ELLIPSE FITTING AND THREE-DIMENSIONAL LOCALIZATION OF OBJECTS BASED ON ELLIPTIC FEATURES

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ABSTRACT

In the area of robotic vision we are interested in developing algorithms for the control of a robot manipulator by means of visual feedback. Image analysis and exploitation of the image features are therefore steps of importance. This paper deals with the problem of three-dimensional localization of objects representing circular patterns. The first part of the paper is concerned with the parametrization of elliptic curves and a second part is dedicated to the estimation of an object's position and orientation.

1. ELLIPSE FITTING

In computer vision, ellipses are commonly viewed as oblique projections of circular features in the image. Such features are typically rare and salient. Hence, they are excellent cues if they can be detected reliably. A further advantage of circular features is that they can constrain the view point to a greater degree than simple line features. Thus they are more reliable in the determination of the object orientation and localization.

There are many methods for ellipse fitting available in the literature. Methods based on the Hough transform [2], [5] are not well suited for our purpose, since they require a large number of data points, do not produce a unique solution and are computationally expensive. The least squares method [3] suffers from severe bias when the arcs are shallow and the noise is not Gaussian. Improvement is achieved by the extended Kalman filter [1], [4], but the problem of bias persists in the case of shallow arcs with high curvature.

An ellipse is represented by the biquadratic form

$$f(x,y) = ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \qquad (1)$$

with the constraint $b^2 - ac < 0$. The set (a,b,c,d,e,f) represents the ellipse parameters. Generally the objective is to find a set of parameters minimizing the error criterion $J = \sum_i f(x_i,y_i)^2$. The bias embodied in the estimation based on the above criterion is due to the assumption that error at the ith point is given by $f(x_i,y_i)$. In our approach we take as error measure the orthogonal distance d_i between the measured point and the fitted ellipse. The fitting process is further enhanced by a preliminary estimation of the center and of the orientation of the ellipse. This information is used to match the coordinate system with the ellipse center and axis. This first estimation is carried out with a simple least squares method applied to the first criterion mentioned above.

1.1. Application of the Extended Kalman Filter

With the normalization $a + c = 1$, (1) becomes:

$$f(x,y) = a(x^2 - y^2) + 2bxy + 2dx + 2ey + f + y^2 = 0 \quad (2)$$

The equation (2) defines a nonlinear relation between the parameter vector $A = (a,b,d,e,f)^t$ and the measurement point $\hat{Y}_i = (x_i, y_i)$

$$F(A, \hat{Y}_i) = 0. \qquad (3)$$

$\hat{Y}_i$ is related to its exact value Y_i by $Y_i = \hat{Y}_i + v_i$, where v_i is the measurement noise with the statistics

$$E(v_i) = 0; \quad E(v_i v_i^t) = L_i = \begin{bmatrix} n^2 & 0 \\ 0 & n^2 \end{bmatrix}$$

Given N points $\hat{Y}_i$, the initial values $\hat{A}_0$ and S_0 of the vector parameters and its covariance matrix, the error criterion is

$$J = (A - \hat{A}_0)^t S_0^{-1} (A - \hat{A}_0) + \sum_{i=1}^{N} d_i^2 \qquad (4)$$

The distance d_i is weighted by the measurement noise covariance matrix

$$d_i^2 = (\hat{Y}_i - Y_i)^t L_i^{-1} (\hat{Y}_i - Y_i) \qquad (5)$$

by means of adequate linearization [16], (4) is written

$$J = (A - \hat{A}_0)^t S_0^{-1} (A - \hat{A}_0) + \sum_{i=1}^{N} \frac{1}{R_i^2} (z_i - h_i^t A)^2 \qquad (6)$$

where

$$z_i = -F(\hat{A}_{i-1}, P\hat{Y}_i) + h_i^t \hat{A}_{i-1} \qquad R^2 = \frac{\partial F}{\partial Y}^t L \frac{\partial F}{\partial Y}$$
$$h_i = \frac{\partial F}{\partial A} - \frac{1}{R_i} \frac{\partial R_i}{\partial A}^t F(\hat{A}_{i-1}, P\hat{Y}_i) \qquad L = PP^t$$

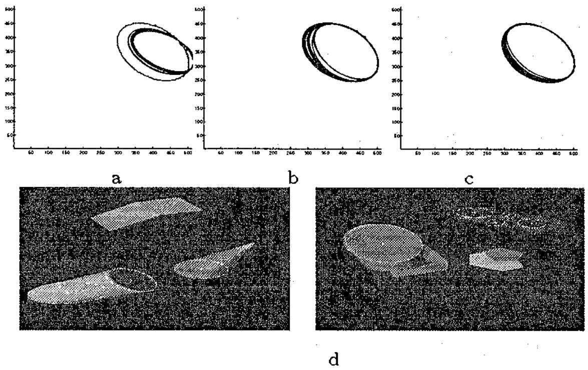


Figure 1: A sets of fitted ellipses generated by the LS (a), The classic EKF (b) and our method (c). The data is a simulated third ellipse arc length to which is added a (0,2) Gaussian noise.Estimated ellipses superimposed on original images (d)

The vector parameter minimizing the above criterion is then determined by the recursive Kalman filter equations :

$$\begin{aligned} K_i &= S_{i-1}h_i(R_i^2 + h_i^t S_{i-1} h_i)^{-1} \\ \hat{A}_i &= \hat{A}_{i-1} + K_i(z_i - h_i^t \hat{A}_{i-1}) \\ S_i &= S_{i-1} - K_i h_i^t S_{i-1} \end{aligned} \tag{7}$$

1.2. Results

Tests have been made on simulated ellipse arcs to which we have added a (0,2) Gaussian noise. A comparison between the LS, EKF and the suggested method is shown in figure 1a,b,c. Tests on real images are shown in figure 1d. We realize that elliptical feature have been extracted and located with reasonable accuracy.

2. OBJECT LOCALIZATION

In model-based localization the problem of estimating object position and orientation can be stated as follows : given a set of object features (points, lines, circles), given their geometric configuration and description in the object frame and given the projections of these features in the image, determine the rigid transformation composed of translation and rotation between the camera frame and the object frame. The translation corresponds to the object position and the rotation corresponds to the object orientation. The solutions provided the literature for this problem can be divided into two main categories: analytical and numeric methods. Analytical solutions are used when the number of fits image features-object model features is reduced. These methods are based on points [6], [8], lines [7],[9] or quadratic curves [10],[11]. They use basic mathematics and they are little time consuming; however they are not sufficiently robust to noise. When the number of correspondences is large, numeric methods may be more efficient, because the pose information content is redundant and because the measurement errors and image noise average out among the image features. In the numeric methods, the solution is generally obtained by the minimization of a nonlinear error criterion [12], [14], [17]; more recent approaches apply linear techniques [13],[15]. The approach suggested in this paper presents both the analytical aspect and the numeric aspect and tries to conserve the advantages of analytical and numeric methods. A closed form solution is used for the determination of the translation between the camera frame and the object frame. This solution is presented in a more general framework concerning three-dimentional locations of circles. Contrary to general closed form solutions, this approach does not make any simplifying assumptions on the perspective projection; furthermore, by exploiting the geometrical relations between features in both image and object model, the location of an object circle can lead to the determination of the object orientation. In this way, a preliminary and valid estimation of the rotation associated with the object orientation is obtained. This first estimation is then used as initialization for the object orientation determination process which uses a numeric method based on Kalman filtering.

2.1. Object Position

The coordinates of the object frame center with respect to the camera frame defines the translation between the object frame $R_o(M_o, \mathbf{u}, \mathbf{v}, \mathbf{w})$ and the camera frame $R_c(O, \mathbf{i}, \mathbf{j}, \mathbf{k})$. The center of the circle corresponding to the most salient circular features is taken as the origin of the object frame, and the vector $\mathbf{w}$ of the object frame is normal to the circle plane. At first, the orientation of the circle plane is determined by using the property which states that the intersection of a such plane with the cone defined by an elliptic base (the perspective projection of the circle in the image plane) and the center of the camera's lens (origin of the camera frame) as vertex, generates a circle. Once the orientation has been computed and given the radius circle, the center of the circle is then determined. The different steps of computation are explained in [7]

2.2. Circle Localization Error

The reliability of the circle localization depends on the error ratio in the ellipse parameters' values. The error ratio depends on two kinds of factors: Intrinsic factors of the image such as the image quality and the image noise and extrinsic factors which are the distance-camera and the angle between the circle plane and the camera image plane. These two factors determine, respectively, the size and the amount of deformation of the elliptic feature.

Two series of measurements have been carried out in order to analyse the effects of these last two factors on the localization of circles. The position error is defined as the distance between the actual circle center and the estimated circle center. The orientation error is defined as the angle in degrees between the actual circle plane orientation and the estimated one. The simulated circle has a ray of 20 cm and its projection in the image is corrupted by a Gaussian noise of two pixels variance. From figure 2.a,2.b we notice that the error position increases almost linearly with the distance. This is due to the fact that the size feature decreases when the circle moves away from the camera and consequently the ratio noise/feature increases. However, the error remains acceptable up to a distance of 60 cm (the computed circle center remains enclosed in the actual circle). The orientation error increases also with the distance,

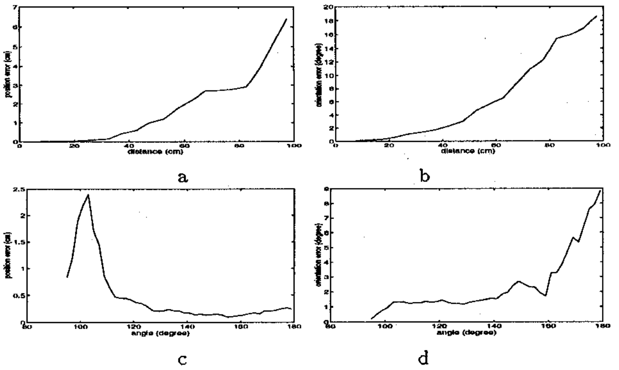


Figure 2: Errors of the position and the orientation mapped in function of the distance camera-object and the angle between camera image plane and circle plane. The errors were averaged over 50 simulations

but in a less sensitive way. Indeed, uo to 70 cm, the orientation deviation is less then 10°. In figure 2c,2d we realize that for an angle close to 90° the position error is relatively significant. This is due to the fact that in the neighbourhood of 90° the ellipse surface is small (close to a segment) and consequently the ratio noise/featue is high. For angles above 110° the error remains less than 5 mm. The error orientation is more significant for angles close 180°. Indeed, for such angles the major axis and the minor axis of the ellipse have close values. This fact causes a high uncertainty on the ellipse orientation in the image.

2.3. Object Orientation

The object orientation is defined as the rotation between the object frame $R_o(M_o, \mathbf{u}, \mathbf{v}, \mathbf{w})$ and the camera frame $R_c(O, \mathbf{i}, \mathbf{j}, \mathbf{k})$. The rotation R is expressed by

$$R = \begin{pmatrix} i_1 & j_1 & k_1 \\ i_2 & j_2 & k_2 \\ i_3 & j_3 & k_3 \end{pmatrix}, \quad R^{-1} = R^t = \begin{pmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{pmatrix} \tag{8}$$

The columns of R are the coordinates of $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ in R_o. The columns of R^t represent the components of $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ in R_c.

Let M_i be an object point with coordinates (X_i, Y_i, Z_i) and (u_i, v_i, w_i) in R_c and R_o respectively. We have the relation

$$OM_i = OM_o + M_oM_i \tag{9}$$

the projection of (9) on the axis Oi et Oj gives

$$\begin{array}{l} X_i = X_o + M_oM_i\mathbf{i} \\ Y_i = Y_o \ + M_oM_i\mathbf{j} \end{array} \tag{10}$$

the combination of (??) with (10) followed by the elimination of $\frac{Z_i}{fc}$ yields

$$x_i(Y_o + M_oM_i\mathbf{j}) - y_i(X_o + M_oM_i\mathbf{i}) = 0 \tag{11}$$

The dot product is independent from the coordinate frame so by expressing M_oM_i, $\mathbf{i}$, and $\mathbf{j}$ in R_o, (11) is written

$$x_i(Y_o + u_ij_1 + v_ij_2 + w_ij_3) - y_i(X_o + u_ii_1 + v_ii_2 + w_ii_3) = 0 \tag{12}$$

the equation (12) defines a linear relation between a parameter vector $a = (i_1, i_2, i_3, j_1, j_2, j_3)^t$, an image point m_i and an object point M_i

$$f(a, m_i, M_i) = 0 \tag{13}$$

Given a sufficient number of (image point, object point) correspondences, the equation (13) provides linear system in which the unknowns are $\mathbf{i}$ and $\mathbf{j}$. Once $\mathbf{i}$ and $\mathbf{j}$ have been computed, the vector $\mathbf{k}$ is obtained by the cross product $\mathbf{i} \wedge \mathbf{j}$. However, because of noise, the resulting matrix R may be not orthonormal so it should be corrected by normalizing $\mathbf{i}$ and $\mathbf{k}$ then replacing $\mathbf{j}$ by the cross product of $\mathbf{k}$ and $\mathbf{i}$.

The observation equation related to (13) is

$$b_i = h_i^t a + n_i$$

with

$$\begin{array}{l} b_i = x_iY_o - y_iX_o \\ h_i^t = (y_iu_i, y_iv_i, y_iw_i, -x_iu_i, -x_iv_i, -x_iw_i) \end{array}$$

n_i is the observation noise with the statistics $(0, W_i)$. The variance W_i is function of b_i,h_i, the current estimation $\hat{a}$ and the image noise covariance matrix L. The vector parameter can then be determined by th recursive Kalman filter equations (7)

2.3.1. Matching Control

The use of the Kalman filter enables us to obtain reasonable matches during the estimation process. Given an object point M_i and the current estimation $\hat{a}$, let a candidate for a match be the image point m_i. $f(\hat{a}, m_i, M_i)$ is an independent random variable with a normal distribution which satisfies

$$E[f] = 0; \quad var[f] = S_i = (\frac{\partial f}{\partial m_i})V(\frac{\partial f}{\partial m_i})^t + (\frac{\partial f}{\partial a})\hat{Q}(\frac{\partial f}{\partial a})^t$$

The fit (m_i, M_i) is estimated true if the Mahalabonis distance $\delta(\hat{a}, m_i, M_i) = f(\hat{a}, m_i, M_i)^t S_i^{-1} f(\hat{a}, m_i, M_i)$ is below a certain threshold. The rank of S_i is equal to one, so δ has a χ^2 distribution with one degree of freedom. The chosen threshold corresponds to a probability of 90%.

2.3.2. Initialization

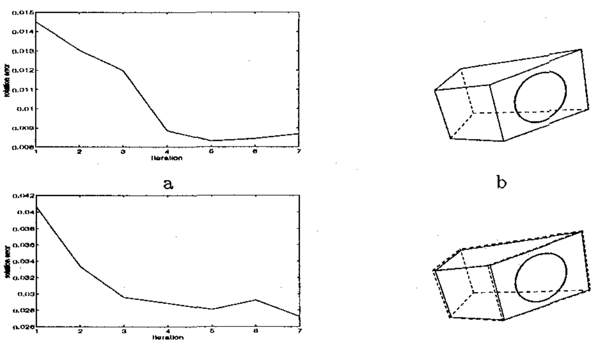
The circle orientation computed in the circle localization process is taken as a first estimation of $\mathbf{w}$ and by expressing the elements of the rotation matrix R in function of the Euler angles (α, β, γ). We obtain the following system

$$\begin{array}{l} i_3 = -\sin\beta \\ j_3 = \cos\beta\sin\alpha \\ k_3 = \cos\alpha\cos\beta \\ f(\alpha, \beta\,\gamma, m_i, M_i) = 0 \end{array} \tag{14}$$

The solution (α, β, γ) of the above system gives a reasonable initialization of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. However, there are two problems. First the solution of this system is not unique. Second the choice of the first match (m_i, M_i) is problematic. For this purpose, we refer to the geometrical relations of the image features to reduce the number of matches. The solutions related to the potential matches are then determined. The potential transformation is applied to the model, if the projected model features do not fit the image features the related correspondence is rejected (figure 3).

a b

Figure 3: Verification of the solutions resulting from the system (14). The projection of the model features with a false transformation (a) does not match the image features. The transformation which fit the best the model projection with the image is taken as initialization (b).



a b c d

Figure 4: convergence of the deviation of the rotation estimate (a,c). the deviation was averaged over 50 simulations. The object model is projected with the estimated transformation on the object image (b,d).

2.4. Experimental tests

Tests have been made on synthetic polyhedral objects presenting a circle on one of their faces. for a given transformation (translation + rotation), the object is projected into the image and Gaussian noise is added to the image points. Two series of tests have been carried out, the first with a Gaussian noise of two pixels variance (figure 4.a,4.b), the second with a noise variance of four pixels (figure 4.c,4.d) .

3. CONCLUSION

This paper proposes an approach for 3D object localization based on elliptic features. Owing to its geometrical ownerships, an ellipse reduces the complexity of the recognition and the amount of matches between the image and the model. Additionally, an ellipse constrains the view points to a greater degree than a simple line. Indeed, the ellipse based circle localization enables to determine the object position and two of its three degrees of freedom. For the cases when the whole transformation is needed, a linear iteration technique is recommended for the the determination of the rotation between the camera frame and the object frame. This technique is based on Kalman filtering and ensures good selection of matches.

The application of the EKF for the minimization of the sum of distances between the measured points and the fitted ellipse reduces the amount of bias in the estimated ellipse parameters and improves therefore the reliability of the ellipse based object localization. However, the efficiency of this technique is limited by extrinsic factors: the distanc of the object from the camera and the relative orientation of the circle plane with respect to the camera image plane. Tests have been carried out in order to determine both the effects of these factors on the localization and the efficiency margin of the circle localization technique.

These series of simulations can be considered as a first step towards a more complete analysis of the sensitivity of localization in terms of the above-mentioned factors. Building a general methodology for 3D localization of objects integrating the elements and the results of such analysis constitutes part of our future work.

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