the image, which can be accomplished more easily and reliably than computing vanishing points and lines. The user needs to identify the two planes by clicking a rectangle on each. After the indication of two planes, the camera intrinsic parameters can be calculated.

The authors first discuss the situation where the two world planes are orthogonal and their common axis of intersection is visible in the image. Then two homography relationships can be obtained. These two homography have two crucial properties: the third column of both the homography relationships are equal up to the scale factor and the first column of both are equal up to the scale factor. Based on these two properties, camera parameters, up to a scale factor, can be acquired. According to the previously computed rotation matrix and translation vector, the scale factor can be eliminated. Then, the authors extend the method to the situation when two planes are orthogonal but the common X axis is not visible. The authors assume the coordinate by assigning a temporary X axis. Then by introducing vanishing points, the effective of X axis can be ignored and the camera intrinsic parameters can be calculated. Besides, the authors also discuss the situation when the two planes are not orthogonal, however, camera calibration routine is not been involved.

The authors perform the experiment by two sets of images. The first set includes 10 images of a laboratory scene. The height of an object is accurately measured. Then, camera calibration and 3D reconstruction are executed to obtain the recovered height, which is to compare with the authentic one. The second set is an outdoor image depicting a human. Similarly, the height of the human is been retrieved and the retrieved data is compared with authentic one.

The authors claim that the average error in the calculation of the height, when dealing with first image set, is 0.9%. When the object is switched to a larger one, the average error is 0.89%. As the second image set, the average error of acquiring a human's height, as claimed by the author, is 0.38%. The authors thus claim that these results demonstrate the applicability and robustness of the method.

The authors claim that they are the first to propose this method. The authors also claim that the proposed method can be accomplished more easily and reliably than computing vanishing points and lines.

3.1.7 *Wilczkowiak [2005]: Using Geometric Constraints Through Parallelepipeds for Calibration and 3D Modeling.* The authors refer to the work of Faugeras [1992] and claim that prior information on the acquisition process or on the scene is required to recover a Euclidean reconstruction. The authors claim that such prior knowledge does not only solve the projective ambiguity in the reconstruction but do also usually stabilize the sensitive reconstruction process. Furthermore, it often leads to simple and direct solutions for the estimation of camera parameters, which may eventually be adjusted non-linearly for higher accuracy. Based on it, the authors propose the problem: camera calibration using geometric constraints through parallelepipeds.

The authors first refer to the work of Tsai [1986] to introduce the approaches based on calibration pattern. The authors claim that such approaches rely on specific acquisition systems and are thus seldom available in general situation. Then the authors refer to the work of Maybank and Faugeras [1992], Triggs [1997], Hartley [1994] and Pollefeys and Van Gool [1997] to introduce the standard self-calibration algorithms. Although efforts have been made to dealt with the self-calibration method's shortcoming: no unique solution, by restraining the camera motions ([Hartley 1997], [De Agapito et al. 1999] and [Armstrong et al. 1994]) or by incorporating prior knowledge on the camera ([Zisserman et al. 1998]) or on the scene, the authors still claim that a large number of images is usually necessary. Next, the authors refer to the work of Hartley and Zisserman [2003] to introduce the approaches based on scene structure and camera motion. However, it decreases the accuracy of the results. At last, the authors introduce the approaches by incorporating Euclidean scene constraints through vanishing points of mutually orthogonal directions as defined by known cubical structures ([Caprile and Torre 1990], [Cipolla and Boyer 1998] and [Chen et al. 1999]) or by dominating scene directions ([Kosecka and Zhang 2002]). The authors claim that such approaches inspire their work. The authors state that their previous work of Wilczkowiak et al. [2001] and Wilczkowiak et al. [2002] has been synthesized in this paper.

The authors claim that a factorization-based approach is used to compute intrinsic parameters. First, the canonical projection matrices are been estimated, then missing data matrices is computed. Next, missing data matrices are been normalized to unit determinant. Then construct the measurement matrix and compute its SVD. Extract the matrices U and V. Then establish a linear equation system Z based on prior knowledge and solve it to least squares. If Z is positive definite, intrinsic matrix can be extracted from UT^{-1} and TV^T using e.g. QR decomposition.

The authors state that parallelepipeds are frequently present in man-made environments and they naturally encode the scene's affine structure. First, the authors analysis the situation when there is one parallelepiped in a single view, then the authors extend it to m parallelepipeds in n views. Then the authors introduce the idea of factorization so that the intrinsic matrix can be retrieved after a series of matrices transformation. The authors also discuss about the singularities brought by the factorization: they list every possible solutions under different environments and then determine and present the certain and unique solution to every situation.

The authors perform two sets of experiments: experiment with synthetic data and experiment with real image. In the experiment with synthetic data, tests, which are under fixed and known environments, are performed for different orientations of the parallelepiped. In the experiment with real images, the authors apply their method to indoor and outdoor scenes. The authors first determine the parallelepipeds contained in every image and then perform the calibration routine. The calibration quality is judged through the reconstruction, which is relied on the calibration result.

The authors claim that in simulation, calibration fails often for orientations with 10^o of the singular ones. However, for the intermediate range of orientations, the relative error of calibration is smaller than 5% when the parallelepiped covers more than 20% of the image. They also claim that results in case of successful calibration are slightly better for the parallelepiped method. To the experiment with real data, the authors claim that the proposed calibration approach gives excellent initial results for general 3D model reconstruction methods.

The authors claim that they are the first to introduce this method. The authors do not compare their work to others.

3.1.8 *Grammatikopoulos [2007]: An Automatic Approach for Camera Calibration from Vanishing Points.* The authors state that the use of vanishing points refers essentially to camera calibration. The authors refer to the work of Brauer-Burchardt and Voss [2001], Caprile and Torre [1990], Cipolla et al. [1999], Liebowitz et al. [1999], Sturm and Maybank [1999] and Van Den Heuvel [1999] to introduce the problem: using three vanishing points of orthogonal directions for camera calibration.

The authors refer to the work of Karras et al. [1993] and Liebowitz et al. [1999] to demonstrate that two vanishing points of orthogonal space directions cannot provide sufficient information to perform full camera calibration. However, if additional information is provided, full camera calibration is possible ([Grammatikopoulos et al. 2004]). The authors claim that their work is against such background. The work proposed by the authors relates particularly to those of Becker and Bove [1995], Brauer-Burchardt and Voss [2001] and Van Den Heuvel [1999].

The authors state that the camera calibration is achieved iteratively through a simultaneous least-squares adjustment of all image points belonging to lines, which intersect in the three dominant vanishing points of the scene, on the assumption that these correspond to orthogonal space directions. Extraction of line segments and their association with particular vanishing points runs automatically based on a modified approach of the work of Rother [2000], while edges are linked anew after the first estimation of lens distortion.

The authors first obtain vanishing points as the common points of concurring lines free from distortion. Then, six image coordinates of the vanishing points of three orthogonal directions supply an equal number of equations, in which the camera intrinsic parameters can be retrieved. The authors also dealt with the situation when there are more than three vanishing points available by introducing the adjustment process. This adjustment process is iterative: the estimated coefficients of radial distortion are used to correct the initial (distorted) extracted edge points, which is used to determine the camera parameters.

The authors perform the experiment to evaluate with two groups of images taken with different cameras. These images are typical terrestrial ones so that vanishing points are far from image centre, which approaches commonly existing practical conditions. First, lines and edges are extracted and grouped. Then, after the extraction of vanishing points, calibration routine has been applied to calibrate the camera. At last, errors of different sets of acquired data have been listed and compared.

The authors claim that calibration data obtained automatically from vanishing points on single images with no geodetic control allows to perform scene reconstruction with a mean accuracy not worse than 1 pixel in object space. Besides, the authors claim that the presented tests indicate that this automatic single image calibration approach through vanishing points shows good consistency and the data acquired from this approach fits for tasks of moderate accuracy requirements.

The authors claim that they are the first to introduce this approach. They do not compare their work with others. They also claim that the accuracy of the approach depends on image noise, possible deviations of lines from orthogonality and parallelism, and the distribution of image lines over the frame, their length and the particular perspective distortion of the imaged scene.

3.1.9 *Wu [2007]: Robust Self-calibration from Single Image Using RANSAC.* The authors refer to the work of Maybank and Faugeras [1992] and Hartley [1994] to introduce the methods of computing the calibration for multiple images. Then the authors propose the alternative method: camera calibration from a single image by using RANSAC algorithm, vanishing points and vanishing lines. The authors claim that the proposed method is more useful in the reality.

The authors refer to the work of Maybank and Faugeras [1992] to introduce the camera calibration method by using Kruppa's equation. Then the authors refer to the work of Hartley [1994] to illustrate how to calibrate a camera from pure rotation which is based on recovered homography. The authors also refer to the work of Triggs [1998] to introduce the camera calibration method through planar scenes constraints. The authors claim that through the proposed method, both intrinsic and extrinsic parameters can be retrieved just from the information of point in single image. Moreover, the authors state that under the assumption of obtaining the image by perspective projection, vanishing line and vanishing point of parallel planes are useful features. Hence the work of Liebowitz and Zisserman [1999] is referred to demonstrate the scene constraints by using vanishing points of orthogonal directions and rectified planes. Besides, the work of Lv et al. [2002] is been introduced, which uses the horizon line and the vanishing point by tracking a moving human in the scene to get an approximate solution under some assumptions. What's more, the authors refer to the work of Krahnstoever and Mendonca [2005] to introduce the attempt to handle the noise models in the data for self-calibration by using homology and Bayesian solution.

The authors propose the camera calibration method by vanishing points and vanishing lines. The vanishing points, the intersection of two lines, can perform an equation set which contains the intrinsic parameters and rotation angles. Besides, the vanishing lines, the intersection of two planes, can perform another equation set which also contains the intrinsic parameters and rotation angles. This ensures the unique solution of the intrinsic parameters. The method also adopts RANSAC algorithm to eliminate the outliers and to cull the input points to improve the accurateness of estimation. Besides, foot-to-head homology is used to converge the symmetric transfer errors.

The authors first acquire the equation set which is to represent the vanishing points. After the transformation, such equation set can be rewritten in Cartesian coordinates. Next, the equation set of the vanishing lines is introduced. These lines are set to be parallel to the ground. Right now the solution for camera calibration can be retrieved. The authors state that the information in the vanishing point, vanishing line and homology are equivalent to the information contained in the classical projection model. Knowledge of one can be used to recover the other. However, the computed result is of sensitivity. Therefore, in order to make the result be robust, RANSAC algorithm, which is a random sampling procedure and is used to eliminate the outliers from a large number of samples, is introduced to estimate the vanishing point, vanishing line and homology.

The authors conduct the experiments with synthetic data and real data. In simulation, some vertical poles are randomly inserted on the ground plane to facilitate the execution of the method. The comparison of estimation and ground truth for different values of the noise standard deviation and the comparison of original estimation and estimation with noise reduction are presented. In the experiment with real data, four images taken from different focal lengths are used. The result is compared with the result taken by the method from Zhang et al. [1997] to verify the accuracy.

The authors claim that in simulation, as the increase of noise, the mean of results have more bias with larger standard deviation. However, the errors of mean and standard deviation after noise reduction are much less than the original ones. The authors thus state that the noise reduction procedure is effective. In the experiment with real data, the authors claim that if the scenes in the images are not perspective enough, a little noise at locations of heads and feet can cause large errors. Therefore, the authors suggest that the images containing perspective scene should be used to obtain a better result.

The authors claim that they are the first to introduce this method. They also claim that their method is robust and effective when the scene of input image is perspective. They do not state the result of the comparison with others' methods.

3.1.10 *Avinash [2008]: Perspective Geometry Based Single Image Camera Calibration.* The authors seem to be the first introducing the method: camera calibration from a single image containing a rectangular prism. The rectangular prism is used to generate two vanishing points for camera calibration.

The authors refer to the work of Slama et al. [1980] to introduce the classical approach by minimizing a nonlinear error function. The authors also refer to the work of Tsai [1987] as the amelioration of Slama et al. [1980] through closed-form solution. Besides, the authors introduce the work of Caprile and Torre [1990], Wang and Tsai [1991] and Brauer-Burchardt and Voss [2001] to illustrate the use of vanishing points of parallel lines drawn on the faces of a cube for camera calibration. Next, the work of Willson and Shafer [1994] is been referred to introduce the idea of enclosing definitions for the centre of focus as the centre of the field of vision. This is also the optical centre of the lens used in the camera for acquiring the image. At last, the authors referred to the work of Zhang [2000] to introduce the most widely used algorithm of calibrating a camera through multi-views of a simple planar pattern.

The authors propose the method of applying a rectangular prism as the target to calibrate the camera. Based on the geometry of the perspective distortion of the edges of the prism from the image, vanishing points can be retrieved. Then fixing up the picture plane and fixing up the station point is carried out. Thus the intrinsic parameters can be calculated from the formed equation.

The authors first bring the image plane into the regular convention by shifting the image with the height of the image. Then the authors manually determine two vanishing points of parallel edges. Then the connection line between these two vanishing points is determined. Through simple triangle geometry the station point can be determined. The focal length is the perpendicular dropped to the picture plane, which is defined by a line parallel to the connection line. The principal point coordinates can also be determined as the intersection of the horizon line and the line which goes through the station point and is perpendicular to the picture plane.

The authors conduct two experiments. The first is by a 45 degree orientation of the calibration object. The second is by a rectangular prism with one face having a checker board. The rectangular prism acts as the calibration object and the checker board pattern acts as the calibration object for the compared algorithm ([Zhang

2000]).

The authors state that the result obtained by proposed method is almost the same compared to the algorithm ([Zhang 2000]), and it is analytical and straightforward and non iterative. The disadvantage of the proposed method, the authors claim, is that such method requires a specific setup: the orientation of the rectangular prism should be known.

The authors claim that they are the first to introduce this idea. They compare their work to the work of Zhang [2000] and claim that the proposed work is fast and requires only one image, however the orientation of the calibration object should be aware of before the calibration, which the work of Zhang [2000] does not need it.

3.2 Calibration Based on Circles

3.2.1 *Fremont [2002]: Direct Camera Calibration Using Two Concentric Circles from a Single Image.* The authors state that the most important stage of the modelling is the calibration stage. The authors claim that assuming zero skew, unit aspect ratio, no distortions and square pixels are appropriate for modern cameras. Then, the authors propose the problem: calibrating the camera using only one image of two concentric circles of known radii.

The authors state that calibration methods are divided into two categories: calibration using calibration pattern and self-calibration methods. The authors refer to the work of Tsai [1987] and Zhang [1999] to illustrate the fist category and define the calibration pattern as a 3D object of known geometry with a known position in the space. Then the authors refer to the work of Triggs [1997] to demonstrate the self-calibration methods and refine such methods as absolute quadric estimation over a sequence of images. The authors criticize the calibration using calibration pattern that when using a calibration pattern, the precision of the parameter estimation is related to the accuracy of the image measurements of the calibration pattern, and the algorithm needs a large set of image points to converge using a non-linear optimization ([Faugeras 1993]).

The authors propose to calibrate the camera using only one image of two concentric circles of known radii. First, the perspective projection of concentric circles, which are ellipses in a classic configuration, are detected automatically using the Hough transform based algorithm ([Kanatani and Ohta 2004]). Then the coefficients of the conic, or say the ellipse, are estimated using standard conic fitting algorithm ([Fitzgibbon et al. 1999] and [Zhang 1997]). The authors state that such coefficients are function of intrinsic and extrinsic parameters as well as the projected circle centre ([Kanatani and Liu 1993]). At last, from the n image points of the estimated ellipse, the system will be over-determined.

The authors first introduce the pin-hole projection model. Then by introducing the ellipse constraint, the ellipse fitting algorithm ([Fitzgibbon et al. 1999]) is applied to estimate the conic coefficients. Next, since the projected circle centre is different from the ellipses' centres, a new projective invariant, which is that the centre of projected concentric circles always lies on a line defined by the ellipses' centres under any projective transformations ([Kim et al. 2002]), for the projection of concentric circles, is been introduced. The authors then define the cross-ratio of four aligned points. This cross-ratio is been used to extract the image coordinates of the projected circle centre. Following is the definition matrix of the principal ellipse ([Kanatani and Liu 1993]). Once the n image points of the estimated ellipse are determined, the over-determined system can be constructed and this system can be solved by the least square pseudo-inverse technique.

The authors perform two kinds of experiments: experiments with synthetic images and experiments with real images. During the experiments, ellipses are fitted by a direct least square method ([Fitzgibbon et al. 1999]). In the simulation, noise is added to the synthetic images and relative errors of intrinsic parameters are acquired. In the experiments with real images, the ellipses are detected using the method provided by Kanatani and Ohta [2004]. The results are compared with the calibration method proposed by Tsai [1987] and these results are used to re-project principal circle.

The authors claim that during the experiments, the proposed method shows comparable performances with the existing calibration algorithm. The authors also claim that this calibration method is well-suited to the 3D reconstruction problem from an image sequence of an object placed on a turn-table by using the table as the calibration pattern.

The authors claim that they are the first to address this problem. They claim that their method is of comparable performances.

3.2.2 *Teramoto [2002]: Camera Calibration by a Single Image of Balls: from Conics to the Absolute Conic.* The authors state that camera calibration is necessary if one is to obtain metric information from images. Then the authors refer to the work of Penna [1991] to introduce the calibration method by using balls. The authors propose the similar problem: camera calibration from a single view which contains three balls.

The authors state that their method belongs to the category of approaches by observing a predesigned calibration object whose geometry is known precisely ([Faugeras 1993], [Tsai 1986] and [Wei and Ma 1993]). The authors refer to the work of Penna [1991] and state that Penna [1991] is the first attempt to use ball to calibrate the camera. The authors also state that Penna [1991] only tries to compute the aspect ratio of the two image axes. The authors then refer to the work of Beardsley et al. [1992] to demonstrate that by rotating objects which have parallel lines, the trajectories of vanishing points are ellipse when calibrating cameras by vanishing lines. Next, the authors refer to the work of Daucher et al. [1994] to introduce a separate effort toward camera calibration, which brings the similar result compared with the work of Beardsley et al. [1992].

The authors state that a ball's image is a ellipse. The authors also state that the camera intrinsic parameters can be represented as the image of the absolute conic (IAC). The authors present an algorithm to optimally compute the camera intrinsic parameters and the relative size and position of each ball from the image of balls. The implementation consists of following steps: (1) extract boundary for each ball and sample 40-60 points for each boundary; (2) fit each point set by a conic; (3) choose the ball that is the forest from the image centre and estimate the focal length; (4) normalize the image (points) with the focal length determined in (3); (5) estimate the relative size and position of each ball; (6) optimally estimate all of the parameters in the normalized image.

The authors assume the camera is a pinhole. Then the authors build up the model which describes the projection of a ball in the scene. The boundary of the ball and the camera's focal point forms a circular cone, whose intersection with the image plane becomes an ellipse. Then the projection equation can be obtained. Assuming that the ball is with the unit radius, the conic can be represented by the equation of the image of the absolute conic and the rotation matrix. From this equation the camera intrinsic parameters can be uniquely determined. Since every ball provides 2 constraints fitting the intrinsic parameters, in order to solve the equation, at least 3 balls are needed to find out the solution for 5 intrinsic parameters. Then the authors use the Leverberg-Marquartd algorithm to optimize the rough intrinsic parameters. The authors claim that the success of convergence to the global minimum depends on good initial estimates of the parameters. What's more, in order to be more numerically robust, the authors normalize the image first by the initial estimate of the intrinsic matrix, then optimally estimate the parameters over the normalized image, and finally transform the obtained intrinsic parameters back to their original domains.

The authors first perform the simulated experiment. The authors generate a simulated scene containing three balls. After successfully obtaining the optimized results, the estimated parameters and their estimated variances are projected back to the original digital image domain. The whole process is implemented separately under three different Gaussian noise levels. The overall results are compared with the authenticated data. Next the authors conduct the experiment with real images. The obtained result is also compared with the authenticated data.

In the simulation, the authors claim that the larger the Gaussian noise, the more the computed values deviate from the true values, and the larger the computed covariance. The authors also claim that the closer the balls are to the image centre, the more the computed values deviate from the true values. In the experiment with real images, the authors claim that the results are promising and the algorithm they proposed is robust. Overall, the authors claim that the proposed algorithm is effective.

The authors claim that they are the first to address this problem. They do not compare their method with the others.

3.2.3 *Chen [2004]: Camera Calibration with Two Arbitrary Coplanar Circles.* The authors claim that for camera calibration, the usage of point corresponding data can be avoided by using geometrical patterns such as straight lines and circles instead. The authors claim that to previous calibration methods by circular patterns, multi-view is needed. The authors thus propose the problem: single image camera calibration by circles.

The authors refer to the work of Meng and Hu [2003] to introduce the method by using a pattern that consists of a circle and straight lines passing through its centre. This method needs at least three different views. Then the authors refer to the work of Kim et al. [2002] and Yang et al. [2000] to demonstrate the method that makes use of planar concentric circles or concentric ellipses. They require two views. Next, the authors refer to the work of Quan [1996] to introduce the method which is to find the correspondence between conics in two views, and to estimate the relative orientation of the optical axis of two views. Besides, the work of Dhome et al. [1990] is referred for the methods of estimating the attitude and the position of a circle from an image assuming known focal length and radius. What's more, the work of Kanatani and Liu [1993] and Liu and Kanatani [1993] is been referred to introduce the method which is to extract 3D information from conics in images when the intrinsic and extrinsic camera parameters are supposed to be known.

The authors propose the method of camera calibration by using only one single image of two coplanar circles with arbitrary radius, even if centres of the circles and/or part of the circles to be occluded. After the viewing of the scene containing two coplanar circles, an image with two ellipses can be obtained. Then two oblique elliptical cones can be formed from the detected ellipses if the focal length is correct. So that the focal length can be determined by making sure each of the formed cones is stable and is similar to the real cone surfaces.

The authors refer to the work of Dhome et al. [1990] to demonstrate the rigorous description for the properties of ellipses and conic surfaces. Then assume the focal length is known, the unit normal vector of the supporting plane and the centre of the circle can be determined from one perspective image. And because these two circles are coplanar, it can perform a constraint, which makes the product of two vectors be one unit. In practical, the focal length is unknown, so that the product of two vectors can be dealt with by approaching to one unit. Through this approach, the focal length and the unit normal vector can be determined and the ambiguity cased by the undetermined sings can be eliminated.

The authors test the method by using simulated images, then using real images. To the simulation, the authors create two images which are full of coplanar circles. Two circles are selected randomly in order to apply the calibration routine. The calibrated data is compared with authentic data. To the experiment with real images, indoor and outdoor scenes are been shot by two different cameras and a video camera. These images are been converted to the vertical view to the supporting plane by assuming a planar scene, in order to see if it resembles the real scene.

The authors demonstrate the accuracy of the simulated experiment by presenting RMS errors, which are 5.52 pixels and 9.21 pixels respectively and standard deviations, which are 9.21 pixels and 11.89 pixels, when the focal length is set to be 200 pixels. To the experiment with real images, some of the converted images do not show a perfect circle. The authors claim that the considerable reasons are the rest parameters which is unknown and the radial distortion of the cameras are not compensated.

The authors claim that they are the first to propose this method. The authors also claim that compared with existing methods, their method can determine the focal length of camera as well as extrinsic camera parameters. Even in the case that the position and the size of the circles are not available, the proposed method, as claimed by the authors, can still give the focal length and the normal vector of the supporting plane.

3.2.4 *Jiang [2005]: Detection of Concentric Circles for Camera Calibration.* The authors claim that using planar patterns for camera calibration is popular for its practical convenience. The authors also claim that planar pattern of pairs of concentric circles is geometrically richer than point-like features. The authors refer to the work of Fremont and Chellali [2002], O'Gorman et al. [1990], Kim et al.

[2005] and Yang et al. [2000] and address the problem that such work identified: camera calibration by using planar pattern of pairs of concentric circles.

The authors refer to the work of Sturm and Maybank [1999], Triggs [1998] and Zhang [2000] to introduce the camera calibration method by using planar patterns. The authors state that different patterns exist. Then the authors refer to the work of Sturm and Maybank [1999], Triggs [1998] and Zhang [2000] to introduce the grid pattern, refer to the work of Chen et al. [2004], Fremont and Chellali [2002], Heikkila [2000], Kim et al. [2005] and Yang et al. [2000] to introduce circular pattern and refer to the work of Matsunaga et al. [2000] to introduce the mixed pattern. The authors claim that most of these works are concentrated on the geometric aspect and the bottleneck of automatic detection and identification of the features for the practise of calibration has not been put sufficient efforts. Next the authors refer to the work of Fremont and Chellali [2002], O'Gorman et al. [1990], Kim et al. [2005] and Yang et al. [2000] to state that some geometric properties of patterns of pairs of concentric circles have been introduced and discussed. The authors then criticize that such properties have geometric derivation which can be much simplified by using the simple fact that the image of a pair of concentric circle intersects at a repeated pair of points. Besides, the authors criticize that the intrinsic special geometric properties of the pattern are not been considered. What's more, the authors state that the feature identification issue is not been considered.

The authors introduce a geometric method that constructs a sequence of points strictly convergent to the image of the circle centre from an arbitrary point. The method automatically detects the points of the pattern features by the construction method, and identifies them by invariants. It then takes advantage of homological constraints to consistently and optimally estimate the features in the image. Then the complete system of practical and automatic calibration method can be constructed.

The authors start the analysis by introducing a geometric observation in Euclidean space that the centre of the circle is always in a strictly decreasing convex area formed by all he chords. Then this construction is been moved into conics in a projective plane. Next, by translating the Euclidean concept of the midpoint of a given segment into a projective notion of harmonic conjugate that could be determined by a pair of conics, the image of the centre of circles from the observation of the pair of conics can be constructed. Next, the planar homography is estimated from the correspondences of the pair of circular points and the centres of circles. In image plane, each image of the centre of the conics is determined as the pole of the vanishing line with respect to each conic. Once an optimized solution to the image of the circular points and the homography is obtained, the intrinsic and extrinsic parameters are extracted in the same way as the plane-based method ([Zhang 2000] and [Sturm and Maybank 1999]) and all camera parameters are globally optimized using Maximum Likelihood Estimation method.

The authors conduct experiments on simulated and real image data. In simulation, the virtual camera is initially placed to be orthogonal to the pattern and the principal axis goes through the centre of circles. The radii of imaged circles are 100 and 60 pixels respectively. In experiments with real images, the authors place an arbitrary seed point in each image and test if the circles can be automatically detected. Then the images with incomplete circles are applied to test the robustness and the accuracy of the system.

The authors claim that their method is robustness and accuracy, which the absolute errors of the ratios are less than 0.005. The authors also claim that their method is efficient at detecting a complete pair of concentric circles; however it does not detect any pair that is even partially occluded. The authors then state that this failure does not limit the method toward the calibration task.

The authors claim that they are the first to propose this method. They do not compare their work with the others.

3.2.5 *Colombo [2006]: Camera Calibration with Two Arbitrary Coaxial Circles.* The authors refer to the work of Wong et al. [2003] and introduce the problem originated from Wong et al. [2003]: camera calibration with two arbitrary coaxial circles. This method is based on the automatic localization of a surface of revolution (SOR) in a single image and its usage as a calibration artifact.

The authors refer to the work of Caprile and Torre [1990] to introduce the scene-based calibration approach which uses the vanishing points of three orthogonal directions. The authors then refer to the work of Daucher et al. [1994] and Agrawal and Davis [2003] to introduce the calibration method of using images of spheres. Besides, the work of Zhang [2000] is referred to introduce the calibration method by using a planar checker board. What's more, the work of Chen et al. [2004] is introduced to illustrate the calibration method of exploiting the image of two arbitrary coplanar circles. For the field of using SORs, work of Wong et al. [2003] is referred as the example of camera calibration from multiple images of SORs. Besides, the work of Wong et al. [2004] and Colombo et al. [2005] is also referred to introduce the single view metric reconstruction derived from SORs. The authors state that the SOR features usable for calibration are the elliptical imaged cross-sections and the apparent contour.

The authors propose their calibration method by providing a framework of extracting a SOR object from image data, and using it as calibration artifact. As the employment of the homology-based curve segmentation strategy, the proposed method can calibrate the intrinsic and extrinsic parameters automatically: the intrinsic parameters can be directly retrieved from the equation set formed by such strategy.

The authors state that the imaged SOR fixed entities are strictly related to the matrix of intrinsic parameters. Since cross sections are parallel circles in 3D, they intersect at the circular points of the families of planes orthogonal to the SOR symmetry axis. Besides, their projections in the image are also related to the image of the absolute conic. So that four linear constraints can be used to compute the equation set which contains four unknowns. The authors claim that the assumption of zero skew factor and unit aspect ratio is made.

The authors conduct the experiments with synthetic and real data. In simulation, increasing Gaussian noise is applied to the data. The results are compared to the predefined data. In the experiment with real images, the reconstructed scenes are presented to test the proposed method.

In simulation, the authors claim the degradation as the noise increases. In the experiment with real images, the authors also claim the noise sensitivity. Besides, the authors claim that the principal point coordinates are more sensitive than the focal length.

The authors claim that they are the first to solve the proposed problem. They do not compare their work with others.

3.3 Calibration Based on the Collaboration of Vanishing Points/Lines and Circles

3.3.1 *Wang [2005]: Camera Calibration and 3D Reconstruction from a Single View Based on Scene Constraints.* The authors refer to the work of Caprile and Torre [1990] and the problem that Caprile and Torre [1990] first addressed: camera calibration from a single image which contains man-made environment. They define the man-made environment as one containing three orthogonal principal directions. They claim that camera calibration from a single image is an improvement compared to the camera calibration from two or more images. They also claim that to solve camera calibration from a single image which contains a man-made environment, a cuboids model is reasonable.

The authors refer to the work of Caprile and Torre [1990] and describe the method proposed by Caprile and Torre [1990]: camera calibration from vanishing points. The authors also refer to the works of Liebowitz et al. [1999], Wilczkowiak et al. [2002] and Liebowitz and Zisserman [1999] and claim that these works are based on Caprile and Torre [1990]. The shortcoming of these works is that their assumption, which is that the pixel is square, may not be applicable to some off-the-shelf digital cameras. Finally, the authors refer to the work of Wilczkowiak et al. [2002] and state that Wilczkowiak et al. [2002] expands the idea to parallelepiped structures. The authors state that their work is based on the idea presented by Caprile and Torre [1990].

The authors propose that two line segments with equal length or a known length ratio on an object can provide an additional independent constraint of the image of the absolute conic. They use this constraint to compute the image of circular points. They also claim that four independent linear constraints on IAC (Image of Absolute Conic) can be obtained from the image of circular points and a vertical vanishing point, which is the image of the direction perpendicular to the plane of IAC. Besides, they claim that a rotation matrix can be recovered from two orthogonal vanishing points and the camera calibration matrix.

The authors present a formal proof of their statements by using the concept of vanishing points and geometry principles. Based on these statements, an equation set, in which camera calibration coefficients can be retrieved, has been acquired. Furthermore, by applying the property of the rotation matrix, the authors show that they can recover the rotation matrix from two orthogonal vanishing points and the camera calibration matrix.

Next, the authors perform two experiments: one is with simulated data and the other is with real images. In a simulated experiment, the authors randomly select a pair of line segments with equal length on X and Y Axes respectively, and calibrate the camera. Then with the estimated intrinsic parameters, the authors compute the extrinsic parameters of the camera. In a practical experiment, the authors calibrate the camera and reconstruct the scene from a set of images by the methods which they propose.

The authors state that in the simulated experiment, their proposed methods are accurate and robust even under level 3 noise when the skew of the camera is zero; in the real experiment, such methods are consistent with the real case and they appear to be realistic.

The authors claim that they are the first to introduce these methods. The authors do not compare their work with others.

3.3.2 *Chen [2008]: Full Camera Calibration from a Single View of Planar Scene.* The authors refer to the work of Ip and Chen [2005] and Chen et al. [2004] and address the problem [Chen et al. 2004] defined: camera calibration from a single image which contains conics. The authors claim that a single image containing two coplanar circles is sufficiently powerful to give a fully automatic calibration framework.

The authors refer to the work of Dhome et al. [1990] to state that conics have been employed to help perform camera calibration before. The authors also refer to the work of Agrawal and Davis [2003] to introduce the calibration strategy of applying spheres as calibration pattern. Next, the authors refer to the work of Caprile and Torre [1990] to state that all intrinsic parameters can be solved from the vanishing points of three mutually orthogonal directions in a single image when the assumption of zero skew and unit aspect ratio is made. Besides, the authors state that multiple patterns or images can be applied to perform calibration when not all three vanishing points are available from a single image. What's more, the work of Ip and Chen [2005] and Chen et al. [2004] are referred by the authors as the closest approach. The work of Ip and Chen [2005] introduces the accurate Euclidean measures from coplanar circles in a calibration-free manner, and the work of Chen et al. [2004] introduces the estimation of the focal length and the camera pose from two coplanar circles in the image.

The authors divide their method into several stages: first, calibration-free planar rectification ([Ip and Chen 2005]) is performed and then extended to recover the vanishing line, the centres of circles and orthogonal vanishing point pairs; second, the distribution of the focal length are computed from all orthogonal vanishing point pairs; third, a statistical optimization routine is designed to estimate the focal length, the principal point and the aspect ratio simultaneously; finally, the calibration result is validated by comparison with the ground truth for synthetic scenes or by augmented reality tests for real scenes.

The authors use the ellipse detecting and fitting ([Fitzgibbon et al. 1999]) and pole-polar computation to solve the vanishing line and a group of orthogonal vanishing point pairs. The orthogonal direction of a given point at infinity can be solved by calculating the intersection of its polar line and the line at infinity. Next, according to the orthogonal property, for each guess of principal point and a set of orthogonal vanishing points, a set of guessed focal length can be computed respectively. Then the downhill simplex method is applied to perform the optimization routine. At last, such guess-and-optimization routine is executed iteratively until reaches the convergence.

The authors conduct the experiment with simulated data and real images to validate their proposed method. In simulation, the result is compared to the predefined data. Besides, a 3D ball model is added into the virtual scene to validate the calibration. In the experiment with the real scene, two reference points are selected on the plane to help build a world coordinate system. Then the authors add a 3D car model into the scene to test the calibration quality.

The authors claim, in simulation, that the method adapts stably to a wide range of principal points and aspect ratios, and all parameters can be estimated with high accuracy. In the experiment with the real image, the authors claim that their method is of validity and stability.

The authors claim that they are the first to fully solve this problem. They claim that their method advantages the others from waiving priori information and restrictions on circle positions.

3.4 Calibration Based on Symmetry

3.4.1 *Hong [2004]: On Symmetry and Multiple-View Geometry: Structure, Pose, and Calibration from a Single Image.* The authors claim that certain relationships among 3D scene features themselves have been, to a large extend, ignored or at least under-studied in computer vision field. Then the authors state that the symmetry can encode 3D information within a single perspective image. Thus the authors propose the problem: camera calibration from a single image which contains the symmetry features.

The authors refer to the work of Mitsumoto et al. [1992] to introduce the study of how to reconstruct a 3D object using the mirror image based on planar symmetry. The authors also refer to the work of Vetter and Poggio [1994] to show the proof that for any reflective symmetric 3D object, one non-accidental 2D model view is sufficient for recognition. Besides, the authors refer to the work of Zabrodsky and Weinshall [1997] to introduce the improvement of 3D reconstruction from image sequences by using bilateral symmetry assumption. What's more, the authors refer to the work of Rothwell et al. [1993], Chain and Cipolla [1996], Liebowitz and Zisserman [1998] and so on to introduce the application of symmetry in 3D object and pose recognition. Then the authors refer to the work of Marola [1989], Mukherjee et al. [1995], Kanatani [1997] and so on to introduce the detection of symmetry from images. The authors state that there is a lack of formal and unified analysis as well as efficient algorithms which would allow people to easily make use of numerous and different types of symmetry that nature offers.

The authors propose the method of calibrating a camera from a single image by adopting the symmetry. There are three subsections of this method. The first is the calibration from translational symmetry. The second subsection is the calibration from reflective symmetry. The authors state that if the reflective symmetry is along three mutually orthogonal directions, this method is similar to the first subsection. The last subsection is the calibration from rotational symmetry.

The authors focus on the fundamental matrix and construct an equation under the epipolar constraints between pairs of hidden images. Then as described above, the authors analyze it in three parts. In the first part, rotation is avoided; and since three mutually orthogonal translations are given, three linear equations can be determined to solve intrinsic parameters. The authors claim that such three orthogonal translations correspond precisely to the notion of "vanishing point" or "vanishing line" defined in Criminisi et al. [2000]. The second part is similar to the first one. In the last part, the calibration matrix satisfies the normalized Kruppa's equation. If only the focal length is unknown, it can be retrieved from the equation derived from the rotational symmetry.

The authors do not conduct any experiment to test the calibration.

The authors claim that they are the first to introduce this method. They do not compare their method to the others.

3.5 Calibration Based on Four Coplanar Control Lines

3.5.1 *Shang [2004]: Analytical Method for Camera Calibration from a Single Image with Four Coplanar Control Lines.* The authors state that most of the existing methods to calibrate the camera need more than six non-coplanar control points or two or more views in different orientations if the control points are coplanar. To improve it, by assuming the awareness of the lens distortion, the authors propose the problem: camera calibration from a single view with four coplanar control lines.

The authors refer to the work of Fischler and Bolles [1981] to introduce the PnP (the perspective of n points) problem. This problem is to find the position and orientation of a camera with respect to a scene object from n corresponding points. Fischler and Bolles [1981] gives a unique solution by using four control points (P4P). The authors claim that the P4P problem with coplanar points can be solved linearly and analytically ([Penna 1991]). The authors also claim that this P4P problem with coplanar points can be solved with one unknown effective focal length ([Abidi and Chandra 1995]). The authors, then, introduce the work of Hu et al. [2001] to introduce the linear method of solving a camera's extrinsic parameters and two intrinsic parameters simultaneously by four coplanar control points.

The authors propose that the camera's extrinsic parameters and two effective focal lengths can be retrieved uniquely, linearly and analytically with four coplanar control lines when the lens distortion and other intrinsic parameters are known. The authors use control lines rather than control points to perform constraints which can be used to compute intrinsic parameters. Using control lines also brings the benefit of suppressing the image noise so that the calibrated result is better than the analogous method by using of control points.

The authors first present the mathematical description of the pin-hole model and then present the equation set related to the perspective homography. Since the four coplanar control lines are obtained, they can be transformed to four constraints with four linear equations, because a planar line is defined by two parameters. Then combined with the previous obtained homography equation sets, the unique solution can be acquired finally.

The authors conduct simulated experiments to test this method. The authors compare the calibration result obtained with control lines with the calibration result obtained analogously with control points. During the experiment, a calibration system is built up and lines are extracted by fitting arithmetic.

The authors claim that the experiment shows comparatively results. Extracting image lines by fitting arithmetic can effectively suppress the image noise. If more coplanar control lines are available, the result can be improved by a least-squares fit. The authors claim that effective focal lengths can be derived from the proposed method when the lens distortion has been corrected and the other required parameters are given precisely. At last, the authors claim that solutions of off-plane values given by the proposed method are not as precise as those of in-plane values: it is because that the off-plane variation cannot be sensitively detected during the calibration.

The authors claim that they are the first to propose this method. They also claim that such method is easy to be implemented and is practical for work site usage. The authors claim that their method brings better calibration precision than methods with control points.

3.6 Calibration Based on Three Known Coplanar Points

3.6.1 *Park [2007]: Quaternion-Based Camera Calibration and 3D Scene Reconstruction.* The author states that despite the sensitivity of solving 3D scene reconstruction from numerical errors, mathematical analysis is one of the basic approaches to better understand 3D scene reconstruction. The author then proposes the problem: camera calibration from a single image, which contains three known points on a single plane.

The author refers to the work of Park and Kim [2002] to introduce the idea of extending the quaternion by adding a translation step.

The author proposes a camera calibration method for a image which contains three coplanar known points. This method focuses on solving a quaternion based set of nonlinear equations. Since three points are known, their coordinates and the co-plane relationship can be used to uniquely solve the nonlinear equations.

The author first introduces the adjusted quaternion. The reason for using quaternion is its ability of representing 3 DOFs. The author then defines the Heptanion as the extension of the quaternion by adding an additional part of translation vector. Therefore the Heptanion can represent 6 DOFs. By projection model, the equation can be derived to describe the camera. The co-plane relationship can provide the mapping between ground and camera image, and combining with three known points the equation can be solved and the intrinsic parameters can be retrieved.

The author conducts the experiment of dealing with a real image. The image is reconstructed with the help of the proposed calibration method. Then the reconstructed image is demonstrated by VRML (Virtual Reality Modelling Language) to test the calibration accuracy.

The author claims that some texture mismatch may be due to either computation error or VRML browser error. Besides, the author claims that the worst height computation error for the above scene is 12%.

The author claims that he is the first to propose this method. The author does not compare his work to the others.

4. CONCLUDING COMMENTS

Caprile and Torre [1990] proposed an innovative idea of using vanishing points to calibrate the camera. It can be used to calibrate the camera from a single image. Since then, many methods, which have been inspired by this idea, have been proposed to address this problem. These methods either extend the original idea, e.g. [Wilczkowiak et al. 2001] and [Wilczkowiak et al. 2005], or modify the implementation of it, e.g. [Guillou et al. 2000] and [Svedberg and Carlsson 2000].

Another direction, camera calibration by circles from a first image, is first proposed by Fremont and Chellali [2002]. After that, several similar methods are proposed [Teramoto and Xu 2002], [Chen et al. 2004], etc.. During it, the feature combination of vanishing points/lines and circles in the images has also been published. The work of Wang et al. [2005] and Chen et al. [2008] is the example.

In recent years, other directions have been proposed: symmetry [Hong et al. 2004], coplanar lines [Shang et al. 2004] and coplanar points [Park 2007] have also been applied to calibrate the camera from a single image.

Future work will be focused on "making more accurate error analysis and treating more complex camera models" [Chen et al. 2008], the improvement of current methods, such as extend the original idea to estimate "image center and the radial distortion parameters" [Chen et al. 2004] and trying to apply "other structural information" [Svedberg and Carlsson 2000] and the expansion of application in other fields, such as "trying to obtain 3D texture models of generic objects" [Colombo et al. 2006].

REFERENCES

Abidi, M.A., Chandra, T. 1995. A new efficient and direct solution for pose estimation using quadrangular targets: algorithm and evaluation. *Transactions on Pattern Analysis and Machine Intelligence*, 17(5). IEEE, 534–538.

Agrawal, M., Davis, L.S. 2003. Camera calibration using spheres: A semi-definite programming approach. *International Conference on Computer Vision*, IEEE, 782–789.

Antone, M.E., Teller, S. 2000. Automatic recovery of relative camera rotations for urban scenes. *Conference on Computer Vision and Pattern Recognition*, vol. 2. IEEE, 282–289.

Armstrong, M., Zisserman, A., Beardsley, P. 1994. Euclidean structure from uncalibrated images. In *Proceedings of British Machine Vision Conference*, 509–518.

Avinash, N., Murali, S. 2008. Perspective Geometry Based Single Image Camera Calibration. *Journal of Mathematical Imaging and Vision*, 30(3). Springer, 221–230.

Beardsley, P., Murray, D., Zisserman, A. 1992. Camera calibration using multiple images. In *Proceedings of the Second European Conference on Computer Vision*, Springer, 312–320.

Becker, S., Bove, V.M. 1995. Semiautomatic 3-D model extraction from uncalibrated 2-D camera views. In *Proceedings of the SPIE Symposium on Electronic Imaging*, 447–461.

Bouguet, J.Y. 2001. Camera calibration for Matlab. *Technical report, available from http://www. vision. caltech. edu/bouguetj/calib doc/*, Intel.

Brauer-Burchardt, C., Voss, K. 2001. Image rectification for reconstruction of destroyed buildings using single views. *Virtual and Augmented Architecture: Proceedings of the International Symposium on Virtual and Augmented Architecture*, Springer, 159–169.

Brillault-O'Mahony, B. 1991. New method for vanishing point detection. *CVGIP: Image Understanding*, 54(2). Academic Press, 289–300.

Caprile, B., Torre, V. 1990. Using vanishing points for camera calibration. *International Journal of Computer Vision*, 4(2). Springer-Verlag, 127–139.

Chain, T.J., Cipolla, R. 1996. Geometric saliency of curve correspondences and grouping of symmetric contours. *European Conference on Computer Vision*, Springer-Verlag, 385–398.

Chen, C., Y, C., H, Y. 1999. New calibration-free approach for augmented reality based on parameterized cuboid structure. *International Conference on Computer Vision*, vol. 1. IEEE, 30-37.

Chen, Q., Wu, H., Wada, T. 2004. Camera calibration with two arbitrary coplanar circles. *Lecture Notes in Computer Science*, Springer, 521–532.

Chen, W., Jiang, B.C. 1991. 3-D camera calibration using vanishing point concept. *Pattern Recognition*, 24(1). Elsevier, 57-67.

Chen, Y., Ip, H., Huang, Z., Wang, G. 2008. Full Camera Calibration from a Single View of Planar Scene. *International Symposium on Advances in Visual Computing*, Springer, 815–824.

Cipolla, R., Boyer, E. 1998. 3D model acquisition from uncalibrated images. In *Proceedings of IAPR Workshop on Machine Vision Applications*, 559–568.

Cipolla, R., Robertson, D. 1999. 3D models of architectural scenes from uncalibrated images and vanishing points. In *Proceedings of IAPR Workshop on Machine Vision Applications*, 824–829.

Cipolla, R., Drummond, T., Robertson, D. 1999. Camera calibration from vanishing points in images of architectural scenes. In *Proceedings of British Machine Vision Conference*, vo1. 2, 382–391.

Colombo, C., Del Bimbo, A., Pernici, F. 2005. Metric 3D reconstruction and texture acquisition of surfaces of revolution from a single uncalibrated view. *Transactions on Pattern Analysis and Machine Intelligence*, 27(1). IEEE, 99–114.

Colombo, C., Comanducci, D., Del Bimbo, A. 2006. Camera calibration with two arbitrary coaxial circles. *Lecture Notes in Computer Science*, vo1. 3951. Springer, 265–276.

Coughlan, J.M., Yuille, A.L. 1999. Manhattan world: Compass direction from a single image by bayesian inference. *International Conference on Computer Vision*, 941–947.

Criminisi, A., Reid, I., Zisserman, A. 2000. Single view metrology. *International Journal of Computer Vision*, vo1. 40. Springer, 2, 123–148.

Daucher, N., Dhome, M., Laprest, J.T. 1994. Camera calibration from spheres images. In *Proceedings of European Conference on Computer Vision*, Springer-Verlag, 449–454.

De Agapito, L., Hartley, R.I., Hayman, E. 1999. Linear self-calibration of a rotating and zooming camera. *Conference on Computer Vision and Pattern Recognition*, vol. 1. IEEE, 15–21.

Debevec, P., Taylor, C.J., Malik, J. 1996. Modeling and rendering architecture from photographs: A hybrid geometry- and imaged-based approach. *Proceedings of the 23rd Annual Conference on Computer Graphics and Interactive Techniques*, ACM, 11–20.

Deutscher, J., Isard, M., MacCormick, J. 2002. Automatic camera calibration from a single Manhattan image. *Lecture Notes in Computer Science*, Springer, 175–188.

Dhome, M., Lapreste, J.T., Rives, G., Richetin, M. 1990. Spatial localisation of modelled objects of revolution in monocular perspective vision. *Computer Vision: Proceedings*, Springer, 475–485.

Faugeras, O.D., Toscani, G. 1987. Camera calibration for 3D computer vision. In *Proceedings of International Workshop on Machine Vision and Machine Intelligence*.

Faugeras, O.D. 1992. What can be seen in three dimensions with an uncalibrated stereo rig?. In *Proceedings of the Second European Conference on Computer Vision*, Springer-Verlag, 563–578.

Faugeras, O.D. 1993. 3D Computer Vision. MIT Press.

Faugeras, O., Robert, L., Laveau, S., Csurka, G., Zeller, C., Gauclin, C., Zoghlami, I. 1998. 3-D reconstruction of urban scenes from image sequences. *Computer Vision and Image Understanding*, 69(3), 292–309.

Fischler, M.A., Bolles, R.C. 1981. Random sample consensus: A paradigm for model fitting with applications to image analysis and automated cartography. *Communications of the ACM*, vol. 24, ACM. 381–395.

Fitzgibbon, A., Pilu, M., Fisher, R.B. 1999. Direct least square fitting of ellipses. *Transactions on Pattern Analysis and Machine Intelligence*, 21(5). IEEE, 476–480.

Francesca, Piero della 1942. *De Prospectiva Pingendi*. Sansoni.

Fremont, V., Chellali, R. 2002. Direct camera calibration using two concentric circles from a single view. *International Conference on Artificial Reality and Telexistence*, 93–98.

Guillou, E., Meneveaux, D., Maisel, E., Bouatouch, K. 2000. Using vanishing points for camera calibration and coarse 3D reconstruction from a single image. *The Visual Computer*, 16(7). Springer-Verlag, 396–410.

Grammatikopoulos, L., Karras, G., Petsa, E. 2004. Camera calibration combining images with two vanishing points. *International Archives of the Photogrammetry, Remote Sensing & Spatial Information Sciences*, 35(5), 99–104.

GRAMMATIKOPOULOS, L., KARRAS, G., PETSA, E. 2007. An automatic approach for camera calibration from vanishing points. *Journal of Photogrammetry and Remote Sensing*, 62(1). Elsevier, 64–76.

GROSSMANN, E., ORTIN, D., SANTOS-VICTOR, J. 2002. Single and multi-view reconstruction of structured scenes. *Asian Conference on Computer Vision*, 93–104.

HARALICK, R.M. 1999. Determining camera parameters from the perspective projection of a rectangle. *Pattern Recognition*, 22(3). Elsevier, 225–230.

HARTLEY, R. 1994. Euclidean Reconstruction from Uncalibrated Views. *Workshop on Applications of Invariance in Computer Vision*, Springer-Verlag, 237.

HARTLEY, R. 1994. Self-calibration from multiple views with a rotating camera. *Lecture Notes in Computer Science*, vol. 800. Springer-Verlag, 471–478.

HARTLEY, R. 1997. Self-calibration of stationary cameras. *International Journal of Computer Vision*, 22(1). Springer-Verlag, 5-23.

HARTLEY, R., ZISSERMAN, A. 2003. *Multiple view geometry in computer vision.* Cambridge University Press.

HEIKKILA, J. 2000. Geometric camera calibration using circular control points. *Transactions on Pattern Analysis and Machine Intelligence*, 22(10). IEEE, 1066–1077.

HONG, W., YANG, A.Y., HUANG, K., MA, Y. 2004. On symmetry and multiple-view geometry: Structure, pose, and calibration from a single image. *International Journal of Computer Vision*, 60(3). Springer, 241–265.

HORRY, Y., ANJYO, K.I., ARAI, K. 1997. Tour into the picture: Using a spidery mesh interface to make animation from a single image. In *Proceedings of the 24th Annual Conference on Computer Graphics and Interactive Techniques*, ACM, 225–232.

HU, Z.Y., LEI, C., WU, F.C. 2001. A short note on P4P problem. *Acta Automatica Sinica*, 27(6), 770–776.

IP, H.H.S., CHEN, Y. 2005. Planar rectification by solving the intersection of two circles under 2D homography. *Pattern Recognition*, 38(7). Elsevier, 1117–1120.

JIANG, B.C., CHEN, W. 1987. Camera calibration technique using vanishing point concept. In *Proceedings of IEEE International Conference on System, Man, and Cyber*, IEEE, 640–662.

JIANG, G., QUAN, L. 2005. Detection of concentric circles for camera calibration. *International Conference on Computer Vision*, vol. 1. IEEE, 333–340.

KANATANI, K. 1992. Statistical analysis of focal-length calibration using vanishing points. *Transactions on Robotics and Automation*, 8(6). IEEE, 767–775.

KANATANI, K., LIU, W. 1993. 3D interpretation of conics and orthogonality. *CVGIP: Image Understanding*, 58(3). Academic Press, 286–301.

KANATANI, K. 1997. Comments on "Symmetry as a continuous feature". *Transactions on Pattern Analysis and Machine Intelligence*, 19(3). IEEE, 246–247.

KANATANI, K., OHTA, N. 2004. Automatic detection of circular objects by ellipse growing. *International Journal of Image and Graphics*, 4(1). Singapore: World Scientific, 35–50.

KARRAS, G., PATIAS, P., PETSA, E. 1993. Experiences with digital rectification on non-metric images when ground control is not available. *XV CIPA International Symposium.*

KIM, J.S., KIM, H.W., KWEON, I.S. 2002. A camera calibration method using concentric circles for vision applications. *ACCV*, 23–25.

KIM, J.S., GURDJOS, P., KWEON, I.S. 2005. Geometric and algebraic constraints of projected concentric circles and their applications to camera calibration. *Transactions on Pattern Analysis and Machine Intelligence*, 27(4), IEEE, 637–642.

KOSECKA, J., ZHANG, W. 2002. Video compass. In *Lecture Notes in Computer Science*, Springer-Verlag, 476–490.

KRAHNSTOEVER, N., MENDONCA, P.R.S. 2005. Bayesian autocalibration for surveillance. *International Conference on Computer Vision*, vol. 2. IEEE, 1858–1865.

KUSHAL, A.M., BANSAL, V., BANERJEE, S. 2002. A simple method for interactive 3D reconstruction and camera calibration from a single view. In *Proceedings of Indian Conference in Computer Vision, Graphics and Image Processing.*

LENZ, R., TSAI, R.Y. 1987. Techniques for calibration of the scale factor and image center for high accuracy 3D machine vision metrology. *IEEE Transactions Pattern Analysis and Machine Intelligence*, 10(5). IEEE, 713–720.

LIEBOWITZ, D., ZISSERMAN, A. 1998. Detecting rotational symmetries using normalized convolution. *International Conference on Computer Vision & Pattern Recognition*, 482–488.

LIEBOWITZ, D., CRIMINISI, A., ZISSERMAN, A. 1999. Creating architectural models from images. *Computer Graphics Forum*, 18(3). Blackwell Publishers Ltd., 39–50.

LIEBOWITZ, D., ZISSERMAN, A. 1999. Camera Calibration and 3D Reconstruction from Single Images Using Parallelepipeds. In *Proceedings of the Seventh IEEE International Conference on Computer Vision*, vol. 1. IEEE, 285–292.

LIEBOWITZ, D., ZISSERMAN, A. 1999. Combining scene and auto-calibration constraints. *International Conference on Computer Vision*, vol. 1. IEEE, 293–300.

LIEDTKE, C.E., GRAU, O., GROWE, S. 1995. Use of explicit knowledge for the reconstruction of 3-D object geometry. *Lecture Notes in Computer Science*, vol. 970. Springer, 580–587.

LIU, W., KANATANI, K. 1993. Interpretation of conic motion and its applications. *International Journal of Computer Vision*, 10(1). Springer, 67–84.

LV, F., ZHAO, T., NEVATIA, R. 2002. Self-calibration of a camera from video of a walking human. *International Conference on Pattern Recognition*, vol. 16, 562–567.

MAGEE, M.J., AGGARWAL, J.K. 1984. Determining vanishing points from perspective images. *Computer Vision, Graphics, and Image Processing*, 26(2). Academic Press, 256–267.

MALIS, E., CIPOLLA, R. 2000. Multi-view constraints between collineations: application to self-calibration from unknown planar structures. *Lecture notes in computer science*, Springer, 610–624.

MAROLA, G. 1989. On the detection of the axes of symmetry of symmetric and almostsymmetric planar images. *Transactions on Pattern Analysis and Machine Intelligence*, 11(1). IEEE, 104–108.

MATSUNAGA, C., KANAZAWA, Y., KANATANI, K. 2000. Optimal grid pattern for automated camera calibration using cross ratio. *Transactions on Fundamentals*, 83(10). IEICE, 1921–1928.

MAYBANK, S.J., FAUGERAS, O.D. 1992. A theory of self-calibration of a moving camera. *International Journal of Computer Vision*, 8(2). Springer-Verlag, 123–151.

MCLACHLAN, G.J., BASFORD, K.E. 1988. *Mixture models: inference and applications to clustering.* Marcel Dekker Press.

MENG, X., HU, Z. 2003. A new easy camera calibration technique based on circular points. *Pattern Recognition*, 36(5). Elsevier, 1155–1164.

MITSUMOTO, H., TAMURA, S., OKAZAKI, K., KAJIMI, N., FUKUI, Y. 1992. 3-D reconstruction using mirror images based on a plane symmetry recovering method. *Transactions on Pattern Analysis and Machine Intelligence*, 14(9). IEEE, 941–946.

MUKHERJEE, D.P., ZISSERMAN, A., BRADY, M. 1995. Shape from symmetry: detecting and exploiting symmetry in affine images. *Philosophical Transactions: Physical Sciences and Engineering*, The Royal Society, 77–106.

NAGY, G., FALSAFI, A. 1988. Using vanishing points to locate objects with six degrees of freedom. *Pattern Recognition in Practice III*, North-Holland, 123.

ODONE, B., CAPRILE, B., GIROSI, F., TORRE, V. 1986. On the understanding of motion of rigid objects. In *SPIE Proceedings of Intelligent Robots and Computer Vision V*, vol. 726, 87–101.

O'GORMAN, L., BRUCKSTEIN, A.M., BOSE, C.B., AMIR, I. 1990. Subpixel registration using a concentric ring fiducial. *International Conference on Pattern Recognition*, vol. 2, 16–21.

PACIOLI 1956. De Divina Proportione. Reprint of *Codice Ambrosiano, Fontes Ambrosiani*, 31.

PARK, J., KIM, S. 2002. Kinematics and constrained joint design using quaternion. *The 2002 International Conference on Imaging Science, Systems, and Technology*, 52–63.

PARK, J. 2007. Quaternion-Based Camera Calibration and 3D Scene Reconstruction. *Computer Graphics, Imaging and Visualization*, 89–92.

PARODI, P., PICCIOLI, G. 1996. 3D shape reconstruction by using vanishing points. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 18(2). IEEE, 211–217.

PENNA, M.A. 1991. Determining camera parameters from the perspective projection of a quadrilateral. *Pattern Recognition*, 6(24). 533–541.

PENNA, M.A. 1991. Camera calibration: a quick and easy way to determine the scale factor. *Translations on Pattern Analysis and Machine Intelligence*, 13(12). IEEE, 1240–1245.

POLLEFEYS, M., VAN GOOL, L. 1997. A stratified approach to metric self-calibration. *Conference on Computer Vision and Pattern Recognition*, IEEE, 407–412.

QUAN, L. 1996. Conic reconstruction and correspondence from two views. *Transactions on Pattern Analysis and Machine Intelligence*, 18(2). IEEE, 151–160.

ROTHER, C. 2000. A new approach for vanishing point detection in architectural environments. In *Proceedings of British Machine Vision Conference*, 382–391.

ROTHWELL, C.A., FORSYTH, D.A., ZISSERMAN, A.P., MUNDY, J.L. 1993. Extracting projective structure from single perspective views of 3-d point sets. In *Proceedings of International Conference on Computer Vision* , 573–582.

SEITZ, S.M., DYER, C.R. 1996. View morphing: Synthesizing 3D metamorphoses using image transforms. *SIGGRAPH*, 21–30.

SHANG, Y., YU, Q., ZHANG, X. 2004. Analytical method for camera calibration from a single image with four coplanar control lines. *Applied Optics*, 43(28). OSA, 5364–5369.

SHIGANG, L., TSUJI, S., IMAI, M. 1990. Determining of camera rotation from vanishing points of lines on horizontal planes. *3rd International Conference on Computer Vision*, IEEE, 499–502.

SHUM, H.Y., HAN, M., SZELISKI, R. 1998. Interactive construction of 3D models from panoramic mosaics. *Conference on Computer Vision and Pattern Recognition*, IEEE, 427–433.

SLAMA, C.C., THEURER, C., HENRIKSEN, S.W. 1980. *Manual of photogrammetry.* American Society of Photogrammetry.

STURM, P., MAYBANK, S.J. 1999. A method for interactive 3d reconstruction of piecewise planar objects from single images. *British Machine Vision Conference*, 265–274.

STURM, P., MAYBANK, S.J. 1999. On plane-based camera calibration: A general algorithm, singularities, applications. *Conference on Computer Vision and Pattern Recognition*, vol. 1. IEEE, 432–437.

SUGIHARA, K. 1986. *Machine interpretation of line drawings.* MIT Press.

SUPER, B.J., BOVIK, A.C. 1995. Shape from texture using local spectral moments. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 17(4). IEEE, 333–343.

SVEDBERG, D., CARLSSON, S. 2000. Calibration, pose and novel views from single images of constrained scenes. *Pattern Recognition Letters*, 21(13-14). Elsevier, 1125–1133.

TAI, A., KITTLER, J., PETROU, M., WINDEATT, T. 1993. Vanishing point detection. *Image and Vision Computing*, 11(4), 240–245.

TERAMOTO, H., XU, G. 2002. Camera calibration by a single image of balls: From conics to the absolute conic. In *Proceedings of Asian Conference on Computer Vision*, 499–506.

TRIGGS, B. 1997. Autocalibration and the absolute quadric. *Conference on CVPR*, IEEE, 609–614.

TRIGGS, B. 1998. Autocalibration from planar scenes. *Lecture Notes in Computer Science*, vol. 1406. Springer, 89–108.

TSAI, R. 1986. An efficient and accurate camera calibration technique for 3d machine vision. In *Proceedings of the CVPR*, vol. 86, 364–374.

TSAI, R. 1987. A versatile camera calibration technique for high-accuracy 3D machine vision metrology using off-the-shelf TV cameras and lenses. *IEEE Journal of Robotics and Automation*, 3(4). IEEE, 323–344.

TSAI, R., LENZ, R. 1989. A new technique for fully autonomous and efficient 3D robotics hand/eye calibration. *IEEE Transactions on Robotics and Automation*, 5(3). IEEE, 345–358.

NAGY, G., FALSAFI, A. 1988. Using vanishing points to locate objects with six degrees of freedom. *Pattern Recognition in Practice III*, 123.

VAN DEN HEUVEL, F.A. 1998. 3D reconstruction from a single image using geometric constraints. *ISPRS Journal of Photogrammetry and Remote Sensing*, 53(6). Elsevier, 354–368.

Van Den Heuvel, F.A. 1999. Estimation of interior orientation parameters from constraints on line measurements in a single image. *International Archives of Photogrammetry and Remote Sensing*, 32(5), 81–88.

Vetter, T., Poggio, T. 1994. Symmetric 3D objects are an easy case for 2D object recognition. *Spatial Ision*, vol. 8, 443–453.

Vetter, T., Poggio, T. 1997. Linear object classes and image synthesis from a single example image. *Transactions on Pattern Analysis and Machine Intelligence*, 19(7). IEEE, 733–742.

Wang, L.L., Tsai, W.H. 1991. Camera calibration by vanishing lines for 3-D computer vision. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 13(4). IEEE, 370-376.

Wang, G., Tsui, H.T., Hu, Z., Wu, F. 2005. Camera calibration and 3D reconstruction from a single view based on scene constraints. *Image and Vision Computing*, 23(3). Elsevier, 311–323.

Wei, G.Q., Ma, S.D. 1993. A complete two-plane camera calibration method and experimental comparisons. *International Conference on Computer Vision*, 439–446.

Wilczkowiak, M., Boyer, E., Sturm, P. 2001. Camera calibration and 3D reconstruction from single images using parallelepipeds. *International Conference on Computer Vision*, 142–148.

Wilczkowiak, M., Boyer, E., Sturm, P. 2002. 3D modelling using geometric constraints: A parallelepiped based approach. *Lecture Notes in Computer Science*, vol. 5. Springer-Verlag, 221–236.

Wilczkowiak, M., Sturm, P., Boyer, E. 2005. Using geometric constraints through parallelepipeds for calibration and 3D modeling. *Transactions on Pattern Analysis and Machine Intelligence*, 27(2), IEEE, 194–207.

Willson, R.G., Shafer, S.A. 1994. What is the center of the image?. *Journal of the Optical Society of America A*, 11(11). OSA, 2946–2955.

Wong, K.Y.K., Mendonça, P.R.S., Cipolla, R. 2003. Camera calibration from surfaces of revolution. *Transactions on Pattern Analysis and Machine Intelligence*, IEEE, 147–161.

Wong, K.Y.K., Mendonça, P.R.S., Cipolla, R. 2003. Reconstruction of surfaces of revolution from single uncalibrated views. *Image and Vision Computing*, 22(10). Elsevier, 829–836.

Wu, Q., Shao, T., Chen, T. 2007. Robust Self-calibration from Single Image Using RANSAC. *Lecture Notes in Computer Science*, vol. 4841. Springer, 230.

Yang, C., Sun, F., Hu, Z. 2000. Planar conic based camera calibration. *International Conference on Pattern Recognition*, vol. 15, 555–558.

Zabrodsky, H., Weinshall, D. 1997. Using bilateral symmetry to improve 3D reconstruction from image sequences. *Computer Vision and Image Understanding*, 67(1). Academic Press, 48–57.

Zhang, L., Dugas-Phocion, G., Samson, J.S., Seitz, S.M. 2002. Single-view modelling of free-form scenes. *The Journal of Visualization and Computer Animation*, 13(4). John Wiley & Sons, Ltd., 990–997.

Zhang, Z. 1997. Parameter estimation techniques: A tutorial with application to conic fitting. *Image and Vision Computing*, 15(1). Elsevier, 59–76.

Zhang, Z., Faugeras, O., Deriche, R. 1997. An effective technique for calibrating a binocular stereo through projective reconstruction using both a calibration object and the environment. *Journal of Computer Vision Research*, 1(1), 58–68.

Zhang, Z. 1999. Flexible camera calibration by viewing a plane from unknown orientations. *International Conference on Computer Vision*, vol. 1, 666–673.

Zhang, Z. 2000. A flexible new technique for camera calibration. *Transactions on Pattern Analysis and Machine Intelligence*, 22(11). IEEE, 1330–1334.

Zisserman, A., Liebowitz, D., Armstrong, M., Faugeras, O., Hartley, R.I., Kanade, T., Mundy, J.L., Triggs, W. 1998. Resolving Ambiguities in Auto-Calibration. *Philosophical Transactions: Mathematical, Physical and Engineering Sciences*, The Royal Society, 1193–1211.