Universität Zürich[UZH]

Network Science

Temporal Bow-Tie Component Analysis of the Bitcoin Transaction Network

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Network Science

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ABSTRACT

Bitcoin has dominated the cryptocurrency market since its theoretical dawn in 2008, giving rise to numerous intriguing studies in different fields. In this project, we present a novel perspective of understanding the dynamics of bitcoin transaction network by performing a bow tie analysis. The evolution of the bow tie, as well as the relationship between different compartments is demonstrated. Furthermore, we introduce a Markov Chain model to formulate the interaction of nodes.

1 INTRODUCTION

Bitcoin is a cryptocurrency backed with public and private key encryption that makes every record of transaction public and verifiable (Nakamoto 2008) Users can make transactions to each other by using Bitcoin (BTC) as the currency. A bitcoin network is hence established, with users as nodes and transactions as weighted directed edges.

Due to the bitcoin peer-to-peer, cryptographic architecture, the network's nodes store a complete list of past transactions. Easily accessible data recorded on public ledgers can then be used to analyze this digital currency. There is some relevant previous work that investigates the bitcoin network with reference to network science. Notably, Lischke and Fabian (2016) and Baumann et al. (2014) explore bitcoin transaction network with basic descriptive statistics and network science specific metrics. Moreover, Kondor et al. (2014) look at microscopic link formation and temporal wealth distribution in the bitcoin network.

Throughout this project, we are interested in extracting meaningful information from the bitcoin network. To achieve this, we perform a temporal bow tie component analysis on our network. What elso do we do, where the data comes form, what is the time frame – complete at the end

In this report we will present several further inspection based on Bow-tie analysis. First we fetch the network at evenly chosen time slots and sort them into compartments according to the Bow-tie categorization algorithm. The distribution of nodes into different compartments. [...]

2 BOW-TIE STRUCTURE OF A NETWORK

Real-world networks are very different form theoretical graphs or random models of networks. Many tools, like centralities and community detection algorithms, can be used to study the inner structure of these complex networks. However, for directed networks, there is one very interesting approach that can be used to depict and describe an overall structure of a complex network. Such an approach is analyzing a network by using a ”bow tie” structure. First to use bow-tie structure were Broder et al. (2000) who analyzed the graph structure of the World Wide Web.

Bow-tie structure consists of a 'giant strongly connected' component (GSCC), 'in' component (IN), 'out' component (OUT). There are also tubes, inner tendrils, outer tendrils and disconnected components. The main components: SCC, IN and OUT are unsurprisingly illustrated as a bow tie with SCC as the knot. Every component except disconnected components are collectively called a giant weakly connected component (GWCC). GSCC is the core of the network where every node is reachable from any other. In IN component there exist a path between any node in IN and GSCC, but IN nodes cannot be reached from GSCC. OUT component nodes are also connected with the core and are reachable from GSCC but not the other way around. Tubes represent nodes which are reachable from IN and 'go' directly to OUT, thus there exist a path between the two compartments but its nodes are not connected to GSCC. Inner tendrils and outer tendrils are nodes that are reachable form IN or OUT respectively, but are not connected to any other component. Finally, disconnected components' nodes are connected only within themselves and do not belong to GWCC.

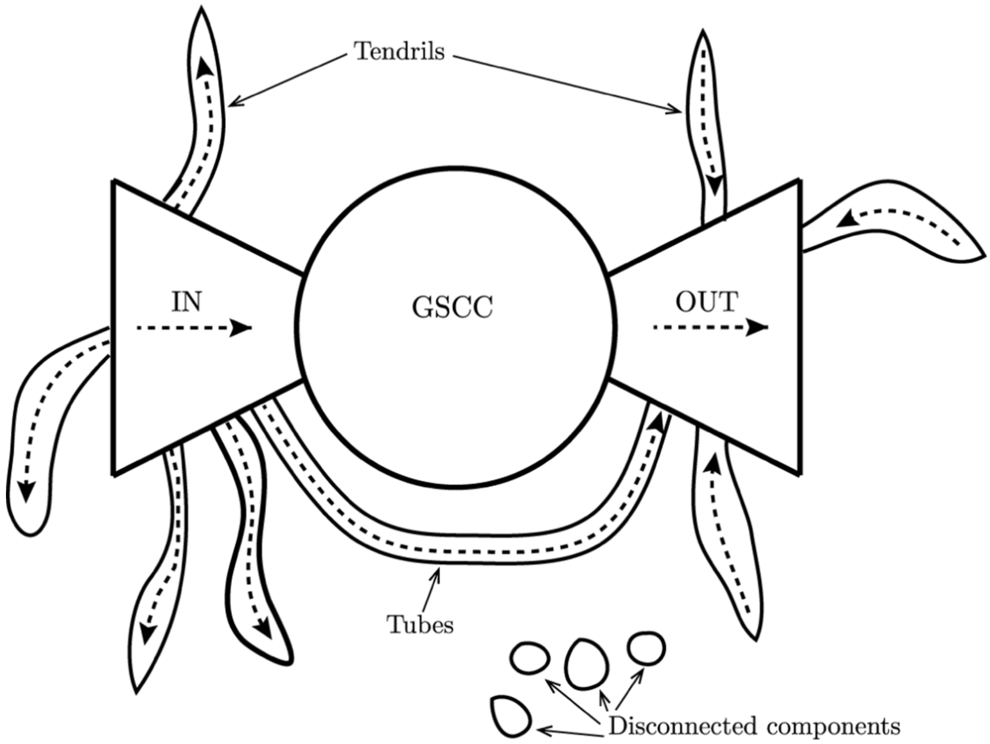

Fig. 1. A graph bow tie structure (Fujita et al. 2019)

Since Broder et al. (2000) the bow tie structure have been applied to explain macroscopic behavior of many complex networks ranging from social, e.g. online dating network (Chen and Nayak 2011) to biological, e.g. bacterial metabolic architecture (Csete and Doyle 2004) Bow tie can be also used to describe microscopic structures, where local bow ties can be found (Mattie et al. 2018; Fujita et al. 2019) It is also worth to mention Vitali et al. (2011) who use the bow tie to analyze control flows of transnational corporations, where the core is composed of a small highly connected clique of financial institutions.

As far as application of bow tie structure to blockchain systems, Fujiwara and Islam (2020) analyze 6-month transaction flows by using Helmholtz-Hodge-Kodaira decomposition and the bow-tie structure. Guo et al. (2019) describe statistics of Ethereum transaction network, including bow tie components. Maesa et al. (2019) evaluate macroscopic properties of bitcoin transaction network by studying the network's connectivity components and their temporal transformation in the bow tie framework. Our work also includes bow tie time analysis of the network components. However, our approach, on top of time series analysis as done in Maesa et al. (2019) includes a Markov Chain model to investigate how bitcoin network actors change their location in the graph topology. Consequently, we can infer how economic actors' activity varies as the network evolves over time.

3 METHOD

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4 RESULTS

4.1 Basic statistics of the network

To analyze basic properties of the network, we use a snapshot of the system at 1 January 2012.

... Nodes and edges and average degree, degree distribution plot, stats + randomized, components table + randomized.

	Orginal Newtork	Randomized Newtork
Global Clustering Coeff.	0.022651	0.035403
Assortativity Coeff.	-0.166781	-0.155987

Table 1. Bitcoin network properties

	Orginal Newtork	Randomized Newtork
SCC	115.0	5273.0
IC	1062.0	0.0
OUT	202.0	0.0
Tubes	65.0	0.0
Tendrils	3211.0	0.0
Fringe	357.0	0.0
Disconnected	1102.0	841.0

Table 2. Components of the bitcoin network and its randomized counterpart.

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4.2 Time series component analysis

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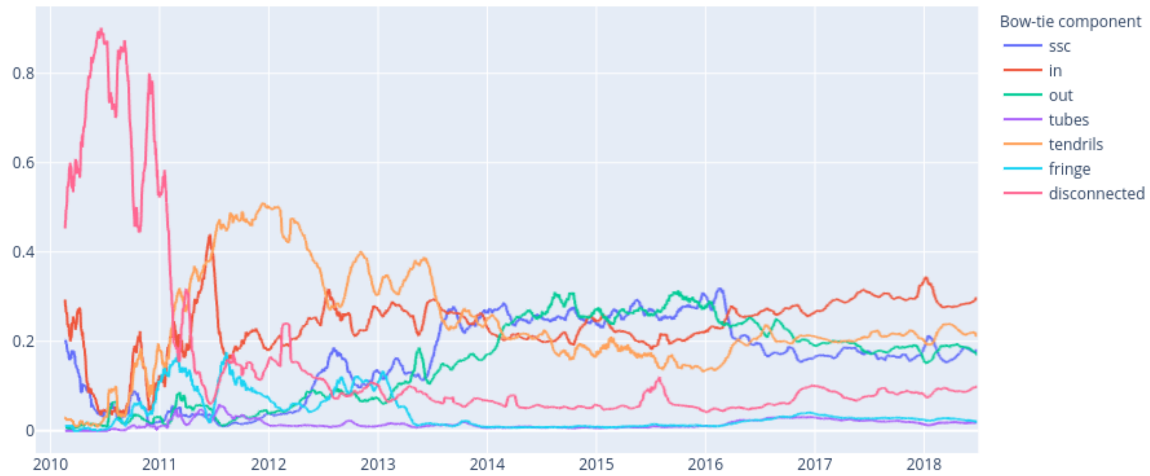

Fig. 2. Bow tie components ratios over time. 30 day moving average.

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4.3 Markov Chain component transitions

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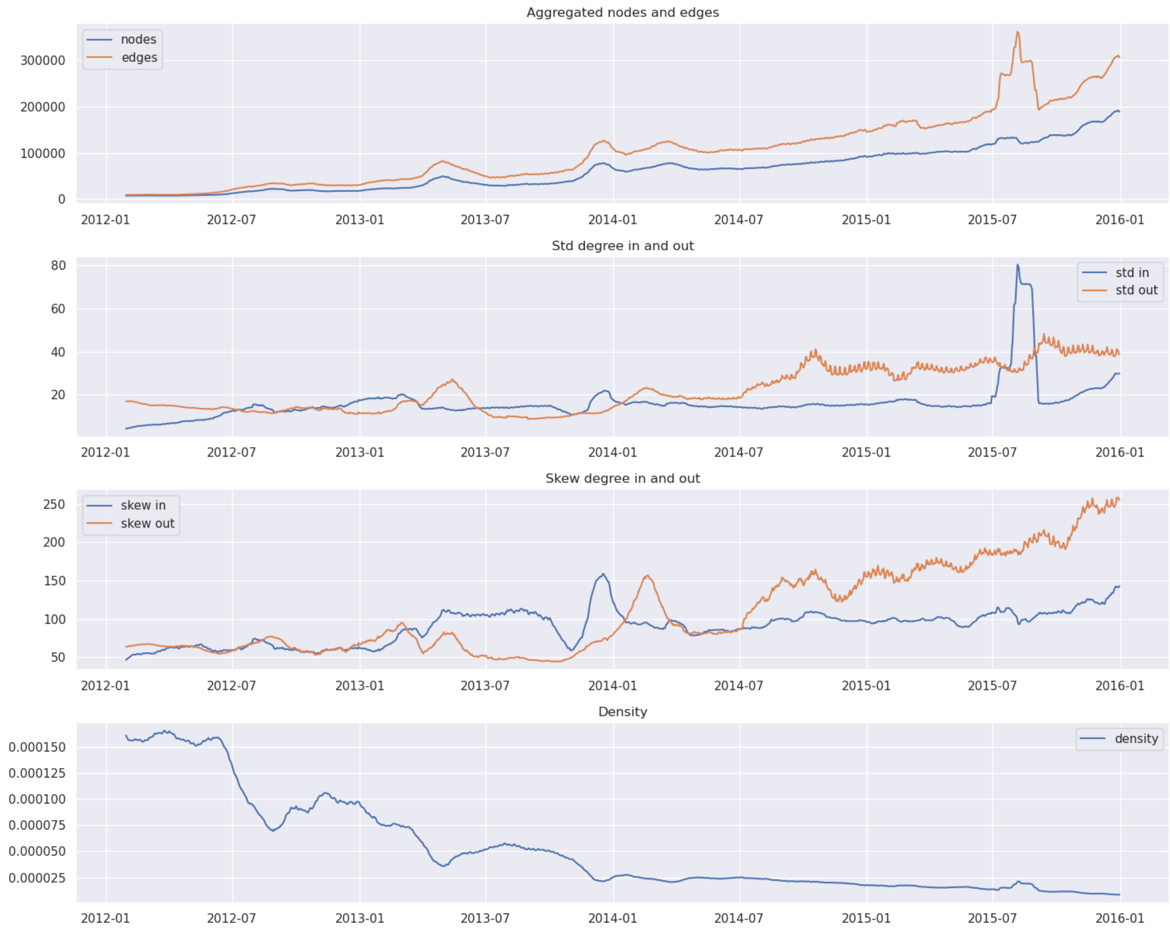

Fig. 3. Nodes, edges and degrees of the network over time. 30 day moving average.

	ssc	in	out	tubes	tendrils	fringe	disconnected
ssc	47.248873	15.875993	26.198685	0.743993	7.805405	0.387624	1.739428
in	15.864009	35.155343	19.446643	1.721531	21.955629	1.065606	4.791239
out	11.071967	28.806202	39.918478	0.590917	14.918899	0.878371	3.815166
tubes	9.757159	21.157759	19.467282	9.877923	34.605220	1.607342	3.527314
tendrils	3.327169	27.061074	14.000744	0.945656	48.543789	1.560581	4.560986
fringe	2.528821	35.138670	7.707061	1.276736	35.404417	6.273357	11.670938
disconnected	4.238323	28.587512	11.090995	0.638962	19.061452	4.234824	32.147932

Fig. 4. Markov Chain state transition matrix for 2009-01-09 to 2018-07-01 in relative numbers. Excluding inactive nodes.

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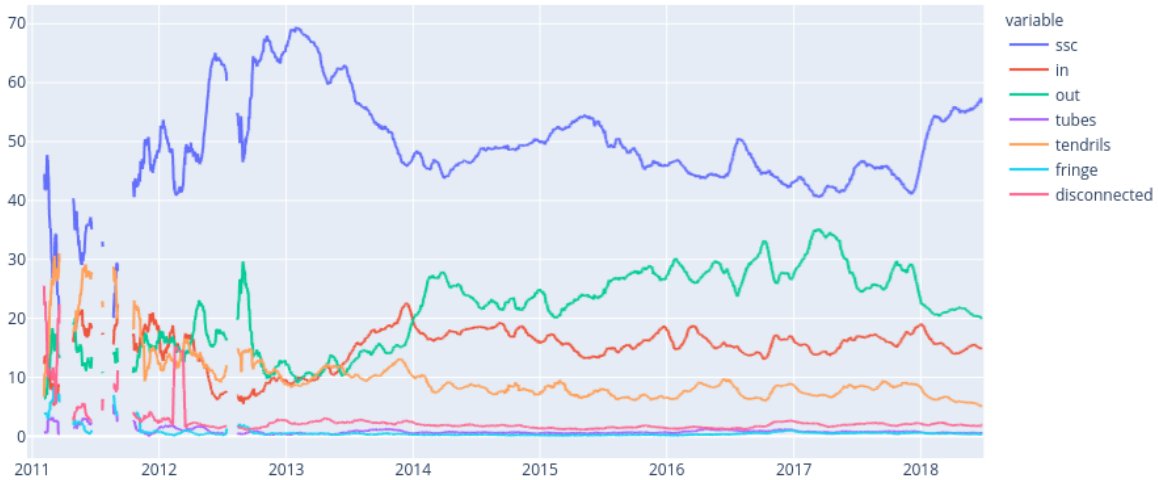

Fig. 5. Markov Chain state probability that a node belongs to a component at t+1 given it is in SCC at t. 30 day moving average.

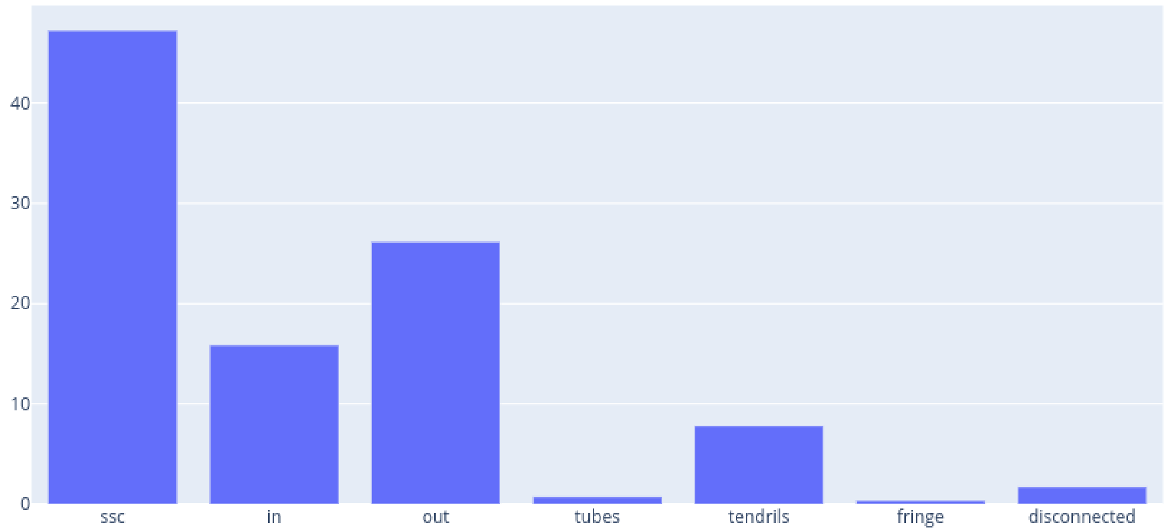

Fig. 6. Probability of a node to be in a different state at t+1 given it is in state SCC at t

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5 CONCLUSION

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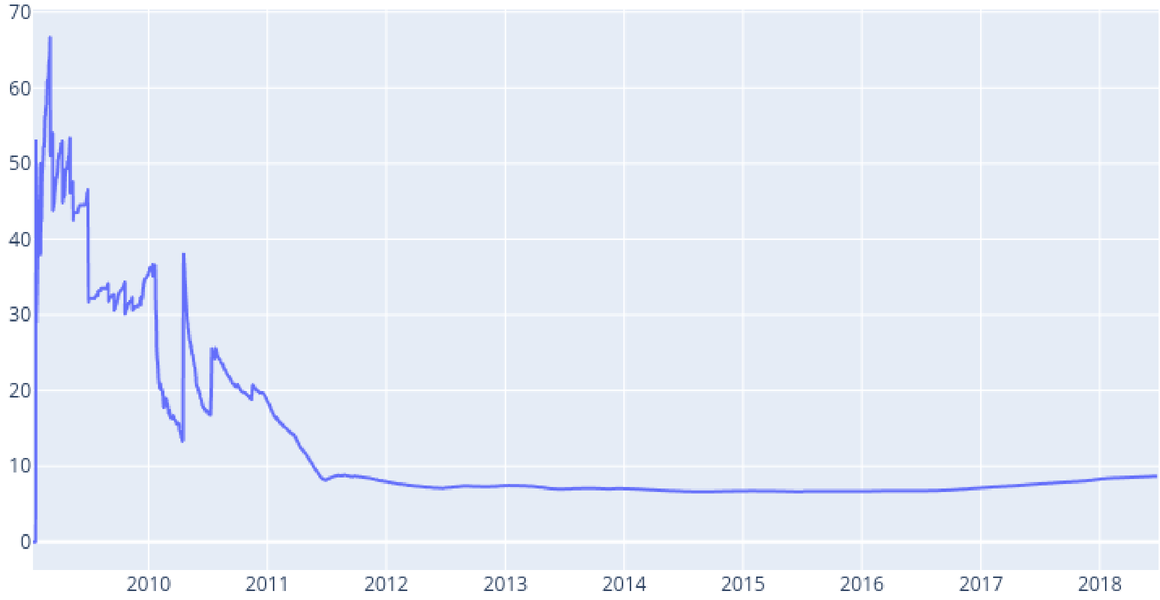

Fig. 7. Ratio of nodes that are present at t and t+1

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