University of Zurich[UZH]
Department of Banking and Finance

Lombard lending: Modeling lending values

Applied Credit Risk Modeling Seminar

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Lombard Lending

A lombard loan

- Type of loan that is backed by assets, or in other words, collateral
- Maximum amount of money that a bank lends to a client that pledges collateral is determined by lending value
    - *Credit = LV × MV of the Collateral*

      OR
    - $X = \lambda V_0$
- *Required Haircut* $= 1 - \lambda$
- *Running haircut* $= (V_t - X_t) / V_t$
- Lending values depend on the quality of pledged assets

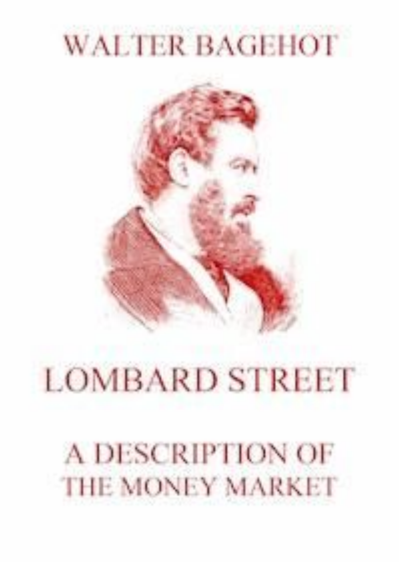
Process

- The bank can *call* part of a loan when the market value of the pledged asset falls below a certain threshold
    - Usually 75% of the initial haircut
- A loan enters a warning stage when haircut erosion is between 0-25%
- Haircut erosion exceeds 25% → margin call state
    - The client has to bring new collateral or limit his exposure within 10 working days
    - If the client doesn’t react, the bank can start liquidating pledged assets

- Closeout period - time between last margin call and liquidation moment
- Maturity of a loan ranges from a week up to 12 months

Lombard lending occurs mostly in money markets

- Secured interbank market for liquidity, i.e. repo market
- Central banks
- Hedge funds
- **Private Banking**



> **Credit Suisse and Nomura warn of losses after Archegos-linked sell-off**
>
> Banks face earnings hit following $20bn stock fire sale by prime brokerage client

Private banking

Why would a client want get a lombard credit?

- Receiving liquidity without selling existing assets
  - Benefits from dividends
  - Taking advantage of a bull market
  - Secured loan → lower risk to the bank → lower interest rate
  - Tax benefits: deferring capital gains tax

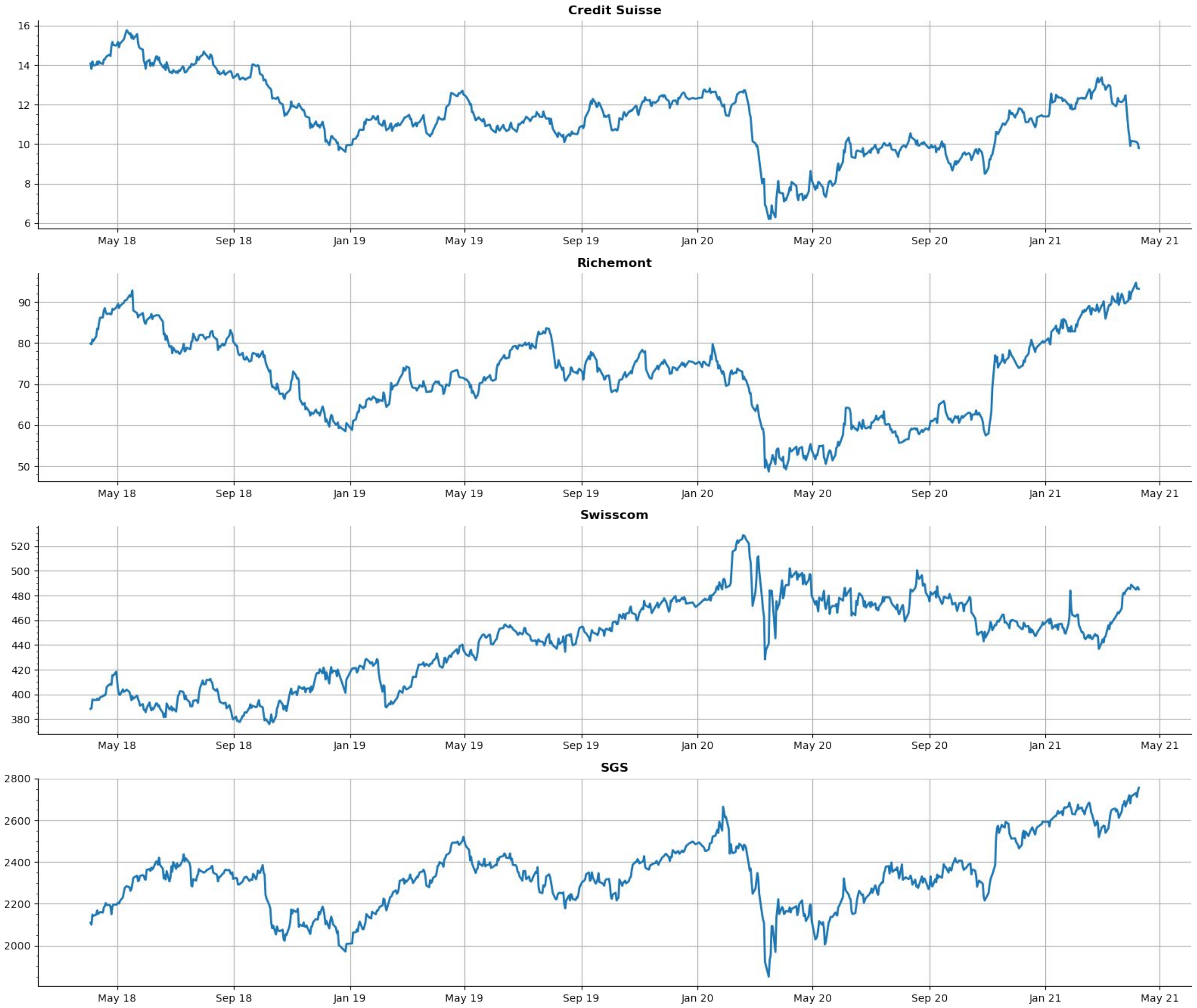
In this project we focus mainly on the market risk component of a lombard credit.

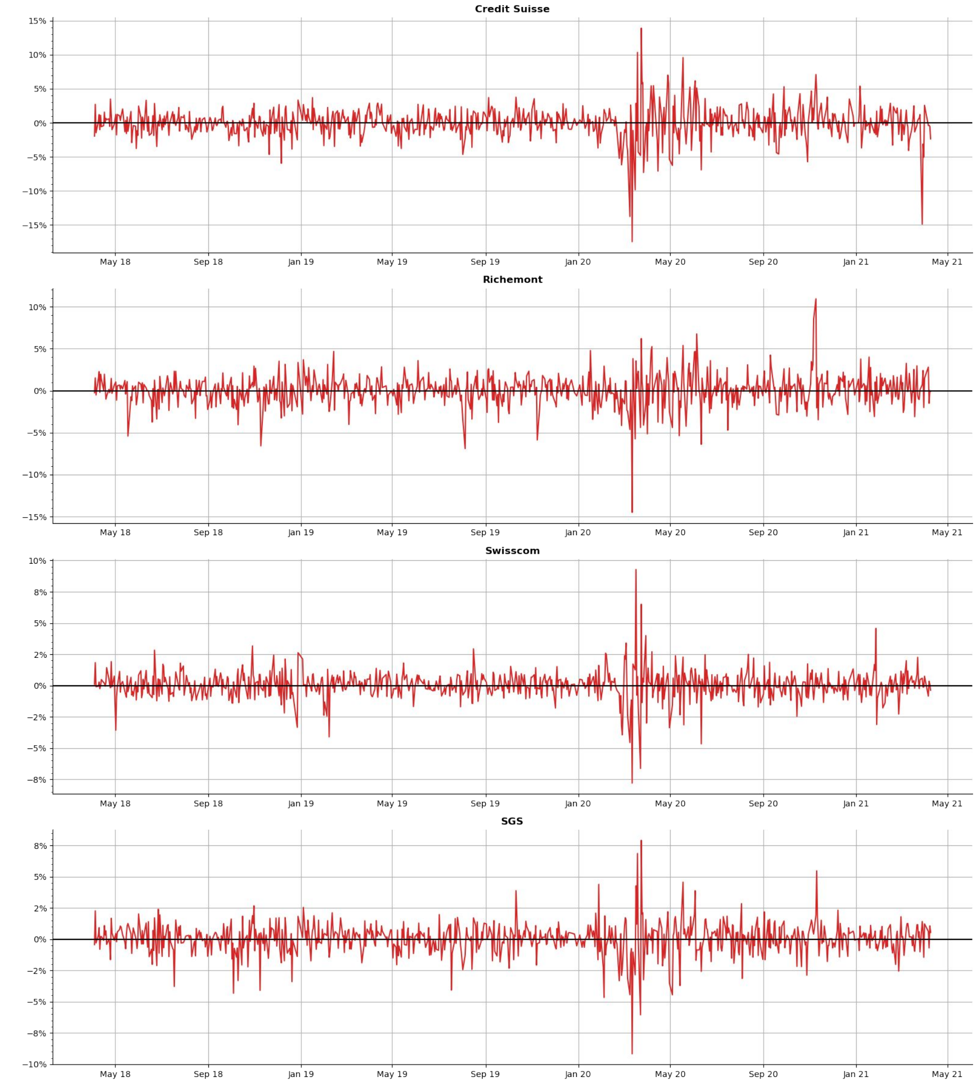
The Data and Basic Statistics

Data

- 20 equities of the SMI index
- Three years of data
  - from 3.04.2018 to 3.04.2021
- Obtained from yahoo finance
- Log returns

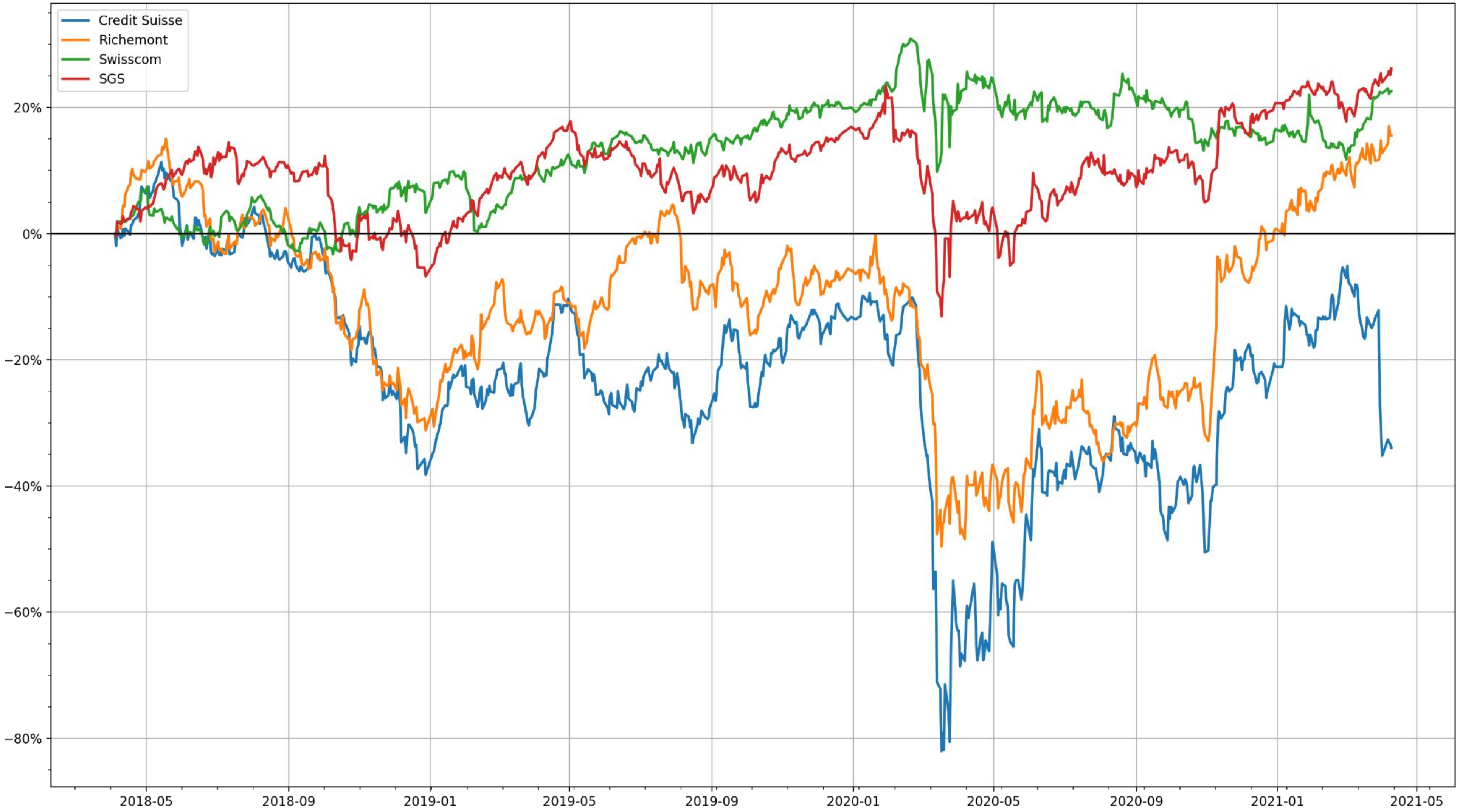
- We specifically focus on 4 stocks in the index:
  - Credit Suisse
  - Richemont
  - Swisscom
  - SGS

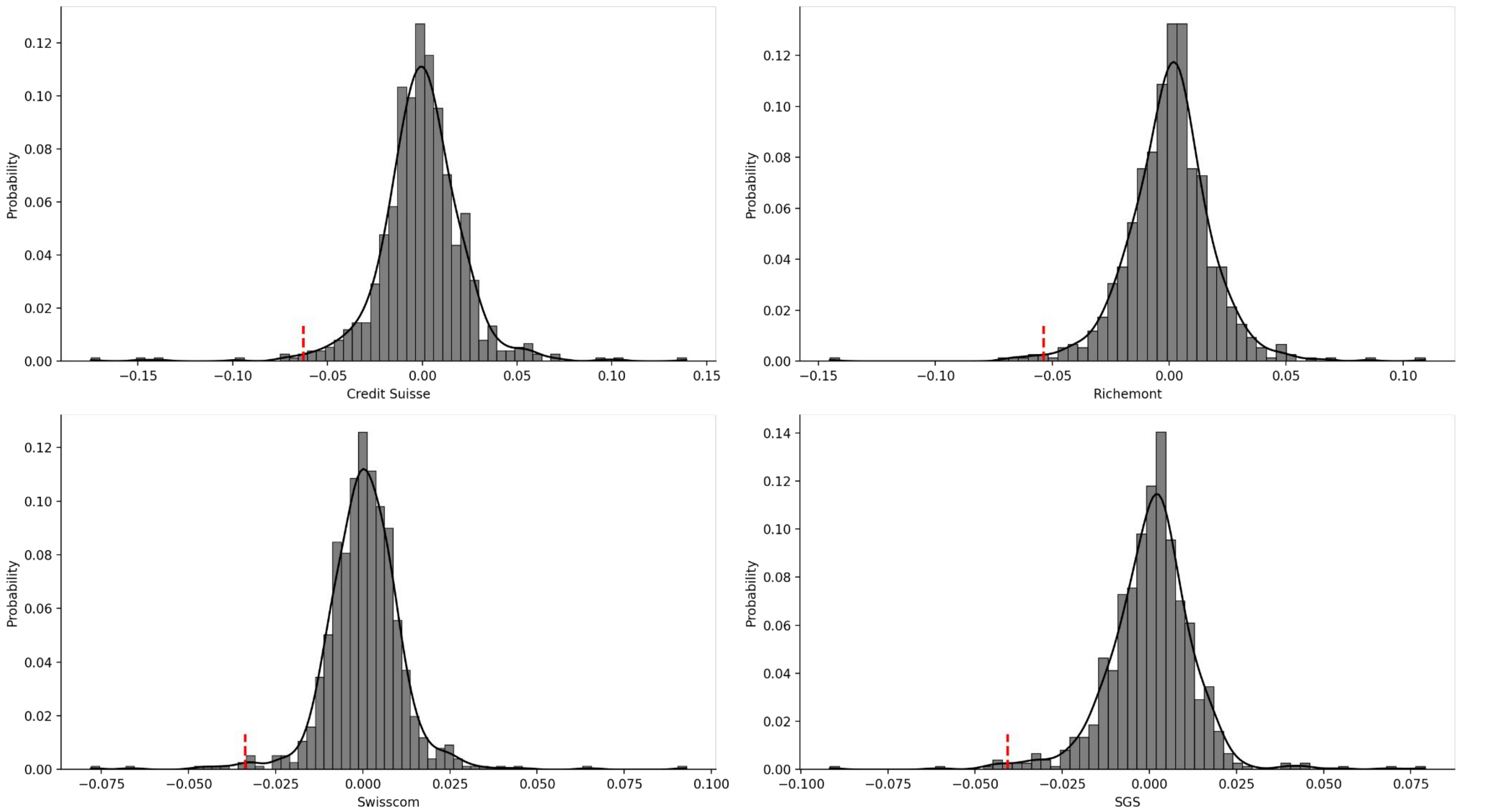

Credit Suisse
16
14
12
10
8
6
Richemont
90
80
70
60
50
Swisscom
520
500
480
460
440
420
400
380
SGS
2800
2600
2400
2200
2000
May 18
Sep 18
Jan 19
May 19
Sep 19
Jan 20
May 20
Sep 20
Jan 21
May 21

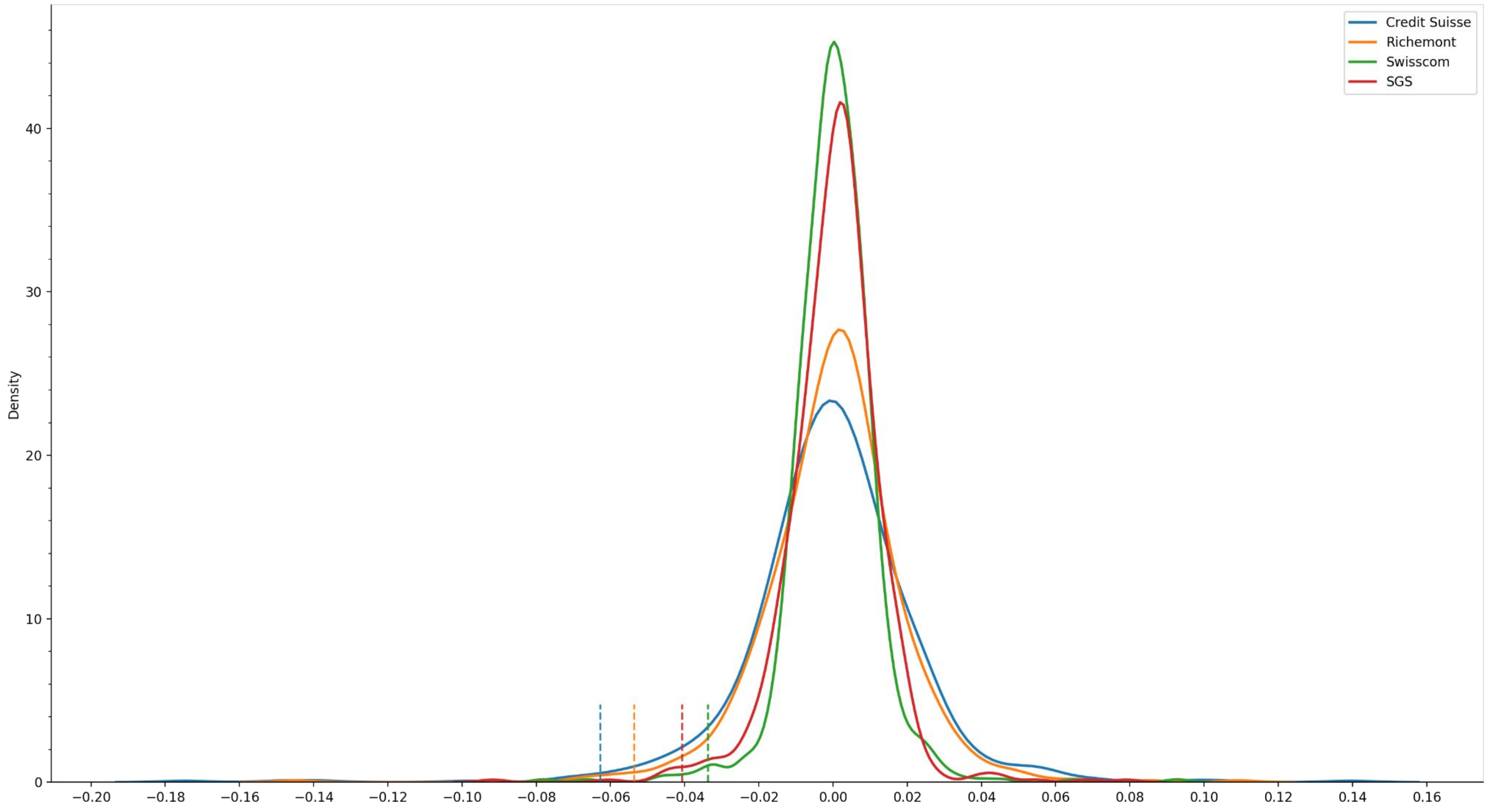

Credit Suisse
15%
10%
5%
0%
−5%
−10%
−15%
Richemont
10%
5%
0%
−5%
−10%
−15%
Swisscom
10%
8%
5%
2%
0%
−2%
−5%
−8%
SGS
8%
5%
2%
0%
−2%
−5%
−8%
−10%
May 18
Sep 18
Jan 19
May 19
Sep 19
Jan 20
May 20
Sep 20
Jan 21
May 21

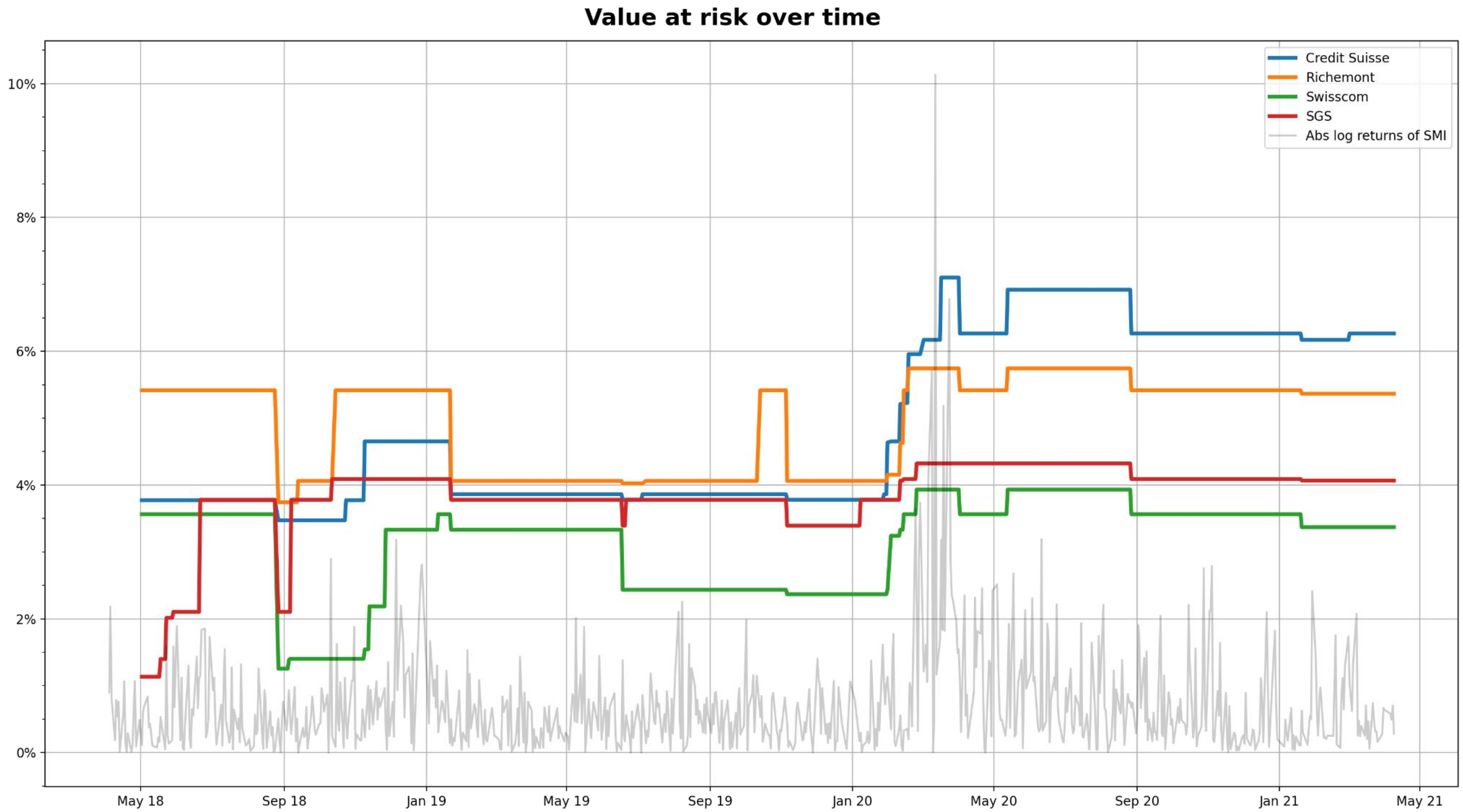
Title

	Credit Suisse	Richemont	Swisscom	SGS
Min	-17.47%	-14.5%	-7.79%	-9.17%
Max	13.92%	10.94%	9.28%	7.91%
Median	0.0%	0.09%	0.02%	0.14%
Mean	-0.05%	0.02%	0.03%	0.04%
Standard Deviation	2.32%	1.87%	1.14%	1.29%
Skewness	-0.8	-0.42	0.07	-0.37
Kurtosis	10.52	7.4	12.23	7.9
Value at Risk	0.06	0.05	0.03	0.04
Expected Shortfall	0.11	0.07	0.05	0.05


Credit Suisse
Richemont
Swisscom
SGS
20%
0%
−20%
−40%
−60%
−80%
2018-05
2018-09
2019-01
2019-05
2019-09
2020-01
2020-05
2020-09
2021-01
2021-05


Probability
Credit Suisse
Probability
Richemont
Probability
Swisscom
Probability
SGS


Credit Suisse
Richemont
Swisscom
SGS
Density
0
10
20
30
40
−0.20
−0.18
−0.16
−0.14
−0.12
−0.10
−0.08
−0.06
−0.04
−0.02
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16


Value at risk over time
Credit Suisse
Richemont
Swisscom
SGS
Abs log returns of SMI
10%
8%
6%
4%
2%
0%
May 18
Sep 18
Jan 19
May 19
Sep 19
Jan 20
May 20
Sep 20
Jan 21
May 21

Kendall correlation coefficient

	Credit Suisse	Richemont	Swisscom	SGS	SMI20	SP500
Credit Suisse	1.00	0.42	0.23	0.32	0.43	0.30
Richemont	0.42	1.00	0.18	0.39	0.46	0.29
Swisscom	0.23	0.18	1.00	0.29	0.37	0.12
SGS	0.32	0.39	0.29	1.00	0.52	0.25
SMI20	0.43	0.46	0.37	0.52	1.00	0.33
SP500	0.30	0.29	0.12	0.25	0.33	1.00

Systematic risk

	Beta	Coskewness	Cokurtosis
Credit Suisse	1.50	-0.96	9.41
Richemont	1.13	-1.04	6.59
Swisscom	0.66	-0.97	8.23
SGS	0.89	-0.84	8.84

The model

Collateral asset process

- Equity as collateral → asset value process follows a geometric brownian motion

$$dV_t = V_t(\mu dt + \sigma dB_t), t \geq 0$$

- Solution

$$V_t = v_0 \exp[(\frac{\mu - \sigma^2}{2})t + \sigma B_t]$$

- For a lognormal random variable

$$drift : (\mu - \frac{\sigma^2}{2})t$$

$$diffusion : \sigma^2 t$$

Inputs

Random Variables

- V_t = market value of collateral at time *t*
- X_t = bank's exposure at time *t*

Parameters

- Haircut erosion that initiates a margin call
  - $\alpha = 0.25$
- Closeout period
  - $\delta = 10$ (days)
- Probability that V < X at the end of the closeout period is at most
  - $\varepsilon = 0.01$

Notation

- λ = lending value
- $\beta = 1-(1-\lambda)*\alpha$, i.e. margin call trigger
- τ_n = margin call times

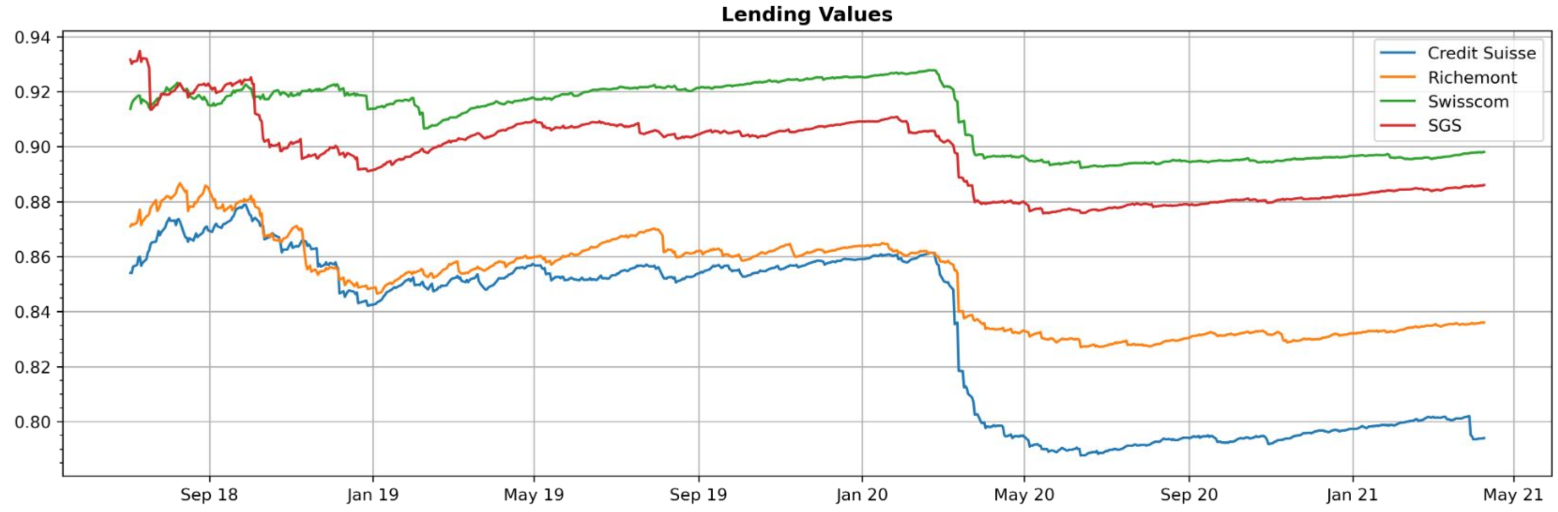
$$P[V_{\tau_n+\delta} \leq X_{\tau_n}] \leq \epsilon$$

$$\Phi\left(\frac{log(\frac{\lambda}{\beta}) - (\mu - \frac{\sigma^2}{2})\delta}{\sigma\sqrt{\delta}}\right) \leq \epsilon$$

Results

Lending values

	Lending Value	Annualized Volatility
Credit Suisse	0.79	0.37
Richemont	0.84	0.30
Swisscom	0.90	0.18
SGS	0.89	0.20


Lending Values
0.94
0.92
0.90
0.88
0.86
0.84
0.82
0.80
Sep 18
Jan 19
May 19
Sep 19
Jan 20
May 20
Sep 20
Jan 21
May 21
Credit Suisse
Richemont
Swisscom
SGS


Returns
20%
0%
−20%
−40%
−60%
−80%
Sep 18
Jan 19
May 19
Sep 19
Jan 20
May 20
Sep 20
Jan 21
May 21
Credit Suisse
Richemont
Swisscom
SGS

	Lending Value	Annualized Volatility
Nestle	0.91	0.17
Swisscom	0.90	0.18
Givaudan	0.90	0.20
Novartis	0.89	0.20
Roche	0.89	0.21
SGS	0.89	0.20
Geberit	0.88	0.22
Zurich Insurance Group	0.87	0.25
Partners Group	0.86	0.26
ABB	0.86	0.26
Sika	0.86	0.28
Lonza	0.85	0.28
Swiss Life Holding	0.85	0.27
LafargeHolcim	0.85	0.28
Swiss Re	0.84	0.29
Richemont	0.84	0.30
UBS	0.83	0.31
Swatch Group	0.83	0.30
Alcon	0.83	0.33
Credit Suisse	0.79	0.37

Backtesting

- Four different collateral assets
- Two contract dates
  - 23.04.2019
  - 23.04.2020
- Three different maturities
  - Three months (64 business days)
  - Six months (126 business days)
  - Twelve months (252 business days)

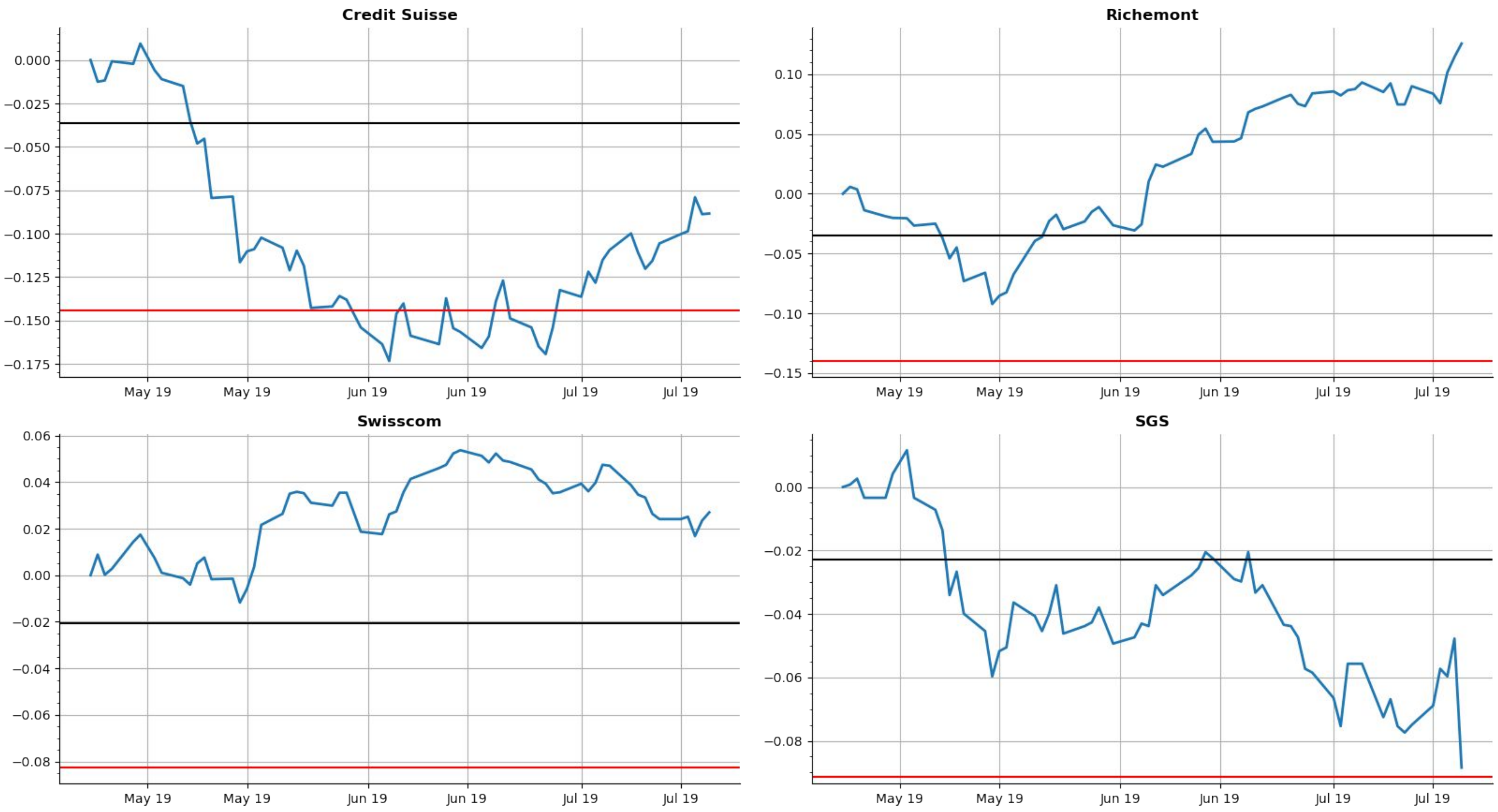
- Assume the borrower doesn’t react on a margin call
- Assume constant bank exposure
- Were there margin calls? If so, how many?
- Was there a default? If so, when?

Contract date: 23.04.2019

Three month loan

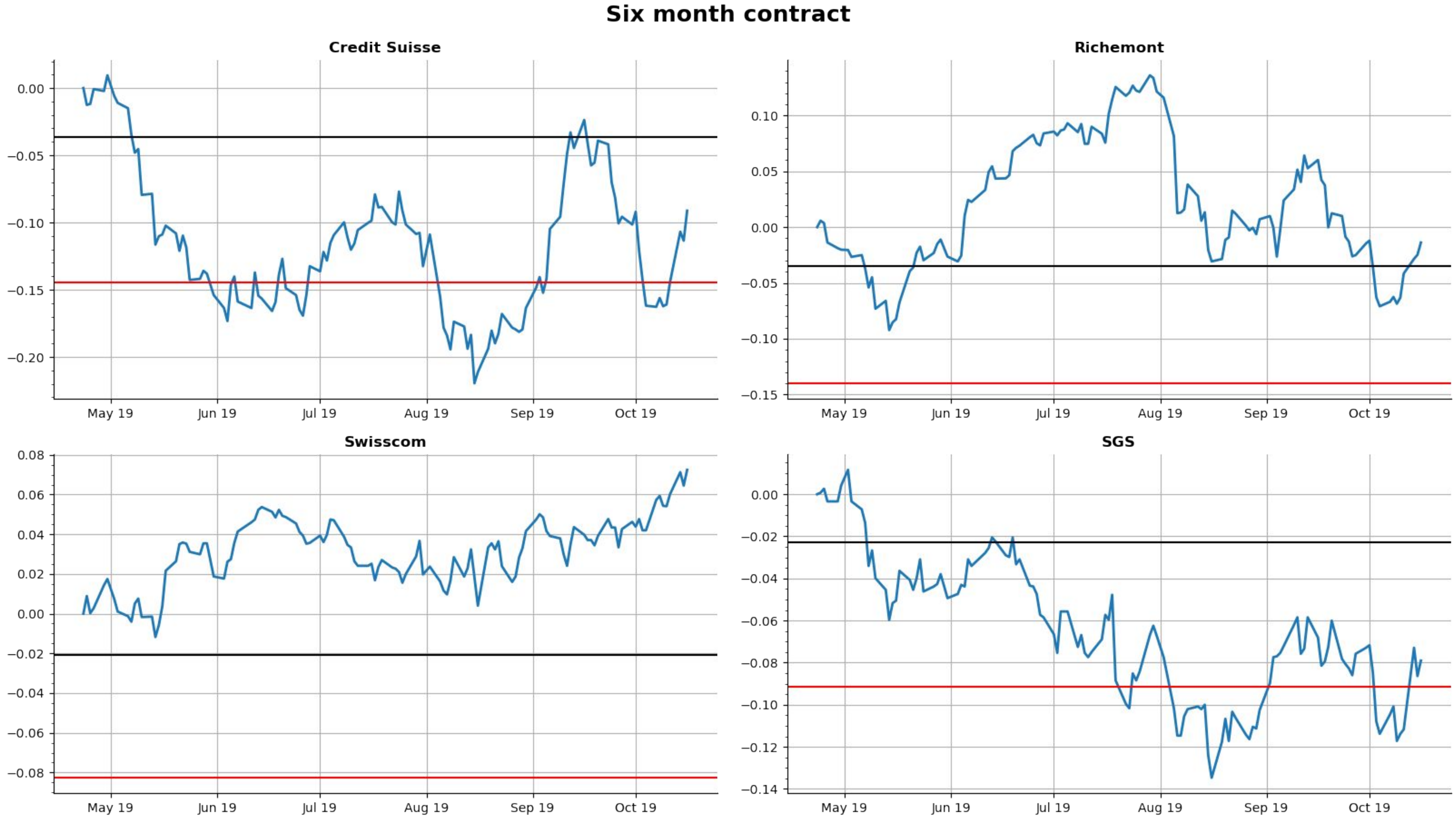
	# of days in MC state	Frequency	Default	Default Date
Credit Suisse	51	0.84	1	21 May 2019
Richemont	11	0.18	1	20 May 2019
Swisscom	0	0.0	0	-
SGS	48	0.79	1	21 May 2019

Three month contract



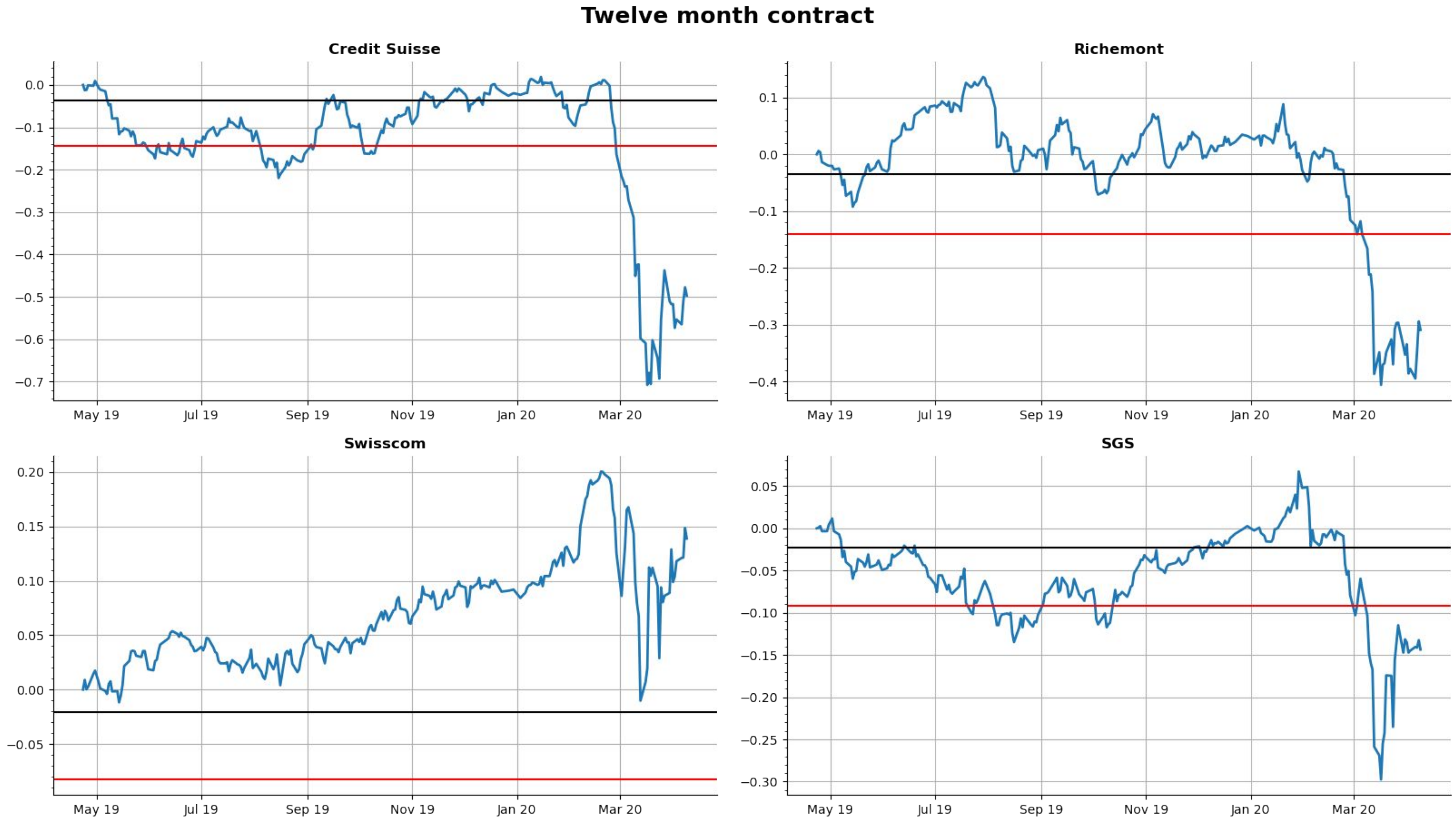
Six month loan

	# of days in MC state	Frequency	Default	Default Date
Credit Suisse	111	0.9	1	21 May 2019
Richemont	19	0.15	1	20 May 2019
Swisscom	0	0.0	0	-
SGS	110	0.89	1	21 May 2019


Six month contract
Credit Suisse
Richemont
Swisscom
SGS
0.00
−0.05
−0.10
−0.15
−0.20
0.10
0.05
0.00
−0.05
−0.10
−0.15
0.08
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
−0.08
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
May 19
Jun 19
Jul 19
Aug 19
Sep 19
Oct 19

Twelve month loan

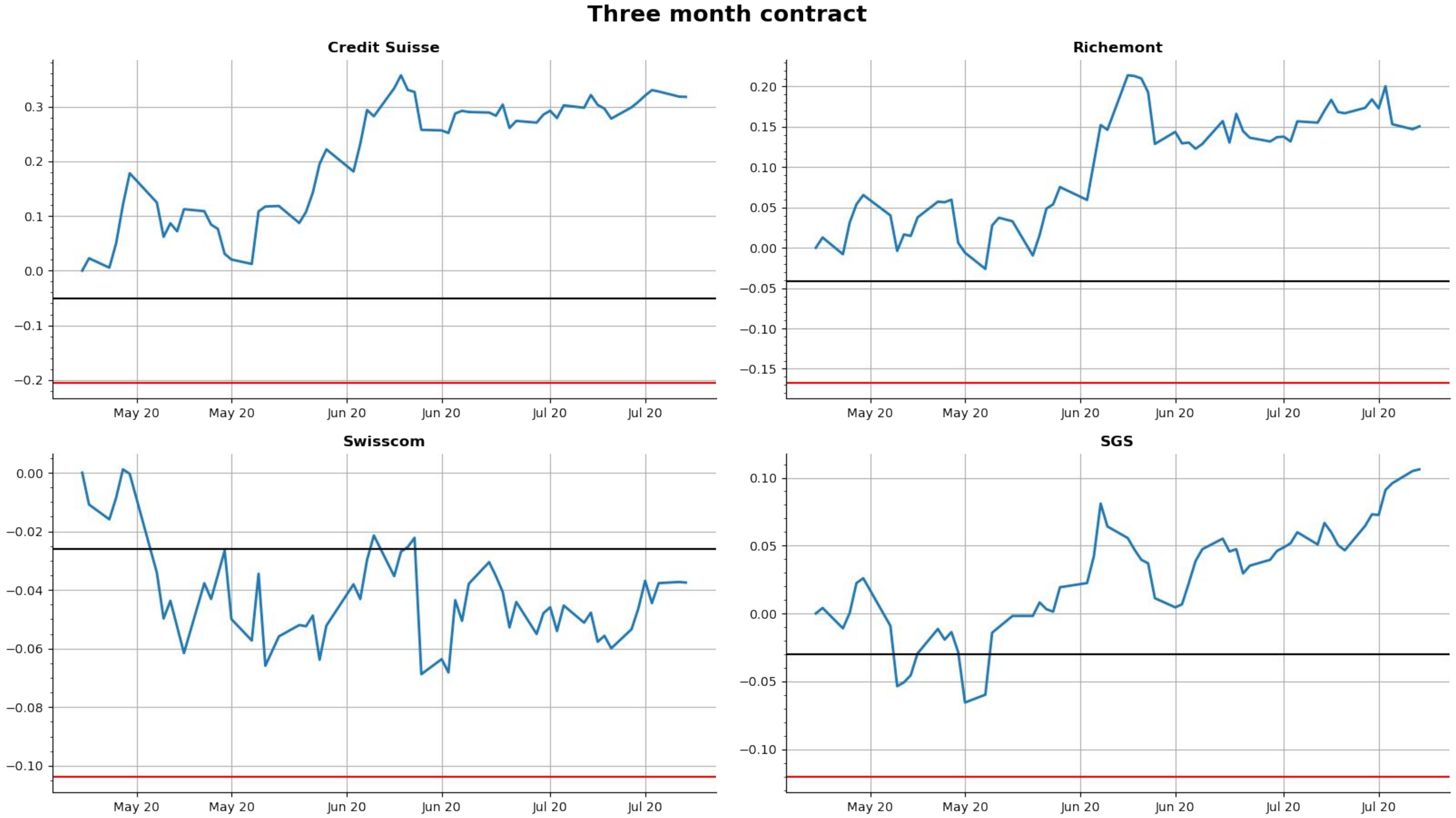
	# of days in MC state	Frequency	Default	Default Date
Credit Suisse	178	0.73	1	21 May 2019
Richemont	54	0.22	1	20 May 2019
Swisscom	0	0.0	0	-
SGS	178	0.73	1	21 May 2019


Twelve month contract
Credit Suisse
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
May 19
Jul 19
Sep 19
Nov 19
Jan 20
Mar 20
Richemont
0.1
0.0
−0.1
−0.2
−0.3
−0.4
May 19
Jul 19
Sep 19
Nov 19
Jan 20
Mar 20
Swisscom
0.20
0.15
0.10
0.05
0.00
−0.05
May 19
Jul 19
Sep 19
Nov 19
Jan 20
Mar 20
SGS
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
May 19
Jul 19
Sep 19
Nov 19
Jan 20
Mar 20

Contract date: 23.04.2020

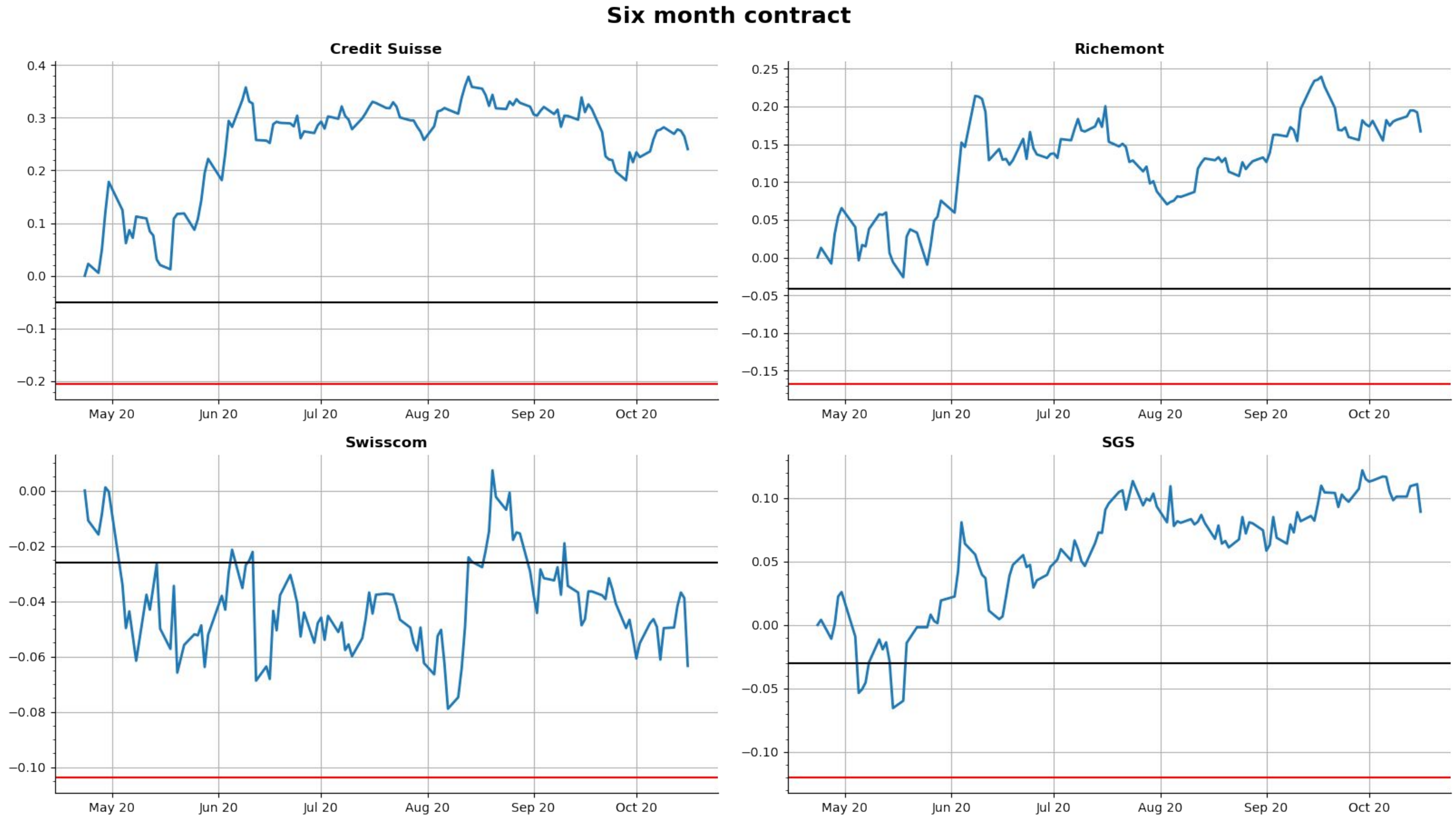
Three month loan

	# of days in MC state	Frequency	Default	Default Date
Credit Suisse	0	0.0	0	-
Richemont	0	0.0	0	-
Swisscom	52	0.85	1	15 May 2020
SGS	5	0.08	0	-


Three month contract
Credit Suisse
0.3
0.2
0.1
0.0
−0.1
−0.2
May 20
May 20
Jun 20
Jun 20
Jul 20
Jul 20
Richemont
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
May 20
May 20
Jun 20
Jun 20
Jul 20
Jul 20
Swisscom
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
May 20
May 20
Jun 20
Jun 20
Jul 20
Jul 20
SGS
0.10
0.05
0.00
−0.05
−0.10
May 20
May 20
Jun 20
Jun 20
Jul 20
Jul 20

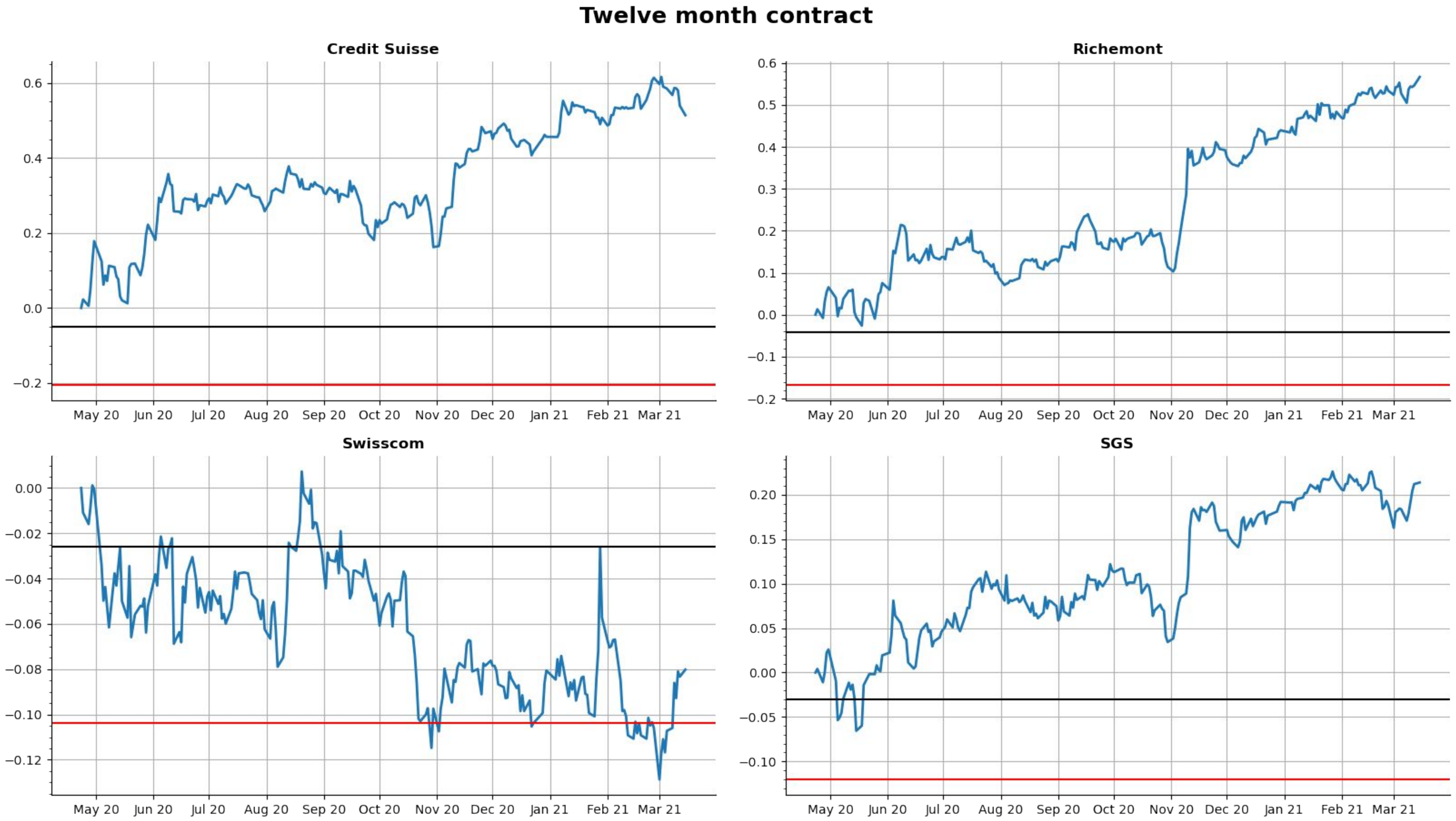
Six month loan

	# of days in MC state	Frequency	Default	Default Date
Credit Suisse	0	0.0	0	-
Richemont	0	0.0	0	-
Swisscom	103	0.83	1	15 May 2020
SGS	5	0.04	0	-


Six month contract
Credit Suisse
Richemont
Swisscom
SGS
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.02
−0.04
−0.06
−0.08
−0.10
May 20
Jun 20
Jul 20
Aug 20
Sep 20
Oct 20

Twelve month loan

	# of days in MC state	Frequency	Default	Default Date
Credit Suisse	0	0.0	0	-
Richemont	0	0.0	0	-
Swisscom	205	0.91	1	15 May 2020
SGS	5	0.02	0	-


Twelve month contract
Credit Suisse
Richemont
Swisscom
SGS
May 20
Jun 20
Jul 20
Aug 20
Sep 20
Oct 20
Nov 20
Dec 20
Jan 21
Feb 21
Mar 21

Adjusting for liquidity

Liquidity Parameter

- Price for share for order flow x, denoted by $V_t(x)$

$$V_t(x) = e^{\gamma x} V_t, \qquad \gamma \geq 0$$

- The Liquidity Parameter γ can be estimated through linear regression:

$$\log\left(\frac{v_{i+1}}{v_i}\right) = \log\left(\frac{V_{t_{i+1}}(x_{i+1})}{V_{t_i}(x_i)}\right) = \gamma \frac{\log(v_{i+1}/v_i)}{\sqrt{t_{i+1} - t_i}} + \left(\mu - \frac{\sigma^2}{2}\right)\sqrt{t_{i+1} - t_i} + \sigma\varepsilon$$

- Alternative simplified formula with ADTV:

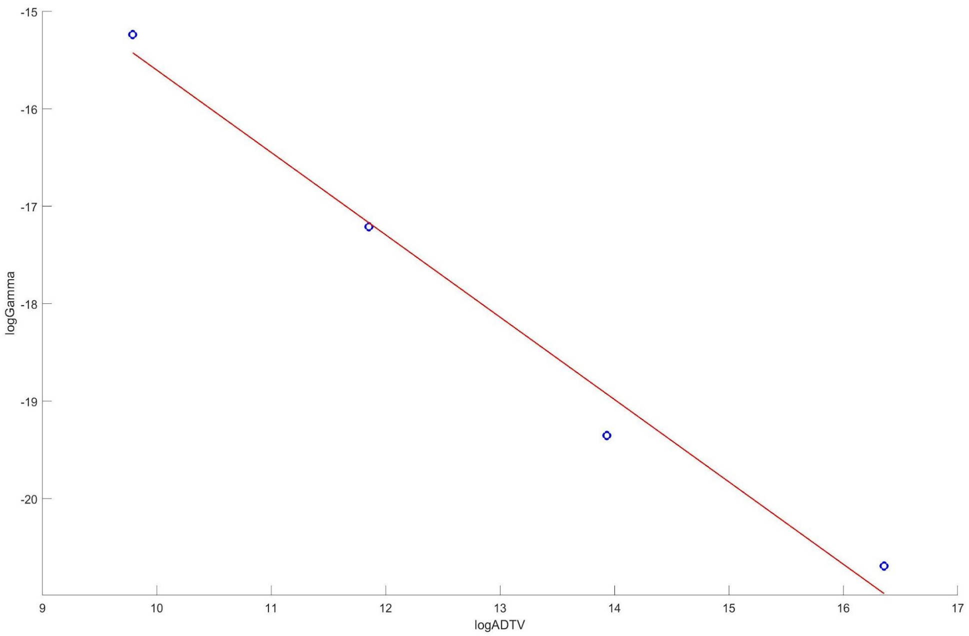
$$\hat{\gamma} = 10^{\hat{a}} \, ADTV^{\hat{b}}$$

Model with liquidity parameter

- Data 3-month tick-data with time step of 1 minute for 4 stocks:

Stock	Gamma
Credit Suisse	1.03140E-09
Richemont	3.95144E-09
Swisscom	3.35750E-08
SGS	2.40886E-07

- The ADTV method delivers a good fit to gamma estimated from tick-data. $(R^2 = 0.983)$
- ADTV method is chosen:
  - Tick-data method requires a huge amount of data
  - High computation effort required for tick-data
  - ADTV method is simple and very good



Liquidity Parameter Estimation

- Liquidity cost with percent :

$$L_t = -\theta x(V_t(-\theta x) - V_t) = -\theta x V_t(e^{-\gamma\theta x} - 1)$$

- Position Value:

$$\tilde{U}_t = U_t - L_t = xV_t + \theta x(e^{-\gamma\theta x} - 1)V_t$$

- For simplicity, we assume $\theta = 1$, the lending value :

$$P[e^{-\gamma x} x V_\tau \leq \lambda x V^*_{\tau-\delta}] \leq \epsilon$$

- V_t is geometric Brownian Motion

$$P[e^{-\gamma x} Z_\delta \leq \lambda \beta^{-1}] \leq \epsilon$$

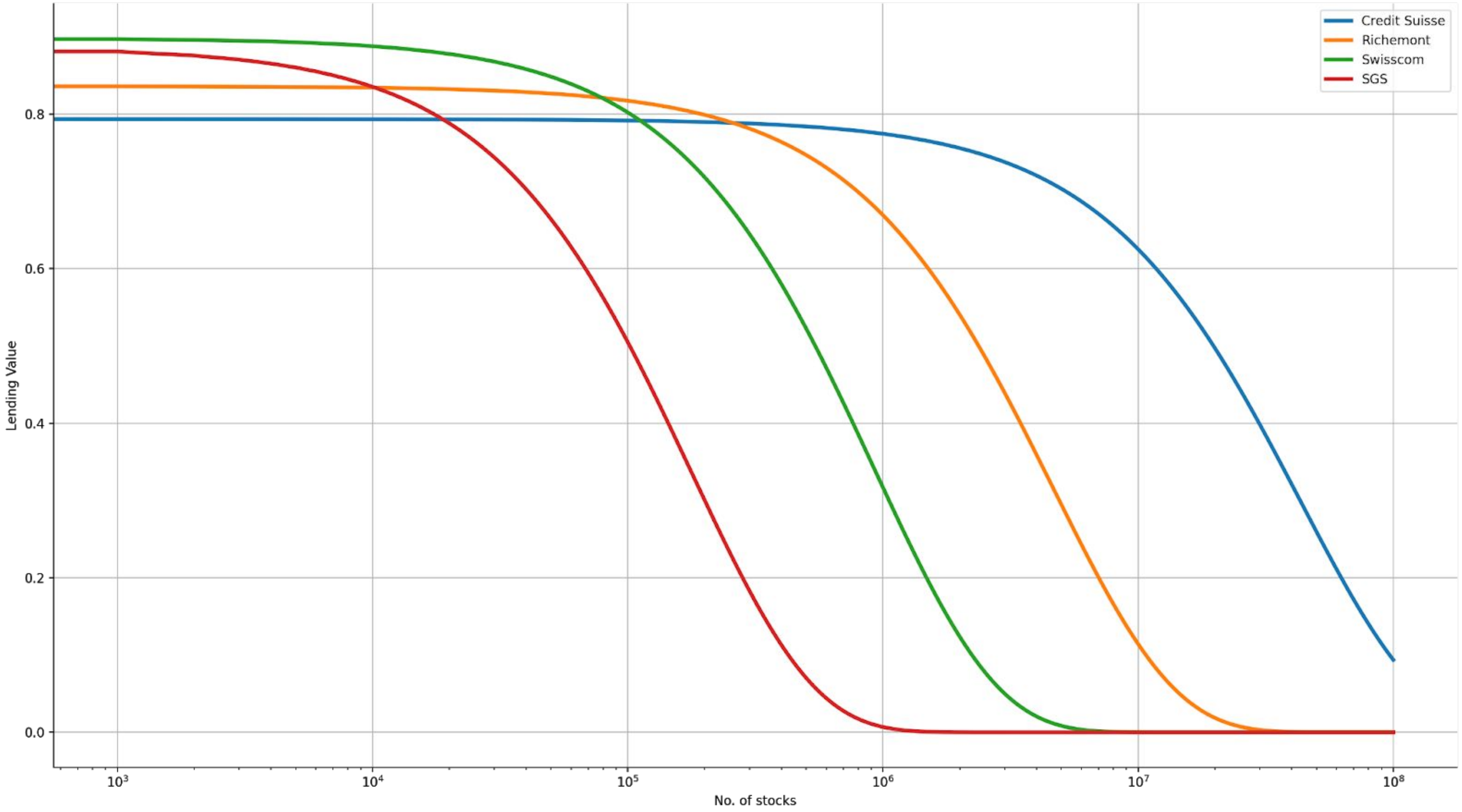
- Margin call trigger $\beta := 1 - (1-\lambda)\alpha$ and Z_δ is $(\mu - \frac{\sigma^2}{2}, \sigma^2\delta)$ - lognormally distributed

$$(1-\alpha)\exp(-\gamma x + \left(\mu - \frac{\sigma^2}{2}\right)\delta + \sigma\sqrt{\delta}\Phi^{-1}(\epsilon))$$

Liquidity Parameter

	ADTV	Gamma
Credit Suisse	22595017	1.931434e-08
Richemont	1379817	1.780821e-07
Swisscom	184674	8.802315e-07
SGS	23095	4.591796e-06

Lending Value with liquidity parameter



Lending Value with liquidity parameter

	x = 0	x = 1 000	x =1 000 000
Credit Suisse	0.79	0.79	0.77
Richemont	0.84	0.84	0.67
Swisscom	0.90	0.90	0.32
SGS	0.89	0.88	0.01

Possible extensions of the model

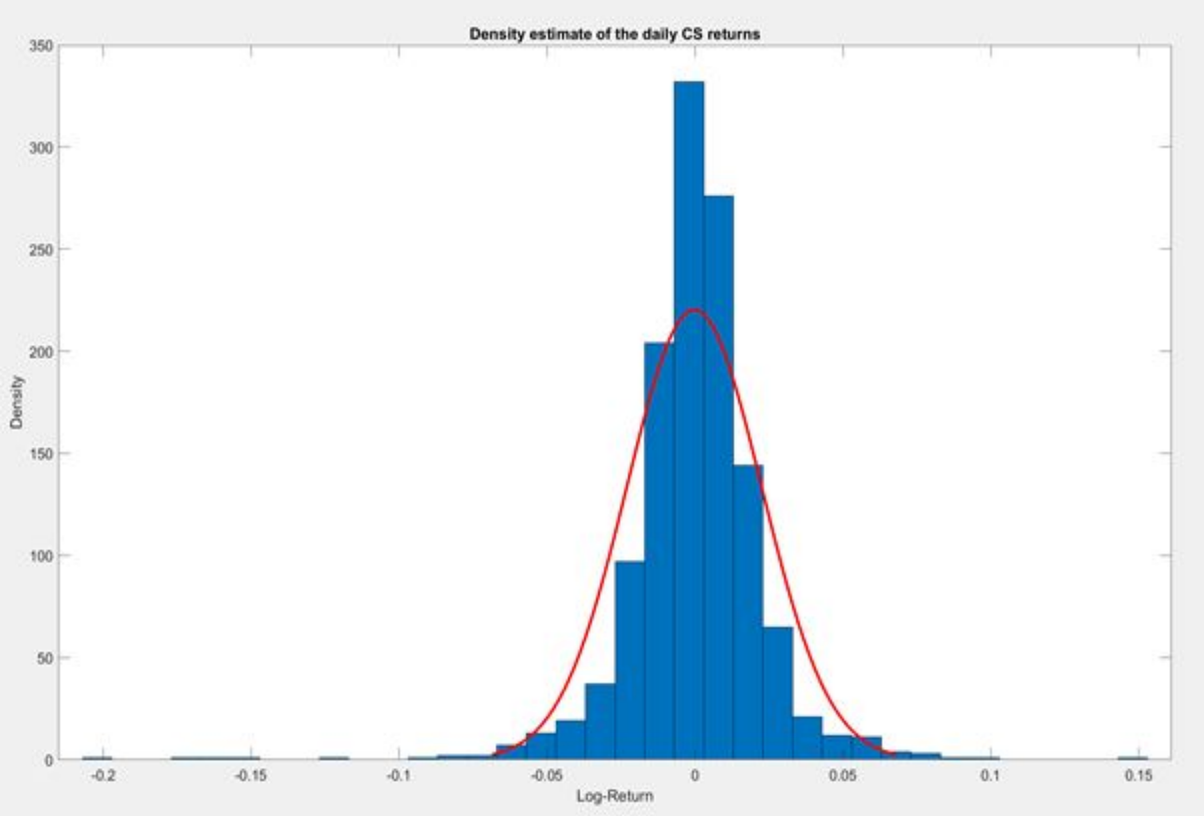
Main Assumptions for collateral

$$dV_t = V_t(\mu dt + \sigma dB_t), \qquad t \geq 0$$
$$V_0 = v_0 > 0.$$

- The collateral follows geometric Brownian motion
- Log-Return is normal-distributed
- Constant drift & volatility

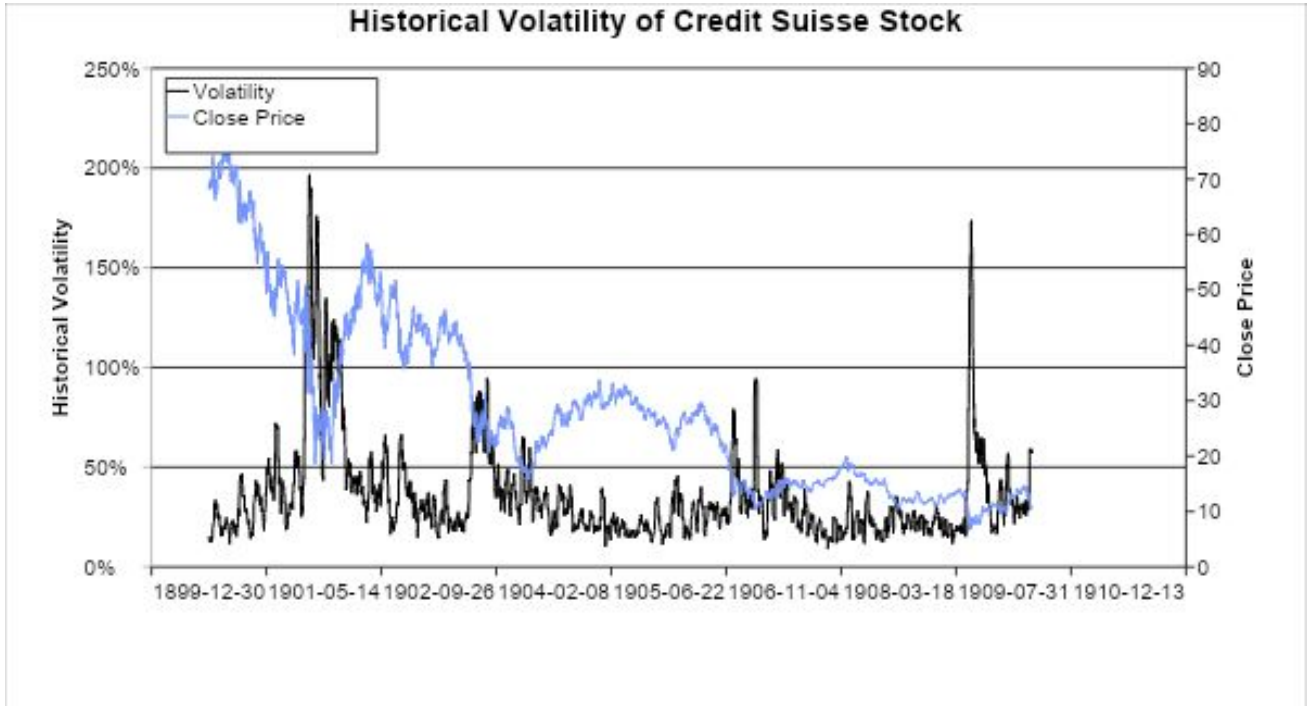
Log-Return is not normal-distributed

- Heavy Tail
- Left-Skewed
- Kurtosis >> 3



Varying Volatility

- High volatility during crises
- Strong negative correlation between stock price and volatility
- Financial Crisis → stock price falls → higher Debt / Equity ratio → the company is more risky → higher volatility



1. Extension – Local Volatility Model

$$dV_t = V_t(\mu(V_t, t)dt + \sigma(V_t, t)dB_t), \qquad t \geq 0$$
$$V_0 = v_0 > 0.$$

- The drift and volatility varies with the stock price and time.
- $\sigma(V_t, t)$ can be almost any function (continuous, positive, Lipschitz & linear growth conditions)
- $\sigma(V_t, t)$ can be determined by through Dupire's formula:

$$\sigma^2(S_t = K, T) = \frac{\frac{\partial C}{\partial T}(K,T) + q_T C(K,T) + (r_T - q_T)K\frac{\partial C}{\partial K}(K,T)}{\frac{1}{2}K^2\frac{\partial^2 C}{\partial K^2}(K,T)}$$

- Numerical Methods needed

2. Extension – Stochastic Volatility

$$dV_t = V_t\left(\mu(V_t, \sigma_t, t)dt + \sqrt{\sigma_t}dB_t\right)$$
$$d\sigma_t = \kappa(\theta - \sigma_t)dt + \eta\sqrt{\sigma_t}d\widetilde{B_t}$$

- Stochastic volatility term.
- σ_t is a mean reverting process.
- To guarantee σ_t, $\eta^2 < 2\kappa\theta$
- Market is not complete, Parameter must be calibrated with market data:
  - Least-Squares approach
  - Relative pricing errors model
  - Implied volatility
  - Weighted implied volatility

3. Extension – Merton model

$$V_t = V_0 exp(L_t)$$

$$L_t = \mu t + \sigma B_t + \sum_{k=1}^{N_t} Y_k$$

- N_t is a poisson process that tells how many jumps occured within a period.
- $P(N_t = k) = e^{-\lambda t} \frac{(\lambda t)^k}{k!}$.
- Y_k is the jump size which is normal distributed

Further Limitation of Geometric Brownian Motion

- Only one stock as collateral
- Not suitable for bonds product as collateral:
  - No upwards trend
  - Mean reverting process
- Difficulties for multicollateral Lombard Loan:
  - Addtional stochastic process – No closed-form analytical solution
  - Correlation between the collateral
  - Liquidation Analysis

Summary

Liquidity has a large impact on the lending value

- Lombard Loan is a credit pledged with relatively liquid assets as stocks, bonds and selected life insurance policies.
- The bank is more exposed to market risk than counterparty default risk.
- Liquidity has a large impact on the lending value of Lombard Loan.
- Limitation of Geometric Brownian Motion and possible extensions.

Thank you!