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Exercise Sheet 3
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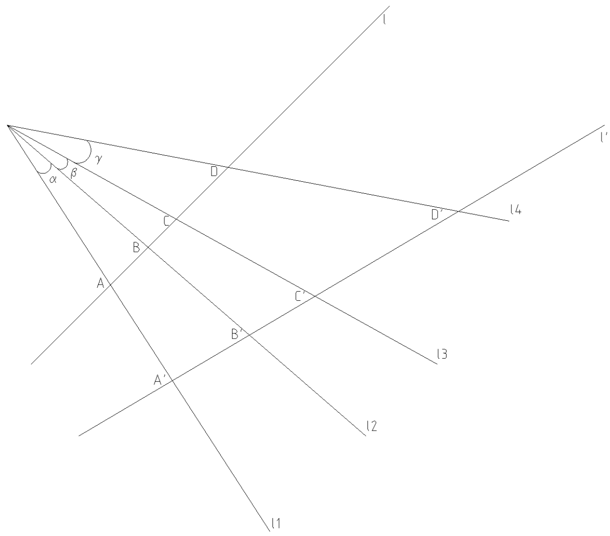
Exercises in 3D Computer Vision I

Exercise 1 Cross Ratio of Angles

In this exercise, you will show that the cross-ratio of four collinear points A, B, C, D is equal to

$$\{A, B; C, D\} = \frac{sin(\alpha + \beta)sin(\beta + \gamma)}{sin(\alpha + \beta + \gamma)sin\beta},$$

as shown in the following figure.



The point definition of the cross-ratio is given by

$$\{A, B; C, D\} = \frac{\overline{AC} \cdot \overline{BD}}{\overline{BC} \cdot \overline{AD}}.$$

a) Show that the area of a triangle PQR is

$$f_{area}(P, Q, R) = \frac{1}{2}\overline{PQ} \cdot \overline{PR} \cdot \sin\theta,$$

where $\overline{PQ}$ denotes the distance between the two points P and Q and θ is the angle between the lines joining the point P to the points Q and R.

b) Define the ratio of three collinear points A, B, C as

$$f_{ratio}(A, B, C) = \frac{\overline{AB}}{\overline{BC}}$$

for some orientation of the line supporting the three points. Show that

$$f_{ratio}(A, B, C) = f_{area}(A, B, O)/f_{area}(B, C, O),$$

where O is some point not lying on this line.

c) Conclude that the cross-ratio $\{A, B; C, D\}$ is given by the formula above. **Tip:** start with the definition of the cross ratio and use the results of the intermediate steps.

Exercise 2 Normalized Direct Linear Transformation (DLT)

Typically, the matrix $\mathbf{A}$ used in the DLT is badly conditioned, meaning that the order of magnitude of its entries varies strongly. The reason for this behavior lies in the image coordinates x_i, y_i, which are in the order of 100, while the homogeneous scale factor w_i is in the order of 1. To avoid numerical instability when solving $\mathbf{Ah} = \mathbf{0}$ for the entries of the homography $\mathbf{H}$, a normalization procedure is required.

The steps of the *normalized* DLT algorithm are as follows:

- Normalize the points $\mathbf{x}_i$ of one image with the transformation $\mathbf{T}$ such that their centroid is at the origin and that the average distance from the origin is equal to $\sqrt{2}$. An average point then has homogeneous coordinates $(1, 1, 1)^\top$. Denote the transformed points by $\tilde{\mathbf{x}}_i$.
- Apply an analogous normalization procedure to the points $\mathbf{x}'_i$ of the other image (this time with the transformation $\mathbf{U}$), giving the transformed points $\tilde{\mathbf{x}}'_i$.
- Use the standard DLT to compute the entries of the homography $\tilde{\mathbf{H}}$ from the transformed point correspondences $\{\tilde{\mathbf{x}}_i \leftrightarrow \tilde{\mathbf{x}}'_i\}$.
- Denormalize the obtained homography $\tilde{\mathbf{H}}$ to get the homography $\mathbf{H}$ relating the original point correspondences $\{\mathbf{x}_i \leftrightarrow \mathbf{x}'_i\}$.

Answer the following questions:

a) Show the explicit form of the matrix $\mathbf{T}$. How do you normalize the points with that transformation matrix?

b) How do you finally obtain the homography $\mathbf{H}$ that relates the unnormalized image points by using $\mathbf{T}$ and $\mathbf{U}$?

Exercise 3 (H) Affine Image Rectification

Recording an image with a camera adds distortion to a scene, i.e. distances, angles, and areas are different (**affine** distortion) as well as parallelism is not preserved (**perspective** distortion). In order to get rid of the distortion in a 2D plane 2D projective geometry offers methods for image rectification. In this exercise you will only undistort the image perspectively, i.e. recover parallelism.

a) Construct the vanishing line in the figure below geometrically (by hand). You don't have to hand in your drawing!

b) Compute the vanishing line of the figure below in MatLab (by clicking on the image). Give outputs which explain what a possible user has to do (respectively, how the user has to click). By using this vanishing line construct a matrix that transforms the image such that parallel lines are indeed parallel. Apply the matrix to the image. The image `skyscraper.jpg` can be found under `http://campar.in.tum.de/Chair/TeachingSs09CV` at the bottom of the website.

Exercise 4 (H) Normalized DLT for Image Mosaicing

In the midterm exam you manually computed the homography relating points in a camera image of a football field to the corresponding points in the real world plane. In this homework, you are asked to implement the DLT algorithm for homography estimation in MATLAB.

In addition, you will explore a special case of homography estimation where you are given *two* camera images of a real world scene. In fact, when two camera views of an arbitrary scene are obtained by only rotating the camera (i.e. no translation), as if it were mounted on a tripod, then the images are related by a homography. Estimating this homography allows to stitch the two images together to obtain a panoramic view of the scene. This technique is called *mosaicing*.

Figure 1: *Left*: Reference image. *Middle*: Image after rotation. *Right*: Stitched image.

a) Take two images of the same scene with only a difference in rotation[1].You can also use the two images `TUfinger01_resampled.jpg` and `TUfinger02_resampled.jpg` on the bottom of our website (see Figure 2).

b) Write a MATLAB function `mydlt` that takes an arbitrary number (≥ 4) of point correspondences and outputs the homography matrix $\mathbf{H}$ that is consistent with the correspondences.

c) Write a MATLAB function `mydlt_norm` that makes use of `mydlt` but implements the *normalized* DLT algorithm. This function should have the same signature as `mydlt`.

d) Manually place at least 4 correspondences (for instance, using `ginput`) in your images and use `mydlt` to compute a homography. Then, repeat the experiment using `mydlt_norm`.

e) Use the backward warping technique to create two mosaics, or panoramic images, from the two images stitched together. Create one mosaic for each of the two homographies you computed in d). Do you see any difference between the mosaics obtained by using the standard DLT and the normalized DLT?

f) Use at least 10 point correspondences and repeat the procedure. Can you now identify a difference between normalized and unnormalized DLT?

[1] Imagine you place your camera on a tripod, take a picture, rotate the camera about 20° and take another picture.