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3-D Object Recognition Using Projective Invariant Relationship by Single-View

Kyoung Sig Roh*, Bume Jae You, and In So Kweon***

*Department of Automation and Design Engineering, Korea Advanced Institute of Science and Technology.207-43, Cheongryangri-dong, Dongdaemoon-gu, Seoul, Korea.

**Division of Electronics and Information Technology, Korea Institute of Science and Technology, P.O. Box 131, Cheongryang, Seoul 130-650, Korea

Abstract: *We propose a new method for recognizing three-dimensional objects using a three-dimensional invariant relationship for a special structure and geometric hashing by single-view. We use a special structure consisting of four co-planar points and any two non-coplanar points with respect to the plane. We derive an invariant relationship for the structure, which is represented by a plane equation. For recognition of 3-D objects using geometric hashing, a set of points on the plane is mapped into a set of points intersecting the plane and the unit sphere, thereby satisfying the invariant relationship. Because the structure is much more general than these previously structures proposed by Rothwell et al. [1] and Zhu et al. [2,3], it gives enough many voting to generate hypotheses.*

Experiments using 3-D polyhedral objects are carried out to demonstrate the feasibility of our method for 3-D object recognition.

1.Introduction

Most of the invariants used in computer vision applications have been based on plane-to-plane mappings. These invariants of the plane projective group have been thoroughly studied and many forms are known. They have also been successfully applied to working vision systems [4-6]. Constructing invariants for 3D structures from their 2D perspective images is much more difficult, and represents the major goal of current research in the application of invariant theory to vision. Burns et al. [7] show that invariants cannot be measured for 3D point sets in a general position from a single view, that is, for sets that contain absolutely no structure.

There are three possible solution to this problem, categorized as follows: In the first category, one basically deals with space projective invariants derived from two images, provided that the epipolar geometry of the two images is determined a priori [8,10,17]. Second, without computing the epipolar geometry, space projective invariants from three images can be determined [11,12]. Third, some special structures can provide projective invariants by one view [1-3].

Among these three categories, the third approach does not require correspondence information between features in each image. Rothwell et al. [1] proposed two special structures from which a one-viewed projective invariant can be derived. One is for points that lie on the vertices of a polyhedron, from which invariants are computed by using an algebraic framework of constraints between points and planes. The other is for objects that are bilateral symmetric. For the first class of objects, a minimum of seven points that lie on the vertices of a six-sided polyhedron are required in order to recover the projective structure. For the second class of objects, a minimum of eight points, or four points and two lines that are bilateral symmetric, are needed. Zhu et al. [2,3] proposed an algorithm to compute an invariant based on a structure of six points on adjacent planes that provided two sets of four coplanar points. The invariant is less constrained than the invariant proposed by Rothwell et. al [1], because it require six points instead of seven.

In this paper, we propose a new invariant relationship for a structure that is even more general than the one proposed by Zhu et al. [2,3]. The structure consists of a set of six points - four coplanar points and two non-coplanar points. In general, this structure provides an invariant by two viewed images for which a priori epipolar geometry is not required [10]. However, we derive an invariant relationship for the structure using just one view. The relationship can be represented as an orthogonal plane for a vector that is computed uniquely from the structure. To recognize three-dimensional objects, we propose a model-base using geometric hashing by using the invariant relationship.

This paper is organized as follows. In section 2, the invariant relationship is derived for a structure that consists of four coplanar points and two non-coplanar points. A method to construct the model-base by using the invariant relationship and geometric hashing is proposed. In section 3, preliminary experiments are tested for feasibility. In section 4, we present experimental results for real three-dimensional objects.

2. Invariant relationship and database structure

In this section, we present a three dimensional projective invariant relationship from a single view, which is based on a structure with six points: four coplanar points and two non-coplanar points. We also present a new model-base using the invariant relationship.

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2.1 Invariant relationship

We derive the invariant for the structure, which consists of four coplanar points and any two other points, using a canonical frame concept [13].

Proposition *Let X_i, $i=1\sim6$ be six points on an object and x_i, $i=1\sim6$ be the corresponding image points, where X_1, X_2, X_3, and X_4 are coplanar points and X_5, X_5 are two any non-coplanar points, shown in Fig.1. Then, the invariant relationship between the point sets of object and corresponding image points becomes a form of a* ***plane equation,*** *as follows:*

$$-\frac{(\mathbf{V}_1\times\mathbf{V}_2)}{\|\mathbf{V}_1\times\mathbf{V}_2\|}\bullet\frac{\mathbf{V}_3}{\|\mathbf{V}_3\|}=0$$

where, $\mathbf{V}_1=(u_5,\ v_5,\ w_5)$, $\mathbf{V}_2=(u_6,\ v_6,\ w_6)$, $\mathbf{V}_3=(\bar{\alpha},\ \bar{\beta},\ \bar{\gamma})$ *and all of these are presented in canonical coordinates.*

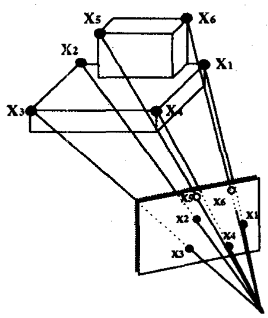


Fig.1 Projection of a set of six points, X_1, X_2, X_3, and X_4 for coplanar sets and X_4, X_5 for two non-coplanar

Proof: We assign canonical projective coordinates to the six points as follows:

$$\mathbf{X}_1\to(1,0,0,0),\ \ \mathbf{X}_2\to(0,1,0,0),\mathbf{X}_3\to(0,0,1,0),\qquad(1)$$
$$\mathbf{X}_4\to(\alpha,\beta,\gamma,0),\mathbf{X}_5\to(0,0,0,1),\mathbf{X}_6\to\ (1,1,1,1)$$

Thus, $\mathbf{X}_i$, i=1~3 and $\mathbf{X}_5$ $\mathbf{X}_6$ form a canonical basis. We can obtain a unique space collineation $\mathbf{A}_{4x4}$, det($\mathbf{A}_{4x4}$) $\neq 0$, which transforms the original five points into the canonical basis. The fourth point is transformed into its projective coordinates, $(\alpha, \beta, \gamma, 0)^T$ by $\mathbf{A}_{4x4}$. For the projections of these six points onto an image, we take $\mathbf{x}_i$, i=1, ..., 4 as the canonical projective coordinates in the image plane space. We can then obtain a unique plane collineation $\mathbf{A}_{3x3}$, det($\mathbf{A}_{3x3}$) $\neq 0$. $\mathbf{A}_{3x3}$ transforms the fifth and sixth points into $(u_5, v_5, w_5)^T$ and $(u_6, v_6, w_6)^T$.

$$\mathbf{x}_1\to(1,0,0),\mathbf{x}_2\to(0,1,0),\mathbf{x}_3,\to(0,0,1)\qquad(2)$$
$$\mathbf{x}_4\to(1,1,1),\mathbf{x}_5\to(u_5,v_5,w_5),\mathbf{x}_6\to(u_6,v_6,w_6)$$

The relationship between the object points and the corresponding image points is

$$\begin{bmatrix}1\,0\,0\,1 & u_5\,u_6\\ 0\,1\,0\,1 & v_5\,v_6\\ 0\,0\,1\,1 & w_5\,w_6\end{bmatrix}=\rho_i\mathbf{T}\begin{bmatrix}1\,0\,0\,\alpha\,0\,1\\ 0\,1\,0\,\beta\,0\,1\\ 0\,0\,1\,\gamma\,0\,1\\ 0\,0\,0\,0\ 1\,1\end{bmatrix},\quad \mathbf{T}=\begin{bmatrix}t_{11} & t_{12} & t_{13} & t_{14}\\ t_{21} & t_{22} & t_{23} & t_{24}\\ t_{31} & t_{32} & t_{33} & t_{34}\end{bmatrix}\qquad(3)$$

The right hand side of Eq. (3) is arranged to

$$\begin{bmatrix}\rho_1t_{11} & \rho_2t_{12} & \rho_3t_{13} & \rho_4(\alpha t_{11}+\beta t_{12}+\gamma t_{13}) & \rho_5t_{14} & \rho_6(t_{11}+t_{12}+t_{13}+t_{14})\\ \rho_1t_{21} & \rho_2t_{22} & \rho_3t_{23} & \rho_4(\alpha t_{21}+\beta t_{22}+\gamma t_{23}) & \rho_5t_{24} & \rho_6(t_{21}+t_{22}+t_{23}+t_{24})\\ \rho_1t_{31} & \rho_2t_{32} & \rho_3t_{33} & \rho_4(\alpha t_{31}+\beta t_{32}+\gamma t_{33}) & \rho_5t_{34} & \rho_6(t_{31}+t_{32}+t_{33}+t_{34})\end{bmatrix}\qquad(4)$$

Therefore, from Eq. (3) and (4), we can obtain each element of the transformation matrix **T** as follows:

$$t_{11}=1/\rho_1,\ t_{22}=1/\rho_2,\ t_{33}=1/\rho_3,\ t_{12}=t_{13}=t_{21}=t_{23}=t_{31}=t_{32}=0$$
$$t_{14}=\frac{u_5}{\rho_5},\ t_{24}=\frac{v_5}{\rho_5},\ t_{34}=\frac{w_5}{\rho_5},\ \text{and}$$
$$\frac{1}{\rho_1}=\frac{\bar{\alpha}}{\rho_4},\ \frac{1}{\rho_2}=\frac{\bar{\beta}}{\rho_4},\ \frac{1}{\rho_3}=\frac{\bar{\gamma}}{\rho_4},\ \text{where }\bar{\alpha}=\frac{1}{\alpha},\ \bar{\beta}=\frac{1}{\beta},\ \bar{\gamma}=\frac{1}{\gamma}.\qquad(5)$$

We can define the invariant relationship from the sixth column in Eq. (4) and the elements computed in Eq. (5),

$$\begin{bmatrix}\bar{\alpha} & u_5 & -u_6\\ \bar{\beta} & v_5 & -v_6\\ \bar{\gamma} & w_5 & -w_6\end{bmatrix}\begin{bmatrix}1/\rho_4\\ 1/\rho_5\\ 1/\rho_6\end{bmatrix}=0\qquad(6)$$

From the condition for a non-trivial solution for the equation, we obtain the relationship,

$$\begin{vmatrix}\bar{\alpha} & u_5 & -u_6\\ \bar{\beta} & v_5 & -v_6\\ \bar{\gamma} & w_5 & -w_6\end{vmatrix}=-(\mathbf{V}_1\times\mathbf{V}_2)\bullet\mathbf{V}_3=0\ ,\qquad(7)$$

$$\text{or }\frac{(\mathbf{V}_1\times\mathbf{V}_2)}{\|\mathbf{V}_1\times\mathbf{V}_2\|}\bullet\frac{\mathbf{V}_3}{\|\mathbf{V}_3\|}=0$$

where, $\mathbf{V}_1=(u_5,\ v_5,\ w_5)$, $\mathbf{V}_2=(u_6,\ v_6,\ w_6)$, $\mathbf{V}_3=(\bar{\alpha},\ \bar{\beta},\ \bar{\gamma})$

From the invariant relationship defined by Eq. (7), $\mathbf{V}_3$, designed from the structured object points, is orthogonal to the cross product of $\mathbf{V}_1$ and $\mathbf{V}_2$, which are extracted from the image. Therefore, all the vectors on the plane orthogonal to $\mathbf{V}_3$ satisfy the above relationship.

If the sixth point $\mathbf{X}_6$ is on the plane constructed by ($\mathbf{X}_3$, $\mathbf{X}_4$, $\mathbf{X}_5$), the structure becomes the same as that proposed by Zhu et al. [2,3]. We can easily derive the invariant for the structure by adding the invariant relation to the coplanar condition.

2.2 Database structure

To use the invariant relationship obtained in the previous section for the recognition of three dimensional polyhedral objects, we must construct an efficient database, or model-base.

Given the invariant for a set of points on a structured object $(\bar{\alpha},\bar{\beta},\bar{\gamma})^T$, we must record information about the structure, a model number and a plane number, and another two points. It is very inefficient, however, to consider all positions on the plane. Thus, we consider a surface on a unit sphere as a structure of a model-base.

Fig.2 shows the proposed model-base structure, where $(\bar{\alpha}, \bar{\beta}, \bar{\gamma})^T$ is the normalized vector for the invariant for object points and the invariant circle (χ) represents the group of vectors orthogonal to $(\bar{\alpha}, \bar{\beta}, \bar{\gamma})^T$.

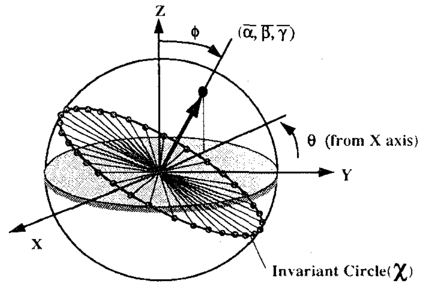


Fig.2 An unit sphere as a structure of a model-base

A vector in the model-base structure can be represented by two parameters (θ, ϕ) as follows:

$$(\bar{\alpha}, \bar{\beta}, \bar{\gamma}) = (\sin\phi\cos\theta,\ \sin\phi\sin\theta,\ \cos\phi) \quad (8)$$

From Eq. (8), we can convert the model-base space to (θ, ϕ)-space:

$$\theta = \tan^{-1}\left(\frac{\bar{\beta}}{\bar{\alpha}}\right),\quad \phi = \cos^{-1}(\bar{\gamma}) \cdot \quad (9)$$

We can compute vectors on the invariant circle by a coordinate transformation: the Z-axis of the new coordinate system is aligned with the old Z-axis and the X-axis is placed on the X-Y plane of the old coordinate system. We then obtain

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} (\cos\phi\cos\theta) & (-\sin\theta) & (\sin\phi\cos\theta) \\ (\cos\phi\ \sin\theta) & (\cos\theta) & (\sin\phi\sin\theta) \\ (-\sin\phi) & 0 & (\cos\phi) \end{bmatrix} \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix}. \quad (10)$$

The vectors on the invariant circle are then

$$(X', Y', Z') = (\cos\varphi,\ \sin\varphi,\ 0),\ \text{for } \varphi = 0 \sim 180 \quad (11)$$

Here, we only consider φ=0~180 because of the symmetric property. These vectors are represented in the (θ, ϕ)−space as

$$\theta' = \tan^{-1}\left(\frac{Y}{X}\right),\quad \phi' = \cos^{-1}(Z) \quad (12)$$

3. Preliminary Test

3.1 Preliminary test using a 3-D polyhedral object.

As a feasibility test for 3-D object recognition, we select a simple three-dimensional object. Fig.3 shows the object. Fig.4(a) shows the (X,Y,Z)-space of the model-base constructed for the structure consisting of four coplanar points (1,2,3,4) and two non-coplanar points (9,12). Fig.4(b) shows (θ, ϕ)−space. For this structure, $(\bar{\alpha}, \bar{\beta}, \bar{\gamma})^T$ is (-0.8966, -0.3472, 0.2747).

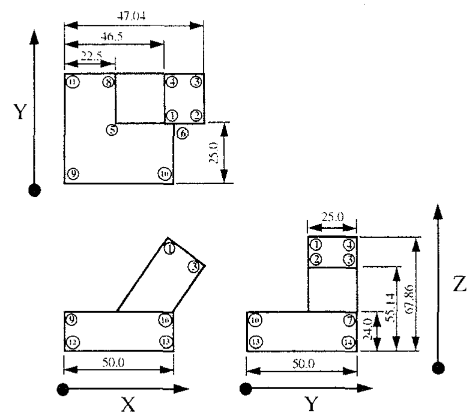
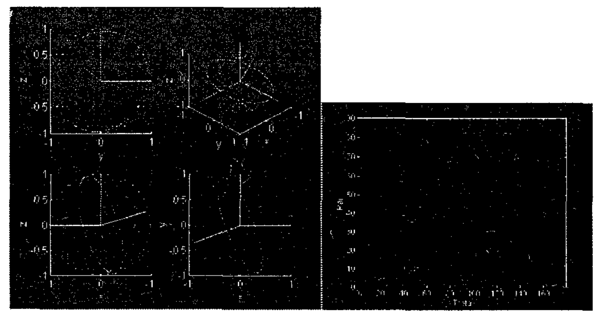


Fig.3 A 3-D object used for experiments



(a) (X, Y, Z)-Space (b) (θ, ϕ)−space.

Fig.4 (a) In (X,Y,Z), (b) in (θ, ϕ) space, the model-base for a structure consisting of four coplanar points(1,2,3,4) and two non-coplanar points space(9,12)

Fig.5 shows seven images of the object from different viewing directions.

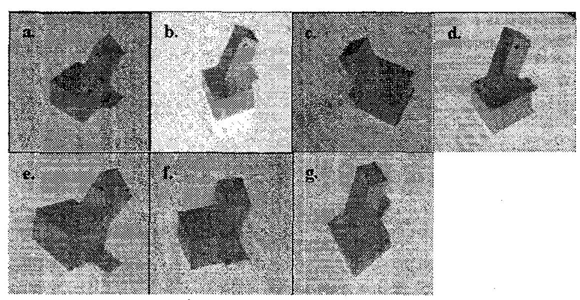


Fig.5 Images of the object from seven different views

Table 1 represents the cross products of two canonical coordinates (or vectors) computed in each image, and the dot product between the cross product vector and $(\bar{\alpha}, \bar{\beta}, \bar{\gamma})^T$, which is computed in advance by using the stored 3-D coordinate values of the object. In this table, error denotes the angle difference between the computed and the true $(\bar{\alpha}, \bar{\beta}, \bar{\gamma})^T$.

Table 1. Extracted indexing vector

	Known $V_3 = (\bar{\alpha}, \bar{\beta}, \bar{\gamma})$=(-0.8966, -0.3472, 0.2747)			
	$V_4 (= V_1 \times V_2)$	(θ', ϕ')	$\cos^{-1}(V_3 \bullet V_4)$	Error
A	-0.42, 0.57,-0.70	126.66,45.25	90.65	0.65
B	-0.43, 0.67,-0.60	122.87,53.45	90.50	0.50
C	-0.43, 0.60,-0.67	125.55,47.68	90.48	0.48
D	-0.45, 0.71,-0.53	122.55,57.70	89.23	0.77
E	-0.43, 0.56,-0.71	127.76,44.80	90.09	0.09
F	-0.44, 0.49,-0.75	131.50,41.06	89.29	0.71
G	-0.45, 0.68,-0.58	123.67,54.64	89.39	0.61

Fig.6 shows indexing by the invariant vector computed by the corresponding points on each image.

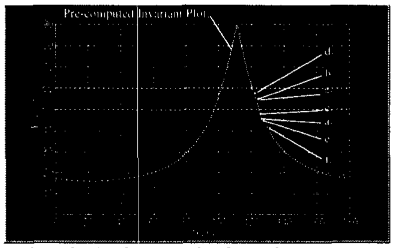


Fig.6 Indexing by the invariant vector

4. Experiments

4.1 Geometric hashing

The geometric invariant provides an indexing function for an efficient model-based object recognition, in which the time complexity is rarely affected by the number of models. The idea of geometric hashing was introduced by Y. Lamdan et. al[14] as a method for the indexing-based object recognition. Its importance and efficiency have also been emphasized in many recognition systems [15,16].

This is based on two stages: the first is an intensive model *preprocessing stage*, done off-line, where transformation invariant features of the models are indexed into a hash table. The second is an actual *recognition stage*, which employs the efficient indexing resulting from the above technique.

The pseudo-code of the algorithm is as follows:

```
for Model I
 for Plane j (be consisted of four points)
  for Point k (excepts four points on plane j)
   for Point l (excepts four points on plane j and k)
    COMPUT (α, β, γ, 0)
    STORE { i, j,k} into the entries in hash table
                            indicated by Eq.(14)
End for end for end for end for
```

The pseudo-code for object recognition is as follows:

```
Given a scene with N point features,
For point i=1~N
For point j=1~N (except i)
For point k=1~N (except i, j)
For point l=1~N (except i, j, k)
For point m=1~N (except i, j, k, l)
CHECK weather a set of the five points is feasible.
If the set is feasible.
For point n=1~N (except i, j, k, l, m)
  COMPUT V1 = (uj,vj,wj), V2 = (uk,vk,wk), and V4 = V1 × V2
  INDEXING into the entry in hash table indicated by V4
  VOTING {model #, plane #, point #}'s in the entry
End for
if # of VOTING > Threshold
  HYPOTHESES GENERATION & VERIFICATION
  if VERIFICATION == Successful. EXIT.
end if end if
end if end for endfor endfor endfor endfor
```

In recognition algorithm, the condition that a set of five points is feasible, is as follows:

For a set of five points, the set is feasible, if the convex hull for four points among five points is four and the other one point is outside of a rectangle constructed by the four points.

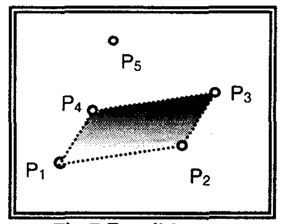
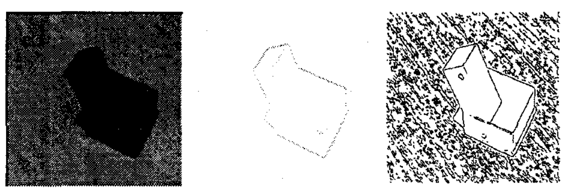


Fig.7 Feasible set

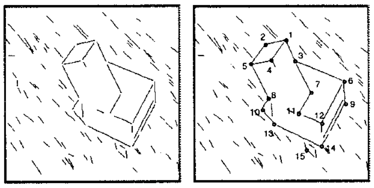
Fig.7 shows one example of the feasible set.

4.2 Image processing and Hypotheses Generation

To reduce the time complexity of hypotheses generation, we search for corner points as well as closed polygons in image processing. We use an algorithm proposed by A. Etemadi [18] to extract corners and polygons. In Fig. 8, if we select point features 1, 2, 5, 4, and 7 as a feasible set, we have the structure proposed by Rothwell[1], consisting of three adjacent planes, (1,2,5,4), (4,5,8,11) and (1,4,11,7). The structure proposed by Zhu [2,3] can also be constructed by two adjacent planes, (1,2,5,4) and (1,4,11,7). Unfortunately, they do not provide sufficient invariants for object recognition. For this particular scene, however, our proposed invariant can be defined up to nine different structures, which can be used to generate many hypotheses for object recognition.



(a)Input image (b)Edge detection (c)Hysteresis T.



(d) Line linking (e) Corner and polygon detection
(●: Polygon, ○: Corner)

Fig.8 Image processing

We compute invariants for points set consisting of (1, 2, 5, 4) and 7, and 3, 6, 8, 9, ..., 15. In addition, we vote the information in the model-base indexed by these invariants, which include information for the plane number and another one point.

Hypotheses, then, are generated if the voted number is greater than a predefined threshold. Table 2 represents ten generated hypotheses for scene features 1, 2, 5, 4, and 7. The point represents the point stored in the model-base as a basis, which is explained in section 3.2.

Table 2. The result of hypotheses generation

	Plane	Point	Vote		Plane	Point	Vote
1st	**1**	**5**	**8**	6th	5	1	7
2nd	1	9	7	7th	5	2	7
3rd	1	13	7	8th	5	3	7
4th	3	12	7	9th	6	1	6
5th	3	13	7	10th	6	2	6

If any polygons are not searched from the result of preprocessing, we assume the feasible sets, which consist of polygon.

4.3 Verification and Registration

For each generated hypothesis, we compute a transformation between the image and the model, and project the model onto the image plane. Then we count points within an error bound, i.e., matching points. We select hypothesis with a maximum number of matching points.

Fig.9 shows the results of transformation for the 1st and 2nd hypotheses. The asterisks (*) represent detected corner points and the circles (O) represent the transformed model corners. Table 3 shows the number of matching points obtained by verification.

From the verification results, the first hypothesis is selected as the true hypothesis with 12 matching points.

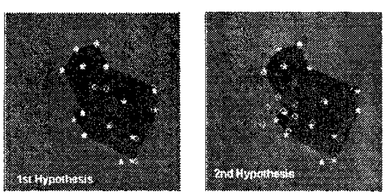


Fig.9 The result of verification for 1st and 2nd hypotheses

Table 3. Verification results

	# of Matching		# of Matching
1st	**13**	6th	6
2nd	8	7th	7
3rd	6	8th	6
4th	6	9th	6
5th	7	10th	6

Fig.10 shows a registration of the three-dimensional model overlaid onto the third image.

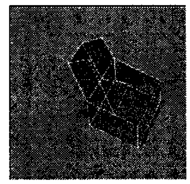
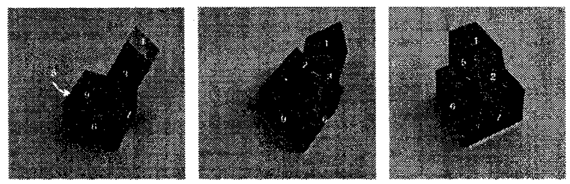


Fig.10 The registration of 3-D object onto the image

4.4 Experiments

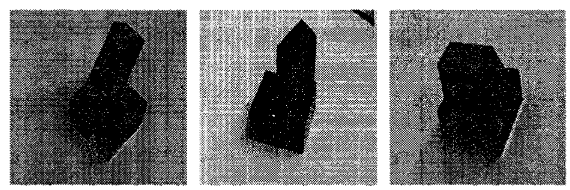
Fig.11 shows three models for testing our algorithm. The numbers in the figure represent the plane number.



(a) Model I. **(b) Model II.** **(c) Model III.**

Fig.11 The Models

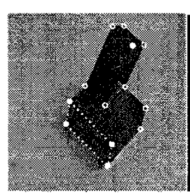
Fig.12 shows three-input images obtained from various camera-views.



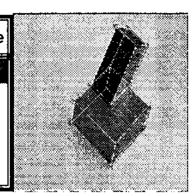
(a) Input I. **(b) Input II.** **(c) Input III.**

Fig.12 Input Images

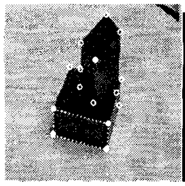
Fig.13 shows the results of preprocessing and hypotheses generation, and verification for three inputs. The numbers of the generated hypotheses are 14, 16, and 24, respectively.



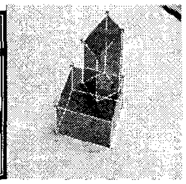
Model	Plane	Point	Vote
1	5	1	8
1	5	2	7
2	1	9	7
:	:	:	:
3	2	15	8
3	6	2	7



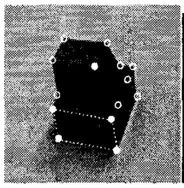
(a) Input I.



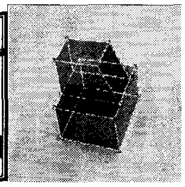
Model	Plane	Point	Vote
1	1	6	7
1	1	13	7
:	:	:	:
2	9	1	8
:	:	:	:
3	6	1	8



(b) Input II.



Model	Plane	Point	Vote
1	2	13	7
1	2	14	7
1	5	1	8
:	:	:	:
3	6	1	8
3	6	2	7



(c) Input III.

Fig.13 Results of recognition for each input image

5. Conclusion

In this paper, we proposed a new 3-D invariant relationship for a special structure consisting of four coplanar points and any two non-coplanar points using only single-view. For some structures, Zisserman and Maybank [10] showed that the invariant can be constructed by two-view without computing the epipolar geometry. However, we derived an invariant relationship by one-view, which is represented as a form of a plane equation. Based on this plane equation, we proposed a method for combining the relationship with a geometric hashing concept for recognizing three-dimensional objects. Since the structure is more general than the previously proposed structures, a hashing based method was feasible for 3-D object recognition. Experiments using real scenes demonstrate that the proposed invariant relationship can be further extended to real 3-D object recognition.

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