Distributed Constraint Resolution as Universal Cognition: A Scale-Free Framework Unifying Physics and Intelligence

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Abstract

We propose a formal framework in which cognition is identified with a scale-free process: the exploration of degrees of freedom followed by convergence through distributed constraint resolution into coherent, goal-stabilizing patterns. *DCR dynamics* is a Markov kernel K_ϵ on the configuration space Ω of a constraint network, satisfying two conditions: uniform minorization (Doeblin exploration) and quantitative drift toward low constraint violation (resolution). We prove geometric convergence to a unique stationary distribution concentrating near the feasible set; a DCR dynamics is *cognitive* when positive edge mutual information emerges at stationarity. For finite state spaces, we prove exact closure under lumpable coarse-graining; for continuous state spaces, approximate closure holds under timescale separation (detailed in appendices). We provide structural mappings suggesting that nonequilibrium self-organization, evolutionary adaptation, and aspects of quantum measurement can be read as structural witnesses of DCR. The stronger ontological identification—the cosmos is cognitive at every scale—is treated as an optional metaphysical interpretation, not a theorem.

Keywords: cognition, intelligence, constraint resolution, free energy principle, integrated information, scale-free, coarse-graining, self-organization

1 Introduction

The search for a general, principled definition of intelligence remains a long-standing open problem across cognitive science, physics, and philosophy of mind. Existing frameworks each illuminate a facet of the problem but fall short of universality:

- The *Free Energy Principle* (FEP) [Friston, 2010, 2019] provides an elegant variational account: any system persisting at nonequilibrium steady state minimizes variational free energy. Yet FEP assumes a Markov blanket separating system from environment and a generative model as primitives [Kirchhoff et al., 2018], limiting its applicability to systems where these structures can be identified.

- *Integrated Information Theory* (IIT) [Tononi, 2004, Tononi et al., 2016] offers a quantitative measure of consciousness (Φ), but in its original formulations (IIT 2.0/3.0) it is a static, state-level measure rather than a process-level account[1], and its computation is intractable for large systems.
- *Autopoiesis* [Maturana and Varela, 1980] captures self-production but lacks formal predictive content beyond the biological domain.
- *Panpsychism* [Chalmers, 1995] attributes experience to fundamental entities but provides no mechanism and no solution to the combination problem—how micro-experiences compose into macro-experiences.

We propose that these limitations stem from a common root: each framework privileges a particular *level of description* (Bayesian inference, information integration, self-production) rather than identifying the *scale-free process* that underlies all of them.

Our central thesis:

> *Intelligent behavior emerges when components explore degrees of freedom and converge through distributed constraint resolution into coherent, goal-stabilizing patterns.*

We call this the **Distributed Constraint Resolution** (DCR) framework. The three components are, informally:

1. *Exploration* — a mechanism that generates variability across the configuration space (condition **(C1)**).
2. *Resolution* — local updates that reduce constraint violations between neighboring components (condition **(C2)**).
3. *Stabilization* — convergence in distribution to a coherent, low-violation attractor (Theorem 2.11).

We treat *goals* purely operationally: a goal is any attractor that is robust under perturbations at the timescale of interest (noise, model mismatch, or environmental variation), not a representation of future states. The key claim is that this triad constitutes the *minimal* structural signature of cognition; we discuss how broadly it appears across physical scales and what would falsify that universality claim in Sections 6 and 7.

The paper is organized as follows. Section 2 formalizes the DCR framework and proves convergence under Doeblin's condition. Section 3 proves exact closure for finite lumpable chains. Section 4 exhibits structural witnesses that fundamental physical processes instantiate the DCR triad. Section 5 relates DCR to FEP and IIT. Section 6 derives falsifiable predictions. Section 7 discusses implications and limitations.

Contributions. Beyond the structural definition of cognition, this paper makes four specific technical contributions: (1) a convergence theorem for DCR systems under Doeblin's condition, and an exact closure theorem for finite lumpable chains (with approximate closure under timescale separation in Appendix B); (2) embedding of the Free Energy Principle as a special case and establishment of DCR coherence as a necessary condition for IIT's $\Phi > 0$, under particular constraint structures and coherence refinements; (3) structural witnesses that physical processes across scales instantiate the DCR structure; and (4) connections to discrete-ontic models of quantum mechanics [Powers et al., 2024] and neural-network universe proposals [Vanchurin, 2020].

[1] IIT 4.0 introduces dynamical considerations that partially address this limitation; see Section 5.2 for discussion.

Remark 1.1 (What is proved vs. what is proposed). Theorems in this paper establish: (i) existence and geometric convergence of the stationary distribution under Doeblin-type exploration and Lyapunov drift (Theorem 2.11); and (ii) exact closure under classical lumpability for finite chains (Theorem 3.2), with approximate closure under explicit timescale-separation assumptions (Theorem B.8). Claims in Section 4 are *structural mappings* that exhibit how standard models can be cast into the DCR template; they are not new derivations of the underlying physics, and they do not resolve foundational questions such as the quantum measurement problem.

Scope and terminology. Throughout this paper, *cognitive* is a defined technical property of a dynamical system (Definition 2.3): a system is cognitive if and only if it instantiates the DCR triad of exploration, distributed constraint resolution, and convergence to coherent attractors. This is not a claim about phenomenal consciousness, folk intelligence, or teleology. DCR does not assert that photons "have experiences" or that convection cells "think"; it asserts that the *formal process* by which these systems evolve—exploring degrees of freedom, resolving constraints locally, stabilizing into coherent patterns—is structurally identical to the process that, at higher cognitive depth (Definition 3.4), underlies what we ordinarily call intelligence. The ontological claim is that this process is fundamental and universal; whether one additionally identifies it with experience is a separate philosophical question that DCR does not adjudicate (see Section 7, Limitation 5). As a universality claim, DCR is falsified by any robust cognitive phenomenon that lacks (i) genuine exploration, (ii) local constraint-processing, or (iii) convergence to a coherent attractor under coarse-graining.

For finite state spaces, exact closure holds under Kemeny–Snell lumpability (Theorem 3.2). For continuous state spaces, approximate closure (Theorem B.8) additionally requires timescale separation between intra-group and inter-group dynamics—a condition that holds widely but is not universal. We regard these as sufficient conditions, not necessary ones; the framework's axioms are themselves scale-free, and it is specifically the composition mechanism that requires additional structure (see Section 7, Limitation 1).

We treat the metaphysical identification—the cosmos is cognitive at every scale—as optional; the formal results hold without it (see Remarks C.1 and C.2 in Appendix C).

2 The DCR Framework

2.1 Constraint Networks

Definition 2.1 (Constraint Network). A *constraint network* is a tuple $\mathcal{N} = (S, \{D_s\}, G, \{v_e\})$ where:

1. S is a finite set of *components*.
2. $\{D_s\}_{s\in S}$ assigns to each component s a measurable *state space* D_s (its degrees of freedom).
3. $G = (S, E)$ is an undirected graph encoding the *interaction topology*.
4. $\{v_e\}_{e\in E}$ assigns to each edge $e = \{s, s'\} \in E$ a measurable *constraint cost function* $v_e : D_s \times D_{s'} \to \mathbb{R}_{\geq 0}$.

The *configuration space* is $\Omega = \prod_{s\in S} D_s$. In the special case of *hard constraints*, one defines $R_e := \{v_e = 0\} \subseteq D_s \times D_{s'}$.

For the convergence results in Section 2.3 and the coarse-graining construction in Section 3, we restrict to compact Polish state spaces D_s (hence compact metrizable Ω) in order to invoke standard Markov-chain stability results [Meyn and Tweedie, 1993]; the definitions above are stated in greater generality to clarify which results depend on compactness and which do not. Compactness is a genuine modeling restriction: many physical systems (e.g., unbounded velocities, Gaussian fields) naturally live on non-compact spaces. For such systems, the Foster–Lyapunov drift machinery of Section 2.3 remains the correct tool (it was designed for non-compact state spaces), but our main theorems are stated in compact-space language for simplicity. Extending the results to locally compact or Polish spaces with appropriate moment conditions is standard but notationally heavier; we do not pursue it here.

Notation. Throughout, K_ϵ denotes the transition kernel of the DCR dynamics (Definition 2.3), R and E denote the resolution and exploration kernels when the decomposition $K_\epsilon = (1-\epsilon)R + \epsilon E$ is used. We write μ_ϵ for stationary distributions, V for constraint cost (Definition 2.2), Coh_E for edge-sum coherence (Definition 2.7), and $\mathcal{F} = \arg\min V$ for the feasible set.

The formal development assumes $|S| < \infty$ and compact Polish state spaces D_s. Continuum field theories should be read as finite-element discretizations; extensions to non-compact or countable component spaces are discussed in Appendix A. The discreteness assumption may be less restrictive than it appears—see Remark C.5 in Appendix C.

Definition 2.2 (Constraint Cost and Feasible Set)**.** The *constraint cost* (or *total violation*) of a configuration $\omega \in \Omega$ is

$$V(\omega) = \sum_{e=\{s,s'\}\in E} v_e(\omega_s, \omega_{s'}), \tag{2.1}$$

where $v_e : D_s \times D_{s'} \to \mathbb{R}_{\geq 0}$. Without loss of generality, we normalize V so that $\min_\Omega V = 0$ (replace V by $V - \min V$; this does not change $\arg\min V$). The *feasible set* is $\mathcal{F} := \{V = 0\} = \arg\min V$, which is non-empty and compact whenever V is continuous on compact Ω.

2.2 DCR Systems

Definition 2.3 (DCR Dynamics at Noise Level ϵ (Doeblin form))**.** A tuple (Ω, V, K_ϵ) satisfies *DCR dynamics at noise level* ϵ if Ω is a compact metrizable configuration space on a constraint network $\mathcal{N}$, V is the constraint cost (Definition 2.2), and K_ϵ is a Markov kernel on Ω satisfying:

(C1) Exploration (Uniform minorization). There exist a probability measure ν on Ω and $\beta > 0$ such that

$$K_\epsilon(\omega, A) \;\geq\; \beta\,\nu(A) \qquad \text{for all } \omega \in \Omega,\; A \text{ measurable.} \tag{2.2}$$

This is Doeblin's condition, a strong sufficient condition for uniform ergodicity (see Appendix A for the weaker Harris-recurrence formulation on general state spaces).

(C2) Resolution (Global drift—strong form). There exist $\alpha \in (0,1)$ and $B < \infty$ such that

$$\mathbb{E}[V(X_{t+1}) \mid X_t = \omega] \;\leq\; (1-\alpha)\,V(\omega) + B\epsilon \qquad \text{for all } \omega \in \Omega. \tag{2.3}$$

The kernel systematically drives the system toward low violation, with residual fluctuation of order ϵ. Most physical systems satisfy only a small-set drift ($\mathbb{E}[V(X_{t+1}) \mid$

$X_t] \leq V(X_t) - \alpha V(X_t) + b\,\mathbf{1}_C(X_t) + B\epsilon$ for a compact set C); see Appendix A for the standard Foster–Lyapunov formulation.

We allow β to depend on ϵ (e.g. $\beta = O(\epsilon)$ in mixture models); α and B may also depend on ϵ, but the sharpest results obtain when α is bounded away from 0 uniformly in ϵ.

Definition 2.4 (Cognitive DCR System)**.** A DCR dynamics (Ω, V, K_ϵ) is *cognitive* if its unique stationary distribution μ_ϵ (whose existence follows from Theorem 2.11) satisfies the *coherence witness*:

(C3) Coherence. There exists an edge $\{s, s'\} \in E$ with

$$I_{\mu_\epsilon}(X_s;\, X_{s'}) > 0. \tag{2.4}$$

Remark 2.5 (Nontriviality)*.* Condition **(C3)** alone is intentionally minimal: it witnesses the presence of nontrivial coordination at stationarity. We regard a DCR system as *cognitively nontrivial* only when $\mathbb{E}_{\mu_\epsilon}[V] = O(\epsilon)$ with bounded constants (Theorem 2.11(ii)), so that the stationary mass concentrates near $\mathcal{F}$ as $\epsilon \to 0$, *and* coherence persists over a non-vanishing range of ϵ. These conditions exclude ordinary interacting ergodic systems where positive edge MI is an incidental consequence of weak coupling rather than maintained constraint resolution. Stronger graded notions (robustness of Coh_E under coarse-graining, scaling of Coh_E with system size) are natural but not needed for the convergence and closure theorems.

Remark 2.6 (Optional nonequilibrium strengthening)*.* The formal conditions (C1)–(C3) do not by themselves exclude reversible (equilibrium) kernels if one chooses V so that drift is satisfied. Our intention is to reserve "cognitive" for systems with maintained far-from-equilibrium drive. A sufficient strengthening is to require that K_ϵ violates detailed balance with respect to μ_ϵ (i.e., has nonzero stationary probability currents). None of the convergence or closure results depend on this strengthening; it is an optional criterion for excluding equilibrium systems from the "cognitive" label.

Definition 2.7 (Edge-Sum Coherence)**.** Fix reference measures $\{\lambda_s\}$ on each D_s (counting measure when D_s is finite; Lebesgue or a natural volume form when D_s is continuous). For a probability measure μ on $\Omega = \prod_{s\in S} D_s$ such that each marginal μ_s and each pairwise marginal $\mu_{s,s'}$ admit densities $f_s = \frac{d\mu_s}{d\lambda_s}$ and $f_{s,s'} = \frac{d\mu_{s,s'}}{d(\lambda_s \otimes \lambda_{s'})}$, define

$$I_\mu(X_s;\, X_{s'}) \;:=\; \int f_{s,s'} \ln \frac{f_{s,s'}}{f_s\, f_{s'}}\, d(\lambda_s \otimes \lambda_{s'}) \;\in\; [0, \infty], \tag{2.5}$$

and the *edge-sum coherence*

$$\mathrm{Coh}_E(\mu) \;:=\; \sum_{\{s,s'\}\in E} I_\mu(X_s;\, X_{s'}). \tag{2.6}$$

If any single edge has $I_\mu(X_s; X_{s'}) > 0$, then $\mathrm{Coh}_E(\mu) > 0$. We assume throughout that $I_\mu(X_s; X_{s'}) < \infty$ for all $\{s, s'\} \in E$; this holds automatically when each D_s is finite, and holds in continuous models under mild regularity (e.g., bounded densities with respect to the product reference measure; see Assumption A.7 for the general case).

Remark 2.8 (Strength of the exploration axiom)*.* Condition **(C1)** (Doeblin's condition) is a strong sufficient condition chosen to obtain clean geometric convergence rates. Most physical and biological systems satisfy only weaker Harris-type irreducibility plus minorization on a small set; the general formulation and proofs under these weaker conditions are given in Appendix A. The main text uses the Doeblin form for notational simplicity; all results extend to the Harris setting with the usual modifications (polynomial rather than geometric rates, V-uniform ergodicity).

Remark 2.9 (Locality vs. Lyapunov drift)*.* The locality of resolution—each update depends only on a neighborhood in G—is a *modeling* constraint that motivates the "distributed" in DCR. The convergence and closure theorems (Theorem 2.11, Theorem 3.2) require only the global drift inequality (2.3); locality is used to interpret V as a constraint-network cost and to motivate the physical mappings in Section 4.

A natural way to *construct* a kernel satisfying **(C1)**–**(C2)** (and, for cognitive systems, also **(C3)**) is the convex combination

$$K_\epsilon \;=\; (1-\epsilon)\,R + \epsilon\,E, \tag{2.7}$$

where R is a *resolution kernel* composed from local updates $\{R_s\}_{s\in S}$—each R_s depends only on site s and its neighbors $\mathcal{N}_G(s)$, and R reduces V in expectation:

$$\int_\Omega V(\omega')\,R(\omega,d\omega') \le V(\omega) \qquad \text{for all } \omega\in\Omega. \tag{2.8}$$

The kernel E is an *exploration kernel* providing uniform noise. If E satisfies Doeblin's condition with measure ν and constant β_E, then K_ϵ inherits it with $\beta = \epsilon\,\beta_E$. Similarly, if R contracts violation by factor $(1-\alpha)$, then K_ϵ satisfies **(C2)** with constants depending on α, ϵ, and the worst-case effect of E on V. The decomposition is useful for modeling but plays no role in the formal results: only the conditions on K_ϵ matter.

The *distributed* qualifier in DCR means that the update rules are local: each R_s depends only on site s and its neighbors. No central controller evaluates a global objective. Constraint *satisfaction*, by contrast, is a genuinely non-local phenomenon: the coherent pattern that emerges involves coordinated states across the entire network. A Lyapunov function V may exist as an *analysis tool* certifying convergence but is not computed or accessed by any component.

Remark 2.10 (Equilibrium systems)*.* Equilibrium Gibbs samplers (Glauber/Metropolis at fixed temperature) can generate high statistical dependence, hence high $\mathrm{Coh}_E(\mu)$, but they do not constitute DCR systems. At equilibrium (detailed balance), any local tendency to move toward lower V is exactly balanced by reverse moves in the stationary flux; there is no maintained nonequilibrium drive that keeps the system near $\mathcal{F}$ while performing work against perturbations. A Metropolis chain does bias toward lower energy outside the typical set, but the long-run behavior is constraint-consistent fluctuation, not maintained far-from-equilibrium stabilization. DCR is intended to capture such maintained, directed stabilization—a nonequilibrium signature—not merely biased sampling. The exclusion is *definitional*: cognition in the DCR sense requires ongoing, directed constraint resolution.

2.3 Convergence

Theorem 2.11 (DCR Convergence)**.** *Let (Ω, V, K_ϵ) satisfy DCR dynamics (Definition 2.3) on compact metrizable Ω. Then:*

- *(i) K_ϵ has a unique stationary distribution μ_ϵ, and convergence is geometric: $\|\mu_0 K_\epsilon^t - \mu_\epsilon\|_{\mathrm{TV}} \le 2(1-\beta)^t$ for any initial distribution μ_0.*
- *(ii) $\mathbb{E}_{\mu_\epsilon}[V] \le (B/\alpha)\,\epsilon$. If $\inf_{\epsilon\le\epsilon_0}\alpha(\epsilon) > 0$ and $\sup_{\epsilon\le\epsilon_0} B(\epsilon) < \infty$, then $\mathbb{E}_{\mu_\epsilon}[V] = O(\epsilon)$ uniformly as $\epsilon\to 0$.*

(iii) For any open $U \supseteq \mathcal{F}$, $\mu_\epsilon(U) \to 1$ as $\epsilon \to 0$.
In particular, if (Ω, V, K_ϵ) is cognitive (Definition 2.4), then $\mathrm{Coh}_E(\mu_\epsilon) > 0$.

Proof. **(C1)** is Doeblin's condition. By the Doeblin coupling theorem (see, e.g., Levin and Peres 2017, Theorem 4.9), K_ϵ admits a unique stationary distribution μ_ϵ with geometric convergence at rate $(1-\beta)$; since $\|\mu_0 - \mu_\epsilon\|_{\mathrm{TV}} \le 2$, this gives (i).

For (ii), integrate the drift condition **(C2)** against μ_ϵ:

$$\int V\, d\mu_\epsilon = \int \mathbb{E}[V(X_{t+1}) \mid X_t = \omega]\, \mu_\epsilon(d\omega) \le (1-\alpha)\int V\, d\mu_\epsilon + B\epsilon.$$

Rearranging: $\alpha \int V\, d\mu_\epsilon \le B\epsilon$.

For (iii), since $\min V = 0$, Markov's inequality gives $\mu_\epsilon(\{V > \eta\}) \le (B/\alpha\eta)\, \epsilon \to 0$ for any fixed $\eta > 0$. For any open $U \supseteq \mathcal{F}$, $\Omega \setminus U$ is compact and disjoint from $\mathcal{F} = \{V = 0\}$, so $m := \inf_{\Omega\setminus U} V > 0$ by continuity. Then $\{V < m\} \subseteq U$, giving $\mu_\epsilon(U) \to 1$.

The coherence claim follows directly from condition **(C3)**.

For the general (non-compact or non-Doeblin) case using Foster–Lyapunov drift and Harris recurrence, see Appendix A. Note: if minorization arises solely from the exploration mixture ($\beta = O(\epsilon)$), the geometric rate correspondingly slows as $\epsilon \to 0$. $\square$

Proposition 2.12 (Coherence for nontrivial pairwise interactions)**.** *Let $\Omega = \prod_s D_s$ be finite with pairwise-interaction Gibbs measure $\pi(\omega) \propto \exp\Big(-\sum_{\{s,s'\}\in E} J_{s,s'}(\omega_s, \omega_{s'})\Big)$ satisfying $\pi(\omega) > 0$ for all ω. If, for some edge $\{s, s'\}$, the pairwise factor $e^{-J_{s,s'}(\omega_s,\omega_{s'})}$ does not factorize as $f_s(\omega_s)\, f_{s'}(\omega_{s'})$, then:*

(a) ***Conditional dependence (unconditional).*** *$I_\pi(X_s; X_{s'} \mid X_{S\setminus\{s,s'\}}) > 0$: the components are conditionally dependent given the remainder of the system.*

(b) ***Marginal dependence (under faithfulness).*** *If the parameters satisfy a* faithfulness *condition—no exact cancellations through the rest of the graph produce marginal independence (a generic condition; see Spirtes et al. 2001)—then $I_\pi(X_s; X_{s'}) > 0$, and for all sufficiently small $\epsilon > 0$ any DCR dynamics K_ϵ on finite Ω with $\mu_\epsilon \to \pi$ in total variation as $\epsilon \to 0$ is cognitive (Definition 2.4).*

Proof. (a) By the Hammersley–Clifford theorem, under the positivity condition a pairwise Gibbs measure satisfies the pairwise Markov property: $X_s \perp\!\!\!\perp X_{s'} \mid X_{S\setminus\{s,s'\}}$ if and only if the pairwise factor on $\{s, s'\}$ factorizes. Hence a non-factorizable edge factor implies $I_\pi(X_s; X_{s'} \mid X_{S\setminus\{s,s'\}}) > 0$.

(b) A non-factorizable edge potential guarantees conditional dependence but not necessarily marginal dependence: parameter cancellations through the rest of the graph can produce $I_\pi(X_s; X_{s'}) = 0$ even when $J_{s,s'} \ne 0$ (e.g., in certain Ising models on cycles where competing paths exactly cancel). The faithfulness assumption excludes such measure-zero parameter coincidences, yielding $I_\pi(X_s; X_{s'}) > 0$. By standard perturbation theory for finite irreducible aperiodic chains, μ_ϵ depends continuously on ϵ in total variation, so $I_{\mu_\epsilon}(X_s; X_{s'}) > 0$ persists for small ϵ. $\square$

2.4 What DCR Excludes

A definition of cognition is only useful if it excludes something. DCR excludes three classes of systems:

1. **Equilibrium systems.** A system at thermodynamic equilibrium exhibits no directed constraint reduction and no maintained far-from-equilibrium stabilization.

(An interacting equilibrium system, e.g., an Ising model below T_c, may have high correlations and even local bias toward lower V, but under detailed balance there is no maintained nonequilibrium drive producing persistent stabilization against perturbations. DCR is intended to capture such maintained directed stabilization, not merely equilibrium sampling; see Remark 2.10.) *A rock at thermal equilibrium is not cognitive.*

2. **Unconstrained stochastic systems.** A system with exploration but no constraint structure ($\mathcal{R} = \emptyset$) undergoes a random walk on Ω with no convergence to coherent patterns. $V \equiv 0$ trivially, and $\mathrm{Coh}_E(\mu) = 0$ for the stationary (uniform) distribution. *Brownian motion in free space is not cognitive.*
3. **Fully deterministic single-trajectory systems.** A system with a single degree of freedom following a deterministic trajectory has no exploration (the path is unique) and no distributed resolution (there is only one component). *A single classical particle in a potential well is not cognitive.*

Cognition in the DCR sense requires the non-trivial intersection: non-equilibrium dynamics, constraint structure, and distributed convergence to coherent attractors.

We emphasize that the boundary between cognitive and non-cognitive is *continuous*, not sharp. A system near equilibrium with weak constraints and small fluctuations satisfies the DCR conditions only marginally: the coherence $\mathrm{Coh}_E(\mu_\epsilon)$ is near zero and the convergence timescale is long. DCR does not impose a binary threshold; rather, it provides a graded measure through the coherence of the attractor and the cognitive depth δ (Definition 3.4). The exclusions above identify the *limiting cases* where one or more components of the triad vanish entirely, not a boundary that systems cross discontinuously.

2.5 Worked Example: Antiferromagnetic Ising Model

Before proceeding to scale-freeness and physics, we ground the framework in a concrete toy model where every quantity can be computed explicitly.

Example 2.13 (Antiferromagnetic Ising lattice)**.** Consider N spins on a graph $G = (S, E)$ (e.g., a d-dimensional lattice) with $D_s = \{-1, +1\}$ for each $s \in S$. This is a finite constraint network with $\Omega = \{-1, +1\}^N$ ($|\Omega| = 2^N$).

Constraints. The antiferromagnetic coupling prefers opposite spins on neighboring sites. For each edge $e = \{s, s'\} \in E$:

$$v_e(\omega_s, \omega_{s'}) = \tfrac{1}{2}(1 + \omega_s\,\omega_{s'}) = \begin{cases} 0 & \text{if } \omega_s \neq \omega_{s'}, \\ 1 & \text{if } \omega_s = \omega_{s'}. \end{cases}$$

The total violation is $V(\omega) = \sum_{\{s,s'\}\in E} v_e(\omega_s, \omega_{s'})$, which counts the number of frustrated (same-spin) edges. The feasible set $\mathcal{F} = \{\omega : V(\omega) = 0\}$ consists of the proper 2-colorings of G (if G is bipartite) or is empty (if G is not bipartite; in the latter case one works with approximate feasibility).

Resolution kernel. Select a site s uniformly at random. If flipping s strictly reduces V, flip it; otherwise keep it. This defines a Markov kernel R satisfying $\int V(\omega')\, R(\omega, d\omega') \leq V(\omega)$ for all ω (violation-reducing, Equation (2.8)). Updates are local: the flip decision for s depends only on ω_s and $(\omega_{s'})_{s'\in\mathcal{N}_G(s)}$.

Exploration kernel. With probability ϵ, resample the entire configuration uniformly: $E(\omega, \cdot) = \mathrm{Unif}(\Omega)$. This global refresh ensures every state is reachable in one step.

Verification of DCR conditions.

- ***(C1)*** *(Minorization):* Since $E(\omega, \cdot) = \mathrm{Unif}(\Omega)$ for all ω, $K_\epsilon(\omega, A) \geq \epsilon\,\mathrm{Unif}(A)$ for every measurable A. This is Doeblin's condition with $\beta = \epsilon$ and $\nu = \mathrm{Unif}(\Omega)$.
- ***(C2)*** *(Drift):* The greedy flip rule satisfies Equation (2.8), and uniform resampling has bounded expected violation, so $\mathbb{E}[V(X_{t+1}) \mid X_t = \omega] \leq (1-\epsilon)\,V(\omega) + \epsilon\,\mathbb{E}_{\mathrm{Unif}}[V]$, which is condition **(C2)** with $\alpha = \epsilon$ and $B = \mathbb{E}_{\mathrm{Unif}}[V]$. In this toy construction, α scales with ϵ because exploration is implemented as a global refresh. A more realistic alternative uses local random flips (with probability ϵ, randomize a single site): on finite Ω, the m-step kernel K_ϵ^m satisfies Doeblin for $m = O(|S|)$, and the resolution rate α is set by the greedy-flip frequency, remaining bounded away from 0 as $\epsilon \to 0$.
- ***(C3)*** *(Coherence):* On bipartite graphs, the ground states have maximally anti-correlated neighbors: $I_{\mu_\epsilon}(X_s; X_{s'}) > 0$ for edges $\{s, s'\}$. By perturbation theory, this persists for small $\epsilon > 0$.

By Theorem 2.11, the chain converges to a unique μ_ϵ with $\mathrm{Coh}_E(\mu_\epsilon) > 0$ and $\mu_\epsilon(\mathcal{F}) \to 1$ as $\epsilon \to 0$ on bipartite graphs. This is a cognitive DCR system of depth 1.

Remark 2.14 (Doeblin via m-step minorization)*.* On finite Ω, it is often easier to verify Doeblin's condition for a skeleton chain: if $K_\epsilon^m(\omega, \cdot) \geq \beta_m\,\nu(\cdot)$ for some $m \geq 1$, then K_ϵ is uniformly ergodic with rate $\|\mu_0 K_\epsilon^t - \mu_\epsilon\|_{\mathrm{TV}} \leq 2(1-\beta_m)^{\lfloor t/m \rfloor}$. The local-flip alternative in the Ising example uses this: one-step Doeblin fails for small ϵ, but K_ϵ^m with $m = O(|S|)$ satisfies it.

3 Scale-Freeness and the Combination Problem

The central mathematical contribution of this paper is a closure result: the DCR form is stable under coarse-graining—cognitive systems at one scale compose into cognitive systems at the next.

For finite state spaces, exact closure follows from classical lumpability (Theorem 3.2). For continuous state spaces, approximate closure holds under timescale separation between intra-group and inter-group dynamics (Appendix B)—a condition that holds in many physical systems (atomic vs. molecular, synaptic vs. network) but is not universal. The framework's axioms (Section 2) are themselves scale-free; it is specifically the composition mechanism that requires additional structure.

3.1 Closure for Finite Chains

A central question for any purportedly scale-free framework is whether its structure is preserved under coarse-graining. For DCR on finite state spaces, exact closure follows from classical lumpability.

Definition 3.1 (Lumpable Partition)**.** Let K_ϵ be a Markov kernel on finite Ω. A surjection $\pi : \Omega \twoheadrightarrow \tilde{\Omega}$ is *lumpable* (in the sense of Kemeny–Snell) if, for every $\tilde{y} \in \tilde{\Omega}$, the transition probability $K_\epsilon(x, \pi^{-1}(\tilde{y}))$ depends on x only through $\pi(x)$.

Theorem 3.2 (Exact Closure Under Lumpability)**.** *Let (Ω, V, K_ϵ) satisfy DCR dynamics (Definition 2.3) on finite Ω, and let $\pi : \Omega \twoheadrightarrow \tilde{\Omega}$ be lumpable for K_ϵ. Define the quotient kernel $\tilde{K}_\epsilon$ on $\tilde{\Omega}$ by*

$$\tilde{K}_\epsilon(\tilde{x}, \tilde{y}) \;=\; K_\epsilon(x, \pi^{-1}(\tilde{y})) \qquad \text{for any } x \in \pi^{-1}(\tilde{x}),$$

and the macro-cost (lower envelope) $\tilde{V}(\tilde{x}) = \inf\{V(x) : \pi(x) = \tilde{x}\}$ *(an infimum, attained since* Ω *is finite). This is a Lyapunov surrogate certifying drift, not necessarily a directly measured macro-observable. Then* $(\tilde{\Omega}, \tilde{V}, \tilde{K}_\epsilon)$ *satisfies DCR dynamics:*

(i) ***Minorization:*** $\tilde{K}_\epsilon(\tilde{x}, \cdot) \geq \beta\, \tilde{\nu}(\cdot)$ *with* $\tilde{\nu} = \pi_\# \nu$.

(ii) ***Drift:*** $\mathbb{E}[\tilde{V}(\tilde{X}_{t+1}) \mid \tilde{X}_t = \tilde{x}] \leq (1-\alpha)\, \tilde{V}(\tilde{x}) + B\epsilon$.

(iii) ***Coherence (conditional):*** *If* $\tilde{\Omega} = \prod_{s' \in S'} \tilde{D}_{s'}$ *inherits a product structure with edges* E'*, then for each macro-edge* $\{a', b'\} \in E'$ *the data-processing inequality gives* $I_{\tilde{\mu}_\epsilon}(\tilde{X}_{a'}; \tilde{X}_{b'}) \leq I_{\mu_\epsilon}(X_A; X_B)$ *where* $A = \pi^{-1}(a')$, $B = \pi^{-1}(b')$*. Coarse-graining can destroy dependence (the inequality is strict in general), so this is a conditional guarantee:* $\mathrm{Coh}_{E'}(\tilde{\mu}_\epsilon) > 0$ *holds whenever the projection preserves at least one edge's mutual information. A sufficient condition is the following. Assume* π *is* componentwise*: it is built from maps* $\pi_s : D_s \to \tilde{D}_{\sigma(s)}$ *for a grouping* $\sigma : S \to S'$*, so that* $\pi(\omega)_{\sigma(s)} = \pi_s(\omega_s)$*. If for some edge* $\{s, s'\} \in E$ *with* $I_{\mu_\epsilon}(X_s; X_{s'}) > 0$*, the induced map* $(\pi_s, \pi_{s'}) : D_s \times D_{s'} \to \tilde{D}_{\sigma(s)} \times \tilde{D}_{\sigma(s')}$ *is injective, then* $I_{\tilde{\mu}_\epsilon}(\tilde{X}_{\sigma(s)}; \tilde{X}_{\sigma(s')}) = I_{\mu_\epsilon}(X_s; X_{s'}) > 0$.

Proof. By Kemeny–Snell [Kemeny and Snell, 1960], lumpability ensures that $\{\pi(X_t)\}$ is a Markov chain with kernel $\tilde{K}_\epsilon$.

Minorization. For any $\tilde{x} \in \tilde{\Omega}$ and $\tilde{A} \subseteq \tilde{\Omega}$, $\tilde{K}_\epsilon(\tilde{x}, \tilde{A}) = K_\epsilon(x, \pi^{-1}(\tilde{A})) \geq \beta\, \nu(\pi^{-1}(\tilde{A})) = \beta\, \tilde{\nu}(\tilde{A})$.

Drift. For any $x \in \pi^{-1}(\tilde{x})$, $\mathbb{E}[\tilde{V}(\pi(X_{t+1})) \mid X_t = x] \leq \mathbb{E}[V(X_{t+1}) \mid X_t = x] \leq (1-\alpha)\, V(x) + B\epsilon$. Taking the minimum over $x \in \pi^{-1}(\tilde{x})$: $(1-\alpha)\, \tilde{V}(\tilde{x}) + B\epsilon$; by lumpability the left-hand side depends only on $\tilde{x}$, giving the drift.

Coherence. The stationary distribution of $\tilde{K}_\epsilon$ is $\tilde{\mu}_\epsilon = \pi_\# \mu_\epsilon$. Let $\tilde{X} = \pi(X)$. By data processing, $I_{\tilde{\mu}_\epsilon}(\tilde{X}_{a'}; \tilde{X}_{b'}) \leq I_{\mu_\epsilon}(X_A; X_B)$ where A, B are the fibers of a', b'. Coarse-graining can destroy dependence (the inequality is strict in general). For the sufficient condition: when π is componentwise and the induced map $(\pi_s, \pi_{s'})$ on $D_s \times D_{s'}$ is injective, the marginal distribution on $(\tilde{X}_{\sigma(s)}, \tilde{X}_{\sigma(s')})$ is a relabeling of $(X_s, X_{s'})$, preserving mutual information exactly. □

Remark 3.3 (Valid coarse-grainings). That coarse-graining can destroy coherence is expected, not a defect: a projection that collapses all interacting components into one state discards the phenomenon it purports to describe. A coarse-graining is *valid* for studying cognition at a given scale only if it preserves enough structure to retain some statistical dependence. The componentwise-injectivity condition in (iii) is one clean sufficient criterion; another (standard in statistical mechanics and the approximate closure of Appendix B) is that the macro-variables act as approximate sufficient statistics for the inter-group interactions.

Approximate closure. For general (possibly continuous) state spaces, closure holds approximately under explicit timescale-separation and macro-sufficiency conditions; see Appendix B for the full development.

3.2 Conditional Composition and the Combination Problem

The combination problem in panpsychism asks: if fundamental entities have experience, how do micro-experiences combine into the unified macro-experience of, say, a human mind? Theorem 3.2 provides a conditional answer: when the micro-dynamics admits a lumpable partition, the DCR dynamics (minorization and drift) are inherited at the macro-scale; coherence transfers if the projection preserves at least one edge's mutual information.

The "combination" is not a separate metaphysical operation but a consequence of the coarse-graining: one level's resolved constraints become the next level's exploration noise, and the closure theorem guarantees that the resulting macro-chain is again DCR—provided the projection is fine enough to retain some statistical dependence.

Under the conditions of the closure theorem, there is no separate substance (experience, qualia) that needs combining—there is the DCR process, recurring at each scale via coarse-graining, *when* the dynamics admits an effective macro-description. What we call "unified experience" at the human scale is the coherent attractor of a DCR system whose components are themselves coarse-grained DCR systems, recursively. For the approximate (continuous state space) version of this argument, see Appendix B.

3.3 The Depth of Cognition

Not all cognitive systems are equally "intelligent." Given a lumpable partition π, write $\Gamma_\pi(\mathcal{C})$ for the coarse-grained system constructed in Theorem 3.2 (in the finite case the partition π itself serves as the compression map; for continuous state spaces a separate compression map g is needed—see Appendix B). We introduce a measure of cognitive depth:

Definition 3.4 (Cognitive Depth). The *cognitive depth* of a system is the number of nested coarse-graining levels k at which the DCR triad is simultaneously active:

$$\delta(\mathcal{C}) = \max\{k \mid \Gamma_{\pi_k} \circ \cdots \circ \Gamma_{\pi_1}(\mathcal{C}) \text{ is cognitive}\}, \tag{3.1}$$

where the maximum is over all hierarchical sequences of lumpable partitions $(\pi_1, \ldots, \pi_k)$ satisfying the conditions of Theorem 3.2 at each level, subject to the *strict reduction* requirement $|S_{i+1}| < |S_i|$ at each level i (i.e., the partition π_i is non-trivial: at least one group contains more than one component). Since $|S|$ is finite, this ensures $\delta(\mathcal{C}) \leq |S| - 1 < \infty$.

The definition provides a qualitative, non-anthropocentric ordering of intelligence without requiring a binary threshold.[2]

4 Physical Instantiations of DCR

We now exhibit structural witnesses that fundamental physical processes instantiate the DCR triad, providing examples consistent with—though not sufficient to establish—the optional identification of DCR with cognition at every scale. These are mappings from established dynamical descriptions into the DCR template; they are not claimed to be novel derivations of the underlying physics. For each example, we identify the five DCR ingredients using the following checklist:

(i) *Components & degrees of freedom* — what are the interacting parts and their local state spaces?

(ii) *Constraints* — what local compatibility conditions (conservation laws, fitness, synaptic weights) couple neighboring components?

[2]Illustrative ordinal rankings: a hydrogen atom has depth ∼1 (quantum constraint resolution at the particle level); a bacterium ∼3–4 (molecular, metabolic, behavioral); a human brain ∼6–8 (ionic, synaptic, columnar, areal, network, behavioral, social). These estimates are based on empirically identifiable organizational levels and should not be read as precise measurements; see Remark C.17.

(iii) *Exploration* — what mechanism generates variability across the configuration space (quantum fluctuations, mutation, stochastic firing)?
(iv) *Resolution* — how do local interactions reduce constraint violation without global coordination?
(v) *Stable coherent attractor* — what is the coherent macroscopic pattern that emerges (completed transaction, convection roll, adapted species, neural representation)?

Example 4.2 carries out a detailed verification sketch (conditions **(C1)**–**(C3)**) for thermodynamic self-organization (see Remark C.15 for caveats on the proof-sketch character); the remaining examples are presented at the level of structural mappings, with a remark (Example 4.2, following) on how the same verification template applies.

4.1 Quantum Mechanics

Consider a system of N interacting quantum particles (or field modes). The standard decoherence account [Zurek, 2003] treats the suppression of off-diagonal coherences as the mechanism by which classical reality emerges, but decoherence alone never yields a definite outcome—it produces an improper mixture, not a selected event. The DCR structure of quantum mechanics is most transparently exhibited by the *Transactional Interpretation* (TIQM) [Cramer, 1986], which builds on the Wheeler–Feynman absorber theory [Wheeler and Feynman, 1945]. In TIQM, every quantum event is a *transaction*—a completed handshake between emitter and absorber mediated by retarded (offer) and advanced (confirmation) waves. We use TIQM as a *structural witness* that quantum dynamics can be read as a distributed constraint-resolution process; DCR does not require endorsing any particular interpretation of quantum mechanics, and no claim is made that DCR solves or dissolves the measurement problem. A second, interpretation-neutral witness based on decoherence and einselection is given in Remark C.12. The TIQM mapping onto the DCR triad is as follows:

- **Components:** Emitter and absorber sites—the vertices of the spacetime interaction graph.
- **Degrees of freedom:** The possible quantum states (energy, momentum, polarization, spin) at each site, drawn from the local Hilbert space $\mathcal{H}_s$.
- **Exploration:** The *offer wave* ψ (retarded wave) propagates from the emitter to all potential absorbers, exploring every possible transaction partner simultaneously. In the Feynman path integral formulation [Feynman, 1948], this is the sum over all paths weighted by $e^{iS/\hbar}$—the emitter explores the entire accessible configuration space.
- **Constraints:** Conservation laws (energy, momentum, angular momentum, charge) at each interaction vertex. For an edge $\{s, s'\}$ in the interaction graph, the constraint $R_{\{s,s'\}}$ requires that the quantum numbers carried by the offer wave from emitter s match those that absorber s' can accept, given s''s own state and the applicable conservation laws.
- **Resolution:** The *confirmation wave* ψ^* (advanced wave) propagates from each potential absorber back to the emitter. The Wheeler–Feynman handshake is distributed constraint resolution: each absorber independently evaluates the offer against its local constraints and responds; a completed transaction corresponds to a locally consistent satisfaction of conservation constraints across endpoints, yielding a definite exchange event in the TIQM narrative. There is no central selector choosing the outcome—the definite result emerges from the mutual satisfaction of local conservation laws,

distributed across spacetime.

- **Stable pattern:** The *completed transaction*—a definite, irreversible transfer of conserved quantities between emitter and absorber. Once formed, the transaction is a classical fact: the stable coherent attractor of the handshake process.

The retrocausal structure is not a defect but a feature: constraints propagate both forward and backward in time, making the resolution genuinely distributed across spacetime rather than confined to a single time-slice. What is conventionally called "wavefunction collapse" can be viewed, at the level of structural analogy, as the convergence of a distributed constraint satisfaction process to its feasible solution—not a derivation of Born-rule outcome selection.

For N interacting particles, the interaction graph G has particles as vertices and pairwise interactions as edges. Each interaction is a potential transaction site where conservation constraints must be locally satisfied. The global physical outcome—the set of completed transactions—can be viewed as a configuration ω satisfying the relevant local conservation constraints for the realized exchange events, reached by distributed resolution without global coordination.

Remark 4.1 (Selection and collapse). Quantum collapse and constraint-mediated selection (Remark C.11) share the abstract template: variation $\to$ constraint filtering $\to$ retention. In both cases, multiple possibilities are explored (superposition / genetic variation), constraints select which possibilities are realized (conservation laws / fitness landscape), and the selection is local and distributed (each absorber / each organism evaluates constraints independently). We reserve "natural selection" for the biological, inheritance-based instance; the shared structure is the DCR triad, differing only in the physical substrate and timescale.

4.2 Thermodynamic Self-Organization

We model Rayleigh–Bénard convection as a DCR-system under a standard lattice discretization, illustrating how the DCR axioms map onto a concrete physical system. The verification is a sketch, not a full proof (see Remark C.15).

Consider a fluid layer between horizontal plates, heated from below (T_H) and cooled from above (T_C), in the Boussinesq approximation [Chandrasekhar, 1961]. We discretize the fluid domain on a regular d-dimensional lattice with N parcels and spacing h.

Example 4.2 (Bénard convection as a DCR witness)**.** We claim that for Rayleigh number $\mathrm{Ra} > \mathrm{Ra}_c$ (above the convective instability threshold), the stochastic lattice Boussinesq system with thermal fluctuations can be modeled as a cognitive system $\mathcal{C} = (\mathcal{N}, \{X_t\}, R, \mathcal{A})$ in the sense of Definition 2.3, under the following standard assumptions: (i) the finite-difference discretization preserves the energy dissipation structure of the continuous Boussinesq equations, (ii) the spectral gap λ_1 of the graph Laplacian on the lattice is positive, and (iii) the Gaussian noise amplitude $\sigma > 0$ is fixed.

Verification sketch. We construct the DCR tuple and verify the general state-space conditions (a)–(e) of Appendix A; see Remark C.15 for caveats on the level of rigor.

Constraint network. Let $S = \{1, \dots, N\}$ index the parcels, with state space $D_s = [-v_{\max}, v_{\max}]^d \times [T_C, T_H]$ for each s (velocity and temperature, truncated to a compact box—this is a *modeling choice*, not a consequence of the Boussinesq equations alone; see Remark C.15). The configuration space $\Omega = \prod_s D_s$ is compact. The interaction graph G is lattice adjacency (connected for $N \geq 2$). For each edge $e = \{s, s'\} \in E$, the constraint cost

v_e encodes the discretized Boussinesq conservation laws at the interface: $v_e(\omega_s, \omega_{s'}) = 0$ iff the discrete incompressibility condition $\nabla^h_{ss'} \cdot \mathbf{v} = 0$, the discrete momentum balance $\nu\, \nabla^{h,2}_{ss'}\mathbf{v} - (\mathbf{v} \cdot \nabla^h_{ss'})\mathbf{v} + \beta(T - T_{\mathrm{ref}})\mathbf{g} = \nabla^h_{ss'}p$, and the discrete energy balance $\kappa\, \nabla^{h,2}_{ss'}T = (\mathbf{v} \cdot \nabla^h_{ss'})T$ are satisfied at the $\{s, s'\}$ interface, where $\nabla^h_{ss'}$ and $\nabla^{h,2}_{ss'}$ denote the standard finite-difference gradient and Laplacian operators, ν is kinematic viscosity, κ is thermal diffusivity, β is the thermal expansion coefficient, and $\mathbf{g}$ is gravitational acceleration.

Violation. The pairwise violation $v_e(\omega_s, \omega_{s'})$ is the sum of squared residuals of the three conservation equations at the $\{s, s'\}$ interface. Then $v_e \geq 0$ with equality iff the steady-state Boussinesq equations are locally satisfied, and V is continuous on compact Ω.

Dynamics. The resolution kernel R is a semi-implicit Gauss–Seidel sweep of the discretized Boussinesq equations: each parcel updates its velocity and temperature using only the current states of its lattice neighbors, satisfying the locality requirement of Equation (2.8). The exploration kernel E adds Gaussian noise of variance $\sigma^2 \propto k_B T_{\mathrm{ref}}/(\rho\, h^d)$ (fluctuating hydrodynamics), projected onto Ω:

$$E(\omega, \cdot) = \mathrm{Law}\Big(\Pi_\Omega(\omega + \sigma\, \boldsymbol{\xi})\Big), \qquad \boldsymbol{\xi} \sim \mathcal{N}(0, I_{N(d+1)}).$$

Verification of conditions.

(a) *Foster–Lyapunov drift.* The energy functional $\mathcal{E}(\omega) = \sum_s \Big[\frac{1}{2}|\mathbf{v}_s|^2 + \frac{1}{2}|T_s - T_{\mathrm{lin},s}|^2\Big]$ (where T_{lin} is the linear conduction profile) satisfies

$$\mathcal{E}(\omega^{(t+1)}) \leq \mathcal{E}(\omega^{(t)}) - \Delta t\Big[\nu\, \|\nabla^h \mathbf{v}\|^2 + \kappa\, \|\nabla^h \theta\|^2\Big] + \Delta t\, \mathrm{Ra}\,(\theta, v_z)$$

under the resolution step, where $\theta = T - T_{\mathrm{lin}}$. By the discrete Poincaré inequality ($\|\nabla^h u\|^2 \geq \lambda_1 \|u\|^2$ with $\lambda_1 > 0$ the spectral gap of the graph Laplacian), the dissipative terms dominate the buoyancy source outside a compact neighborhood C of the convection roll attractor, yielding a Foster–Lyapunov drift condition (Remark A.2) for $L = V + 1$ with small set C. (The transfer from $\mathcal{E}$ to V near the attractor follows from the comparability of the respective Hessians under the linearized Boussinesq operator—a standard result in the stability theory of discretized Navier–Stokes systems.)

(b) *φ-irreducibility.* The Gaussian kernel E has positive density on $\mathrm{int}(\Omega)$, so $K(\omega, A) \geq \epsilon\, E(\omega, A) > 0$ for any A with positive Lebesgue measure $\lambda(A) > 0$. The chain is λ-irreducible.

(c) *Minorization.* Choose c large enough that the sublevel set $C = \{\omega \in \mathrm{int}(\Omega) : V(\omega) \leq c\}$ is visited infinitely often under the drift (condition (a)). Since C is compactly contained in the interior of Ω, the (unprojected) Gaussian exploration density is bounded below by some $\delta > 0$ on $C \times C$. Hence $K(\omega, A) \geq \epsilon\, \delta\, \lambda(A \cap C)$ for $\omega \in C$, establishing the minorization condition (c) with $\nu = \lambda(\cdot \cap C)/\lambda(C)$. (The boundary of Ω, where the projection Π_Ω distorts densities, is avoided by taking $C \subset \mathrm{int}(\Omega)$; see Remark C.15.)

(d) *Weak Feller.* The Boussinesq update is polynomial in ω and neighbor states (hence continuous); the Gaussian density is smooth. Their convex combination K is weak Feller.

(e) *Full exploration support.* The Lebesgue-equivalent irreducibility measure charges every open set, in particular those meeting $\mathcal{F}$.

Attractor and coherence. For $\mathrm{Ra} > \mathrm{Ra}_c$, the feasible set $\mathcal{F}$ (steady-state solutions of the discretized Boussinesq equations) includes the convection roll patterns [Cross and Hohenberg, 1993]. The Gauss–Seidel update of parcel s makes $(\mathbf{v}_s^{(t+1)}, T_s^{(t+1)})$ depend on the states of neighbors s' through the discretized momentum and energy equations, so the unique μ_ϵ has non-product (s, s')-marginals. By Theorem 2.11, the chain converges to a unique μ_ϵ with $\mathrm{Coh}_E(\mu_\epsilon) > 0$, and the convection rolls constitute the stable coherent attractor $\mathcal{A}$. □

The convection pattern is *not* at equilibrium (DCR correctly excludes equilibrium), and it arises without any central controller selecting the pattern. It is a cognitive system of depth 1.

4.3 Biological Adaptation

- **Components:** Organisms in a population.
- **Degrees of freedom:** Genotype/phenotype space.
- **Exploration:** Mutation, recombination, developmental noise.
- **Constraints:** Environmental fitness landscape, inter-organism competition, predator–prey relations.
- **Resolution:** Natural selection propagates constraints locally (each organism's survival depends on its local fitness, not a global optimization). This is inherently distributed.
- **Stable pattern:** Adapted species occupying fitness peaks—the coherent attractor of the evolutionary dynamics [Kauffman, 1993].

Biological evolution is a cognitive system of depth ≥ 2: the organisms themselves are cognitive systems (metabolic constraint resolution), and the population-level dynamics is a second layer of cognition.

4.4 Neural Cognition

- **Components:** Neurons (or neural populations).
- **Degrees of freedom:** Firing rates, membrane potentials, synaptic states.
- **Exploration:** Spontaneous activity, noise, stochastic neurotransmitter release.
- **Constraints:** Synaptic weights, lateral inhibition, top-down priors encoded in connectivity.
- **Resolution:** Local integration-and-fire dynamics; each neuron resolves its inputs against its threshold. Constraint propagation is distributed across the network.
- **Stable pattern:** Perceptual representations, motor plans, decisions—coherent attractors of the neural dynamics [Seth, 2021].

Neural cognition achieves high depth because the components (neurons) are themselves biochemical cognitive systems, embedded in circuits that form cognitive systems, embedded in areas, and so on up to whole-brain dynamics.

5 Relationship to Existing Frameworks

5.1 Free Energy Principle as a Special Case

Proposition 5.1 (FEP Recovery)**.** *The Free Energy Principle is a special case of DCR obtained when:*

1. *The constraint network is bipartite, partitioned into "internal" and "external" components with a Markov blanket boundary.*

2. *The constraints encode a generative model $p(\tilde{s}, \psi \mid m)$ relating external causes ψ to sensory observations $\tilde{s}$.*
3. *The* stabilization criterion *uses variational free energy F as the Lyapunov functional (in place of $V + 1$). Note that $-F$ does not play the same mathematical role as* Coh*: it is not nonnegative and does not vanish on product measures. The analogy is functional (both witness "convergence to an attractor"), not structural.*
4. *The resolution dynamics is gradient descent on F (recognition dynamics).*

Under these specializations, DCR's "explore–resolve–stabilize" reduces to FEP's "prediction error minimization via active inference."

Proof sketch (full proof in Appendix C.1). The Markov blanket condition gives a bipartite-through-blanket constraint graph G. Setting $v_{\{s,s'\}} = -\ln p(\tilde{s}_{s'} \mid \mu_s)$ defines the per-edge violation. Under a conditional independence assumption (the generative model factorizes over blanket components conditioned on internal states), the total violation reduces to a conditional negative log-likelihood of sensory data given internal states: $V = -\ln p(\tilde{s} \mid \mu)$. (This is *not* the FEP's marginal surprisal $-\ln p(\tilde{s})$, but a conditional analogue.) The variational free energy F serves as the Lyapunov function for the FEP specialization: gradient descent on F provides the resolution kernel, and since F is bounded below with compact sublevel sets, this yields a Foster–Lyapunov drift condition. (The bound $F \geq -\ln p(\tilde{s})$ ensures that concentration near low-F regions entails concentration near low-surprisal regions, but F itself—not V—serves as the Lyapunov function.) Active inference supplies the exploration kernel. The attractor is the set where $q(\psi) \approx p(\psi \mid \tilde{s})$, which is coherent through the shared generative model. □

DCR is strictly more general than FEP in two ways: (1) it does not require a bipartite structure with a Markov blanket, and (2) it does not require the constraints to be expressible as a generative model. Physical constraint resolution (e.g., decoherence, convection) need not involve "inference" in any Bayesian sense.

5.2 Relationship to Integrated Information Theory

Proposition 5.2 (DCR coherence as a necessary condition for IIT's Φ)**.** *Under the following specializations, positive DCR coherence is a necessary condition for positive integrated information Φ in the sense of IIT 2.0 [Tononi, 2004]:*

1. *The constraint network encodes the transition probability matrix (TPM) of IIT: for each edge $\{s, s'\}$, the constraint $R_{\{s,s'\}}$ encodes which state transitions of s are compatible with the current state of s'.*
2. *The analysis is restricted to a single time step (the TPM acts once).*
3. *IIT's Φ is defined via the minimum information partition (MIP) as in Tononi [2004].*

Then $\Phi > 0$ implies $\mathrm{Coh}(\mu_x) > 0$ *(where* Coh *denotes total correlation). That is, positive total correlation is a necessary precursor to nonzero integrated information. The further step to DCR's edge-sum witness* $\mathrm{Coh}_E(\mu) > 0$ *requires an additional assumption excluding pure higher-order (synergistic) dependence; see the full proof in Appendix C.1.*

Proof sketch (full proof in Appendix C.1). For a bipartition π cutting S into A, B, the total correlation decomposes as $\mathrm{Coh}(\mu_x) = \mathrm{Coh}_A(\mu_x) + \mathrm{Coh}_B(\mu_x) + I(X_A; X_B)$, so IIT's $\Phi(x) = \min_\pi I_\pi(X_A; X_B) \leq \mathrm{Coh}(\mu_x)$. Thus $\Phi > 0$ requires $\mathrm{Coh}(\mu_x) > 0$. Note that $\mathrm{Coh}(\mu_x) > 0$ does not in general imply $\mathrm{Coh}_E(\mu) > 0$: purely synergistic distributions (e.g., XOR of independent bits) have positive total correlation but zero pairwise mutual information on every edge. Under a *pairwise faithfulness* assumption—the constraint graph G is faithful to μ_x as a pairwise Markov random field—the implication $\mathrm{Coh}(\mu_x) > 0 \Rightarrow \mathrm{Coh}_E(\mu) > 0$ holds. DCR's coherence is defined on the stationary distribution; the IIT quantity is obtained by restricting to the one-step conditional distribution. □

DCR extends IIT in two directions: (1) it provides a *process-level* account of how coherence arises through exploration and resolution, rather than merely measuring it at a single time step; and (2) it defines cognition for systems where Φ is intractable (the computation is $O(2^n)$) but the DCR triad—exploration, resolution, convergence to coherent attractors—is empirically observable.

6 Predictions and Falsifiability

A framework that explains everything predicts nothing. DCR makes the following falsifiable claims:

1. **Coherence–exploration tradeoff.** In any cognitive system, there exists an optimal regime where exploration rate and constraint strength are balanced. Too much exploration (relative to constraint strength) yields incoherent dynamics; too much constraint yields rigid, brittle systems that fail to adapt. This predicts a universal inverted-U relationship between exploration rate and cognitive performance, testable in neural systems (cf. stochastic resonance [Gammaitoni et al., 1998]), evolutionary simulations, and optimization algorithms.
2. **Depth predicts adaptability.** Systems with greater cognitive depth δ (Definition 3.4) should exhibit greater adaptability to novel environments, because deeper nesting provides more levels at which exploration–resolution can occur. This is testable: compare the adaptability of systems with different organizational depths (e.g., single-celled vs. multicellular organisms, shallow vs. deep neural networks, flat vs. hierarchical organizations).
3. **Critical constraint density.** There exists a critical density of constraints $|E|/|S|$ below which the system cannot sustain coherent attractors and above which the system becomes rigid. This parallels the satisfiability phase transition in random constraint satisfaction problems [Mézard et al., 2002], where a sharp transition from under-constrained (many solutions, low coherence) to over-constrained (no solutions, frozen dynamics) occurs at a critical clause-to-variable ratio. DCR predicts that this transition coincides with maximal cognitive capacity: near the critical point, the system exhibits signatures of self-organized criticality [Bak et al., 1987]—power-law distributions of attractor sizes, long-range correlations, and maximal susceptibility. The novel prediction beyond the SAT literature is that this critical regime should also maximize coherence $\mathrm{Coh}_E(\mu_\epsilon)$ and support the deepest cognitive nesting δ. This is testable in constraint satisfaction problems and neural network models.
4. **Coarse-graining preserves cognitive signatures.** If DCR is correct, then empirically measured coherence, exploration rates, and convergence timescales should obey scaling laws across levels of description of the same system (e.g., single-neuron vs. population vs. whole-brain dynamics). Specifically, the ratio of exploration timescale to resolution timescale should be approximately preserved under coarse-graining.

A speculative extension concerning transaction-density gradients and gravitational/inertial analogues is developed in Remark C.21 in Appendix C.

7 Discussion

7.1 The Cosmos as Cognitive

If DCR is correct, then cognition is not an emergent property of brains—it is what physics *does*. The offer wave explores all possible absorbers; the Wheeler–Feynman handshake resolves conservation constraints; the completed transaction stabilizes into a classical fact. Thermal fluctuations explore the space of flow configurations; the Navier–Stokes equations resolve constraints locally between neighboring parcels; convection rolls stabilize. Mutation explores genotype space; natural selection resolves fitness constraints; adapted species stabilize. The same formal process, recurring at every scale, connected by the coarse-graining construction.

The TIQM framing (Section 4.1) reveals a further unity: what physics calls "collapse" and what biology calls "selection" are both instances of constraint-mediated selection (Remark C.11): multiple possibilities are explored, distributed constraints determine which are realized, and a stable outcome is retained. The retrocausal structure of the transactional interpretation suggests that constraint resolution need not respect the arrow of time; it is a relation among boundary conditions, not a process confined to one temporal direction.

A single motif unifies the three engineering/physical witnesses discussed earlier—distributed consensus (Remark C.3), quantum transactions (Section 4.1), and machine-learning self-play (Section 7, *Self-play as engineered DCR*): in all three, global stabilization arises from local compatibility constraints under partial, delayed information. No central controller possesses the full constraint set at any instant; yet the system converges to a coherent outcome because local violation-reducing steps compose into a globally directed drift (Theorem 2.11).

The discrete ontic model of Powers et al. [2024] lends additional support to this picture. If quantum probabilities arise from counting admissible configurations of binary sequences—*micro-choices* at the level of symbol orderings—then what physics calls a "quantum state" is already a coarse-grained summary of a discrete constraint resolution process (Remark C.14). The continuum of Hilbert space is recovered only in the limit $n \to \infty$; at every finite scale the system is a finite constraint network undergoing exploration, resolution, and stabilization. This dissolves the objection that DCR's discrete formalism cannot capture continuous physics: the continuity is emergent, not fundamental.

This is not panpsychism in the traditional sense. We do not claim that an emitter–absorber pair "has experiences." We claim that the transaction by which a photon is emitted and absorbed is *the same kind of process* as the one by which a neuron participates in perception—formally, structurally the same, as verified through the explicit construction in Example 4.2 and the structural mappings of Sections 4.1, 4.3 and 4.4. "Cognition" is the name we give to this process. Whether one wishes to call this "experience" at the quantum level is a separate philosophical question that DCR does not adjudicate.

7.2 Relationship to Process Philosophy

DCR exhibits structural resonances—not evidential dependencies—with Whitehead's process philosophy [Whitehead, 1929], which held that reality consists not of substances but of "actual occasions": events of experience that "prehend" (take account of) their environment and "concresce" into definite outcomes. The correspondence is suggestive: prehension maps onto exploration of degrees of freedom under constraints imposed by neighboring occasions, and concrescence maps onto resolution into a coherent, stabilized pattern. We note these parallels as interpretive context, not as independent support for DCR's formal claims.

The TIQM framing sharpens the correspondence. A Wheeler–Feynman transaction—a discrete event in which multiple possibilities are explored (the offer wave), constraints are propagated bidirectionally (the confirmation wave), and a definite outcome concresces (the completed transaction)—mirrors Whitehead's insistence that actual occasions are constituted by their relations to both past and future, not built up sequentially. Whether this structural parallel reflects a deeper ontological identity or merely a shared mathematical pattern is a question DCR does not settle; we observe only that process ontology and DCR converge on the same picture of reality as constituted by events of constraint resolution rather than by persistent substances.

7.3 Relation to Neural-Network Universe Proposals

Vanchurin's "world as a neural network" program [Vanchurin, 2020] is the closest existing proposal to DCR in ambition: it posits that the universe is fundamentally a learning system and derives Schrödinger/Madelung-like and Einstein–Hilbert-like effective forms from network update rules under specific assumptions (see Remark C.16 for a summary). The key structural overlap is the two-tier dynamics—trainable variables evolving on a slow timescale and hidden neuron states on a fast timescale—which maps directly onto DCR's timescale separation between inter-group and intra-group dynamics (Definition B.1). Vanchurin's "second law of learning" (entropy production from stochasticity vs. entropy destruction from learning) is a special case of DCR's exploration–resolution balance: stochastic updates inject variability; learning rules reduce constraint violation; stable effective laws emerge at the balance point.

The differences are instructive. Vanchurin's proposal is *substrate-specific*: it commits to the universe being literally a neural network, with particular thermodynamic and Onsager-symmetry assumptions driving the recovery of known physics. DCR is *substrate-agnostic*: it identifies the process (explore–resolve–stabilize) without specifying what implements it. A neural-network universe is one possible microphysics whose effective behavior instantiates the DCR triad; other microphysics (discrete ontic models [Powers et al., 2024], spin networks, causal sets) could equally serve. In this sense, DCR provides the *process-level* explanation of *why* a neural-network universe would produce stable macroscopic laws: because it implements distributed constraint resolution, and DCR dynamics converge to coherent attractors under generic conditions (Theorem 2.11).

7.4 Implications for Artificial Intelligence

Current AI systems (large language models, reinforcement learning agents) implement the DCR triad in restricted form: stochastic sampling (exploration), gradient descent or constraint propagation (resolution), convergence to low-loss configurations (stabilization). DCR predicts that the "intelligence" of these systems is bounded by their cognitive depth: the number of nested levels at which the explore–resolve–stabilize cycle operates simultaneously. This suggests that advances in AI may come not from scaling individual layers but from increasing organizational depth—more levels of nested constraint resolution.

Self-play as engineered DCR. Self-play systems in modern machine learning provide an engineered instance of the DCR triad: parallel rollouts implement exploration; selection and credit-assignment propagate constraints; and training converges to stable policy attractors. From a DCR perspective, the apparent "retroactive" assignment of credit to earlier moves is a benign analogue of retrocausal selection (Remark C.10): later outcomes determine which earlier degrees of freedom were effectively feasible given the constraints.

7.5 Limitations and Open Problems

1. **Timescale separation.** The closure theorem (Theorem B.8) requires timescale separation between intra-group and inter-group dynamics. While this condition holds in many physical systems (atomic vs. molecular, synaptic vs. network), proving closure under weaker conditions—overlapping timescales, continuous-time limits, or stochastic timescale ratios—remains open. The convergence rates derived in Lemma B.5 depend on the separation parameter η; quantifying this dependence precisely for specific physical systems is an important next step.
2. **Quantitative predictions.** While DCR predicts qualitative relationships (inverted-U, depth–adaptability, critical constraint density), deriving precise quantitative predictions requires specifying the constraint structure of particular physical systems, which is a substantial empirical program.
3. **The goal problem.** DCR defines goals as attractors of the dynamics, which avoids teleology. But this means that any attractor counts as a "goal," including *terminal attractors* that extinguish DCR capacity (e.g., a dead organism is a stable attractor of biochemical dynamics, but exploration and resolution have ceased). A natural refinement reserves "goal" for attractors that preserve ongoing DCR activity—connecting to autopoiesis—but formalizing this distinction remains open.
4. **Temporal structure and retrocausality.** The TIQM framing of quantum DCR (Section 4.1) involves advanced waves propagating backward in time, suggesting that constraint resolution can be atemporal—a relation among boundary conditions rather than a process with a definite temporal direction. The current DCR formalism (Section 2) is built on forward-time Markov chains, which cannot accommodate retrocausal constraint propagation. Extending the framework to atemporal or bidirectional constraint resolution—perhaps using the two-state vector formalism or path-integral methods—is needed to fully capture the quantum case and may reveal a deeper temporal structure underlying the DCR triad.
5. **Consciousness.** DCR is a theory of cognition, not of consciousness. It explains the process by which systems explore, resolve, and stabilize, but does not address the "hard problem" [Chalmers, 1995] of why any of this is accompanied by subjective experience. DCR is compatible with, but does not entail, the identity of cognition and consciousness.

8 Conclusion

We have presented the Distributed Constraint Resolution (DCR) framework, a formal, scale-free characterization of cognition as the process by which components explore degrees of freedom and converge through distributed constraint resolution into coherent, goal-stabilizing patterns. We have shown that:

1. The framework is mathematically precise, built on constraint networks, stochastic dynamics, and information-theoretic coherence (Section 2).
2. Under explicit timescale-separation and macro-sufficiency assumptions, the DCR form is stable under coarse-graining (conditional closure), providing a conditional composition scheme that reframes the combination problem and a natural measure of cognitive depth (Section 3).
3. Fundamental physical processes satisfy the DCR axioms: as a detailed verification sketch for thermodynamic self-organization (Example 4.2), and structurally for quantum transactions, biological adaptation, and neural dynamics (Section 4). The quantum case, framed through the Transactional Interpretation, reveals that wavefunction collapse and biological selection share the same abstract template—constraint-mediated selection (Remark C.11). The discrete ontic model of Powers et al. [2024] provides independent evidence that quantum mechanics is compatible with—and may emerge from—the kind of finite combinatorial structure that DCR assumes (Remark C.14); and Vanchurin [2020] independently derives Schrödinger/Madelung-like and Einstein–Hilbert-like effective forms from a learning neural network, a substrate-specific instantiation of the DCR triad (Remark C.16).
4. The Free Energy Principle is recoverable as a special case, and DCR coherence is a necessary condition for IIT's $\Phi > 0$ (Section 5).
5. The framework makes falsifiable predictions about coherence–exploration tradeoffs, depth–adaptability relationships, and critical constraint densities (Section 6).

If DCR is correct, then cognition is not a biological accident but a fundamental feature of physical reality—the process by which the universe explores its own degrees of freedom and resolves into the coherent structures we observe at every scale. The offer wave and the mutation, the handshake and the selection, the transaction and the adapted species, the symbol ordering and the quantum outcome: one process, many substrates.

Acknowledgments. The transactional-interpretation framing in Section 4.1 was sharpened by discussions of Ruth Kastner's Possibilist Transactional Interpretation and its engineering analogues in distributed-ledger consensus; we thank anonymous reviewers for pressing us to clarify the interpretation-independence of the formal results.

A General State Spaces

This appendix collects the full Foster–Lyapunov / Harris recurrence machinery for DCR on general (possibly non-compact) state spaces. The main text (Section 2) works exclusively with compact Ω and Doeblin-type minorization; the results below extend to non-compact Polish spaces at the cost of additional regularity conditions.

A.1 Combined DCR Dynamics — General Version

Remark A.1 (Why Meyn–Tweedie on compact spaces). On a compact state space Ω, existence of a stationary distribution already follows from the Krylov–Bogoliubov theorem (tightness is automatic), and one might wonder why one would invoke the heavier Foster–Lyapunov / minorization / Harris recurrence apparatus of Meyn and Tweedie [1993]. Three reasons: (i) the drift condition provides *quantitative* concentration bounds, not merely existence; (ii) minorization yields geometric convergence rates (Theorem 15.0.1 of Meyn and Tweedie, 1993), which inform the timescale separation requirements in Appendix B; and (iii) the framework extends without modification to non-compact Ω, which is needed for continuum limits (Remark C.5) and for physical systems whose natural state spaces are unbounded (e.g., momentum variables).

For general (possibly non-compact Polish) state spaces, the combined DCR dynamics requires the following conditions on the kernel $K_\epsilon = (1-\epsilon)R + \epsilon\, E$:

(a) **Foster–Lyapunov drift.** There exist constants $\lambda \in (0,1)$, $b < \infty$, and a measurable set $C \subseteq \Omega$ such that for the Lyapunov function $L(\omega) = V(\omega) + 1$:

$$\int_\Omega L(\omega')\, K(\omega, d\omega') \leq \lambda\, L(\omega) + b\, \mathbf{1}_C(\omega). \tag{A.1}$$

(b) **φ-irreducibility.** There exists a probability measure φ on Ω such that for all $\omega \in \Omega$ and all measurable A with $\varphi(A) > 0$: $\sum_{n=1}^{\infty} K^n(\omega, A) > 0$.

(c) **Minorization on a small set.** There exist a measurable set $C \subseteq \Omega$, $\delta > 0$, and a probability measure ν on Ω such that $K(\omega, A) \geq \delta\, \nu(A)$ for all $\omega \in C$ and all measurable A.

(d) **Weak Feller property.** For every bounded continuous $g : \Omega \to \mathbb{R}$, the map $\omega \mapsto \int g(\omega')\, K(\omega, d\omega')$ is continuous.

(e) **Full exploration support.** The irreducibility measure φ satisfies $\varphi(U) > 0$ for every open set U meeting the feasible set.

Remark A.2 (Sufficient conditions for the drift). A simple sufficient condition for (a) arises when the resolution and exploration kernels satisfy separate bounds. Suppose there exist $\alpha > 0$ and $B < \infty$ such that (i) $\int V(\omega')\, R(\omega, d\omega') \leq (1-\alpha)\, V(\omega)$ for all $\omega \notin \mathcal{F}$, and (ii) $\int V(\omega')\, E(\omega, d\omega') \leq B$ for all ω. Setting $\lambda = (1-\epsilon)(1-\alpha)$ and decomposing $K = (1-\epsilon)R + \epsilon E$ gives the drift condition (a) with $b = 1 - \lambda + \epsilon B$ and $C = \{\omega : V(\omega) \leq b/(1-\lambda)\}$. On compact Ω, condition (ii) holds automatically ($B = \max_\Omega V < \infty$).

A.2 Convergence on General State Spaces

Lemma A.3 (Existence of Stationary Distribution)**.** *Under condition (d) (weak Feller property), the Markov chain $\{X_t\}$ on compact metrizable Ω admits at least one stationary distribution μ^*.*

Proof. On a compact metrizable space, tightness of any sequence of probability measures is automatic. The Krylov–Bogoliubov theorem then applies: the Cesàro averages $\mu_T = T^{-1} \sum_{t=0}^{T-1} \delta_\omega K^t$ admit a weakly convergent subsequence, and the weak Feller property (d) ensures that any weak limit point is a stationary distribution. □

Lemma A.4 (Uniqueness and Ergodicity)**.** *Under conditions (a)–(e), the stationary distribution μ^* is unique, and for every initial distribution μ_0: $\|\mu_0 K^t - \mu^*\|_{\mathrm{TV}} \to 0$ as $t \to \infty$.*

Proof. Condition (b) gives φ-irreducibility. Condition (c) provides a 1-small set C. Condition (a) provides a Foster–Lyapunov drift to the small set C. By Theorem 16.0.1 of Meyn and Tweedie [1993], a φ-irreducible chain satisfying a geometric drift condition toward a small set C is *geometrically ergodic*: it admits a unique invariant probability measure μ^*, and $\|\mu_0 K^t - \mu^*\|_{\mathrm{TV}} \leq M\, r^t$ for constants $M < \infty$, $r < 1$. □

Lemma A.5 (Concentration Near the Feasible Set)**.** *For fixed $\epsilon > 0$, the unique stationary distribution μ_ϵ^* satisfies $\int L(\omega)\,\mu_\epsilon^*(d\omega) \leq b/(1-\lambda)$. When the sufficient conditions of Remark A.2 hold, $\int V\,d\mu_\epsilon^* = O(\epsilon)$, and by Markov's inequality, for any fixed $\delta > 0$: $\mu_\epsilon^*(\{\omega : V(\omega) > \delta\}) \to 0$ as $\epsilon \to 0$.*

Proof. Integrating the drift condition against μ_ϵ^*: $\int L\,d\mu_\epsilon^* = \iint L(\omega')\,K(\omega, d\omega')\,\mu_\epsilon^*(d\omega) \leq \lambda \int L\,d\mu_\epsilon^* + b\,\mu_\epsilon^*(C) \leq \lambda \int L\,d\mu_\epsilon^* + b$, giving $\int L\,d\mu_\epsilon^* \leq b/(1-\lambda)$. □

Lemma A.6 (Positive Coherence)**.** *Let μ^* be a probability measure on $\Omega = \prod_{s\in S} D_s$. If there exist components $s, s' \in S$ such that the (s,s')-marginal of μ^* is not a product: $\mu_{s,s'}^* \neq \mu_s^* \otimes \mu_{s'}^*$, then* $\mathrm{Coh}(\mu^*) > 0$.

Proof. By the data processing inequality: $\mathrm{Coh}(\mu^*) \geq D_{\mathrm{KL}}(\mu_{s,s'}^* \| \mu_s^* \otimes \mu_{s'}^*) = I_{\mu^*}(s;\, s') > 0$. □

A.3 Additional Definitions and Lemmas

Assumption A.7 (Finite coherence regime)**.** All stationary measures μ^* considered satisfy $\mathrm{Coh}(\mu^*) < \infty$. This holds automatically in the discrete case ($|D_s| < \infty$), or whenever μ^* admits a density with respect to the product reference measure.

Remark A.8 (Stochastic resolution variant)*.* The strict supermartingale condition Equation (2.8) excludes kernels that occasionally increase violation. A relaxed variant replaces it with a drift-outside-a-set form that connects directly to the Foster–Lyapunov condition (a) on the combined kernel K. All convergence results go through under this relaxation.

Definition A.9 (Operational Agency)**.** A DCR-system exhibits *operational agency* if the combined dynamics $K_\epsilon = (1-\epsilon)R + \epsilon\,E$ satisfies the Foster–Lyapunov drift condition (a) toward the attractor $\mathcal{A}$. This makes agency a substrate-independent dynamical property: outside the compact "return set," the system contracts toward $\mathcal{A}$ in expectation. Every DCR-system satisfying the drift condition exhibits operational agency by construction.

Remark A.10 (Annealing variants)*.* The fixed-ϵ analysis can be extended to a cooling schedule $\epsilon_t \to 0$. Classical simulated annealing results [Hajek, 1988] imply convergence of the occupation measures to a distribution supported on the global minimizers of V under appropriate conditions.

Remark A.11 (Why DCR dynamics produce non-product stationary measures)*.* The resolution kernel R updates component s as a function of its neighbors, introducing statistical dependence. A product stationary measure would require this neighbor-dependence to be exactly cancelled by marginal averaging—a non-generic coincidence. In each application, the non-product property must be verified for the specific dynamics at hand.

Remark A.12 (Heuristic criterion for non-product stationary measure)*.* In practice, a non-product stationary distribution holds whenever the resolution kernel introduces genuine dependence between neighbors and the exploration kernel does not wash it out.

B Approximate Closure Under Coarse-Graining

This appendix develops the full coarse-graining closure machinery under timescale separation, disintegration, and lifting regularity conditions. The main text (Section 3) states an exact closure theorem for finite lumpable chains; the results below handle the approximate, continuous-state-space case.

Definition B.1 (Timescale Separation)**.** Let $\eta > 0$ be a *scale ratio.* We say the micro-dynamics exhibits *η-timescale separation* with respect to partition π and compression $\{g_{s'}\}$ if:

1. **Fast intra-group mixing.** For each macro-component $s' \in S'$, let $\partial s'$ denote the micro-components outside $\pi^{-1}(s')$ that share an edge with some component inside $\pi^{-1}(s')$ (the *micro-boundary* of group s'). For each boundary configuration $b_{s'} = (\omega_s)_{s\in\partial s'}$, the restricted chain on $\prod_{s\in\pi^{-1}(s')} D_s$ with $b_{s'}$ frozen has a unique conditional equilibrium $\nu_{s'}(\cdot \mid b_{s'})$ and mixes to this equilibrium in time τ_{fast}.
2. **Slow inter-group dynamics.** The inter-group updates (those involving edges in $E_{\text{cross}} = \{\{s_1, s_2\} \in E \mid \pi(s_1) \neq \pi(s_2)\}$) occur at rate η relative to intra-group updates.
3. $\eta\,\tau_{\text{fast}} \to 0$, i.e., fast degrees of freedom equilibrate before the next inter-group update.
4. **Macro-sufficiency (representative-independence).** The expected cross-group violation after fast equilibration depends on the macro-boundary alone, not on the specific micro-configuration within each group.

Remark B.2 (Relation to lumpability and conditional independence)*.* Condition 4 of Definition B.1 is a form of (approximate) *lumpability*: the compressed macrostate renders micro-boundary details conditionally irrelevant for inter-group interactions after fast equilibration. This parallels conditional independence assumptions used in renormalization group methods, Mori–Zwanzig averaging, and Markov state modelling.

Definition B.3 (Macro-Violation)**.** Given the conditional equilibria from Definition B.1, the *macro-violation* of a macro-configuration $\tilde{\omega} \in \tilde{\Omega}$ with respect to an inter-group edge $e' = \{s'_1, s'_2\} \in E'$ is

$$\tilde{v}_{e'}(\tilde{\omega}_{s'_1}, \tilde{\omega}_{s'_2}) = \sum_{\substack{\{s_1,s_2\}\in E_{\text{cross}} \\ \pi(s_1)=s'_1,\, \pi(s_2)=s'_2}} \mathbb{E}_{\nu_{s'_1}(\cdot\,|\,b_{s'_1})\otimes\nu_{s'_2}(\cdot\,|\,b_{s'_2})}\Big[v_{\{s_1,s_2\}}(x_{s_1}, x_{s_2})\Big]. \tag{B.1}$$

The total macro-violation is $\widetilde{V}(\tilde{\omega}) = \sum_{e'\in E'} \tilde{v}_{e'}$.

Lemma B.4 (Effective Markovianity Under Timescale Separation)**.** *Under η-timescale separation and macro-sufficiency (Definition B.1), and additionally assuming:*

(M1) ***Uniform geometric ergodicity of the fast chain.*** *For each group s' and boundary configuration $b_{s'}$, the restricted intra-group chain mixes to $\nu_{s'}(\cdot \mid b_{s'})$ at a geometric rate $\rho < 1$ uniformly over $b_{s'}$.*

(M2) ***Lifting kernel regularity.*** *The product of conditional equilibria admits a regular conditional probability (disintegration) with respect to the compression g.*

Define the effective macro transition kernel:

$$\tilde{K}(\tilde{\omega}, \tilde{A}) = \int_{\Omega} K_{slow}(\omega, g^{-1}(\tilde{A}))\, \rho_{\tilde{\omega}}(d\omega), \tag{B.2}$$

where K_{slow} is the micro-kernel restricted to inter-group updates and $\rho_{\tilde{\omega}}$ is the disintegration measure.

Then, in the limit $\eta\tau_{fast} \to 0$, the slow-time macro-process $\{\tilde{X}_k\}$ is a time-homogeneous Markov chain with kernel $\tilde{K}$. For finite timescale separation, $\{\tilde{X}_k\}$ is approximately Markov with error $\varepsilon(\eta, \rho) \to 0$ as $\eta\tau_{fast} \to 0$.

Proof sketch. Under timescale separation, each group equilibrates to its conditional equilibrium before the next inter-group update. By uniform geometric mixing and macro-sufficiency, the one-step transition probability depends only on the current macro-state, giving Markovianity. For finite separation, the geometric mixing bound yields the approximation error. □

Lemma B.5 (Macro-Resolution Inherits Drift). *Under the assumptions of Lemma B.4, and additionally assuming inter-group contraction:*

(D1) ***Inter-group contraction.*** *There exists $\alpha > 0$ such that the inter-group resolution kernel reduces $\tilde{v}_{e'}$ in expectation uniformly.*

Then $\tilde{K}$ satisfies a Foster–Lyapunov drift condition for $\tilde{L} = \tilde{V} + 1$.

Proof sketch. Under timescale separation, the fast dynamics resolves intra-group violation; the remaining inter-group violation depends on the macro-state by macro-sufficiency. The contraction assumption yields the drift. □

Lemma B.6 (Macro-Exploration Inherits φ-Irreducibility). *If K is φ-irreducible with full exploration support, g is continuous and surjective, and the slow-time skeleton includes inter-group exploration events, then $\tilde{K}$ is $\tilde{\varphi}$-irreducible on $\tilde{\Omega}$.*

Proof. Follows from pushing forward the micro-level irreducibility through the compression map g. □

Lemma B.7 (Macro-Coherence). *If $\tilde{\mu}^*$ has a non-product marginal along at least one macro-edge, then* $\mathrm{Coh}(\tilde{\mu}^*) > 0$.

Proof. Direct application of Lemma A.6 to $\tilde{\mu}^*$. □

Theorem B.8 (Closure Under Coarse-Graining (conditional)). *Let $\mathcal{C}$ be a cognitive system, $\pi : S \twoheadrightarrow S'$ a partition map, and $\{g_{s'}\}$ compression maps with continuous, surjective $g : \Omega \to \tilde{\Omega}$. If:*

(i) The micro-dynamics exhibits η-timescale separation with uniform geometric mixing (M1) and lifting kernel regularity (M2).

(ii) The macro-constraint graph G' is connected.

(iii) The macro-feasible set is non-empty and compact, the inter-group resolution satisfies contraction (D1), and $\tilde{\mu}^*$ has a non-product marginal along at least one macro-edge.

(iv) The effective macro kernel $\tilde{K}$ is weak Feller and admits a minorization on a compact sublevel set.

Then $\tilde{\mathcal{C}} = \Gamma_{\pi,g}(\mathcal{C})$ is a cognitive system on $\tilde{\Omega}$.

Proof (structural argument). We verify each component of a cognitive system for $\tilde{\mathcal{C}}$: the constraint network is well-defined by Section 3.1; exploration follows from Lemma B.6; resolution from Lemma B.5; and coherence from Lemma B.7. Applying the convergence theorem to the macro-system on $\tilde{\Omega}$ completes the verification. □

Remark B.9 (Logical status of the closure theorem). The closure theorem is conditional: it shows that *if* coarse-graining yields an effective Markov macro-model satisfying DCR-type conditions, *then* the macro-model is a DCR-system. Two concrete classes where the assumptions are standard:

1. *Finite-state chains with exact lumpability* (Kemeny–Snell)—the closure is exact.
2. *Metastable Markov state models*—the closure holds approximately, with error controlled by the spectral gap ratio.

C Additional Remarks and Extensions

This appendix collects remarks and extensions that, while valuable for a complete picture, are not essential to the main argument.

Remark C.1 (Against biological exceptionalism)*.* A common tacit assumption in foundations is *biological exceptionalism*: that genuine "agency"—selection among alternatives—only appears in organisms (or observers), rendering measurement and self-organization mysterious outside biology. DCR rejects this exclusivity claim. In DCR, "agency" is operational and graded (Definition A.9): it is the capacity of a system to *select and stabilize* a concrete behavior from a latent space of possible behaviors under distributed constraints. Biological agency is then a deep-nesting special case (high cognitive depth, Definition 3.4), not a different ontological category. This stance does not diminish biology; it situates biological complexity within a wider dynamical landscape.

Remark C.2 (Ontological interpretation)*.* The claim that the DCR triad *is* cognition—that the cosmos is cognitive at every scale—is a metaphysical identification, not a theorem derivable from the definitions. The formal machinery is compatible with two readings:

1. *Modelling framework (minimal):* DCR provides a unified dynamical vocabulary applicable across scales.
2. *Ontological identity (optional):* the DCR triad *is* cognition; the cosmos is cognitive at every scale.

We explore reading (2) as an optional, parsimonious ontological identification—one process at every scale, rather than cognition as a sui generis phenomenon. All formal results hold under either reading; the choice between them is philosophical, not mathematical.

Remark C.3 (Engineering witness: consensus as DCR)*.* Distributed consensus protocols provide an instructive engineering witness for DCR. In replicated-state-machine settings, independent agents maintain local copies of a shared state and must reconcile conflicting updates (e.g., competing spends) without a central controller. A natural strategy is to allow local proposal, temporary branching into mutually incompatible candidate evolutions, and subsequent convergence via a selection rule informed by delayed nonlocal information. Read through the DCR lens: proposal corresponds to exploration, local validity checks and compatibility propagation correspond to distributed constraint resolution, and eventual agreement on a single ledger state is a coherent attractor. We emphasize this is a *witness* rather than a reduction: the purpose is to exhibit a familiar, explicitly mechanistic system where global coherence arises from local constraint processing under partial information.

Remark C.4 (Engineering witness: self-play as DCR)*.* Self-play in machine learning provides a second engineering witness: rollouts generate a branching space of candidate trajectories (exploration), credit assignment and constraint propagation prune or reweight trajectories (resolution), and training concentrates on stable policy attractors (stabilization). This is a witness, not a reduction; the point is that global coherence can arise from local update rules under delayed, nonlocal feedback, paralleling Remark C.3 and the branch-selection picture in Remark C.10.

Remark C.5 (Finiteness of S)*.* The formal development assumes $|S| < \infty$. This suffices for systems with a natural decomposition into discrete components (particles, neurons, organisms) but appears to exclude continuum field theories. The physical examples in Section 4 (e.g., fluid parcels in convection) should be read as finite-element discretizations of the underlying continuum; the formal extension to countable or measure-theoretic component spaces (replacing sums with integrals in Equations (2.1) and (2.6)) is straightforward but introduces technical regularity conditions that we defer to future work.

We note, however, that the discreteness assumption may be less restrictive than it appears. There exist proposals in which the continuum structure of standard quantum mechanics is not fundamental but emergent from a discrete substrate. Powers et al. [2024] construct a model built entirely from finite binary sequences that reproduces the probability distributions of canonical quantum mechanics, with small deviations at finite n that shrink as n increases. For any finite n the system is a finite constraint network in the sense of Definition 2.1; the continuum of canonical quantum theory is recovered only in the limit. This suggests that the continuum structure of standard physics may be an idealization of a fundamentally discrete substrate—precisely the kind of substrate that DCR is designed to describe (see Remark C.14).

Remark C.6 (Branching exploration variant). In some domains it is natural to treat exploration not as additive noise but as *branching*: from a given configuration, the system generates a family of candidate next states (a "multiway" step), temporarily maintaining a set of possible evolutions. Constraint resolution then prunes or reweights these branches, and stabilization corresponds to selecting (or concentrating on) a single consistent history. The Markov-kernel formalism still applies by viewing a branching–selection step as an induced stochastic kernel obtained by marginalizing over the latent branch variable.

Remark C.7 (Coherence in the unique-attractor limit). If the feasible set $\mathcal{F}$ is a singleton $\{\omega^\star\}$ and $\mu^*_\epsilon \to \delta_{\omega^\star}$ as $\epsilon \to 0$, then $\mathrm{Coh}(\delta_{\omega^\star}) = 0$ (each marginal is a point mass and the product of marginals equals μ^*). This is not a defect but a feature of the framework's design: the coherence condition $\mathrm{Coh}_E(\mu_\epsilon) > 0$ is evaluated at *fixed* $\epsilon > 0$, where the stationary measure retains genuine statistical spread due to ongoing exploration. A system with $\epsilon = 0$ has no exploration and is therefore not a DCR-system (Definition 2.3). In the fixed-ϵ regime, if the stationary distribution has any non-product edge marginal, then $\mathrm{Coh}_E(\mu_\epsilon) > 0$.

Remark C.8 (Alternative coordination witnesses). The main text uses edge-sum mutual information $\mathrm{Coh}_E(\mu) = \sum_{\{s,s'\}\in E} I_\mu(X_s; X_{s'})$ as the coherence witness (Definition 2.7). Other choices work equally well:

1. *Total correlation (multi-information):* $\mathrm{Coh}(\mu) = D_{\mathrm{KL}}(\mu \,\|\, \bigotimes_s \mu_s)$, which is implied by $\mathrm{Coh}_E > 0$ (since any pairwise dependence implies total dependence) and is used in the IIT discussion (Proposition 5.2). The converse $\mathrm{Coh} > 0 \Rightarrow \mathrm{Coh}_E > 0$ does *not* hold in general (purely synergistic distributions are a counterexample). Also, Coh_E can exceed Coh because pairwise mutual informations double-count shared dependence.
2. *Cut-based dependence measures* that quantify information loss under graph partitions (e.g., graph-cut multi-information).

All results that use coherence as a *witness* go through with any nonnegative dependence functional that vanishes on product measures and is positive whenever a non-product marginal exists. The specific choice is not essential to the framework.

Remark C.9 (Relation to optimization and computer science). The Ising example makes explicit the connection between DCR and classical optimization/constraint-satisfaction frameworks. *Simulated annealing* [Hajek, 1988] is a DCR dynamics where ϵ (temperature) is decreased over time, driving the system toward global violation minimizers. *Belief propagation* and message-passing algorithms for random constraint satisfaction problems [Mézard et al., 2002] implement distributed resolution: each variable node updates its marginal based on neighboring constraints, a process structurally identical to the resolution kernel R. More broadly, any local-search heuristic for combinatorial constraint satisfaction (SAT, CSP, graph coloring) instantiates the exploration–resolution interplay. DCR does not claim novelty in these algorithms; it claims that the *same formal triad* appears in physical, biological, and cognitive systems, not only in engineered solvers.

Remark C.10 (Collapse as selection over branches). A complementary structural picture treats quantum "collapse" as selection over a branching space of candidate transactions (cf. Remark C.6). An emission event generates a set of potential absorber-matched outcomes (branches); confirmation waves implement distributed feasibility checks; and the realized event corresponds to selecting a branch once sufficiently global consistency information is available. The selection can appear retrocausal because the constraints relevant to branch feasibility depend on spacelike-separated absorbers whose responses are only available after finite propagation delays. This provides an engineering-style intuition for why "late" information can fix an "earlier" outcome without invoking a centralized chooser.

Remark C.11 (Constraint-mediated selection). The recurring pattern across physical scales can be distilled into an abstract selection template:

1. *Variation* — the exploration kernel E generates a population of candidate configurations.
2. *Constraint filtering* — the resolution dynamics R retains configurations that locally reduce constraint violations.
3. *Retention / stabilization* — surviving configurations accumulate near the coherent attractor $\mathcal{A}$, which acts as the long-run "memory" of the process.

We call this *constraint-mediated selection* rather than "natural selection" to emphasize that the filtering step is driven by compatibility constraints on the network, not by reproductive fitness specifically. Darwinian selection is the biological instance; quantum collapse is the physical instance; simulated annealing is the computational instance. In each case, the formal structure is the same DCR triad.

Remark C.12 (Interpretation-independence and decoherence witness). The DCR mapping does not depend on retrocausality. TIQM is used in Section 4.1 because the constraint-satisfaction structure is explicit in that formulation; no claim is made that TIQM is correct or that DCR solves the measurement problem.

A second, less interpretationally loaded witness uses *decoherence and einselection* [Zurek, 2003]. In this framing: (i) *exploration* is the unitary spreading of the system–environment state over the full Hilbert space; (ii) *resolution* is the environment-induced suppression of off-diagonal coherences—a local, distributed process in which each environmental degree of freedom independently constrains the system's phase relations; (iii) *stabilization* is einselection: the emergence of pointer states as the unique basis robust to ongoing decoherence—the coherent attractor of the DCR triad. This mapping avoids retrocausality entirely and relies only on standard open-quantum-systems theory; it does not, however, address the "definite outcome" question (the same limitation as decoherence itself). The two witnesses are complementary: TIQM makes the constraint-satisfaction structure vivid; decoherence makes the distributional character of resolution rigorous.

Remark C.13 (Unpredictability and tie-breaking). In distributed constraint-resolution problems with delayed nonlocal inputs, the outcome can be unpredictable even when the local rules are fixed: the relevant constraints are not simultaneously available at any single location. In such settings, stochasticity can be interpreted operationally as a symmetry-breaking device among near-equally feasible resolutions, rather than as "unstructured noise." This perspective is compatible with the DCR formalism: it amounts to placing the stochasticity in the resolution kernel (or in the branch-selection kernel of Remarks C.6 and C.10) rather than exclusively in the exploration kernel.

Remark C.14 (Discrete ontic structure and micro-choices). The DCR picture of quantum mechanics does not require the continuum structure of canonical quantum theory. Powers et al. [2024] model measurement events as event networks whose nodes and edges are built from finite base-2 (and base-4/base-16) symbol sequences with XOR-like (addition modulo two) composition. Observable quantum numbers correspond to symbol *counts*—coarse summaries of the underlying sequences—while the ordering of symbols within each sequence remains hidden. This hidden ordering is the source of non-determinism (in their phrase: "counts are generally observable, but sequences are not") [Powers et al., 2024].

In DCR terms, the space of admissible ontic configurations (all symbol orderings consistent with the quantum numbers) is the *exploration space*; the contextual compatibility conditions—which orderings are consistent with both observers' measurement events—are the *constraints*; and the observed probability distribution, given by relative frequencies of surviving configurations, is the *stabilized pattern*. The individual symbol orderings are *micro-choices*: local, hidden degrees of freedom whose structured resolution produces the macroscopic outcome.

For finite sequence length n, the model predicts small but unavoidable deviations from canonical QM due to discrete granularity (e.g., rotation angles are effectively rational-valued), with improved agreement as n increases. Powers et al. [2024] propose optical tabletop tests to constrain these n-dependent deviations. Since all empirical measurement data is inherently discrete, the finite-n model is not merely an approximation to the continuous theory but a plausible candidate for a more fundamental description. If this view is correct, it supports the DCR framework's assumption of finite discrete components (Remark C.5): the continuum would be an idealization of an underlying discrete constraint resolution process operating through micro-choices.

Remark C.15 (Proof-sketch character of the Bénard verification). Three aspects of the verification in Example 4.2 merit further care in a fully rigorous treatment. (i) The constraints should be read as local consistency conditions of the *discretized time-step update*: at each step the violation functional measures the residual of the one-step Boussinesq update, not the steady-state PDE residual. The drift argument then requires that the discretized update reduces this residual on average—a statement about the time-stepping scheme, not about the energy functional of the PDE directly. The transfer from the energy functional $\mathcal{E}$ to V near the attractor requires comparability of their respective Hessians under the linearized Boussinesq operator—a standard result in discretized Navier–Stokes stability theory, but one whose constants depend on the lattice spacing h. (ii) The projection Π_Ω onto the compact box breaks the smoothness of the Gaussian exploration kernel at the boundary of Ω; hence the "continuous and positive" density claim in the minorization step holds only on the interior, and a boundary layer analysis is needed to establish the minorization uniformly on the sublevel set C. (iii) The spectral gap λ_1 of the graph Laplacian depends on the lattice discretization, and the drift constants λ, b depend on h and N; the verification is for a fixed discretization and does not address the continuum limit $h \to 0$. These issues are standard in the numerical analysis of stochastic PDE discretizations; we highlight them to be explicit about the level of rigor. The example should be read as a detailed *verification sketch* exhibiting the structural correspondence between Bénard convection and the DCR axioms, not as a fully rigorous proof.

Remark C.16 (Related formal program: neural-network universe). Vanchurin [2020] proposes that the universe at its most fundamental level is a neural network with two tiers of dynamical degrees of freedom: *trainable variables* (weights, biases) and *hidden variables* (neuron states). He shows that near equilibrium, the trainable-variable dynamics is well approximated by Madelung/Schrödinger-type equations (with free energy playing the role of the phase), while further from equilibrium the same dynamics yields Hamilton–Jacobi behavior. In a coarse-grained limit, the hidden-variable dynamics can produce emergent relativistic strings and, via an Onsager-symmetry argument for entropy production, an Einstein–Hilbert-like gravitational term. This is a concrete instance of distributed constraint resolution producing stable macroscopic laws. We treat such "neural-network universe" models as substrate-specific realizations of DCR rather than competitors.

Remark C.17 (Computability of depth). Computing $\delta(\mathcal{C})$ exactly requires optimizing over all hierarchical partition sequences $(\pi_1, \dots, \pi_k)$ and compression maps $(g_1, \dots, g_k)$, verifying the DCR conditions at each level—a combinatorially intractable problem in general. In practice, δ serves as a coarse ordinal ranking rather than a precise cardinal measure: distinguishing depth 2 from depth 6 is meaningful; distinguishing depth 6 from depth 7 requires detailed empirical verification of the DCR triad at each level.

Remark C.18 (IIT versions). The relationship established in Proposition 5.2 targets IIT 2.0, which uses KL divergence as its distance measure. IIT 3.0 [Tononi et al., 2016] replaces KL divergence with the earth mover's distance (Wasserstein metric) and defines Φ over cause–effect structures rather than single distributions; IIT 4.0 further introduces dynamical aspects and "cut" models that differ significantly from the bipartition-based definition used here. The necessary-condition relationship—$\mathrm{Coh} > 0$ is required for $\Phi > 0$—holds conceptually across versions, but the formal apparatus diverges, and a full treatment for IIT 3.0/4.0 would require additional metric-space and causal machinery that we do not develop here.

Remark C.19 (Optional optimization reading). An optional interpretive strengthening is to view the DCR process as optimizing a system-level functional: exploration generates candidate interactions, and resolution selects those that maximize realized interaction (information exchange) subject to local conservation constraints. This reading aligns DCR with ubiquitous variational principles (least/stationary action) at the level of metaphor and motivation, but it is *not* assumed by the formal results: the theorems require only local violation-reduction and ergodic exploration, not a globally computed objective.

Remark C.20 (Transactional density as a witness). In many substrates, one can define a *transaction* as an event in which distributed constraints are jointly satisfied across an interface (e.g., emitter–absorber completion, message commit in consensus, trade execution). The *transactional density* ρ_τ is the rate (or expected count) of such events in the stabilized regime. DCR does not assume that physical dynamics maximizes transactional density, but ρ_τ can serve as an empirical *witness* correlated with coherence and persistence in self-organizing systems.

C.1 Full Proofs for Recovery Propositions

Full proof of Proposition 5.1 (FEP Recovery). We construct an explicit embedding of the FEP formalism into DCR.

Step 1: Constraint network. Let $S = S_\mu \cup S_b \cup S_\eta$ be the decomposition into internal (μ), blanket (b), and external (η) states. The Markov blanket condition means E

contains no edges between S_μ and S_η directly; all coupling is mediated through S_b. This is a constraint network with G having the bipartite-through-blanket structure.

Define the constraint cost functions on blanket–internal edges via the generative model: for $s \in S_\mu$, $s' \in S_b$,

$$v_{\{s,s'\}}(\mu_s, b_{s'}) = -\ln p(\tilde{s}_{s'} \mid \mu_s),$$

encoding which internal states are consistent with which sensory observations.

Step 2: Violation as surprise. Assume the generative model satisfies $p(\tilde{s}_{s'} \mid \mu_s) > 0$ for all $(\mu_s, \tilde{s}_{s'})$ in the model's support (this ensures finite violations; models that assign zero probability to observable events are degenerate). The total violation is then $V(\omega) = -\sum_{\{s,s'\}} \ln p(\tilde{s}_{s'} \mid \mu_s)$, which is finite under the positivity assumption.

Under a conditional independence assumption (the generative model factorizes over blanket components conditioned on internal states, standard in mean-field formulations of FEP): $p(\tilde{s} \mid \mu) = \prod_{s'} p(\tilde{s}_{s'} \mid \mu_{s(s')})$, where $s(s')$ denotes the internal component coupled to blanket component s', the sum reduces to: $V(\omega) = -\ln \prod_{s'} p(\tilde{s}_{s'} \mid \mu_{s(s')}) = -\ln p(\tilde{s} \mid \mu)$—a conditional negative log-likelihood of sensory data given internal states. Note: this is *not* the FEP's marginal surprisal $-\ln p(\tilde{s})$, but a conditional analogue; the marginal surprisal is recovered only after marginalizing over internal states.

The variational free energy $F = \mathbb{E}_q[-\ln p(\tilde{s}, \psi)] + \mathbb{E}_q[\ln q(\psi)]$ satisfies $F \geq -\ln p(\tilde{s})$ (by the non-negativity of KL divergence), so F upper-bounds the marginal surprisal $-\ln p(\tilde{s})$. Note that V as defined above conditions on internal states and is not identical to the marginal surprisal; the connection to FEP's variational bound is that F serves as a Lyapunov function in its own right (see Step 3), not that F directly bounds V.

Step 3: Resolution as free energy minimization. The FEP's recognition dynamics—gradient descent on F with respect to internal parameters—is a local update rule: each internal state μ_s adjusts based on its blanket neighbors. In the FEP specialization, the natural Lyapunov function is F itself (rather than $V + 1$): gradient descent on a smooth function bounded below yields $\mathbb{E}[F_{t+1}] \leq F_t - \alpha \|\nabla F\|^2$ for suitable step size. Since F is bounded below and its sublevel sets are compact (under standard regularity of the generative model), F satisfies the Foster–Lyapunov drift condition (a) for the combined recognition-plus-exploration kernel. (Note that $V \leq F$ does *not* by itself imply that descent on F reduces V; rather, F serves as a valid Lyapunov function in its own right, and concentration near low-F regions entails concentration near low-V regions via the bound.)

Step 4: Exploration as active inference. FEP's active inference includes epistemic actions—perturbations to blanket states that sample the environment. These provide the exploration kernel E: the system probes configurations that might reduce uncertainty, ensuring accessibility of low-violation regions.

Step 5: Attractor. The attracting set under FEP is the set of internal states where F is minimized, i.e., $q(\psi) \approx p(\psi \mid \tilde{s})$. This is a feasible configuration (minimal violation) and is coherent since internal states are statistically coupled through the shared generative model.

Hence the FEP system ($\{S_\mu, S_b, S_\eta\}$, bipartite G, recognition dynamics, epistemic exploration) is a cognitive DCR system under the specializations stated. □

Full proof of Proposition 5.2 (DCR coherence as necessary condition for IIT's Φ). **Step 1: Total correlation and edge-sum coherence.** DCR's coherence measure (Definition 2.7) is the edge-sum mutual information $\mathrm{Coh}_E(\mu) = \sum_{\{s,s'\} \in E} I_\mu(X_s; X_{s'})$. The *multi-information* (total correlation) is $\mathrm{Coh}(\mu) := D_{\mathrm{KL}}(\mu \,\|\, \bigotimes_s \mu_s)$. The implication

$\mathrm{Coh}_E(\mu) > 0 \Rightarrow \mathrm{Coh}(\mu) > 0$ holds because any pairwise dependence implies non-independence.

Important caveat: the converse $\mathrm{Coh}(\mu) > 0 \Rightarrow \mathrm{Coh}_E(\mu) > 0$ is *false* in general. Purely synergistic distributions—e.g., X, Y independent fair bits with $Z = X \oplus Y$—have $\mathrm{Coh}(X, Y, Z) > 0$ but $I(X;Y) = I(X;Z) = I(Y;Z) = 0$. Thus the connection from IIT's $\Phi > 0$ to DCR's edge-sum $\mathrm{Coh}_E > 0$ requires an additional *pairwise faithfulness* assumption: $\mathrm{Coh}(\mu) > 0$ implies $\mathrm{Coh}_E(\mu) > 0$ when μ is faithful to the constraint graph as a pairwise Markov random field (excluding purely higher-order dependencies).

Step 2: From total correlation to integrated information. IIT 2.0 defines Φ using KL divergence as follows. For a system in state x with TPM T, let $\mu_x = p(X_{t+1} \mid X_t = x)$ be the one-step conditional distribution over successor states. For a bipartition π that cuts S into parts A and B, define the partitioned distribution $p_\pi(X_{t+1} \mid x) = p(X^A_{t+1} \mid x) \otimes p(X^B_{t+1} \mid x)$, which severs all inter-part dependencies. Then:

$$\Phi(x) = \min_{\pi \in \mathcal{P}} D_{\mathrm{KL}}\Big(\mu_x \parallel p_\pi(\cdot \mid x)\Big),$$

where $\mathcal{P}$ is the set of bipartitions. By the standard decomposition of total correlation across a bipartition,

$$\mathrm{Coh}(\mu_x) = \mathrm{Coh}_A(\mu_x) + \mathrm{Coh}_B(\mu_x) + I(X_A; X_B),$$

so $D_{\mathrm{KL}}(\mu_x \| p_\pi(\cdot \mid x)) = I_{\mu_x}(X_A; X_B)$ and

$$\Phi(x) = \min_{\pi \in \mathcal{P}} I_{\mu_x}(X_A; X_B).$$

Since $I_{\mu_x}(X_A; X_B) \le \mathrm{Coh}(\mu_x)$ for every bipartition, we have $\Phi(x) \le \mathrm{Coh}(\mu_x)$. Thus $\Phi > 0$ requires $\mathrm{Coh}(\mu_x) > 0$.

Remark on the relationship. The expression above shows that Φ as defined in IIT 2.0 equals the minimum mutual information between the parts of a bipartition of the successor-state distribution. This is a well-defined quantity related to—but not identical with—the more elaborate cause–effect structures and "cut" models used in later IIT versions (3.0, 4.0). The relationship established here is: $\Phi > 0 \Rightarrow \mathrm{Coh}(\mu_x) > 0$ (total correlation is a necessary condition). The further implication $\mathrm{Coh}(\mu_x) > 0 \Rightarrow \mathrm{Coh}_E(\mu) > 0$ requires pairwise faithfulness and does not hold for purely synergistic distributions.

Step 3: Static vs. dynamic. IIT computes Φ at a single time step. DCR's coherence is defined on the stationary distribution μ^*, which integrates over the full dynamical trajectory. The IIT quantity is obtained by restricting μ to the conditional distribution at a single step.

Hence Φ is bounded by total correlation: $\Phi > 0 \Rightarrow \mathrm{Coh}(\mu_x) > 0$, but $\mathrm{Coh} > 0$ does not imply $\Phi > 0$ (dependencies may be fully decomposable across some bipartition). The further step from $\mathrm{Coh}(\mu_x) > 0$ to DCR's edge-sum $\mathrm{Coh}_E(\mu) > 0$ requires pairwise faithfulness (Step 1). In this sense, IIT imposes a *stricter* criterion than either DCR coherence measure. □

Remark C.21 (Transaction-density gradients and gravitational analogues). In substrate models where "exploration" is mediated by a sea of uncollapsed potential interactions (transactions), accelerated motion can induce anisotropies in the accessible interaction field (via horizon-like cutoffs), producing effective resistance-to-acceleration terms and mutual "shielding" effects between nearby bodies. We state this as an explicit conjecture:

Conjecture (Gravity and inertia from transaction-density gradients). Let $\rho_\tau(\mathbf{x})$ denote the local transactional density (completed constraint-satisfying couplings per unit volume per unit time) in a DCR substrate. Two bodies mutually shield the transaction field along their connecting axis, producing a ρ_τ-gradient and hence a net drift toward each other; an accelerating body creates a horizon-like cutoff behind it, producing a ρ_τ-anisotropy opposing acceleration. If gravity and inertia admit such a reformulation, then (i) the gravitational "constant" is a derived function of the ambient ρ_τ, and (ii) inertial mass equals the integrated ρ_τ-anisotropy at fixed acceleration.

This is speculative and untested; we record it because it illustrates how far the DCR+optimization reading might extend, and because it generates concrete simulation targets.

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