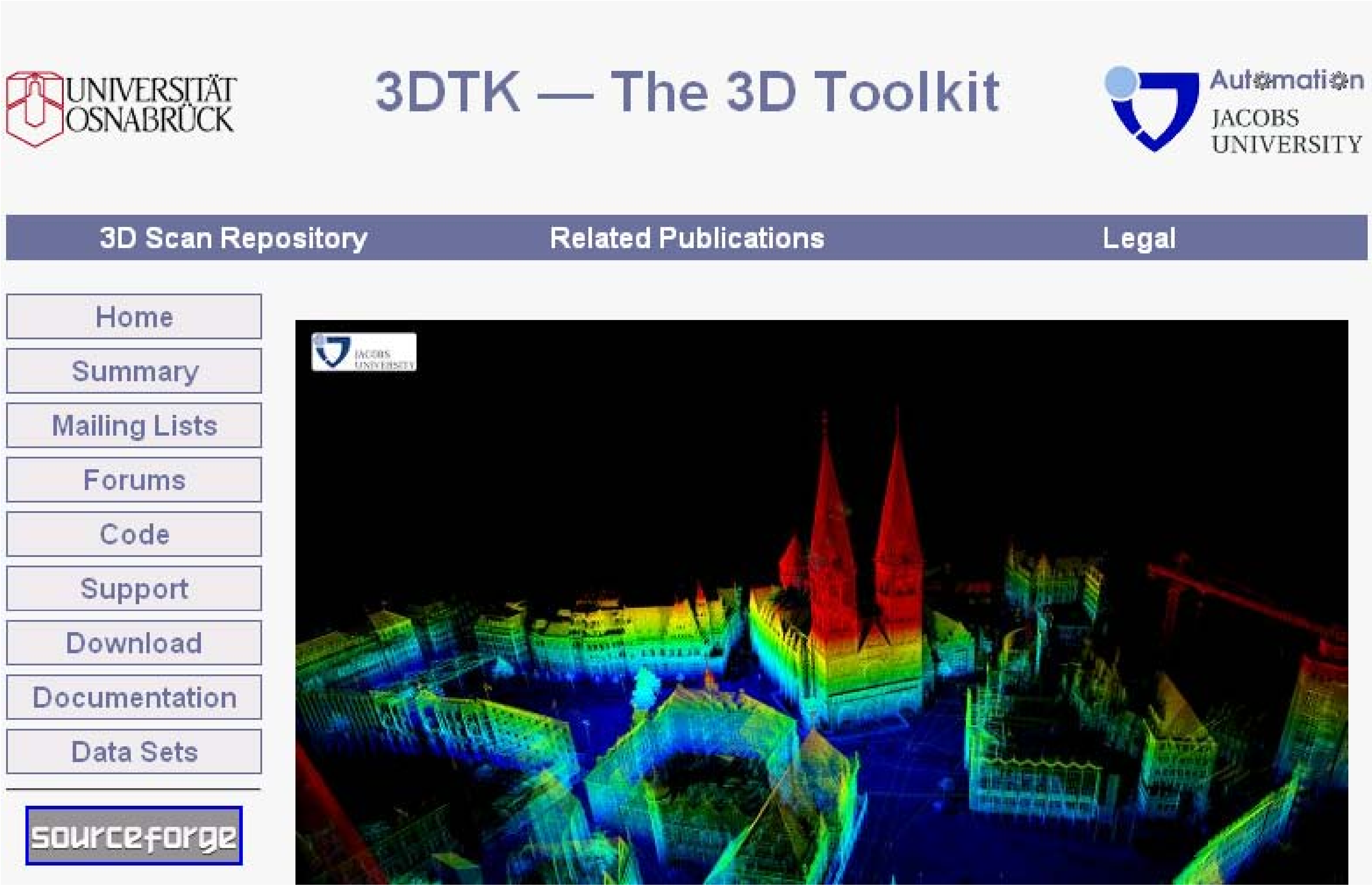

A Tutorial

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3DTK – Introduction (1)

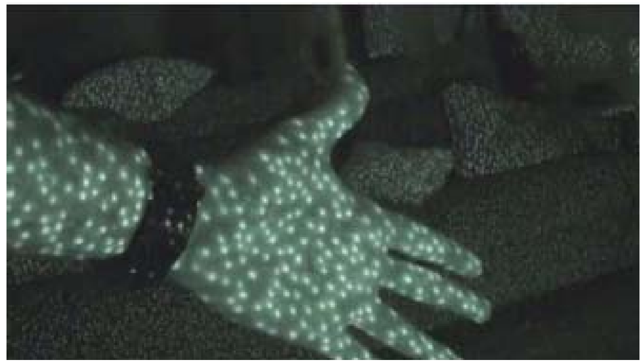
- 3DTK – The 3D Toolkit is a set of compter programs that efficiently processes 3D point cloud data
- An essential part is registration. It was initially developed in a robotics context, thus it focused on robot pose estimates using **six degree of freedom**, thus **6D SLAM**
- Next, will consider 3D laser scans as data
- Agenda
  1. Brief Introduction and Topic Statement
  2. Scan Matching
  3. Global Relaxation

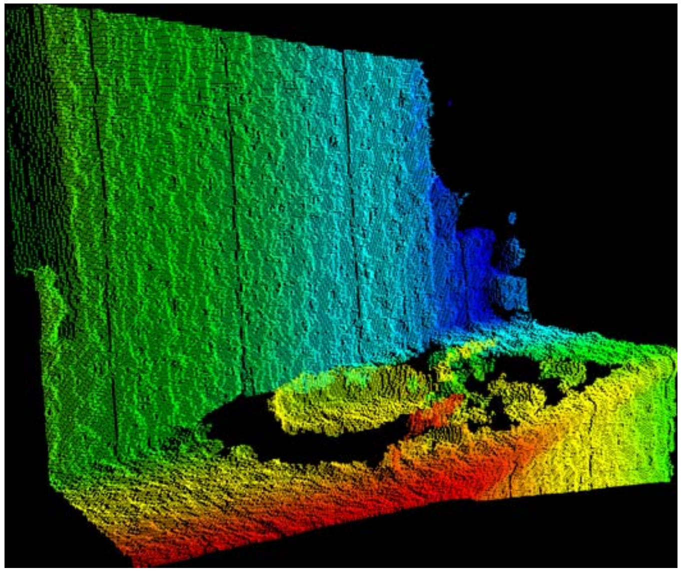
3DTK – Introduction (2)

Microsoft Kinect

- Video 30 Hz
- RGB video: 8-bit VGA resolution (640 × 480 Pixe
- Monochrome Video Stream

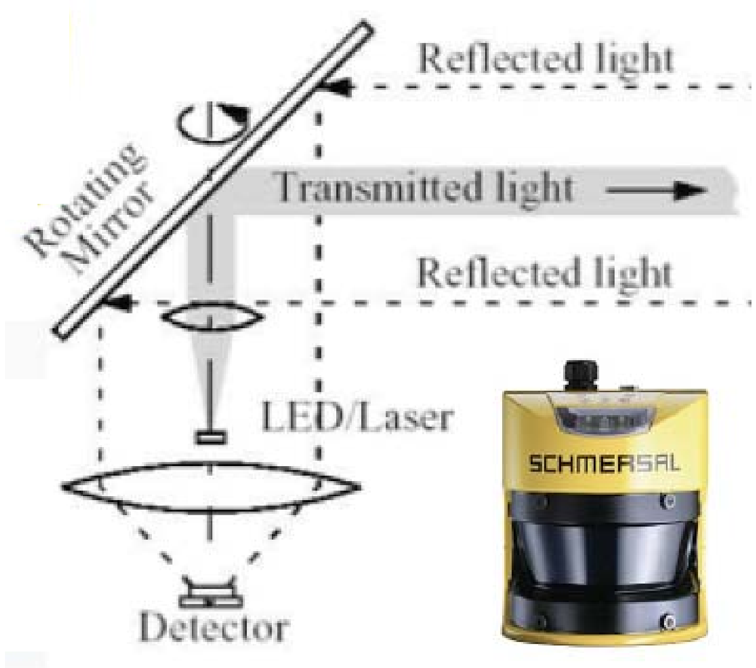
  (depth information): 11-bit VGA
  2048 depth values
- Depth: 1,2 – 3,5 m, (enhanced: 0,7 – 6 m)
- FOV: 57° (h) × 43°(vert)
- Tilt unit 27°
- Cost effective





3DTK – Introduction (3)

c = 299.792.458 m/s (Vacuum), also
d = 299.792.458 [m/s] x *t*/2 (*d* Distance[m], *t* time-of-flight[s])



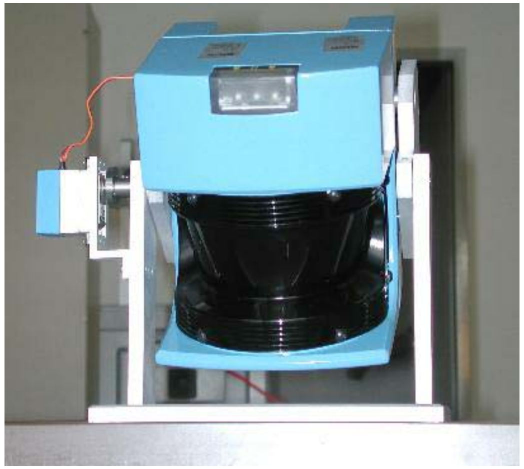
(2D laser scan)

***c* ≈ 0,3 mm/ps**

➥ With a resolution of 10mm: Precision of the time-of-flight measurement in the order of pico seconds **(10^{-12} s)** needed!

3DTK – Introduction (4)

- 3D laser scanner for mobile robots based on SICK LMS



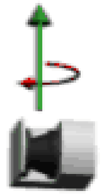
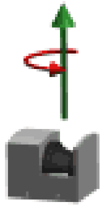
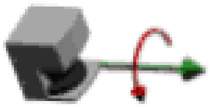
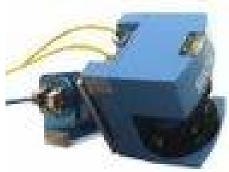
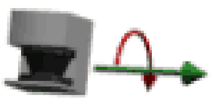
- Based on a regular (e.g., SICK LMS-200) laser scanner
- Relatively cheap sensor
- Controlled pitch motion (120° v)
- Various resolutions and modi, e.g., reflectance measurement {181, 361, 721} [h] x {128, …, 500} [v] points
- Fast measurement, e.g., 3.4 sec (181x256 points)

Mounted on mobile robots for 3D collision avoidance and building 3D maps.

(Video Crash)

(Video NoCrash)

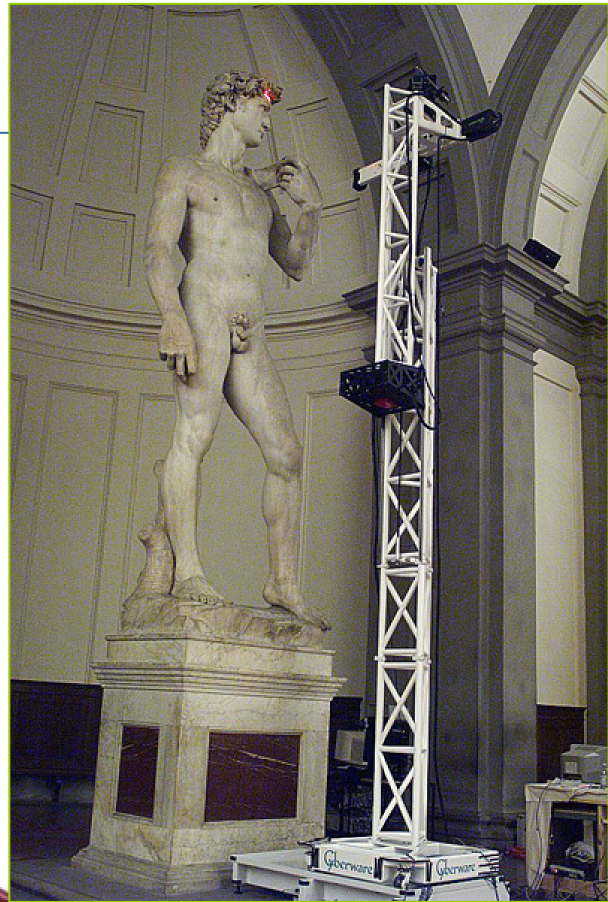
3DTK – Introduction (5)

Mode	Symbol	Cont. rotating	pivoting	Advantages
Yaw				+ Complete 360° scans + Good point arrangements - High point density at top
Yaw-Top				+ Fast scanning (half rot.) - High point density at top - Ground not measured
Roll				+ Fast scanning (half rot.) + High point density in front - Unusual point arrangement
Pitch				- High point density at the sides - Small apex angle + Good point arrangements + Easy to build

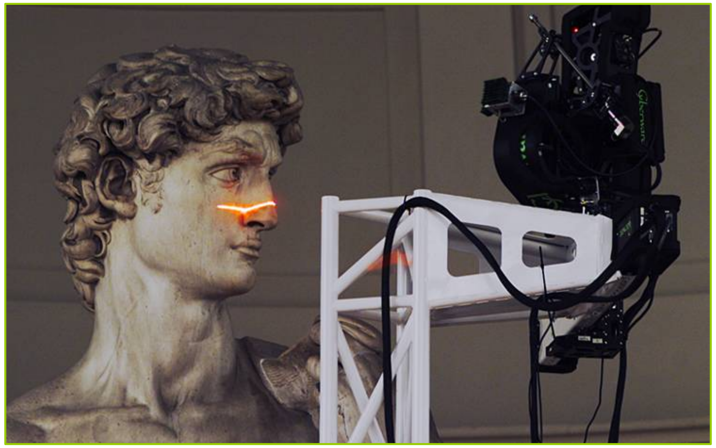
http://www.rts.uni-hannover.de/index.php/%C3%9Cbersicht_der_m%C3%B6glichen_Scannerkonfigurationen

(video)

3DTK – Introduction (6)



- Professional 3D scanners
  - Structured light (close range)

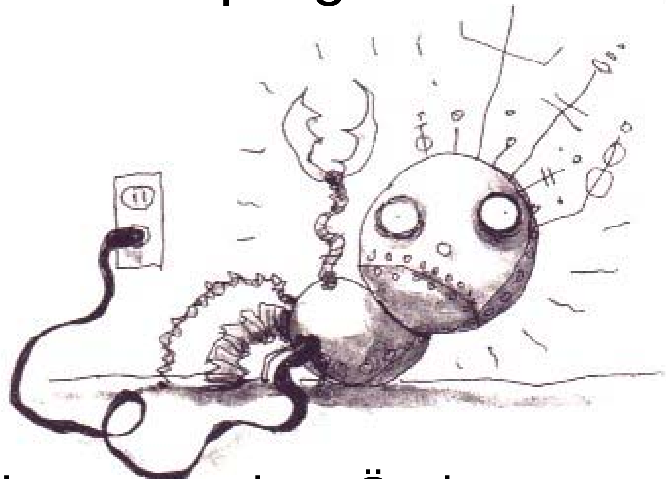


  - pulsed laser vs. time-of-flight (mid and long range)

Laserscanner
LEICA HDS3000

3DTK – Hands-on-experience (1)

- What you should learn now, using the **show** program
  - Most robotic data sets acquired by a rotating SICK scanner contain some outliers (it is worse with the kinect)
  - Data sets of professional scanners can be very large



- Things to try
  - Viewing a single 3D scan acquired in the kvartorp mine, Örebro
    `bin/show –s 1 –e 1 –f old path-to/kvarntorp`
  - Viewing a single 3D scan acquired in the kvartorp mine, Örebro
    `bin/show –s 1 –e 1 –f old –m 2500 path-to/kvarntorp`
  - Viewing multiple 3D scans
    `bin/show –s 1 –e 5 –f old path-to/kvarntorp`
  - Viewing a high resolution outdoor 3D scan
    `bin/show –s 0 –e 0 –f riegl_txt bremen_city`

6D SLAM – The ICP Algorithm (1)

Scan registration Put two independent scans into one frame of reference

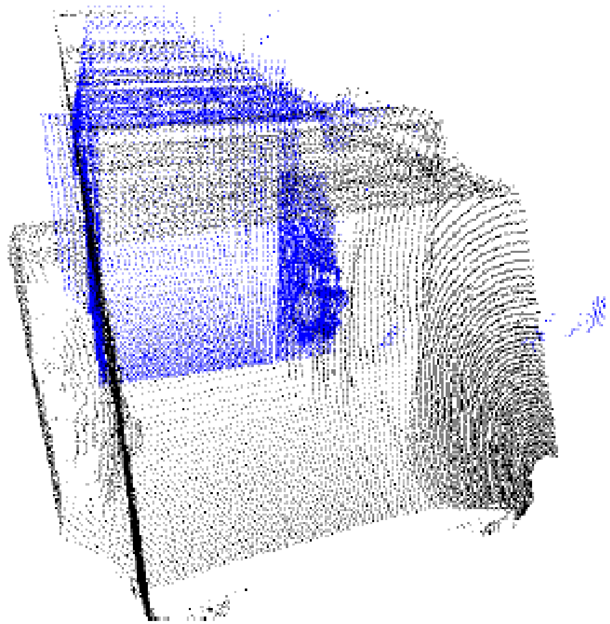
Iterative Closest Point algorithm [Besl/McKay 1992]

For prior point set M ("model set") and data set D

1. Select point correspondences $w_{i,j}$ in {0,1}
2. Minimize for rotation $\mathbf{R}$, translation $\mathbf{t}$

$$E(\mathbf{R},\mathbf{t}) = \sum_{i=1}^{N_m}\sum_{j=1}^{N_d} w_{i,j} \left|\left|\mathbf{m}_i - (\mathbf{R}\mathbf{d}_j + \mathbf{t})\right|\right|^2$$

3. Iterate **1.** and **2.**



SVD-based calculation of rotation

- works in 3 translation plus 3 rotation dimensions

$\Rightarrow$ 6D SLAM with closed loop detection and global relaxation.

6D SLAM – The ICP Algorithm (2)

Closed form (one-step) solution for minimizing of the error function

1. **Cancel the double sum:**

$$E(\mathbf{R},\mathbf{t}) = \sum_{i=1}^{N_m}\sum_{j=1}^{N_d} w_{i,j} \left|\left|\mathbf{m}_i - (\mathbf{R}\mathbf{d}_j + \mathbf{t})\right|\right|^2$$

$$\propto \frac{1}{N}\sum_{i=1}^{N} \left|\left|\mathbf{m}_i - (\mathbf{R}\mathbf{d}_i + \mathbf{t})\right|\right|^2,$$

2. **Compute centroids of the matching points**

$$\mathbf{c}_m = \frac{1}{N}\sum_{i=1}^{N}\mathbf{m}_i, \qquad \mathbf{c}_d = \frac{1}{N}\sum_{i=1}^{N}\mathbf{d}_j$$

$$M' = \{\mathbf{m}'_i = \mathbf{m}_i - \mathbf{c}_m\}_{1,\ldots,N}, \qquad D' = \{\mathbf{d}'_i = \mathbf{d}_i - \mathbf{c}_d\}_{1,\ldots,N}.$$

3. **Rewrite the error function**

$$E(\mathbf{R},\mathbf{t}) = \frac{1}{N}\sum_{i=1}^{N} \left|\left|\mathbf{m}'_i - \mathbf{R}\mathbf{d}'_i - \underbrace{(\mathbf{t} - \mathbf{c}_m + \mathbf{R}\mathbf{c}_d)}_{=\tilde{\mathbf{t}}}\right|\right|^2$$

6D SLAM – The ICP Algorithm (3)

Closed form (one-step) solution for minimizing of the error function

3. Rewrite the error function

$$E(\mathbf{R},\mathbf{t}) = \frac{1}{N}\sum_{i=1}^{N} ||\mathbf{m}'_i - \mathbf{R}\mathbf{d}'_i - \underbrace{(\mathbf{t} - \mathbf{c}_m + \mathbf{R}\mathbf{c}_d)}_{=\tilde{\mathbf{t}}}||^2$$

$$= \frac{1}{N}\sum_{i=1}^{N} ||\mathbf{m}'_i - \mathbf{R}\mathbf{d}'_i||^2 - \frac{2}{N}\tilde{\mathbf{t}} \cdot \sum_{i=1}^{N} (\mathbf{m}'_i - \mathbf{R}\mathbf{d}'_i) + \frac{1}{N}\sum_{i=1}^{N} ||\tilde{\mathbf{t}}||^2 .$$

- **Minimize only the first term! (The second is zero and the third has a minimum for $\tilde{\mathbf{t}} = 0$).**

$$E(\mathbf{R},\mathbf{t}) = \sum_{i=1}^{N} ||\mathbf{m}'_i - \mathbf{R}\mathbf{d}'_i||^2 .$$

Arun, Huang und Blostein suggest a solution based on the singular value decomosition.

K. S. Arun, T. S. Huang, and S. D. Blostein. Least square fitting of two 3-d point sets. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 9(5):698 – 700, 1987.

6D SLAM – The ICP Algorithm (4)

Theorem: Given a 3 x 3 correlation matrix

$$\mathbf{H} = \sum_{i=1}^{N} \mathbf{m}_i'^T \mathbf{d}_i' = \begin{pmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{pmatrix}$$

with $S_{xx} = \sum_{i=1}^{N} m_{ix}' d_{ix}',\ S_{xy} = \sum_{i=1}^{N} m_{ix}' d_{iy}',\ \ldots$ **,then the optimal solution for** $E(\mathbf{R}, \mathbf{t}) = \sum_{i=1}^{N} \left|\left|\mathbf{m}_i' - \mathbf{R}\mathbf{d}_i'\right|\right|^2$ **is** $\mathbf{R} = \mathbf{V}\mathbf{U}^T$ **with** $\mathbf{H} = \mathbf{U}\Lambda\mathbf{V}^T$ **from the SVD.**

Proof:

$$E(\mathbf{R}, \mathbf{t}) = \sum_{i=1}^{N} \left|\left|\mathbf{m}_i' - \mathbf{R}\mathbf{d}_i'\right|\right|^2 .$$

Rewrite

$$E(\mathbf{R}, \mathbf{t}) = \sum_{i=1}^{N} \left|\left|\mathbf{m}_i'\right|\right|^2 - 2\sum_{i=1}^{N} \mathbf{m}_i' \cdot \mathbf{R}\mathbf{d}_i' + \sum_{i=1}^{N} \left|\left|\mathbf{d}_i'\right|\right|^2 .$$

Rotation is length preserving, i.e., maximize the term

$$\sum_{i=1}^{N} \mathbf{m}_i' \cdot \mathbf{R}\mathbf{d}_i' = \sum_{i=1}^{N} \mathbf{m}_\mathbf{i}'^T \mathbf{R}\mathbf{d}_i'$$

6D SLAM – The ICP Algorithm (5)

Theorem: Given a 3 x 3 correlation matrix

$$\mathbf{H} = \sum_{i=1}^{N} \mathrm{m}_i'^T \mathrm{d}_i' = \begin{pmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{pmatrix}$$

with $S_{xx} = \sum_{i=1}^{N} m_{ix}' d_{ix}',\ S_{xy} = \sum_{i=1}^{N} m_{ix}' d_{iy}',\ \ldots$ **,then the optimal solution for** $E(\mathrm{R},\mathrm{t}) = \sum_{i=1}^{N} \left|\left|\mathrm{m}_i' - \mathrm{R}\mathrm{d}_i'\right|\right|^2$ **is** $\mathrm{R} = \mathbf{V}\mathbf{U}^T$ **with** $\mathbf{H} = \mathbf{U}\Lambda\mathbf{V}^T$ **from the SVD.**

Proof: $$\sum_{i=1}^{N} \mathrm{m}_i' \cdot \mathrm{R}\mathrm{d}_i' = \sum_{i=1}^{N} \mathrm{m}_\mathrm{i}'^T \mathrm{R}\mathrm{d}_i'$$

Rewrite using the trace of a matrix

$$\mathrm{Trace}\left(\sum_{i=1}^{N} \mathrm{R}\mathrm{d}_i' \mathrm{m}_\mathrm{i}'^T\right) = \mathrm{Trace}\left(\mathrm{R}\mathbf{H}\right)$$

Lemma: For all positiv definite matrices AA^T **and all orthonormal matrices** B **the following equation holds:** $\mathrm{Trace}\left(\mathbf{A}\mathbf{A}^T\right) \geq \mathrm{Trace}\left(\mathbf{B}\mathbf{A}\mathbf{A}^T\right)$

$\square$

6D SLAM – The ICP Algorithm (6)

Theorem: Given a 3 x 3 correlation matrix

$$\mathbf{H} = \sum_{i=1}^{N} \mathrm{m}_i'^T \mathrm{d}_i' = \begin{pmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{pmatrix}$$

with $S_{xx} = \sum_{i=1}^{N} m'_{ix} d'_{ix}, \; S_{xy} = \sum_{i=1}^{N} m'_{ix} d'_{iy}, \; \ldots$ **,then the optimal solution for** $E(\mathrm{R}, \mathrm{t}) = \sum_{i=1}^{N} \left|\left| \mathrm{m}_i' - \mathrm{R}\mathrm{d}_i' \right|\right|^2$ **is** $\mathrm{R} = \mathbf{V}\mathbf{U}^T$ **with** $\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^T$ **from the SVD.**

Proof: Suppose the singular value decomposition of H **is** $\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^T$

U **and** V **are orthonormal 3 x 3 and** Λ **a diagonal matrix without negative entries .**

$$\mathrm{R} = \mathbf{V}\mathbf{U}^T.$$

R **is orthonormal and**

$$\begin{aligned} \mathrm{R}\mathbf{H} &= \mathbf{V}\mathbf{U}^T\mathbf{U}\mathbf{\Lambda}\mathbf{V}^T \\ &= \mathbf{V}\mathbf{\Lambda}\mathbf{V}^T \end{aligned}$$

And using the lemma it is $\operatorname{Trace}(\mathrm{R}\mathbf{H}) \geq \operatorname{Trace}(\mathrm{B}\mathrm{R}\mathbf{H})$.

Therefore R **maximizes**

$$\sum_{i=1}^{N} \mathrm{m}_\mathrm{i}'^T \mathrm{R}\mathrm{d}_i'$$

□

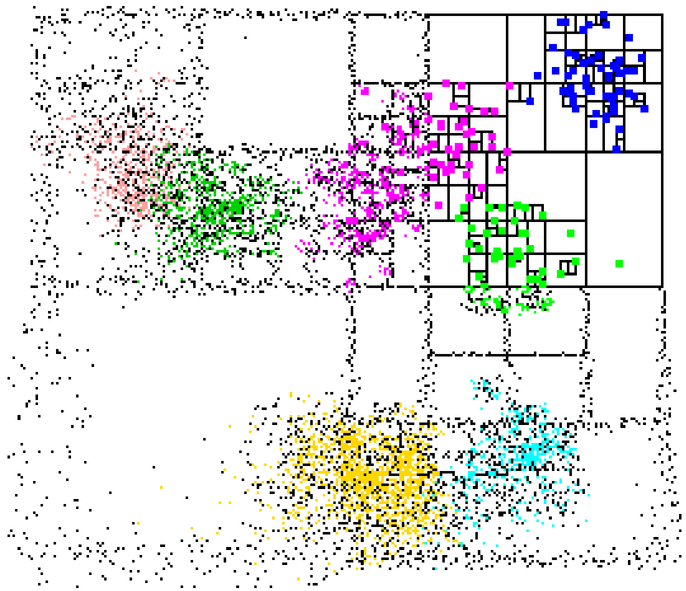
6D SLAM – The ICP Algorithm (7)

- Estimating the transformation can be accomplished very fast O(n)

- Closest point search
  - Naïve $O(n^2)$, i.e., brute force
  - K-d trees for searching in logarithmic time

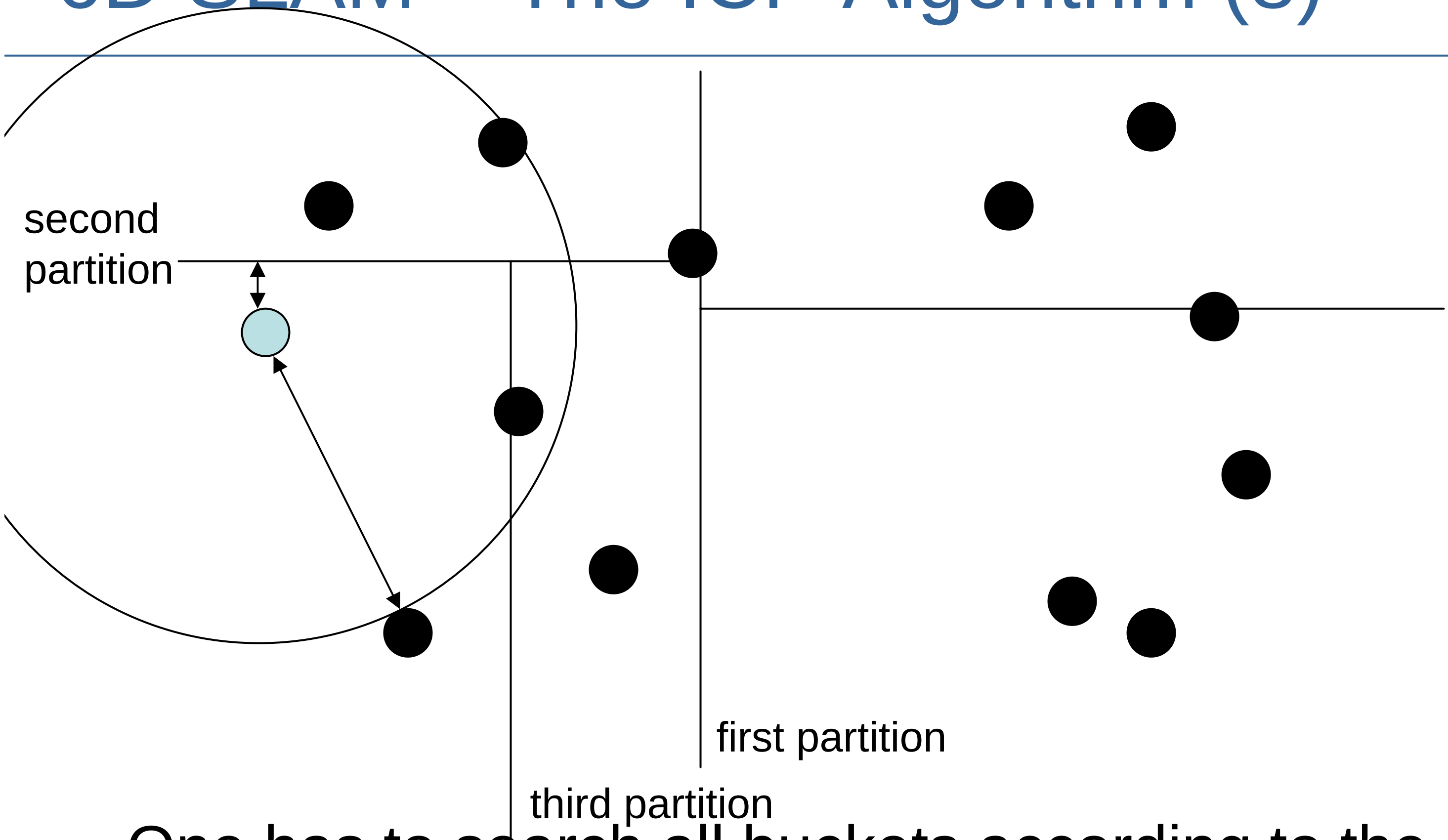
    Recommendation: Start with

    ANN: A Library for Approximate Nearest Neighbor Searching by David M. Mount and Sunil Arya (University of Maryland)
    - Easy to use
    - Many different methods are available
    - Quite fast



`http://www.cs.umd.edu/~mount/ANN/`

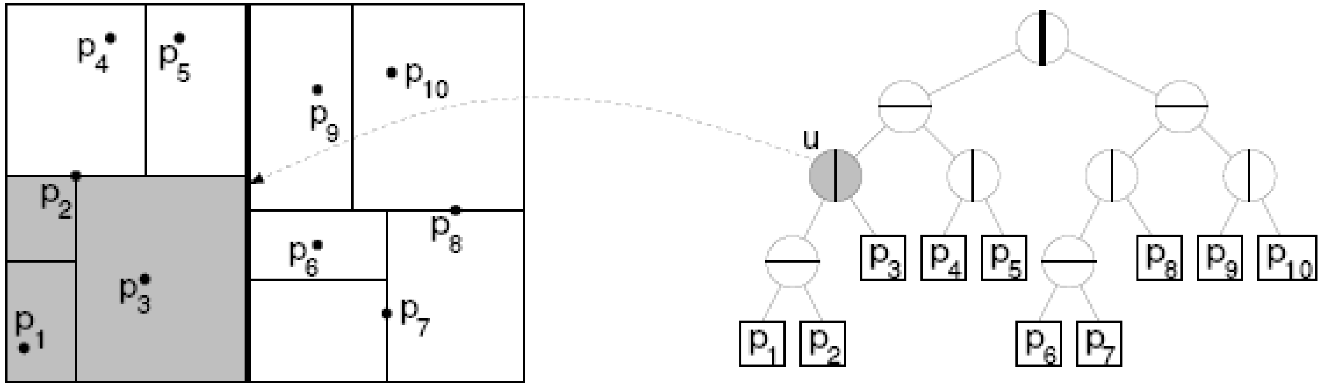
6D SLAM – The ICP Algorithm (8)



- One has to search all buckets according to the ball-within-bounds-test. ⇨ Backtracking
- Approximation in the ANN package represents a method for not-evaluating leafs, taking small errors into account.

6D SLAM – The ICP Algorithm (9)

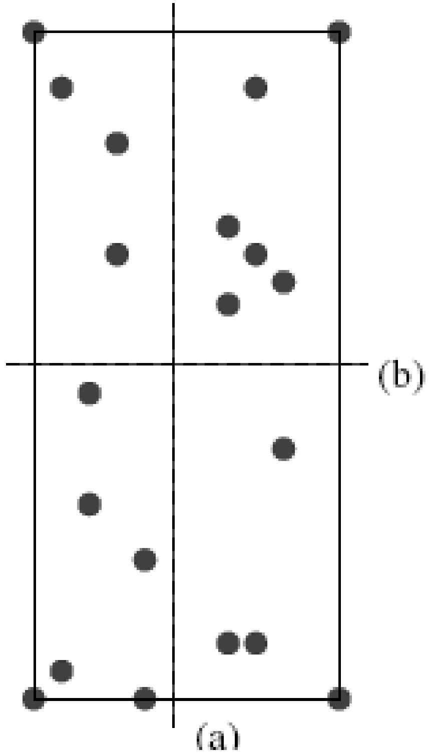
- How to split a k-d tree during construction?



1. Splitting at median
   - Fast calculation of median is needed (accomplishable in O(n)???)
   - Cells may have an arbitrary aspect ratio
   - Final tree has size $\lceil \log_2 n \rceil$
2. Midpoint splitting rule
   - Fast and easy to compute
   - Guarantees aspect ratio, but may result in trivial splits
3. Midpoint splitting rule that reverts to splitting at media to avoid degeneration.

6D SLAM – The ICP Algorithm (10)

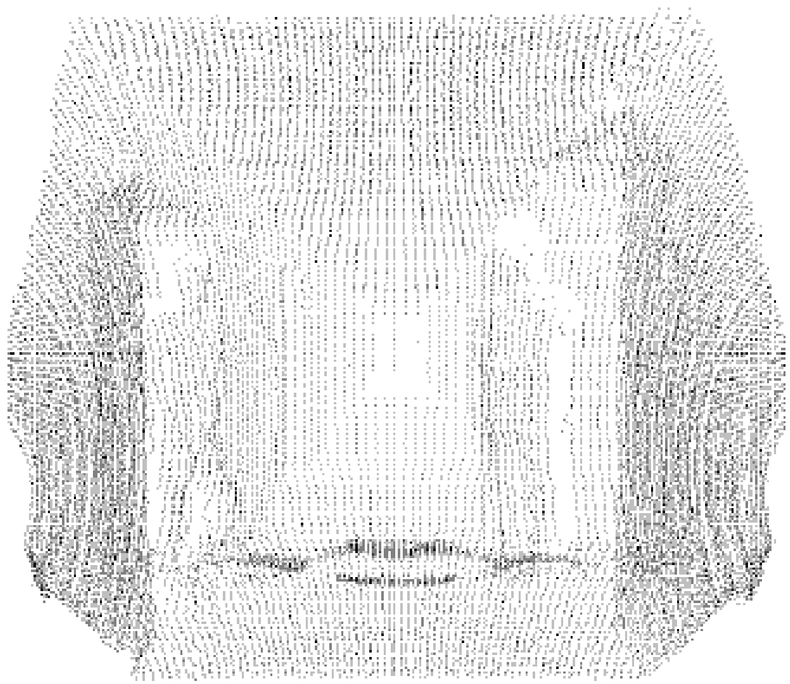
- Other methods are implemented in ANN as well
- Best performance is achieved by the so-called optimized k-d tree

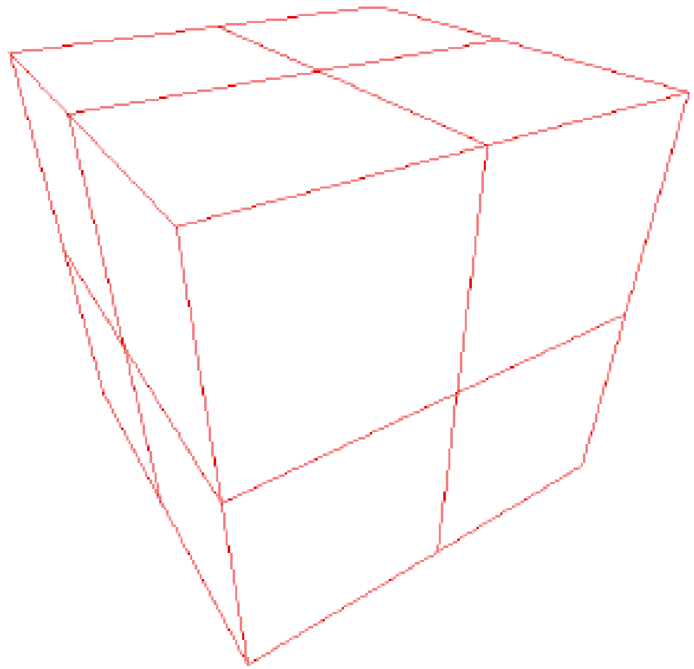


Choose (b) over a, since it reduces the total amount of backtrackung.

6D SLAM – The ICP Algorithm (11)

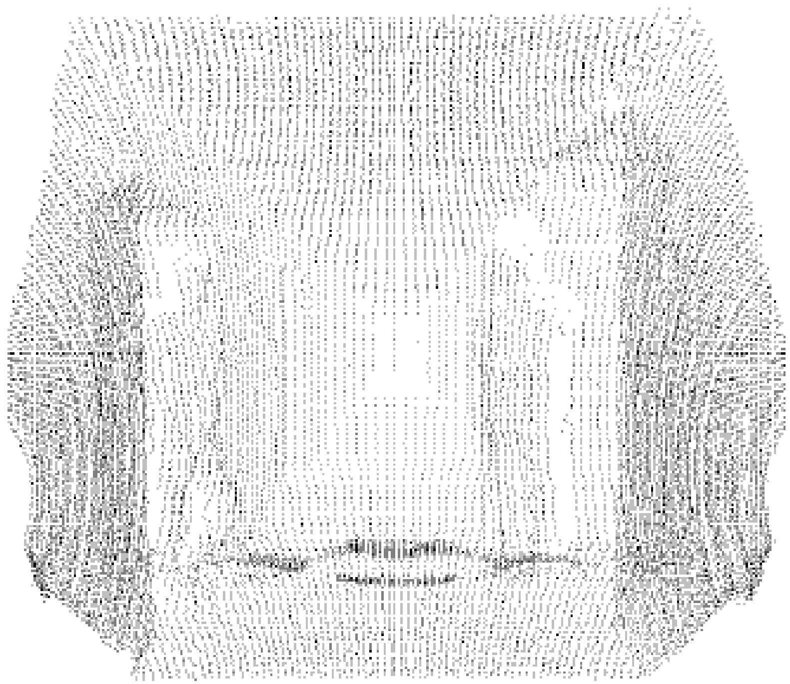
- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud

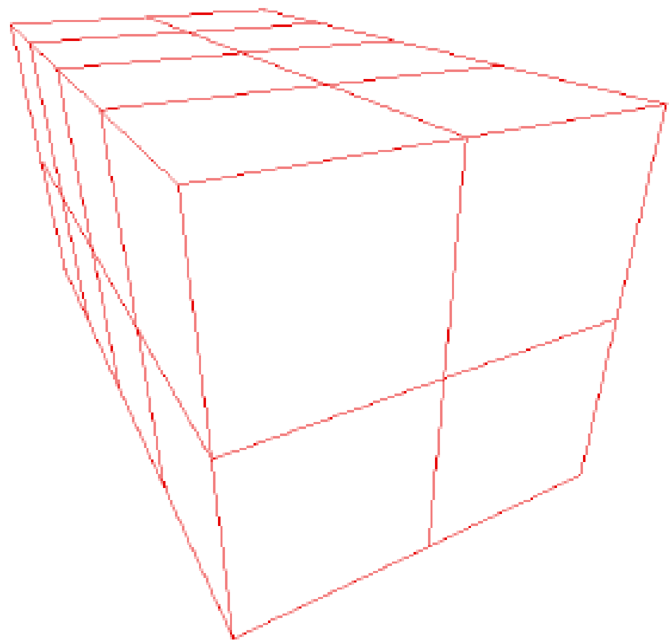




6D SLAM – The ICP Algorithm (12)

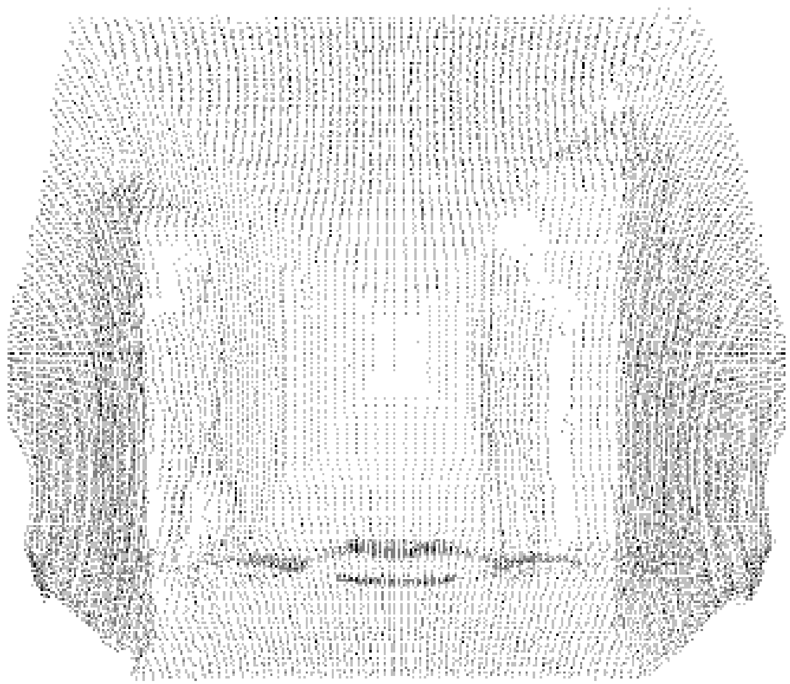
- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud

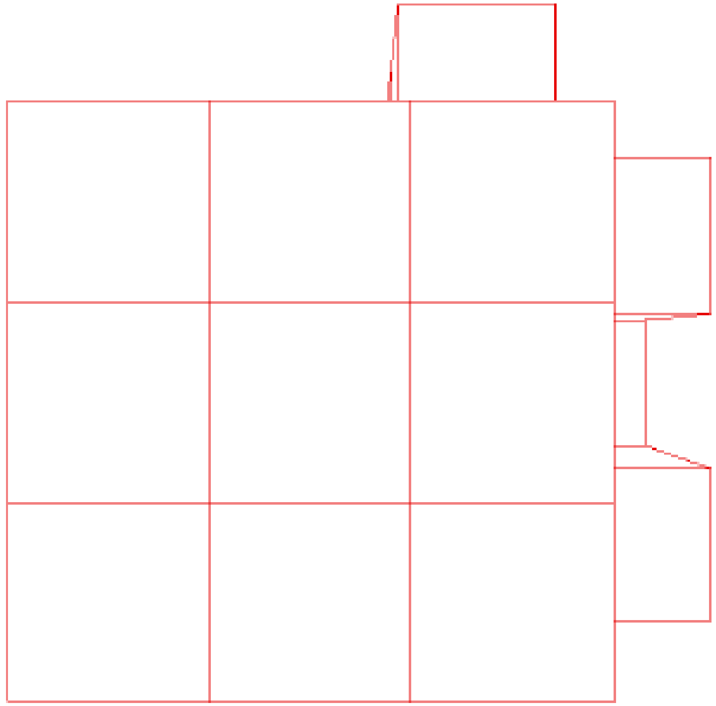




6D SLAM – The ICP Algorithm (13)

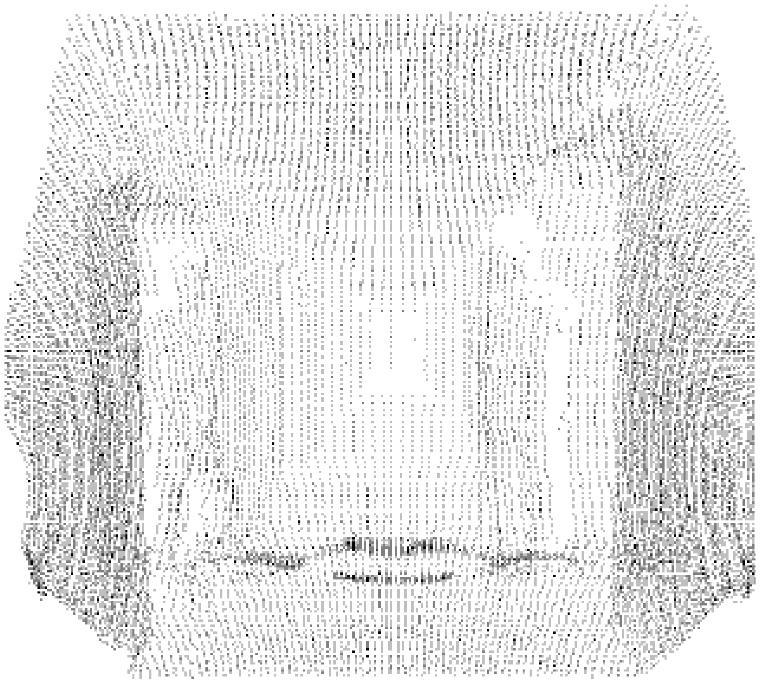
- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud

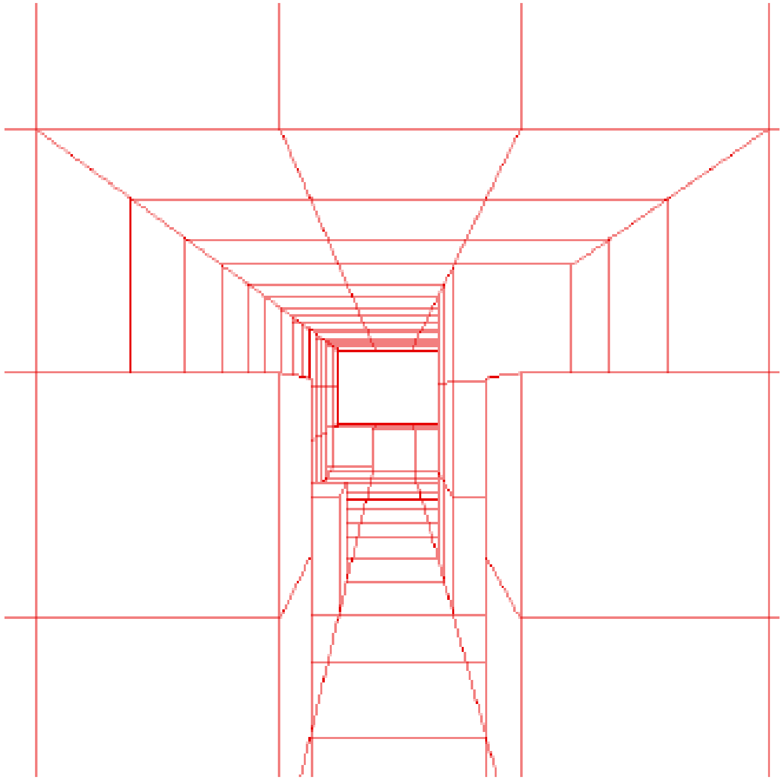




6D SLAM – The ICP Algorithm (14)

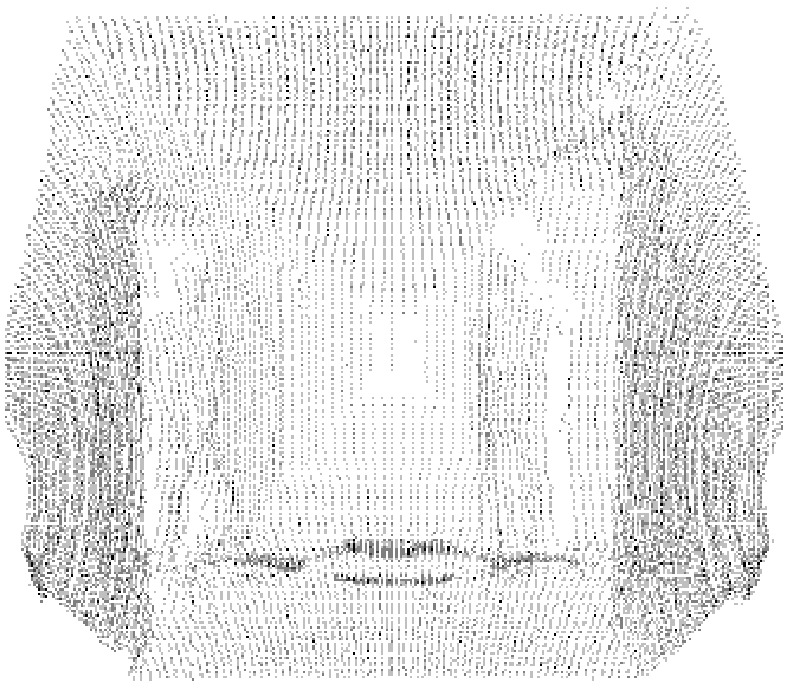
- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud

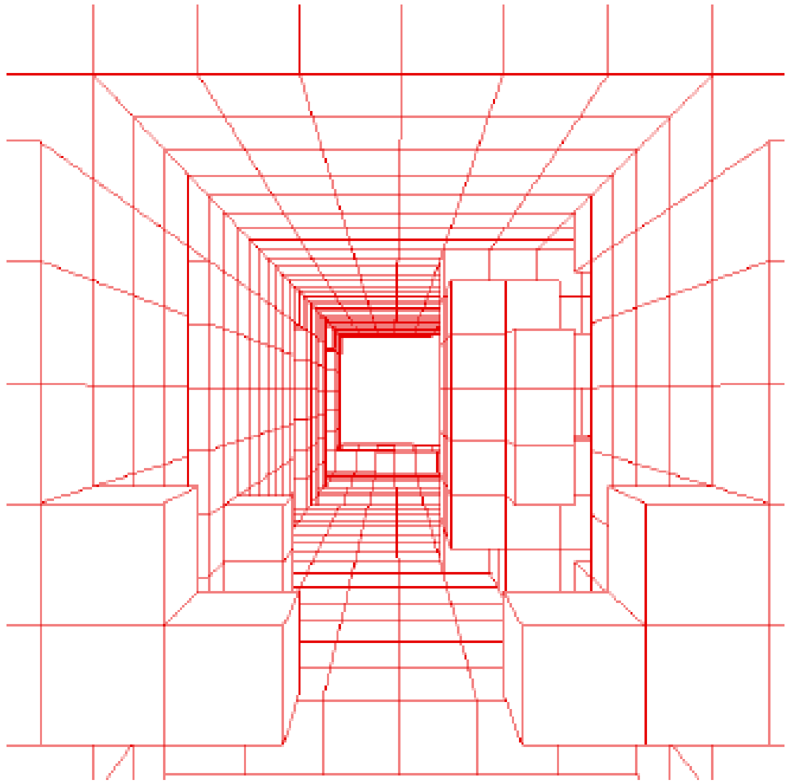




6D SLAM – The ICP Algorithm (15)

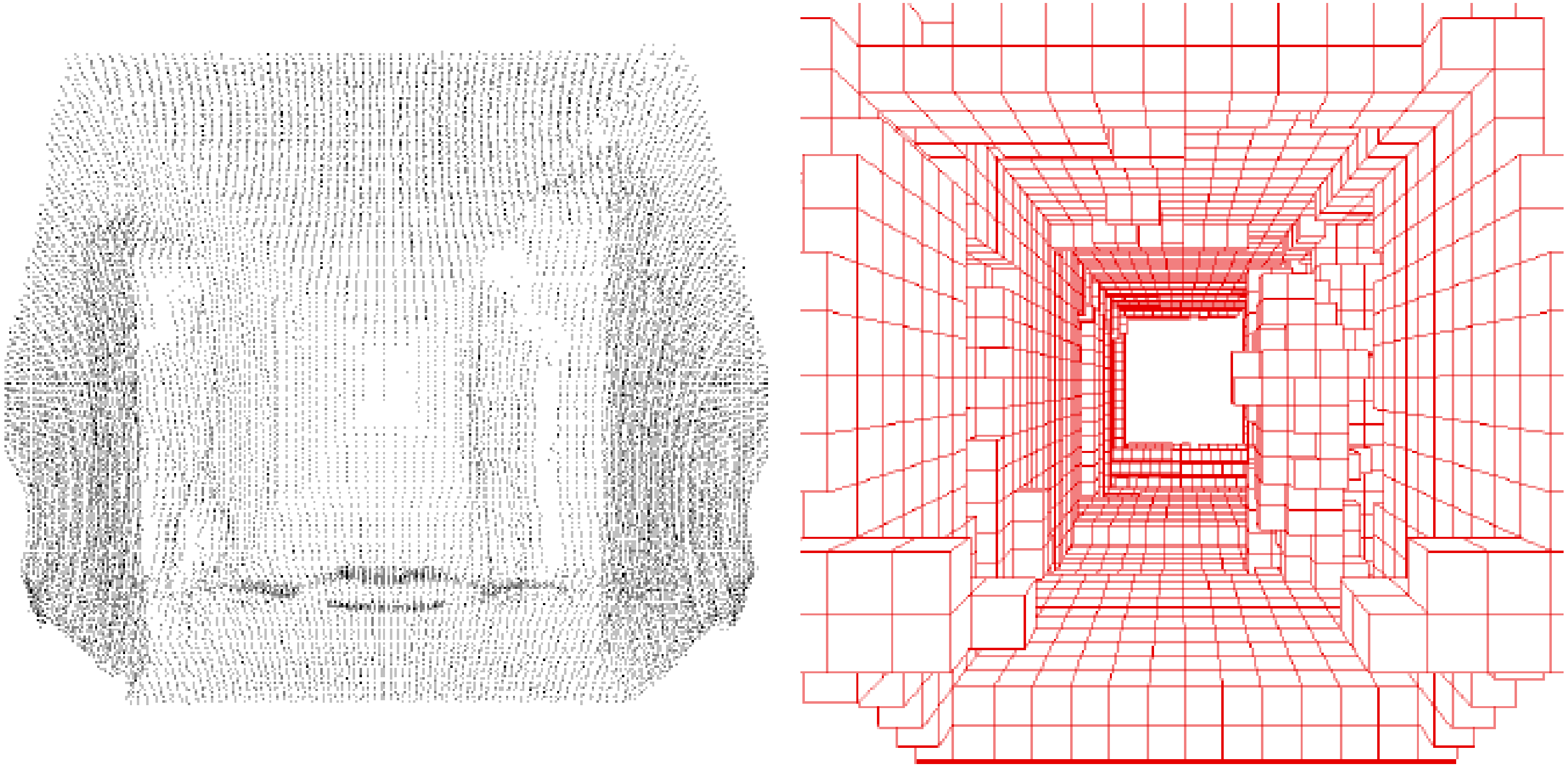
- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud





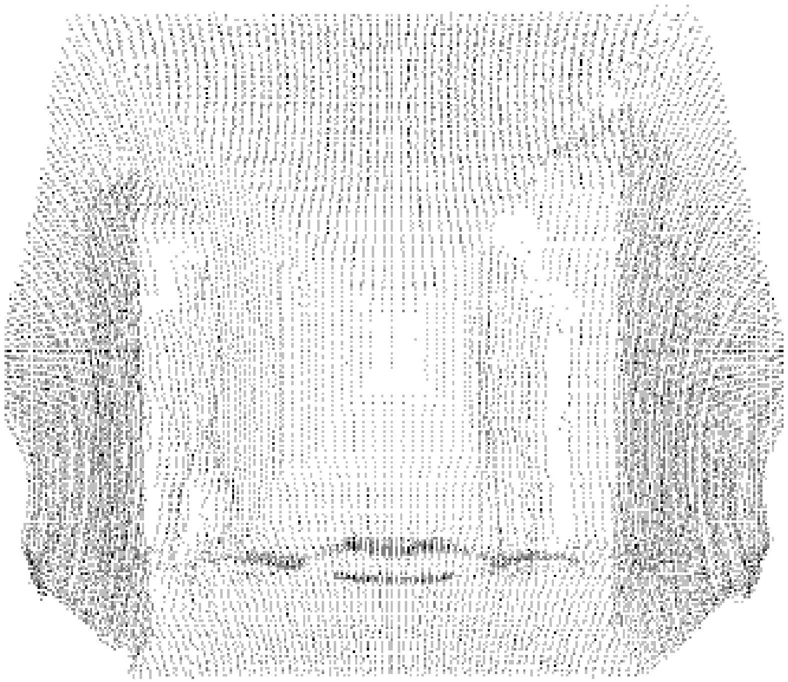
6D SLAM – The ICP Algorithm (16)

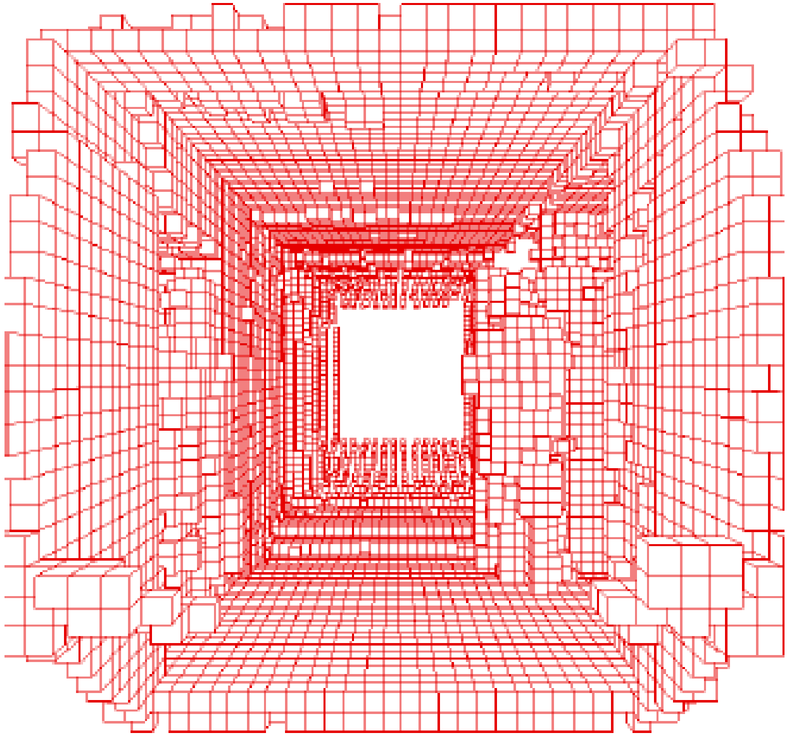
- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud



6D SLAM – The ICP Algorithm (17)

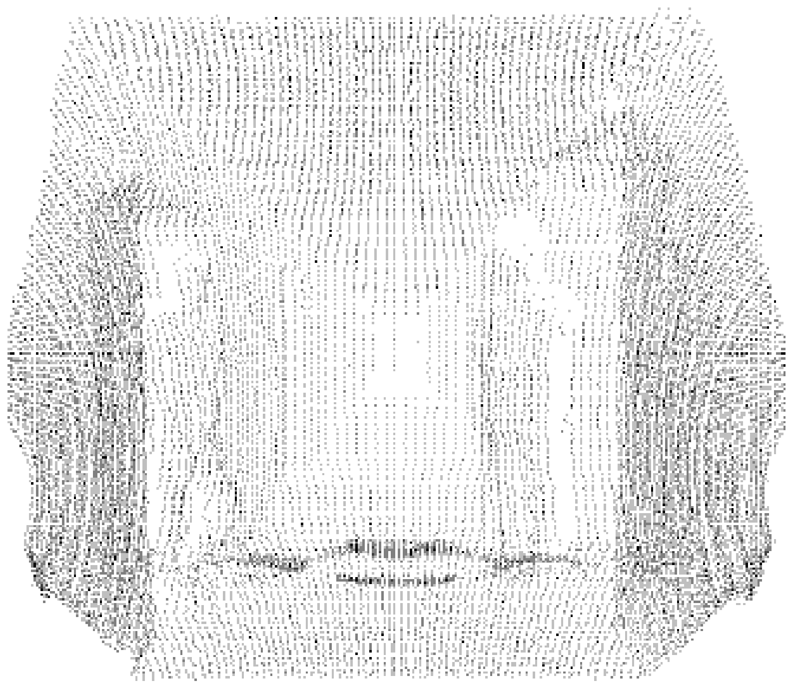
- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud

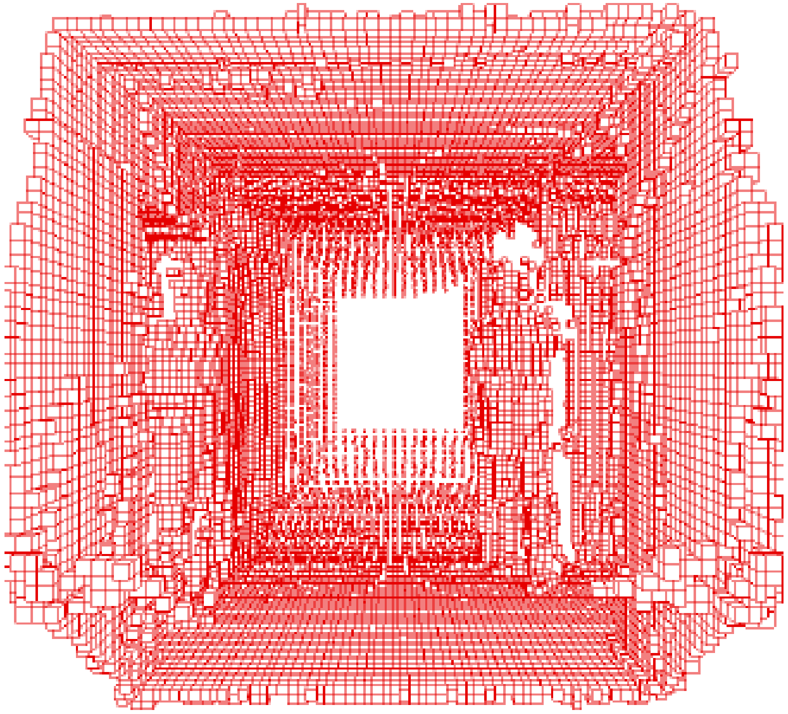




6D SLAM – The ICP Algorithm (18)

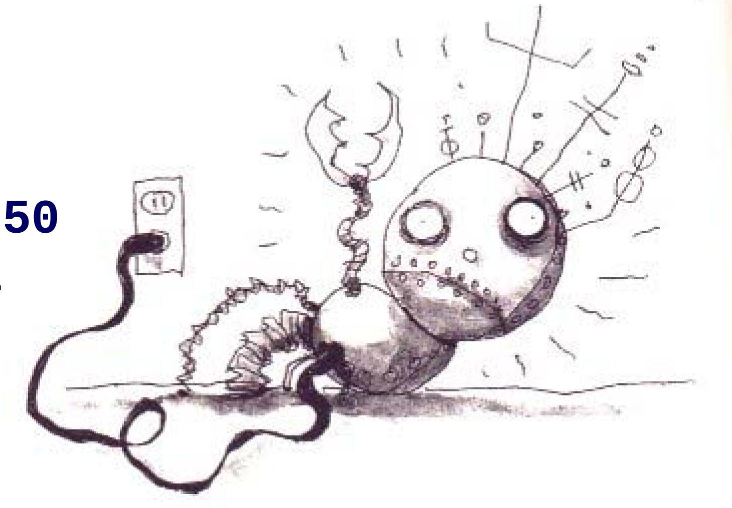
- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud





3DTK – Hands-on-experience (2)

- Things to try
  - Odometry extrapolation and ICP on the mine data set



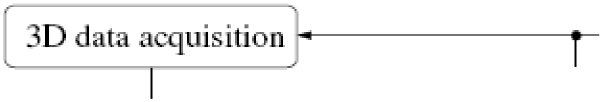
```
bin/slam6D -s 1 -e 10 -r 10 -m 3000 -d 50
     -i 1000 --epsICP=0.000001 --anim=1
     -f old path-to/kvarntorp
bin/show –s 0 –e 10 –m 3000
     -f old path-to/kvarntorp
```

  - Change the above call to **`-e 75`**

  - Odometry extrapolation and ICP on a large loop (Univ. Hannover)

```
bin/slam6D –s 1 –e 75 –r 10 –i 100 ––epsICP=0.00001 –d 150
     path-to/hannover
bin/show –s 1 –e 75 path-to/hannover
```

Closed Loop Detection and Global Relaxation



6D SLAM – Global Relaxation (1)

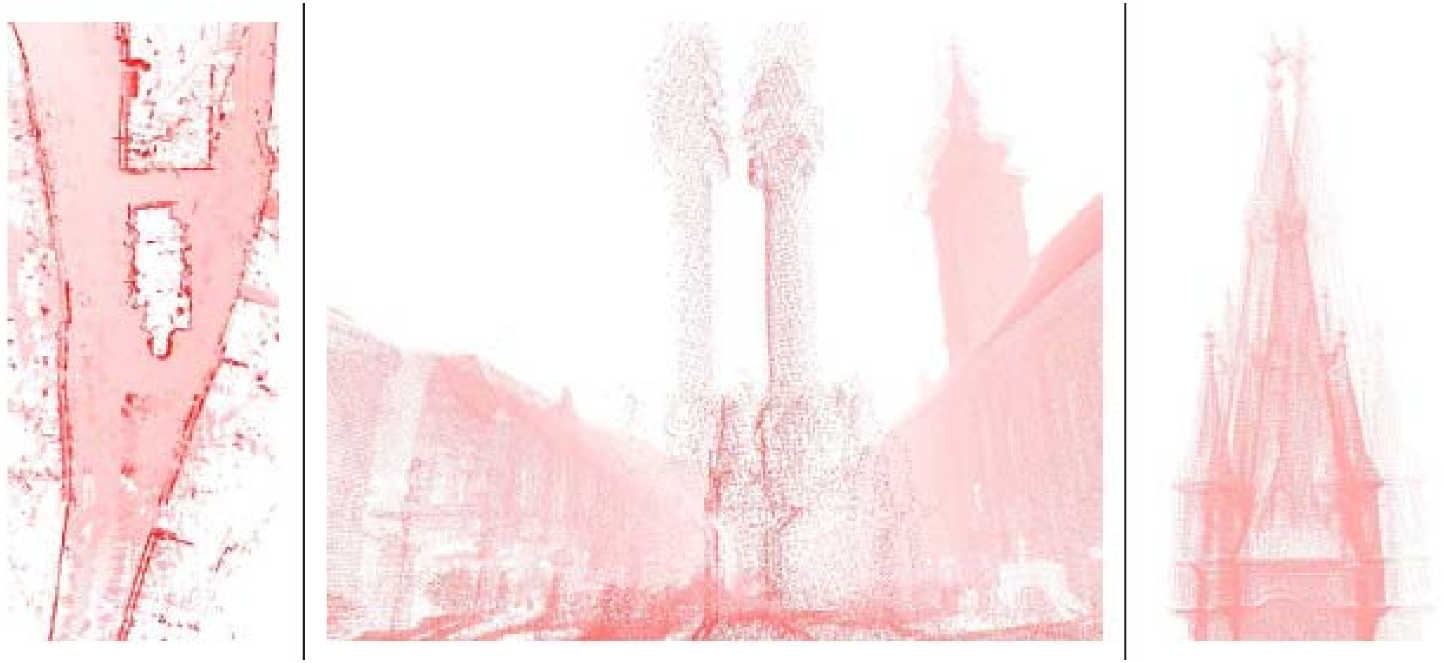
- In SLAM loop closing is the key to build consistent maps
  - Notice: Consistent vs. correct or accurate

- GraphSLAM
  1. Graph Estimation
  2. Graph Optimization

1. Graph Estimation
   - Simple strategy: Connect poses with graph edges that are close enough
   - Simple strategy: Connect poses, they have enough point pairs (closest points)

6D SLAM – Global Relaxation (1)

- Consecutive Scan Matching with ICP results in an erroneous map, since small matching errors sum up.

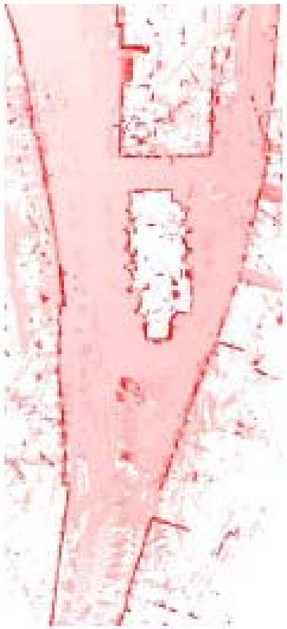


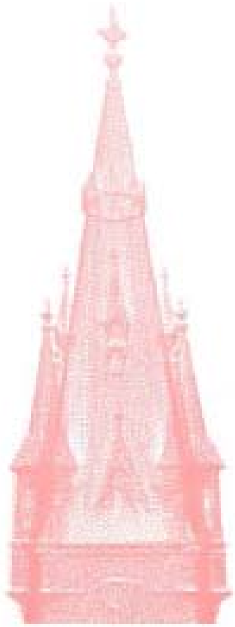
6D SLAM – Global Relaxation (2)

- Consecutive Scan Matching with ICP results in an erroneous map, since small matching errors sum up.

6D SLAM – Global Relaxation (3)

- Consecutive Scan Matching with ICP results in an erroneous map, since small matching errors sum up.





6D SLAM – Global Relaxation (4)

- Consecutive Scan Matching with ICP results in an erroneous map, since small matching errors sum up.

⇨ Replace the ICP error function by a global one, i.e.,

$$D_{i,j} = X_i - X_j$$

$$W = \sum_{(i,j)} (D_{i,j} - \bar{D}_{i,j})^T C_{i,j}^{-1} (D_{i,j} - \bar{D}_{i,j})$$

where $\bar{D}_{i,j} = D_{i,j} + \Delta D_{i,j}$ models random Gaussian noise, added to the unknown exact pose $D_{i,j}$ and $C_{i,j}$ the co-variance matrix of the overlapping scans computed from **closest point pairs**.

(Video Uni Hannover)

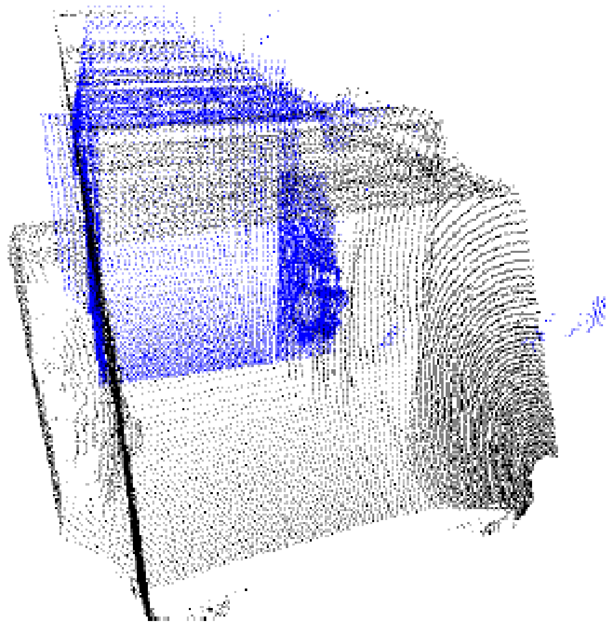
(Video courtesy Riegl)

(Video 1) (Video 2) (Video 3)

6D SLAM – The ICP Algorithm
The global ICP Algorithm (1)

Scan registration Put two independent scans into one frame of reference

Iterative Closest Point algorithm [Besl/McKay 1992]



For prior point set M (“model set”) and data set D

1. Select point correspondences $w_{i,j}$ in {0,1}
2. Minimize for rotation $\mathbf{R}$, translation $\mathbf{t}$

$$E(\mathbf{R},\mathbf{t}) = \sum_{i=1}^{N_m} \sum_{j=1}^{N_d} w_{i,j} \, ||\mathbf{m}_i - (\mathbf{R}\mathbf{d}_j + \mathbf{t})||^2$$

3. Iterate **1.** and **2.**

Four closed form solution for the minimization

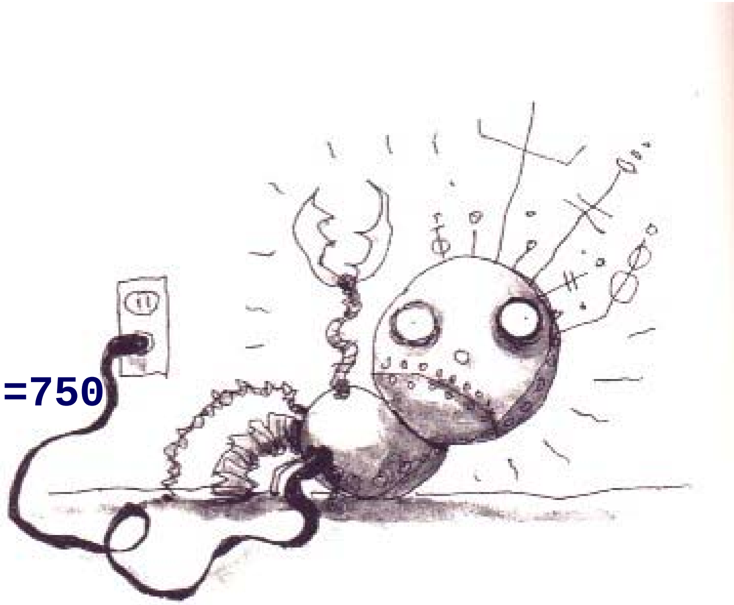
Global consistent registration

$$E = \sum_{j \to k} \sum_{i} |\mathbf{R}_j \mathbf{m}_i + \mathbf{t}_j - (\mathbf{R}_k \mathbf{d}_i + \mathbf{t}_k)|^2$$

Minimize for all rotations $\mathbf{R}$ and translations $\mathbf{t}$ at the same time

3DTK – Hands-on-experience (3)

- Things to try
  - Odometry extrapolation and ICP and loop detection and global relaxation on a large loop



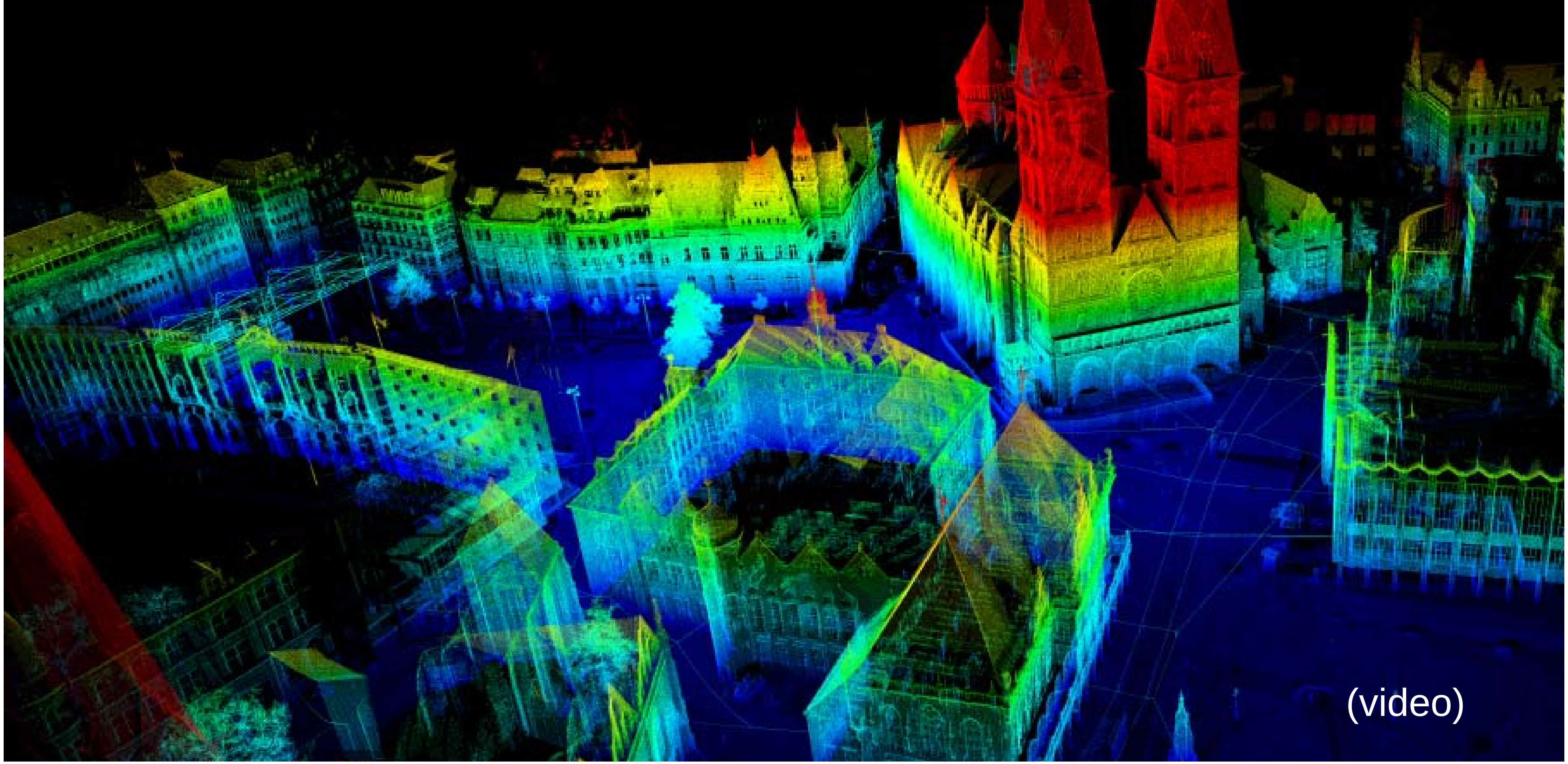
```
bin/slam6D -s 1 -e 65 -r 10 -i 100 -d 75
     --epsICP=0.00001 -D 250 -I 50 --cldist=750
     -L 0 -G 1 path-to/dat_hannover1
bin/show -s 1 -e 65 path-to/dat_hannover1
```

  - Closed loop detection, using the mining data set

```
bin/slam6D -s 1 -e 76 -r 10 -m 3000 -d 50 -i 1000 --epsICP=0.000001
           -I 50 -D 75 --clpairs=5000 -f old path-to/kvarntorp/
bin/show -s 1 -e 76 -m 3000 -f old path-to/kvarntorp/
```

6D SLAM – Hands-on-experience (4)

```
bin/scan_red -s 0 -e 12 -r 10 path-to//bremen_city
bin/slam6D -a 2 -f uos path-to/bremen_city/reduced
    -d 150 -s 0 -e 12 --anim=1 -n path-to/bremen_city/bremen.net
    -G 1 -D 100 -i 0 -I 50 -p --epsSLAM=0.0
bin/show -s 0 -e 12 /home/nuechter/dat/bremen_city/reduced
```



(video)