Using AS-AD framework and VAR model to [...] Interwar US

Quantitative Economic History - Applications
Spring Term 2021

Hubert Mrugala

Banking and Finance

hubert.mrugala@uzh.ch

19764265

1 Introduction

Aaa.

2 Data

The dataset consists of 252 monthly observations of American Industrial Production (IP) and Consumer Index Prices (CPI) from January 1919 to December 1939. The data can be split into two periods, i.e. pre-great depression and post great depression. The first one I define as the time between the beginning of the data sample, January 1919, to the upper-turning point in May 1929, the second one follows from that point to the end of the time-series. After the war there is a sharp increase in the price level index from 57 in the beginning of 1919 to 70 in mid 1920s. Then, prices deflate to about 59 (Depression of 1920-1921) and stay around that level until February 1931, when CPI falls below 55. Not surprisingly years 1931-33 are considered to be the most severe time of the Great Depression as in this period prices were declining each month until reaching the lowest point of 43.7 in April 1933. Pre-Great Depression Industrial Production, except the time of 1920-21 depression, was growing steadily. At the peak, IP was about 40 and it took almost 7 years of depression and recovery to come back to that level. At the end of the inter-war period we can also see a very sudden collapse of IP. It is the Recession of 1937-38 - third-worst recession of the twentieth century. CITE

At the beginning of the period CPI is around 58, then rises and stays about that level untill Febuary 1931 when the metric falls below 55. One exception The average value of IP for the whole period is 29.94, and it's standard deviation equals 6.11. CPI for the same period shows similar volatility with standard deviation of 6.80 and mean of 55.55.

Table 1: Yearly data

Year	IP	IP (% change)	CPI	CPI (% change)
1919	24.3	NaN	57.6	NaN
1920	28.1	15.6	67.2	16.7
1921	19.2	-31.7	66.3	-1.3
1922	21.3	10.9	59.0	-11.0
1923	28.4	33.3	58.5	-0.8
1924	28.6	0.7	60.1	2.7
1925	30.0	4.9	60.3	0.3
1926	32.0	6.7	62.5	3.6
1927	31.9	-0.3	61.0	-2.4
1928	31.6	-0.9	60.1	-1.5
1929	36.1	14.2	59.5	-1.0
1930	33.4	-7.5	59.6	0.2
1931	25.9	-22.5	55.4	-7.0
1932	21.3	-17.8	49.8	-10.1
1933	19.0	-10.8	44.9	-9.8
1934	23.5	23.7	46.0	2.4
1935	27.9	18.7	47.4	3.0
1936	31.7	13.6	48.2	1.7
1937	39.4	24.3	49.1	1.9
1938	27.4	-30.5	49.6	1.0
1939	34.0	24.1	48.6	-2.0

Table 2: Basic Statistics

| | Pre-Great Depression | | Great Depression | | All Data: 1919-1939 | |
	IP	CPI	IP	CPI	IP	CPI
count	125.0	125.0	127.0	127.0	252.0	252.0
mean	29.2	61.2	30.7	50.0	29.9	55.5
median	29.9	60.1	29.7	48.7	29.8	58.4
std	5.1	3.3	6.9	4.3	6.1	6.8
min	18.8	56.3	17.6	43.7	17.6	43.7
25%	26.1	59.4	25.5	47.5	25.9	48.7
50%	29.9	60.1	29.7	48.7	29.8	58.4
75%	33.2	61.7	35.5	50.8	33.9	60.2
max	40.8	72.9	44.2	60.4	44.2	72.9

Table 3: Restricted Long-run Responses

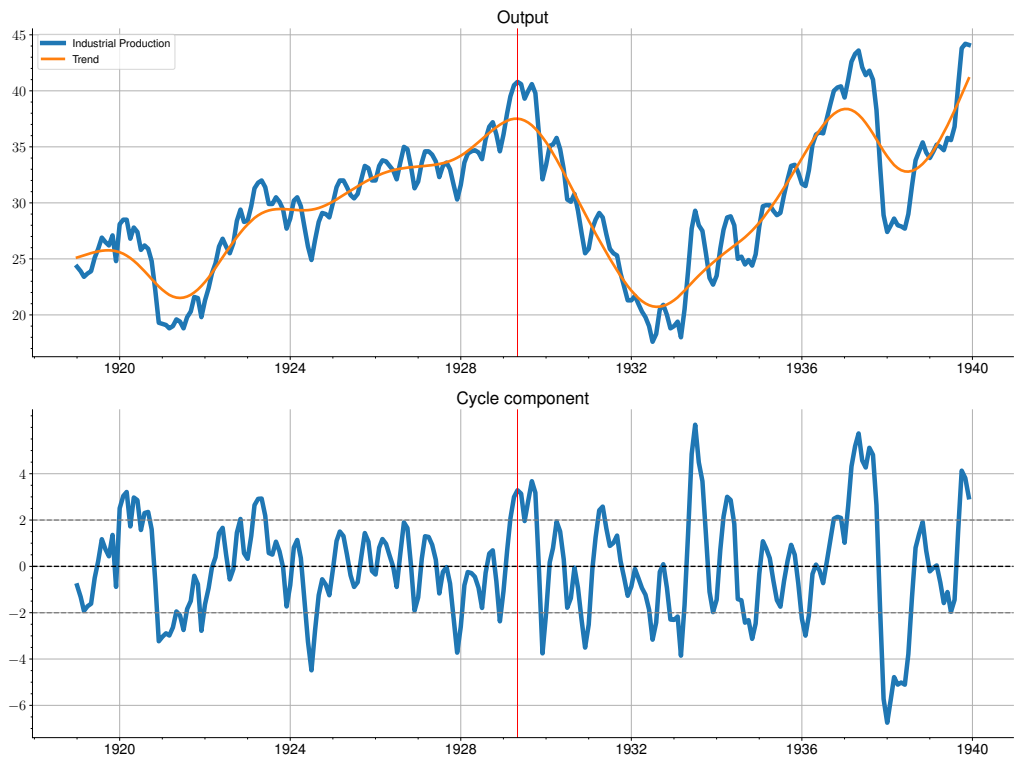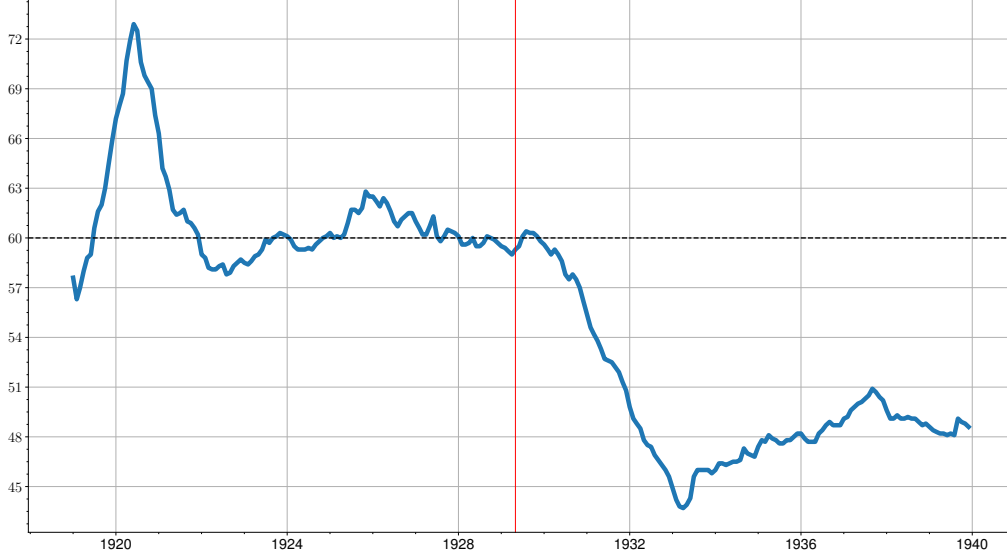|	Supply	Demand
Output	4.82	0.00
Prices	0.70	1.08

Figure 1: Industrial Production

Figure 2: Consumenr Price Index

3 Method

3.1 Vector Autoregressive Model

To analyze relationship between two endogenous time-series, price level $\mathbf{M}$ and output $\mathbf{Y}$, a vector autoregression (VAR) model is used.

$$\mathbf{y_t} = \mathbf{c} + \mathbf{A_1 y_{t-1}} + \mathbf{A_2 y_{t-2}} + \cdots + \mathbf{A_p y_{t-p}} + \mathbf{e_t} \tag{1}$$

In the formula above $\mathbf{c}$ is a constant, $\mathbf{A_i}$ are coefficient matrices and $\mathbf{u_i}$ are error terms. When analyzing monetary transmission mechanism, VAR can be also expressed in the following form (Favero).

$$\mathbf{G} \begin{pmatrix} \mathbf{Y}_t \\ \mathbf{M}_t \end{pmatrix} = \mathbf{K}(L) \begin{pmatrix} \mathbf{Y}_{t-1} \\ \mathbf{M}_{t-1} \end{pmatrix} + \mathbf{F} \begin{pmatrix} \epsilon_{\mathbf{Y,t}} \\ \epsilon_{\mathbf{M,t}} \end{pmatrix} \tag{2}$$

In the structural model above matrix $\mathbf{G}$ represents contemporaneous relations among the variables and $\mathbf{K(L)}$ is the matrix of finite-order lag polynomial. The model is not directly observable, however, a reduced form can be

estimated.

$$
\begin{pmatrix} \mathbf{Y}_t \\ \mathbf{M}_t \end{pmatrix} = \mathbf{G}^{-1}\mathbf{K}(L) \begin{pmatrix} \mathbf{Y}_{t-1} \\ \mathbf{M}_{t-1} \end{pmatrix} + \mathbf{G}^{-1}\mathbf{F} \begin{pmatrix} \epsilon_{\mathbf{Y},t} \\ \epsilon_{\mathbf{M},t} \end{pmatrix} \tag{3}
$$

In this paper, vector autoregression is used to obtain impulse responses and forecast error decompositions in the AS-AD framework. To do this, I use Blanchard-Quach decomposition where we assume that aggregate demand shocks has no long-run effect on real output. The assumption helps to solve the identification problem by imposing restictions on the impact matrix $\mathbf{S} = \mathbf{G}^{-1} - \mathbf{F}$

$$
\mathbf{u_t} = \mathbf{S}\epsilon_{\mathbf{t}} \tag{4}
$$

To find S first lets start with the moving average representations of reduced form VAR equation 1 and the structural model eq2.

$$
\begin{aligned}
\mathbf{y_t} &= \mathbf{B(L)u_t}, \\
\mathbf{y_t} &= \mathbf{C(L)}\epsilon_{\mathbf{t}}
\end{aligned} \tag{5}
$$

After substituting for u t and simplifying

$$
\mathbf{B}(L)\mathbf{S} = \mathbf{C}(L) \tag{6}
$$

The long-run coefficient matrix is $\mathbf{C(1)} = \mathbf{B(1)S}$, and so to get $\mathbf{S}$, long-run $\mathbf{C(1)}$ is needed. Postmultiplying C(1) with its transpose gives

$$
\mathbf{B}(1)\mathbf{SS'B}(1)' = \mathbf{C}(1)\mathbf{S}^{-1}\mathbf{SS'}\left(\mathbf{S'}\right)^{-1}\mathbf{C}(1)' = \mathbf{C}(1)\mathbf{C}(1)'. \tag{7}
$$

Cholesky decomposition of the expression above produces the restricted long-run coefficient matrix $\mathbf{C(1)}$, that allows to calculate matrix $\mathbf{S}$.

3.2 Impulse Responses

Impact matix S connects structural shocks e t to the reduced form residuals
u t

$$\mathbf{u_t} = \mathbf{S}\epsilon_\mathbf{t} \tag{8}$$

To find S first lets start with the moving average representations of reduced
form VAR equation 1 and the structural model eq2.

$$\begin{aligned}
\mathbf{y_t} &= \mathbf{B(L)u_t}, \\
\mathbf{y_t} &= \mathbf{C(L)}\epsilon_\mathbf{t}
\end{aligned} \tag{9}$$

After substituting for u t and simplifying

$$\mathbf{B}(L)\mathbf{S} = \mathbf{C}(L) \tag{10}$$

The long-run coefficient matrix is $\mathbf{C(1)} = \mathbf{B(1)S}$, and so to get $\mathbf{S}$, long-run
$\mathbf{C(1)}$ is needed. Postmultiplying C(1) with its transpose gives

$$\mathbf{B}(1)\mathbf{S}\mathbf{S}'\mathbf{B}(1)' = \mathbf{C}(1)\mathbf{S}^{-1}\mathbf{S}\mathbf{S}'\left(\mathbf{S}'\right)^{-1}\mathbf{C}(1)' = \mathbf{C}(1)\mathbf{C}(1)'. \tag{11}$$

Cholesky decomposition of the expression above produces the restricted long-
run coefficient matrix $\mathbf{C(1)}$, that allows to calculate matrix $\mathbf{S}$.

3.3 Forecast Error Variance Decomposition

Forecast error variances for two variables $(k = 1, 2)$ can be expressed as

$$\mathrm{E}\left[(y_{k,t+1} - \hat{y}_{k,t+1})^2\right] = \mathbf{e}_k\mathbf{\Sigma}\mathbf{e}_k' = \mathbf{e}_k\mathbf{S}\mathbf{S}'\mathbf{e}_k' = \mathbf{e}_k\mathbf{C}_0\mathbf{C}_0'\mathbf{e}_k'. \tag{12}$$

Then contribution of the the shock l to the forecast error variances of vari-
able k at time h can be, generally, written in a for of the following three
dimentional array

$$\theta_{h,kl} = \frac{\sum_{j=0}^{h-1} \left(\mathbf{e}_k \mathbf{C}_j \mathbf{e}_l' \right)^2}{\sum_{j=0}^{h-1} \mathbf{e}_k \mathbf{C}_j \mathbf{C}_j' \mathbf{e}_k'} \tag{13}$$

4 Results

- Price index:

  - 1700-1823: Schumpeter (1938), in Mitchell and Deane (1971, p. 468-469)

  - 1823-1913: Mitchell (2003, p. 863-864)

- Industrial production: Crafts and Harley (1992, p. 725-727)

Figure 3: Impulse Responses
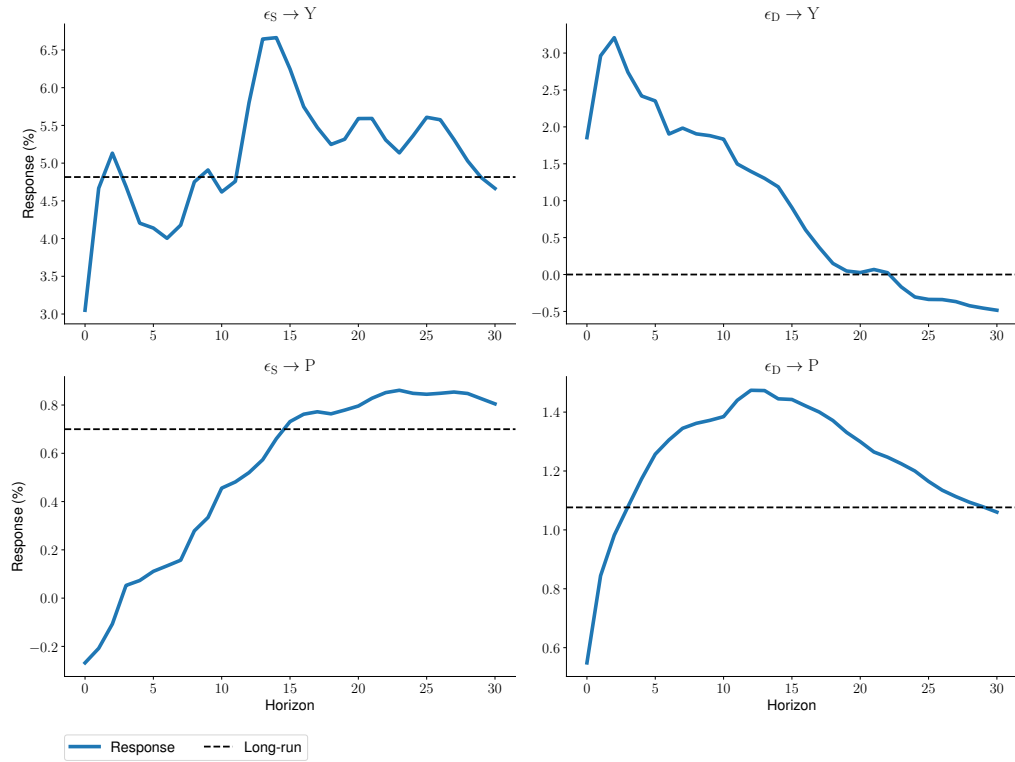
To be discussed.

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