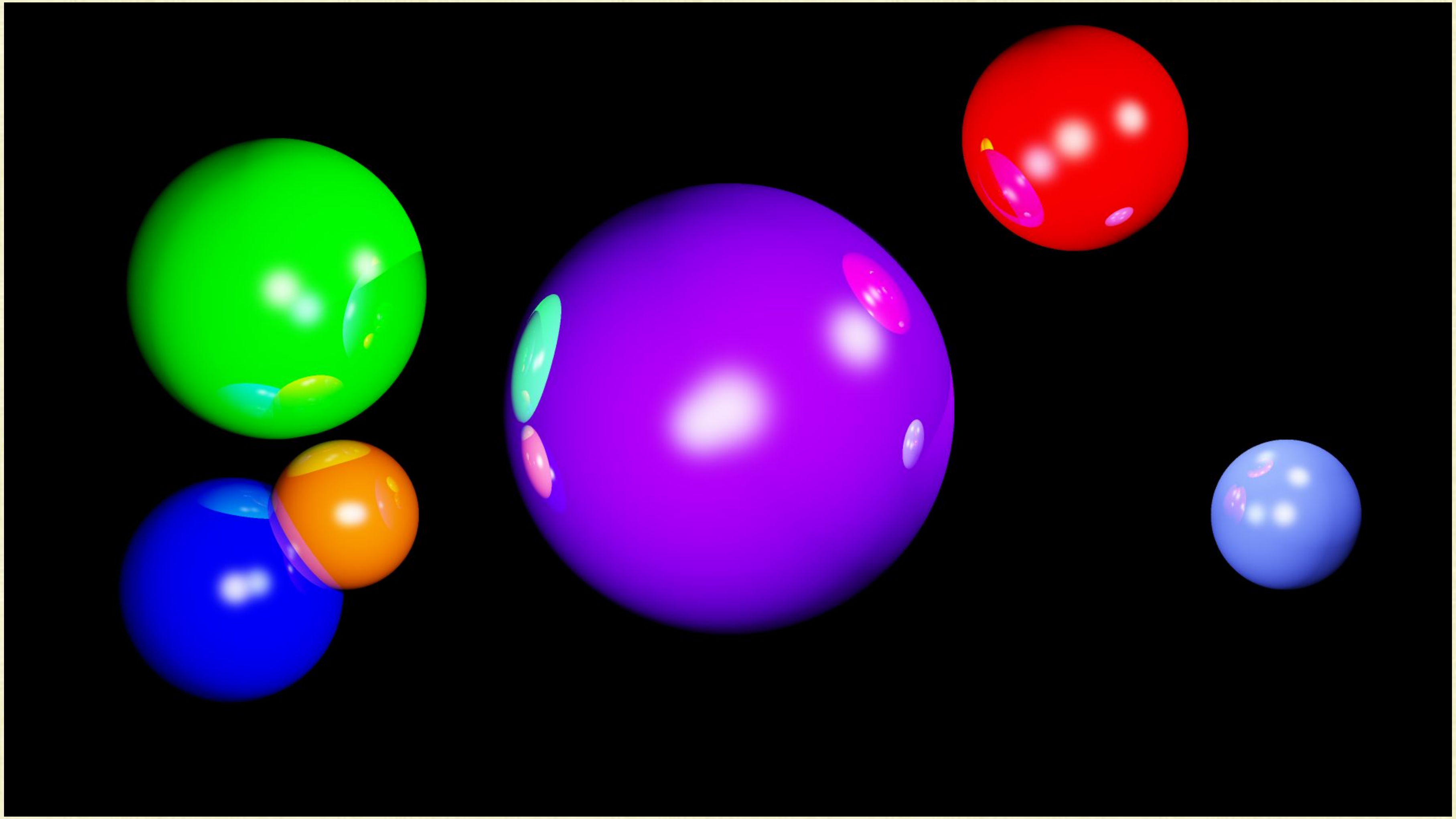
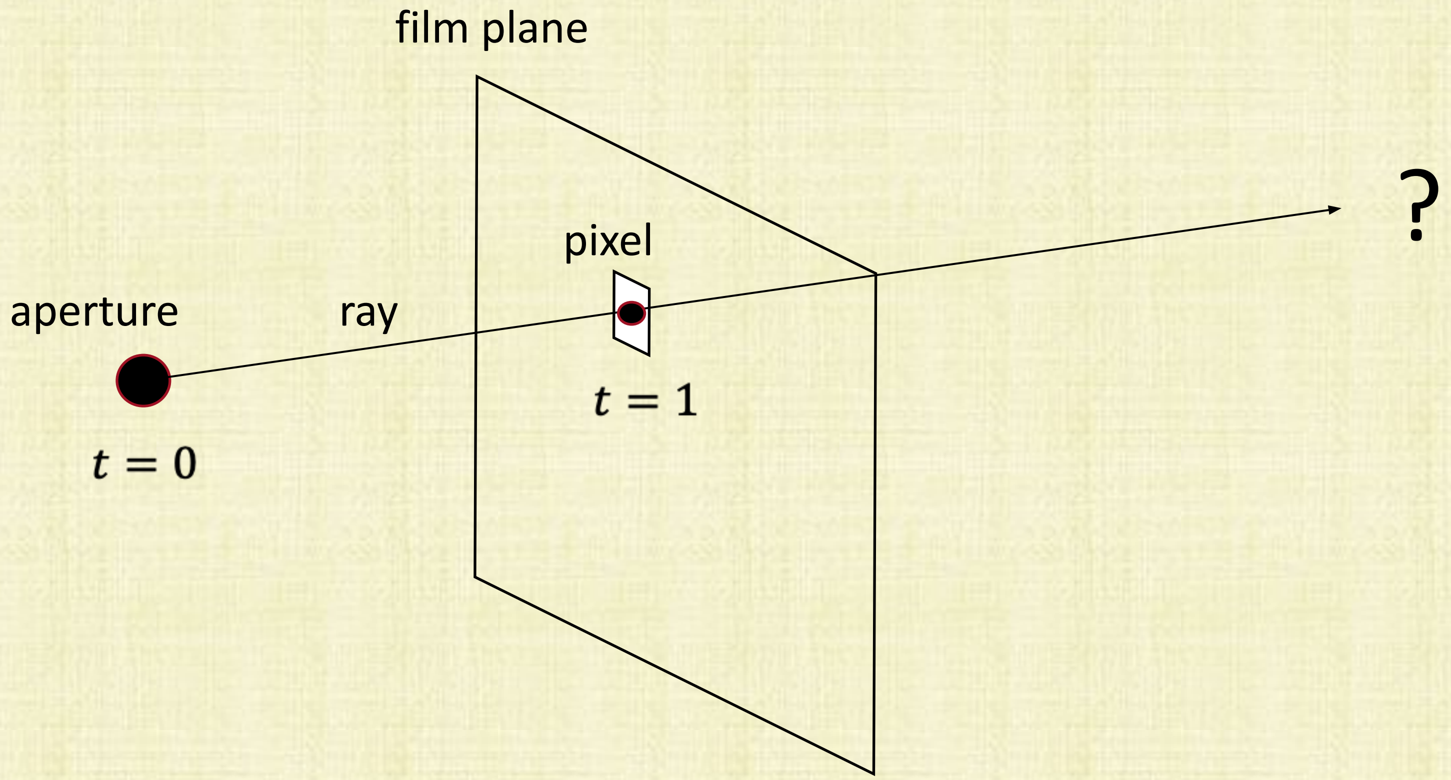
# Ray Tracing



# Constructing Rays

- For each pixel, make a ray and intersect it with objects in the scene
- The first intersection is used to determine a color for the pixel
- The ray is $R(t) = A + (P - A)t$ where $A$ is the aperture and $P$ is the pixel location
- The ray is defined by $t \in [0, \infty)$, although only $t \in [1, t_{far}]$ will be inside the viewing frustum
- We only care about the first intersection with $t \geq 1$



# Parallelization

- Ray tracing is a per pixel operation (scanline rendering is a per triangle operation)
- Ray tracing is inherently parallel, since the ray for each pixel is independent of the rays for other pixels

- Can utilize modern parallel CPUs/Clusters/GPUs to significantly accelerate a ray tracer
    - Threading (e.g., Pthread, OpenMP) distributes rays across CPU cores
    - Message Passing Interface (MPI) distributes rays across CPUs on different machines (unshared memory)
    - OptiX/CUDA distributes rays on the GPU
- Memory coherency helps when distributing rays to various threads/processors
    - Assign spatially neighboring rays (passing through neighboring pixels) to the same core/processor
    - These rays tend to intersect with the same objects in the scene, and thus tend to access the same memory

- For the sake of comparison, scanline rendering is a per triangle operation, and is parallelized to handle one triangle at a time (usually on a GPU)

# Ray-Triangle Intersection

- Given the enormous number of triangles, many approaches have been implemented and tested in various software/hardware settings:

- Triangles are contained in planes, so it is often useful to look at Ray-Plane intersections first
- A Ray-Plane intersection yields a point, and a subsequent test determines whether that point is inside the triangle (or not)
  - Both the triangle and the point can be projected into 2D, and the 2D triangle rasterization test (to the left of all 3 rays, discussed last week) can be used to determine "inside"
    - The projection can be done into the xy, xz, yz plane by merely dropping the z, y, x coordinate (respectively) from the triangle vertices and the point
    - The most robust coordinate to drop is the one with the largest component in the triangle's normal (so that the projected triangle has maximal area)
  - Alternatively, there is a fully 3D version of the 2D rasterization

- One can skip the Ray-Plane intersection and consider the Ray-Triangle intersection directly
  - Thematically, this approach is similar to how ray tracing works for non-triangle geometry (ray tracers handle non-triangle geometry better than scanline rendering does)

# Ray-Plane Intersection

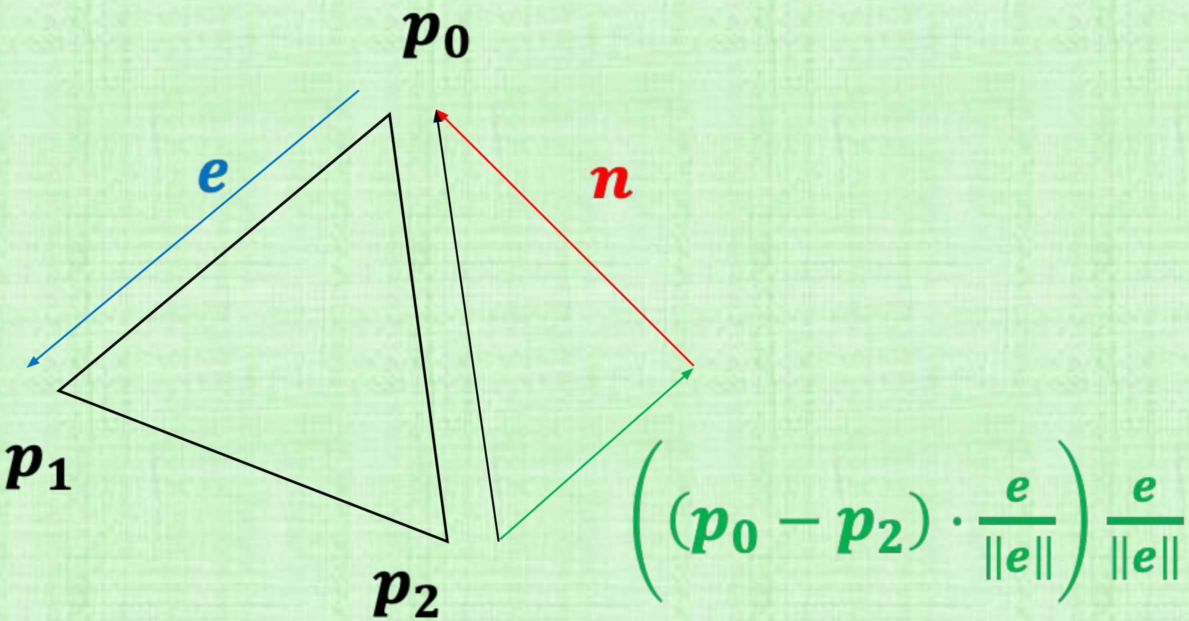
- A plane is defined by a point $p_o$ (on it) and a normal direction $N$
- A point $p$ is on the plane if $(p - p_o) \cdot N = 0$
- A ray $R(t) = A + (P - A)t$ intersects the plane when $(R(t) - p_o) \cdot N = 0$ for some $t \geq 0$
- That is, $(A + (P - A)t - p_o) \cdot N = 0$ or $(A - p_o) \cdot N + (P - A) \cdot Nt \; = 0$
- So, $t = \frac{(p_o - A) \cdot N}{(P - A) \cdot N}$
- Note: The length of $N$ cancels (so it need not be unit length)

- As always, if $t \notin [1, t_{far}]$ or there is another intersection with a smaller $t$ value, then this intersection is ignored

- Note: the (non-unit length) triangle normal can be computed by taking the cross product of any two edges (as long as the triangle does not have zero area)
- Note: Any triangle vertex can be used as a point on the plane

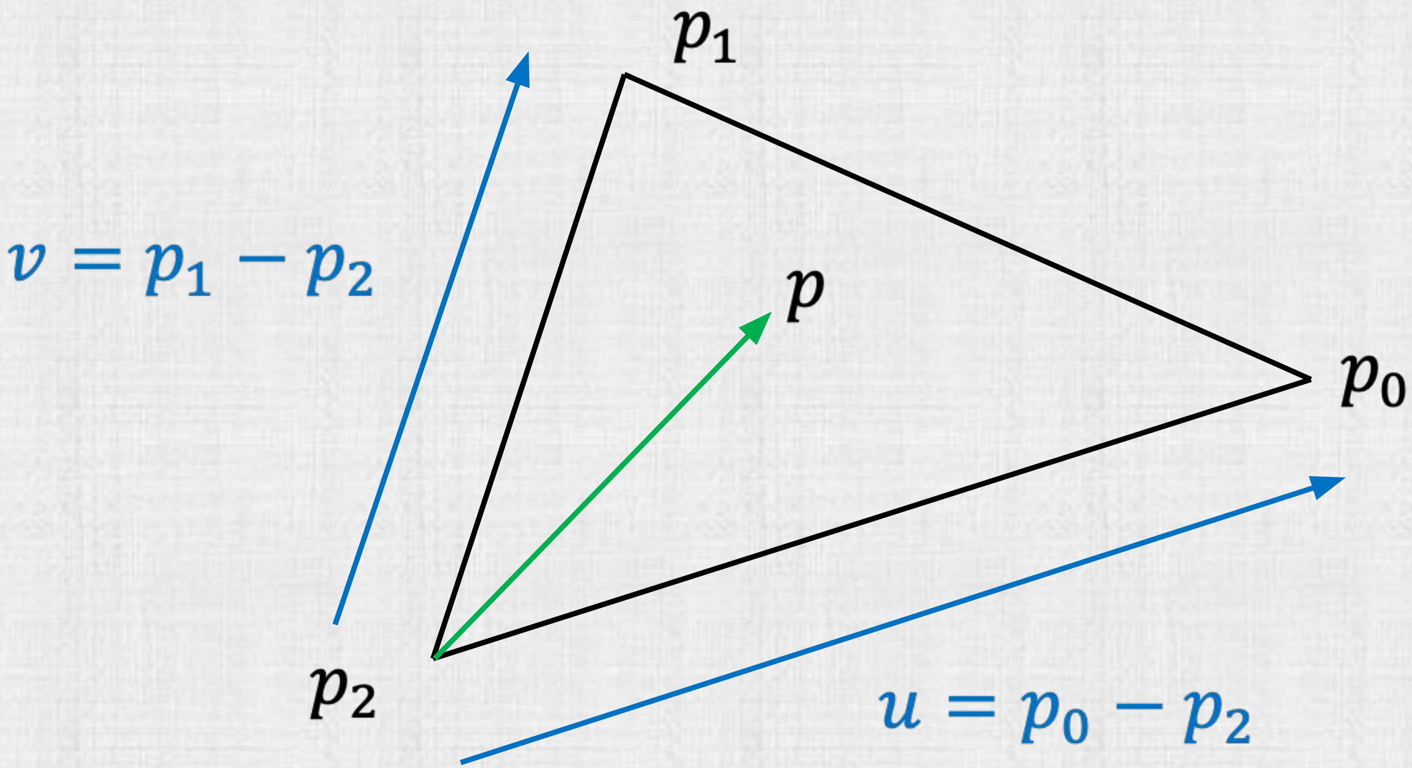
# 3D Point Inside a 3D Triangle

- Given $t_{int} = \frac{(p_o - A)\cdot N}{(P-A)\cdot N}$, evaluate $R(t_{int}) = R_o$ to find the intersection point
- Then, given a directed edge of the triangle $e = p_1 - p_0$, compute a normal to that edge (in the plane of the triangle) via $n = (p_0 - p_2) - \left((p_0 - p_2)\cdot \frac{e}{\|e\|}\right)\frac{e}{\|e\|}$
- As usual, $R_o$ is interior to ray $e$ when $(R_o - p_0)\cdot n < 0$
- As usual, if $R_o$ is interior to all three edges, it is interior to the triangle



# Recall: Triangle Basis Vectors

- Compute edge vectors $u = p_0 - p_2$ and $v = p_1 - p_2$
- Any point $p$ interior to the triangle can be written as $p = p_2 + \beta_1 u + \beta_2 v$ with $\beta_1, \beta_2 \in [0,1]$ and $\beta_1 + \beta_2 \leq 1$
- Substitutions and collecting terms gives $p = \beta_1 p_0 + \beta_2 p_1 + (1 - \beta_1 - \beta_2) p_2$ implying the equivalence: $\alpha_0 = \beta_1,\ \alpha_1 = \beta_2\ , \alpha_2 = 1 - \beta_1 - \beta_2$



# Direct Ray-Triangle Intersection

- (Recall) Triangle Basis Vectors: $p = p_2 + \beta_1 u + \beta_2 v$ with $\beta_1, \beta_2 \in [0,1]$ and $\beta_1 + \beta_2 \leq 1$
- Points on the ray have $R(t) = A + (P - A)t$
- So an intersection point has $A + (P - A)t = p_2 + \beta_1 u + \beta_2 v$
- Or $(u \quad v \quad A - P)\begin{pmatrix}\beta_1 \\ \beta_2 \\ t\end{pmatrix} = A - p_2$ where $(u \quad v \quad A - P)$ is a 3x3 matrix and $A - p_2$ is a 3x1 vector (3 equations with 3 unknowns)
- This 3x3 system is degenerate when the columns of the 3x3 matrix are not full rank
- This happens when the triangle has zero area or the ray direction, $P - A$, is perpendicular to the plane's normal
- Otherwise, there is a unique solution
- $R(t_{int})$ is inside the triangle, when the unique solution has: $\beta_1, \beta_2 \in [0,1]$ and $\beta_1 + \beta_2 \leq 1$

- As always, if $t \notin [1, t_{far}]$ or there is another intersection with a smaller $t$ value, then this intersection is ignored

# Solving with Cramer's Rule

- Solving the 3x3 system with Cramer's Rule allows for code optimization:

- First compute the determinant of the 3x3 coefficient matrix $\Delta = |(u \quad v \quad A - P)|$, which is nonzero when a solution exists
- Then compute $t = \frac{\Delta_t}{\Delta}$ where the numerator is the determinant: $\Delta_t = |(u \quad v \quad A - p_0)|$
- When $t \notin \left[1, t_{far}\right]$ or there is an earlier intersection, one can quit early (ignoring this intersection)

- Compute $\beta_1 = \frac{\Delta_{\beta_1}}{\Delta}$ where $\Delta_{\beta_1} = |(A - p_0 \quad v \quad A - P)|$
- When $\beta_1 \notin [0,1]$ one can quit early

- Compute $\beta_2 = \frac{\Delta_{\beta_2}}{\Delta}$ where $\Delta_{\beta_2} = |(u \quad A - p_0 \quad A - P)|$
- When $\beta_2 \in [0,1 - \beta_1]$ the intersection is marked as true

# Ray-Object Intersections

- As long as a ray-geometry intersection routine can be written, ray tracing can be applied to any representation of geometry
- This is in contrast to scanline rendering where objects need to be turned into triangles
- In addition to triangle meshes, ray tracers may use: analytic descriptions of geometry, implicitly defined surfaces, parametric surfaces, etc.

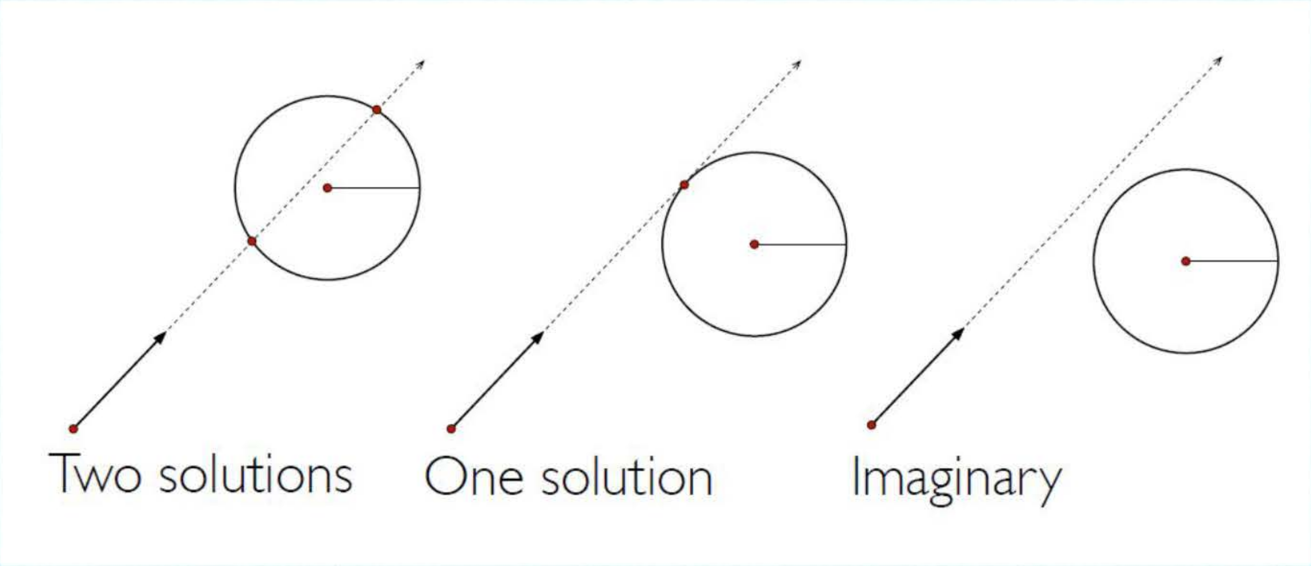
- The surfaces of many objects can be written as functions
- E.g., $f(p) = 0$ if and only if $p$ is on the surface (e.g. the equation for a plane)
- Sometimes there are additional constraints (such as on the barycentric weights for triangles)
- One broad/useful class of such objects are <u>implicit surfaces</u> (covered later in the class)
- The ray-object intersection routines often proceed down a similar path:
  - substitute the ray equation in for the point, i.e. $f(R(t)) = 0$
  - solve for $t$
  - check the solution against any additional constraints

# Ray-Sphere Intersections

- A point $p$ is on a sphere with center $C$ and radius $r$ when $\|p - C\|_2 = r$
  - Or (squaring both sides), when $(p - C) \cdot (p - C) = r^2$
- Substitute $R(t) = A + (P - A)t$ in for $p$ to get a quadratic equation in $t$:

$$(P - A) \cdot (P - A)t^2 + 2(P - A) \cdot (A - C)t + (A - C) \cdot (A - C) - r^2 = 0$$

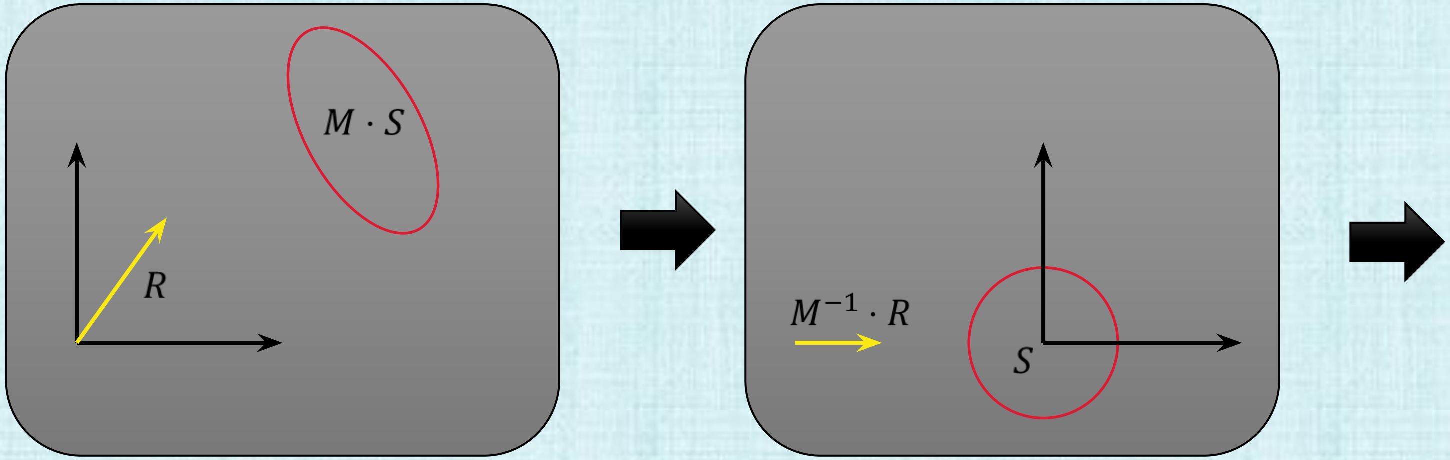
- When the discriminant of this quadratic equation is positive, there are two solutions (choose the one the ray hits first)
- When the discriminant is zero, there is one solution (the ray tangentially grazes the sphere)
- When the discriminant is negative, there are no solutions



# Transformed Objects

- Geometry is often stored/represented in a convenient object space
- The object space can make the geometry simpler to deal with
  - E.g., spheres can be centered at the origin, objects are not sheared, coordinates may be non-dimensionalized for numerical robustness, there may be (auxiliary) geometric acceleration structures, more convenient color and texture information, etc.

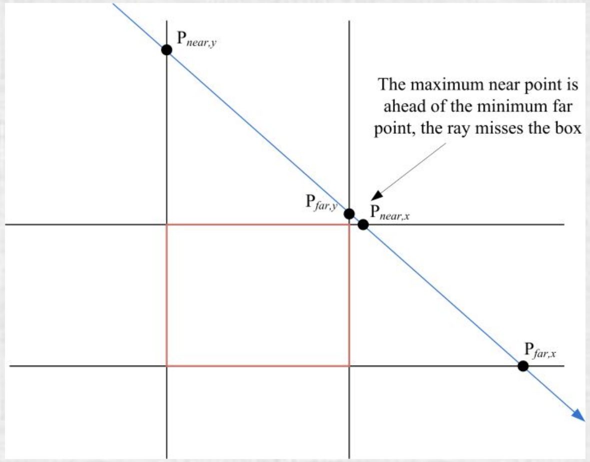
- We often prefer to ray trace in this convenient object space, rather than world space
- Transform the ray into object space and find the ray-object intersection, then transform the relevant information back to world space

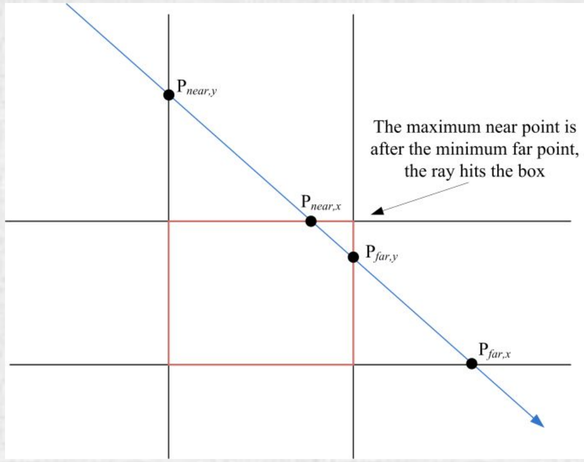


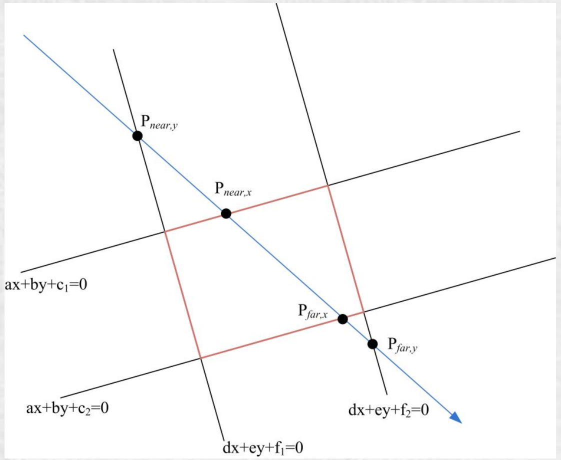

$$\boldsymbol{M} \cdot \boldsymbol{R}_{\boldsymbol{object}}(\boldsymbol{t}_{\boldsymbol{int}})$$

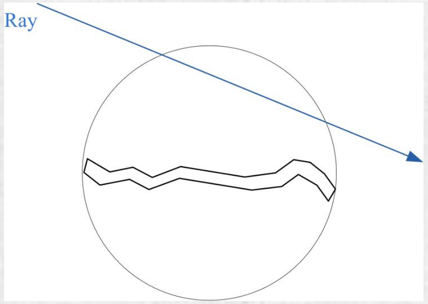
# Aside: Code Acceleration

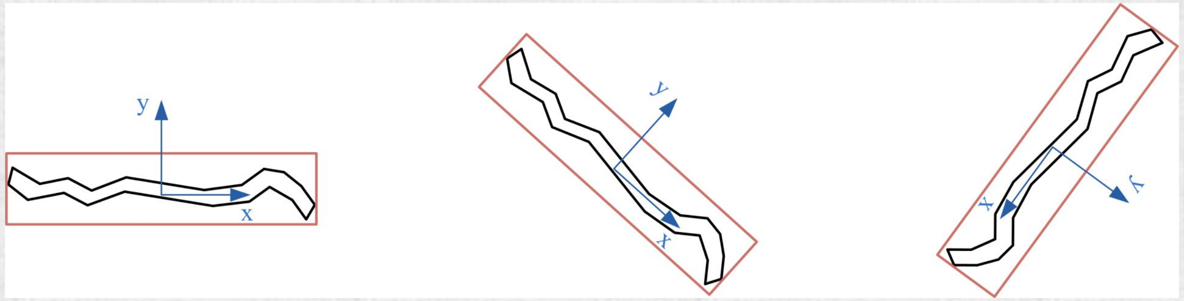
- Ray-Object intersections can be expensive
- So, put complex objects inside simpler objects, and first test for intersections against the simpler object (potentially skipping tests against the complex object)
- Simple <u>bounding volumes</u>: spheres, axis-aligned bounding boxes (AABB), or oriented bounding boxes (OBB)



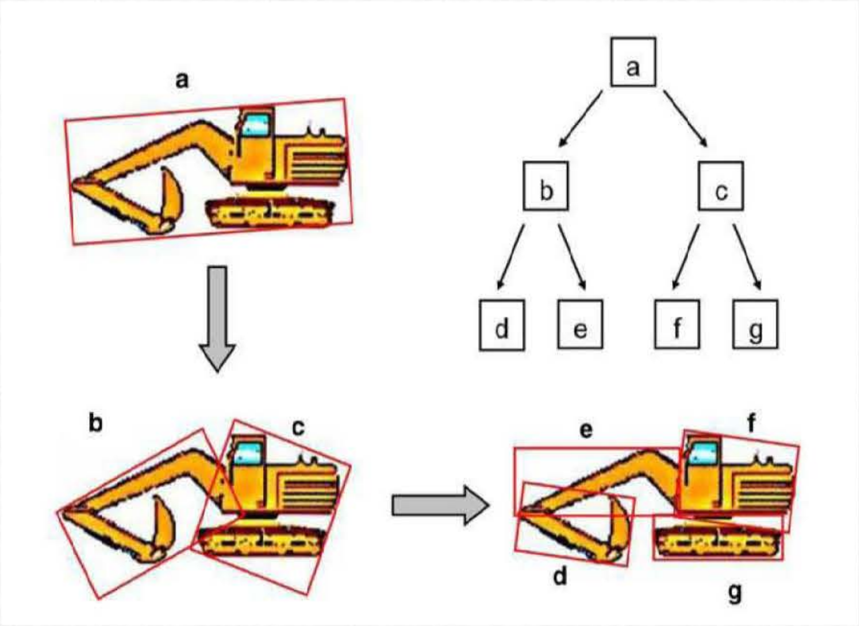



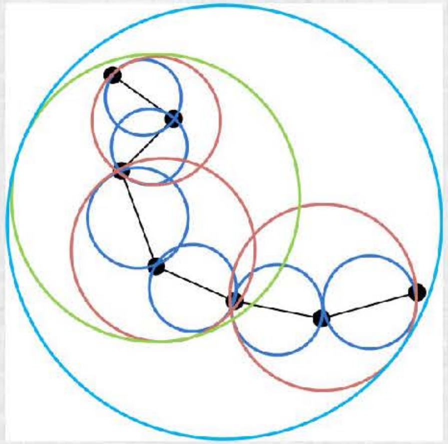



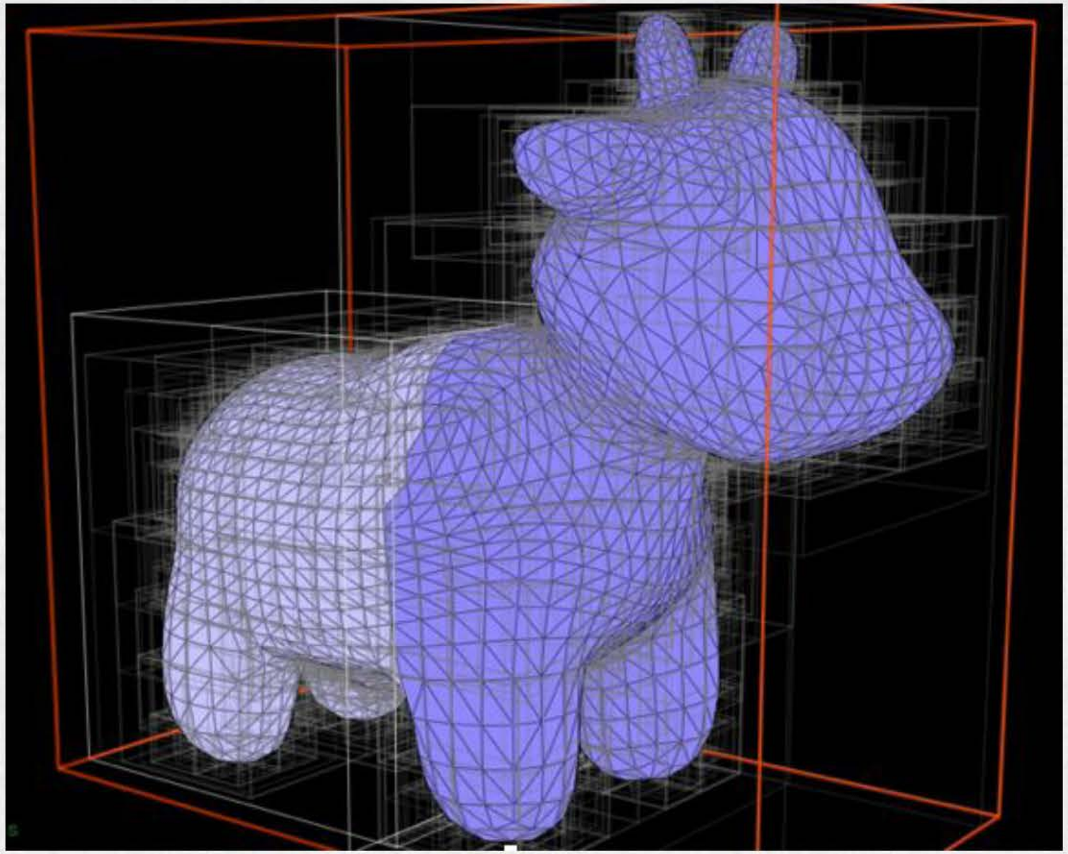



# Aside: Code Acceleration

- For complex objects, build a <u>hierarchical tree structure</u> in object space
- The lowest levels of the tree contain the primitives used for intersections (and have simple geometry bounding them); then, these are combined hierarchically into a $\log n$ height tree
- Starting at the top of a Bounding Volume Hierarchy (BVH), one can prune out many nonessential (missed) ray-object collision checks

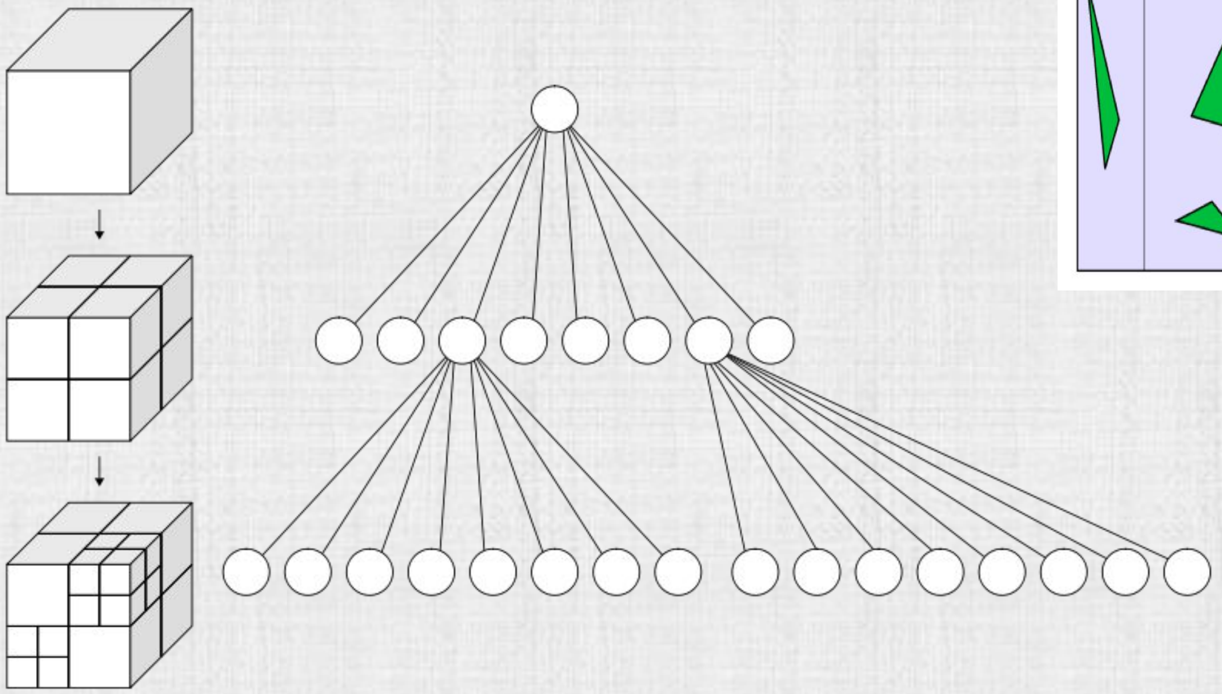


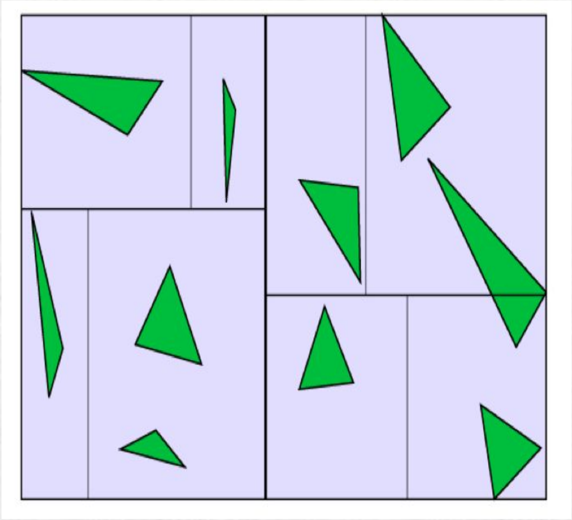



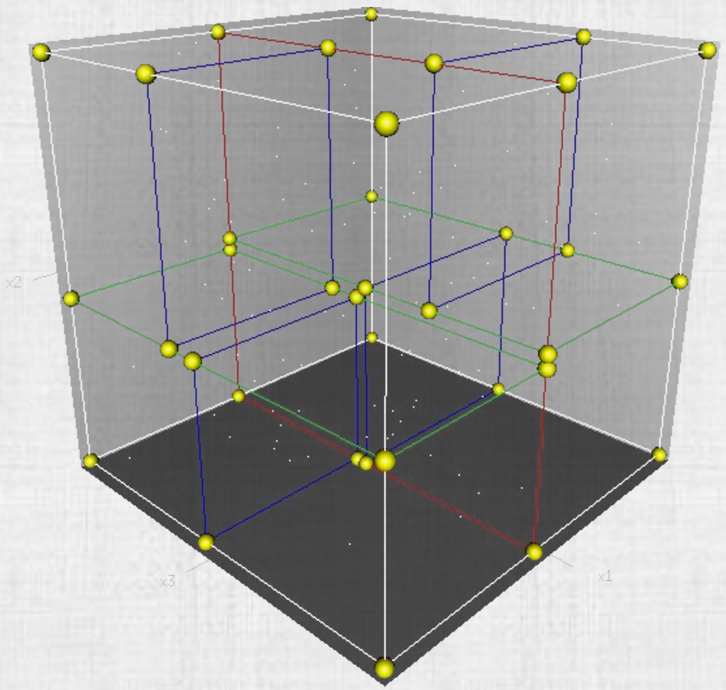


# Aside: Code Acceleration

- Instead of a bottom-up bounding volume hierarchy approach, <u>octrees</u> and <u>K-D trees</u> take a top-down approach to hierarchically partitioning objects (and space)







# Normals

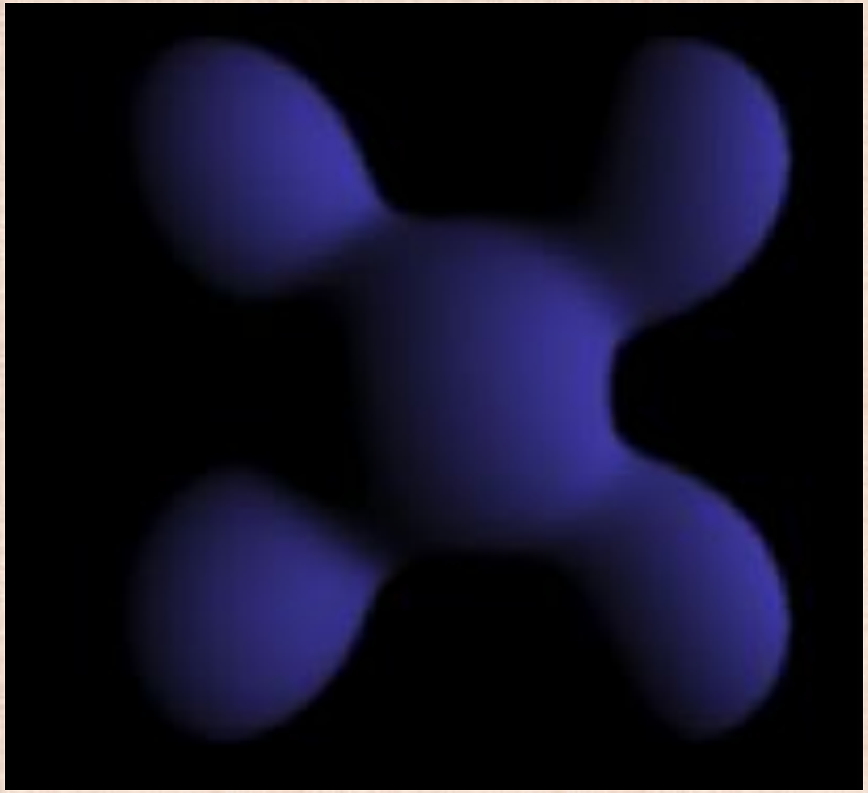
- We need to model how the intersected geometry interacts with incoming light
- Objects tilted towards the light are bombarded with more photons than those tilted away from the light

- The surface normal at the point $R(t_{int})$ can be used to approximate a plane (locally) tangent to the surface
- Compare the (unit) incoming light direction $\hat{L}$ with the local (unit) normal $\hat{N}$ to approximate the titling angle via: $-\hat{L} \cdot \hat{N} = \cos\theta$
- Incoming light with intensity $I$ is <u>scaled down</u> to $I \max(0, \cos\theta)$
  - the max with 0 prunes triangles facing away from the light
- If $(k_R, k_G, k_B)$ is the RGB color of a triangle ($k_R, k_G, k_B \in [0,1]$ are reflection coefficients), then the pixel color is $(k_R, k_G, k_B)\, I \max(0, \cos\theta)$

# Ambient vs. Diffuse Shading

- <u>Ambient shading</u> colors a pixel when its ray intersects the object
- <u>Diffuse shading</u> attenuates object color based on how far the local unit normal is tilted away from the light source (note how your eyes/brain work backwards and imagine a 3D shape)

**Ambient**



**Diffuse**

# Computing Unit Normals

- The unit normal to a plane is used in the plane's definition, and is thus readily accessible
  - although it might need to be normalized to unit length

- The unit normal to a triangle can be computed by normalizing the cross product of two edges
- Be careful with the ordering in the cross product to make sure the normal points outwards from the object (as opposed to inwards)

- For more general objects, one needs to provide a function that returns an (outward) unit normal for the point of intersection
- E.g., a sphere with intersection point $R(t_{int})$, has an (outward) unit normal of:

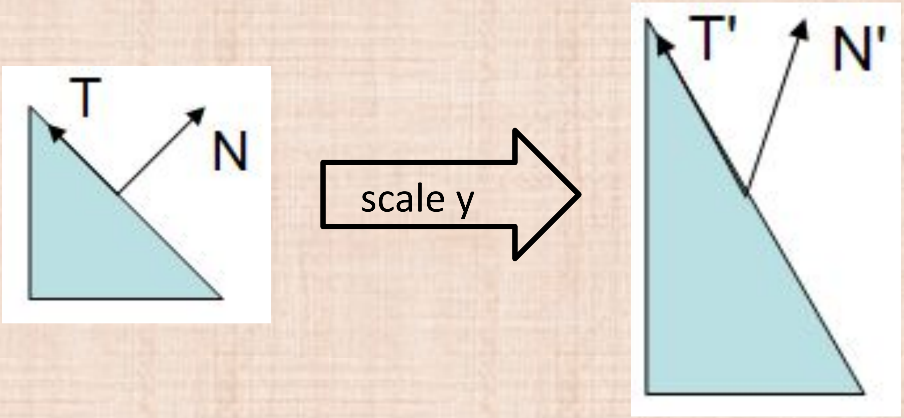
$$\hat{N} = \frac{R(t_{int}) - C}{\|R(t_{int}) - C\|_2}$$

# Transformed Objects

- When ray tracing geometry in object space, the object space normal needs to be transformed back into world space along with the intersection point

- Let $u$ and $v$ be edge vectors of a triangle in object space, and $Mu$ and $Mv$ be their corresponding world space versions
- The object space normal $\widehat{N}$ is transformed to world space via $M^{-T}\widehat{N}$
  - Note: $Mu \cdot M^{-T}\widehat{N} = (Mu)^T M^{-T}\widehat{N} = u^T M^T M^{-T}\widehat{N} = u^T\widehat{N} = u \cdot \widehat{N} = 0$, and $Mv \cdot M^{-T}\widehat{N} = 0$
  - Note: $M^{-T}\widehat{N}$ needs to be normalized to make it unit length

- Careful, DO NOT USE $M\widehat{N}$ as the world space normal:




N’ is not the normal

# Shadows

- The incoming light intensity $I$ needs to be reduced, when photons are blocked by other objects or parts of the same object
- Shadow rays determine if photons from a light can directly hit a point under examination
- A shadow ray is cast from the intersection point $R(t_{int})$ in the direction of the light $-\hat{L}$:

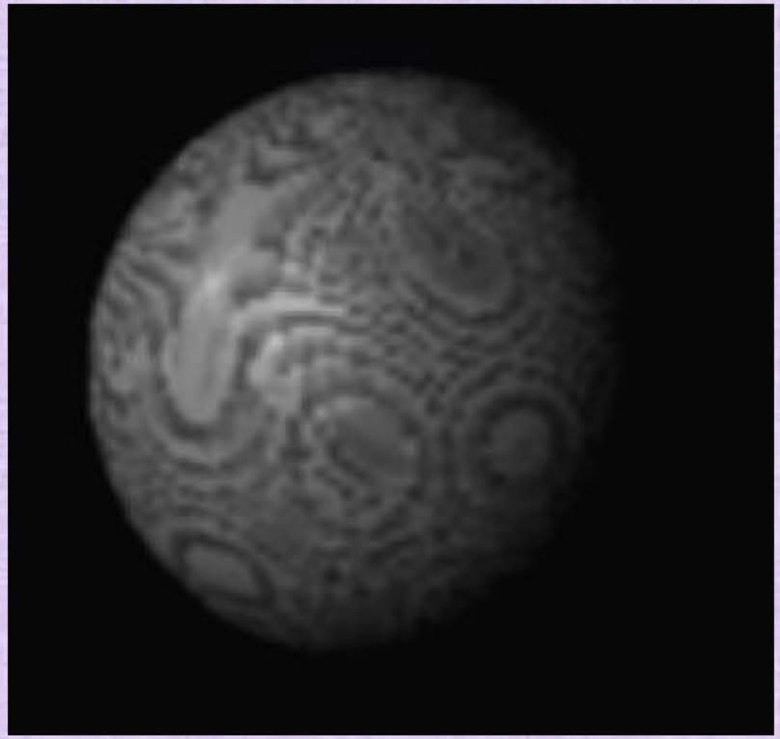
$$S(t) = R(t_{int}) - \hat{L}t \text{ with } t \in (0, t_{light})$$

- If no intersections are found for $0 < t < t_{light}$, then the light source is unobscured
- Otherwise, the point is shadowed (by whatever intersected the shadow ray), and photons from the light source are not used to color the pixel

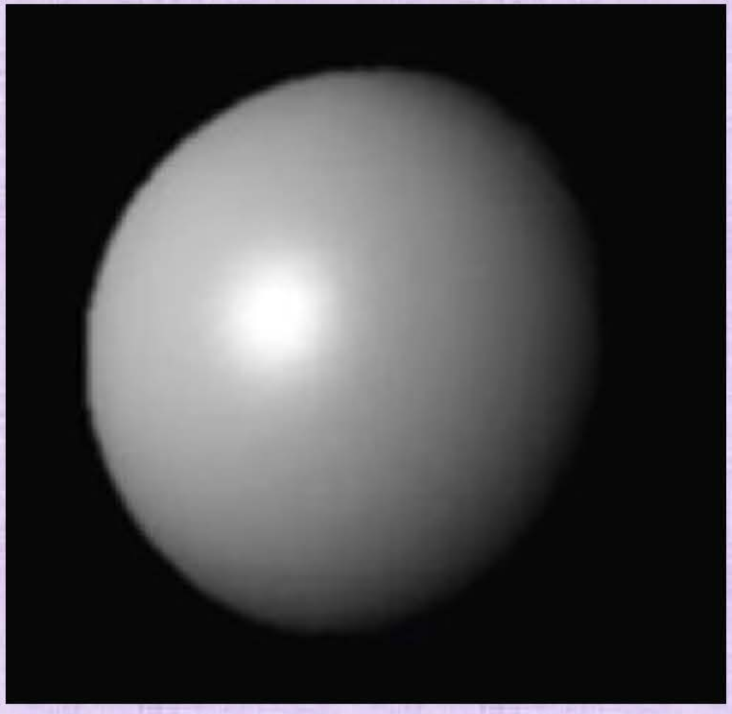
- Note: every light source in the scene is checked with a separate shadow ray
- Note: one often includes low intensity ambient shading for points in complete shadow, so that they are not completely black

# Spurious Self-Occlusion

- Note: $t = 0$ is not included in $t \in (0, t_{light})$ in order to avoid incorrectly computing an intersection with the same object near $R(t_{int})$
  - This can happen because of issues with numerical precision
- Note: shadow rays cannot simply ignore the object in question (when aiming to avoid spurious self-intersection) because that prevents objects from correctly self-shadowing



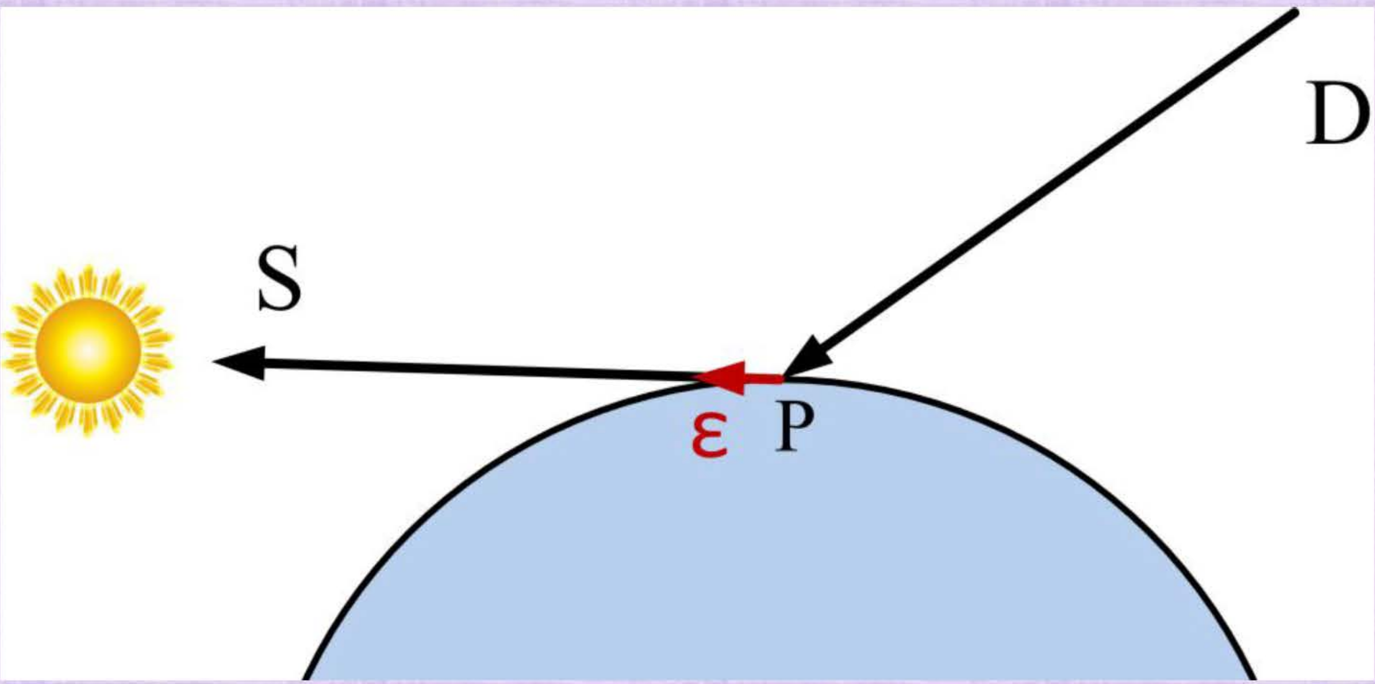
**incorrect self-shadowing**



**correct shading**

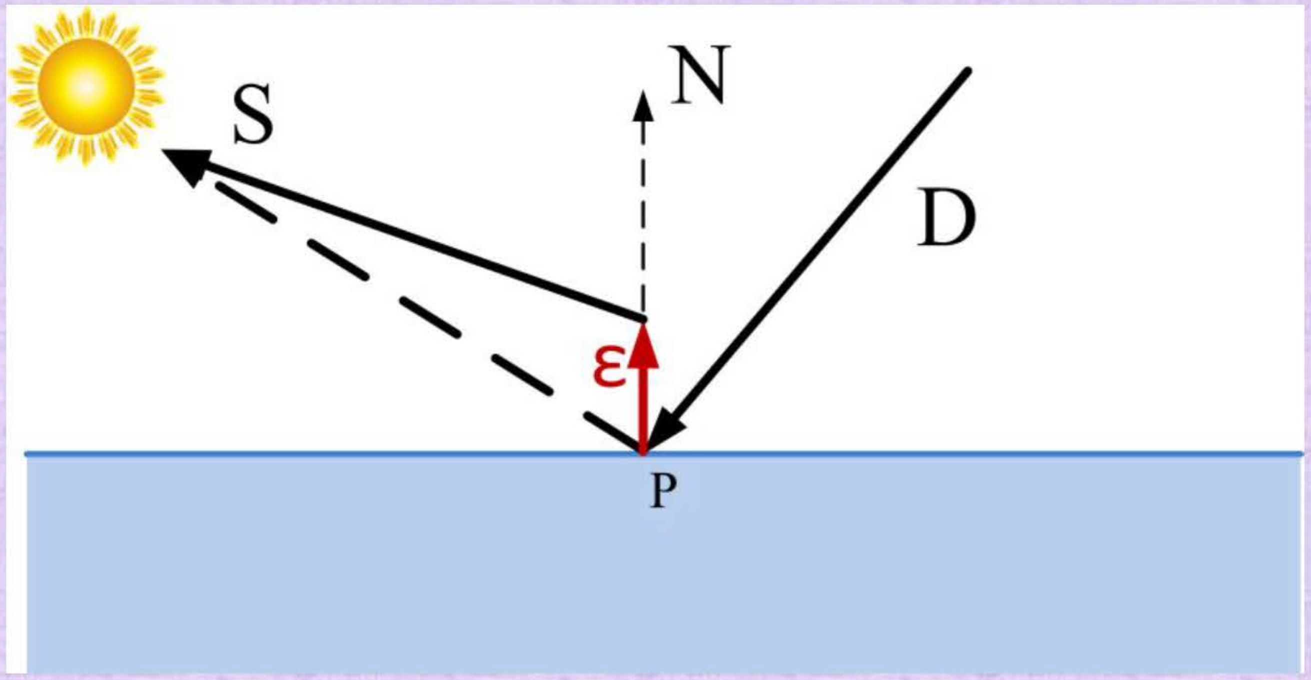
# Spurious Self-Occlusion

- A simple solution is to use $t \in (\epsilon, t_{light})$ for some $\epsilon > 0$ large enough to guarantee that the ray does not incorrectly re-intersect the same object
- This works well for many cases
- However, grazing shadow rays (near an object's silhouette) may still incorrectly re-intersect the object



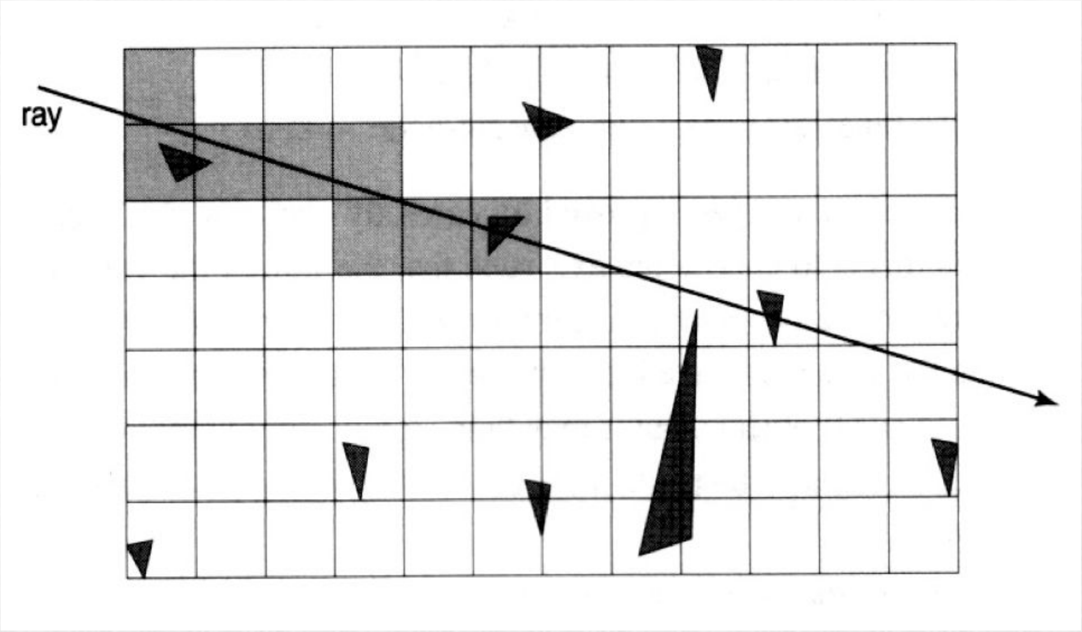
# Spurious Self-Occlusion

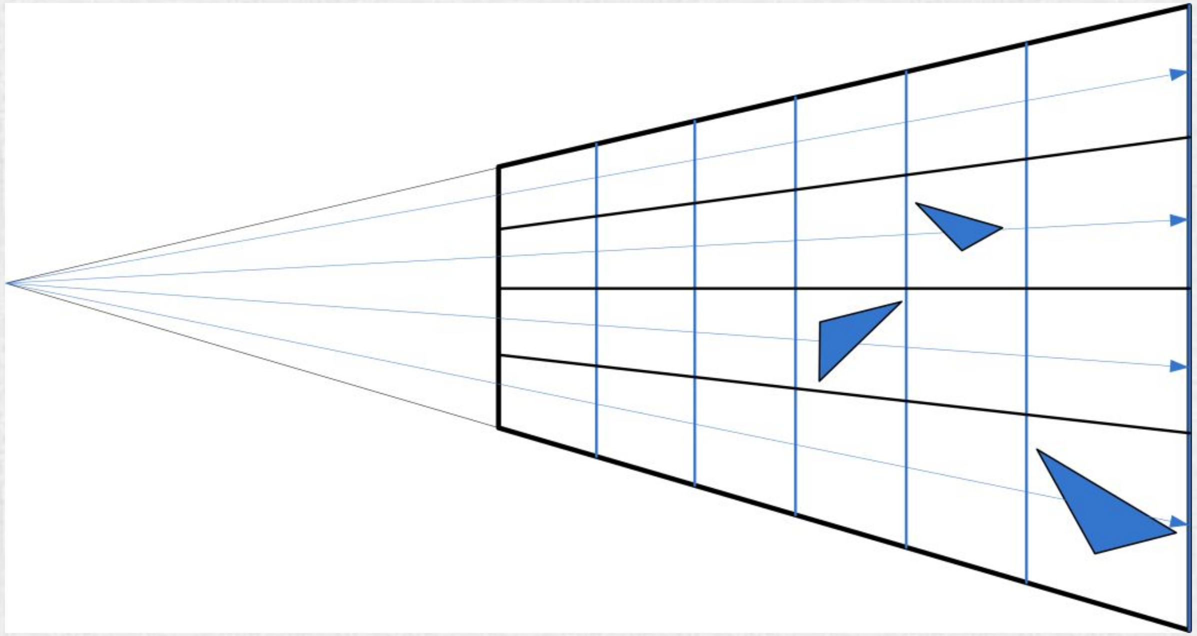
- Another option is to perturb the starting point of the shadow ray to be slightly away from the object (typically in the normal direction), e.g. from $R(t_{int})$ to $R(t_{int}) + \epsilon\hat{N}$
- The light direction also needs to be modified, to go from the light to $R(t_{int}) + \epsilon\hat{N}$
- The new shadow ray is $S(t) = \left(R(t_{int}) + \epsilon\hat{N}\right) - \hat{L}_{mod}t$ where $t \in [0, t_{light})$
- This works well, but one needs to take care that the new starting point $R(t_{int}) + \epsilon\hat{N}$ does not fall inside (or too close to) any nearby geometry



# Aside: Code Acceleration

- When there are many objects in the scene, checking rays against all of their top level simple bounding volumes can become expensive
- Thus, world space bounding volume hierarchies, octrees, and K-D trees are used
- Also useful (but flat instead of hierarchical) are <u>uniform spatial partitions</u> (uniform grids) and <u>viewing frustum partitions</u>



# Aside: Code Acceleration

•There are many variants: <u>rectilinear grids</u> with movable lines, hierarchies of uniform grids, and a structure proposed by [Losasso et al. 2006] that allows <u>octrees to be allocated inside the cells of a uniform spatial partition</u>