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SoSe 2011
Exercise Sheet 7
June 30, 2011

Exercises in 3D Computer Vision I

Exercise 1 Error Propagation

Let $\mathbf{v}$ be an m-dimensional random vector with mean $\overline{\mathbf{v}}$ and covariance matrix $\mathbf{C}$. $f : \mathbb{R}^m \to \mathbb{R}^n$ is a function differentiable in the neighborhood of $\mathbf{v}$. The mapped random variable $\mathbf{v}' = f(\mathbf{v})$ can then be approximated by

$$f(\mathbf{v}) \approx f(\overline{\mathbf{v}}) + \mathbf{J}(\mathbf{v} - \overline{\mathbf{v}}), \tag{1}$$

where $\mathbf{J}$ is the Jacobian matrix (i.e. the matrix of the first partial derivatives of f), evaluated at $\overline{\mathbf{v}}$. The mean of $\mathbf{v}'$ is now $f(\overline{\mathbf{v}})$ and its covariance matrix is $\mathbf{JCJ}^\top$.

For the following exercises, consider $\mathbf{x} = [x, y]^\top$, a two-dimensional random variable with mean $\overline{\mathbf{x}}$ and covariance matrix $\mathbf{C}$:

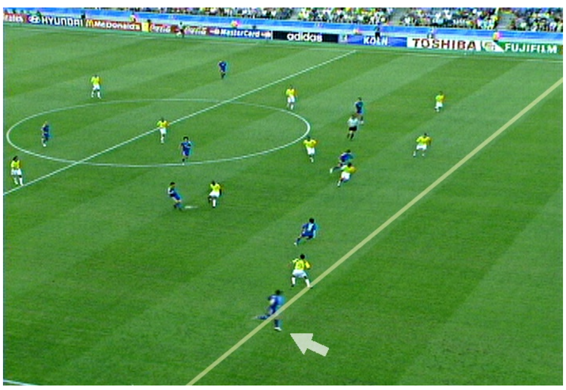
$$\overline{\mathbf{x}} = [3, 4]^\top, \qquad \mathbf{C} = \begin{pmatrix} 2 & 0 \\ 0 & 9 \end{pmatrix}.$$

a) Let $f(\mathbf{x}) = \|\mathbf{x}\|$. What are the mean and standard deviation of $x' = f(\mathbf{x})$?

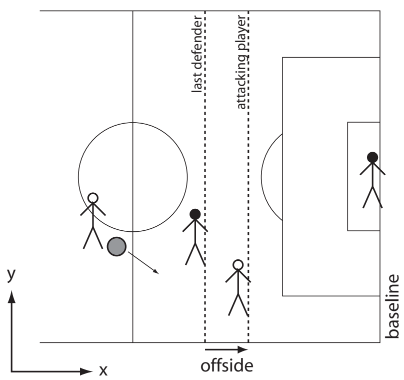
b) Let $g(\mathbf{x}) = \|\mathbf{x}\|\mathbf{x}$. What are the mean and covariance of $\mathbf{x}' = g(\mathbf{x})$?

Exercise 2 Error Propagation on a Football Field

In this exercise, we would like to create a software that helps to detect offside situations in football games. For the scope of this exercise, it is sufficient to know that an offside situation occurs when the attacking player is closer to the baseline than the last player of the other team at the moment the ball is kicked by a third person (Figure b).



(a)



(b)

The software receives a picture (taken with an arbitrarily positioned camera) at the moment the ball is kicked, and a user has to click on the two players involved. From the positions on the screen, the distance between the two players along the x-axis of the field is computed (see Figures). As a given offside would seriously interrupt the game, one wants to be really sure that it is an offside, and for that the error distribution of the measurements (clicks) is taken into account.

a) Before the actual game, we need to determine how precisely the user is able to click on a point on the screen. For that, he is asked to repeatedly click on the same point. This results in the following 2D coordinates:

$$(4,1)^\top,(0,7)^\top,(6,3)^\top,(7,6)^\top,(2,8)^\top,$$
$$(4,4)^\top,(3,2)^\top,(8,9)^\top,(10,4)^\top,(6,6)^\top$$

Compute the mean $\bar{\mathbf{p}}$ and the covariance $\mathbf{C}$ of the point distribution.

b) Assume that, during a real game, the user marks the position of two players by clicking on the two 2D points $\mathbf{p}_1 = (4,5,1)^\top$ and $\mathbf{p}_2 = (2,2,1)^\top$. You are also given a homography

$$\mathbf{H} = \begin{pmatrix} 2 & -2 & 0 \\ 2 & -5 & 1 \\ 3 & -2 & -4 \end{pmatrix}$$

that converts screen to field coordinates. Derive a function f for error propagation that maps any point $\mathbf{p}$ to the transformed (by $\mathbf{H}$) inhomogeneous point $\mathbf{p}'$.

c) Compute the difference in the x-coordinate between the two players in the football field.

d) Forward propagate the covariance of the two clicked image points to the corresponding points in the football field. Assume that both image points have equal covariance $\mathbf{C}$, obtained in a). *Hint*: Derive the Jacobian of f with respect to x, y and think of which parts of the Jacobian are of interest in this problem.

e) You now have the *individual* variances of the x-coordinates of the two players in the football field. How do you obtain the desired variance of the *difference* in the x-coordinate?

<u>Exercise 3</u> (H) Error Propagation for Homographies

Suppose that we would like to find the center of a certain template (Figure 1, left) in a target image (Figure 1, right) and the uncertainty of the process of finding it. We suppose also that we do not have the dimensions of the images.

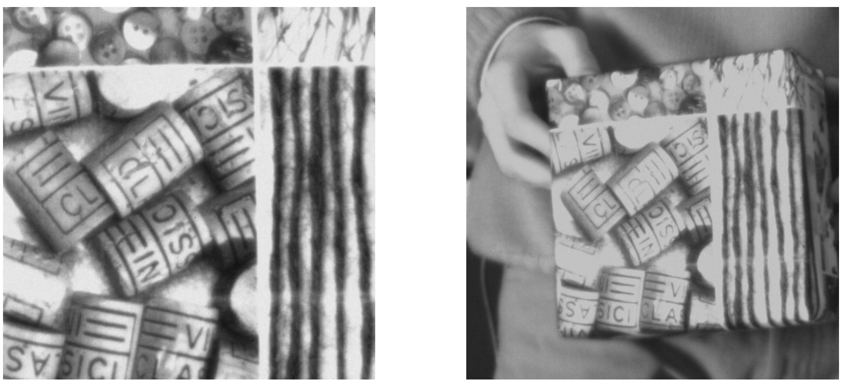


Figure 1: Test images for error propagation. *Left*: Template. *Right*: Target.

The only thing we have is the homography $\mathbf{H}$ that transforms the template into the target image:

$$\mathbf{H} = \begin{bmatrix} 0.923535 & 0.02394 & 131.650330 \\ -0.098678 & 1.008938 & 120.014473 \\ -0.000134 & -0.000013 & 1.050149 \end{bmatrix}$$

The process is the following: We click 10 points on the template that could be the center. We compute the mean and the covariance of the clicked points. Then, we propagate the result to the target image.

Write a MATLAB program that accomplishes the following procedure:

- Load the 2 images `im1.pgm` and `im2.pgm` available on the lecture website.
- Click 10 points on the template image that might be its center (use the function `ginput`).
- Compute the mean and the covariance of the clicked points. Plot the center by a cross and the covariance by an uncertainty ellipse (use the function `error_ellipse` provided on the lecture website).
- Given the homography above, propagate the result to the second image and plot it.

The output of your program should look like in Figure 2.

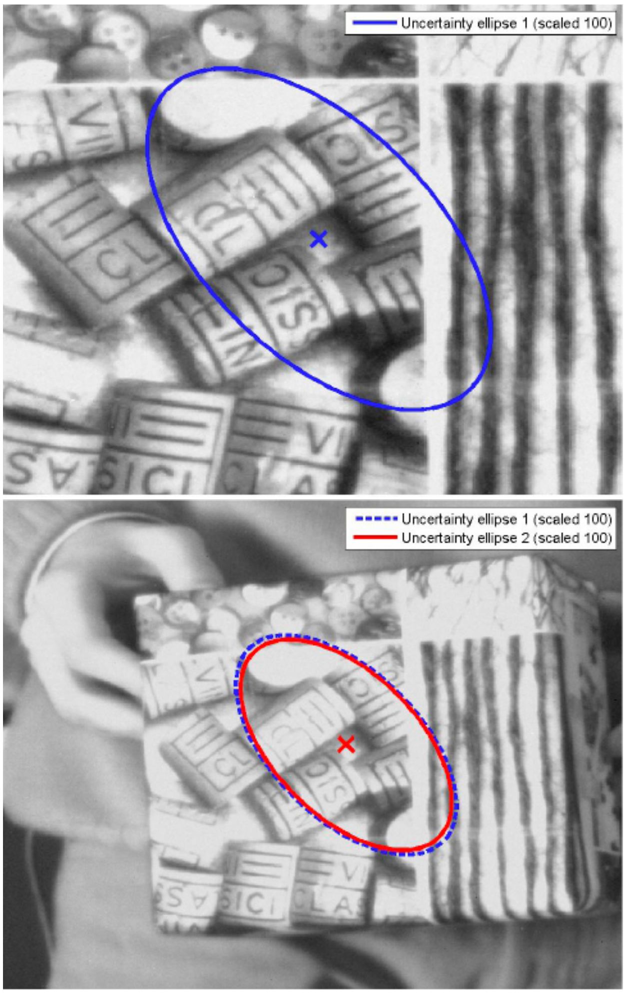


Figure 2: Result expected from the MATLAB program.