
LAB 7: Spectrogram Analysis - The Short Time Fourier Transform

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Governing Equation

$$X(k, m) = \sum_{n=0}^{N-1} x(mM + n)W_N^{kn}$$

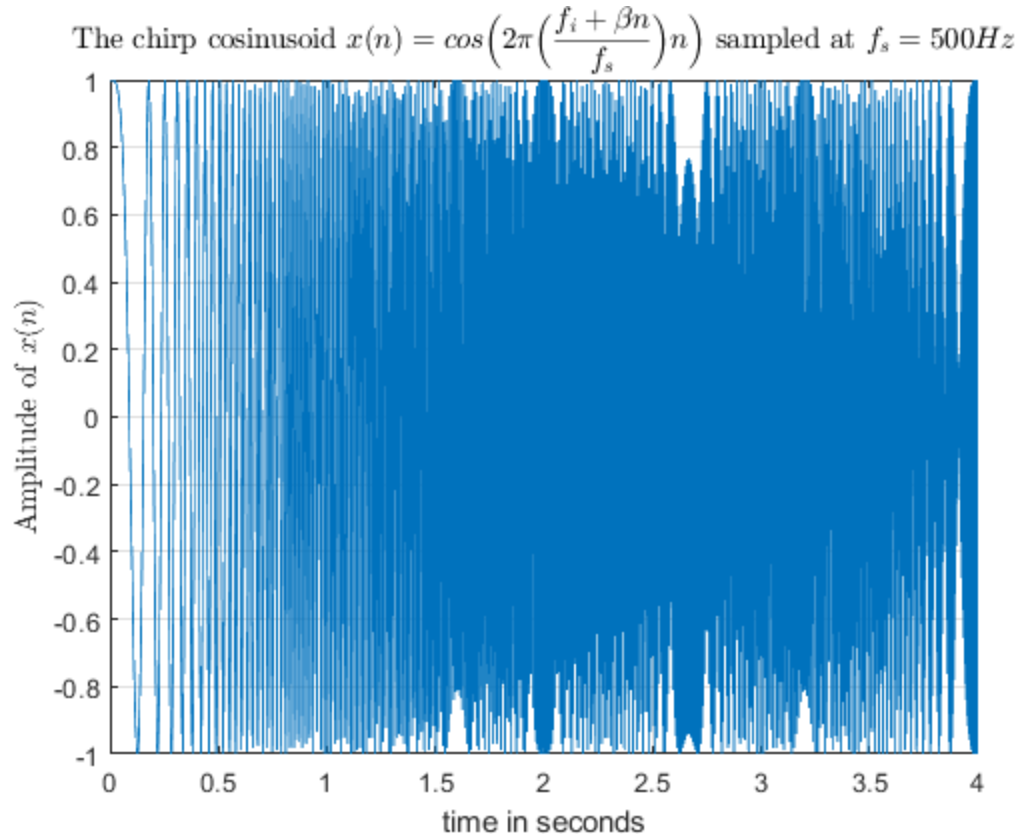
Main Namespace

```
function main()
```

Outputs

```
clear the terminal.
```

```
clc;  
clear;  
output_chirp = question1();
```

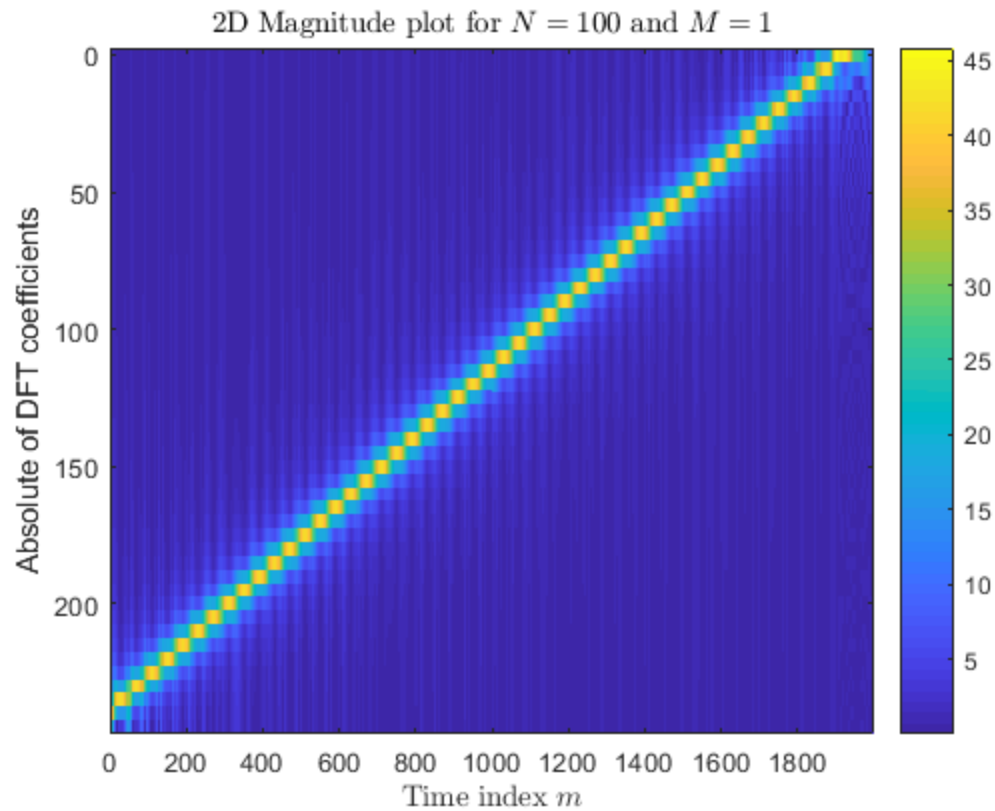


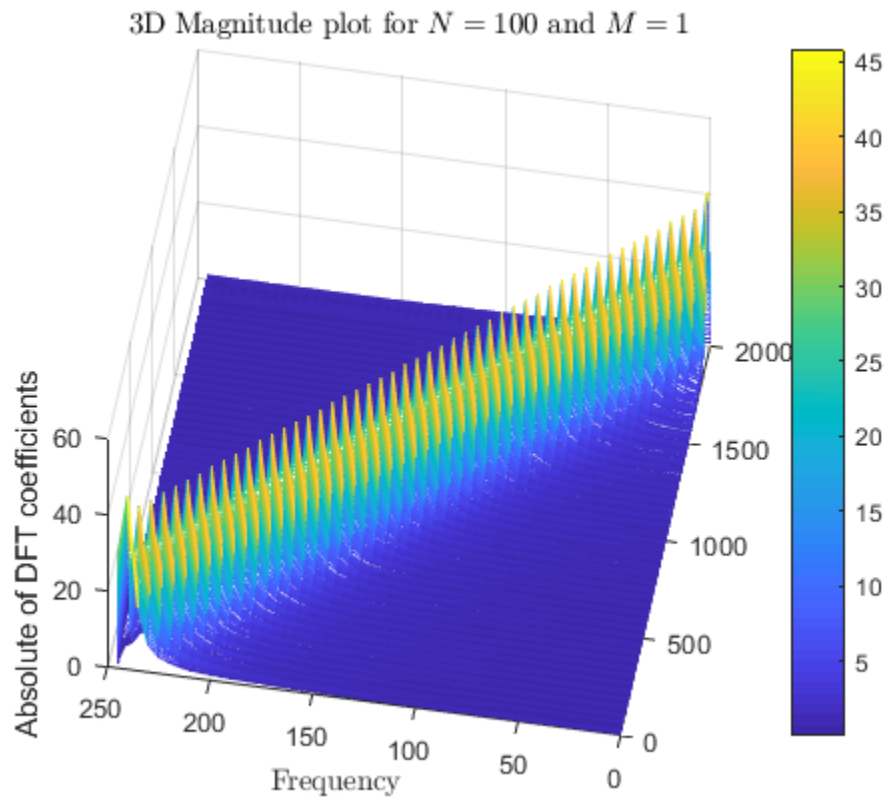
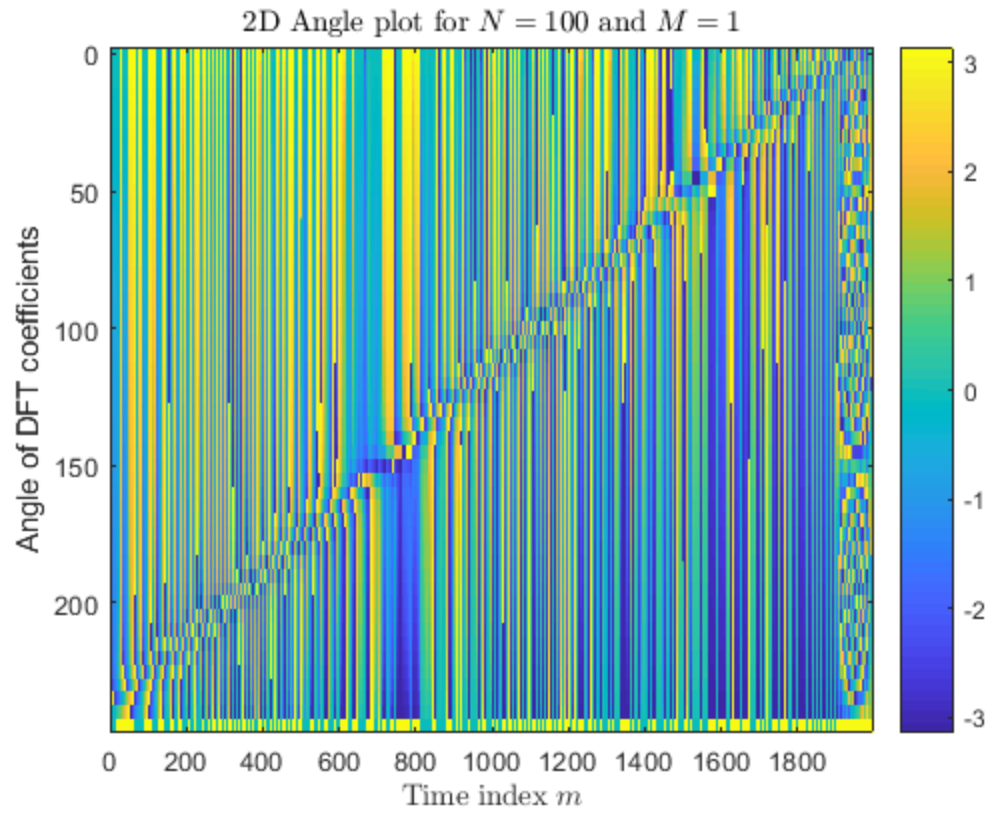
Plot of 2D and 3D graph for N=100 and M=1

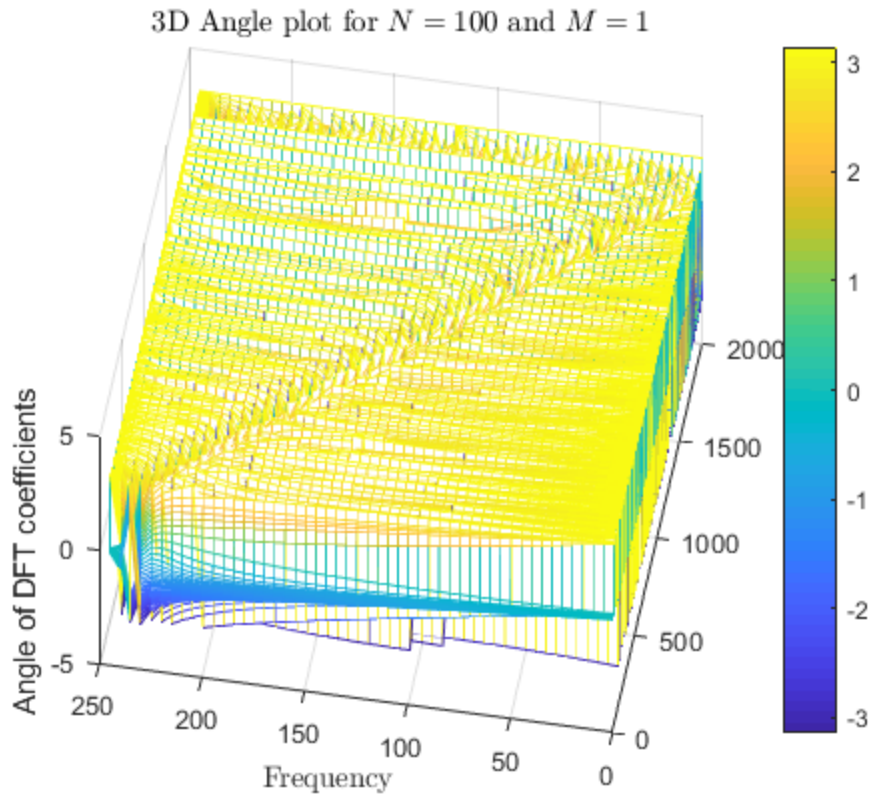
```
% run the block_it function on the above output.
mtx_N100_M1 = block_it(output_chirp, 100, 1);
dftN100M1 = colDFT(100, 1, mtx_N100_M1);
% since DFT is symmetric around a unit circle, either take
% first 50 or last 50 to plot the graph.
figure;
imagesc([0:1999], [49*5:-5:0], abs(dftN100M1(1:50, :)));
xlabel('Time index $m$', 'interpreter', 'latex');
ylabel('Absolute of DFT coefficients');
title('2D Magnitude plot for $N=100$ and
$M=1$', 'interpreter', 'latex');
colorbar;
figure;
imagesc([0:1999], [49*5:-5:0], angle(dftN100M1(1:50, :)));
xlabel('Time index $m$', 'interpreter', 'latex');
ylabel('Angle of DFT coefficients');
title('2D Angle plot for $N=100$ and
$M=1$', 'interpreter', 'latex');
colorbar;

figure;
mesh([0:1999], [49*5:-5:0], abs(dftN100M1(1:50, :)));
colorbar;
```

```
view(-80, 60);  
xlabel('Time index  $m$ ', 'interpreter', 'latex');  
ylabel('Frequency', 'interpreter', 'latex');  
zlabel('Absolute of DFT coefficients');  
title('3D Magnitude plot for  $N=100$  and  
 $M=1$ ', 'interpreter', 'latex');  
figure;  
mesh([0:1999], [49*5:-5:0], angle(dftN100M1(1:50, :)));  
colorbar;  
view(-80, 60);  
xlabel('Time index  $m$ ', 'interpreter', 'latex');  
ylabel('Frequency', 'interpreter', 'latex');  
zlabel('Angle of DFT coefficients');  
title('3D Angle plot for  $N=100$  and  
 $M=1$ ', 'interpreter', 'latex');
```



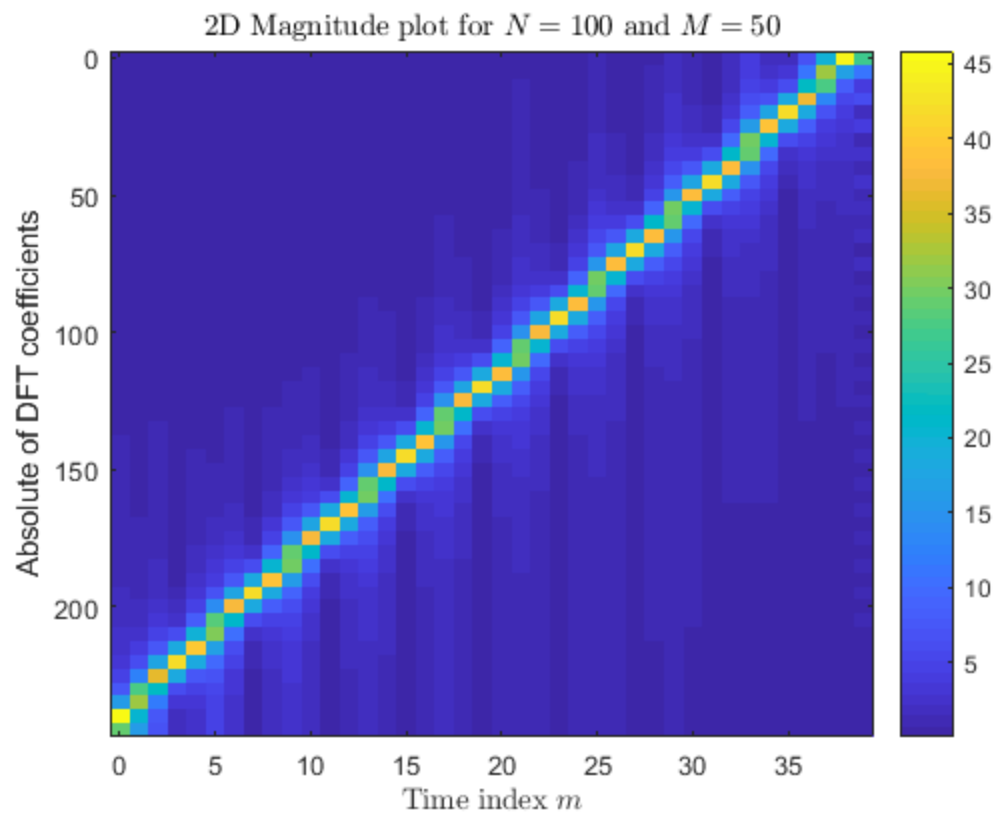


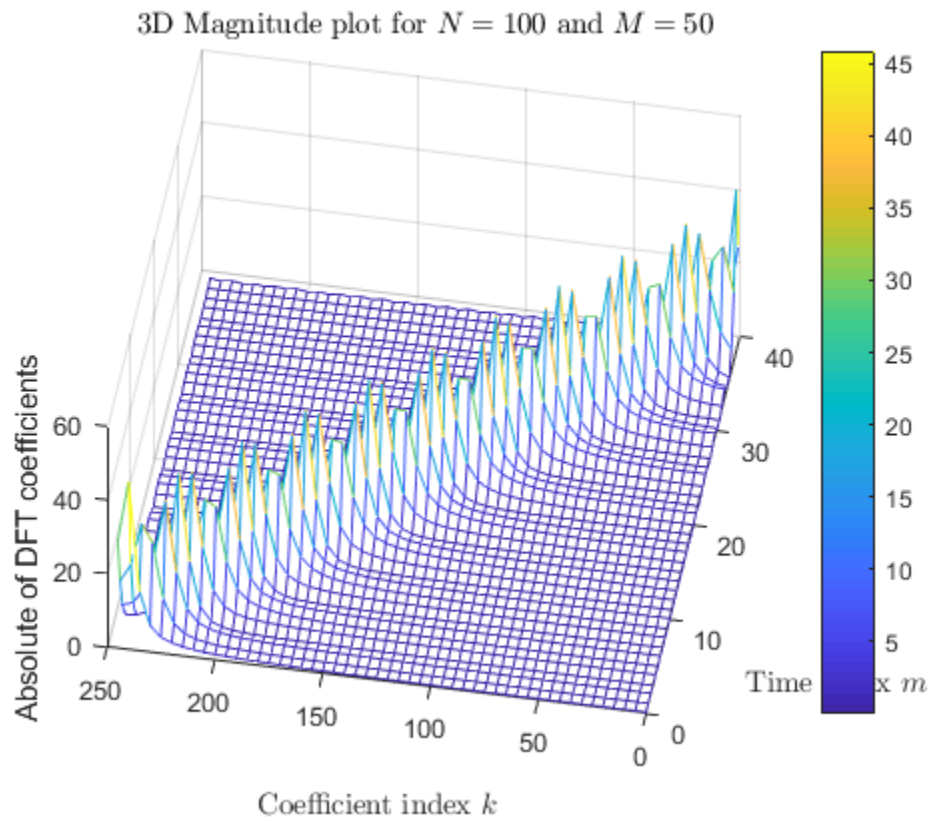
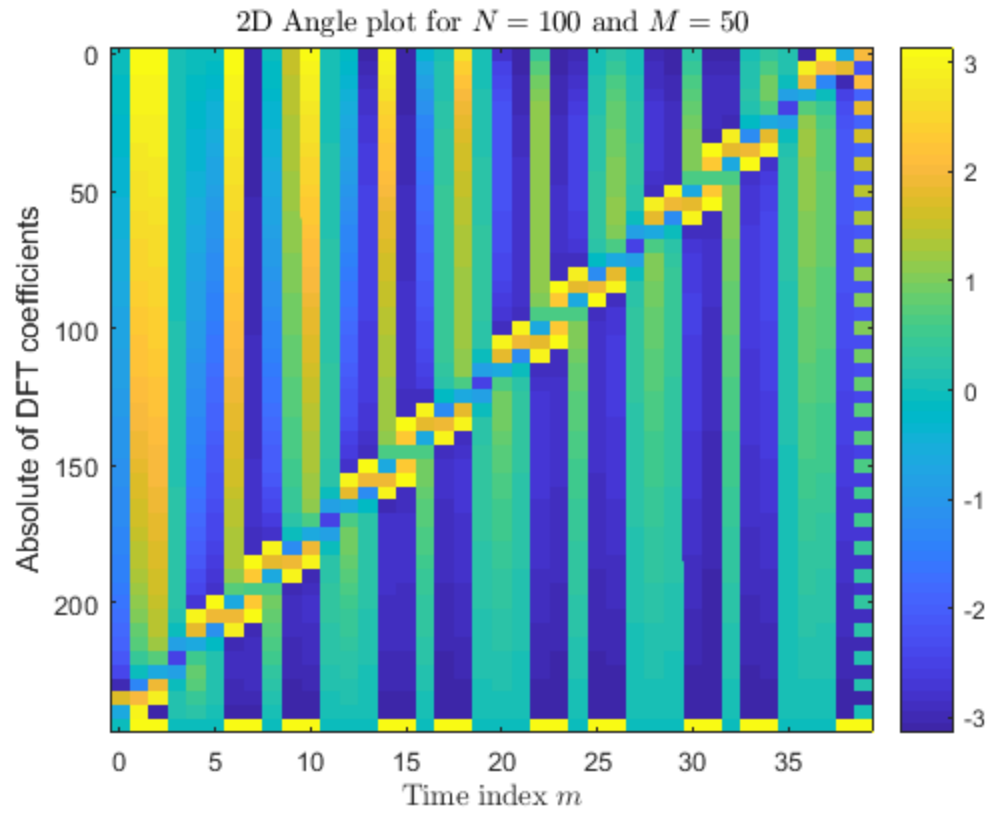


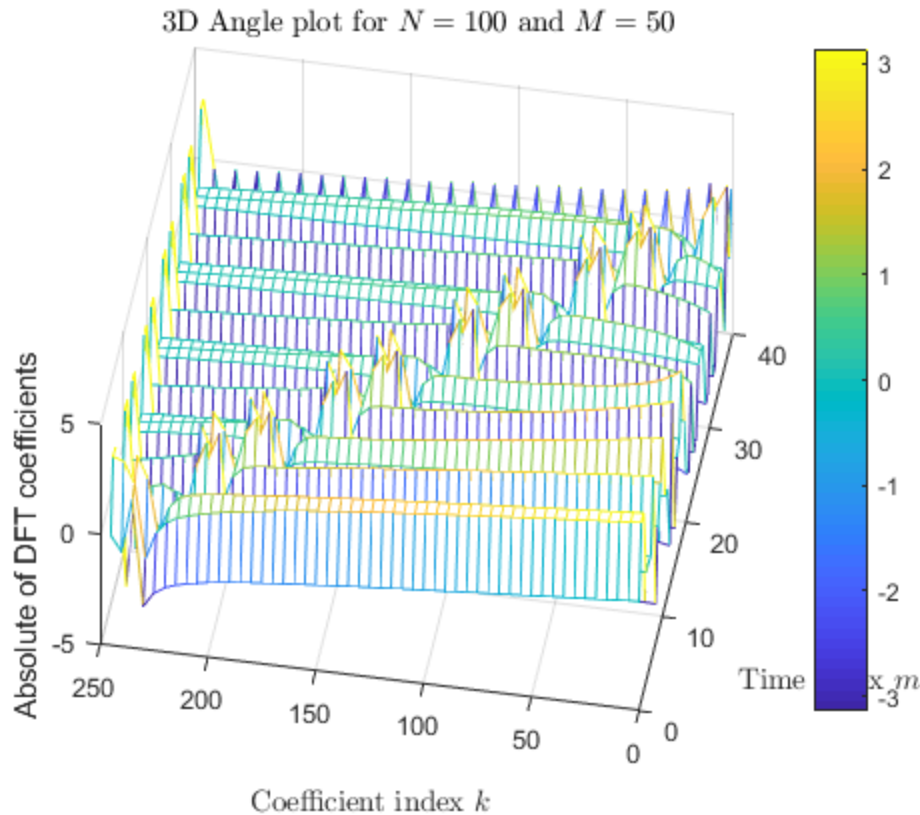
Plot of 2D and 3D graph for $N=100$ and $M=50$

```
% run the block_it function on the above output.
mtx_N100_M50 = block_it(output_chirp, 100, 50);
dftN100M50 = colDFT(100, 1, mtx_N100_M50);
% since DFT is symmetric around a unit circle, either take
% first 50 or last 50 to plot the graph.
figure;
imagesc([0:39], [49*5:-5:0], abs(dftN100M50(1:50, :)));
xlabel('Time index  $m$ ', 'interpreter', 'latex');
ylabel('Absolute of DFT coefficients');
title('2D Magnitude plot for  $N=100$  and  
 $M=50$ ', 'interpreter', 'latex');
colorbar;
figure;
imagesc([0:39], [49*5:-5:0], angle(dftN100M50(1:50, :)));
xlabel('Time index  $m$ ', 'interpreter', 'latex');
ylabel('Absolute of DFT coefficients');
title('2D Angle plot for  $N=100$  and  
 $M=50$ ', 'interpreter', 'latex');
colorbar;
figure;
mesh([0:39], [49*5:-5:0], abs(dftN100M50(1:50, :)));
colorbar;
view(-80, 60);
```

```
xlabel('Time index  $m$ ', 'interpreter', 'latex');  
ylabel('Coefficient index  $k$ ', 'interpreter', 'latex');  
zlabel('Absolute of DFT coefficients');  
title('3D Magnitude plot for  $N=100$  and  
 $M=50$ ', 'interpreter', 'latex');  
figure;  
mesh([0:39], [49*5:-5:0], angle(dftN100M50(1:50, :)));  
colorbar;  
view(-80, 60);  
xlabel('Time index  $m$ ', 'interpreter', 'latex');  
ylabel('Coefficient index  $k$ ', 'interpreter', 'latex');  
zlabel('Absolute of DFT coefficients');  
title('3D Angle plot for  $N=100$  and  
 $M=50$ ', 'interpreter', 'latex');
```





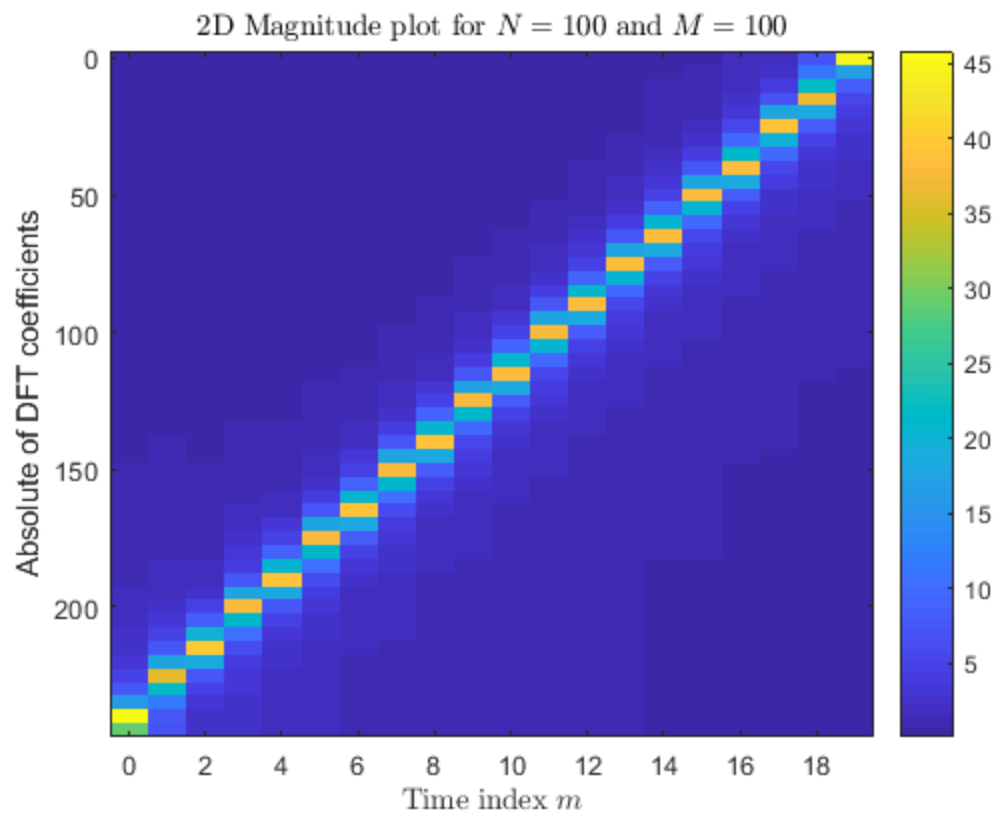


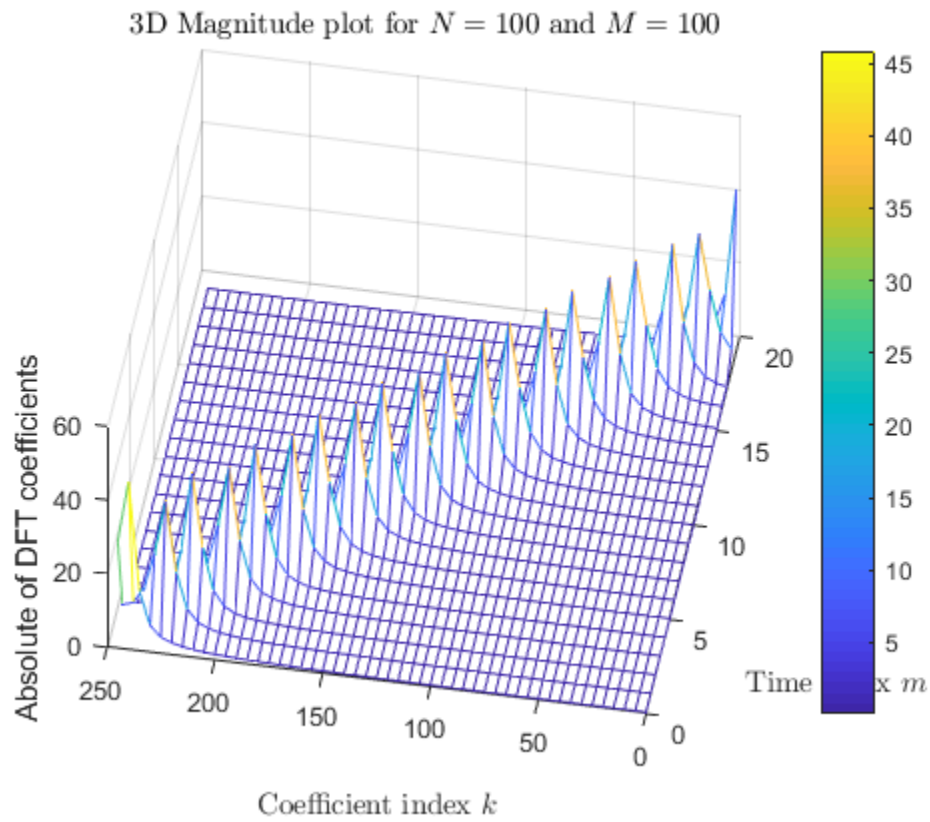
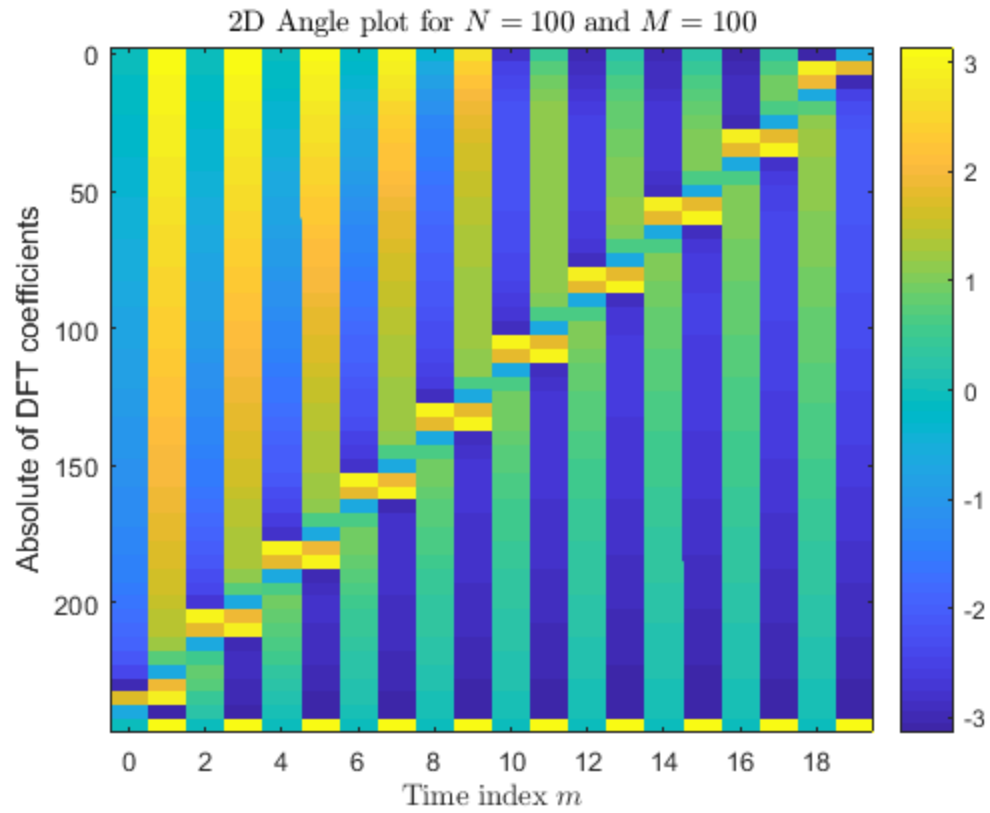
Plot of 2D and 3D graph for $N=100$ and $M=100$

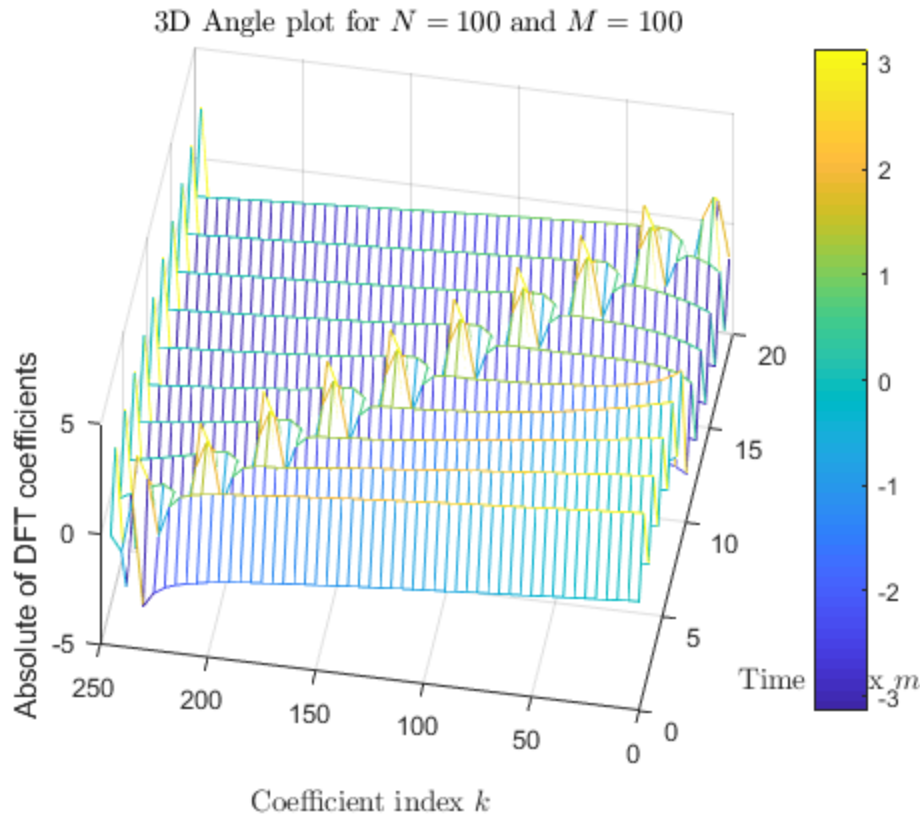
```
% run the block_it function on the above output.
mtx_N100_M100 = block_it(output_chirp, 100, 100);
dftN100M100 = colDFT(100, 1, mtx_N100_M100);
% since DFT is symmetric around a unit circle, either take
% first 50 or last 50 to plot the graph.
figure;
imagesc([0:19], [49*5:-5:0], abs(dftN100M100(1:50, :)));
xlabel('Time index  $m$ ', 'interpreter', 'latex');
ylabel('Absolute of DFT coefficients');
title('2D Magnitude plot for  $N=100$  and
 $M=100$ ', 'interpreter', 'latex');
colorbar;
figure;
imagesc([0:19], [49*5:-5:0], angle(dftN100M100(1:50, :)));
xlabel('Time index  $m$ ', 'interpreter', 'latex');
ylabel('Absolute of DFT coefficients');
title('2D Angle plot for  $N=100$  and
 $M=100$ ', 'interpreter', 'latex');
colorbar;
figure;
mesh([0:19], [49*5:-5:0], abs(dftN100M100(1:50, :)));
colorbar;
view(-80, 60);
```



```
xlabel('Time index  $m$ ', 'interpreter', 'latex');  
ylabel('Coefficient index  $k$ ', 'interpreter', 'latex');  
zlabel('Absolute of DFT coefficients');  
title('3D Magnitude plot for  $N=100$  and  
 $M=100$ ', 'interpreter', 'latex');  
figure;  
mesh([0:19], [49*5:-5:0], angle(dftN100M100(1:50, :)));  
colorbar;  
view(-80, 60);  
xlabel('Time index  $m$ ', 'interpreter', 'latex');  
ylabel('Coefficient index  $k$ ', 'interpreter', 'latex');  
zlabel('Absolute of DFT coefficients');  
title('3D Angle plot for  $N=100$  and  
 $M=100$ ', 'interpreter', 'latex');
```



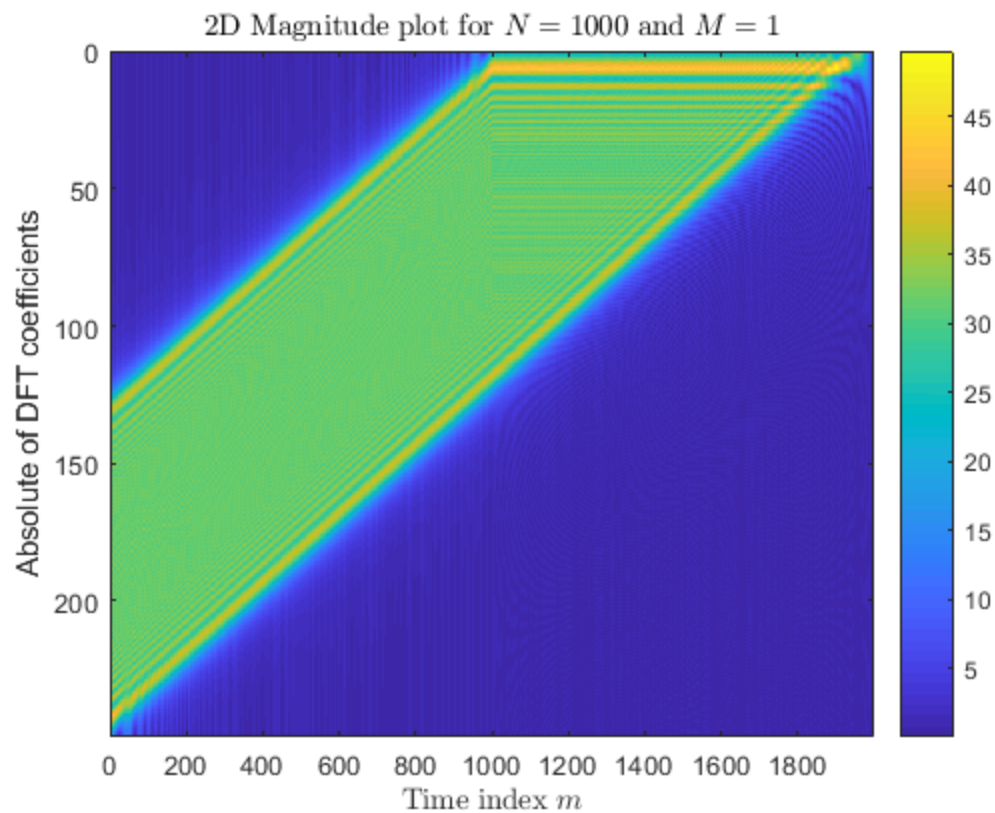


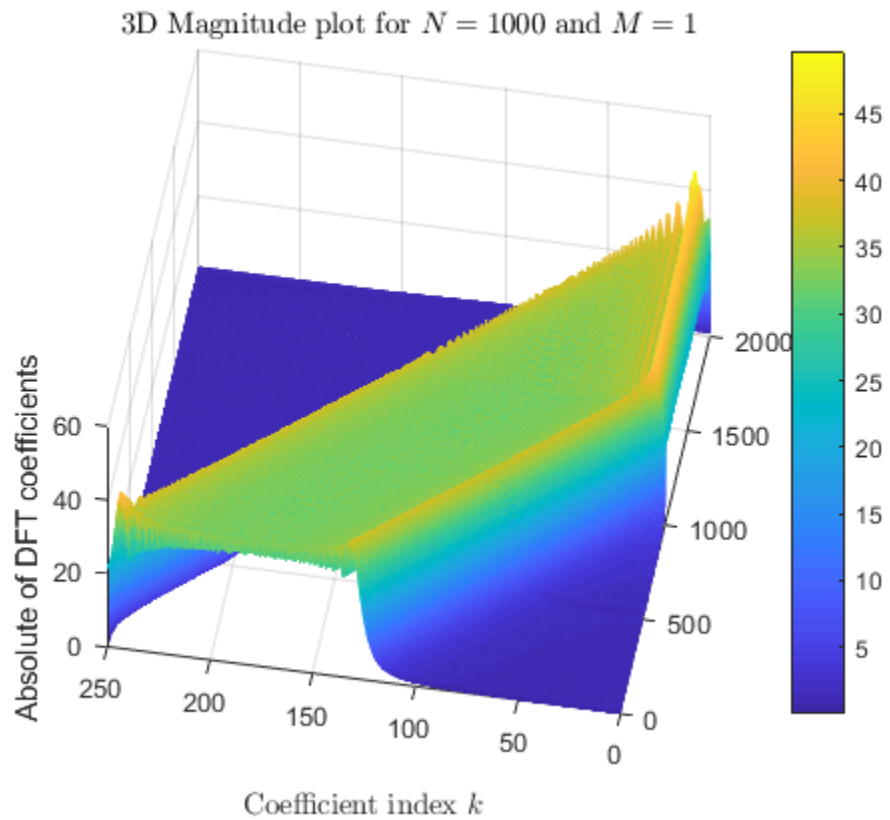
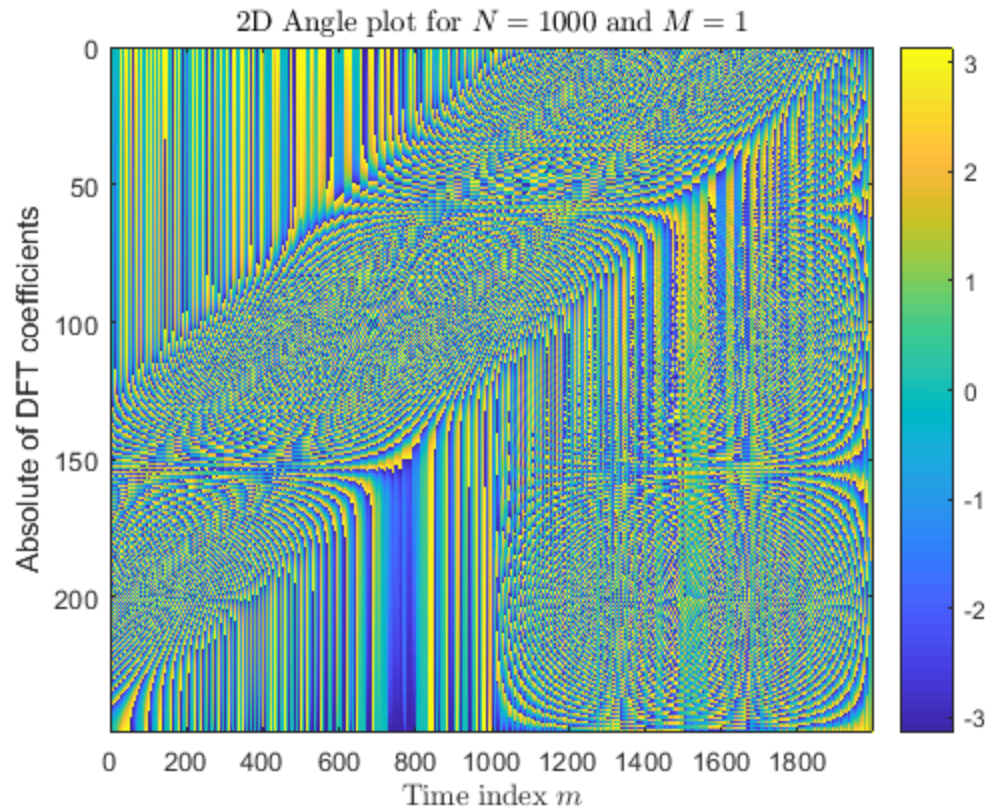


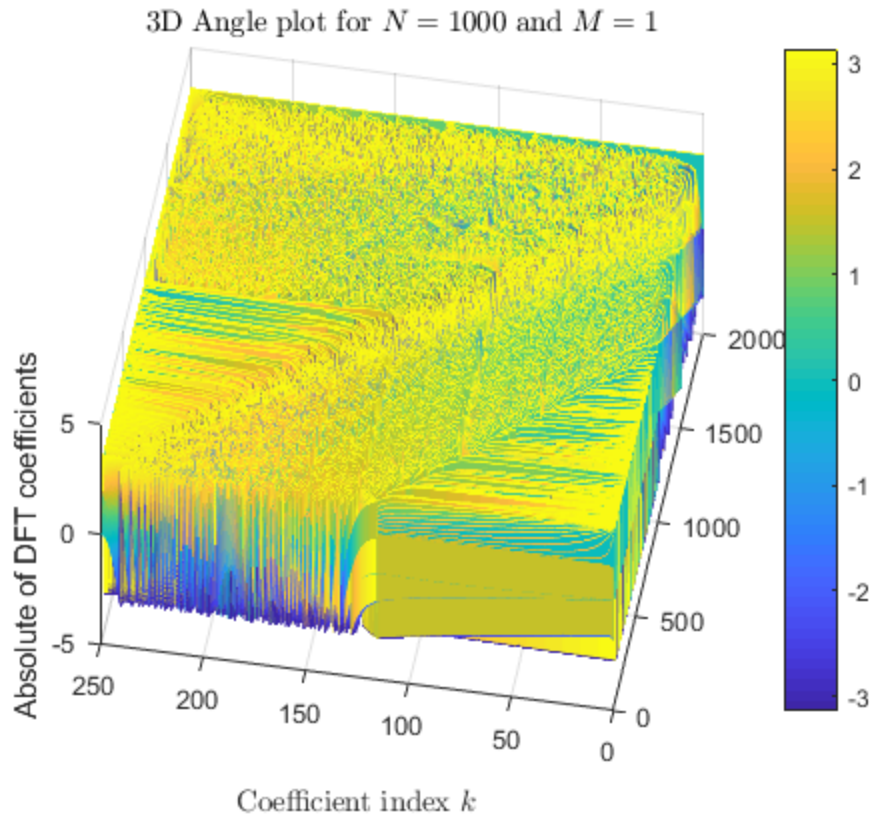
Plot of 2D and 3D graph for $N=1000$ and $M=1$

```
% run the block_it function on the above output.
mtx_N1000_M1 = block_it(output_chirp, 1000, 1);
dftN1000M1 = colDFT(1000, 1, mtx_N1000_M1);
% since DFT is symmetric around a unit circle, either take
% first 50 or last 50 to plot the graph.
figure;
imagesc([0:1999], [499*0.5:-0.5:0], abs(dftN1000M1(1:500, :)));
xlabel('Time index  $m$ ', 'interpreter', 'latex');
ylabel('Absolute of DFT coefficients');
title('2D Magnitude plot for  $N=1000$  and
 $M=1$ ', 'interpreter', 'latex');
colorbar;
figure;
imagesc([0:1999], [499*0.5:-0.5:0], angle(dftN1000M1(1:500, :)));
xlabel('Time index  $m$ ', 'interpreter', 'latex');
ylabel('Angle of DFT coefficients');
title('2D Angle plot for  $N=1000$  and
 $M=1$ ', 'interpreter', 'latex');
colorbar;
figure;
mesh([0:1999], [499*0.5:-0.5:0], abs(dftN1000M1(1:500, :)));
colorbar;
view(-80, 60);
```

```
xlabel('Time index  $m$ ', 'interpreter', 'latex');  
ylabel('Coefficient index  $k$ ', 'interpreter', 'latex');  
zlabel('Absolute of DFT coefficients');  
title('3D Magnitude plot for  $N=1000$  and  
 $M=1$ ', 'interpreter', 'latex');  
figure;  
mesh([0:1999], [499*0.5:-0.5:0], angle(dftN1000M1(1:500, :)));  
colorbar;  
view(-80, 60);  
xlabel('Time index  $m$ ', 'interpreter', 'latex');  
ylabel('Coefficient index  $k$ ', 'interpreter', 'latex');  
zlabel('Absolute of DFT coefficients');  
title('3D Angle plot for  $N=1000$  and  
 $M=1$ ', 'interpreter', 'latex');
```







inputSignal.mat

first step is to load the signal.

```
signal = load('inputSignal.mat', '-mat');
signal = signal.inputSignal;

% plot the signal as well.
figure;
plot(signal);
grid on;
title('Signal from inputSignal.mat file');
xlabel('Samples (n)');
ylabel('Signal Amplitude');

% run the block_it function on the above signal for N = 100 and M
= 1.
sig_N100_M1 = block_it(signal, 100, 1);
sigdftN100M1 = colDFT(100, 1, sig_N100_M1);
% since DFT is symmetric around a unit circle, either take
% first 50 or last 50 to plot the graph.
figure;
imagesc([0:1999], [49*5:-5:0], abs(sigdftN100M1(1:50, :)));
xlabel('Time index $m$', 'interpreter', 'latex');
ylabel('Absolute of DFT coefficients');
```

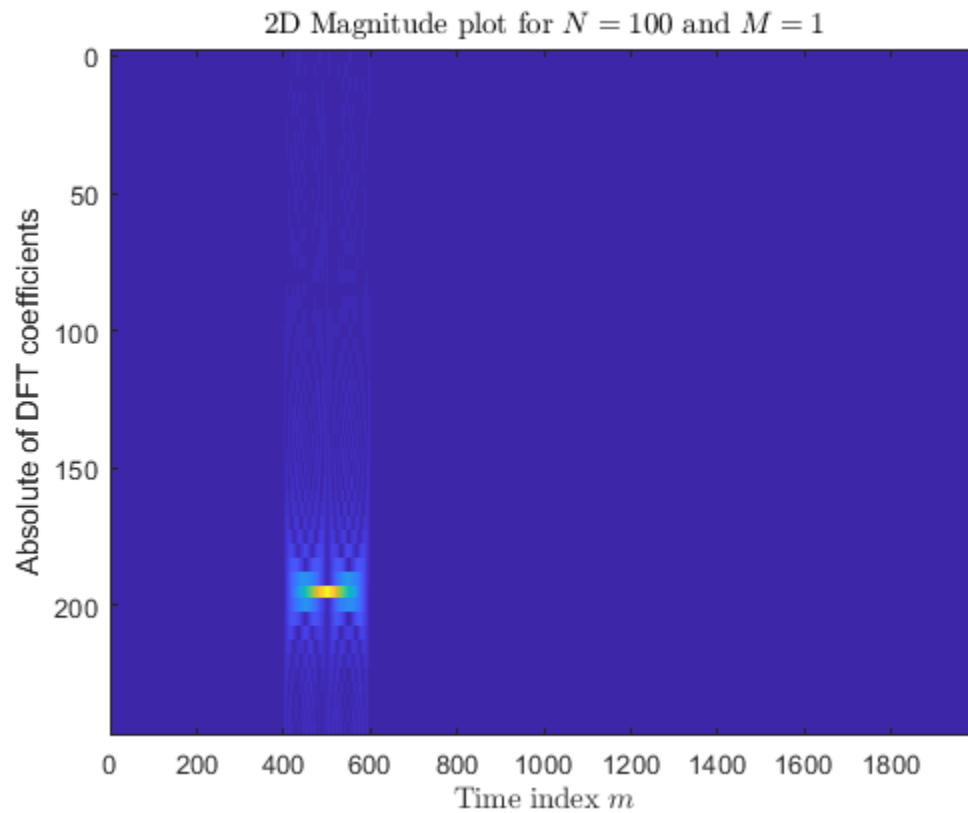
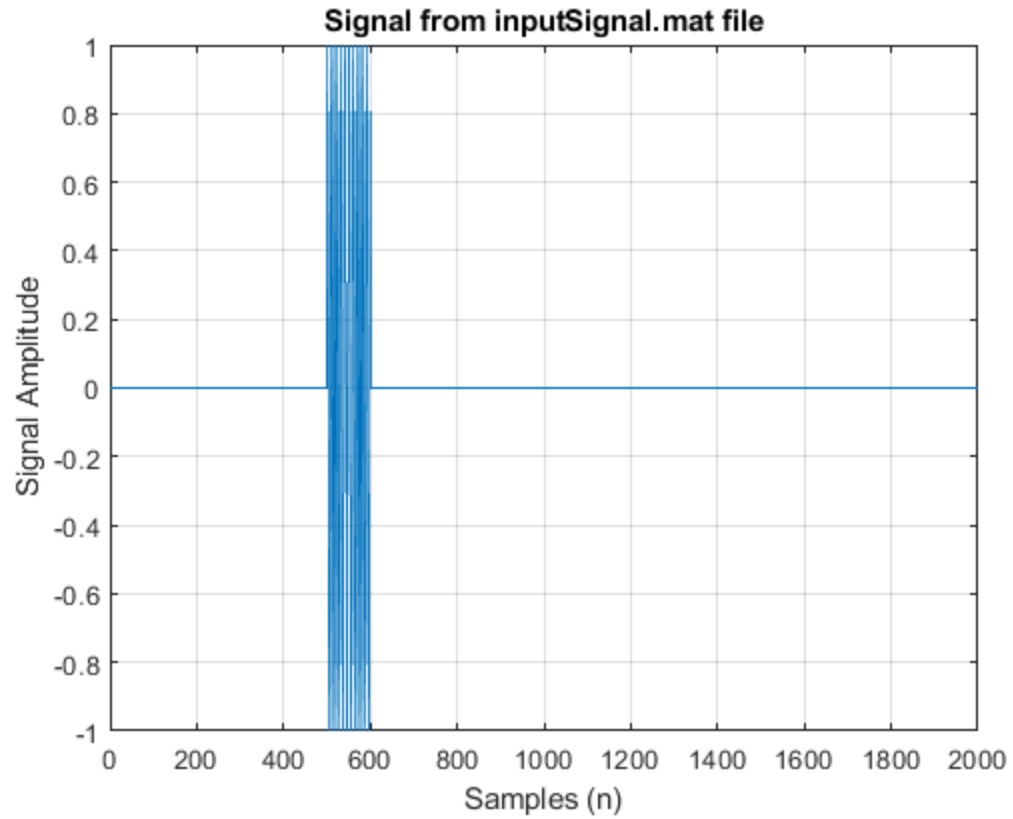
```
title('2D Magnitude plot for $N=100$ and  
$M=1$', 'interpreter', 'latex');  
figure;  
imagesc([0:1999], [49*5:-5:0], angle(sigdftN100M1(1:50, :)));  
xlabel('Time index $m$', 'interpreter', 'latex');  
ylabel('Absolute of DFT coefficients');  
title('2D Angle plot for $N=100$ and  
$M=1$', 'interpreter', 'latex');  
colorbar;  
figure;  
mesh([0:1999], [49*5:-5:0], abs(sigdftN100M1(1:50, :)));  
colorbar;  
xlabel('Time index $m$', 'interpreter', 'latex');  
ylabel('Coefficient index $k$', 'interpreter', 'latex');  
zlabel('Absolute of DFT coefficients');  
title('3D Magnitude plot for $N=100$ and  
$M=1$', 'interpreter', 'latex');  
figure;  
mesh([0:1999], [49*5:-5:0], angle(sigdftN100M1(1:50, :)));  
colorbar;  
xlabel('Time index $m$', 'interpreter', 'latex');  
ylabel('Coefficient index $k$', 'interpreter', 'latex');  
zlabel('Absolute of DFT coefficients');  
title('3D Angle plot for $N=100$ and  
$M=1$', 'interpreter', 'latex');  
  
% run the block_it function on the above signal for N = 100 and M  
= 50.  
sig_N100_M50 = block_it(signal, 100, 50);  
sigdftN100M50 = coldFT(100, 50, sig_N100_M50);  
% since DFT is symmetric around a unit circle, either take  
% first 50 or last 50 to plot the graph.  
figure;  
imagesc([0:39], [49*5:-5:0], abs(sigdftN100M50(1:50, :)));  
xlabel('Time index $m$', 'interpreter', 'latex');  
ylabel('Absolute of DFT coefficients');  
title('2D Magnitude plot for $N=100$ and  
$M=50$', 'interpreter', 'latex');  
colorbar;  
figure;  
imagesc([0:39], [49*5:-5:0], angle(sigdftN100M50(1:50, :)));  
xlabel('Time index $m$', 'interpreter', 'latex');  
ylabel('Absolute of DFT coefficients');  
title('2D Angle plot for $N=100$ and  
$M=50$', 'interpreter', 'latex');  
colorbar;  
figure;  
mesh([0:39], [49*5:-5:0], abs(sigdftN100M50(1:50, :)));  
colorbar;  
view(-80, 60);  
xlabel('Time index $m$', 'interpreter', 'latex');  
ylabel('Coefficient index $k$', 'interpreter', 'latex');  
zlabel('Absolute of DFT coefficients');
```

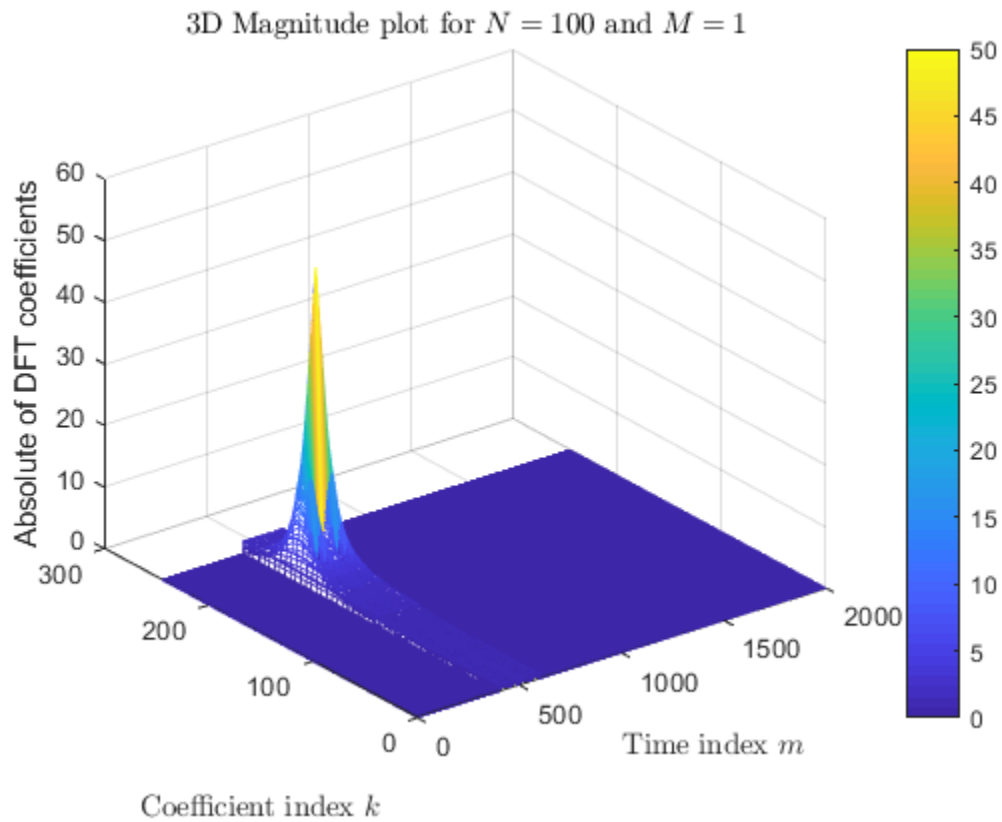
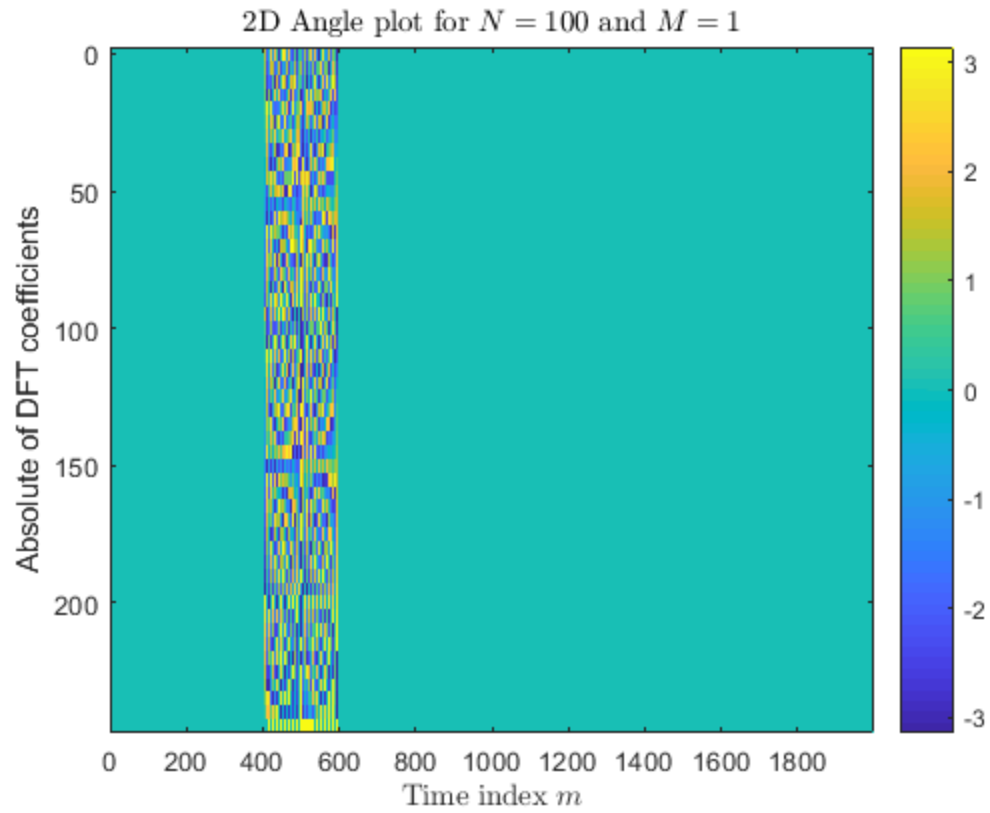


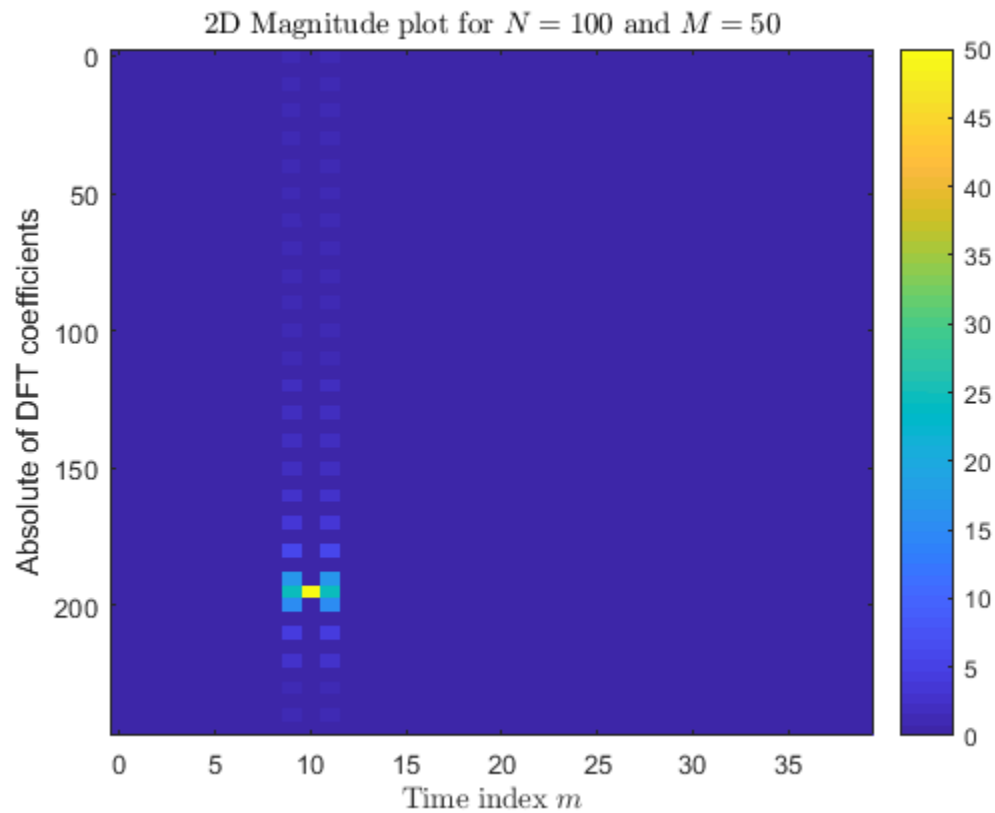
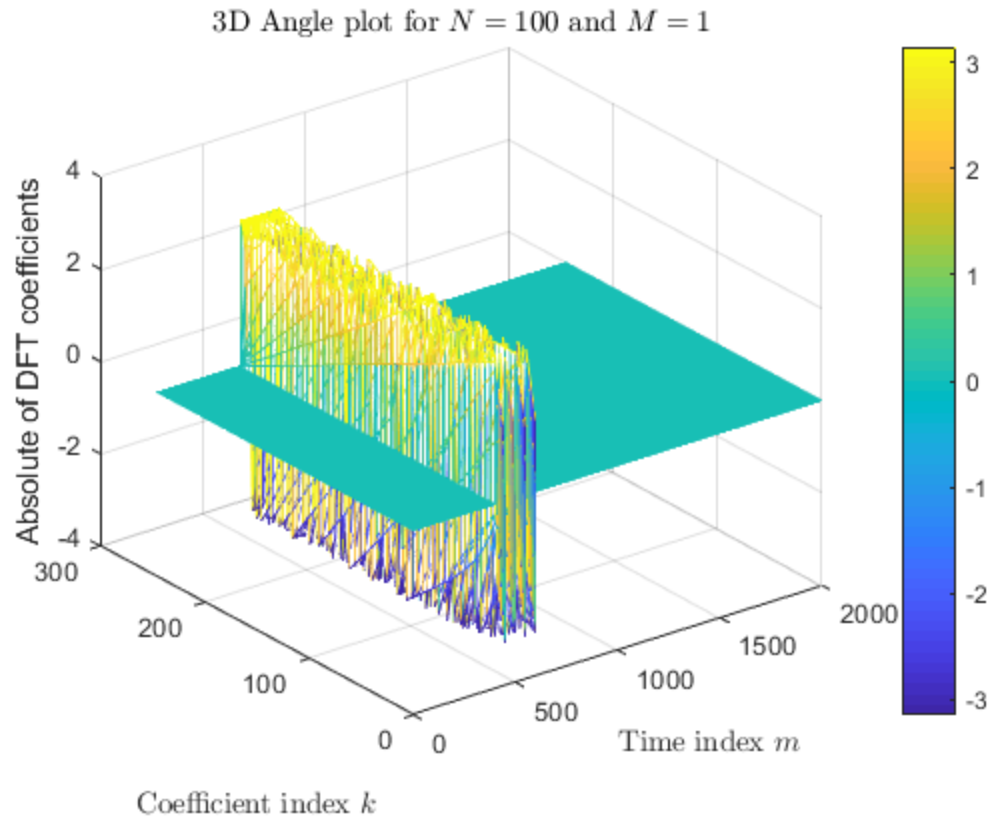
```
title('3D Magnitude plot for $N=100$ and  
$M=50$', 'interpreter', 'latex');  
figure;  
mesh([0:39], [49*5:-5:0], angle(sigdftN100M50(1:50, :)));  
colorbar;  
view(-80, 60);  
xlabel('Time index $m$', 'interpreter', 'latex');  
ylabel('Coefficient index $k$', 'interpreter', 'latex');  
zlabel('Absolute of DFT coefficients');  
title('3D Angle plot for $N=100$ and  
$M=50$', 'interpreter', 'latex');  
  
% run the block_it function on the above signal for N = 100 and M  
= 100.  
sig_N100_M100 = block_it(signal, 100, 100);  
sigdftN100M100 = coldFT(100, 100, sig_N100_M100);  
% since DFT is symmetric around a unit circle, either take  
% first 50 or last 50 to plot the graph.  
figure;  
imagesc([0:19], [49*5:-5:0], abs(sigdftN100M100(1:50, :)));  
xlabel('Time index $m$', 'interpreter', 'latex');  
ylabel('Absolute of DFT coefficients');  
title('2D Magnitude plot for $N=100$ and  
$M=100$', 'interpreter', 'latex');  
colorbar;  
figure;  
imagesc([0:19], [49*5:-5:0], angle(sigdftN100M100(1:50, :)));  
xlabel('Time index $m$', 'interpreter', 'latex');  
ylabel('Absolute of DFT coefficients');  
title('2D Angle plot for $N=100$ and  
$M=100$', 'interpreter', 'latex');  
colorbar;  
figure;  
mesh([0:19], [49*5:-5:0], abs(sigdftN100M100(1:50, :)));  
colorbar;  
view(-80, 60);  
xlabel('Time index $m$', 'interpreter', 'latex');  
ylabel('Coefficient index $k$', 'interpreter', 'latex');  
zlabel('Absolute of DFT coefficients');  
title('3D Magnitude plot for $N=100$ and  
$M=100$', 'interpreter', 'latex');  
figure;  
mesh([0:19], [49*5:-5:0], angle(sigdftN100M100(1:50, :)));  
colorbar;  
view(-80, 60);  
xlabel('Time index $m$', 'interpreter', 'latex');  
ylabel('Coefficient index $k$', 'interpreter', 'latex');  
zlabel('Absolute of DFT coefficients');  
title('3D Angle plot for $N=100$ and  
$M=100$', 'interpreter', 'latex');  
  
% run the block_it function on the above signal for N = 1000 and M  
= 1.  
sig_N1000_M1 = block_it(signal, 1000, 1);
```

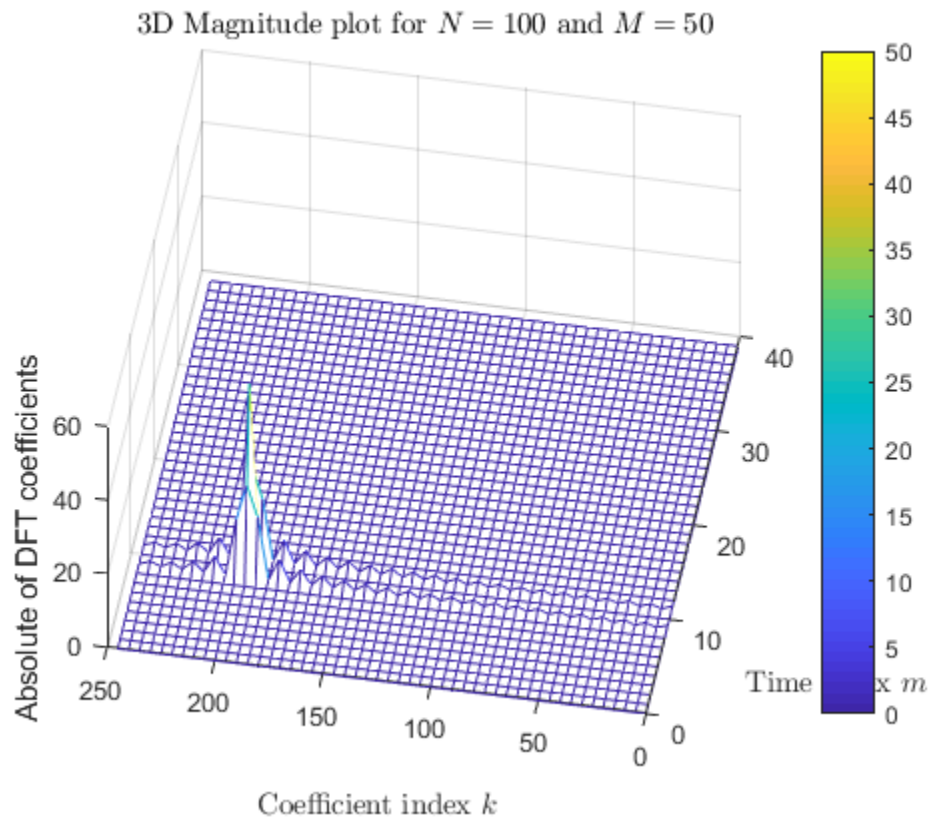
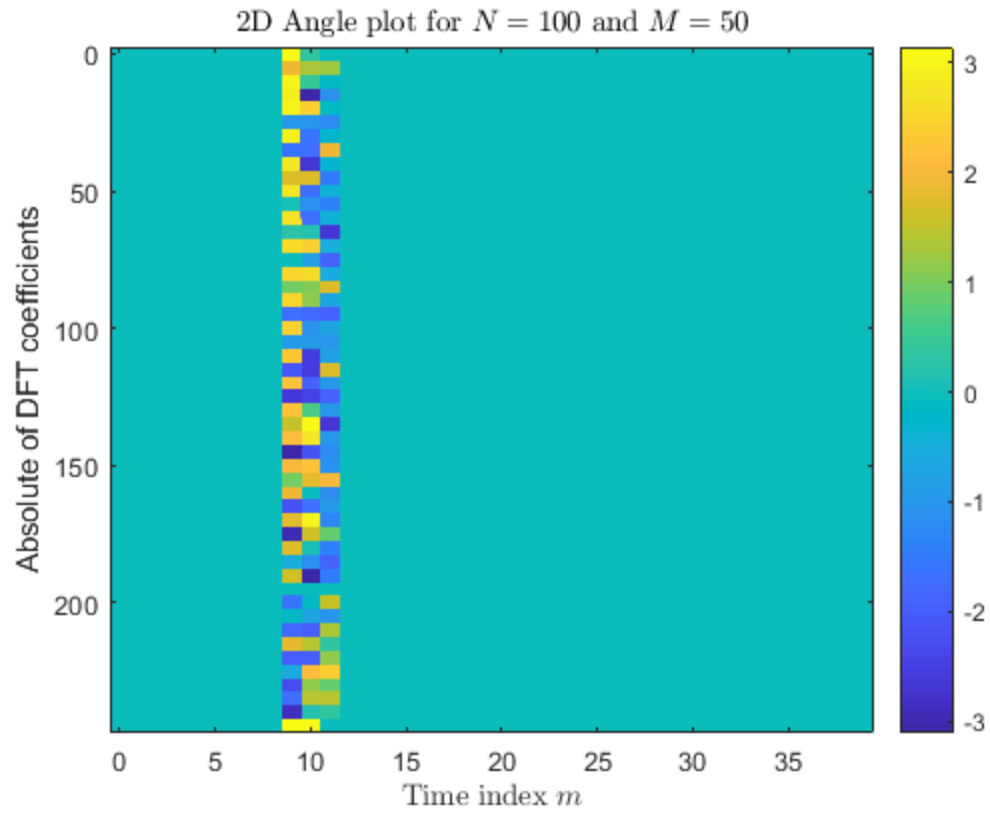


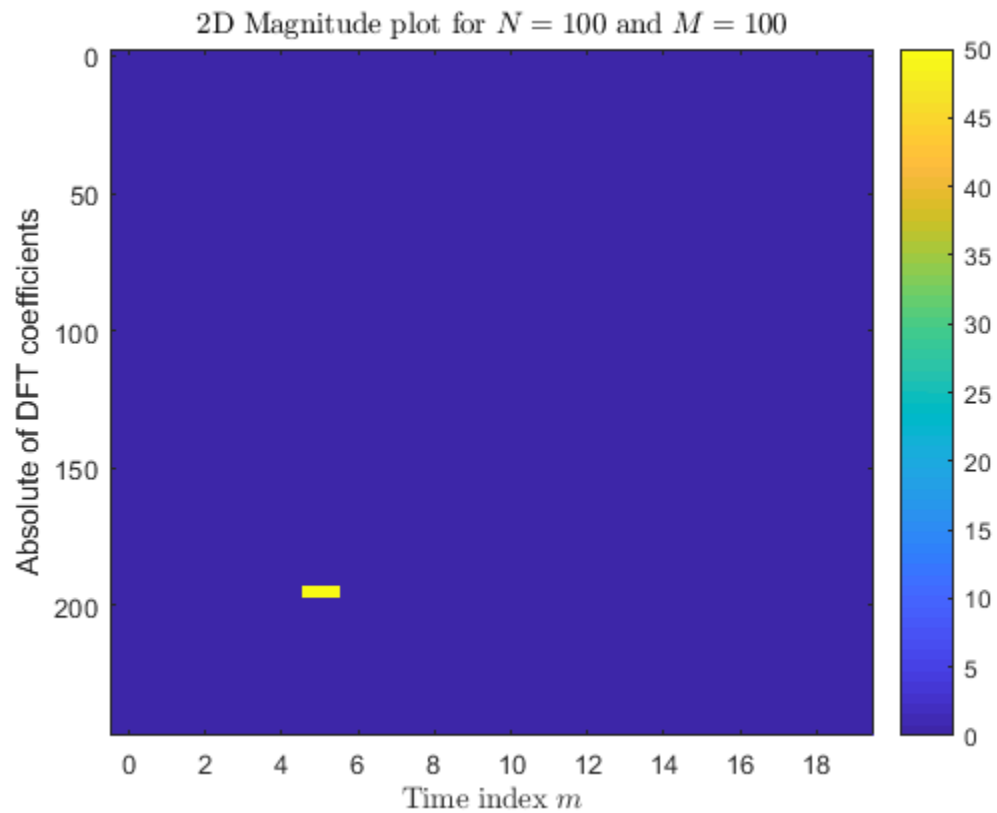
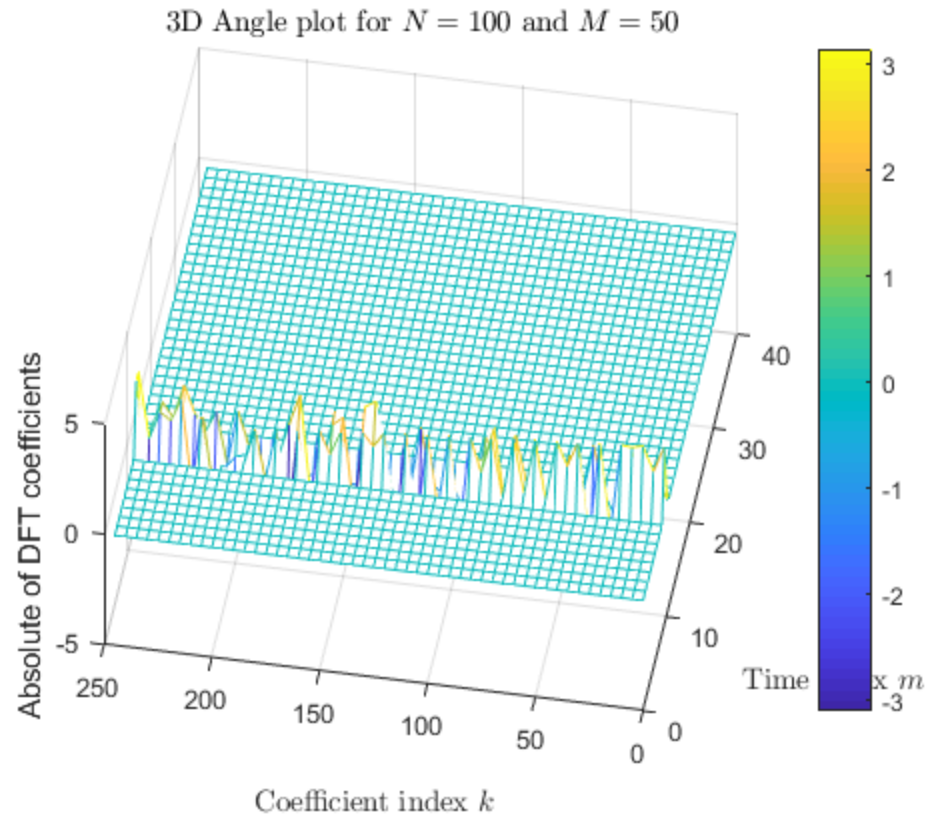
```
sigdftN1000M1 = colDFT(1000, 1, sig_N1000_M1);  
% since DFT is symmetric around a unit circle, either take  
% first 50 or last 50 to plot the graph.  
figure;  
imagesc([0:1999], [499*0.5:-0.5:0], abs(sigdftN1000M1(1:500, :)));  
xlabel('Time index  $m$ ', 'interpreter', 'latex');  
ylabel('Absolute of DFT coefficients');  
title('2D Magnitude plot for  $N=1000$  and  
 $M=1$ ', 'interpreter', 'latex');  
colorbar;  
figure;  
imagesc([0:1999], [499*0.5:-0.5:0],  
angle(sigdftN1000M1(1:500, :)));  
xlabel('Time index  $m$ ', 'interpreter', 'latex');  
ylabel('Absolute of DFT coefficients');  
title('2D Angle plot for  $N=1000$  and  
 $M=1$ ', 'interpreter', 'latex');  
colorbar;  
figure;  
mesh([0:1999], [499*0.5:-0.5:0], abs(sigdftN1000M1(1:500, :)));  
view(-80, 60);  
xlabel('Time index  $m$ ', 'interpreter', 'latex');  
ylabel('Coefficient index  $k$ ', 'interpreter', 'latex');  
zlabel('Absolute of DFT coefficients');  
title('3D Magnitude plot for  $N=1000$  and  
 $M=1$ ', 'interpreter', 'latex');  
figure;  
mesh([0:1999], [499*0.5:-0.5:0], angle(sigdftN1000M1(1:500, :)));  
view(-80, 60);  
xlabel('Time index  $m$ ', 'interpreter', 'latex');  
ylabel('Coefficient index  $k$ ', 'interpreter', 'latex');  
zlabel('Absolute of DFT coefficients');  
title('3D Angle plot for  $N=1000$  and  
 $M=1$ ', 'interpreter', 'latex');
```

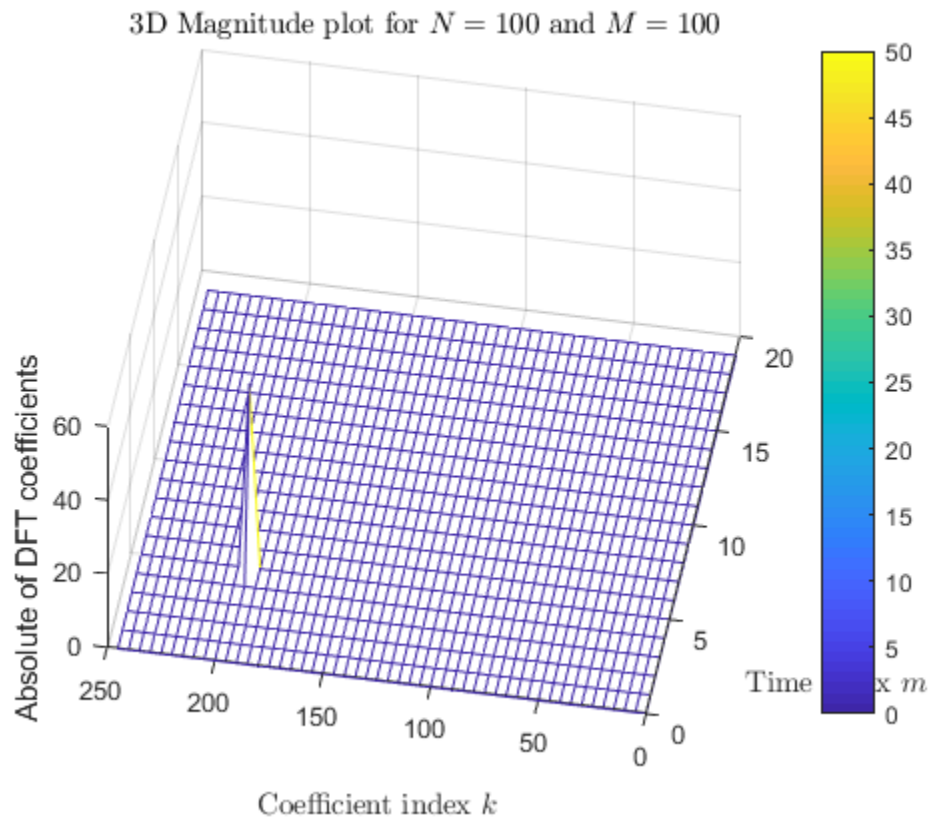
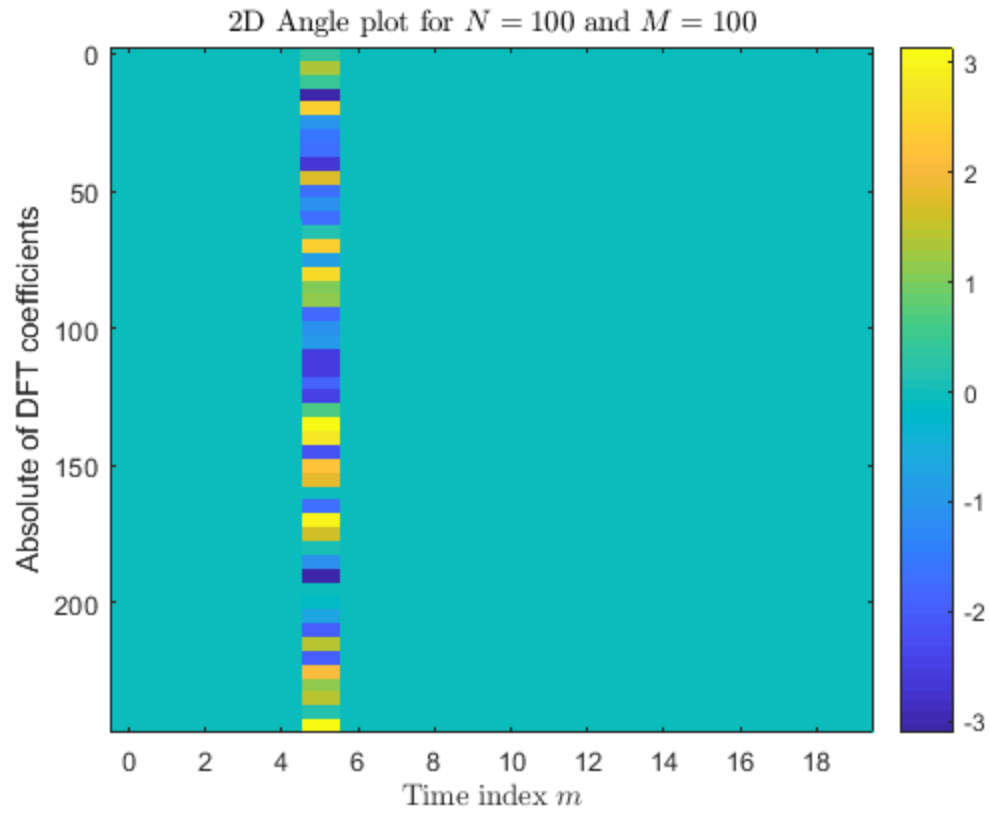


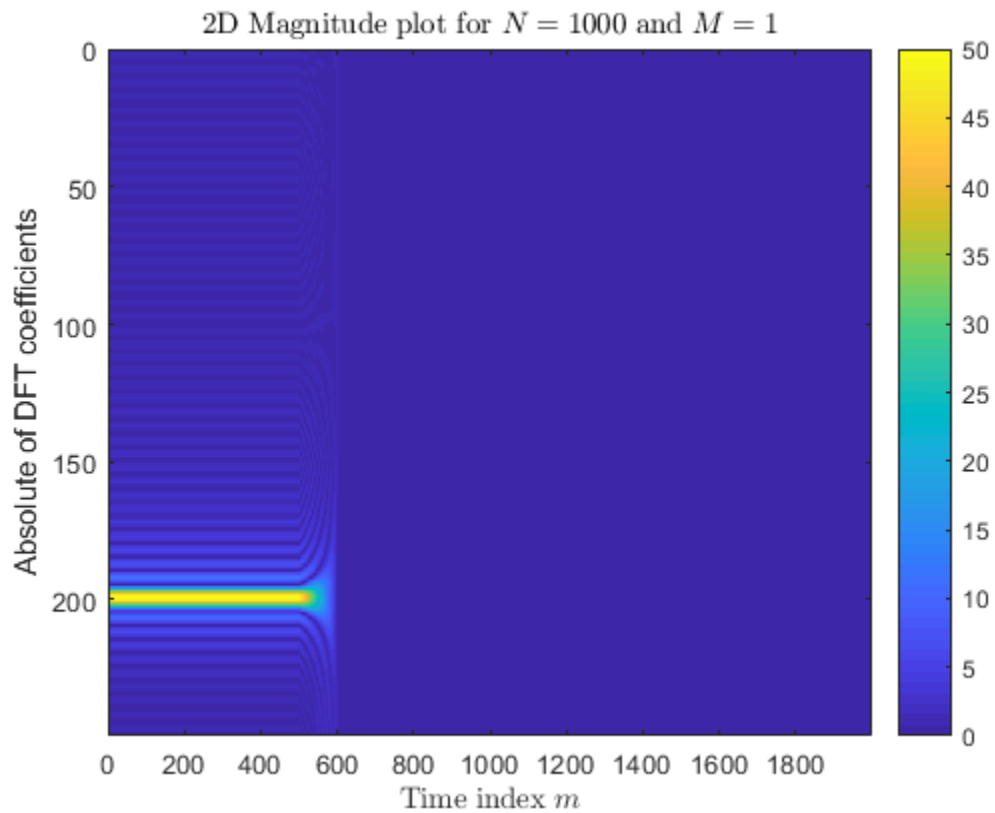
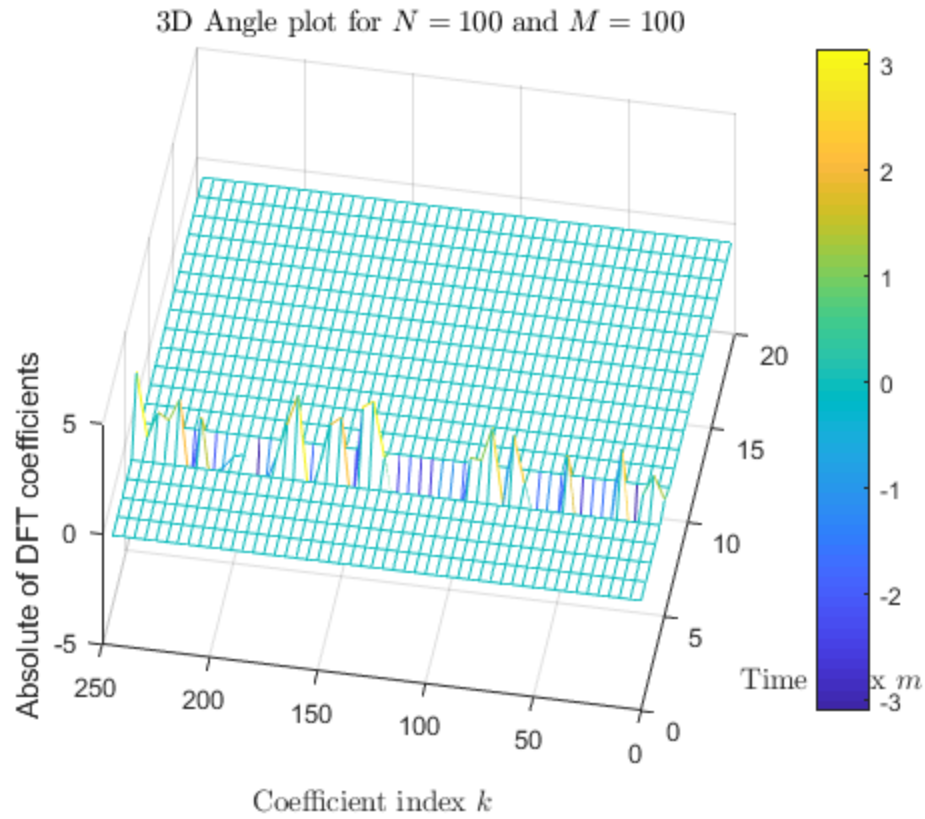


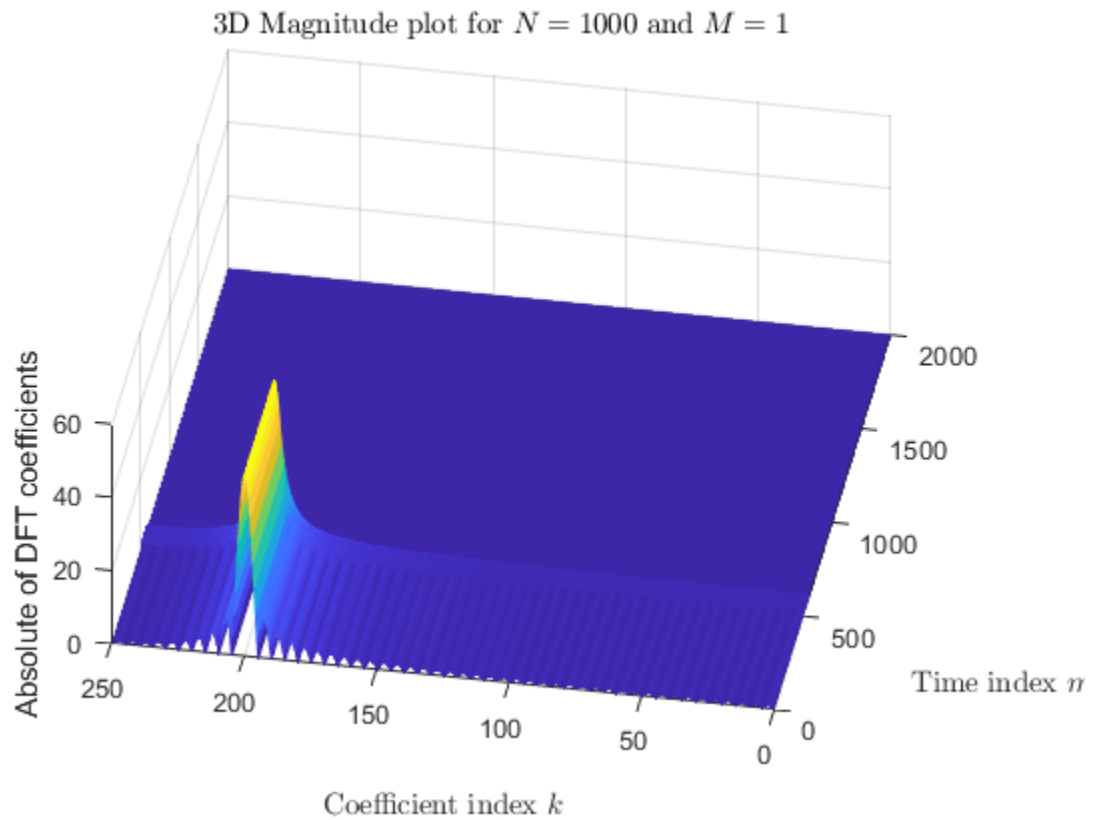
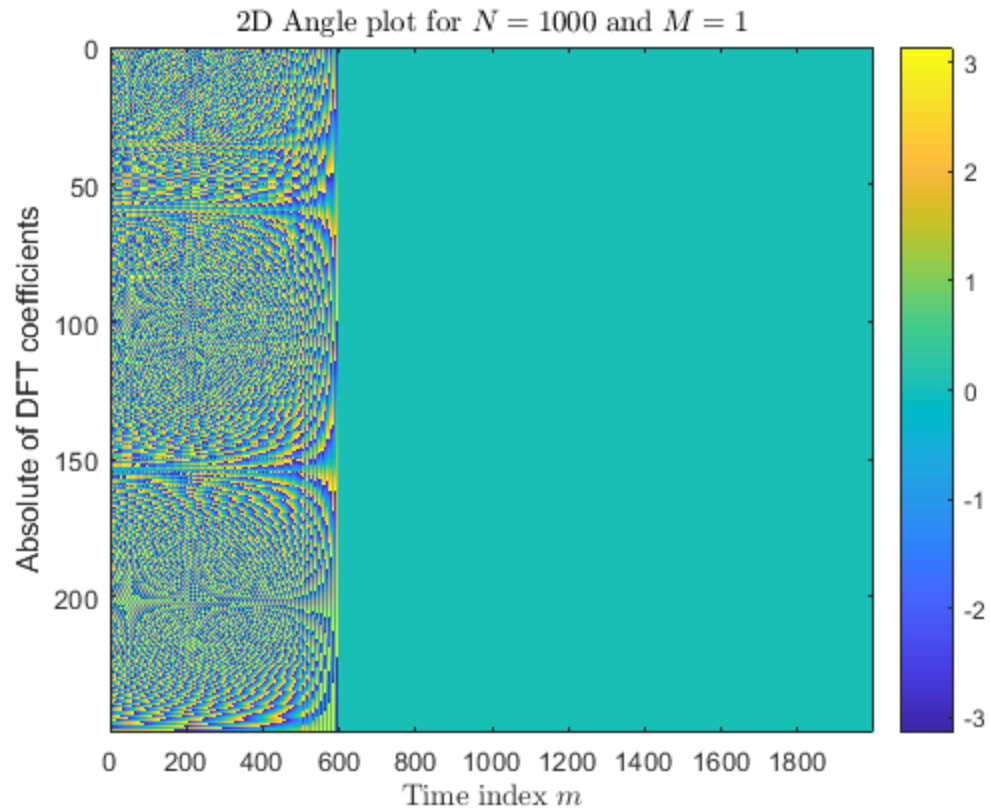


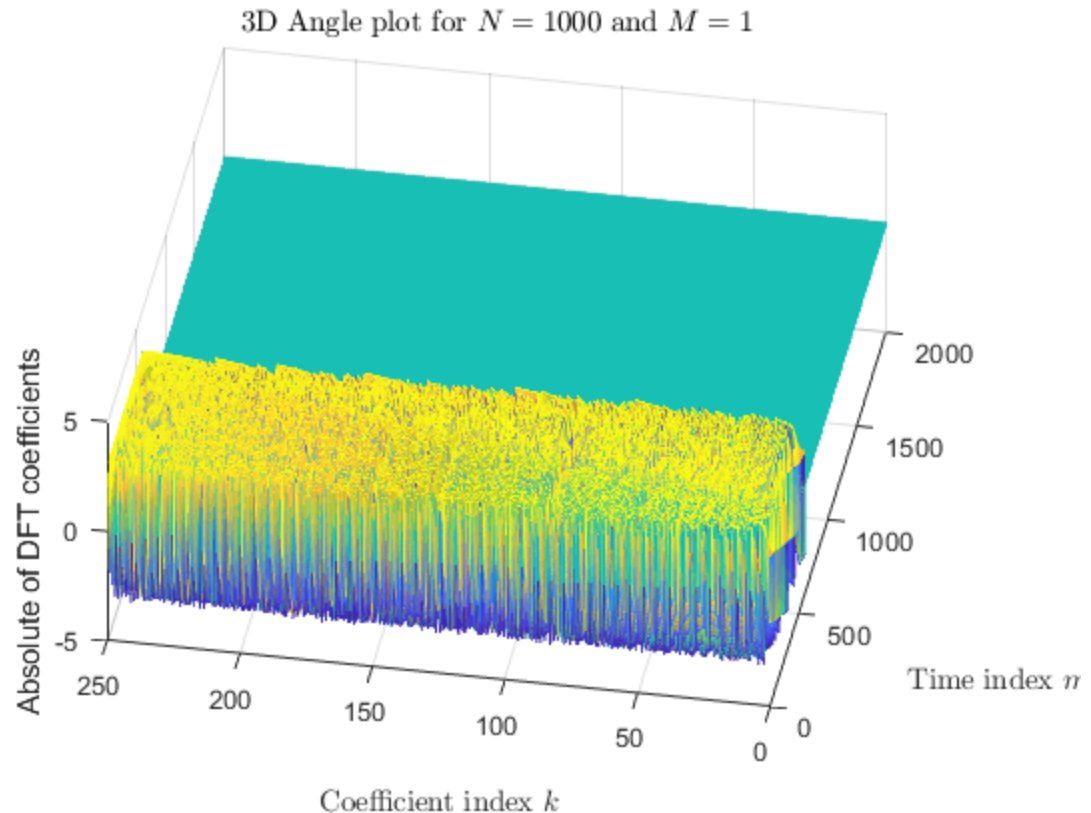












Function for Columnwise DFT.

define a function which will calculate the DFTs for each column and store it as another matrix.

```
function dftOut = colDFT(N, M, matr)
    % find the N pt DFT matrix (using the user defined DFT matrix
    generator).
    D = UDFT(N);
    % find the number of columns in the matrix.
    numcols = size(matr, 2);
    % initialize a blank dftOut matrix.
    dftOut = [];
    % now iterate over the columns.
    for col = 1:numcols
        current_column = matr(:, col);
        % multiply by the DFT matrix and store the result as column
        in the dftOut matrix.
        dftOut(:, size(dftOut,2)+1) = D*current_column;
    end
    % the final result is dftOut.
end
```

User Defined DFT matrix generator

```
function dftmtxout = UDFT(N)
```

```
% define a dft matrix dftmtxout.
dftmtxout = [];

% now iterate over rows which represent 'k' in the DFT matrix
and then iterate over columns which represent
% 'n' values in the DFT matrix.
for k = 1:N
    row = [];
    for n = 1:N
        row = [row exp(-j*2*pi*(k-1)*(n-1)/N)];
    end
    dftmtxout = [dftmtxout; row];
end
end
```

Function to show Chirp Function Usage - Question 1

```
function y = question1()
% we require 2000 samples and the sampling freq = fs = 500 Hz.
% 2000 X (1/500.)s = 4 sec
fs = 500;
t = 0: 1/fs: 4- (1/fs);

% use chirp now.
y = chirp(t, 0, 2, 125);
% plot this figure.
figure;
subplot(1,1,1);
plot(t, y);
grid on;
xlabel('time in seconds');
ylabel('Amplitude of  $x(n)$ ', 'interpreter', 'latex');
title('The chirp cosinusoid  $\displaystyle x(n) = \cos\Big(2\pi$   
 $\Big(\frac{f_{i}}{f_{s}} + \beta n\Big)f_{s} n\Big)$  sampled at  
 $f_{s}=500\text{Hz}$ ', 'interpreter', 'latex');

end
```

Function for dividing Signal into overlapping blocks - Question 2

```
function block_matrix = block_it(signal, N, M)
% duration of the signal.
L = length(signal);

% compute the zeros needed to be added.
zeros_needed = N - rem(L, M);
signal = [signal zeros(1, zeros_needed)];
```

```
% count the number of iterations.
uplimit = L/M;
% define a matrix.
block_matrix = [];

% initialise the iterations.
for i = 1:uplimit
    % get the current block.
    start_index = (i-1)*M + 1;
    stop_index  = (i-1)*M + N;

    current_block = signal(start_index:stop_index);

    % add this as a row the matrix.
    block_matrix = [block_matrix; current_block];

    % exit the loop after all the iterations are completed.
end
% take the transpose of the matrix.
block_matrix = transpose(block_matrix);
end
end
```

Comments on the STFT of the Chirp Cosinusoid

It is clear from the 2D plots that as we increase the value of M from 1 to 100, the quality of the line in the spectrogram decreases. That happens because when $M = 1$, we are having the maximum overlap and that's why we get a sharper resolution for the change in frequency over time. **M is similar to the time derivative in the case of continuous time.** The more smaller the M is, the more precise rate of change can be observed in the frequency of the cosinusoid. If $M = 100$, which is the case of least overlap, we are actually missing out the important changes in the frequency. So, we won't observe those changes which might have occurred in between. However, that is the important part. We might not always need to make $M = 1$, because that might not even be required. What is required is that we are able to tune M in range 1 to N (here, 100) to get the required graph for change in frequency over time. Now when $N = 1000$ and $M = 1$, we are actually dividing the signal into 1000 rows and each time we are observing the change in frequency by setting the time slot $M = 1$ for good precision. Since the chirp is a linear increase in the frequency of cosinusoid (atleast in this case), we observe that the 2D and 3D plots are straight lines, i.e., the DFT coefficient magnitude increase with time and therefore the **center frequency** also increases, this is the implication over here.

Comments on STFT of the inputSignal.mat signal.

This new signal has a single frequency or some set of frequencies at certain time. Most of the time the frequency present in it is 0 Hz or D.C. value. **The comments made about chirp cosinusoid made above apply same here as well.** What's new here is that there is only a single frequency present in the signal starting at sample value 500 and then the same frequency sharply decreases to 0 Hz again, that is the D.C. value again and remains that throughout. That's why we see a single dot like spectrum at only a particular

m value which is a particular instant of time. Also, one more interesting feature to note here is that when $N = 100$, as we increase M from 1 to 100, the degree of precision decreases in the sense ripples in the plot decreases. That is there are more ripples in $M = 1$ and almost no ripples when $M = 100$.

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