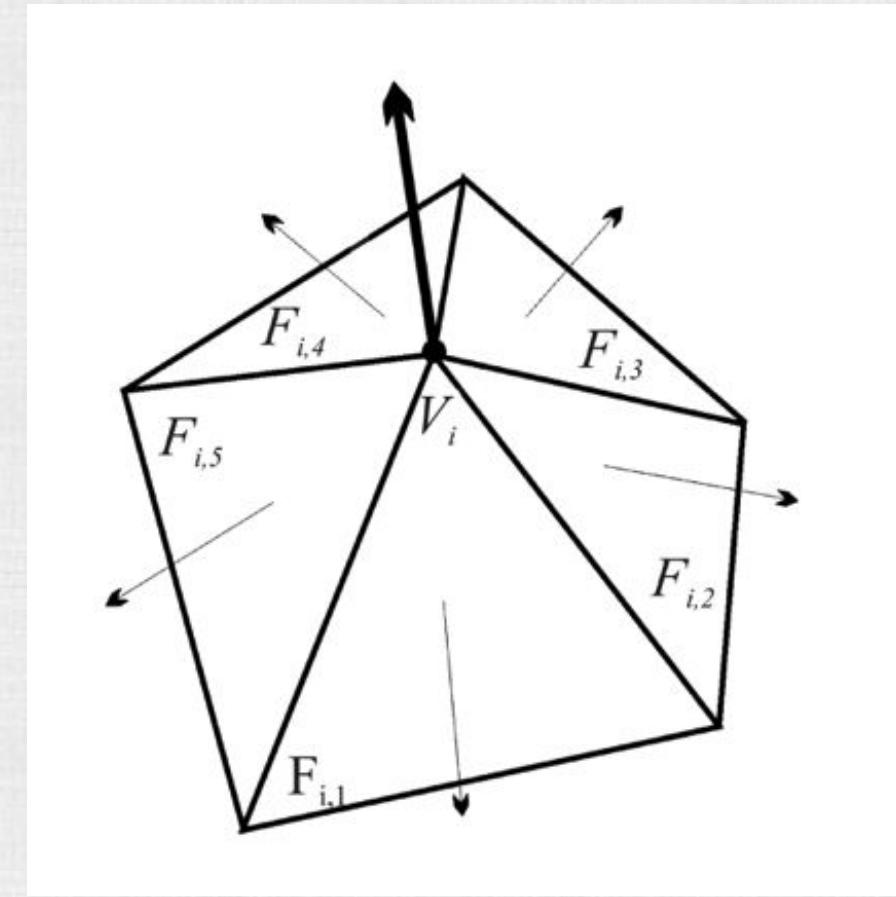
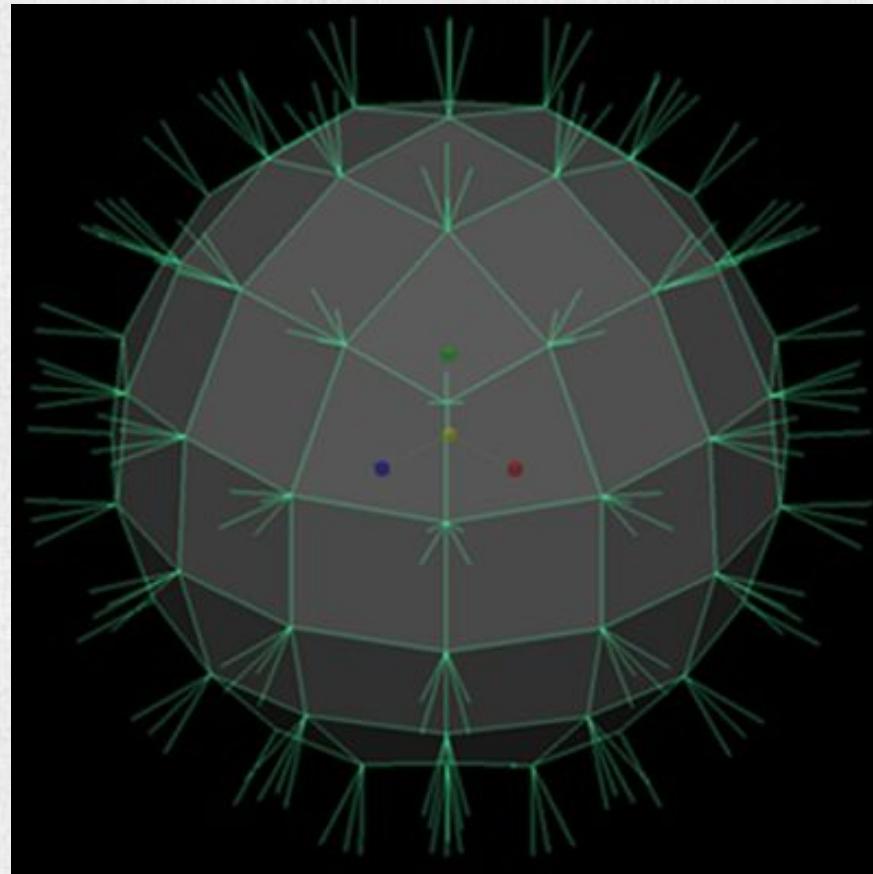


More Texture Mapping

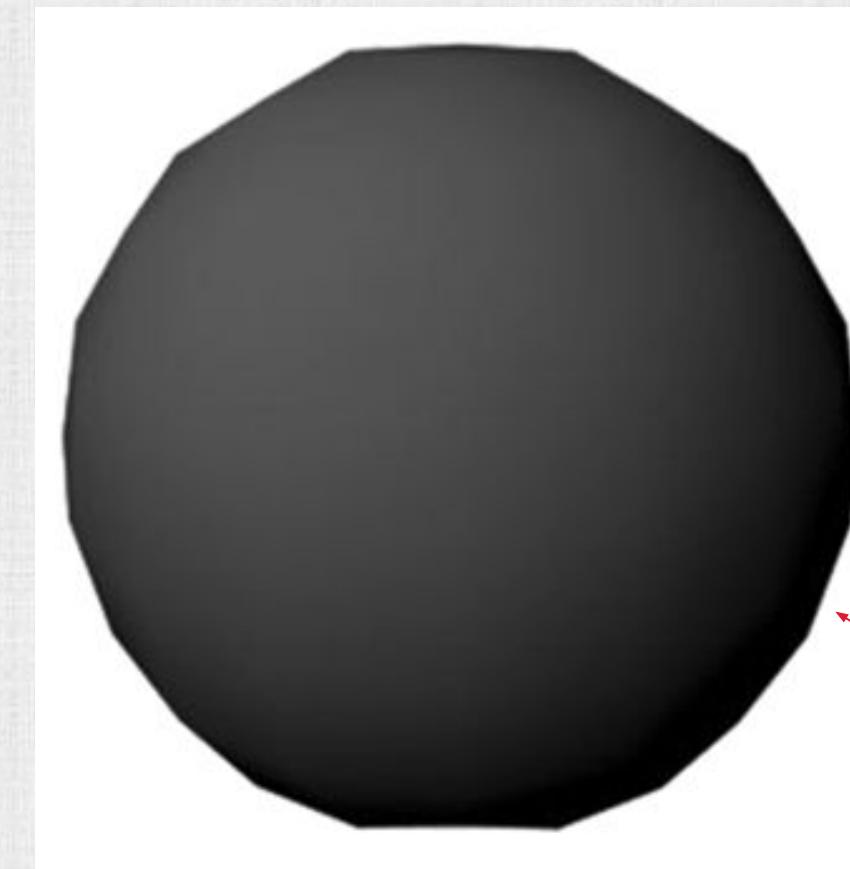
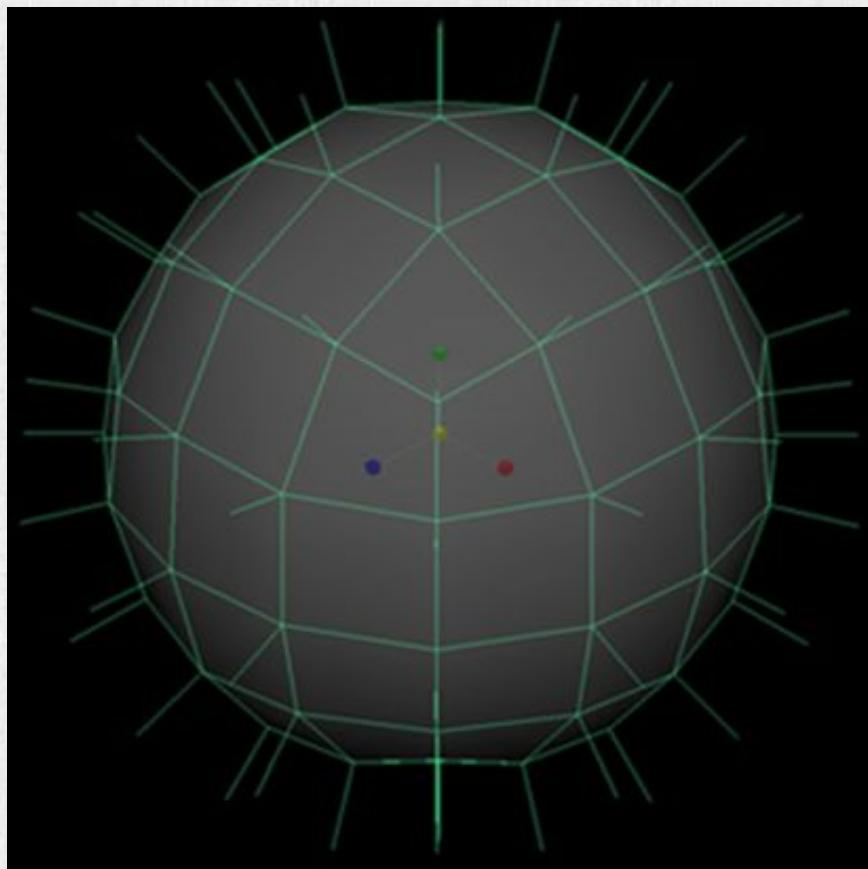


Recall: (Averaged) Vertex Normals

- Each vertex has a number of incident triangles, each with their own normal
- Averaging those face normals (or a weighted average based on: area, angle, etc.) gives a unique normal for each vertex



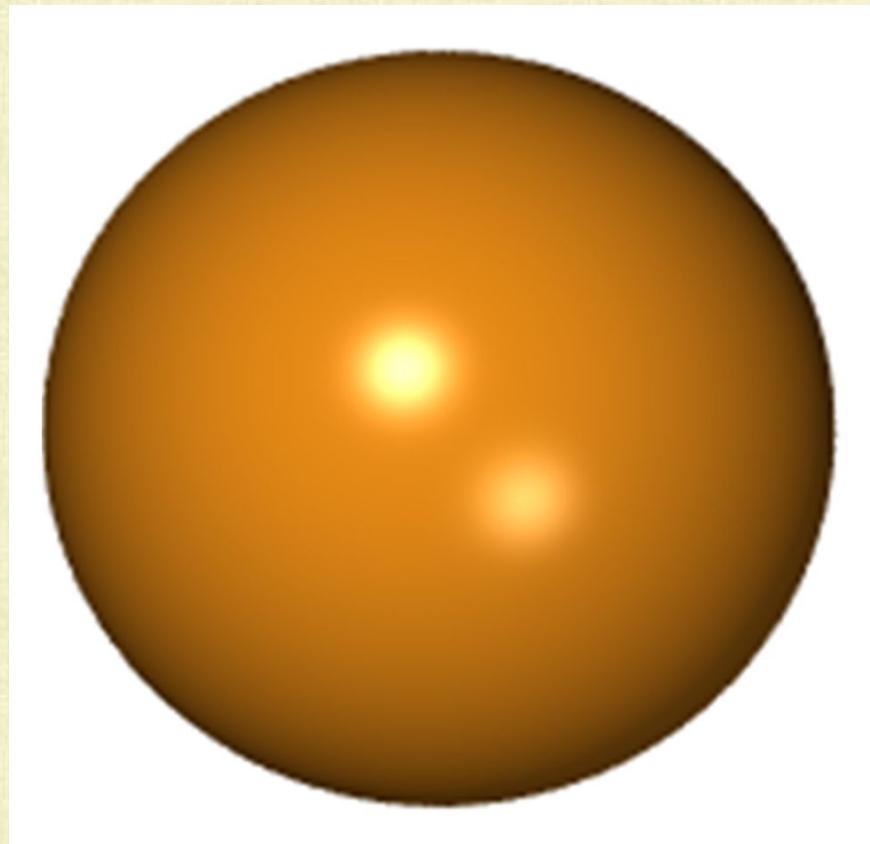
Recall: Smooth Shading



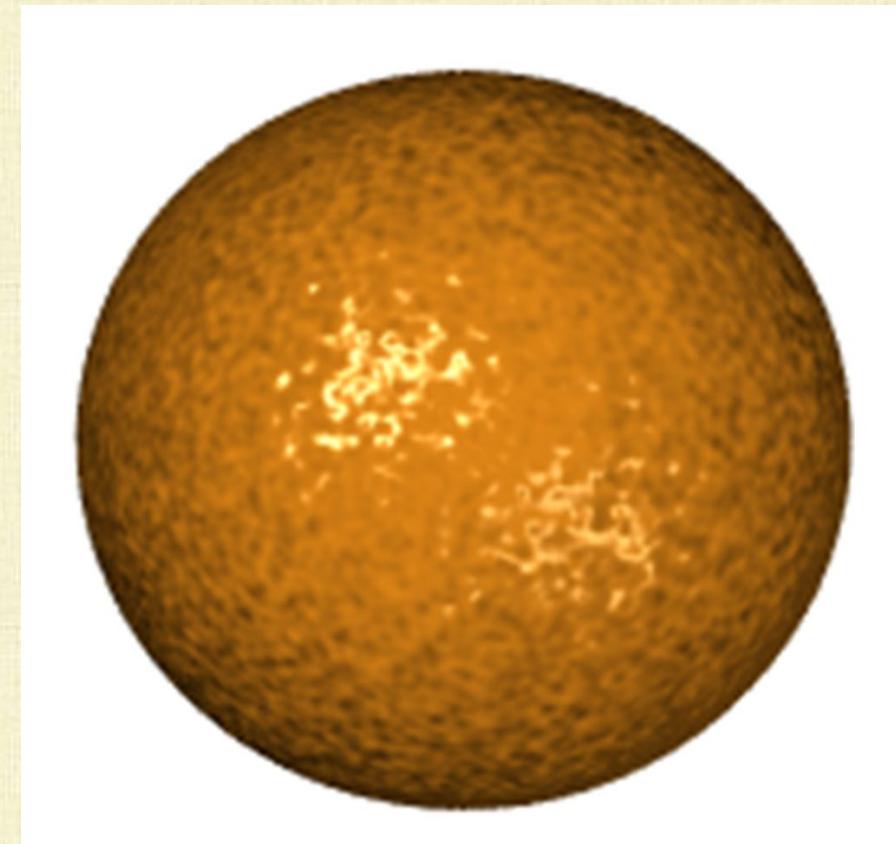
faceted
silhouette

Perturbing the Normal

- Store a normal vector in the texture (instead of a color)
- This perturbed normal can “fake” geometric details



using real normal



using fake normal

Bump Map

- Single-channel (grey-scale) height map h_{ij} , representing the height at location (u_i, v_j)

- The tangent plane at a point (u_i, v_j, h_{ij}) is given by:

$$-\frac{\partial h(u_i, v_j)}{\partial u}(u - u_i) - \frac{\partial h(u_i, v_j)}{\partial v}(v - v_j) + (h - h_{ij}) = 0$$

- So, the outward (non-unit) normal is $\left(-\frac{\partial h(u_i, v_j)}{\partial u}, -\frac{\partial h(u_i, v_j)}{\partial v}, 1\right)$

- Partial derivatives are computed via finite differences: $\frac{\partial h(u_i, v_j)}{\partial u} = \frac{h_{i+1,j} - h_{i-1,j}}{u_{i+1} - u_{i-1}}$ and $\frac{\partial h(u_i, v_j)}{\partial v} = \frac{h_{i,j+1} - h_{i,j-1}}{v_{j+1} - v_{j-1}}$



Normal Map

- A normalized vector has each component in $[-1,1]$, so one can convert back and forth to a color via:

$$(R, G, B) = 255 \frac{\vec{N} + (1,1,1)}{2} \quad \text{and} \quad \vec{N} = \frac{2}{255}(R, G, B) - (1,1,1)$$

- Normal maps use more storage than bump maps , but require less computation

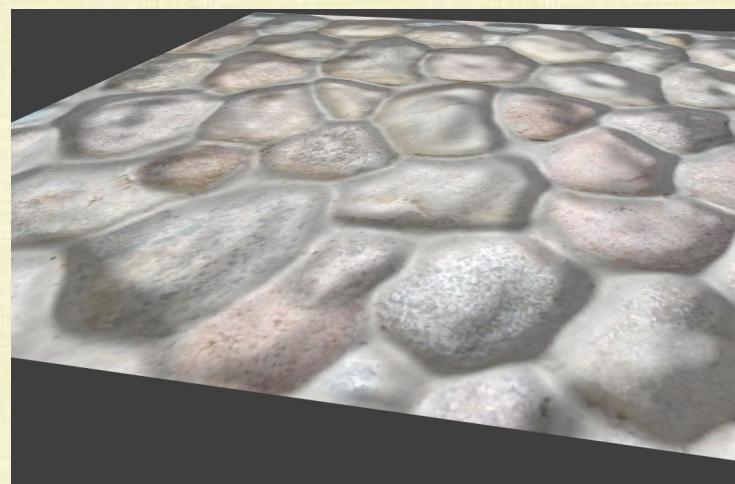
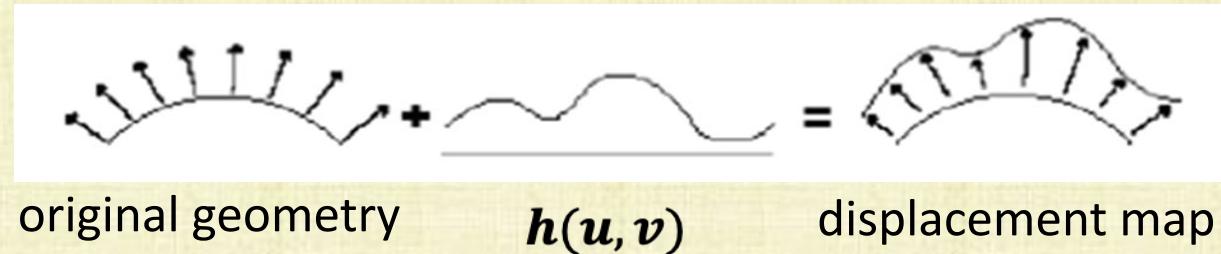


normal mapping on a plane

(note the variation in specular highlights created by variation of the

Displacement Mapping

- Subdivide geometry at render time, and use a height map $h(u, v)$ to (actually) perturb vertices in the normal direction
- Pros: self-occlusion, self-shadowing, correct silhouettes
- Cons: expensive, requires adaptive tessellation, still need bump/normal map for sub-triangle (fake) detail



bump map



displacement map

Displacement Mapping



bump map



displacement map

Recall: Measuring Incoming Light

- Light Probe: a small reflective chrome sphere
- Photograph it, in order to record the incoming light (at its location) from all directions



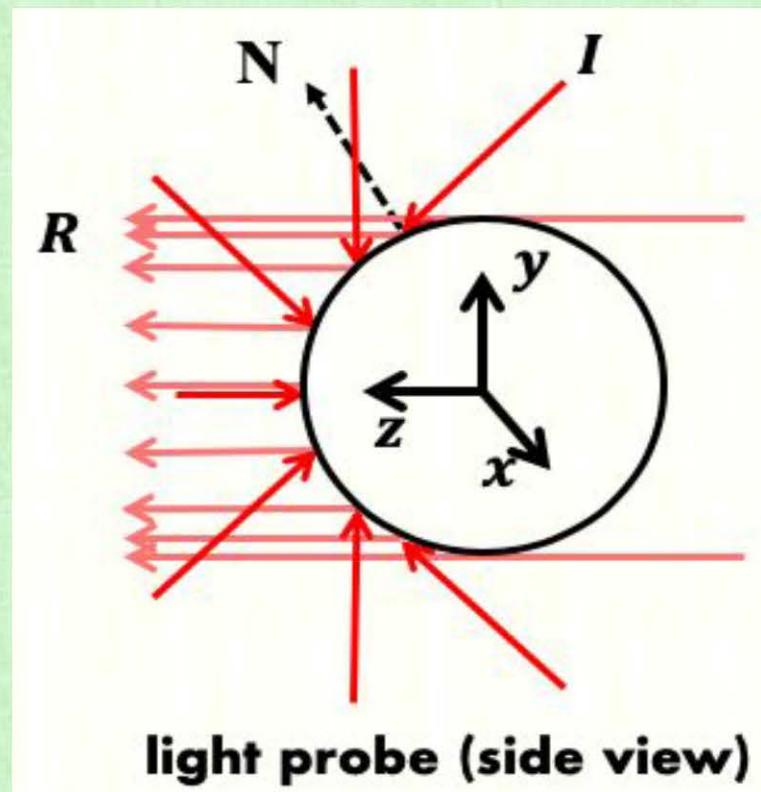
Recall: Using the (measured) Incoming Light

- The (measured) incoming light can be used to render a synthetic object (with realistic lighting)



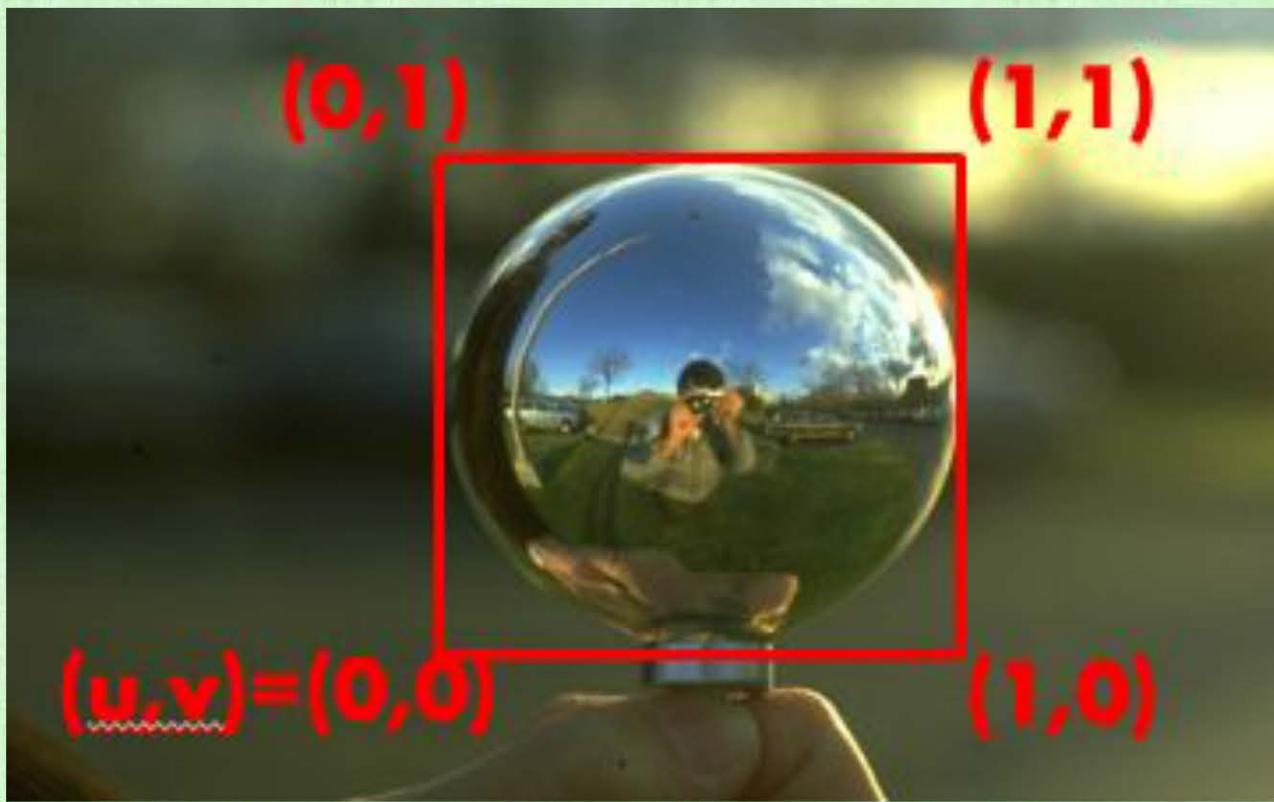
Environment Mapping

- Place a coordinate system at the center of the sphere, so the surface normal is: $\mathbf{N} = \frac{1}{\sqrt{x^2+y^2+z^2}}(x, y, z)$
- \mathbf{R} is the direction from the light probe to the camera
- Since \mathbf{I} and \mathbf{R} are equal-angle from \mathbf{N} (because of mirror reflection), \mathbf{N} has a one-to-one correspondence with \mathbf{I}
- Note: assuming that the intensity of incoming light depends only on incoming direction \mathbf{I} (not on position)



Environment Mapping

- Given a normal on geometry (to be rendered)
- Use n_x and n_y (which are in the range [-1, 1]) to obtain texture coordinates $(u, v) = \frac{1}{2}(n_x + 1, n_y + 1)$
- Then, look up the incoming light on a picture of the light probe

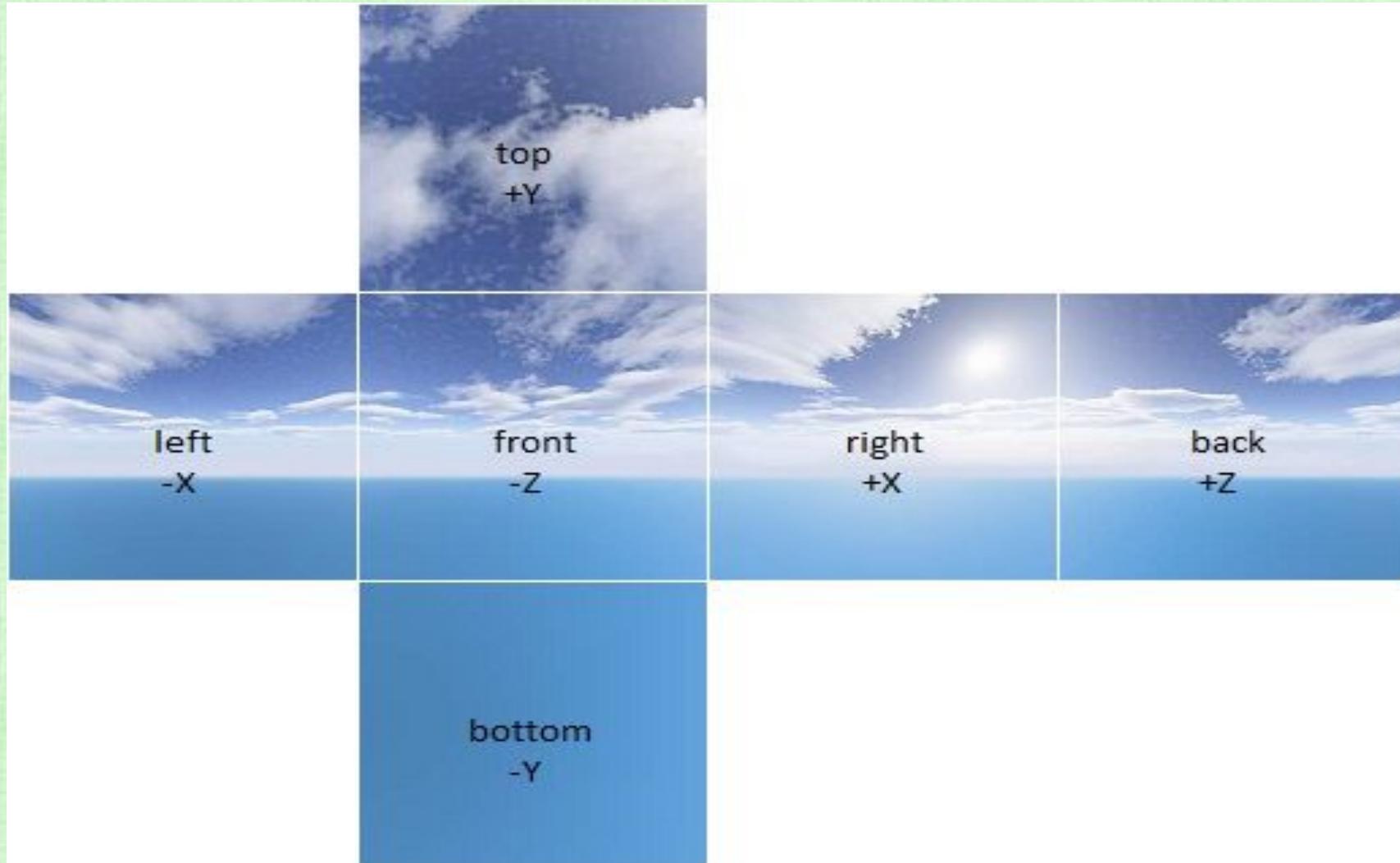


Environment Mapping

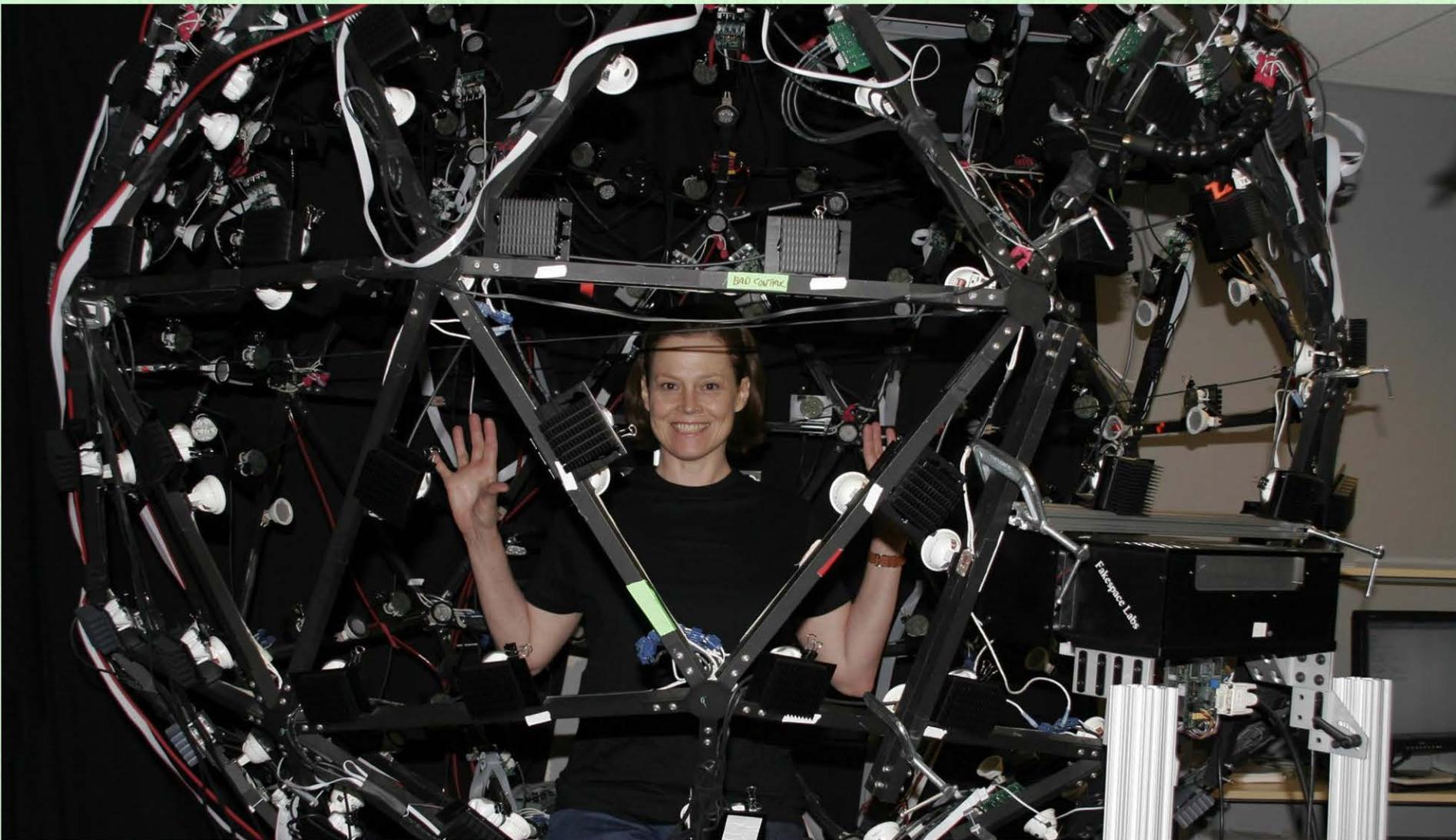


Sky Boxes

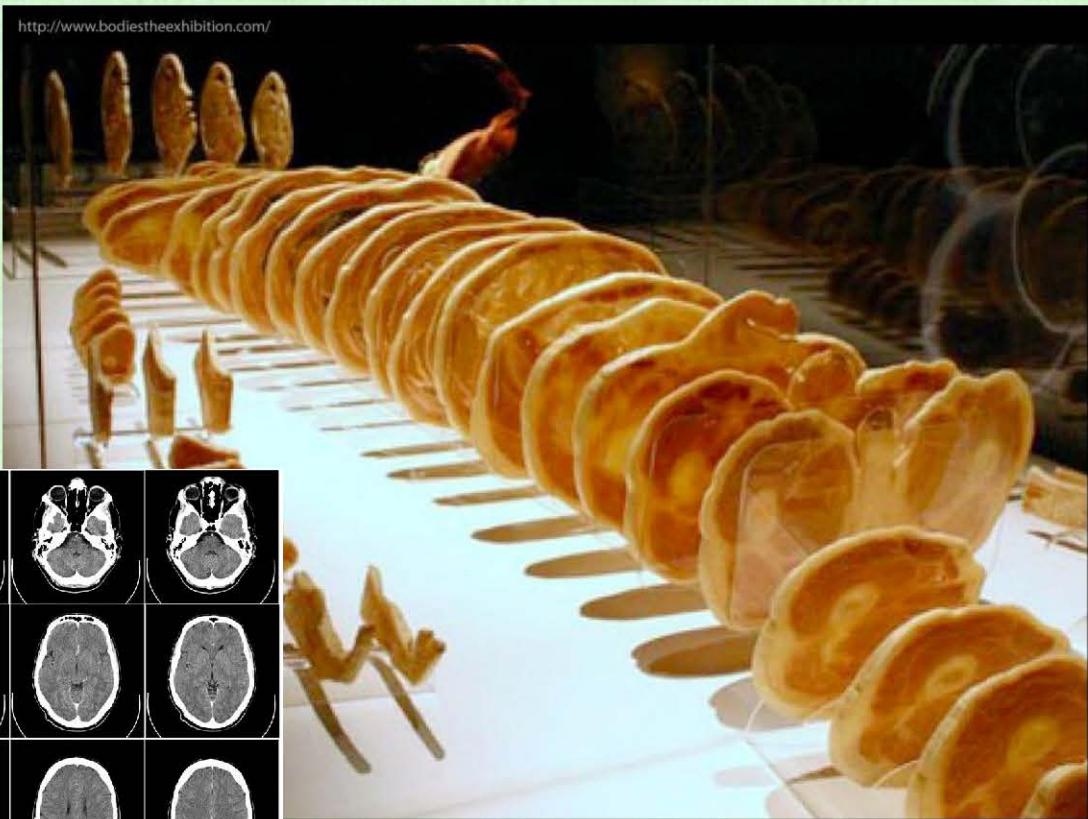
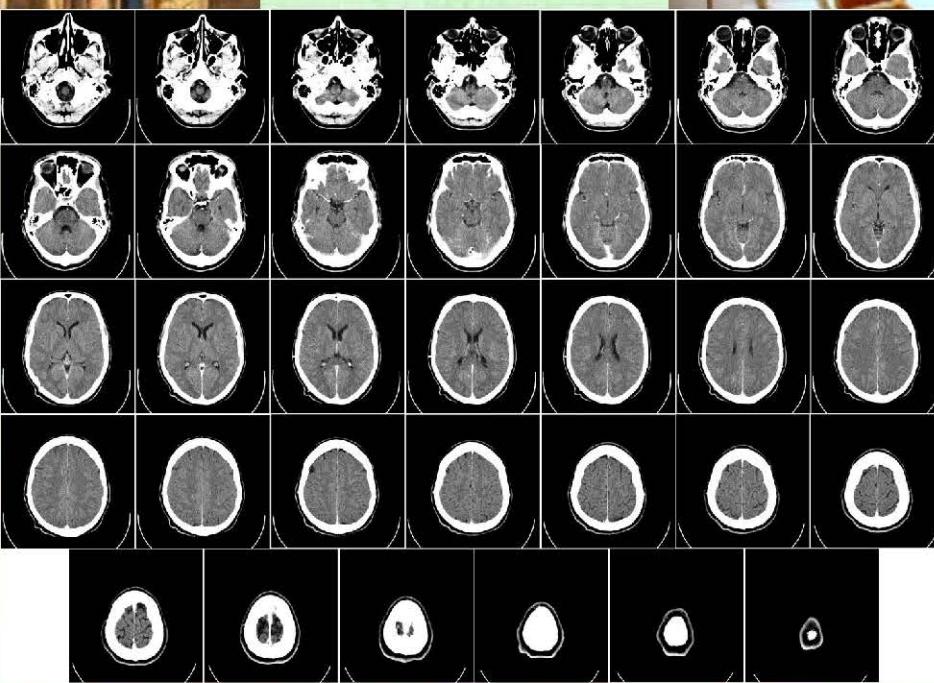
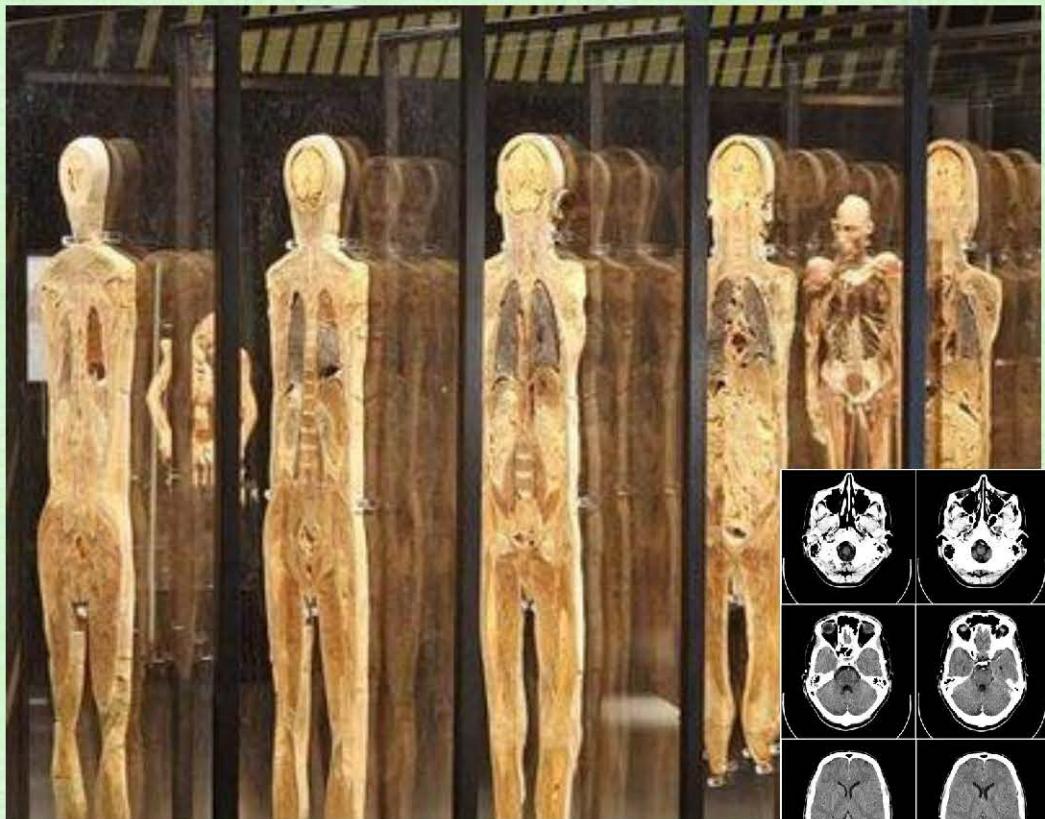
- Approximate the sky with a texture on the inside of geometry.



Texture Acquisition via Imaging

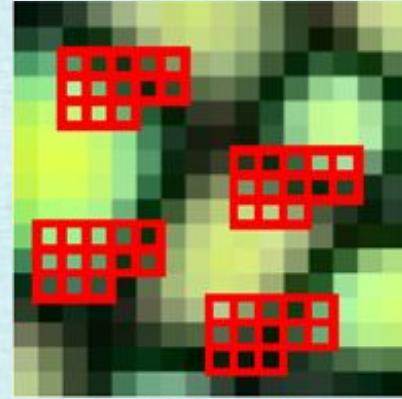
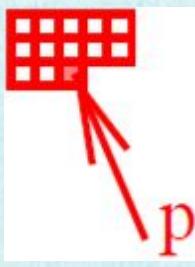


Texture Acquisition via Imaging



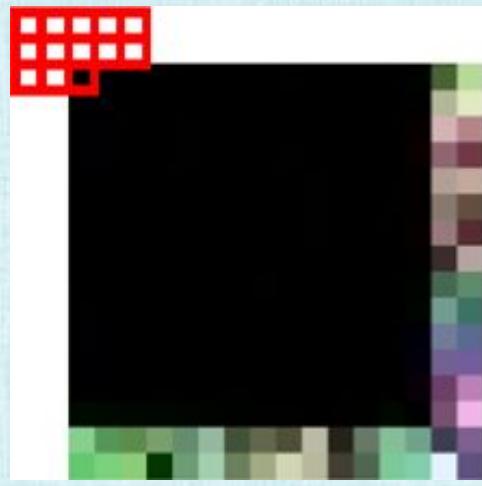
Texture Synthesis: Pixel Based

- Create a large non-repetitive texture (one pixel at a time) from a small sample (by using its structural content)
- Generate the texture in a raster scan ordering
- To generate the texture for pixel p
 - compare p 's neighboring pixels in the (red) stencil to all potential choices in the sample
 - choose the one with the smallest difference to fill pixel p
- When the **stencil** needs values outside the domain, use periodic boundary conditions (so, fill the last few rows/columns with random values)



stencil

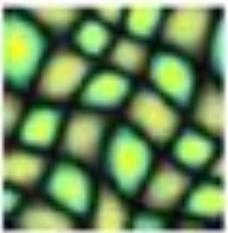
texture sample



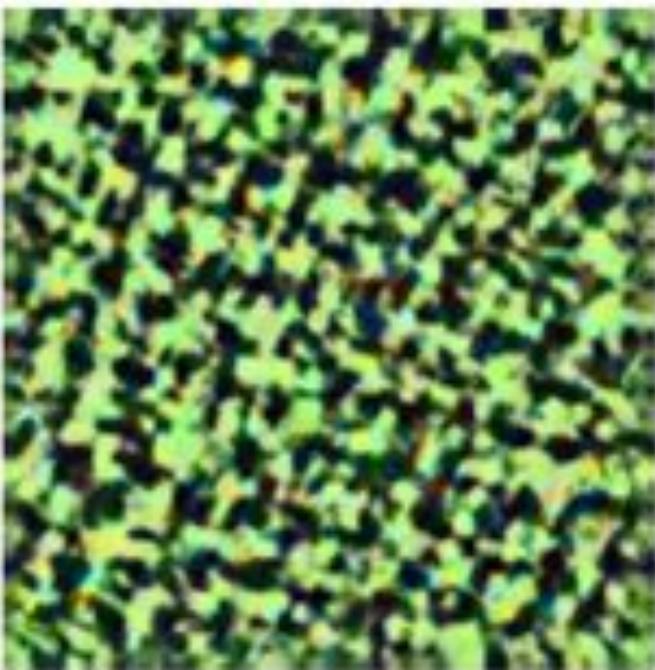
raster scan ordering (with randomly generated periodic boundaries)



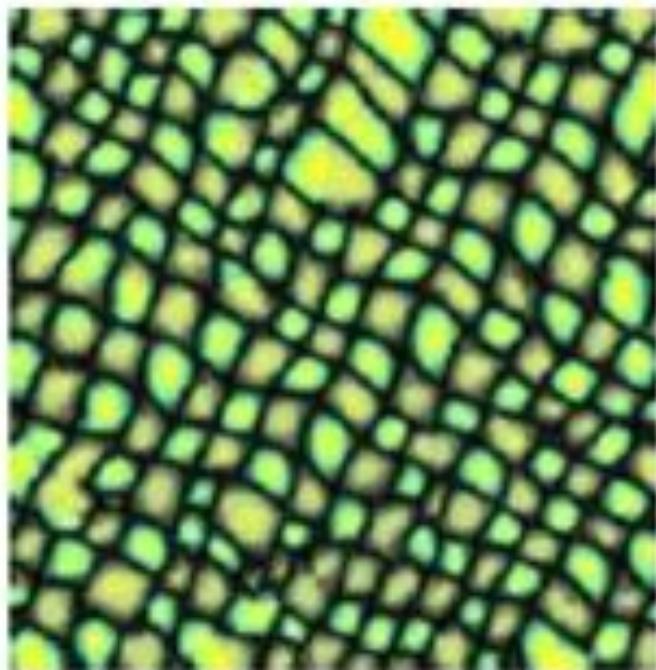
Texture Synthesis: Pixel Based



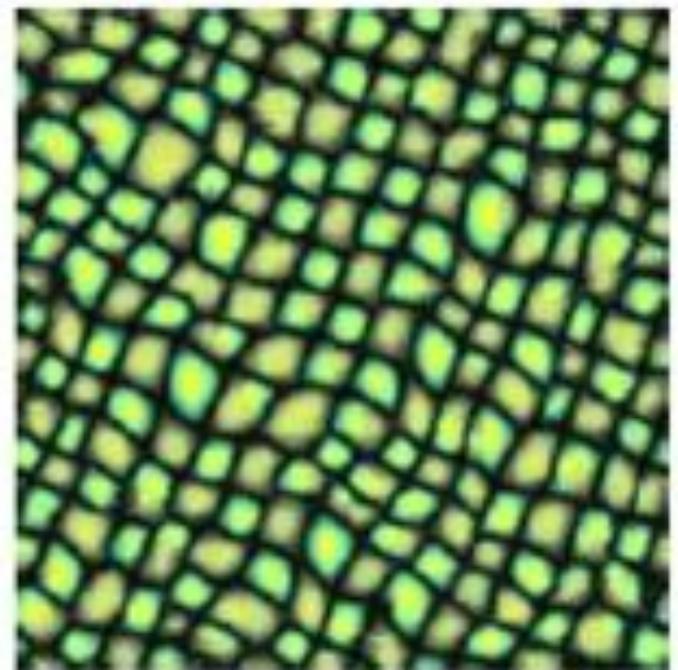
Sample



Heeger and Bergen



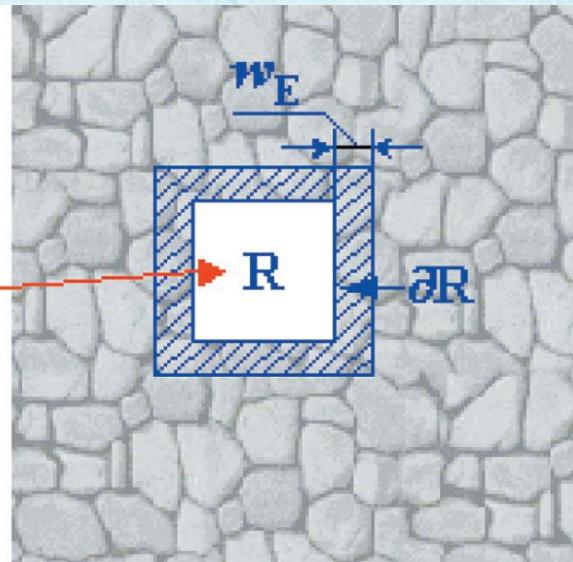
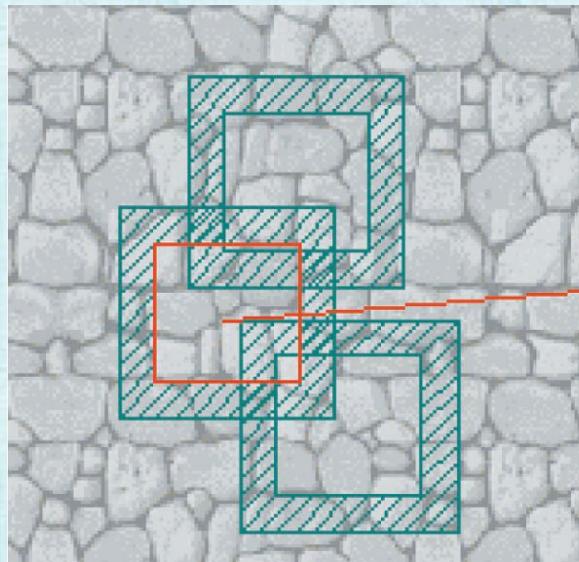
Efros and Leung



Wei and Levoy

Texture Synthesis: Patch Based

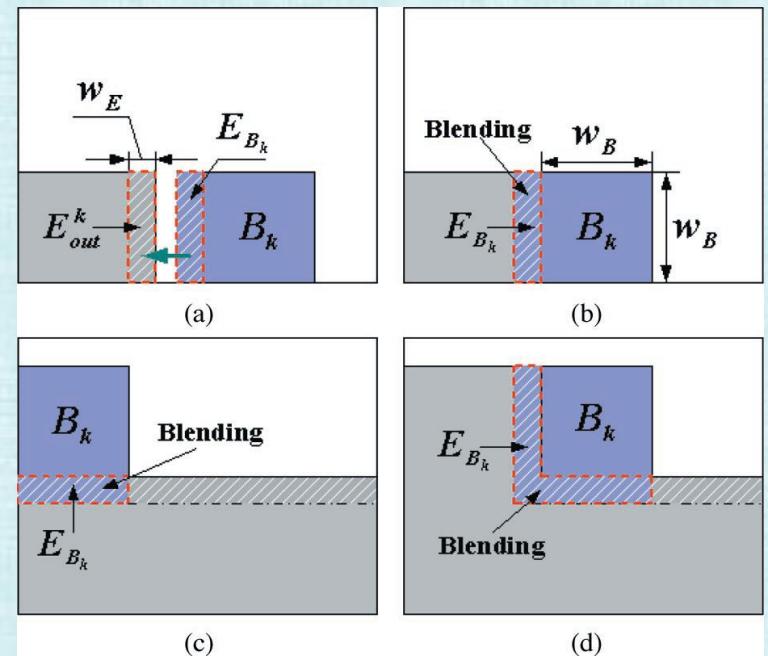
- For each patch being considered:
 - search the original sample to find candidates which best match the overlap regions (on the boundaries)
 - choose the best candidates
 - blends overlapped regions to remove “seams”



sample

texture

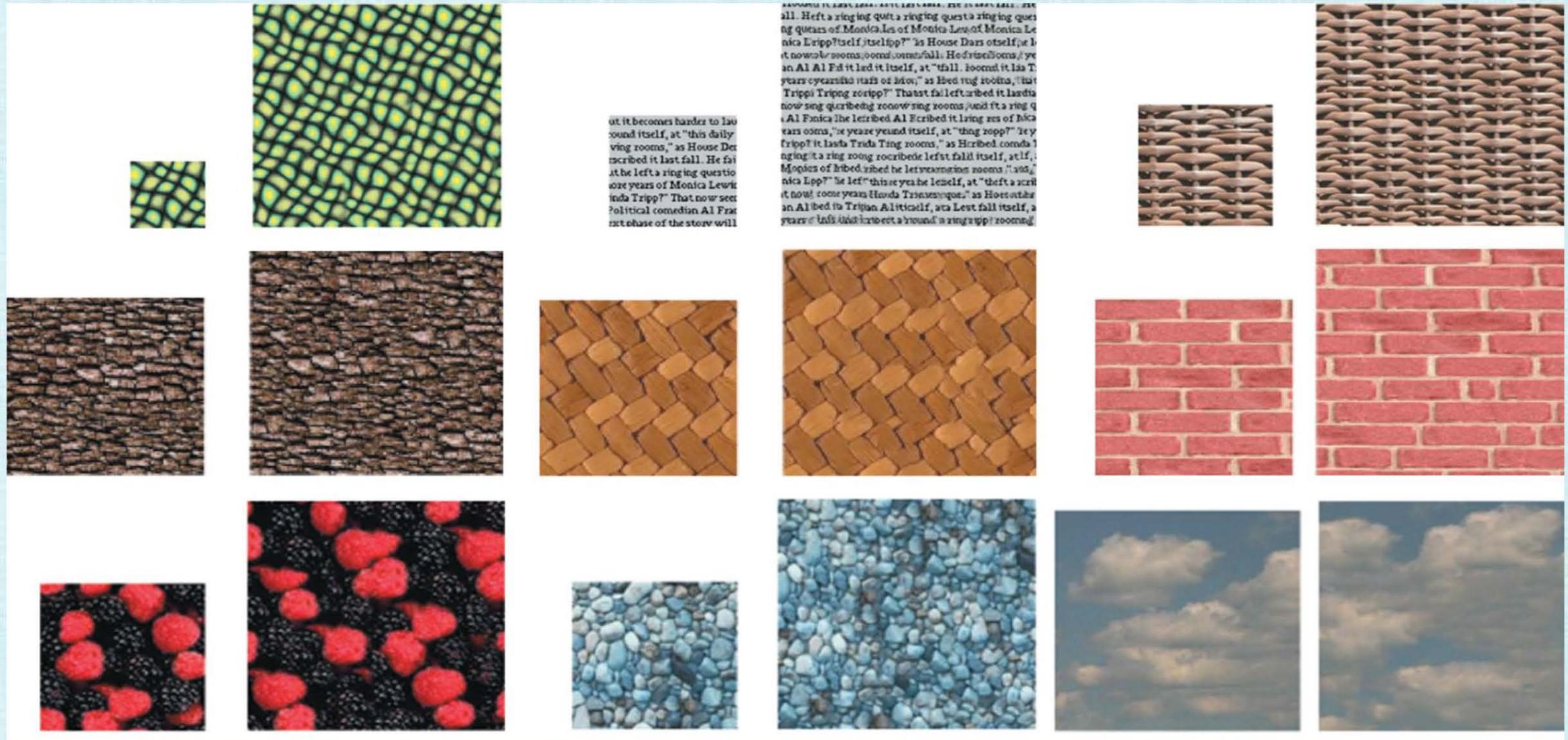
matching
boundary
regions



(c)

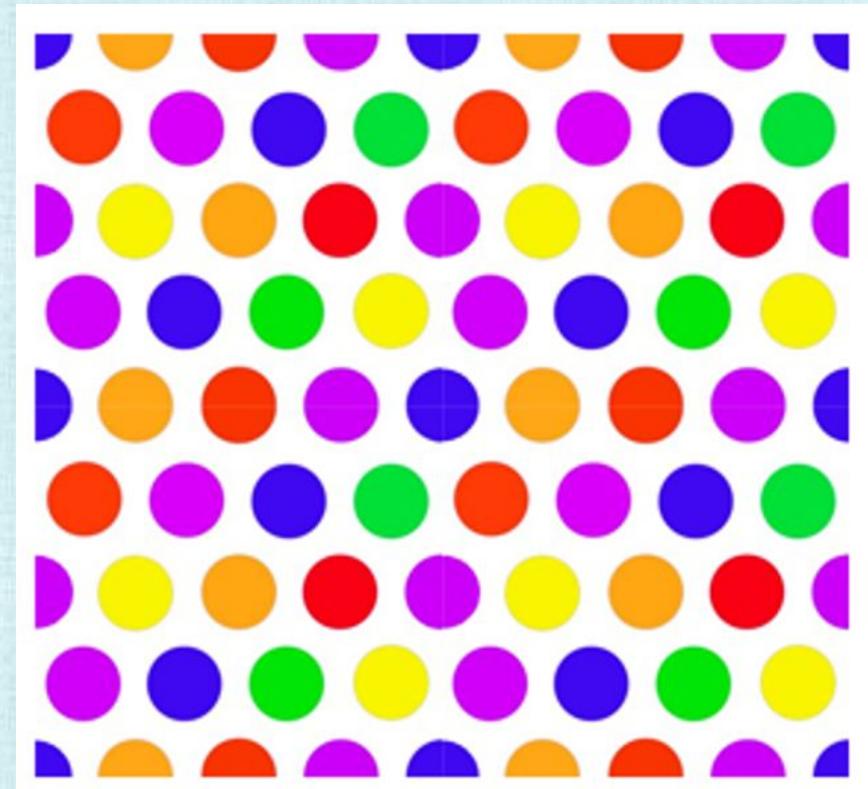
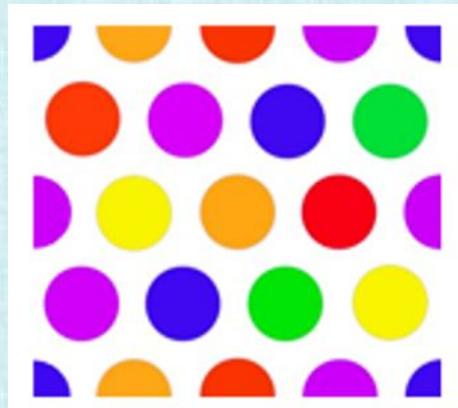
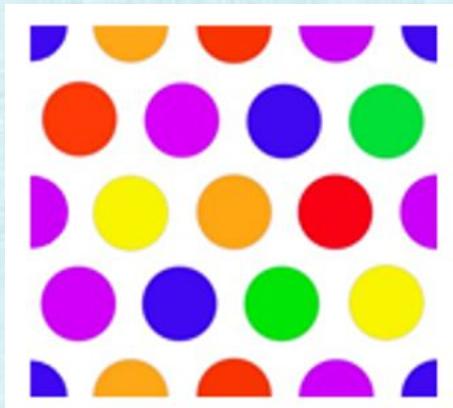
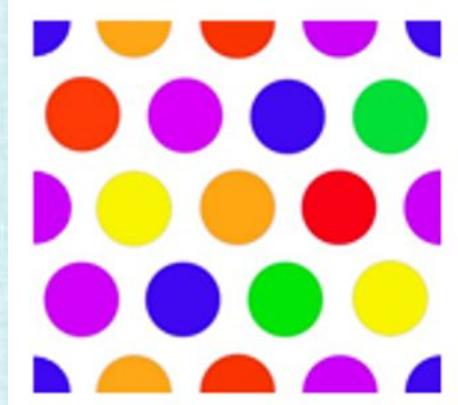
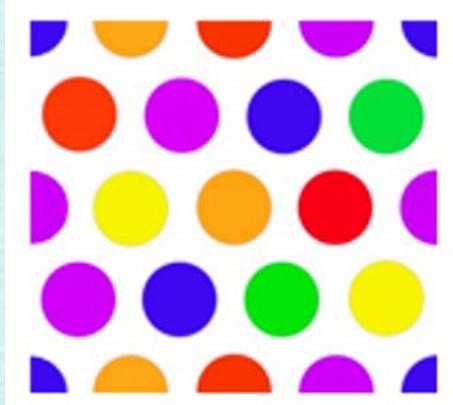
(d)

Texture Synthesis: Patch Based



Don't Stretch Textures!

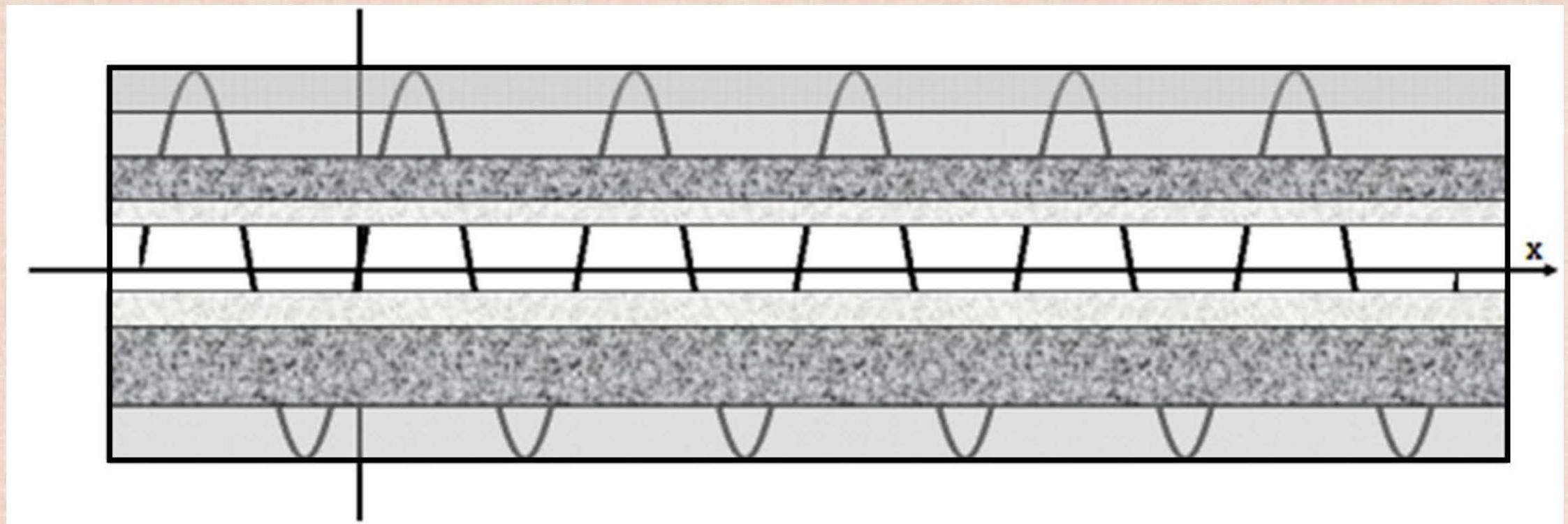
- Stretching out 10 bricks to cover an entire wall of a building is going to look unrealistic!
- Can instead **tile textures** if the small tiles are made with periodic boundaries



Marble Texture

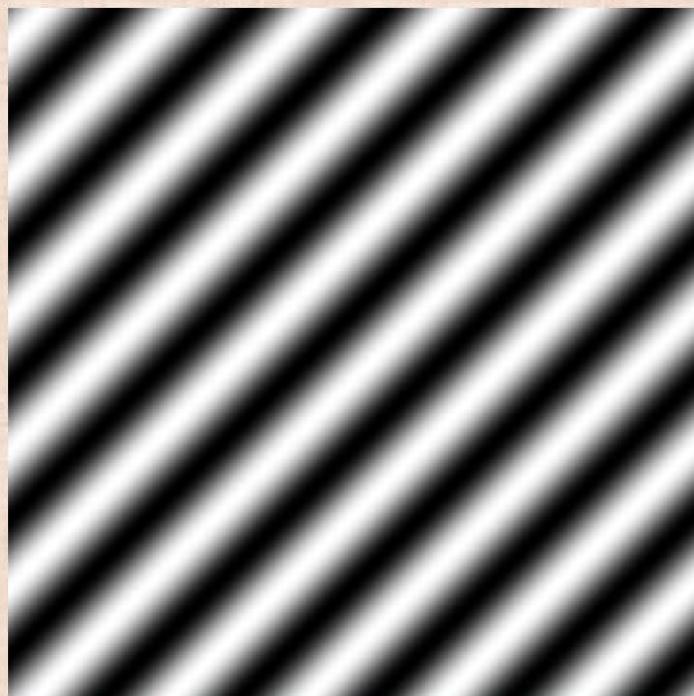
- Predefine layers of different colors
- Use a function to map (u, v) texture locations to layers
- For example:

$$\text{marbleColor}(u, v) = \text{LayerColor}(\sin(k_u u + k_v v))$$

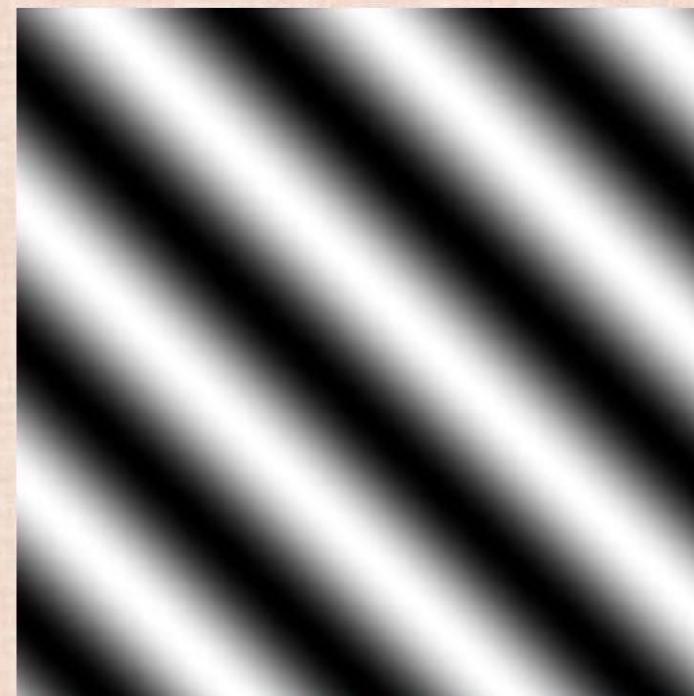


Marble Texture

- k_u and k_v are spatial frequencies
- (k_u, k_v) determines the direction, and $\frac{2\pi}{\sqrt{k_u^2+k_v^2}}$ determines the periodicity
- Too regular (need to add noise/randomness)



higher frequency



lower frequency

Perlin Noise

- Noise should have both coherency and structure in order to look more natural
- Ken Perlin proposed a specific (and amazing!) method for doing this

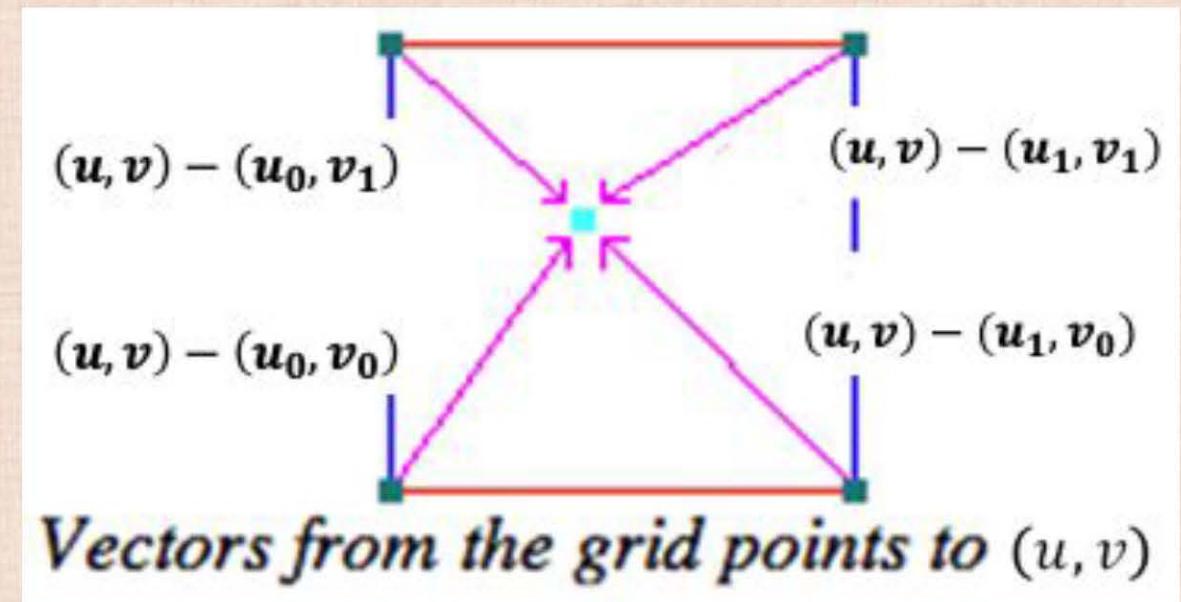
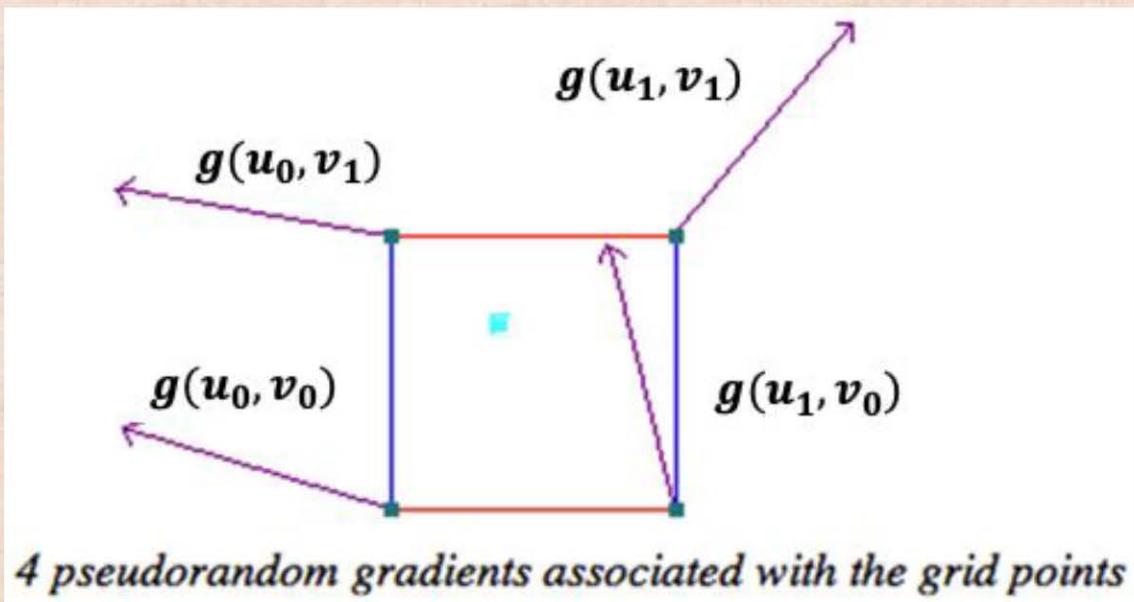


Perlin Noise

- Place a 2D grid over the image, and assign a random (unit) gradient $g(u_i, v_j)$ to each grid point
- For each pixel, compute the dot-products between vectors from the grid corners and the corresponding gradient
- Take a weighted average of the result:

$$\text{noise}(u, v) = \sum_{i=0,1; j=0,1} w\left(\frac{u - u_i}{\Delta u}\right) w\left(\frac{v - v_j}{\Delta v}\right) g(u_i, v_j) \cdot (u - u_i, v - v_j)$$

- Cubic weighting function: $w(t) = 2|t|^3 - 3|t|^2 + 1$ for $-1 < t < 1$



Multiple Scales

- Many natural textures contain a variety of feature sizes
- Mimic this by adding together noises with different frequencies and amplitudes:

$$perlin(u, v) = \sum_k noise(frequency(k) * (u, v)) * amplitude(k)$$

- Each successive noise function is twice the frequency of the previous one:

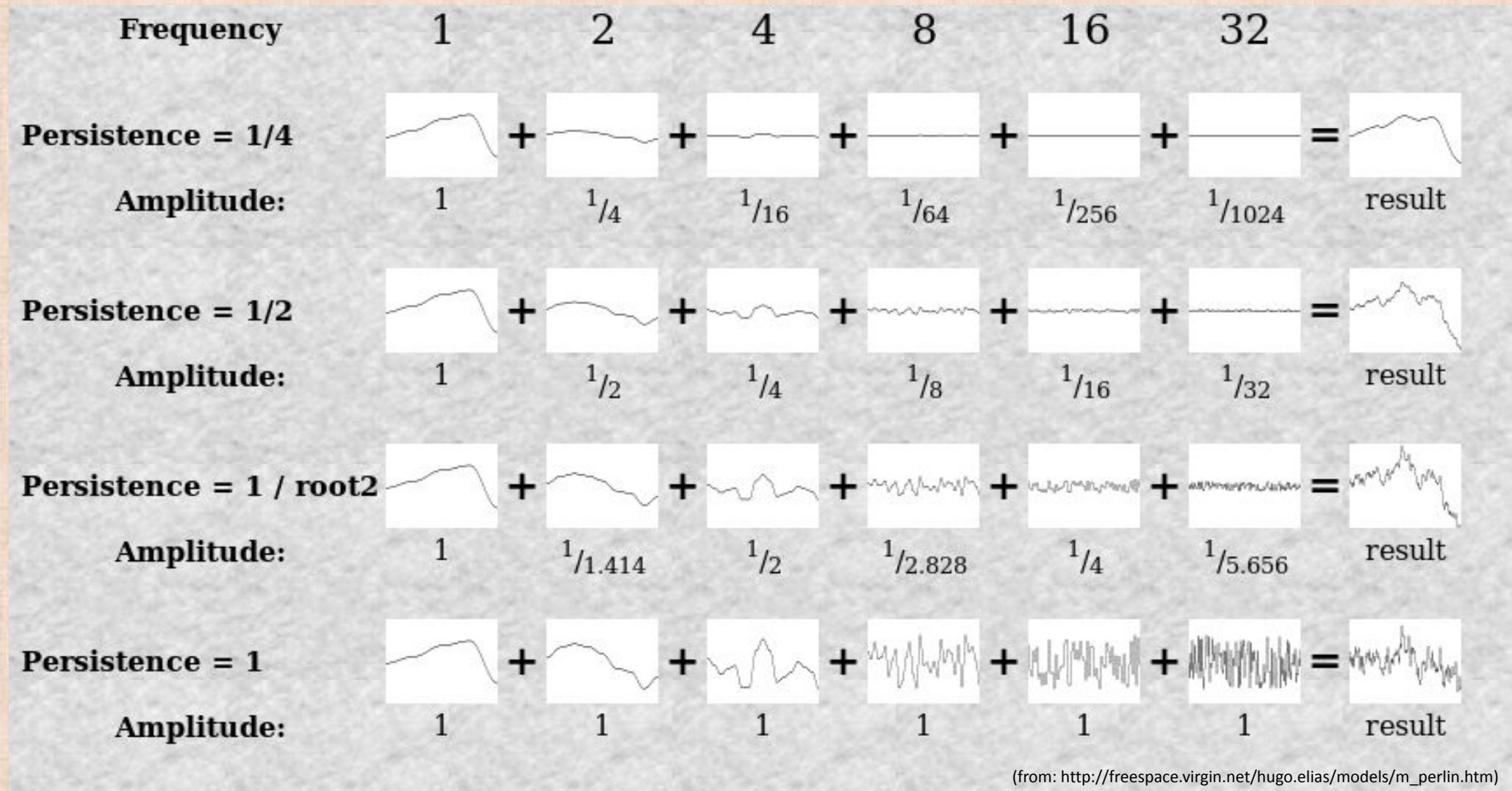
$$frequency(k) = 2^k$$

- The amplitude of higher frequencies is measured by a persistence parameter (≤ 1)
- Higher frequencies have a diminished contribution:

$$amplitude(k) = persistence^k$$

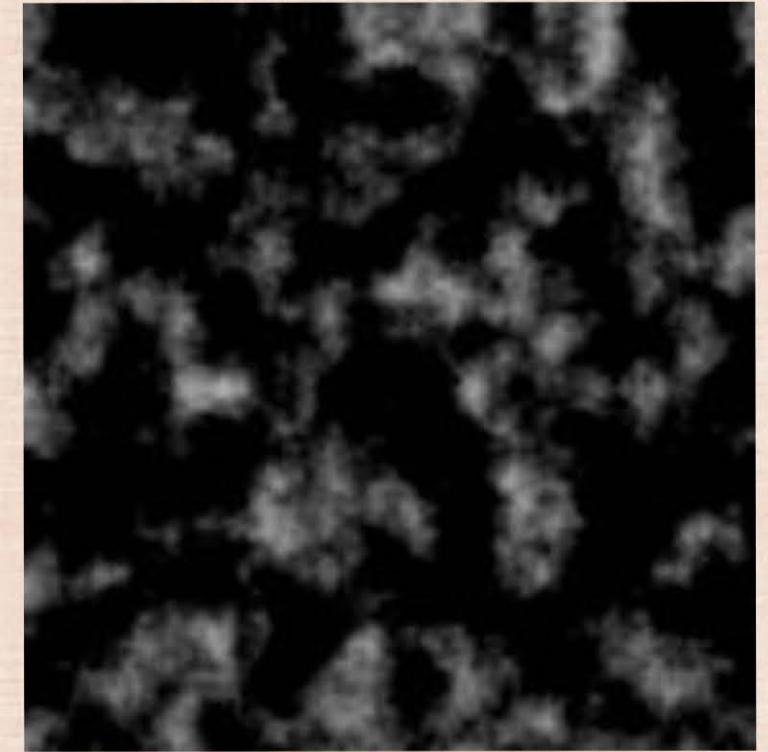
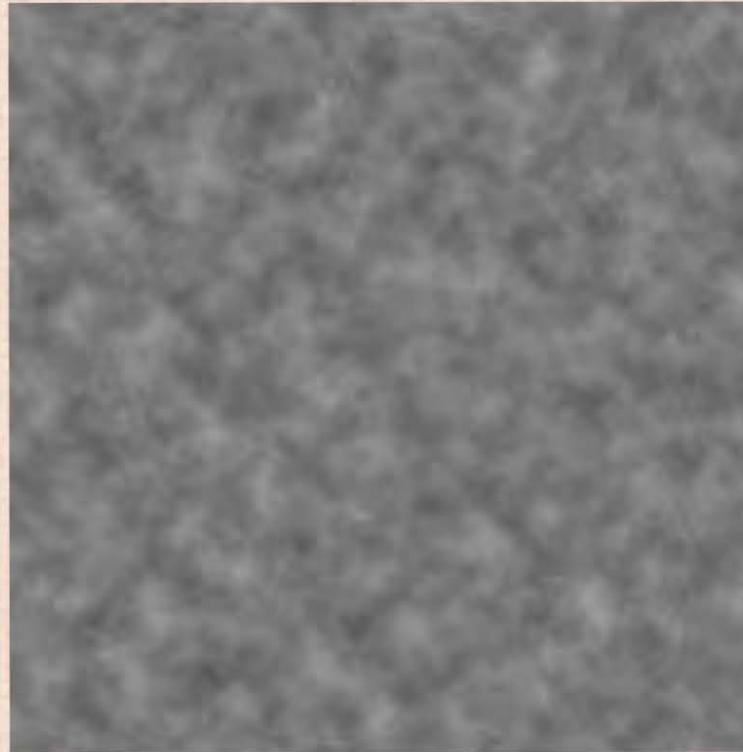
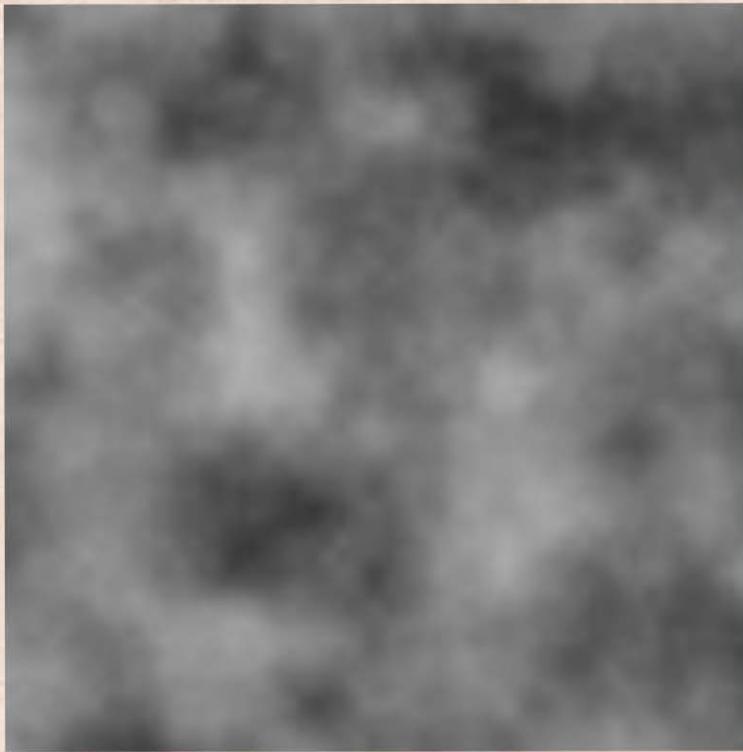
1D Examples

- Smaller persistence -> less higher frequency noise -> smoother result



(from: http://freespace.virgin.net/hugo.elias/models/m_perlin.htm)

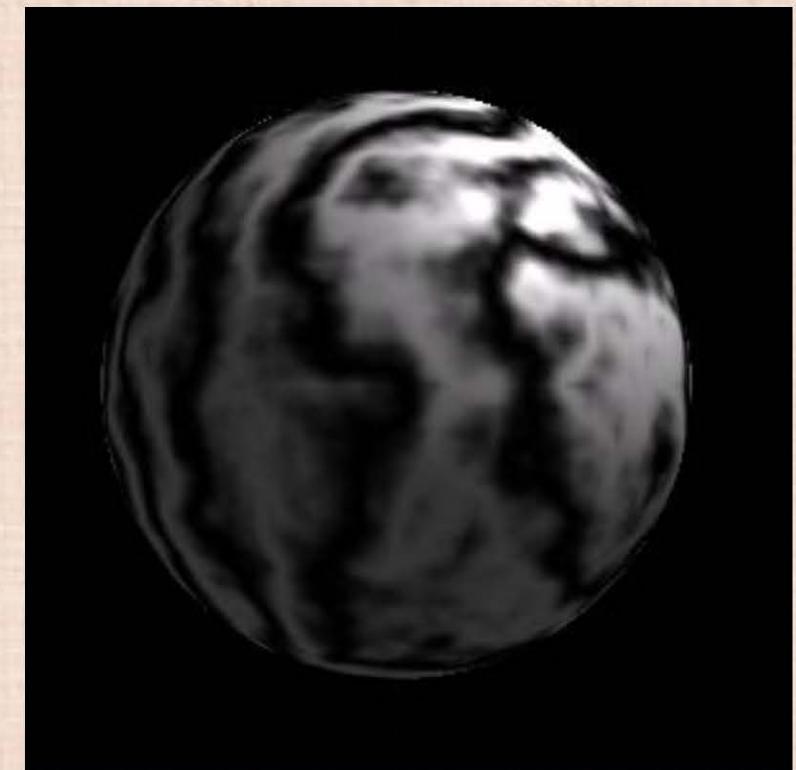
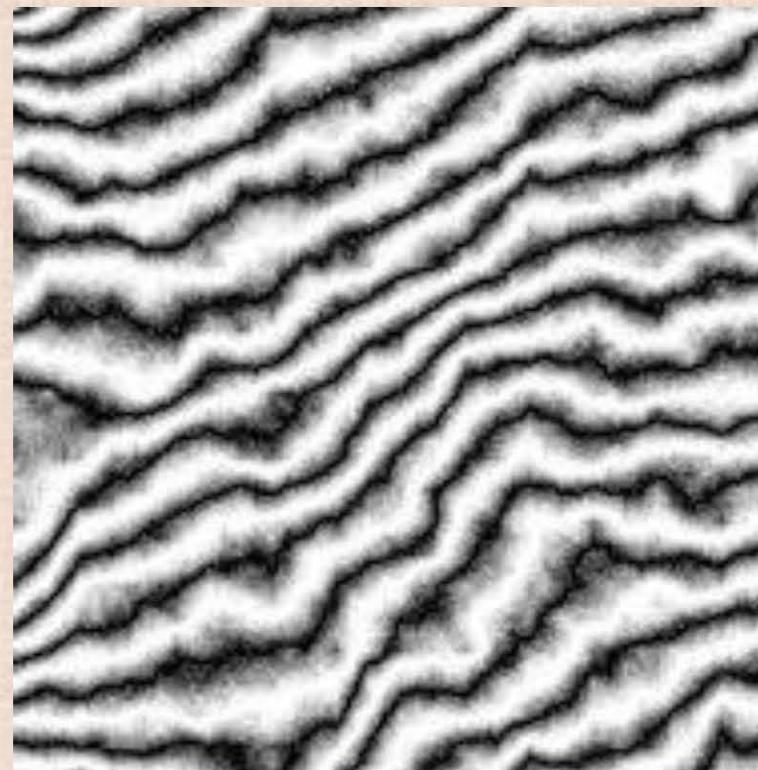
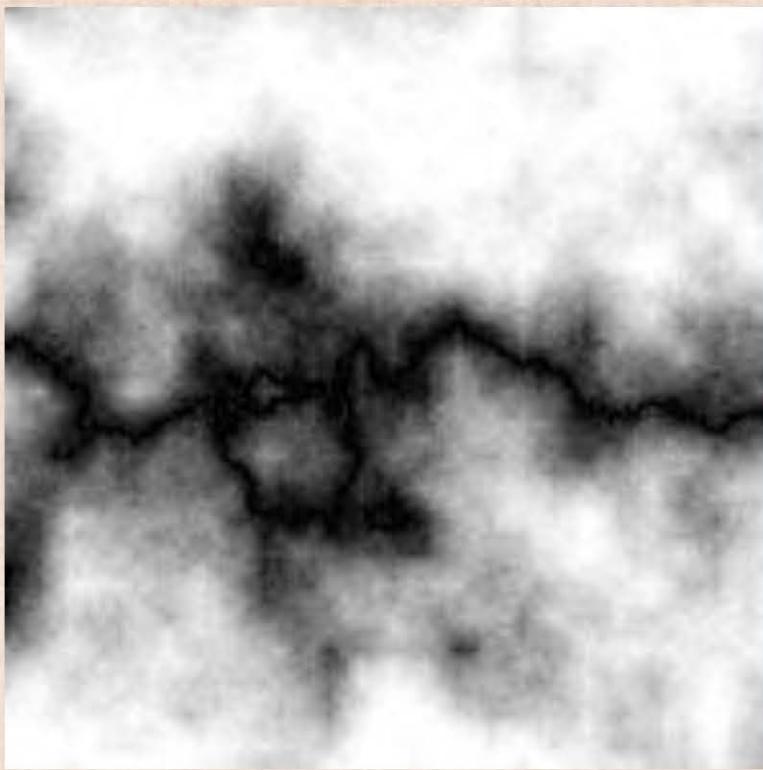
2D Examples



Marble Texture + Perlin Noise

- Set the value of A to scale the amount of noise:

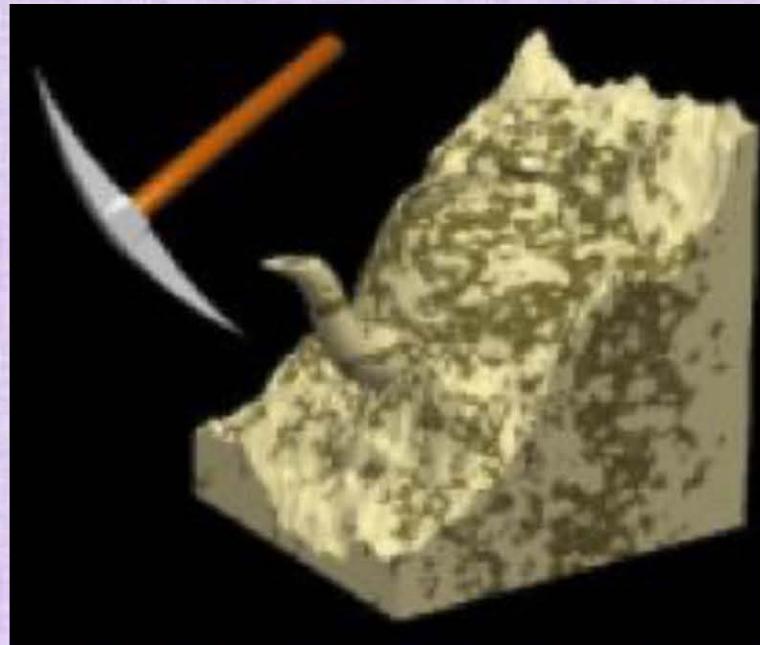
$$\text{marbleColor}(u, v) = \text{LayerColor} \left(\sin(k_u u + k_v v + A * \text{perlin}(u, v)) \right)$$



3D Marble Texture

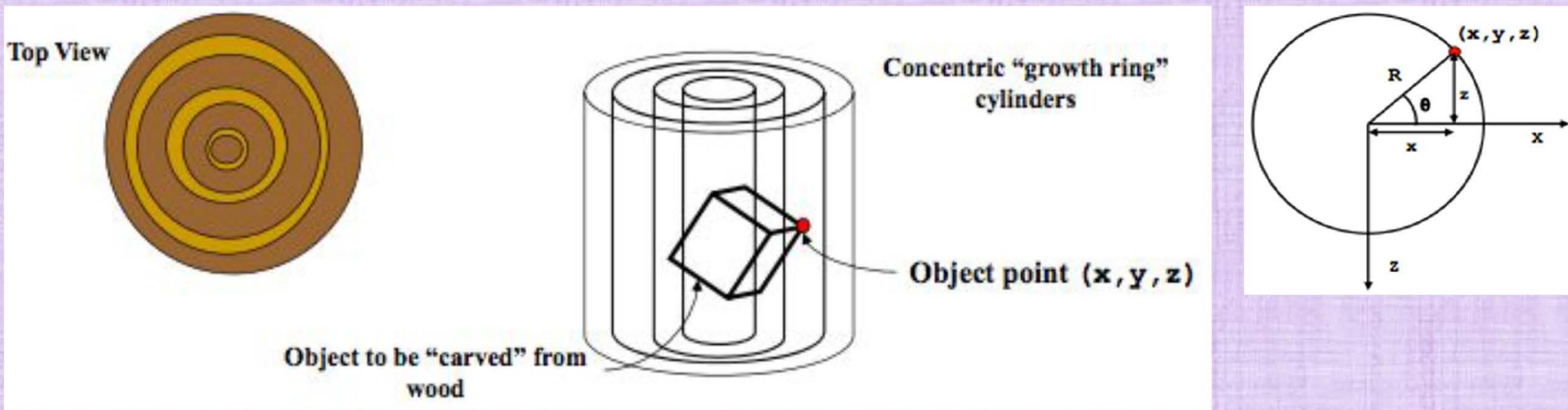
- Carve an object out of a 3D texture (eliminating the need to gift-wrap a complex 3D object)
- Marble texture function w/Perlin noise (for 3D):

$$\text{marbleColor}(u, v, w) = \text{LayerColor} \left(\sin(k_u u + k_v v + k_w w + A * \text{perlin}(u, v, w)) \right)$$

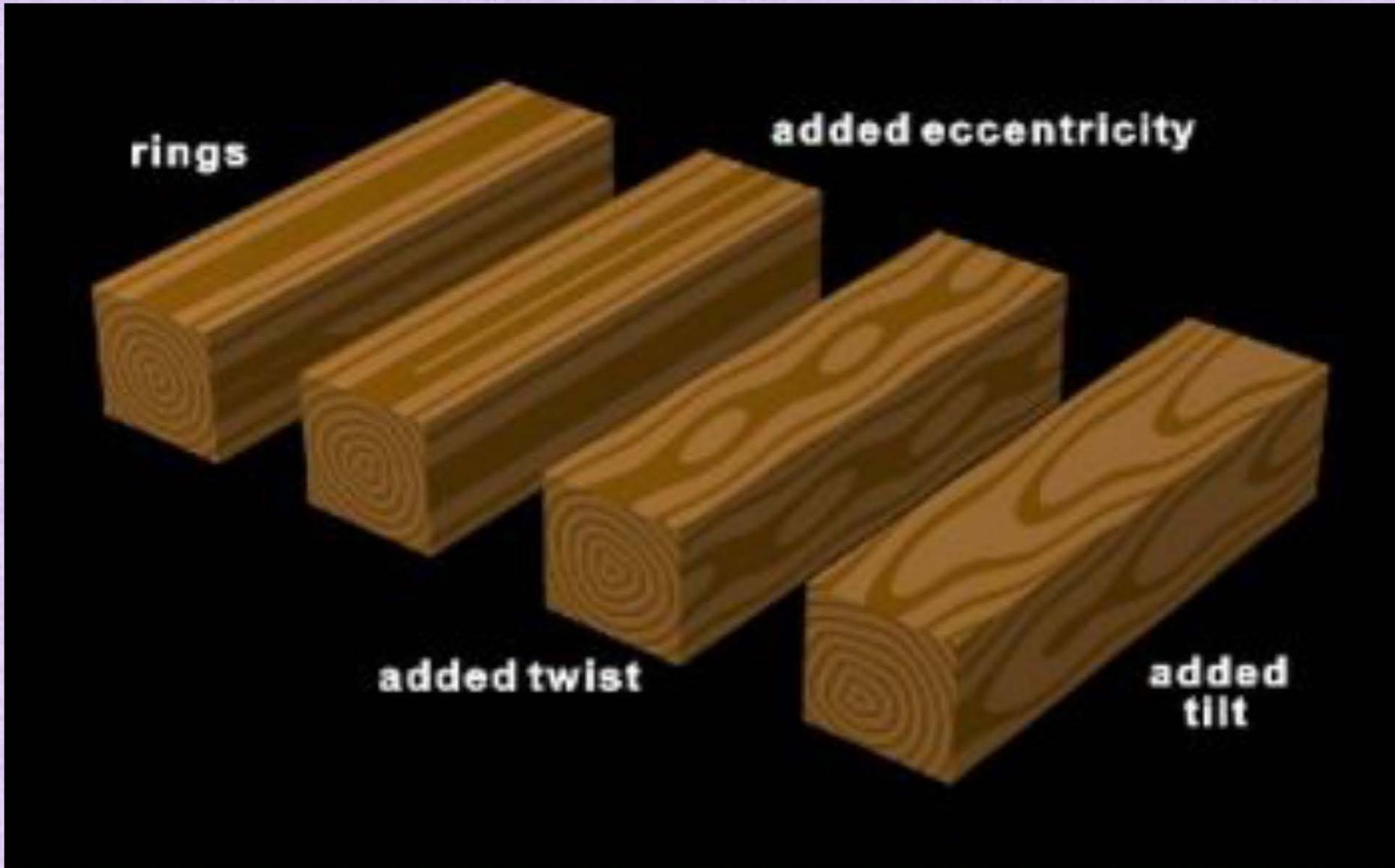


3D Wood Texture

- Procedurally generate tree rings (and cut the object out of the 3D texture)
- Cylindrical coordinates for (x, y, z) object points: $H = y, R = \sqrt{x^2 + z^2}, \theta = \tan^{-1} \left(\frac{z}{x} \right)$



3D Wood Texture

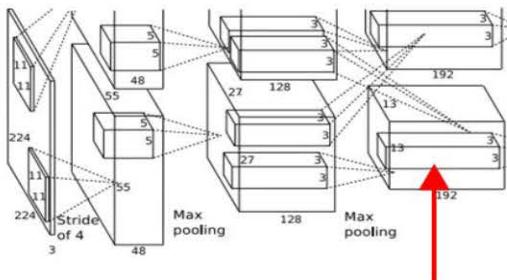


Machine Learning

Neural Texture Synthesis: Gram Matrix



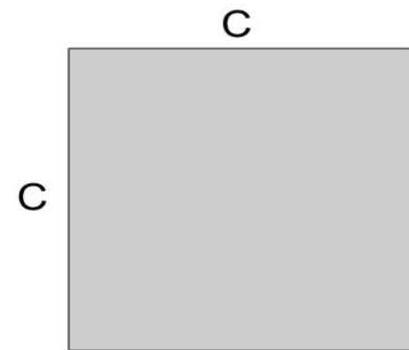
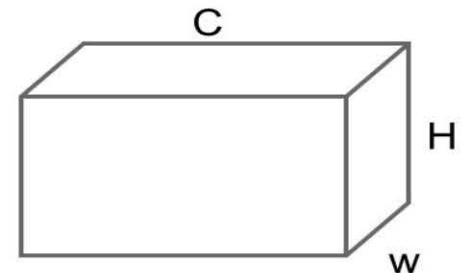
[This image](#) is in the public domain.



Each layer of CNN gives $C \times H \times W$ tensor of features; $H \times W$ grid of C -dimensional vectors

Outer product of two C -dimensional vectors gives $C \times C$ matrix measuring co-occurrence

Average over all HW pairs of vectors, giving **Gram matrix** of shape $C \times C$



Efficient to compute; reshape features from

$C \times H \times W$ to $=C \times HW$

then compute $G = FF^T$

Machine Learning

Neural Texture Synthesis

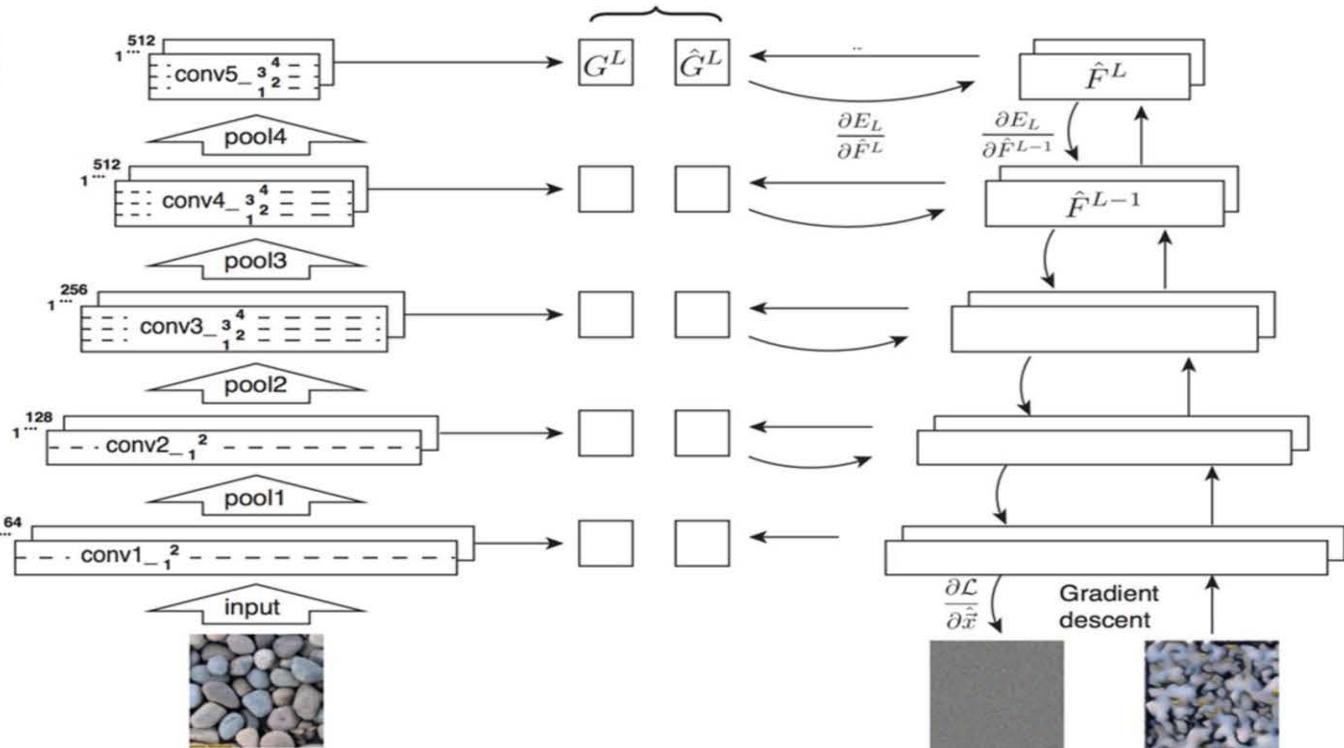
1. Pretrain a CNN on ImageNet (VGG-19)
2. Run input texture forward through CNN, record activations on every layer; layer i gives feature map of shape $C_i \times H_i \times W_i$
3. At each layer compute the *Gram matrix* giving outer product of features:

$$G_{ij}^l = \sum_k F_{ik}^l F_{jk}^l \text{ (shape } C_i \times C_i\text{)}$$

4. Initialize generated image from random noise
5. Pass generated image through CNN, compute Gram matrix on each layer
6. Compute loss: weighted sum of L2 distance between Gram matrices
7. Backprop to get gradient on image
8. Make gradient step on image
9. GOTO 5

Gatys, Ecker, and Bethge, "Texture Synthesis Using Convolutional Neural Networks", NIPS 2015
Figure copyright Leon Gatys, Alexander S. Ecker, and Matthias Bethge, 2015. Reproduced with permission.

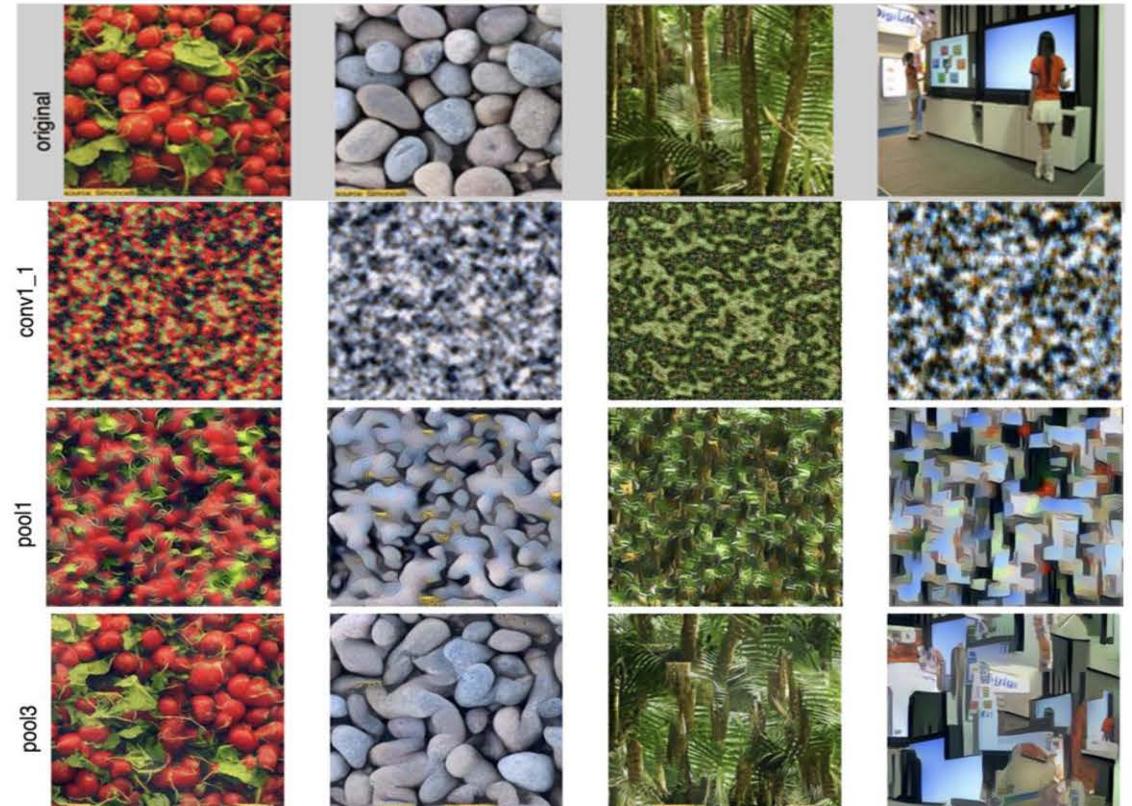
$$E_l = \frac{1}{4N_l^2 M_l^2} \sum_{i,j} (G_{ij}^l - \hat{G}_{ij}^l)^2 \quad \mathcal{L}(\vec{x}, \hat{\vec{x}}) = \sum_{l=0}^L w_l E_l$$



Machine Learning

Neural Texture Synthesis

Reconstructing texture
from higher layers recovers
larger features from the
input texture



Gatys, Ecker, and Bethge, "Texture Synthesis Using Convolutional Neural Networks", NIPS 2015
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Machine Learning



Machine Learning

