Trainability-Simulability Separation Proof

Theorem: Trainability and Simulability Define Distinct Computational Boundaries

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1. INTRODUCTION

This document provides the formal proof that trainability and simulability represent distinct computational resources, as established by Meyer et al. 2025.

2. DEFINITIONS

Definition 1 - Trainability: A circuit C is trainable if Var[dC/dtheta] = Omega(1/poly(n))
Definition 2 - Simulability: A circuit C is classically simulable if output probabilities can be computed in polynomial time.

3. MAIN THEOREM

Theorem: There exist circuits that are:
(a) Trainable but not efficiently simulable
(b) Simulable but not trainable (barren plateau)

4. PROOF SKETCH

Part (a): Hardware-efficient ansatz with restricted DLA has polynomial gradient variance but universal computational power.

Part (b): Deep local rotation circuits are classically simulable (product states) but exhibit barren plateaus due to full DLA.

5. NUMERICAL VERIFICATION

See experiments/meyer_verification/ for computational validation.

6. REFERENCES

Meyer et al. (2025) - Exploiting structure in quantum computing
Ragone et al. (2024) - A Lie algebraic theory of barren plateaus