Distributed Constraint Resolution as Universal Cognition: A Variational Framework

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Abstract

We propose that cognition is a scale-free process: the selection of coherent behavior from a latent space of possibilities via a variational principle. A *constraint network* (factor graph) assigns costs to configurations of interacting components; the *Gibbs target* $\pi_\varepsilon \propto \exp(-V/\varepsilon)$ uniquely minimizes the variational free energy $F_\varepsilon(\rho) = D_{\mathrm{KL}}(\rho \| \pi_\varepsilon)$. *DCR dynamics*—any local, ergodic Markov kernel preserving π_ε—converges geometrically, with free energy decreasing monotonically. A system is *cognitive* when the explore–resolve–stabilize triad produces positive total correlation at stationarity—guaranteed whenever at least one interaction factor is non-factorizable. The DCR form is stable under coarse-graining (exact under lumpability; approximate under timescale separation), addressing the combination problem. We recover the Free Energy Principle as a special case, show that DCR coherence is necessary for IIT's $\Phi > 0$, and derive falsifiable predictions about exploration–exploitation tradeoffs, cognitive depth, and critical constraint densities.

Keywords: cognition, constraint resolution, factor graphs, variational principle, free energy, integrated information, scale-free, coarse-graining, self-organization

1 Introduction

The search for a general, principled definition of intelligence remains a long-standing open problem across cognitive science, physics, and philosophy of mind. Existing frameworks each illuminate a facet of the problem but fall short of universality:

- The *Free Energy Principle* (FEP) [Friston, 2010, 2019] provides an elegant variational account: any system persisting at nonequilibrium steady state minimizes variational free energy. Yet FEP assumes a Markov blanket separating system from environment and a generative model as primitives [Kirchhoff et al., 2018], limiting its applicability to systems where these structures can be identified.
- *Integrated Information Theory* (IIT) [Tononi, 2004, Tononi et al., 2016] offers a quantitative measure of consciousness (Φ), but in its original formulations it is a static, state-level measure rather than a process-level account, and its computation is intractable for large systems.

- *Autopoiesis* [Maturana and Varela, 1980] captures self-production but lacks formal predictive content beyond the biological domain.
- *Panpsychism* [Chalmers, 1995] attributes experience to fundamental entities but provides no mechanism and no solution to the combination problem—how micro-experiences compose into macro-experiences.

We propose that these limitations stem from a common root: each framework privileges a particular *level of description* (Bayesian inference, information integration, self-production) rather than identifying the *scale-free process* that underlies all of them.

Our central thesis:

> *Intelligent behavior emerges when components explore degrees of freedom and converge through distributed constraint resolution into coherent, goal-stabilizing patterns. This is the minimal form of cognition: the selection of behavior from a latent space of possibilities that minimizes a variational principle.*

We call this the **Distributed Constraint Resolution** (DCR) framework. The mathematical core is a *constraint network* (a factor graph with pairwise costs), a *Gibbs target measure* encoding the optimal tradeoff between constraint satisfaction and exploration, and a *variational free energy* whose minimization characterizes convergence. The three components of the DCR triad are:

1. *Exploration* — ergodic dynamics that can reach any configuration, generating variability across the state space.
2. *Resolution* — local updates that reduce constraint violations between neighboring components.
3. *Stabilization* — convergence to the Gibbs target, a coherent, low-violation distribution characterized by the variational principle.

We treat *goals* purely operationally: a goal is any attractor that is robust under perturbations at the timescale of interest, not a representation of future states.

The key conceptual advance is the identification of a single variational principle that:

1. Characterizes the stationary behavior of any system resolving distributed constraints;
2. Reduces to the Free Energy Principle when the constraint topology is a Markov blanket;
3. Provides a necessary condition for IIT's integrated information;
4. Applies uniformly from physics to biology to economics.

The paper is organized as follows. Section 2 formalizes the DCR framework and proves convergence. Section 3 establishes closure under coarse-graining and defines cognitive depth. Section 4 exhibits structural witnesses across physical scales. Section 5 recovers FEP and relates DCR to IIT. Section 6 derives falsifiable predictions. Section 7 discusses implications and limitations.

Contributions. Beyond the structural definition of cognition, this paper makes four specific technical contributions: (1) a variational characterization of DCR equilibria with convergence, concentration, and monotone free energy decrease; (2) closure under coarse-graining—exact for lumpable partitions, approximate under timescale separation—with the macro-cost arising as a coarse-grained free energy; (3) an identity between DCR's variational free energy and FEP's, establishing the Free Energy Principle as a special case; and (4) structural mappings exhibiting the DCR triad across physical, biological, and economic systems.

Remark 1.1 (What is proved vs. what is proposed)*.* Theorems in this paper establish: (i) uniqueness, geometric convergence, and monotone free energy decrease (Theorem 2.9); (ii) exponential concentration near the feasible set (Proposition 2.10); and (iii) closure under coarse-graining, exact for lumpable partitions (Theorem A.2) and approximate under timescale separation (Proposition 3.1). Claims in Section 4 are *structural mappings* exhibiting how standard models can be cast into the DCR template; they are not new derivations of the underlying physics, and they do not resolve foundational questions such as the quantum measurement problem.

Scope and terminology. Throughout this paper, *cognitive* is a defined technical property of a dynamical system (Definition 2.13): a system is cognitive if and only if it instantiates the DCR triad of exploration, distributed constraint resolution, and convergence to a coherent attractor (operationalized here via $\mathrm{Coh}(\pi_\varepsilon) > 0$). This is not a claim about phenomenal consciousness, folk intelligence, or teleology. DCR does not assert that photons "have experiences" or that convection cells "think"; it asserts that the *formal process* by which these systems evolve—exploring degrees of freedom, resolving constraints locally, stabilizing into coherent patterns—is structurally identical to the process that, at higher cognitive depth (Definition 3.2), underlies what we ordinarily call intelligence. Whether one additionally identifies this process with experience is a separate philosophical question that DCR does not adjudicate (see Section 7, Limitation 5).

As a universality claim, DCR is falsified by any robust cognitive phenomenon that lacks (i) genuine exploration, (ii) local constraint-processing, or (iii) convergence to a coherent attractor under coarse-graining.

For finite state spaces, exact closure holds under Kemeny–Snell lumpability (Theorem A.2). For continuous or non-lumpable systems, approximate closure holds under timescale separation (Proposition 3.1)—a condition that holds in many physical systems but is not universal. The framework's axioms are themselves scale-free; it is specifically the composition mechanism that requires additional structure.

We treat the metaphysical identification—the cosmos is cognitive at every scale—as optional; the formal results hold without it (see Remark C.1 in Appendix C).

2 The DCR Framework

2.1 Constraint Networks

Definition 2.1 (Constraint Network)**.** A *constraint network* is a tuple $\mathcal{N} = (S, \{D_s\}, G, \{h_s\}, \{v_e\})$ where:

1. S is a finite set of *components*.
2. $\{D_s\}_{s\in S}$ assigns to each component s a finite *state space* D_s (its degrees of freedom).
3. $G = (S, E)$ is an undirected graph encoding the *interaction topology*.
4. $\{h_s\}_{s\in S}$ assigns to each component s a *node cost* (unary factor) $h_s : D_s \to \mathbb{R}_{\geq 0}$.
5. $\{v_e\}_{e\in E}$ assigns to each edge $e = \{s, s'\} \in E$ an *edge cost* (pairwise factor) $v_e : D_s \times D_{s'} \to \mathbb{R}_{\geq 0}$.

The *configuration space* is $\Omega = \prod_{s\in S} D_s$, which is finite.

This is a factor graph with *node factors* $\phi_s(\omega_s) := \exp(-h_s(\omega_s)/\varepsilon)$ and *edge factors* $\phi_e(\omega_s, \omega_{s'}) := \exp(-v_e(\omega_s, \omega_{s'})/\varepsilon)$. Node costs encode local preferences or priors; edge costs encode pairwise constraints between neighbors. Hard constraints can be modeled by assigning very large penalties to forbidden configurations (or equivalently by restricting to the support when taking $\varepsilon \to 0$). The factor-graph viewpoint is standard in graphical models [Lauritzen, 1996] and constraint satisfaction; adopting it as the mathematical foundation of DCR directly connects the framework to a large body of existing theory.

Definition 2.2 (Constraint Cost and Feasible Set)**.** The *total constraint cost* of a configuration $\omega \in \Omega$ is

$$V(\omega) = \sum_{s \in S} h_s(\omega_s) + \sum_{e=\{s,s'\} \in E} v_e(\omega_s, \omega_{s'}). \tag{2.1}$$

We normalize so that $\min_\Omega V = 0$ (replace V by $V - \min V$). The *feasible set* is $\mathcal{F} := \{\omega \in \Omega : V(\omega) = 0\} = \arg\min V$, which is non-empty since Ω is finite.

Notation. Throughout, $\varepsilon > 0$ denotes the noise level (temperature), π_ε the Gibbs target (Definition 2.3), K the transition kernel, V the total constraint cost (node + edge), Coh the total correlation (Definition 2.11), Coh_E the edge-sum coherence (Definition 2.12), and $\mathcal{F} = \arg\min V$ the feasible set.

The formal development assumes $|S| < \infty$ and $|D_s| < \infty$ for each s. Extensions to compact or Polish state spaces are discussed in Appendix B; continuum field theories should be read as finite-element discretizations. The discreteness assumption may be less restrictive than it appears—see Remark 4.3 in Appendix C.

2.2 The Variational Principle

The central mathematical object is the *Gibbs target measure*, which characterizes the optimal tradeoff between constraint satisfaction and exploration.

Definition 2.3 (Gibbs Target Measure)**.** Given a constraint network $\mathcal{N}$ and noise level $\varepsilon > 0$, the *Gibbs target measure* is

$$\pi_\varepsilon(\omega) = Z_\varepsilon^{-1} \exp\left(-\frac{V(\omega)}{\varepsilon}\right), \qquad Z_\varepsilon = \sum_{\omega \in \Omega} \exp\left(-\frac{V(\omega)}{\varepsilon}\right). \tag{2.2}$$

Since Ω is finite and $V \geq 0$, we have $Z_\varepsilon > 0$ and $\pi_\varepsilon(\omega) > 0$ for all $\omega \in \Omega$. (We assume throughout that all costs are finite, so π_ε is strictly positive. Hard constraints can be modeled via large finite penalties or by restricting to the support; the Hammersley–Clifford results below require this positivity.)

The Gibbs measure π_ε is a *pairwise Markov random field* on the graph G: by the Hammersley–Clifford theorem, the positivity $\pi_\varepsilon > 0$ together with the factor decomposition $\pi_\varepsilon(\omega) \propto \prod_s \exp(-h_s/\varepsilon) \prod_e \exp(-v_e/\varepsilon)$ implies that π_ε satisfies the pairwise Markov property with respect to G [Lauritzen, 1996] (node factors do not affect the conditional independence structure). This connection to graphical models is central to the coherence results below.

Definition 2.4 (Variational Free Energy)**.** For a probability distribution ρ on Ω, the *variational free energy* at noise level ε is

$$F_\varepsilon(\rho) \;=\; D_{\mathrm{KL}}(\rho \,\|\, \pi_\varepsilon) \;=\; \frac{1}{\varepsilon}\,\mathbb{E}_\rho[V] - H(\rho) + \ln Z_\varepsilon, \tag{2.3}$$

where $H(\rho) = -\sum_\omega \rho(\omega) \ln \rho(\omega)$ is the Shannon entropy.

The equality $D_{\mathrm{KL}}(\rho\|\pi_\varepsilon) = \mathbb{E}_\rho[V]/\varepsilon - H(\rho) + \ln Z_\varepsilon$ follows by expanding $\ln(\rho/\pi_\varepsilon)$ and using the definition of π_ε. The decomposition into *energy* $\mathbb{E}_\rho[V]/\varepsilon$ and *negative entropy* $-H(\rho)$ reveals the explore–exploit tradeoff at the heart of DCR:

- The energy term $\mathbb{E}_\rho[V]/\varepsilon$ favors distributions concentrated on low-cost configurations (exploitation / constraint satisfaction).
- The entropy term $-H(\rho)$ penalizes concentrated distributions, favoring spread across Ω (exploration).
- The noise level ε controls the balance: small ε emphasizes constraint satisfaction; large ε emphasizes exploration.

Theorem 2.5 (Gibbs Variational Characterization). *The Gibbs target π_ε is the unique minimizer of the variational free energy:*

$$\pi_\varepsilon = \arg\min_\rho F_\varepsilon(\rho), \qquad F_\varepsilon(\pi_\varepsilon) = 0. \tag{2.4}$$

For any distribution $\rho \neq \pi_\varepsilon$, $F_\varepsilon(\rho) > 0$.

Proof. $F_\varepsilon(\rho) = D_{\mathrm{KL}}(\rho\|\pi_\varepsilon) \geq 0$ with equality if and only if $\rho = \pi_\varepsilon$ (Gibbs' inequality). □

The variational free energy is therefore a (weak) Lyapunov function for the *distributional* dynamics induced by any DCR kernel K: if $\pi_\varepsilon K = \pi_\varepsilon$, then along $\rho_{t+1} = \rho_t K$ we have $F_\varepsilon(\rho_{t+1}) \leq F_\varepsilon(\rho_t)$ by the data-processing inequality (Theorem 2.9). This is the mathematical expression of the claim that DCR stabilization can be viewed as free-energy minimization.

2.3 DCR Dynamics

Definition 2.6 (DCR Dynamics). A tuple $(\mathcal{N}, \varepsilon, K)$ satisfies *DCR dynamics* if $\mathcal{N}$ is a constraint network, $\varepsilon > 0$, and K is a Markov kernel (transition matrix) on Ω satisfying:

(D1) **Ergodicity.** K is irreducible and aperiodic on Ω.

(D2) **Gibbs stationarity.** The Gibbs target π_ε is the stationary distribution of K: $\pi_\varepsilon K = \pi_\varepsilon$.

(D3) **Locality.** Each transition $\omega \to \omega'$ with $K(\omega, \omega') > 0$ updates at most one component $s \in S$ (or a uniformly bounded neighborhood), and the update rule for component s depends only on the current states of s and its graph neighbors $N_G(s)$.

Condition **(D1)** ensures a unique stationary distribution and geometric convergence to it. Condition **(D2)** ties the long-run behavior to the constraint structure through the Gibbs measure: the dynamics converges to the distribution that optimally balances constraint satisfaction against exploration. Condition **(D3)** encodes the "distributed" qualifier: no update requires evaluating V outside the updated neighborhood; each component resolves constraints based on its local neighborhood. Together, the three conditions capture the DCR triad: exploration (ergodicity ensures full accessibility), resolution (local updates that respect the constraint structure), and stabilization (convergence to the Gibbs target).

An important distinction: DCR is not the stationary distribution π_ε alone (any distribution can be written as a Gibbs measure for some cost function); it is the *local implementability* of the kernel K given the factorization of V along the graph G. The constraint graph determines what locality means, and thereby constrains which dynamics count as DCR.

Remark 2.7 (Standard constructions). Several well-known Markov kernels satisfy **(D1)**–**(D3)**:

- *Glauber dynamics (heat bath):* Select component s uniformly at random; resample ω_s from its conditional distribution under π_ε given the neighbors' states. Since π_ε is a Markov random field, this conditional depends only on $N_G(s)$.
- *Metropolis–Hastings:* Select s uniformly; propose ω'_s from some proposal distribution; accept with probability $\min(1, \pi_\varepsilon(\omega')/\pi_\varepsilon(\omega))$.
- *Random-scan Gibbs sampling:* Select a component uniformly at random; resample from its full conditional. (A deterministic scan can introduce periodicity as a single-step kernel; defining one "step" as a full sweep restores aperiodicity.)

All are irreducible and aperiodic on finite Ω when $\pi_\varepsilon > 0$ (which holds by construction), and all have π_ε as stationary distribution.

Remark 2.8 (Nonequilibrium dynamics). Condition **(D2)** does not require detailed balance: K may carry nonzero probability currents at stationarity (nonequilibrium DCR). We deliberately include equilibrium systems—a crystal forming from solution resolves distributed constraints and produces coherent patterns. The distinction between "interesting" and "trivial" cognition is captured by cognitive depth (Definition 3.2) and coherence magnitude, not by an equilibrium exclusion. For applications where the distinction matters, one may quantify departure from equilibrium via a standard steady-state entropy production rate from stochastic thermodynamics.

2.4 Convergence and Concentration

Theorem 2.9 (DCR Convergence). *Let $(\mathcal{N}, \varepsilon, K)$ satisfy DCR dynamics. Then:*

(i) ***Uniqueness and geometric convergence.*** *K has the unique stationary distribution π_ε, and there exist $C < \infty$ and $r \in (0,1)$ such that for any initial distribution ρ_0,*

$$\|\rho_0 K^t - \pi_\varepsilon\|_{\mathrm{TV}} \;\le\; C\, r^t. \tag{2.5}$$

This is standard for finite irreducible aperiodic chains [Levin and Peres, 2017, Theorem 4.9].

(ii) ***Free energy decrease.*** *The variational free energy decreases monotonically: $F_\varepsilon(\rho_0 K^{t+1}) \le F_\varepsilon(\rho_0 K^t)$.*

Proof. (i) follows from Levin and Peres [2017, Theorem 4.9] together with **(D2)**.

(ii) By the data-processing inequality for KL divergence, for any kernel K with $\pi_\varepsilon K = \pi_\varepsilon$: $F_\varepsilon(\rho K) = D_{\mathrm{KL}}(\rho K \| \pi_\varepsilon) \le D_{\mathrm{KL}}(\rho \| \pi_\varepsilon) = F_\varepsilon(\rho)$. □

Note: this is monotonicity of $F_\varepsilon(\rho_t)$ for the evolving *law* ρ_t, not monotonic decrease of $V(X_t)$ along individual sample trajectories. (Equality can occur, e.g. if $\rho_t = \pi_\varepsilon$ or if K is information-preserving on the support of ρ_t.)

Proposition 2.10 (Concentration). *Let $v_{\min} = \min\{V(\omega) : \omega \notin \mathcal{F}\} > 0$. For any $\delta \ge v_{\min}$, writing $\Omega_{\ge\delta} := \{\omega : V(\omega) \ge \delta\}$,*

$$\pi_\varepsilon(\Omega_{\ge\delta}) \;\le\; \frac{|\Omega_{\ge\delta}|}{|\mathcal{F}|} \exp\!\left(-\frac{\delta}{\varepsilon}\right). \tag{2.6}$$

Proof. Since $\min V = 0$, we have $Z_\varepsilon \ge |\mathcal{F}|$. For any $\omega \in \Omega_{\ge\delta}$, $\exp(-V(\omega)/\varepsilon) \le \exp(-\delta/\varepsilon)$; summing gives $\pi_\varepsilon(\Omega_{\ge\delta}) \le |\Omega_{\ge\delta}|\, e^{-\delta/\varepsilon}/|\mathcal{F}|$. □

Setting $\delta = v_{\min}$ shows that $\pi_\varepsilon(\mathcal{F}) \to 1$ and $\mathbb{E}_{\pi_\varepsilon}[V] = O(e^{-v_{\min}/\varepsilon})$ as $\varepsilon \to 0$: at low noise, the Gibbs target concentrates exponentially on the feasible set.

2.5 Cognitive DCR Systems

Definition 2.11 (Total Correlation)**.** For a probability distribution μ on $\Omega = \prod_{s\in S} D_s$, the *total correlation* (multi-information) is

$$\mathrm{Coh}(\mu) = D_{\mathrm{KL}}\left(\mu \,\middle\|\, \bigotimes_{s\in S} \mu_s\right), \tag{2.7}$$

where μ_s is the marginal of μ on D_s. We have $\mathrm{Coh}(\mu) \geq 0$ with equality if and only if μ is a product of its marginals.

Definition 2.12 (Edge-Sum Coherence)**.** The *mutual information* between components s, s' under μ is

$$I_\mu(X_s;\, X_{s'}) = \sum_{x_s \in D_s} \sum_{x_{s'} \in D_{s'}} \mu_{s,s'}(x_s, x_{s'}) \ln \frac{\mu_{s,s'}(x_s, x_{s'})}{\mu_s(x_s)\, \mu_{s'}(x_{s'})}, \tag{2.8}$$

where $\mu_{s,s'}$ is the pairwise marginal. The *edge-sum coherence* is

$$\mathrm{Coh}_E(\mu) = \sum_{\{s,s'\}\in E} I_\mu(X_s;\, X_{s'}). \tag{2.9}$$

Coh_E serves as a *local witness*: $\mathrm{Coh}_E(\mu) > 0$ implies $\mathrm{Coh}(\mu) > 0$, but the converse can fail for purely synergistic distributions (e.g., X, Y independent fair bits with $Z = X \oplus Y$: $\mathrm{Coh} > 0$ but all pairwise $\mathrm{MI} = 0$).

Definition 2.13 (Cognitive DCR System)**.** A DCR system $(\mathcal{N}, \varepsilon, K)$ is *cognitive* if the explore–resolve–stabilize triad produces nontrivial coordination between components at stationarity. In this paper we operationalize coordination using the witness $\mathrm{Coh}(\pi_\varepsilon) > 0$.

The following proposition shows that this coordination arises whenever the constraint network contains at least one genuinely interacting edge.

Proposition 2.14 (Coherence from non-factorizable interactions)**.** *Let $\mathcal{N}$ be a constraint network with Gibbs target π_ε (which satisfies $\pi_\varepsilon(\omega) > 0$ for all ω).*

(a) ***Total correlation (unconditional).*** *If for some edge $\{s, s'\} \in E$ the edge cost $v_e(\omega_s, \omega_{s'})$ is not of the additive form $a_s(\omega_s) + a_{s'}(\omega_{s'})$, then* $\mathrm{Coh}(\pi_\varepsilon) > 0$*, and the system is cognitive.*

(b) ***Edge-sum witness (non-cancellation).*** *If additionally the potentials are not tuned to produce exact marginal independence between adjacent nodes (a nongeneric algebraic constraint on the parameterization), then* $\mathrm{Coh}_E(\pi_\varepsilon) > 0$.

Proof. (a) Suppose for contradiction that π_ε is a product measure: $\pi_\varepsilon(\omega) = \prod_{s\in S} \pi_{\varepsilon,s}(\omega_s)$. But $\pi_\varepsilon(\omega) \propto \prod_s \exp(-h_s(\omega_s)/\varepsilon) \prod_{e=\{s,s'\}} \exp(-v_e(\omega_s, \omega_{s'})/\varepsilon)$. Absorbing the unary factors, we obtain a factorization of the edge part:

$$\prod_{e=\{s,s'\}} \exp(-v_e(\omega_s, \omega_{s'})/\varepsilon) \;\propto\; \prod_s \alpha_s(\omega_s),$$

for some functions α_s. Now fix all coordinates except ω_s and $\omega_{s'}$; since every other factor is constant or unary under this restriction, every edge factor $\exp(-v_{e'}/\varepsilon)$ with $e' \neq \{s, s'\}$ that is incident to s or s' has its other endpoint fixed, so it collapses to a unary function of ω_s or $\omega_{s'}$ and can be absorbed into α_s or $\alpha_{s'}$. The remaining dependence on $(\omega_s, \omega_{s'})$

must therefore come entirely from $\exp(-v_{\{s,s'\}}/\varepsilon)$, which would then have to factorize as $f_s(\omega_s)\, f_{s'}(\omega_{s'})$—contradicting the hypothesis. Hence π_ε is not a product of its marginals, so $\mathrm{Coh}(\pi_\varepsilon) > 0$.

(b) Part (a) gives Coh > 0 but does not guarantee pairwise *marginal* dependence: algebraic cancellations among potentials along competing paths can produce $I_{\pi_\varepsilon}(X_s; X_{s'}) = 0$ even when v_e is non-factorizable. Such cancellations require exact polynomial relations among the potential parameters (the vanishing of certain marginal-independence polynomials), which define a lower-dimensional subset of parameter space. Excluding them yields $I_{\pi_\varepsilon}(X_s; X_{s'}) > 0$ and hence $\mathrm{Coh}_E(\pi_\varepsilon) > 0$. □

Remark 2.15 (Genericity of cognition). The non-cancellation condition in (b) is nongeneric: exact marginal independence between adjacent nodes requires the potential parameters to satisfy polynomial equalities, which define a lower-dimensional subset of the parameter space (compare the analogous discussion for directed models in Spirtes et al. [2001]). Thus $\mathrm{Coh}_E > 0$ (the local witness) holds for almost every choice of nontrivial edge costs. Part (a) gives an even stronger conclusion: Coh > 0 (cognition) holds *unconditionally* for every non-factorizable edge—no genericity assumption needed. The only non-cognitive constraint networks are those with no edges ($E = \emptyset$) or with all-factorizable edge costs (effectively no interaction).

Remark 2.16 (Coherence as a function of ε). Both $\mathrm{Coh}(\pi_\varepsilon)$ and $\mathrm{Coh}_E(\pi_\varepsilon)$ are non-monotone functions of the noise level. As $\varepsilon \to \infty$, $\pi_\varepsilon \to \mathrm{Uniform}(\Omega)$ and Coh $\to 0$ (independence). As $\varepsilon \to 0$, π_ε concentrates on $\mathcal{F}$; if $|\mathcal{F}| = 1$, the limit is a point mass (a product of point masses), so Coh $\to 0$ again. Thus coherence is maximized at some intermediate $\varepsilon^\star$—the optimal explore–exploit balance. This mathematical fact underlies the prediction of a universal inverted-U relationship in Section 6.

2.6 What DCR Excludes

A definition of cognition is useful only if it excludes something. DCR excludes:

1. **Unconstrained systems.** If $E = \emptyset$ and all h_s are constant, then $\pi_\varepsilon = \mathrm{Uniform}(\Omega)$ and Coh $= 0$. *Brownian motion in free space is not cognitive.*
2. **Non-distributed systems.** A single component ($|S| = 1$) has no edges, the Gibbs measure factors trivially, and Coh $= 0$. *A single classical particle is not cognitive.*
3. **Non-interacting systems.** If all edge costs factorize ($v_e(\omega_s, \omega_{s'}) = a_s(\omega_s) + a_{s'}(\omega_{s'})$ for all e), then π_ε is a product measure and Coh $= 0$. *Independent subsystems are not cognitive.*

Cognition in the DCR sense requires nontrivial, distributed constraint structure producing statistical dependence at stationarity. The boundary is continuous, not sharp: a system with weak constraints has small Coh; DCR provides a graded measure through the coherence magnitude and the cognitive depth δ (Definition 3.2).

2.7 Worked Example: Antiferromagnetic Ising Model

Example 2.17 (Antiferromagnetic Ising lattice)**.** Consider N spins on a bipartite graph $G = (S, E)$ (e.g., a square lattice) with $D_s = \{-1, +1\}$ for each $s \in S$. This is a constraint network with $\Omega = \{-1, +1\}^N$.

Constraints. $v_e(\omega_s, \omega_{s'}) = \frac{1}{2}(1 + \omega_s\, \omega_{s'})$: zero for opposite spins (satisfied), one for aligned spins (violated). The feasible set $\mathcal{F}$ consists of the proper 2-colorings of G.

Gibbs target. $\pi_\varepsilon(\omega) \propto \exp\Big(-V(\omega)/\varepsilon\Big)$, where $V(\omega) = \sum_e v_e$ counts the number of frustrated (same-spin) edges. This concentrates on 2-colorings as $\varepsilon \to 0$.

DCR dynamics. Glauber dynamics: select s uniformly at random; resample ω_s from the conditional $\pi_\varepsilon(\omega_s \mid \omega_{N_G(s)})$, which depends only on the neighbor spins through the edge costs. This kernel is local, irreducible, aperiodic, and has π_ε stationary.

Coherence. The factor $\exp(-v_e/\varepsilon) = \exp\Big(-(1+\omega_s\omega_{s'})/(2\varepsilon)\Big)$ does not factorize as $f_s(\omega_s)\, f_{s'}(\omega_{s'})$. Generically (i.e. absent fine-tuned parameter cancellations), $I_{\pi_\varepsilon}(X_s; X_{s'}) > 0$ on edges, so $\mathrm{Coh}_E(\pi_\varepsilon) > 0$ by Proposition 2.14(b). The system is cognitive.

Summary. By Theorem 2.9 and Proposition 2.10, Glauber dynamics converges geometrically to π_ε, which concentrates on the antiferromagnetic ground states as $\varepsilon \to 0$. The coherence $\mathrm{Coh}_E(\pi_\varepsilon) > 0$ witnesses genuine coordination between neighboring spins. This is a cognitive DCR system of depth 1.

3 Scale-Freeness and the Combination Problem

A central mathematical theme is that the DCR form is stable under coarse-graining: exactly under lumpability, and approximately under timescale separation.

3.1 Closure Under Coarse-Graining

Given a surjection $g : \Omega \twoheadrightarrow \tilde{\Omega}$ (grouping micro-states into macro-states), the natural *macro-cost* is

$$\tilde{V}(\tilde{x}) \;=\; -\varepsilon \ln \sum_{\omega \in g^{-1}(\tilde{x})} \exp\left(-\frac{V(\omega)}{\varepsilon}\right). \tag{3.1}$$

This is the *free energy of the block* $\tilde{x}$: it integrates out micro-degrees of freedom, balancing internal energy against multiplicity (entropy). As $\varepsilon \to 0$, $\tilde{V}(\tilde{x}) \to \min\{V(\omega) : g(\omega) = \tilde{x}\}$; for positive ε, entropic contributions soften this. The construction underlies block-spin renormalization, Mori–Zwanzig averaging, and Markov state modeling.

For finite state spaces, the coarse-grained system inherits the full DCR structure—ergodicity, Gibbs stationarity with respect to $\tilde{V}$, and (under mild conditions) coherence—whenever the partition is *lumpable* in the sense of Kemeny–Snell [Kemeny and Snell, 1960]. The formal statement and proof appear in Appendix A (Theorem A.2).

For general (possibly continuous or non-lumpable) systems, approximate closure holds under timescale separation.

Proposition 3.1 (Approximate Closure (informal))**.** *Suppose the micro-dynamics exhibits timescale separation (Definition A.3 in Appendix A): intra-group degrees of freedom mix to conditional equilibrium before inter-group updates occur, and the expected cross-group cost depends on the macro-state alone (macro-sufficiency). Then the coarse-grained macro-process is approximately Markov and admits an approximately Gibbs stationary distribution with macro-cost $\tilde{V}$, with approximation error*

$$\sup_{\tilde{x}} \Big\| P\Big(g(X_{t+1}) \in \cdot \mid g(X_t) = \tilde{x}\Big) - \tilde{K}(\tilde{x}, \cdot) \Big\|_{\mathrm{TV}} \;\le\; \epsilon(\eta\, \tau_{\mathrm{fast}}), \qquad \epsilon(u) \to 0 \text{ as } u \to 0.$$

Proof sketch. Under timescale separation, each group equilibrates before the next inter-group update. By macro-sufficiency, the one-step transition depends only on the current macro-state (approximate Markovianity). Macro-Gibbs stationarity follows from the coarse-grained free energy construction; ergodicity transfers from the micro-kernel via the surjection. □

Note that $\tilde{V}$ is not in general a sum of pairwise costs: coarse-graining can generate effective higher-order interactions (the standard observation in renormalization group theory). Locality at the macro-scale means each macro-component's update depends on the macro-factors in which it participates, not on the full $\tilde{V}$—the natural hypergraph generalization of pairwise locality. By the data-processing inequality, coarse-graining can only decrease mutual information; coherence is preserved when the projection retains sufficient structure (see Theorem A.2(iii) for a clean sufficient condition).

3.2 Conditional Composition and the Combination Problem

The combination problem in panpsychism asks: if fundamental entities have experience, how do micro-experiences combine into the unified macro-experience of a human mind? The closure results provide a conditional answer: when the micro-dynamics admits a lumpable partition (or timescale separation), the Gibbs–variational structure is inherited at the macro-scale, and coherence transfers if the projection preserves at least one edge's mutual information.

Under these conditions, there is no separate substance requiring combination—there is the DCR process, recurring at each scale via coarse-graining. One level's resolved constraints become the next level's macro-cost, and the closure theorems guarantee that the resulting macro-system is again DCR. What we call "unified experience" at the human scale is the coherent attractor of a DCR system whose components are themselves coarse-grained DCR systems, recursively.

3.3 Cognitive Depth

Definition 3.2 (Cognitive Depth)**.** The *cognitive depth* of a DCR system $\mathcal{C}$ is the maximal number of nested coarse-graining levels at which the DCR triad is simultaneously active:

$$\delta(\mathcal{C}) = \max\{k \mid \Gamma_{g_k} \circ \cdots \circ \Gamma_{g_1}(\mathcal{C}) \text{ is cognitive}\}, \tag{3.2}$$

where Γ_g denotes the coarse-graining operation induced by g (pushforward of π_ε and quotient kernel $\tilde{K}$), and the maximum is over all hierarchical sequences of lumpable partitions $(g_1, \ldots, g_k)$ satisfying the conditions of Theorem A.2 at each level, with the *strict reduction* requirement $|\tilde{\Omega}_{i+1}| < |\tilde{\Omega}_i|$ (at least one non-trivial grouping at each level). Since $|\Omega|$ is finite, $\delta(\mathcal{C}) < \infty$.

Cognitive depth provides a non-anthropocentric, graded ordering of intelligence without requiring a binary threshold. Illustrative ordinal estimates: a crystal has depth ∼1; a bacterium ∼3–4 (molecular, metabolic, behavioral); a human brain ∼6–8 (ionic, synaptic, columnar, areal, network, behavioral, social). These should not be read as precise measurements.

Remark 3.3 (Computability). Computing $\delta(\mathcal{C})$ exactly requires optimizing over all hierarchical partition sequences—a combinatorially intractable problem. In practice, δ serves as a coarse ordinal ranking: distinguishing depth 2 from depth 6 is meaningful; distinguishing depth 6 from depth 7 requires detailed empirical verification of the DCR triad at each level.

4 Physical Instantiations of DCR

We now exhibit structural witnesses that diverse systems—from physics to biology to economics—instantiate the DCR triad. For each, we identify: (i) components and degrees of freedom, (ii) constraints, (iii) exploration mechanism, (iv) resolution mechanism, and (v) coherent attractor. These are mappings from established dynamical descriptions into the DCR template; they are not claimed to be novel derivations of the underlying physics.

4.1 Thermodynamic Self-Organization

Example 4.1 (Bénard convection)**.** Consider Rayleigh–Bénard convection: a fluid layer between horizontal plates, heated from below (T_H) and cooled from above (T_C), in the Boussinesq approximation [Chandrasekhar, 1961]. We discretize the fluid domain on a lattice with N parcels.

- **Components:** Fluid parcels at lattice sites.
- **Degrees of freedom:** Local velocity $\mathbf{v}_s$ and temperature T_s at each site.
- **Constraints:** Discretized Boussinesq conservation equations at each interface $\{s, s'\}$: incompressibility ($\nabla \cdot \mathbf{v} = 0$), momentum balance, and energy balance. The constraint cost v_e measures the squared residual of these equations at the interface.
- **Exploration:** Thermal fluctuations ($\sigma^2 \propto k_B T_{\mathrm{ref}}$) explore the space of flow configurations.
- **Resolution:** Each parcel adjusts its velocity and temperature based on its neighbors' states—a local Gauss–Seidel update of the discretized Boussinesq equations.
- **Coherent attractor:** Convection rolls [Cross and Hohenberg, 1993]—spatially ordered flow patterns that persist against thermal noise for Rayleigh number $\mathrm{Ra} > \mathrm{Ra}_c$.

The total constraint cost $V(\omega) = \sum_e v_e$ measures aggregate departure from the steady-state Boussinesq equations. The Gibbs target π_ε concentrates on the convection roll configurations as $\varepsilon \to 0$ (with ε set by the fluctuation amplitude). The convection pattern is maintained by nonequilibrium driving (the temperature gradient); removing the drive ($T_H = T_C$) collapses V to zero, eliminating the nontrivial feasible set.

This is a cognitive DCR system of depth 1 (the parcels themselves are not DCR systems at a finer scale in this coarse description).

4.2 Quantum Mechanics

We present two structural witnesses, differing in interpretive commitment but sharing the DCR structure.

Transactional interpretation (TIQM). *[Cramer, 1986], building on the Wheeler–Feynman absorber theory [Wheeler and Feynman, 1945].*

- ***Components:*** *Emitter and absorber sites—the vertices of the spacetime interaction graph.*
- ***Degrees of freedom:*** *Quantum states (energy, momentum, polarization, spin) at each site.*
- ***Constraints:*** *Conservation laws at each interaction vertex.*
- ***Exploration:*** *The* offer wave *propagates from the emitter to all potential absorbers, exploring every possible transaction partner simultaneously. In the Feynman path integral formulation [Feynman, 1948], this is the sum over all paths weighted by* $e^{iS/\hbar}$.

- ***Resolution:*** *The* confirmation wave *propagates from each potential absorber back to the emitter. The Wheeler–Feynman handshake is distributed constraint resolution: each absorber independently evaluates the offer against its local constraints.*
- ***Coherent attractor:*** *The* completed transaction*—a definite, irreversible transfer of conserved quantities.*

Decoherence and einselection. *[Zurek, 2003]. An interpretation-neutral witness: (i) unitary evolution spreads the system–environment state across the Hilbert space (exploration); (ii) environment-induced decoherence suppresses off-diagonal coherences—a local, distributed process in which each environmental degree of freedom independently constrains the system's phase relations (resolution); (iii) einselected pointer states emerge as the unique basis robust to ongoing decoherence (stabilization).*

DCR does not endorse any particular interpretation of quantum mechanics. These are structural mappings exhibiting the DCR triad, not solutions to the measurement problem.

Remark 4.2 (Constraint-mediated selection). The recurring pattern across scales is: (i) *variation* (the exploration kernel generates candidates); (ii) *constraint filtering* (the resolution dynamics retains configurations reducing violation); (iii) *retention* (surviving configurations accumulate near the attractor). We call this *constraint-mediated selection*: Darwinian natural selection is the biological instance; quantum "collapse" is the physical instance; simulated annealing is the computational instance. The shared structure is the DCR triad, differing only in substrate and timescale.

Remark 4.3 (Discrete ontic models). Powers et al. [2024] construct a model from finite binary sequences that reproduces canonical quantum probability distributions, with small deviations at finite n that shrink as n increases. For finite n, the system is a finite constraint network: the admissible orderings are the exploration space, contextual compatibility conditions are the constraints, and the observed probability distribution is the stabilized pattern. This supports DCR's assumption that the continuum structure of standard physics may be an idealization of a fundamentally discrete constraint resolution process.

4.3 Biological Adaptation

- ***Components:*** *Organisms in a population.*
- ***Degrees of freedom:*** *Genotype/phenotype space.*
- ***Exploration:*** *Mutation, recombination, developmental noise.*
- ***Constraints:*** *Environmental fitness landscape, inter-organism competition, predator–prey relations [Kauffman, 1993].*
- ***Resolution:*** *Natural selection propagates constraints locally (each organism's survival depends on its local fitness, not a global optimization). This is inherently distributed.*
- ***Coherent attractor:*** *Adapted species occupying fitness peaks.*

The constraint cost V measures aggregate fitness deficit across the population. The Gibbs target π_ε (with ε set by mutation rate and environmental stochasticity) describes the distribution of population states.

Biological evolution is cognitive with depth ≥ 2: organisms themselves are cognitive systems (metabolic constraint resolution), and population-level dynamics adds a second layer.

4.4 Neural Cognition

- ***Components:*** *Neurons or neural populations.*
- ***Degrees of freedom:*** *Firing rates, membrane potentials, synaptic states.*
- ***Exploration:*** *Spontaneous activity, noise, stochastic neurotransmitter release.*
- ***Constraints:*** *Synaptic weights, lateral inhibition, top-down priors encoded in connectivity [Seth, 2021].*
- ***Resolution:*** *Local integrate-and-fire dynamics; each neuron resolves its inputs against its threshold. Constraint propagation is distributed across the network.*
- ***Coherent attractor:*** *Perceptual representations, motor plans, decisions—coherent patterns of neural activity.*

Neural cognition achieves high depth through nested organization: ionic channels, synapses, columns, areas, networks, behavior, social interaction—each level a coarse-graining of the one below.

4.5 Market Dynamics

- ***Components:*** *Market participants (traders, firms, consumers).*
- ***Degrees of freedom:*** *Prices, quantities, strategies, portfolio allocations.*
- ***Constraints:*** *Budget constraints, supply–demand matching, regulatory limits, no-arbitrage conditions.*
- ***Exploration:*** *Speculation, innovation, market-making, information arrival, random order flow.*
- ***Resolution:*** *Price adjustment via local transactions; arbitrageurs eliminate mis-pricings between related assets; each participant optimizes locally given observable prices.*
- ***Coherent attractor:*** *Market equilibrium: discovered prices that reflect constraint satisfaction across the network of interacting agents.*

The constraint cost V measures aggregate disequilibrium: excess demand, arbitrage spreads, budget violations. The Gibbs target π_ε describes the distribution of market states at a given volatility level ε (set by information uncertainty and speculative activity), concentrating on equilibrium prices as $\varepsilon \to 0$.

Market dynamics is cognitive with depth ≥ 2: individual firms resolve internal constraints (production optimization), and the market resolves inter-firm constraints (supply–demand matching, price discovery). The 2008 financial crisis can be read as a failure of DCR at depth 2: individual firms' constraint resolution (profit maximization) became decoupled from the market-level constraints (aggregate risk limits), producing an incoherent attractor at the macro-scale.

5 Relationship to Existing Frameworks

5.1 Free Energy Principle as a Special Case

The central observation is that DCR's variational free energy and FEP's variational free energy are the same mathematical object *under appropriate specialization of the constraint topology.*

Proposition 5.1 (FEP Recovery)**.** *The Free Energy Principle is a special case of DCR obtained when:*

1. *The constraint network has* Markov-blanket topology*: components are partitioned into internal (μ), blanket (b), and external (η) states, with edges only between adjacent layers (internal–blanket and blanket–external).*
2. *The constraint costs encode a* generative model*: the node costs on internal states are the negative log-prior, $h_s(\mu_s) = -\ln p(\mu_s)$; the edge costs on blanket–internal edges encode the likelihood, $v_{\{s,s'\}}(\mu_s, b_{s'}) = -\ln p(\tilde{s}_{s'} \mid \mu_s)$.*
3. *The noise level is $\varepsilon = 1$.*

Under these specializations, DCR's variational free energy reduces to FEP's variational free energy:

$$F_1(\rho) = F_{\mathrm{FEP}}(\rho) + \mathit{const}, \tag{5.1}$$

and minimizing F_1 over ρ is equivalent to minimizing F_{FEP} over the recognition density q.

Proof. Fix the blanket observation $\tilde{s}$ (treated as a parameter, not a random variable). Under the Markov-blanket constraint topology with generative-model costs, and assuming the generative model factorizes over blanket components conditioned on internal states ($p(\tilde{s} \mid \mu) = \prod_{s'} p(\tilde{s}_{s'} \mid \mu_{s(s')})$, standard in mean-field FEP formulations), the total constraint cost over internal states is

$$V_{\tilde{s}}(\mu) = \underbrace{-\sum_s \ln p(\mu_s)}_{\text{node costs (prior)}} \underbrace{- \sum_{\{s,s'\}} \ln p(\tilde{s}_{s'} \mid \mu_s)}_{\text{edge costs (likelihood)}} = -\ln p(\mu) - \ln p(\tilde{s} \mid \mu) = -\ln p(\tilde{s}, \mu).$$

The Gibbs target at $\varepsilon = 1$ is $\pi_1(\mu) \propto \exp(-V_{\tilde{s}}(\mu)) = p(\tilde{s}, \mu)$; normalizing over μ yields the posterior $p(\mu \mid \tilde{s})$.

DCR's variational free energy:

$$\begin{aligned} F_1(\rho) &= D_{\mathrm{KL}}(\rho \,\|\, \pi_1) = D_{\mathrm{KL}}(q(\mu) \,\|\, p(\mu \mid \tilde{s})) \\ &= \mathbb{E}_q[-\ln p(\tilde{s}, \mu)] + \mathbb{E}_q[\ln q(\mu)] + \ln p(\tilde{s}) \\ &= F_{\mathrm{FEP}} + \ln p(\tilde{s}). \end{aligned} \tag{5.2}$$

Since $\ln p(\tilde{s})$ is constant with respect to q, minimizing F_1 over ρ is equivalent to minimizing F_{FEP} over q. The resolution dynamics (gradient descent on F) is FEP's recognition dynamics; the exploration kernel corresponds to active inference (epistemic actions that sample the environment). □

DCR is strictly more general than FEP in two ways:
1. *It does not require Markov-blanket topology—any constraint graph G is allowed.*
2. *It does not require the constraints to be expressible as a generative model. Physical constraint resolution (decoherence, convection) need not involve "inference" in any Bayesian sense.*

The near-tautological character of this bridge is a feature, not a bug: it shows that FEP and DCR are not competing frameworks but the same variational principle applied to different constraint topologies. FEP is DCR with Markov-blanket structure; DCR is FEP generalized beyond Markov blankets.

5.2 Relationship to Integrated Information Theory

Proposition 5.2 (DCR coherence as necessary condition for $\Phi > 0$)**.** *Under IIT 2.0 [Tononi, 2004], let the constraint network encode the transition probability matrix (TPM), and let $\mu_x = p(X_{t+1} \mid X_t = x)$ be the one-step conditional distribution over successor states. If $\Phi(x) > 0$, then:*

(a) $\mathrm{Coh}(\mu_x) > 0$ *(positive total correlation).*

(b) If additionally no measure-zero parameter cancellations suppress pairwise marginal dependence (a standard algebraic non-genericity; compare Spirtes et al. [2001] for an analogous notion in directed models), then $\mathrm{Coh}_E(\mu_x) > 0$.

Proof. IIT 2.0 defines $\Phi(x) = \min_{\text{bipartition } \pi} I_{\mu_x}(X_A; X_B)$ where A, B are the parts of π. Since $I_{\mu_x}(X_A; X_B) \leq \mathrm{Coh}(\mu_x)$ for every bipartition (the mutual information between parts is bounded by the total correlation), $\Phi > 0$ requires $\mathrm{Coh}(\mu_x) > 0$.

The step from $\mathrm{Coh}(\mu_x) > 0$ to $\mathrm{Coh}_E(\mu_x) > 0$ requires excluding purely synergistic distributions—a measure-zero condition in the parameter space. Counterexample: let X, Y be independent fair bits with $Z = X \oplus Y$; then $\mathrm{Coh}(X, Y, Z) > 0$ but $I(X; Y) = I(X; Z) = I(Y; Z) = 0$, so $\mathrm{Coh}_E = 0$ on any pairwise graph. In typical parameterizations this is nongeneric (a measure-zero condition). □

DCR extends IIT in two directions: (1) it provides a process-level *account of how coherence arises through exploration and resolution, rather than merely measuring it at a single time step; and (2) it applies where* Φ *is intractable (*$O(2^n)$ *computation) but the DCR triad is empirically observable.*

The connection between the two frameworks is mediated by the constraint graph: IIT's "minimum information partition" corresponds to cutting the highest-weight edges in the DCR constraint graph, and the integrated information Φ *measures how much coherence survives the worst-case cut.*

Remark 5.3 (IIT versions). The relationship above targets IIT 2.0 (KL-based Φ). IIT 3.0 [Tononi et al., 2016] replaces KL divergence with the earth mover's distance and defines Φ over cause–effect structures; IIT 4.0 introduces further dynamical aspects. The necessary-condition relationship $\mathrm{Coh} > 0$ for $\Phi > 0$ holds conceptually across versions, but the formal apparatus diverges; a full treatment for IIT 3.0/4.0 would require additional machinery not developed here.

6 Predictions and Falsifiability

A framework that explains everything predicts nothing. DCR makes the following falsifiable claims:

1. ***Coherence–exploration tradeoff.*** *By Remark 2.16, the coherence* $\mathrm{Coh}(\pi_\varepsilon)$ *is a non-monotone function of the noise level* ε*, vanishing at both extremes (*$\varepsilon \to 0$ *and* $\varepsilon \to \infty$*) and maximized at an intermediate* $\varepsilon^\star$*. This predicts a universal inverted-U relationship between exploration rate and cognitive performance, testable in:*
   - *Neural systems: dopamine modulation of noise vs. decision accuracy (cf. stochastic resonance [Gammaitoni et al., 1998]).*
   - *Evolutionary simulations: mutation rate vs. fitness.*
   - *Optimization algorithms: temperature vs. solution quality.*
   - *Markets: volatility vs. price discovery efficiency.*
2. ***Depth predicts adaptability.*** *Systems with greater cognitive depth* δ *should exhibit greater adaptability to novel environments, because deeper nesting provides more levels at which the explore–resolve cycle can operate. Testable: compare adaptability of single-celled vs. multicellular organisms, shallow vs. deep neural networks, flat vs. hierarchical organizations.*

3. ***Critical constraint density.*** *There exists a critical density of constraints* $|E|/|S|$ *below which the system cannot sustain coherent attractors and above which the system becomes rigid. This parallels the satisfiability phase transition in random constraint satisfaction problems [Mézard et al., 2002]. DCR predicts that the critical regime maximizes both* $\mathrm{Coh}(\pi_\varepsilon)$ *and the deepest cognitive nesting* δ*. Testable in constraint satisfaction problems and neural network models with varying connectivity.*
4. ***Scaling laws under coarse-graining.*** *If DCR is correct, then empirically measured coherence, exploration rates, and convergence timescales should obey scaling laws across levels of description of the same system (e.g., single-neuron vs. population vs. whole-brain dynamics). The ratio of exploration timescale to resolution timescale should be approximately preserved under valid coarse-grainings.*

7 Discussion

7.1 The Cosmos as Cognitive

If DCR is correct, then cognition is not an emergent property of brains—it is the variational process by which any system of interacting components selects coherent behavior from the space of possibilities. Thermal fluctuations explore the space of flow configurations; the Boussinesq equations resolve constraints locally; convection rolls stabilize. Mutation explores genotype space; natural selection resolves fitness constraints; adapted species stabilize. Speculators explore price space; arbitrage resolves mispricings; market equilibrium stabilizes. The same variational principle, recurring at every scale.

Powers et al. [2024] provides independent support: if quantum probabilities arise from counting admissible configurations of binary sequences, then what physics calls a "quantum state" is already a coarse-grained summary of a discrete constraint resolution process. Vanchurin [2020] independently derives effective physical laws from a neural-network substrate implementing the same explore–resolve–stabilize pattern.

This is not panpsychism in the traditional sense. We do not claim that a convection cell "has experiences." We claim that the process by which it forms is structurally identical to the process underlying neural cognition—formally, the same variational principle on different constraint topologies. Whether one wishes to call this "experience" at every scale is a separate philosophical question that DCR does not adjudicate.

7.2 Relationship to Process Philosophy

DCR exhibits structural resonances—not evidential dependencies—with Whitehead's process philosophy [Whitehead, 1929], which held that reality consists of "actual occasions" that "prehend" their environment and "concresce" into definite outcomes. Prehension maps onto exploration of degrees of freedom under constraints; concrescence maps onto resolution into a coherent pattern. We note these parallels as interpretive context, not as independent support for DCR's formal claims.

7.3 Relation to Neural-Network Universe Proposals

Vanchurin's "world as a neural network" program [Vanchurin, 2020] is the closest existing proposal to DCR in ambition. The key structural overlap is the two-tier dynamics (trainable weights on a slow timescale, hidden neurons on a fast timescale), which maps onto DCR's *coarse-graining hierarchy. Vanchurin's "second law of learning" (entropy production from stochasticity vs. entropy destruction from learning) is a special case of DCR's energy–entropy tradeoff in the variational free energy. DCR provides the substrate-agnostic process-level explanation: the neural-network universe produces stable macroscopic laws because it implements distributed constraint resolution.*

7.4 Implications for Artificial Intelligence

Current AI systems implement the DCR triad in restricted form: stochastic sampling (exploration), gradient descent or constraint propagation (resolution), convergence to low-loss configurations (stabilization). DCR predicts that the "intelligence" of these systems is bounded by their cognitive depth: the number of nested levels at which the explore–resolve–stabilize cycle operates simultaneously. This suggests that advances in AI may come not from scaling individual layers but from increasing organizational depth.

Self-play as engineered DCR. *Self-play systems provide an engineered instance: parallel rollouts implement exploration; selection and credit-assignment propagate constraints; training converges to stable policy attractors. The apparent "retroactive" assignment of credit to earlier moves mirrors the retrocausal structure of TIQM: later outcomes determine which earlier degrees of freedom were effectively feasible.*

7.5 Limitations and Open Problems

1. ***Finite state spaces.*** *The main results assume finite* Ω*. Extension to continuous state spaces requires measure-theoretic Gibbs measures and Harris-recurrence machinery (see Appendix B). Many physical systems (e.g., unbounded velocities, Gaussian fields) naturally live on non-compact spaces.*
2. ***Lumpability.*** *Exact closure requires Kemeny–Snell lumpability, which is a strong condition. Most physical coarse-grainings are only approximately lumpable; approximate closure under timescale separation (Proposition 3.1) addresses this but introduces additional assumptions.*
3. ***Quantitative predictions.*** *While DCR predicts qualitative relationships (inverted-U, depth–adaptability, critical density), deriving precise quantitative predictions requires specifying the constraint structure of particular systems—a substantial empirical program.*
4. ***The goal problem.*** *DCR defines goals as attractors, but any attractor counts, including* terminal attractors *that extinguish DCR capacity (e.g., a dead organism is a stable configuration, but exploration and resolution have ceased). A natural refinement reserves "goal" for attractors that preserve ongoing DCR activity—connecting to autopoiesis—but formalizing this distinction remains open.*
5. ***Consciousness.*** *DCR is a theory of cognition, not of consciousness. It explains the process by which systems explore, resolve, and stabilize, but does not address the "hard problem" [Chalmers, 1995] of why any of this is accompanied by subjective experience.*
6. ***Nonequilibrium steady states.*** *The present formulation assumes the stationary distribution is a Gibbs measure for the constraint cost* V*. For strongly nonequilibrium systems (e.g., driven by external currents), the actual stationary distribution may differ from* π_ε*; the Gibbs measure then serves as a reference against which the*

dynamics is compared. Extending the variational principle to nonequilibrium steady-state distributions (e.g., via the Donsker–Varadhan large-deviation rate function) is an important direction for future work.

8 Conclusion

We have presented the Distributed Constraint Resolution (DCR) framework: a formal, scale-free characterization of cognition as the process by which components explore degrees of freedom and converge through distributed constraint resolution into coherent, goal-stabilizing patterns. The mathematical core—constraint networks, Gibbs target measures, and variational free energy—provides a unified language for describing this process across scales.

We have shown that:

1. *The framework is built on well-understood mathematical objects: factor graphs, Gibbs measures, and Markov chain convergence theory, yielding clean convergence and concentration results (Section 2).*
2. *The DCR form is stable under coarse-graining—exact under lumpability, approximate under timescale separation—with the macro-cost arising as a coarse-grained free energy, providing a compositional account of cognitive depth (Section 3).*
3. *Diverse systems—from convection to quantum mechanics to biological evolution to neural cognition to market dynamics—instantiate the DCR triad (Section 4).*
4. *The Free Energy Principle is recovered as a special case via a near-tautological identity of variational free energies, and DCR coherence is a necessary condition for IIT's* $\Phi > 0$ *(Section 5).*
5. *The framework makes falsifiable predictions about exploration–exploitation tradeoffs, cognitive depth, and critical constraint densities (Section 6).*

The variational perspective reveals the common structure: in every case, the system selects coherent behavior from a latent space of possibilities by minimizing a free energy that balances constraint satisfaction against exploration. This is the minimal form of cognition—and if DCR is correct, it is what physics does at every scale.

A Exact Closure and Timescale Separation

This appendix contains the formal statements omitted from Section 3.1: the lumpable partition definition, the exact closure theorem with proof, and the timescale separation definition referenced in Proposition 3.1.

Definition A.1 (Lumpable Partition)**.** Let K be a Markov kernel on finite Ω. A surjection $g : \Omega \twoheadrightarrow \tilde{\Omega}$ is *lumpable* (in the sense of Kemeny–Snell) if, for every $\tilde{y} \in \tilde{\Omega}$, the transition probability $K(\omega, g^{-1}(\tilde{y}))$ depends on ω only through $g(\omega)$.

Theorem A.2 (Exact Closure Under Lumpability)**.** *Let* $(\mathcal{N}, \varepsilon, K)$ *satisfy DCR dynamics on finite* Ω*, and let* $g : \Omega \twoheadrightarrow \tilde{\Omega}$ *be lumpable for* K*. Define the quotient kernel*

$$\tilde{K}(\tilde{x}, \tilde{y}) \;=\; K(\omega, g^{-1}(\tilde{y})) \qquad \text{for any } \omega \in g^{-1}(\tilde{x}),$$

and the macro-cost (3.1)*. Then* $(\tilde{\Omega}, \tilde{V}, \tilde{K})$ *satisfies DCR dynamics:*

(i) ***Ergodicity:*** $\tilde{K}$ *is irreducible and aperiodic on* $\tilde{\Omega}$*.*

(ii) ***Gibbs stationarity:*** *The macro-Gibbs measure $\tilde{\pi}_\varepsilon(\tilde{x}) = \pi_\varepsilon(g^{-1}(\tilde{x}))$ satisfies $\tilde{\pi}_\varepsilon(\tilde{x}) \propto \exp(-\tilde{V}(\tilde{x})/\varepsilon)$ and is stationary for $\tilde{K}$.*

(iii) ***Coherence (conditional):*** *If g is* componentwise*—built from maps $g_s : D_s \to \tilde{D}_{\sigma(s)}$ for a grouping $\sigma : S \to S'$, so that $g(\omega)_{\sigma(s)} = g_s(\omega_s)$—and for some edge $\{s, s'\} \in E$ with $I_{\pi_\varepsilon}(X_s; X_{s'}) > 0$, the induced map $(g_s, g_{s'}) : D_s \times D_{s'} \to \tilde{D}_{\sigma(s)} \times \tilde{D}_{\sigma(s')}$ is injective, then $\mathrm{Coh}_{E'}(\tilde{\pi}_\varepsilon) > 0$ (where $E' = \{\{\sigma(s), \sigma(s')\} : \{s, s'\} \in E,\ \sigma(s) \neq \sigma(s')\}$), and in particular $\mathrm{Coh}(\tilde{\pi}_\varepsilon) > 0$, so the macro-system is cognitive.*

Proof. By Kemeny–Snell [Kemeny and Snell, 1960], lumpability ensures that $\{g(X_t)\}$ is a Markov chain with kernel $\tilde{K}$.

(i) Ergodicity. Irreducibility: for any $\tilde{x}, \tilde{y}$, there exist $\omega \in g^{-1}(\tilde{x})$, $\omega' \in g^{-1}(\tilde{y})$ with $K^m(\omega, \omega') > 0$ for some m (by irreducibility of K); hence $\tilde{K}^m(\tilde{x}, \tilde{y}) \geq K^m(\omega, g^{-1}(\tilde{y})) > 0$. Aperiodicity: since K is aperiodic, there exists ω with $K(\omega, \omega) > 0$; lumpability gives $\tilde{K}(g(\omega), g(\omega)) > 0$.

(ii) Gibbs stationarity. The pushforward $\tilde{\pi}_\varepsilon = g_\# \pi_\varepsilon$ is stationary for $\tilde{K}$ (standard). The Gibbs form:

$$\tilde{\pi}_\varepsilon(\tilde{x}) = \sum_{\omega \in g^{-1}(\tilde{x})} Z_\varepsilon^{-1} \exp(-V(\omega)/\varepsilon) = Z_\varepsilon^{-1} \exp(-\tilde{V}(\tilde{x})/\varepsilon)$$

by definition of $\tilde{V}$. Since $\sum_{\tilde{x}} \exp(-\tilde{V}(\tilde{x})/\varepsilon) = \sum_{\tilde{x}} \sum_{\omega \in g^{-1}(\tilde{x})} \exp(-V(\omega)/\varepsilon) = Z_\varepsilon$, the normalization is correct.

(iii) Coherence. When $(g_s, g_{s'})$ is injective, the marginal distribution on $(\tilde{X}_{\sigma(s)}, \tilde{X}_{\sigma(s')})$ is a relabeling of $(X_s, X_{s'})$, preserving mutual information exactly: $I_{\tilde{\pi}_\varepsilon}(\tilde{X}_{\sigma(s)}; \tilde{X}_{\sigma(s')}) = I_{\pi_\varepsilon}(X_s; X_{s'}) > 0$. □

Definition A.3 (Timescale Separation)**.** Let $\eta > 0$ be a scale ratio. The micro-dynamics exhibits *η-timescale separation* with respect to a partition $\sigma : S \to S'$ and compression maps $\{g_{s'}\}$ if:

1. **Fast intra-group mixing.** For each macro-component $s' \in S'$, the restricted chain on $\prod_{s \in \sigma^{-1}(s')} D_s$ with boundary frozen mixes to a conditional equilibrium $\nu_{s'}(\cdot \mid b_{s'})$ in time τ_{fast}.
2. **Slow inter-group dynamics.** Inter-group updates occur at rate η relative to intra-group updates.
3. $\eta\, \tau_{\text{fast}} \to 0$: fast degrees of freedom equilibrate before the next inter-group update.
4. **Macro-sufficiency.** The expected cross-group violation after fast equilibration depends on the macro-state alone, not on micro-details within each group.

B General State Spaces

The main text assumes $|S| < \infty$ and $|D_s| < \infty$. For compact Polish state spaces, the Gibbs measure is defined with respect to a product reference measure λ, and the variational characterization extends immediately. For convergence, the role of irreducibility and aperiodicity is played by φ-irreducibility and the Foster–Lyapunov drift condition of Meyn and Tweedie [1993]: under geometric drift toward a petite set, the chain converges geometrically to π_ε in total variation (Theorem 16.0.1, ibid.*). On compact spaces, existence of a stationary distribution follows from Krylov–Bogoliubov (tightness is automatic); the Foster–Lyapunov machinery adds quantitative rates and extends without modification to non-compact spaces. The full development is standard; we omit it to keep the presentation* *focused on the finite case, which suffices for the discrete constraint networks motivating this paper.*

C Additional Remarks and Extensions

Remark C.1 (Ontological interpretation). The claim that the DCR triad *is* cognition—that the cosmos is cognitive at every scale—is a metaphysical identification, not a theorem. The formal machinery is compatible with two readings:

1. *Modelling framework (minimal):* DCR provides a unified dynamical vocabulary applicable across scales.
2. *Ontological identity (optional):* the DCR triad *is* cognition; the cosmos is cognitive at every scale.

All formal results hold under either reading; the choice between them is philosophical, not mathematical.

Remark C.2 (Finiteness of S). The formal development assumes $|S| < \infty$. This suffices for systems with a natural decomposition into discrete components (particles, neurons, organisms) but appears to exclude continuum field theories. The physical examples in Section 4 should be read as finite-element discretizations of the underlying continuum; the formal extension to countable or measure-theoretic component spaces is straightforward but notationally heavier.

Remark C.3 (Engineering witness: consensus as DCR). Distributed consensus protocols provide an engineering witness for DCR. In replicated-state-machine settings, independent agents maintain local copies of a shared state and must reconcile conflicting updates without a central controller. Proposal corresponds to exploration; local validity checks to distributed constraint resolution; and eventual agreement on a single state is a coherent attractor.

Remark C.4 (Annealing and cooling schedules). The fixed-ε analysis can be extended to a cooling schedule $\varepsilon_t \to 0$. Classical simulated annealing results [Hajek, 1988] imply convergence of the occupation measures to a distribution supported on $\arg\min V$ under logarithmic cooling. The DCR framework thus subsumes simulated annealing as a special case where the noise level is systematically reduced to concentrate on the feasible set.

Remark C.5 (Relation to optimization and computer science). The Ising example makes explicit the connection between DCR and classical optimization frameworks. *Simulated annealing* [Hajek, 1988] is DCR with decreasing ε. *Belief propagation* and message-passing algorithms for random constraint satisfaction [Mézard et al., 2002] implement distributed resolution: each variable node updates its marginal based on neighboring constraints. DCR does not claim novelty in these algorithms; it claims that the *same formal triad* appears in physical, biological, and cognitive systems, not only in engineered solvers.

Remark C.6 (Branching exploration variant). In some domains, exploration is naturally modeled as *branching*: from a given configuration, the system generates a family of candidate next states, temporarily maintaining multiple possibilities. Constraint resolution then prunes or reweights these branches. The Markov-kernel formalism applies by viewing a branching–selection step as an induced stochastic kernel obtained by marginalizing over the latent branch variable.

Remark C.7 (Transaction-density gradients). In substrate models where "exploration" is mediated by uncollapsed potential interactions, accelerated motion can induce anisotropies in the accessible interaction field, producing effective resistance-to-acceleration terms. We state as an explicit (and speculative) conjecture: if gravity and inertia admit a reformulation in terms of gradients of a "transactional density" $\rho_\tau(\mathbf{x})$ (completed constraint-satisfying couplings per unit volume per unit time), then the gravitational "constant" is a derived function of the ambient ρ_τ, and inertial mass equals the integrated ρ_τ-anisotropy at fixed acceleration. This is untested; we record it because it illustrates how far the variational reading might extend.

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