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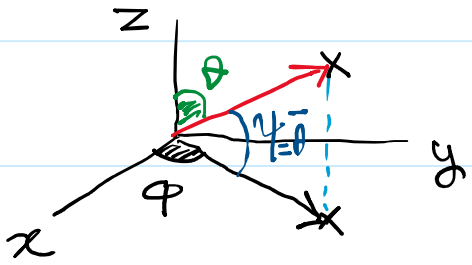
Optimal Array Processing:

* Array is used to filter signal in space-time field.
* The spherical coordinate is used.

$$\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\varphi \end{cases}$$



$$\psi = \bar{\theta} = \frac{\pi}{2} - \theta$$

θ: Polar angle

$\psi = \bar{\theta}$: broadside angle.

Position of elements :

Arrays in one dimention arange on z-axis.

$$P_{z_n} = z_n = \left(n - \frac{N-1}{2}\right)d \; ; \quad n = 0, 1, \ldots, N-1$$

- z coordinate of the nth element
- interelement spacing
- Number of Sensor.

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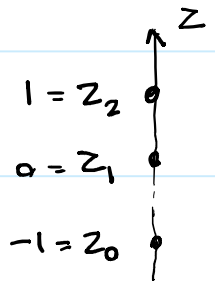
Why $\frac{N-1}{2}$? It's caused center of array put into $z=0$

$N=3 \Rightarrow z_n = (n-1)$; $n = 0, 1, 2$

$z_0 = -1$, $z_1 = 0$, $z_2 = 1$ $\Rightarrow$



$$z_n = \left(n - \frac{N-1}{2}\right) d \quad n = 0, 1, \ldots, N-1$$

$\frac{N-1}{2}$ lead to symmetric location of array. It's important because in Dsp mathematics many complex imaginary part remove & then computation become simple.

computation time delay:

When the plane wave hit to sensors that come from far away. plane wave rich to each sensor in different time.

Consider center of array as time reference $t=0$ then time delay for sensor nth is given

$$\tau_n = \frac{P_{zn} \cos\theta}{c}$$

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C: light velocity.

P_{zn}: sensor position on z-axis

$\cos\theta$: Projection of Place Vector on propagation direction of Wave

τ_n: delay time at Sensor nth.

convert time delay to difference phase.

$$\psi_n = w\tau_n = 2\pi f\left(\frac{P_{zn}\cos\theta}{c}\right)$$

$c = \lambda f$

$$\psi_n = 2\pi f\left(\frac{P_{zn}\cos\theta}{c}\right)$$

$$\psi_n = \frac{2\pi P_{zn}\cos\theta}{\lambda}$$

This is the physical fingerprint that each angle leaves on the Sensor array.

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Broadside Direction: Phase-Domain Analysis.

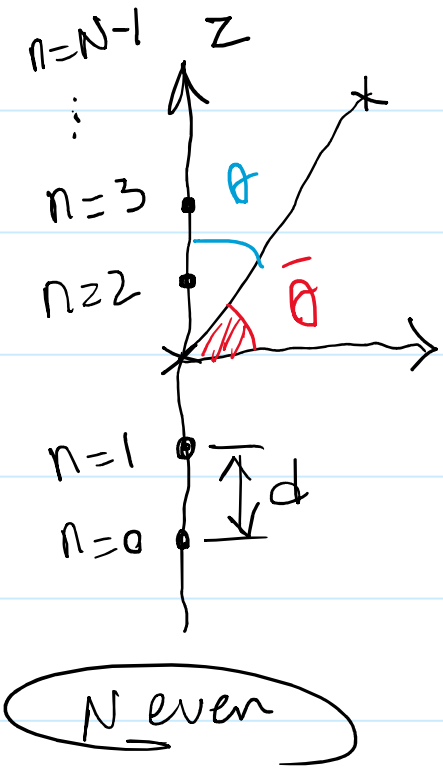
$\theta \rightarrow \bar{\theta}$

$$\Psi_n = \frac{2\pi P_{2n} \sin\bar{\theta}}{\lambda}$$

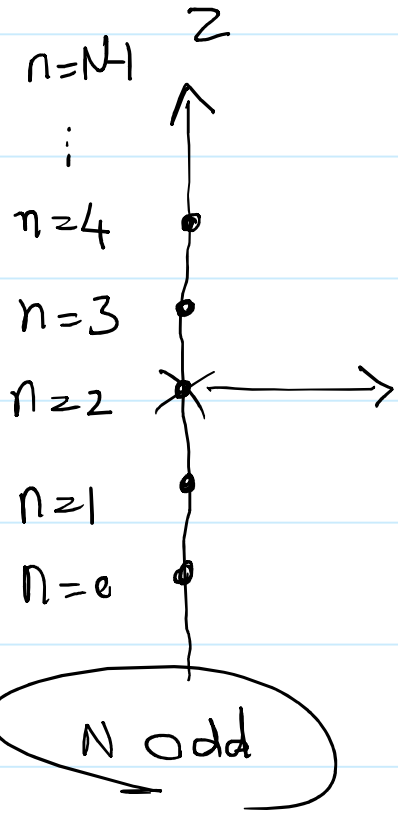
If $\bar{\theta} = 0 \Rightarrow \Psi_n = 0 \Rightarrow$

difference phase will be zero for all sensors. It means that wavefront rich to all sensors simitanously.

If $\bar{\theta} = \pi/2$ or end-fire, maximum phase difference will be happend.



N even



N odd

This is linear array with equal spacing between element.

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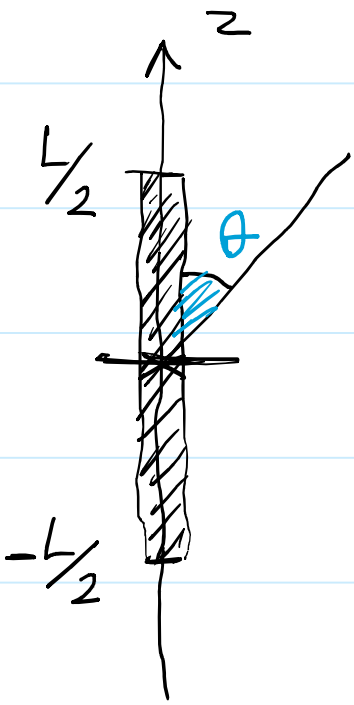
This is shows a linear array with unequal spacing between elements.



This is shows a linear aperture