

In recent decades topology has repeatedly emerged as a source of fascinating discoveries in condensed matter physics. In particular, topological defects have underlied diverse phenomena from fractional electric charge¹ to the superfluid transition in liquid helium-4². Even more recently, the importation of ideas from topology in electronic systems to photonic systems has led to a flood of new discoveries and potential devices³. My Ph.D. research will focus on the prediction of novel properties of topological defects in photonic and electronic systems, with an emphasis on both the **fundamental physics** and **device applications** of these defects.

Intellectual Merit: I have past experience researching a particularly striking property of topological defects. Since July I have been doing research full-time under Professor Claudio Chamon at Boston University, where **in a recent publication (*Science*, in review) we have proposed to use topological defects to realize non-Abelian exchange statistics in photonic systems**⁴. The defect occurs as a vortex in the order parameter describing a certain lattice distortion in two-dimensional honeycomb lattices, and leads to the formation of localized states which gain non-Abelian geometric phases upon their exchange. Our proposal is the first to predict non-Abelian statistics in photonic systems, as opposed to the delicate electronic systems their search was previously confined to, and could thus lead to their first conclusive experimental detection.

My past research exemplifies what I find fascinating about topological defects in electronic and photonic systems - exotic phenomena, with the potential for real-world realization and application. My future work will continue to espouse these traits. I propose two projects. In project (1) I will extend the defect states studied in my previous work to a family of states obeying a **broader class of non-Abelian exchange statistics**, and use these states to propose a **novel topological nonreciprocal optical device**. In project (2) I will separately extend these defect states to Weyl semimetals and their photonic analogues, and show that they can sustain **dissipationless transport** in both systems. Further, I will address **how these topological defects could naturally arise in Weyl semimetals** using a field theoretic approach. I describe each project in more detail below.

Project (1) will seek to realize novel forms of non-Abelian statistics, which when implemented in photonics could lead to a strongly nonreciprocal optical device. While the statistics I previously studied respect reciprocity (light passed through an exchange in one direction will gain the same phase shift as in the opposite direction), I have discovered a simple, exactly solvable, family of topological defect states with “nonreciprocal” statistics. Importantly, the Hamiltonian giving rise to these states breaks a particular symmetry present in the previous, “reciprocal”, Hamiltonian. Unfortunately, these particular defects would be difficult to realize in photonics experiment. My research will thus take clues from this simple model and focus on **using symmetry classification to discover new nonreciprocal extensions of these topological defect states**, especially those which are **experimentally feasible**. Breaking symmetries of the system will bring us into a different symmetry class, in which case I will determine whether nonreciprocity and topological protection of our state can coexist. I will be aided by **international collaboration** with Professor Chamon’s colleague Doctor Christopher Mudry at the Paul Scherrer Institute in Switzerland, an expert in symmetry classification of topological defects. In determining which photonic systems are feasible, I will benefit from **current collaboration with engineering and experimental photonics groups at Boston University and MIT** working to realize the experiment proposed in my previous publication. Beyond device applications, providing a physical realization of this class of non-Abelian statistics would constitute a **major advancement** in fundamental physics.

In Project (2) I will pursue a separate extension of the same topological defect states into both electronic and photonic systems, in which I propose to use a 3D generalization of these

states to achieve **topologically-protected dissipationless transport in Weyl semimetals**. Weyl semimetals have a low-energy electronic theory similar to the systems I previously researched⁵, and are thus capable of sustaining 3D analogs of the same defect states. Crucially however, these formerly-2D states will now extend in the third direction, and can carry current in that direction. **I have already analytically shown that such states would have a chiral dispersion relation**, leading to a lack of backscattering and thus dissipationless transport. To substantiate these claims, in the first part of project (2) I will determine the position-space changes in hopping needed to create these topological defects, using common tight-binding models of Weyl semimetals. As in my previous work, I will use these hoppings to perform exact diagonalization simulations to quantitatively probe the resulting defect states, and verify their chiral dispersion relation. Experimental verification of these properties may be easiest in photonic realizations of Weyl semimetals³, for which I would adapt my model to the photonic setting and find estimates of experimental conditions needed for the states' realization, similar to my previous work.

The second part of my project (2) will address the fact that, in electronics, the conductance of a single defect would be small even with dissipationless transport. To remedy this, I will search for a **field theory of the order parameter describing the appropriate hopping distortions, in which the creation of a macroscopic number of topological defects would occur**. The starting point must be a nonzero mean field value of this order parameter. To determine the conditions for this to occur, I will introduce the appropriate order parameter couplings into the electronic field theory, and integrate out the electron degrees of freedom to derive an effective action for the order parameter. Assuming a nonzero mean field value, one would anticipate a massless Goldstone mode associated with fluctuations in the order parameter phase. It would then be an open problem to determine how topological defects in this Goldstone mode might form, although pursuing connections to vortex formation elsewhere in condensed matter may be fruitful. My extensive graduate-level coursework in field theory well prepares me for this component of the project.

Broader Impacts: Both of my proposed projects feature topologically-protected states with large potential societal impacts through use in photonic and/or electronic devices. Project (1) proposes to use the geometric phase of a topological defect state to uniquely achieve strongly nonreciprocal photonic devices, usable in essential optical elements such as isolators and circulators. This could **greatly reduce undesirable optical loss** compared to traditional nonreciprocal devices, which rely on weakly (relative to my proposal) nonreciprocal materials such as ferrite or strong permanent magnets. Additionally, my proposal would not require breaking time-reversal symmetry, opening the door to **nonreciprocal devices using only conventional materials!**

Project (2) seeks to realize defect states with chiral dispersion in Weyl systems, with potential impact in both photonics and electronics. In photonics, these modes could function as **lossless optical lines, robust against imperfections** due to their topological origin. Such topological "one-way waveguides" have been realized in photonic quantum hall edge states, and have a plethora of novel device applications³. In electronics, the accumulation of a macroscopic number of these modes would lead to **massive conductance**, with clearly tremendous device implications. Assessing the applications and limitations of such devices would require a theory for how this accumulation of topological defects occurs, highlighting the need for the second component of my project (2). All together, this array of broader impacts makes the investigation of topological defects in photonics and electronics well worth pursuing. The NSF GRFP will allow me to do so.

[1] R. Jackiw, et al., *Phys. Rev. D* (1976). [2] J. M. Kosterlitz, et al., *Journal of Physics C* (1973). [3] L. Lu, et al., *Nature Photonics* (2014). [4] T. Iadecola, et al., (2015). [5] X. Wan, et al., *Physical Review B* (2011).