

INTRO TO DATA SCIENCE

LECTURE 8: LOGISTIC REGRESSION

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LAST TIME:

- PROBABILITY**
- BAYESIAN INFERENCE**
- NAIVE BAYES CLASSIFICATION**

QUESTIONS?

I. LOGISTIC REGRESSION

HANDS-ON: LOGISTIC REGRESSION

I. LOGISTIC REGRESSION

Q: What is logistic regression?

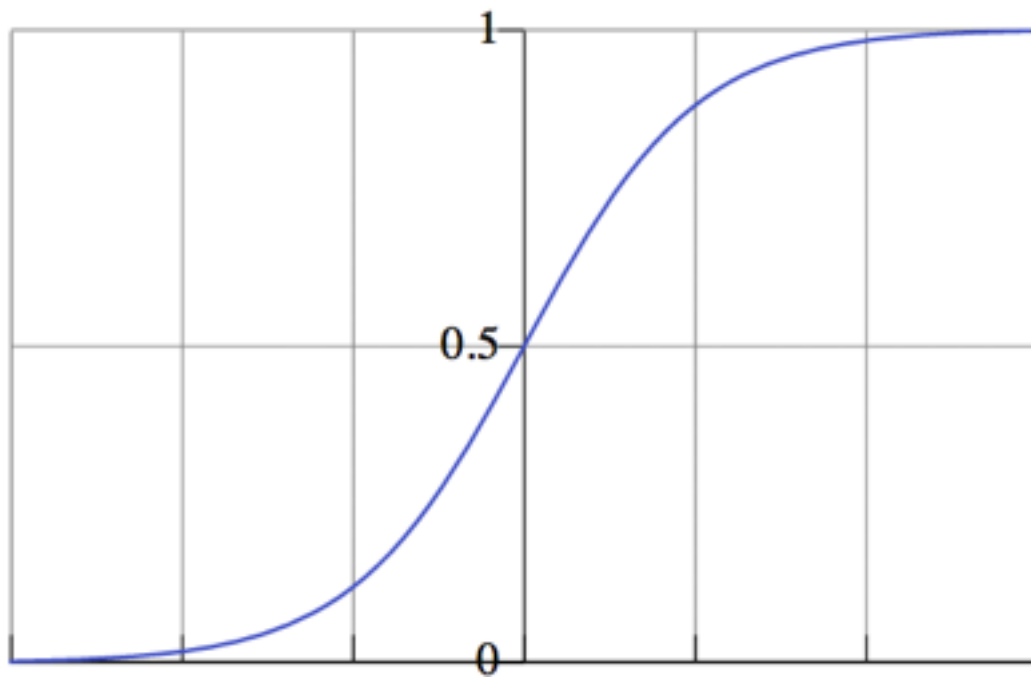
A: A generalization of the linear regression model to classification problems.

*In **linear regression**, we used a set of features to predict the value of a (continuous) outcome variable.*

*In **logistic regression**, we use a set of features to predict **probabilities of (binary) class membership**.*

These probabilities are then mapped to class labels, thus solving the classification problem.

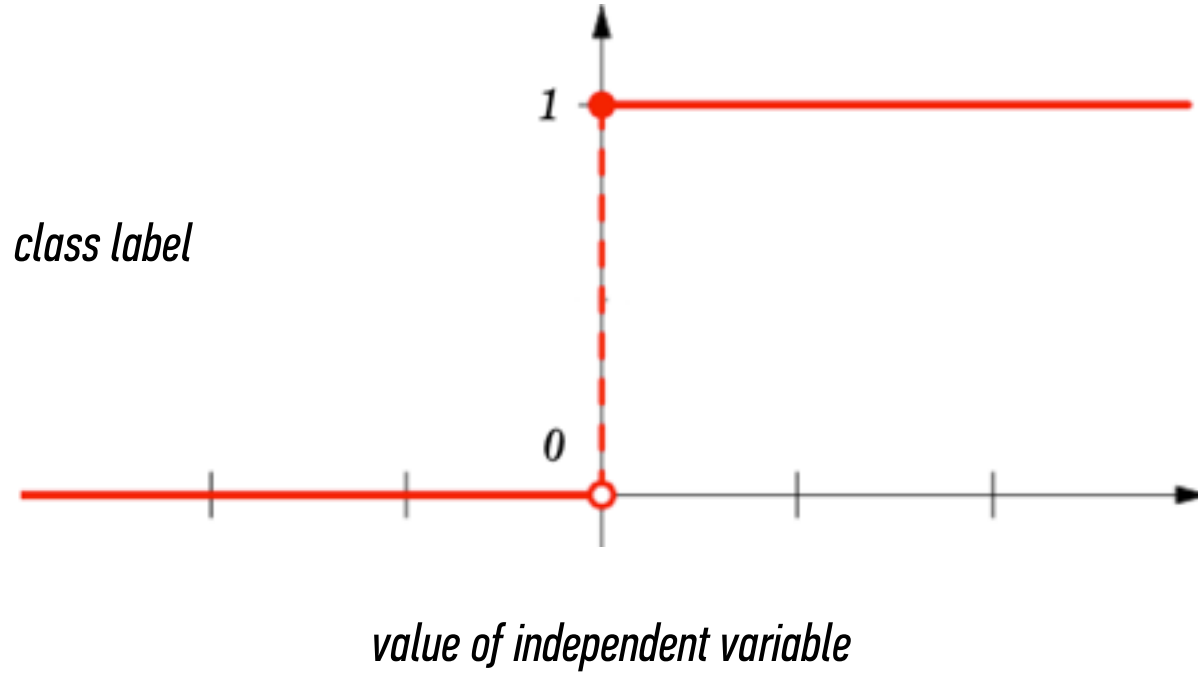
*probability of
belonging to
class*



value of independent variable

NOTE

Probability predictions look like this.



NOTE

Probabilities are “snapped” to class labels (eg by thresholding at 50%).

The logistic regression model is an extension of the linear regression model, with a couple of important differences.

The main difference is in the outcome variable.

*The key variable in any regression problem is the **conditional mean** of the outcome variable y given the value of the feature(s) x :*

$$E(y|x)$$

In linear regression, we assume that this conditional mean is a linear function taking values in $(-\infty, +\infty)$:

$$E(y|x) = \alpha + \beta x$$

In logistic regression, we've seen that the conditional mean of the outcome variable takes values only in the unit interval $[0, 1]$.

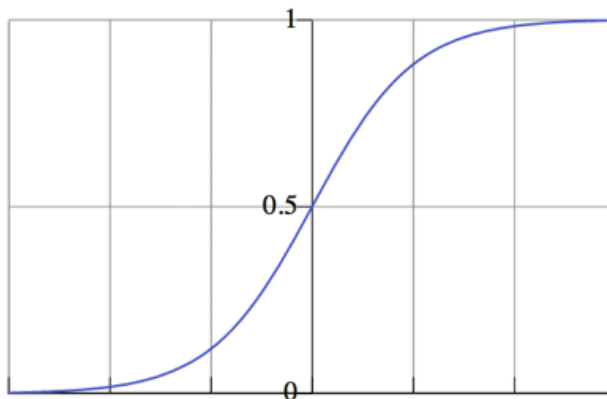
The first step in extending the linear regression model to logistic regression is to map the outcome variable $E(y | x)$ into the unit interval.

Q: How do we do this?

*A: By using a transformation called the **logistic function**:*

$$E(y|x) = \pi(x) = \frac{e^{\alpha+\beta x}}{1+e^{\alpha+\beta x}}$$

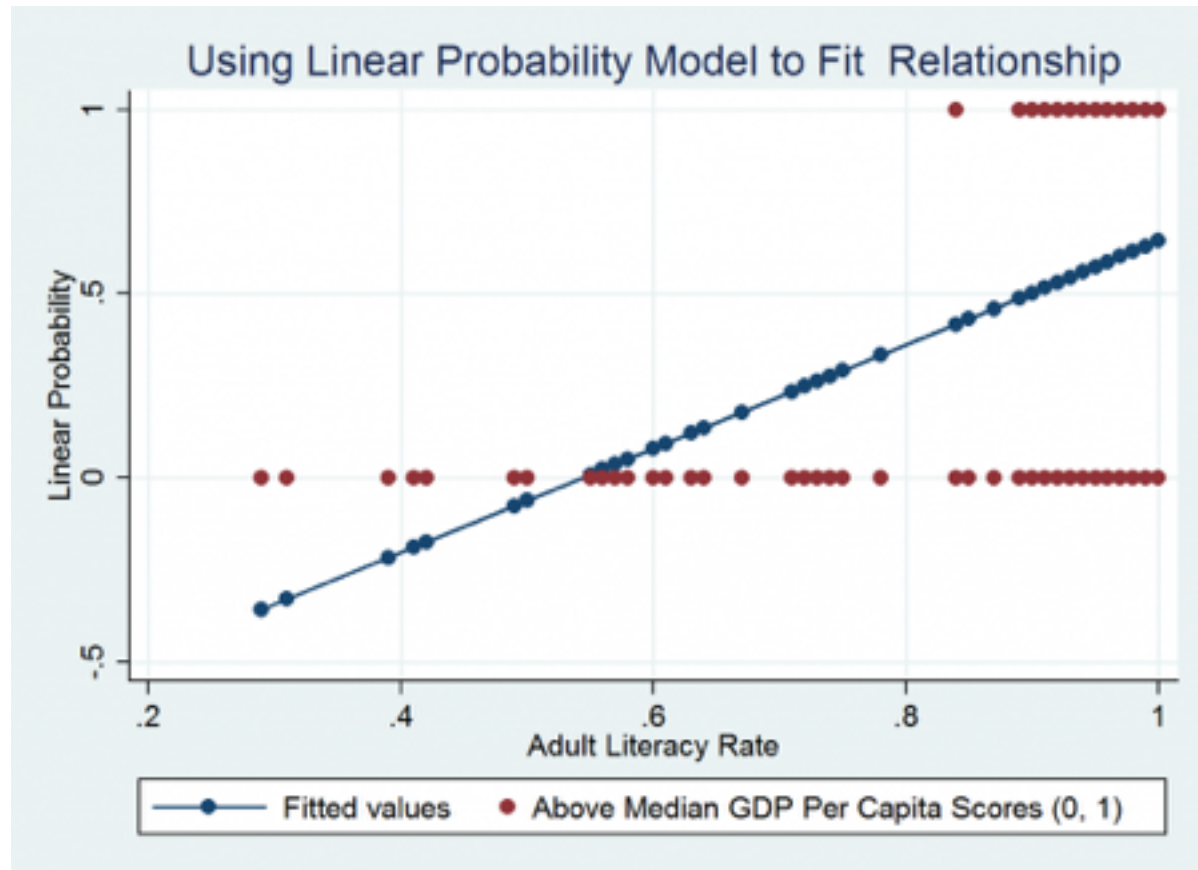
We've already seen what this looks like:



NOTE

For any value of x , y is in the interval $[0, 1]$

This is a nonlinear transformation!



*The **logit function** is an important transformation of the logistic function. Notice that it returns the linear model!*

$$g(x) = \ln\left(\frac{\pi(x)}{1-\pi(x)}\right) = \alpha + \beta x$$

NOTE

This name hints at its usefulness in interpreting our results.

We will see why shortly.

*The logit function is also called the **log-odds function**.*

We can now state the following:

$$e^{g(x)} = OR = e^{\alpha + \beta x}$$

So that if,

$$e^{\beta_i} = n$$

then the odds ratio is increased by a factor of n for a unit increase of x_i

- ▶ *Classification Problems - Often good as a baseline method*
- ▶ *When we need an estimate of class likelihood (“probabilistic classifier”)*
- ▶ *Many attributes*

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