Single View Correspondence Matching for Non-Coplanar Circles Using Euclidean Invariants

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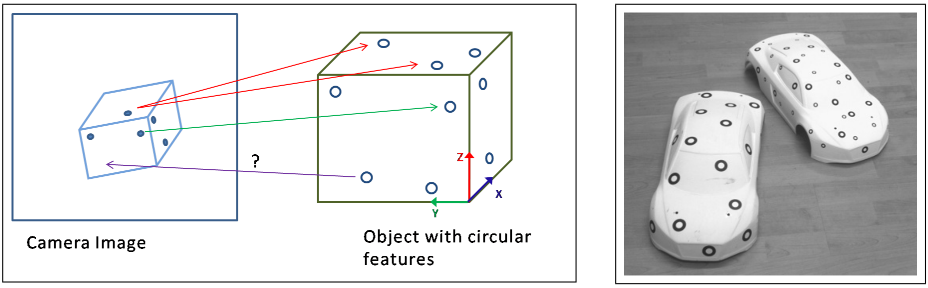
Abstract

In this work we introduce a method to determine 2D-3D correspondence for non-coplanar circles using a single image, given that the 3D information is known. The core idea of our method is to compute 3D information from 2D features, thereby transforming a 2D-3D problem to a 3D-3D problem. Earlier researchers suggested that a pair of non-coplanar circles preserves Euclidean invariants under perspective projection. These invariants can be extracted from their image projections, but with a two fold ambiguity. In this paper, we propose *Conic pair descriptor* based on the Euclidean invariants. The proposed descriptor computes unique Euclidean invariants from known 3D model and Euclidean invariants with two fold ambiguity from its image projections. The proposed matching approach follows three steps to obtain correspondences between the circular features against the ambiguity. In this paper, we have included a detailed account of factors affecting the computation of invariants from conic projections. We have conducted experiments on real and synthetic models, in order to evaluate the proposed method. The experiment with synthetic images focuses on showing the impact of the size and plane orientation of the circles on the success of descriptor matching. We prepared 3D models with artificial circular features and obtained the ground truth 3D data with a Photogrammetric measurement system. The results of the correspondence matching algorithm are evaluated against the ground truth. We also show that our method is robust against false positives and capable of supporting real-time applications.

1 Introduction

Correspondence matching is one of the key problems in computer vision. Pose estimation and object recognition applications require accurate knowledge of correspondence relation $\cancel{(m_i \leftrightarrow M_i)}$ $m_i \leftrightarrow M_i$ between 3D model features $\cancel{(M_i)}$ M_i and 2D image features $\cancel{(m_i)}$ m_i. In case of monocular systems the $\cancel{m_i \leftrightarrow M_i}$ 2D-3D correspondence problem becomes more challenging as the depth information is lost. Various Computer Vision and Augmented Reality applications demand correspondence matching from a single view. Popular approach to achieve this is to compute projective invariant from features ~~like~~ such as points, lines and conics [7]. Such features are ~~selected~~ preferred because they are easier to detect from the images. Invariants are extensively studied topic in vision community, Forsyth et al. [5] and Gros [6] covered a detailed study on ~~projective invariant descriptors~~ projective invariants and their stability under projective transformation. In this paper we will focus on a specific



(a) (b)

Figure 1: Matching problem for non-coplanar circles ; (a) A primitive example explaining the problem when image features have same properties and correspondence relation with model is ambiguous (b) A realistic model prepared with circular fiducials used for Photogrammetric measurements in Industry [Comments: (1) better to add caption to (a) and (b) so that the readers can easily get what they are. (2) better to save all figures as vector image files as .eps and .pdf so that the figures can keep its visual even when they are zoomed up.]

class of features, that is *circles*. Circles are one of the most primitive features~~,~~ . In case of industrial scenario circles are widely present on the model as natural features or circular fiducial are used for Photogrammetric measurements [10] (see Fig. 1). In such cases it is favourable to build tracking approach depending on such features. A single view correspondence matching problem with coplanar circles and ellipses is widely studied [8]~~,~~ . However matching problem with non-coplanar circles has not been addressed. Figure 1 shows a basic example of the problem, where multiple identical circular features exist on different planes of a 3D model. In this case features existing on the model is an advantage, but coplanar invariants can not be used for correspondence matching. In out work we focus on solving this problem of single view matching of multiple non coplanar conics.

3D objects retain Euclidean invariants rather than projective invariants [5]. In case of 3D objects Euclidean invariants are difficult to extract from images due to perspective mapping. Circles have a special property to retain depth information under projective transformation. A world circle always produces an elliptical curve on the image plane. If size of circle is known orientation of circle plane can be defined in 3D (camera coordinates) with a two fold ambiguity [5] [16]. Further, Forsyth et al. [5] proposed that up to three projective invariant can be computed from a non-coplanar pair of circles. They explain that angle between circle planes (angle between surface normals) and distance between centre of the circles are invariant quantities. The concept was proposed in early 90s, however these invariants have remained unexplored. We propose using these invariants to solve correspondence problem when multiple 3D circular features exist on a model. In this approach we bring problem from 2D to 3D by computing 3D invariants from image features, then solve 3D-3D matching problem with model. We introduce *ConicPairDescriptor*, which encapsulate invariants computed from elliptical image features to solve the conic ambiguity and provide accurate matching with 3D features. The proposed method is first attempt to use these invariants, therefore we also carried out simulations to show stability of invariants against change of perspective.

Our contribution is a new method to accurately identify image correspondences when multiple identical 3D circular features exist in the scene. We assume that calibration of camera is known and 3D information of features is available. Often in Industry based model tracking applications 3D-CAD data is known. Our matching method is suitable for tracking any 3D objects having known circles on different planes. In close range photogrammetry multiple circular markers (Figure. 1) are placed on 3D models for surface measurements [10]. These measurements include computation of surface normal and 3D position of each marker. This process involves taking multiple images of the model with additional presence of encoded markers in the scene to solve correspondence. Once 3D measurements are done our method can be extremely useful to support tracking application without using coded patterns. Similarly various industrial parts having natural circles can be identified and tracked with this method. We prepared two car models with circular markers for evaluating our matching method. The proposed method can find corresponding circular marker from a single image, with high accuracy. We also show that our method is stable against false positives detected from the scene. Our method is fast enough to support real-time tracking applications.

2 Related Work

Object detection and pose estimation from conic features is widely studied in 3D vision literature [1][16] [19] [13]. Circular shape is also a popular choice for designing artificial fiducial. Detection of contour points from image and fitting ellipse is a well studied topic [4]. Quan [14] proposed a two view approach for finding correspondence and 3D reconstruction with conic section. Authors have proposed methods to compute invariants for coplanar conics [5] [3]. A 3D problem is simplified to 2D when coplanar features are recovered and used for correspondence. Ying et al. [20] use a coplanar pair for camera calibration. Uchimaya et al. [18] developed invariant descriptors from multiple coplanar circles, and extended the work for deformable model [17]. Work of [9] [11] [12] propose a circular marker for 6D pose estimation, no invariants are computed as correspondence is solved by using unique coded pattern around the circle. [9][12] use circular shape to define circle plane in 3D, Additionally use a coded pattern is used encode 6D pose without ambiguity.(Fig). Luhmann [10] provides detailed account of methods using point circular fiducials in close range Photogrammetry. The current state of the art methods coded patterns are introduced to simplify correspondence problem. [GOM][AICON] are one of the industrial supplies for close range photogrammetry measurement equipments.

[Refer Thesis: Comment on catalogue based methods,]

Literature study suggests that, existing methods either provide a solution for coplanar circular features or non-coplanar coded circular features. The novelty of our method is that it addresses 2D-3D matching problem for non planar circles present in the scene.

3 Method

We will assume that both the 2D and 3D data are already available and focus on the matching method in detail. The 3D data includes the surface normal N_i, centre position M_i and size R_i (diameter) of the circles present on the model. The 2D data includes centre points m_i and conic matrices C_i recovered from the undistorted image. The camera intrinsics (K) and distortion parameters are known. We have followed the approach presented by Naimark

[11] for ellipse detection and Farin et al. [2] for computation of conic matrices from ellipse parameters.

3.1 Conic Invariants : Theory and Computation

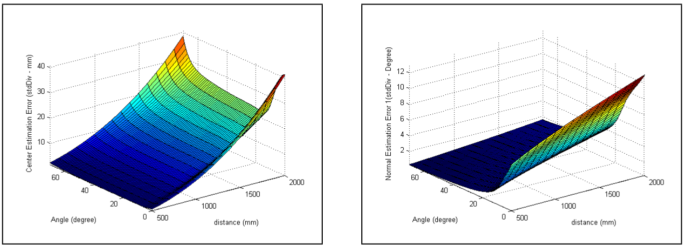
In this part we will explain the theory and computational aspects behind generating Euclidean invariants from the image conics. Let the camera projection centre be assumed as the vertex of a cone having the world circle is as its base. Now the image plane can be considered as a cutting plane π, which always creates an elliptical cross section of the cone. A rotation transformation R_c can be applied to the camera coordinate system such that the new image plane π' intersects the cone as a perfect circle, with the z-axis passing through the centre of both the circles. The normal of the plane (π') is same as the base of the cone, therefore a normal Nc_i to the plane π can be computed using R_c. Similarly, using R_i, distance between the plane π' and the base of the cone can be computed. Additionally, 3D position of circle centre Mc_i can be computed in the original camera coordinate frame. The normal Nc_i and the point Mc_i are sufficient to define the orientation of the circle in camera coordinate system. R_c is a combination of two rotations, one of the two has $\pm\phi$ rotation ambiguity, which in turn introduces a two fold ambiguity in the solution (See Eq.1). We will refer the ambiguity as *Conic ambiguity* and the method to obtain plane orientation as *Ellipse backprojection*. The mathematical model for *Ellipse backprojection* is extensively covered by both Forsyth et al. [5] and Diego [9].

$$\text{Ellipse Backprojection}(m_i, C_i) \rightarrow \pi({Nc_i}^1, {Mc_i}^1), \pi'({Nc_i}^2, {Mc_i}^2) \tag{1}$$
$$\text{Ellipse Backprojection}(m_j, C_j) \rightarrow \pi({Nc_j}^1, {Mc_j}^1), \pi'({Nc_j}^2, {Mc_j}^2)$$

Forsyth et al. [5] explained that in case of three dimensional objects invariant descriptors consists of Euclidean invariants rather than projective invariants. Various invariants can be computed from a pair of non-coplanar circles based on Ellipse backprojection. We use following invariants for our method,

- **Angle between planes** (θ) : It is same as the angle between the surface normals (i.e. $\angle(Nc_i, Nc_j)$). θ can be recovered from the image conics without knowledge of the circle size , however due to *Conic ambiguity* θ will have 4 solutions out of which only one is correct.
- **Distance between circle centres** : This is the distance between Mc_i and Mc_j. The length of the vector d_c is invariant (object scale should be known) and consistent despite of the ambiguity.

The quality of recovered plane from *Ellipse backprojection* depends on distance from the camera r and the viewing angle η(angle between the image plane and the circle plane)[19]. We performed simulations to understand behaviour of Ellipse Backprojection with respect to both r and η. The parameter r is varied from 0.5 to 2 m and η from 0-70° in step wise manner, while recording 100 iterations at each step (Fig. 2). Realistic values have been assumed for camera intrinsics and size of circle (R_i = 5,8,12 mm). Figure. 2 shows results recorded with $R_i = 12$, and image noise as $\sigma = 0.3$. The results suggest, at low viewing angles 0-10° both normal and centre estimation errors are high, at any given distance. This can be explained by the fact that for lower values of η image projection of a circle is circular and not elliptical, therefore recovery of elliptical parameters may have errors. We also observe



(a) (b)

Figure 2: Ellipse backprojection results at different camera distance and viewing angles,Image noise = $\sigma = 0.3, \phi = 12mm$; (a) Error in Centre (Mc_i) (b) Error in Normal (Nc_i)

that the estimation error grows with camera distance, however the error in normal recovery appears less sensitive to camera distance than the error in centre recovery. The results obtained with $R_i = 5, 8$ *mm* show similar pattern, however the magnitude of error is higher as the image projection become smaller in the image. We also computed and compared the results of estimated centres ${Mc_i}^1$ and ${Mc_i}^2$. The maximum distance recored between the two is ≤ 0.1 mm for $R_i = 12$ mm, this suggests that ambiguity can be neglected for the recovered centre position. The centre recovery is prone to higher errors with higher camera distances, however it produces a consistent solution for invariant d_c. After performing *Ellipse backprojection* we have essentially transformed our problem from image space to three dimensional space.

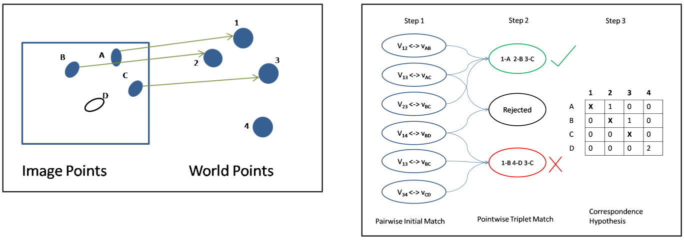
3.2 Descriptor Generation

This part mainly discusses generation of *Conic pair descriptor* from Euclidean invariants. The invariants for 3-D model are computed from available 3D data (M_i, N_i) without any ambiguity (See Eq. 2). The same set of invariants can be computed from corresponding image features, where the recovered d_c component is unique and θ component has 4 solutions (See Eq. 3). In our approach we pursue the idea that the existence of multiple features on the model can be used to overcome the Conic ambiguity problem. The principle idea is to generate descriptors from conic invariants to perform a descriptor matching to obtain $m_i \leftrightarrow M_i$ correspondence. The proposed *Conic pair descriptor* structure,

$$Conic\ pair\ descriptor_{model} = V_p \langle d_c, \theta \rangle_{i,j} \tag{2}$$

$$Conic\ pair\ descriptor_{image} = v_q \langle d_c, \theta_{11}, \theta_{12}, \theta_{21}, \theta_{22} \rangle_{i,j} \tag{3}$$

where V_p represents world circles i, j and v_q represents image conic pair i, j. The reader should note that given a set of points and their corresponding normals in 3D camera space, PFH descriptor [15] also computes similar invariants, however in our case the concept can not be applied as the *Conic ambiguity* restricts us from computing a unique set of invariants. Unlike popular methods, a *Conic pair descriptor* represents two features at same time. In order to uniquely represent a single conic using Euclidean invariants, at least more than two conic features are required. Addition of each conic feature adds 3 wrong solutions of θ in the descriptor, additionally the matching must rely on detection of all the conics used for descriptor computation. We propose matching $v \leftrightarrow V$ first, thereby finding a corresponding conic pair, further we solve individual correspondence $m_i \leftrightarrow M_i$ problem. Descriptor



(a) (b)

Figure 3: Matching problem and the overview of the method to generate correspondence hypothesis ; (a) A problem showing four image points, and their correspondence with world points (1-A,2-B,C-3) and one false positive (D) (b) Image shows the simplified matching process for the problem suggested in (a), partial results of all 3 steps are shown.

$v_{\{1..q\}}$ are computed among each pair of detected n image conics, where $q = \binom{n}{2}$. $V_{\{1...p\}}$ are computed off-line as the 3D data is already available. In this case for l world circles $p \leq \binom{l}{2}$, as pairs not likely to appear in same image can be rejected. After computing the *Conic pair descriptors* the following 3 step matching approach is used to achieve $m_i \leftrightarrow M_i$ correspondences.

3.3 Step 1: Pairwise Initial Matching

In the first stage of the matching process we compare the *Conic pair descriptors*. The objective is to reduce complexity of the problem by finding the possible pair correspondences $(v \leftrightarrow V)$. First the unique component(d_c) between the descriptors is compared, if matched then (θ) component of V is compared with all 4 values of θ component in v. T_{d_c} and T_θ are the threshold values used to compare the respective components. The stage may result in *one to many* type of relation between descriptors. This can be either due to similar feature orientation on object or due to presence of the *Conic ambiguity*. The reader should note that a descriptor represents a pair of conics, therefore the stage is called pairwise matching. [REFER IMAGE]

3.4 Step 2: Pointwise Triplet Matching

[REFER IMAGE] In this stage we simplify the problem further and obtain hypothesis on point wise matching $(m_i \leftrightarrow M_i)$ by performing a verification on $v \leftrightarrow V$ matching results. The first objective is to compare the results of step 1 to identify and reject false descriptor matches. We seek three $v \leftrightarrow V$ results, such that they complement each other to form a unique $m_i \leftrightarrow M_i$ hypothesis for three points. A simple two stage approach is proposed to generate a triplet matching hypothesis,

1 Find any two results of Pairwise Initial Matching in which both the image and the model descriptors represent one and only one common conic. If such results exist then an initial triplet matching hypothesis can be proposed.

$$V_{12} \leftrightarrow v_{AB}, V_{13} \leftrightarrow v_{AC} \xrightarrow{\text{Triplet Hypothesis}} [1\ 2\ 3] \leftrightarrow [A\ B\ C]$$

In the example above we can see that world conic 1 and image conic A is common among the two solutions. We form a 3 point matching hypothesis with these results.

Goal : Find all possible V_p similar to v_q ;
Initialisation : $T_{d_c} = 10$, $T_\theta = 5$;
forall the 3D Feature Descriptors (V), $p \leftarrow 0$ **to** n **do**
    forall the 2D Feature Descriptors (v), $q \leftarrow 0$ **to** l **do**
        if compared_c (V_p,v_q) $< T_{d_c}$ **then** // compares d_c component
            if compareθ (V_p,v_q) $< T_\theta$ **then** // compares θ component
                // All 4 solutions of θ in v_q are checked
                ;
                SavePairResult(p,q) // Save matching descriptor pair
                ;
            end
        end
    end
end

Algorithm 1: Pairwise Initial Matching algorithm

2 Find a new descriptor matching pair which can verify the triplet matching hypothesis formed in the previous stage (e.g. $V_{23} \leftrightarrow v_{BC}$).

If a triplet is verified then the result is saved for further processing. This step may also contain false triplet matches (Ref. 3). If x number of results are generated in step 1, the number of pairs compared in this stage is $\binom{x}{3}$.

3.5 Step 3: Correspondence Hypothesis

In this final step results of Pointwise Triplet Matching are combined and a voting matrix is generated (Ref. 3). A pair having maximum votes in the matrix is proposed as a correspondence hypothesis. In case of conflicting votes the respective $m_i \leftrightarrow M_i$ relation is not considered. A minimum of 3 correspondence are required to compute the pose of the object [8]. We recommend computing pose of the object by selecting top 3 correspondence results. The computed pose can be used to further verify and modify other relations correspondences. This implies that for images having only 3 conic features matching results verification is not possible and results may not be reliable.

4 Evaluation

In this section we will cover experiments (simulations and real) carried out to comment on accuracy and robustness of the algorithm. To the best of our knowledge no other application has attempted using the euclidean invariants generated by circles. The reader should note that the problem of achieving single image correspondence with non-coplanar circles is not addressed earlier. Therefore, alternative methods for comparison are not available. We prepared a test scenario by attaching uncoded circular markers of size ϕ = 12 mm ($M_i = 20$) and 5mm ($M_i = 26$) to two identical car models. The markers are attached randomly and coplanar placement is avoided. Measurement of 3D data is done with state of the art metrology systems, and ground truth is established by giving each model point a unique ID in database.

Table 1: Descriptor Matching Analysis

ϕ	θ	Camera Distance (mm)	Matching Success(%)
12	0-40	500-2000	80-95
	40-90	500-2000	50-80
	0-40	2000-4000	40-60
	40-90	2000-4000	20-40
5	0-40	500-1200	75-90
	40-90	500-1200	40-75
	0-40	1200-2000	55-80
	40-90	1200-2000	35-55
20	0-80	500-2000	100
20	80-90	500-2000	30-90

Images of the models are captured from different perspective with a calibrated camera (5 Mpx).

4.1 Experiment 1 : Descriptor Matching vs Marker Orientation

This experiment is carried out to understand the factors affecting descriptor matching stage with respect to orientation of the circles. A MATLAB simulation is performed to position two circles in different orientations and captured images (noise $\sigma = 0.3$) from 1000 different camera positions for each orientation. The control parameters are distance between circle centres (d_c) is varied from 10 - 150 and plane angle (θ)is varied from 10 - 90. Realistic values are used for camera intrinsics, T_{d_c} = 10 and T_θ = 5 are kept constant through the experiment. Rotation (R) and Translation (T) parameters for camera position are random, only T_z parameters is limits are different from smaller and bigger markers.

The table above provides summary of key observations of the experiment. Matching result shown is AND operation between d_c match and θ match (Sec. 3.3). The key learning from this experiment is that descriptor matching is influenced more by angle between planes than distance between circles at fixed threshold values. It is also seen that success of matching can be improves significantly with increased size of circles, even at higher values of θ. This experiments suggest that when features placement is possible in scene it is advised to chose larger circles or surfaces with lower plane angles for better matching.

4.2 Correspondence Matching vs Threshold Settings

The first experiment has recommendation regarding choice of features and placement of features. In a realistic scenario this option may not be available, therefore aim of the experiment is to understand the role of threshold values (T_{d_c}, T_θ) in overall correspondence matching results. In order to perform this experiment we took 75 images of car model with 12 mm markers (Distance Range 0-2000 mm). All the results (Table. 2) are verified with ground truth data manually. The images have at least 4 conics detected in each image. If hypothesis proposes less than 3 $m_i \leftrightarrow M_i$ results we consider it as *Not Converged*. *Correct* results are divided in two categories purely to provide detailed overview of algorithm performance. *Complete* refers to $m_i \leftrightarrow M_i$ hypothesis being 100% correct, *Partial* results suggest that some of the matching results (excluding top 3 voted pairs) are not correct.

Table 2: Correspondence matching with varying threshold settings

Settings		NC	Correct		Wrong
T_θ	T_{d_c}		Complete	Partial	
5	5	13	62	0	0
5	10	8	67	0	0
5	15	4	67	0	4
3	5	28	47	0	0
3	10	21	50	4	0
3	15	19	52	1	3

Table 3: Time Analysis

Algorithm Stage	CAM 1		CAM 2
	Model	Model + FP	Model
Image Undistortion	39.53%	12	11.79
Marker Detection	38.51	34	30.75
Correspondence Matching	0.35	44.2	1.34
Pose Estimation	21.61	9.8	56.12

In Sec. 3.1, we learned that d_c recovery is weak and therefore a flexible threshold may be appropriate for matching. The results (Table. 2) show that flexible T_{d_c} with stringent T_θ may allow more Pairwise matching results, but due to *conic ambiguity* of θ probability of wrong matching goes higher. On the other hand, very stringent thresholds lead to higher non convergence. Therefore, a right balance of threshold can be selected to achieve balanced results. Our preferred settings for experiments is $T_{d_c} = 10$ and $T_\theta = 5$, as 90% of images show maximum 100% success with (0%) wrong matching. The selection may require change based on density of the features.

4.3 Robustness against false positives

4.4 Time Analysis

In this experiment we focus on analysing time consumed by the matching method when introduced into a tracking application. Two cameras CAM 1 (2560 x 1920) and CAM 2 (640x480) are used for tracking, the results presented are averaged over 100 frames. The results show that our method takes $\leq 1\%$ time from in the tracking pipeline. In terms of frame rates we achieve 2-3 FPS with CAM 1 and 7-8 FPS with CAM 2. Additionally, an exhaustive experiment with 90-140 false positives in the scene shows that matching consumes maximum time in the pipeline (0.7 FPS). Limited tracking range (< 500 mm) of CAM 2 does not allow experiment with such large number of false positives.

5 Conclusion & Future work

In this paper we have demonstrated a successful approach for solving 2D-3D correspondence matching problem for non-coplanar circular features from a single image. We propose a new *Conic descriptor* which represents euclidean invariants generated by a pair of non-coplanar circles. Our method can successfully define correspondences for more than 3 circular features are present in the scene. The proposed method is the first to address the correspondence matching using these invariants since its introduction in the 90s. Our contribution also includes providing detailed understanding of behaviour of invariants with respect to orientation and size of the circles. The major factors affecting matching are also discussed to optimize the method for best possible matching results based on application (threshold settings, circle size, camera distance). The results of the experiments support our claim, that the method is fast, reliable and robust against false positives. Our method can be used for object tracking or object identification in Industrial scenario, where natural or artificial circular features exist on the models. However, the method is generic and can be used for any application dealing with non coplanar circles. The 3D information of the features on the model and camera calibration are the only prerequisites for matching. The reader should also note that algorithm may not perform well with symmetric or coplanar arrangement of circular features.

In context of future work, we would like to improve the method to be able to handle features of different sizes simultaneously. Also a faster matching strategy is required to handle large number of feature points and false positives. We would like to use the same invariants to compute 2D-2D correspondence matching between two images in order to generate the 3D data which is a prerequisite now. We also consider using such matching algorithm to support creating 3D markers for monocular Augmented Reality applications. This can be a cheap alternative to conventionally used 3D spherical markers.

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