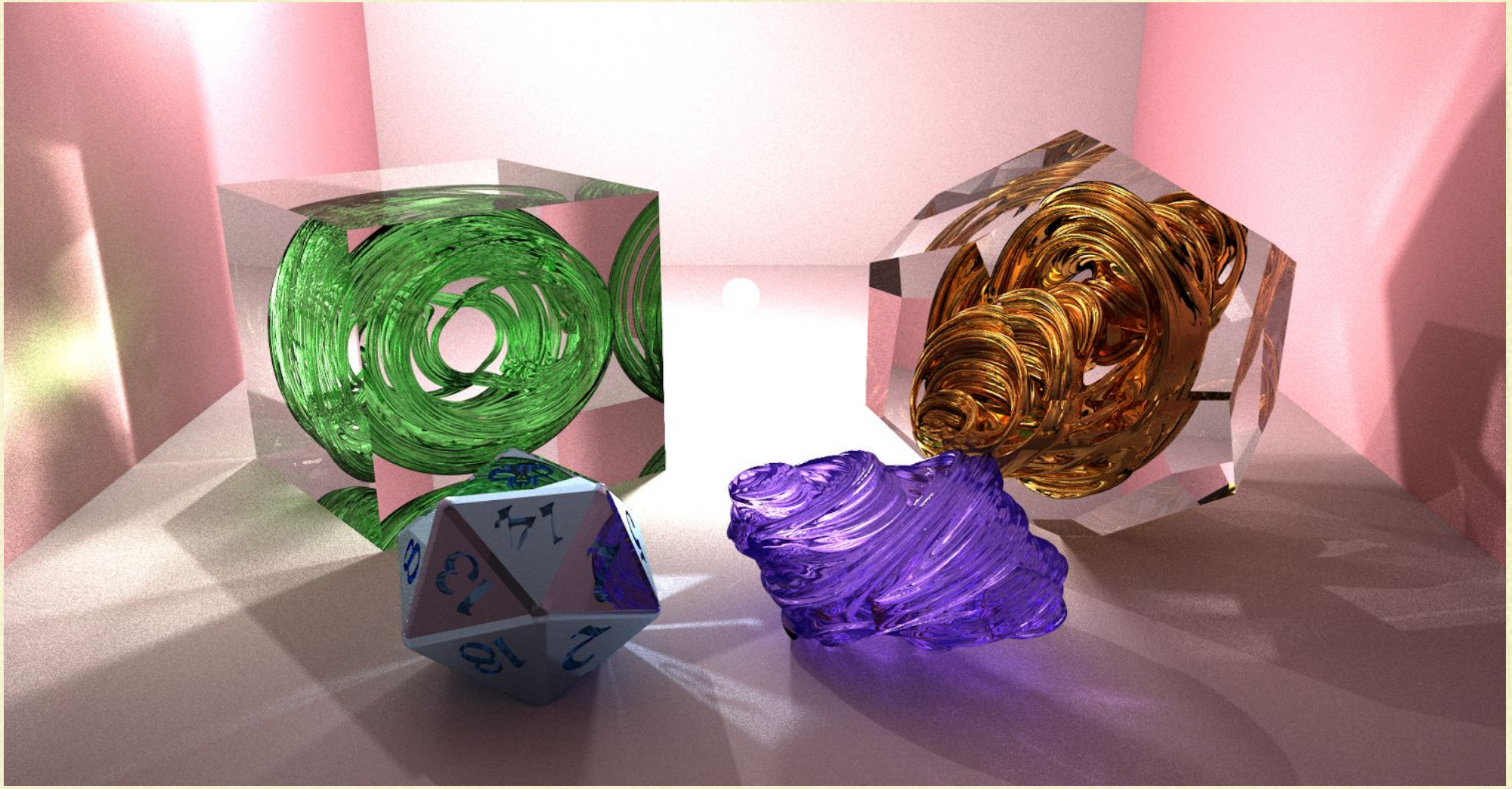
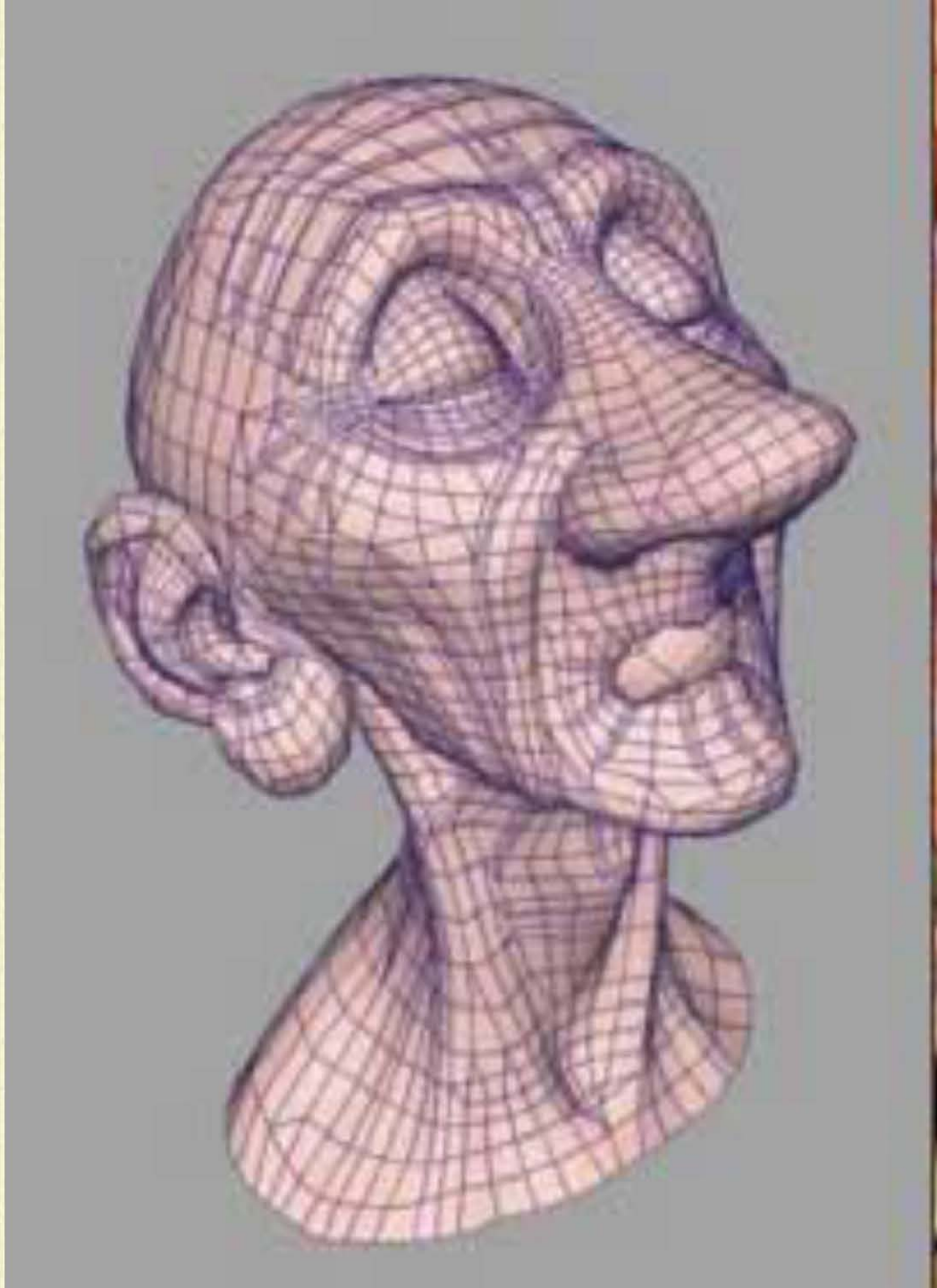
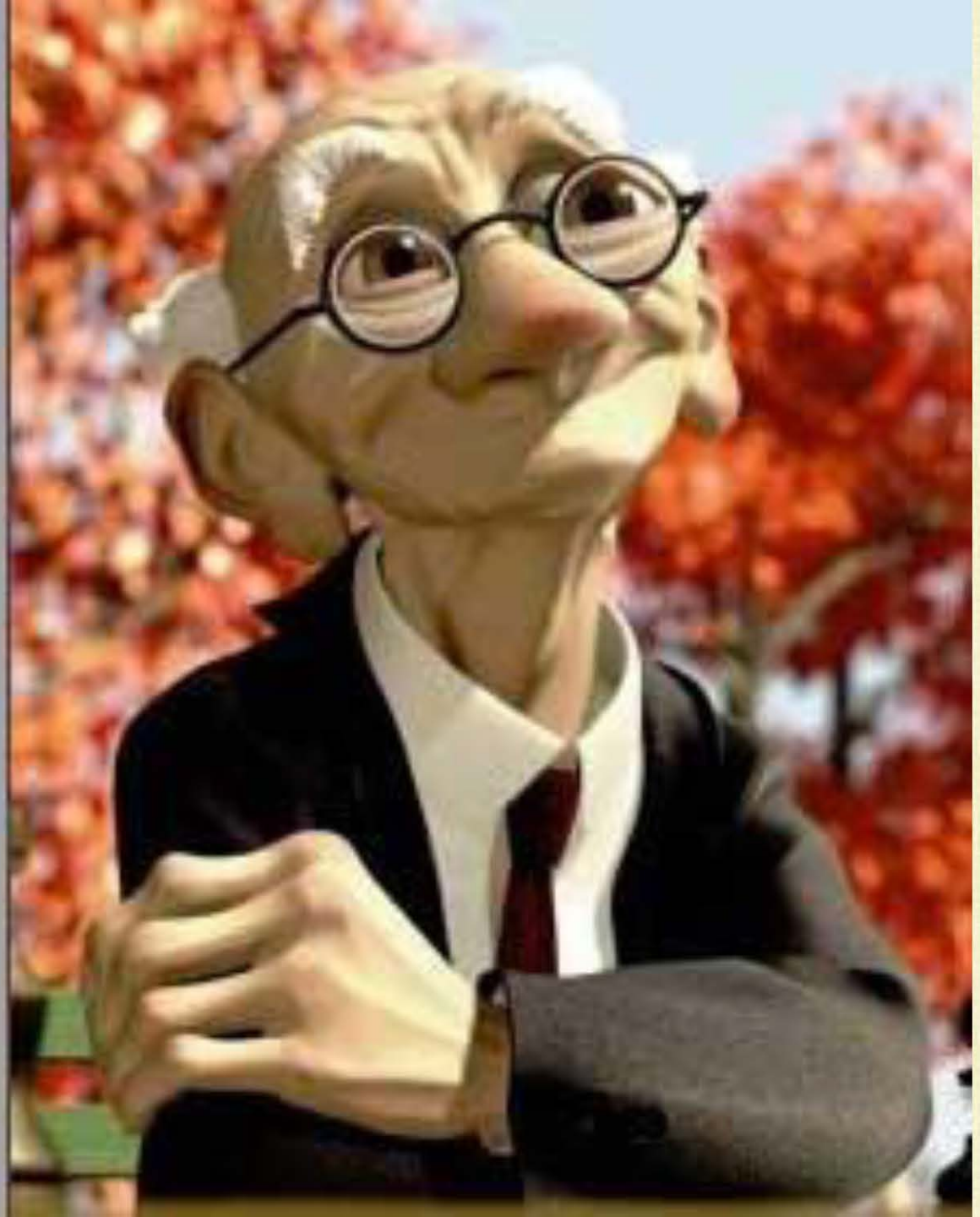
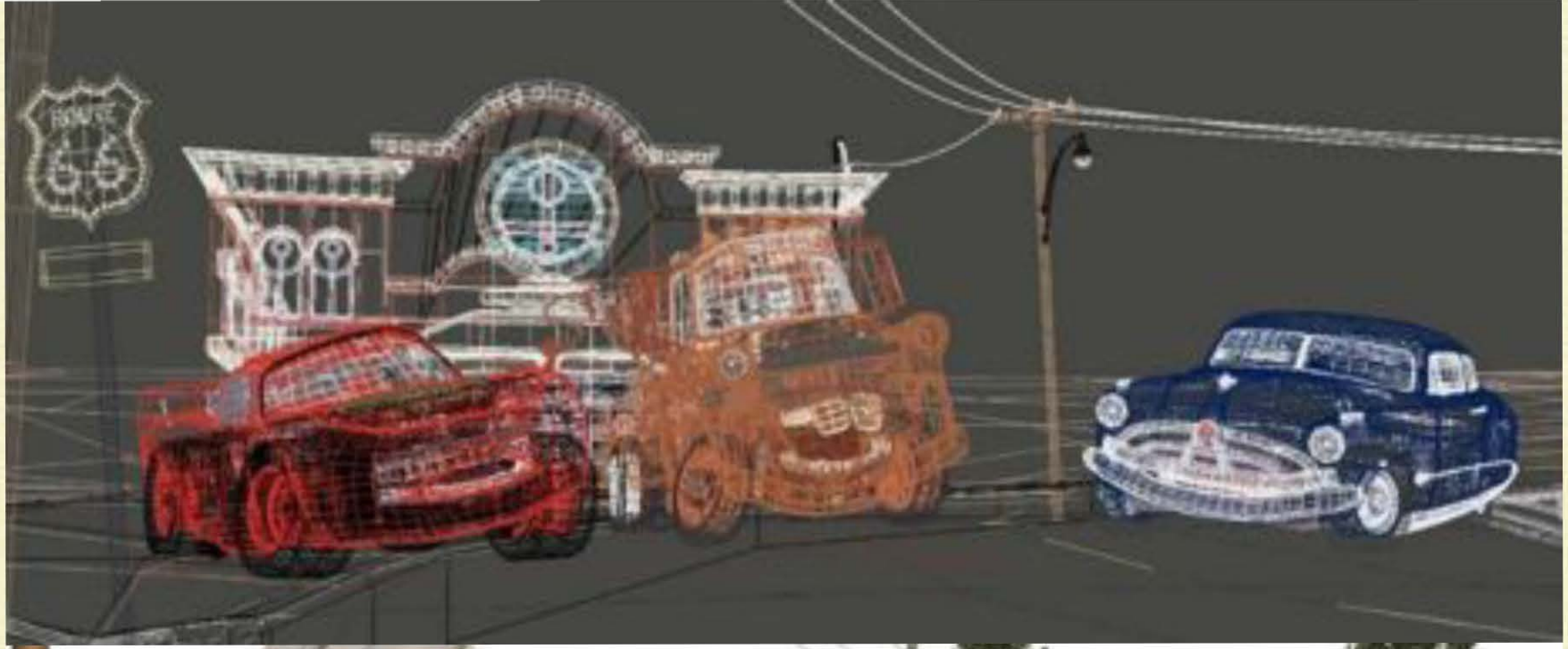
Geometric Modeling



Examples

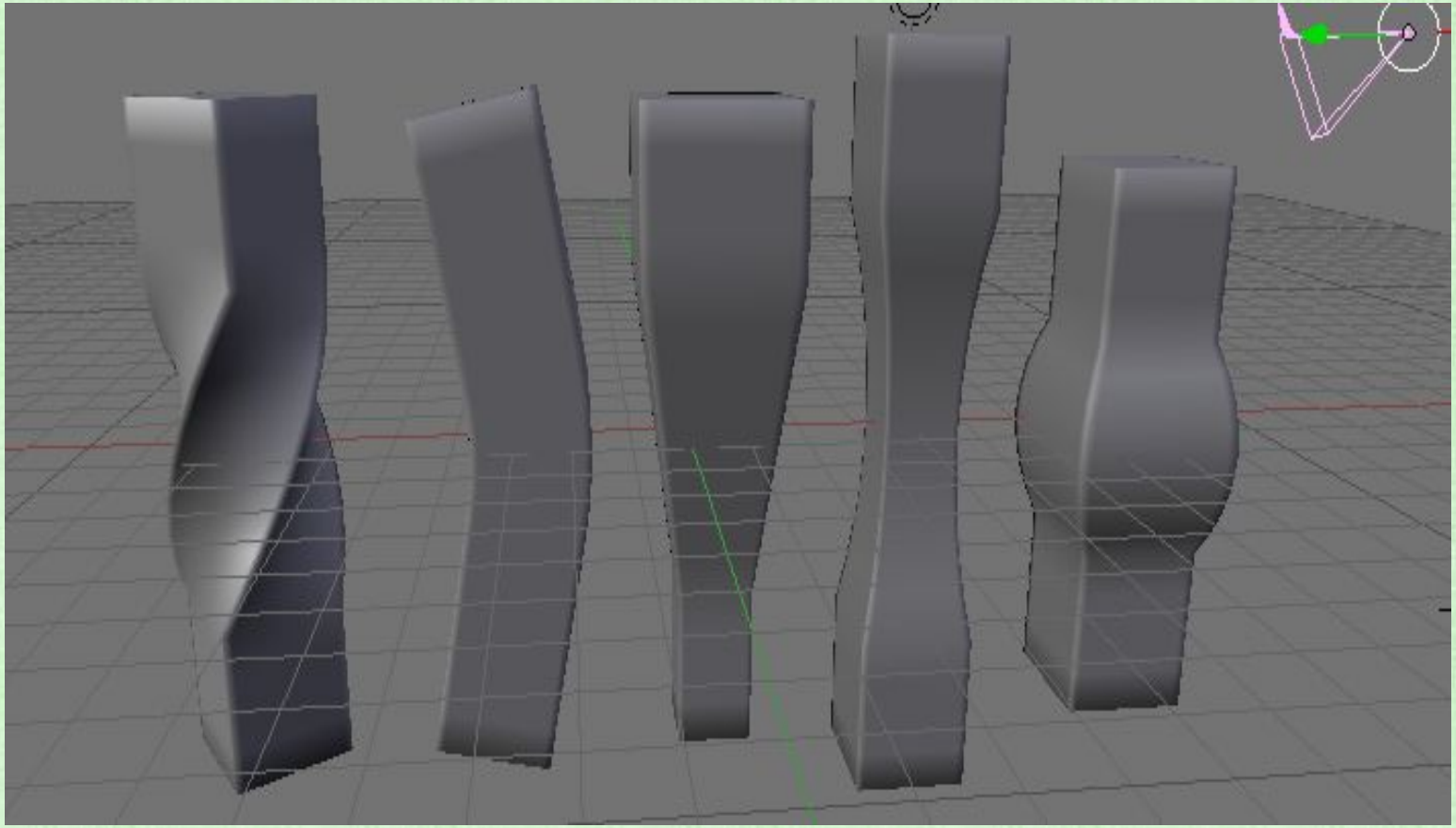






Mesh Editing

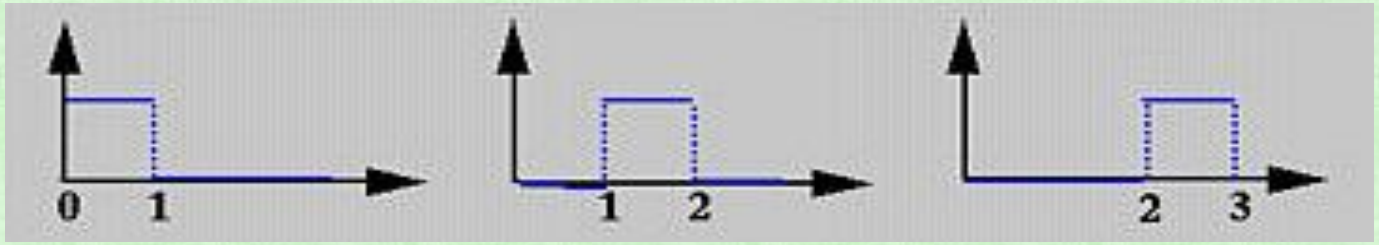
- Manipulate a few control/mesh points and use an algorithm to procedurally deform the mesh (e.g., twist, bend, stretch, etc.)



B-Spline Basis Functions

- Knots: node locations $\{u_0, u_1, \ldots u_m\}$ in parameter space
- Lowest level building blocks are piecewise constant (m of them):

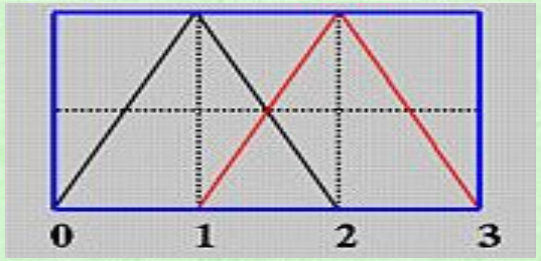
$$N_{i,0}(u) = \begin{cases} 1 & if\ u_i \le u < u_{i+1} \\ 0 & otherwise \end{cases}$$



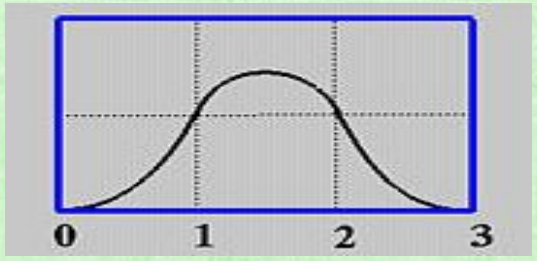
0-th order: $N_{0,0}$ $N_{1,0}$ $N_{2,0}$

- Higher order building blocks are defined recursively (i-th basis function of order j is…):

$$N_{i,j}(u) = \frac{u - u_i}{u_{i+j} - u_i} N_{i,j-1}(u) + \frac{u_{i+j+1} - u}{u_{i+j+1} - u_{i+1}} N_{i+1,j-1}(u)$$



1st order: $N_{0,1}$ $N_{1,1}$

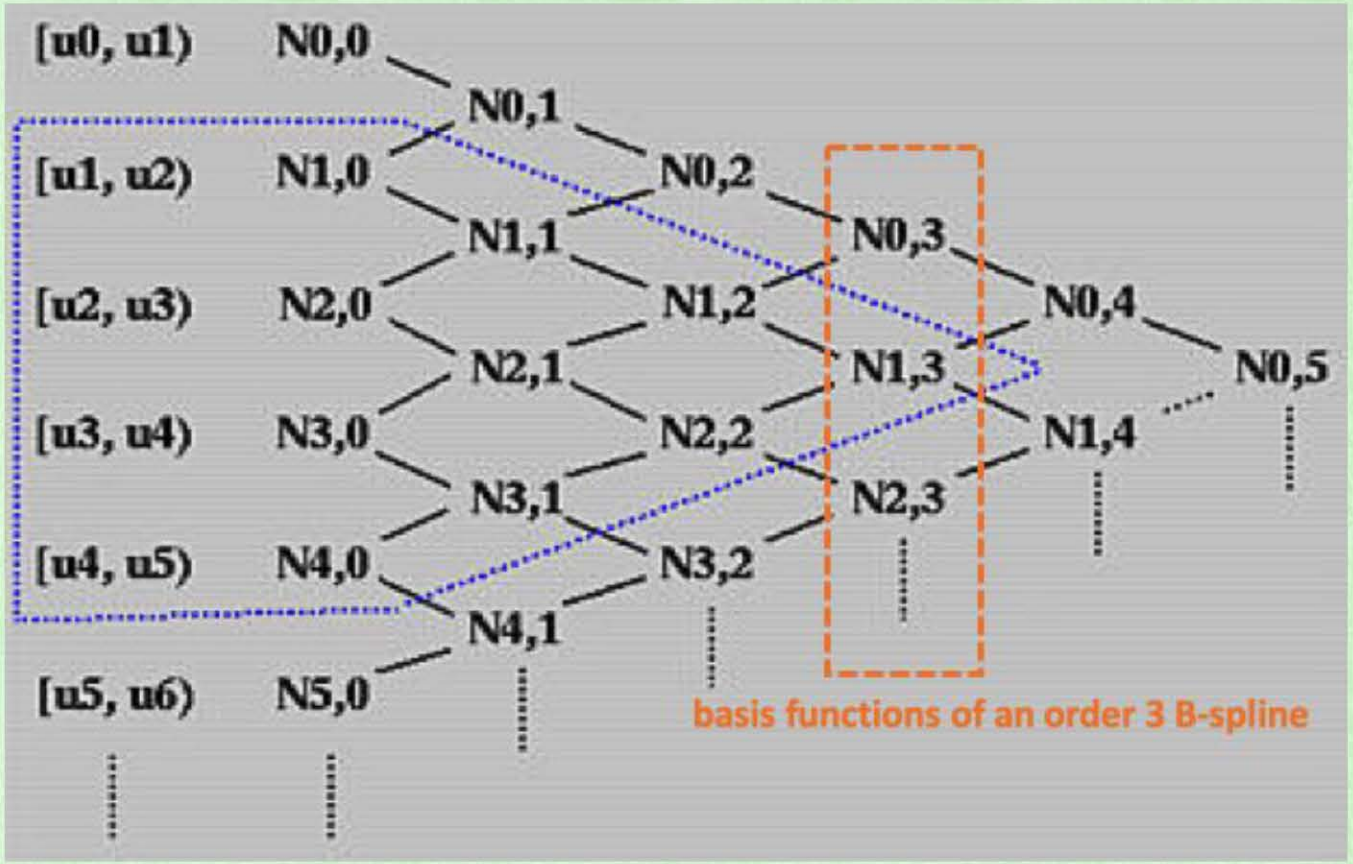


2nd order: $N_{0,2}$

B-Spline Basis Functions

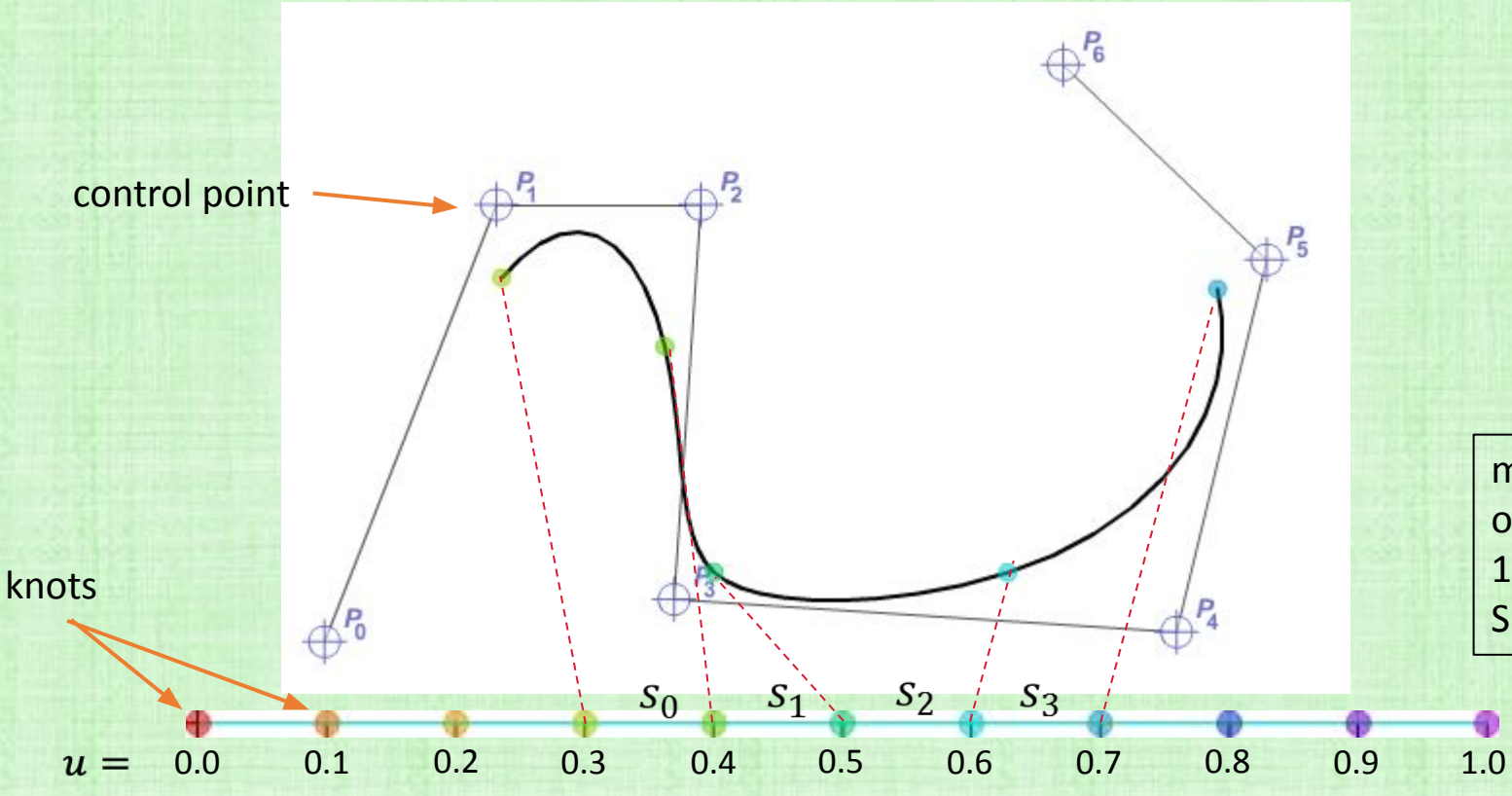
- The basis functions for a B-spline of order p are $N_{i,p}(u)$
- Let $\boldsymbol{P}_0, \boldsymbol{P}_1, \ldots, \boldsymbol{P}_n$ be control points (in 2D or 3D)
- The B-spline curve is defined as: $C(u) = \sum_{i=0}^{n} N_{i,p}(u)\boldsymbol{P}_i$

- Given *m* intervals, a B-spline of order *p* requires *m-p* control points $\boldsymbol{P}_i$
- E.g., the *m=6* intervals (to the right) with order *p=3* requires *m-p=3* (orange box) control points $\boldsymbol{P}_i$
- The summation (above) goes from 0 to 2 to include 3 control points



B-Spline

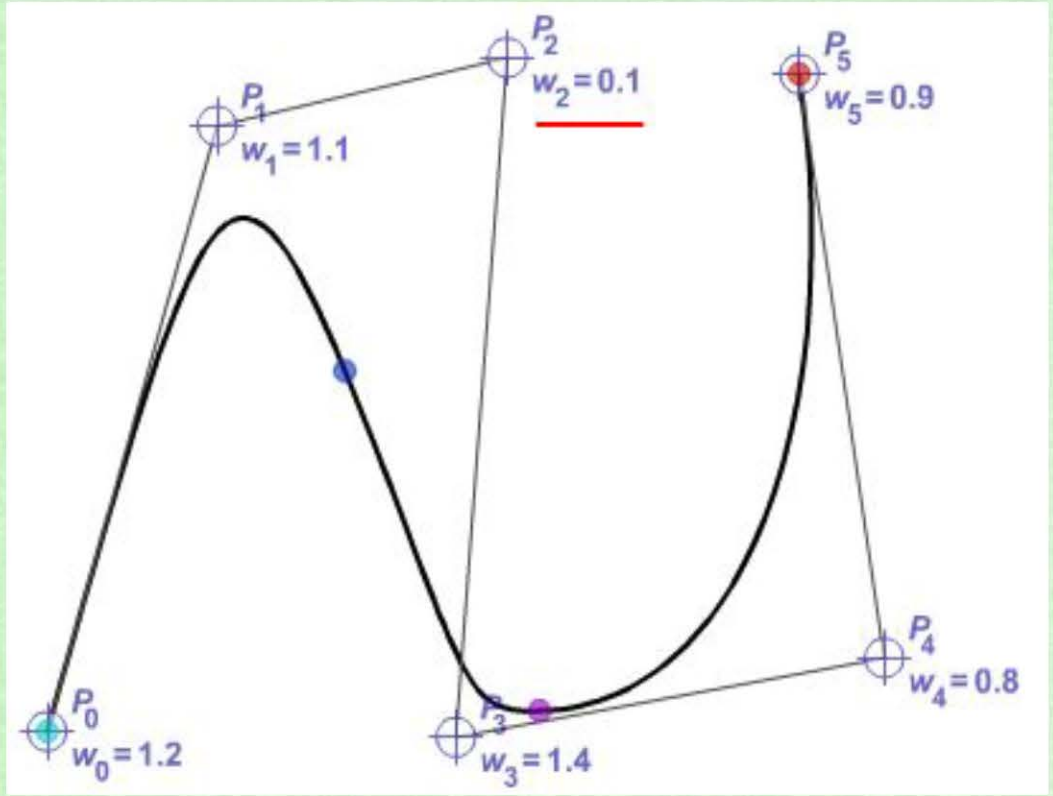
- Ignore boundaries (where there are only 1, 2, 3 cubic basis functions $N_{i,3}$):
  - 3 parametric domain intervals are lost off of each side (of the domain)
  - [u3,u4) is the first interval to have a full representation in the orange box (of prior slide): $N_{0,3}, N_{1,3}, N_{2,3}, N_{3,3}$
- Then, 4 control points define the shape of each curve segment
  - e.g., s_0 = (.3,.4) is controlled P_0, P_1, P_2, P_3

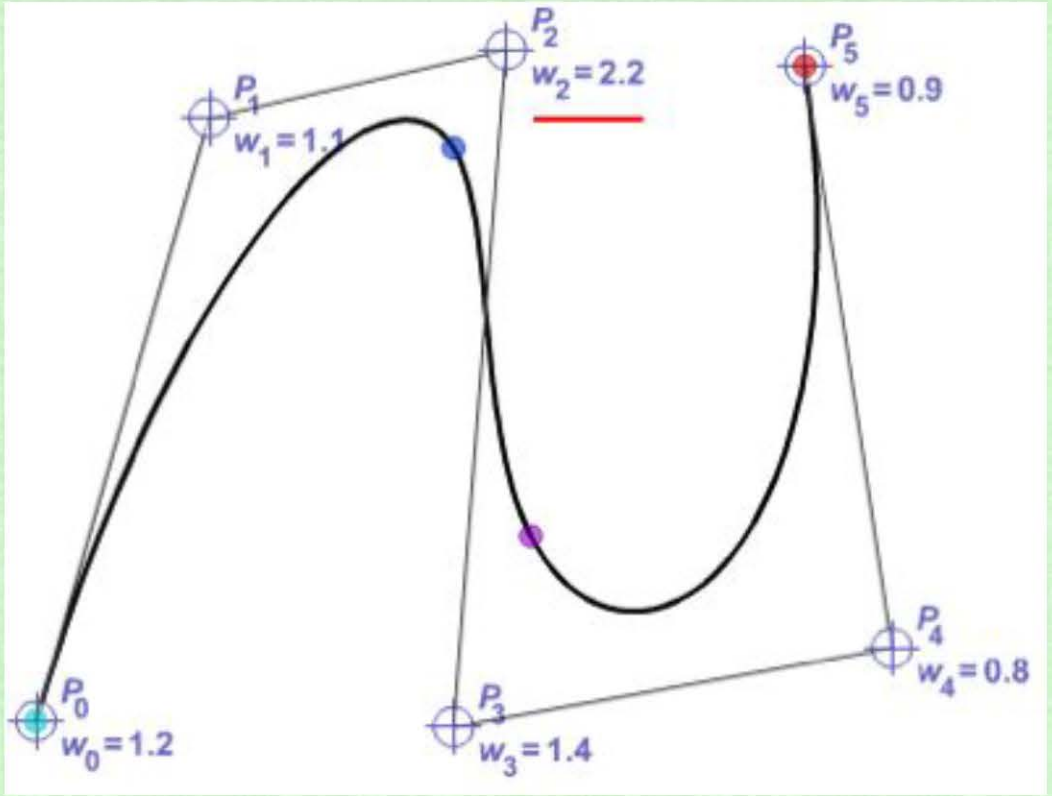


m=10 intervals
order p=3
10-3=7 control points
Sum: i = 0 to 6 (n=6)

NURBS Curve

- Non-Uniform Rational B-Spline (NURBS): $C(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u) w_i \boldsymbol{P}_i}{\sum_{i=0}^{n} N_{i,p}(u) w_i}$
- Increasing the weight w_i on point $\boldsymbol{P}_i$ pulls the curve closer to $\boldsymbol{P}_i$
- Decreasing w_i releases the curve to move farther away from $\boldsymbol{P}_i$

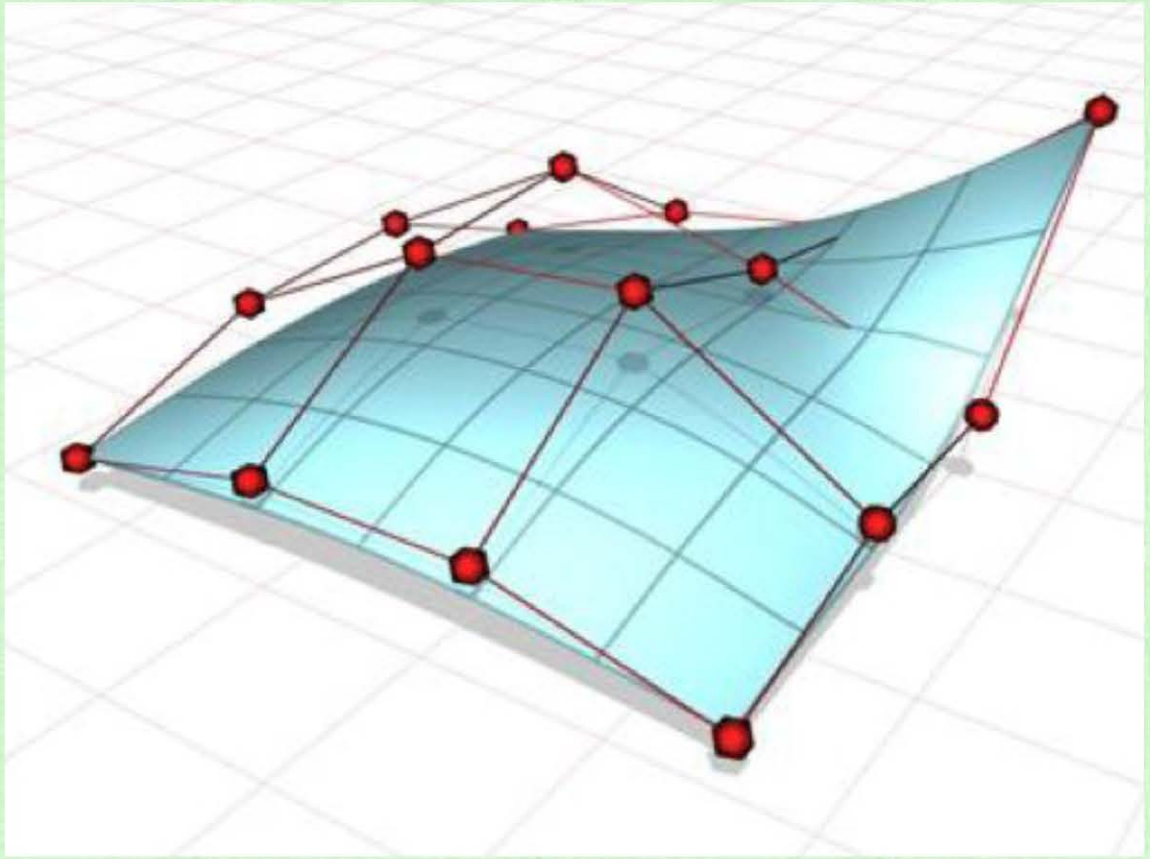




NURBS Surface

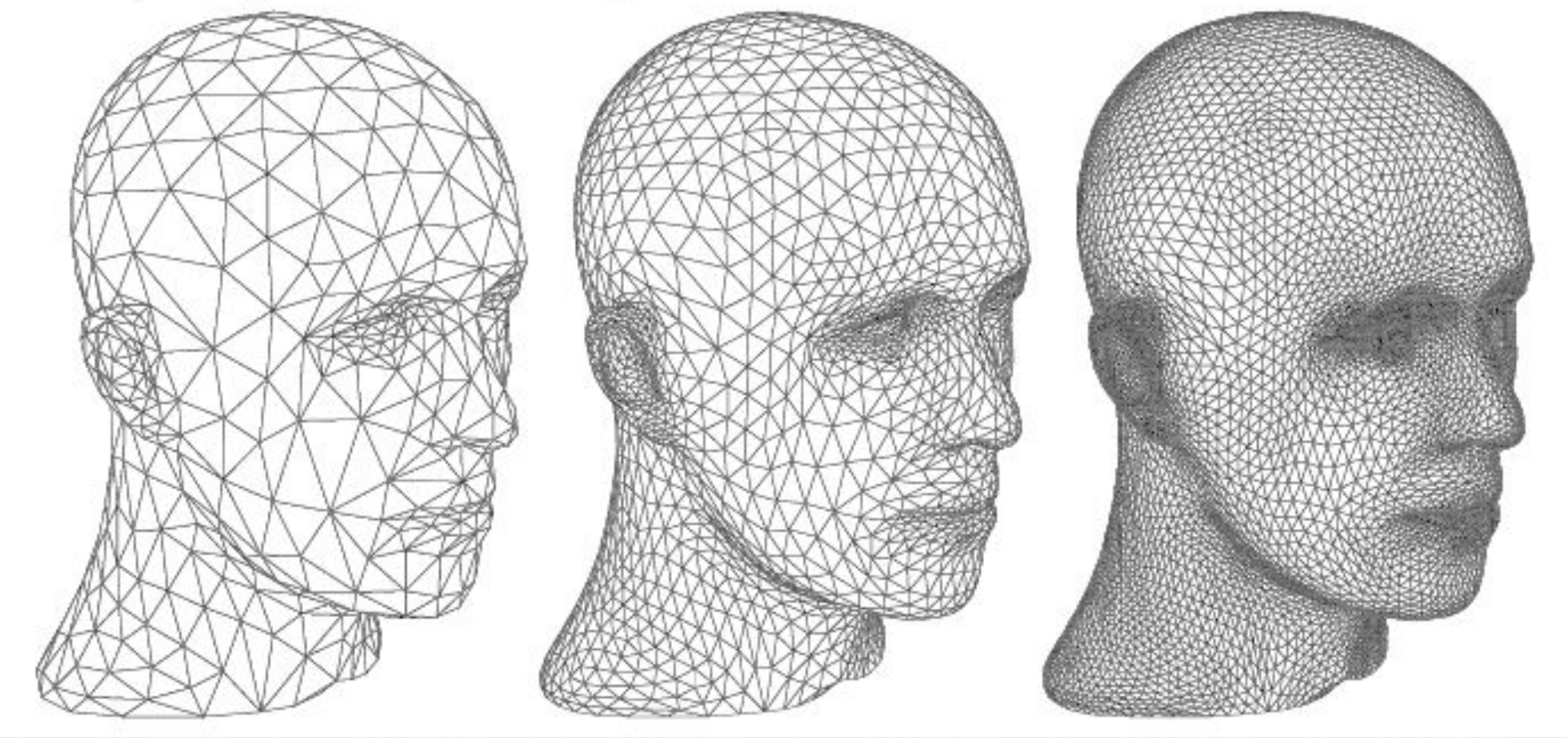
- Extending the ideas from curves to surfaces:

$$S(u,v) = \frac{\sum_{i=0}^{m}\sum_{j=0}^{n} N_{i,p}(u)N_{j,q}(v)w_{i,j}\boldsymbol{P}_{i,j}}{\sum_{i=0}^{m}\sum_{j=0}^{n} N_{i,p}(u)N_{j,q}(v)w_{i,j}}$$

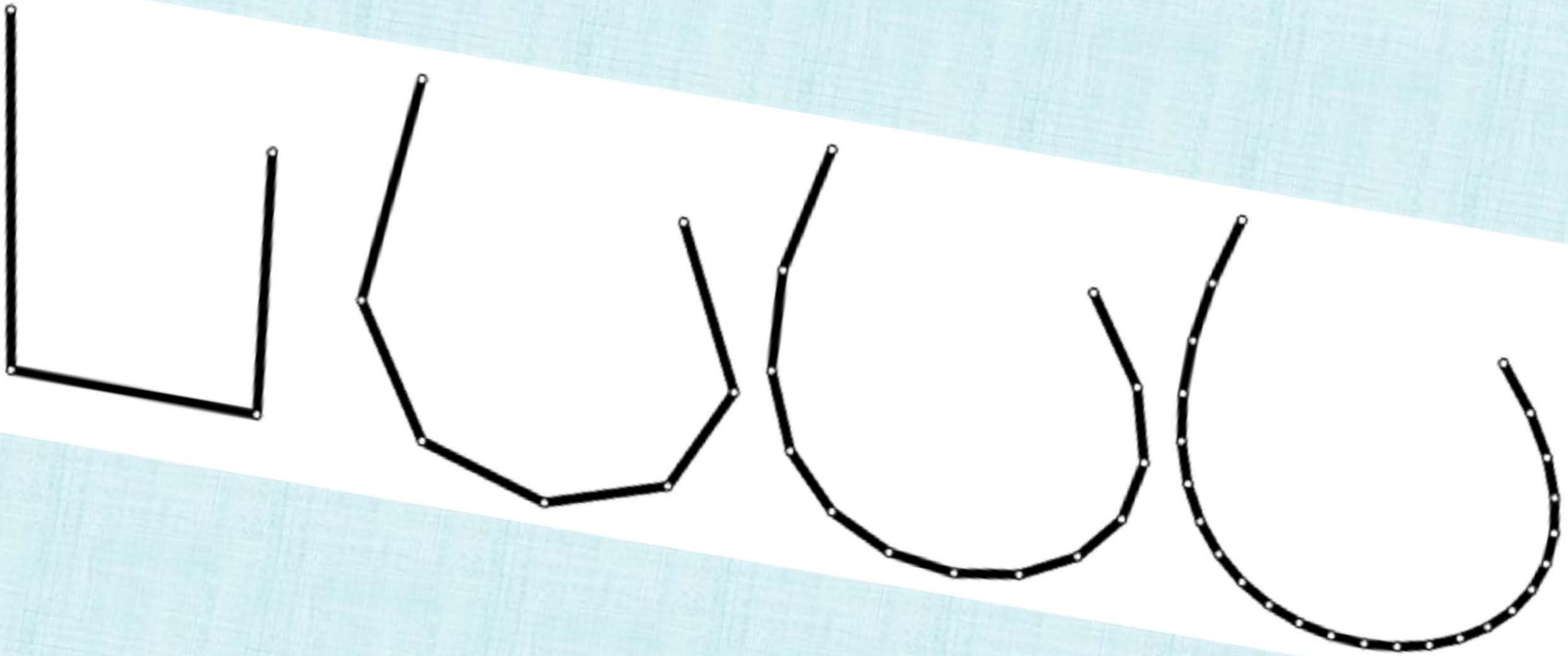


Subdivision

- Automatically generate a finer (and smoother) mesh from a coarser geometric model
- Typically requires only a few refinements

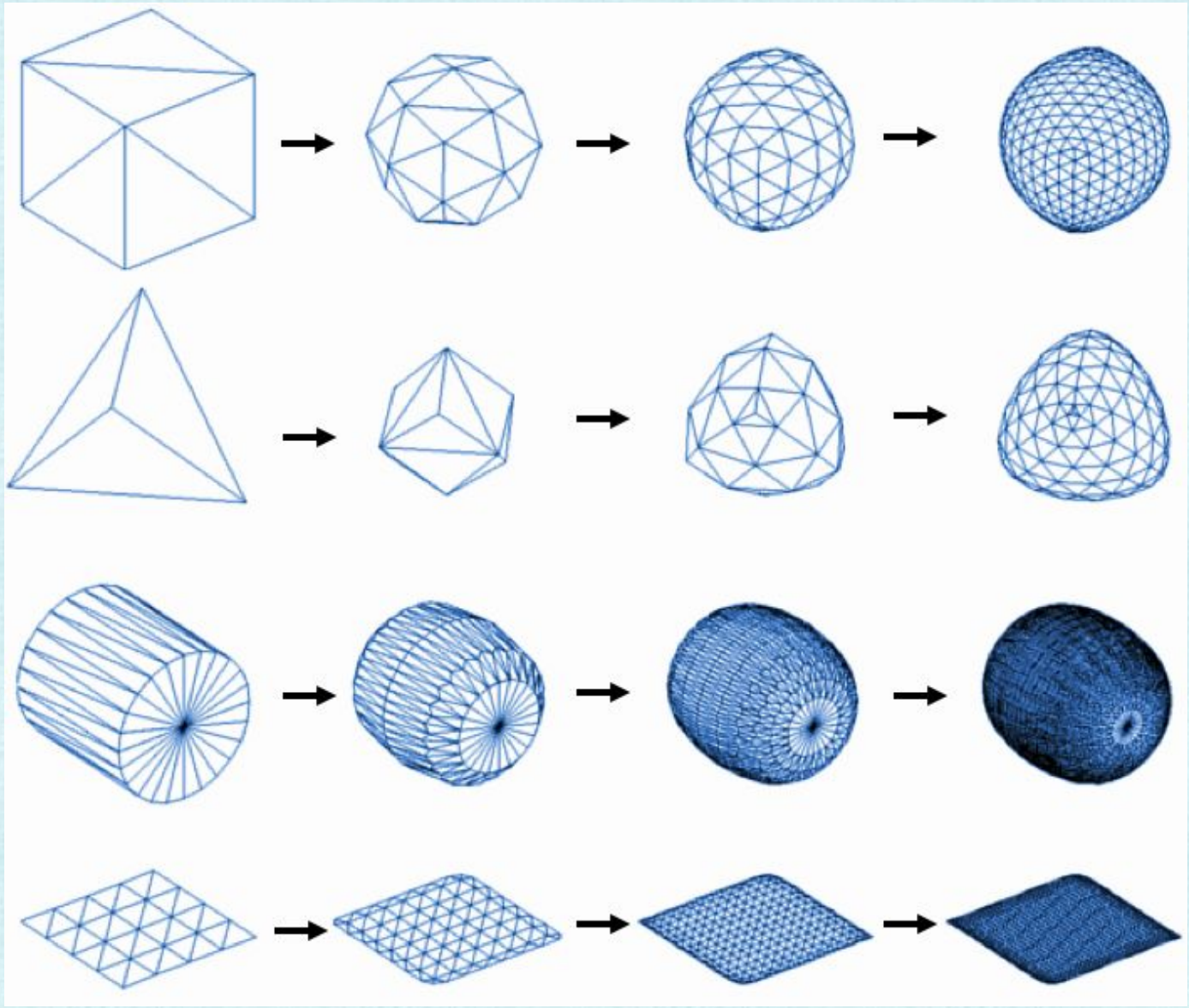


Subdivision Curves



Subdivision Surfaces

- Subdivide each triangle into 4 sub-triangles
- Move all the old and new vertices
- Repeat (if desired)



Charles Loop

Smooth Subdivision Surfaces Based on Triangles

by

Charles Teorell Loop

A thesis submitted to the faculty of
the University of Utah
in partial fulfillment of the requirements for the degree of

Master of Science

Department of Mathematics

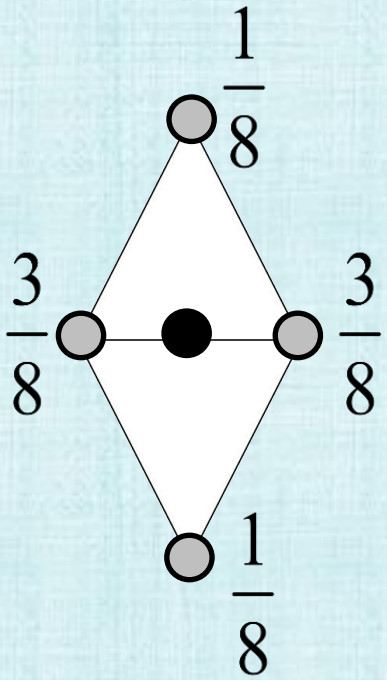
The University of Utah

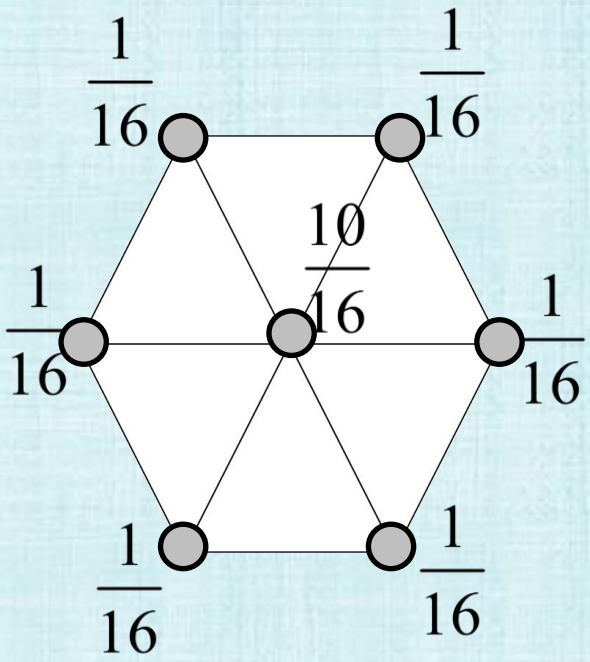
August 1987

2256 citations (for a MS thesis!)

Move Old/New Vertices

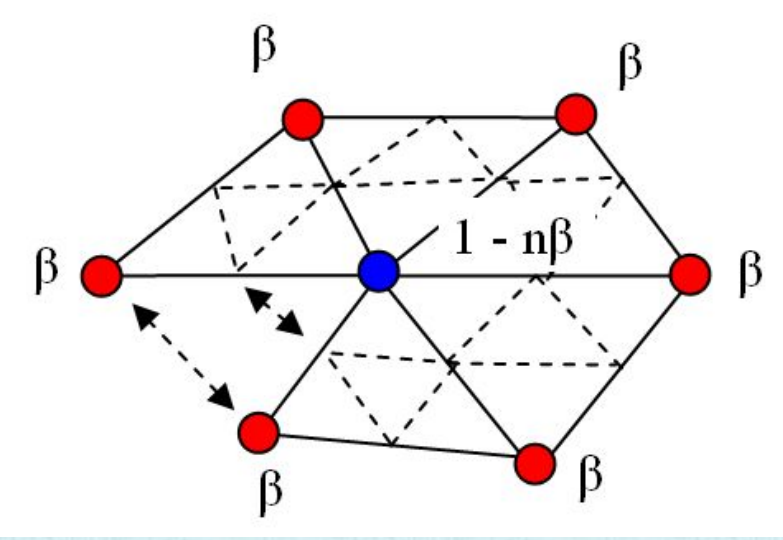
- Set the position of a new vertex (shown in black) using a weighted average of the four nearby original vertices (shown in grey)
- Change the position of each regular original vertex (shown in grey) using a weighted average of the six adjacent original vertices (shown in grey)

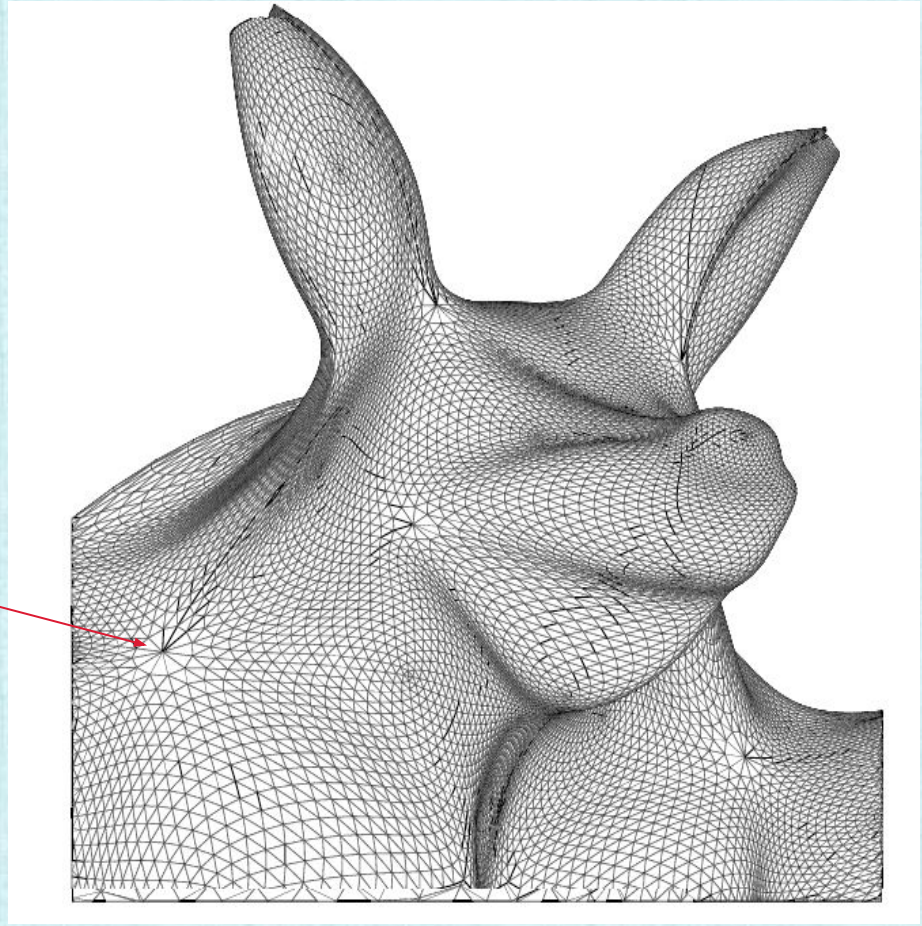




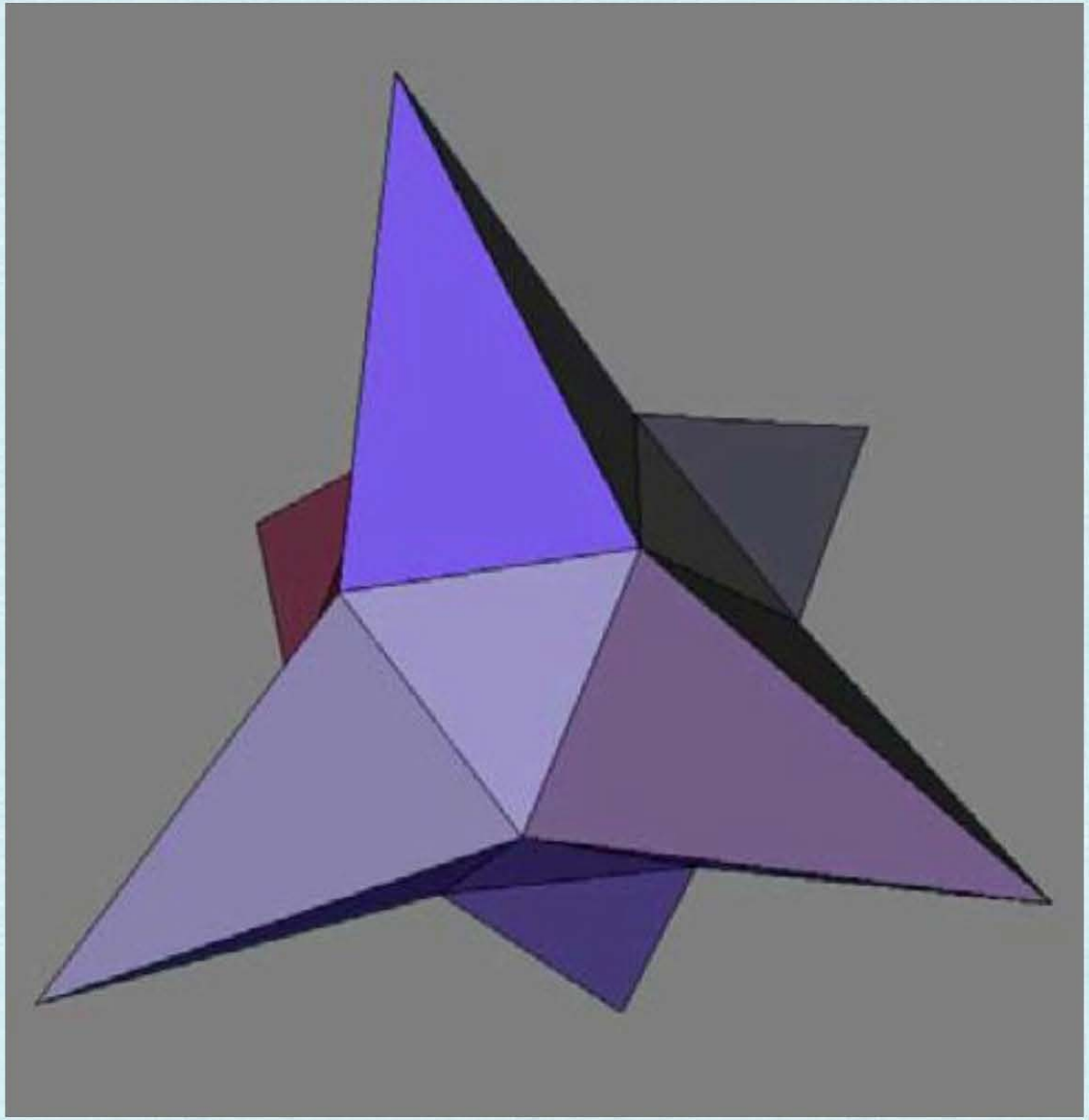
Extraordinary Points

- Most vertices are regular (degree 6)
- If a mesh is topologically equivalent to a sphere, not all vertices can have degree 6
- At extraordinary points, special weights are used:
  - If number of neighbors is $n = 3, \beta = \frac{3}{16}$
  - else $\beta = \frac{3}{8n}$

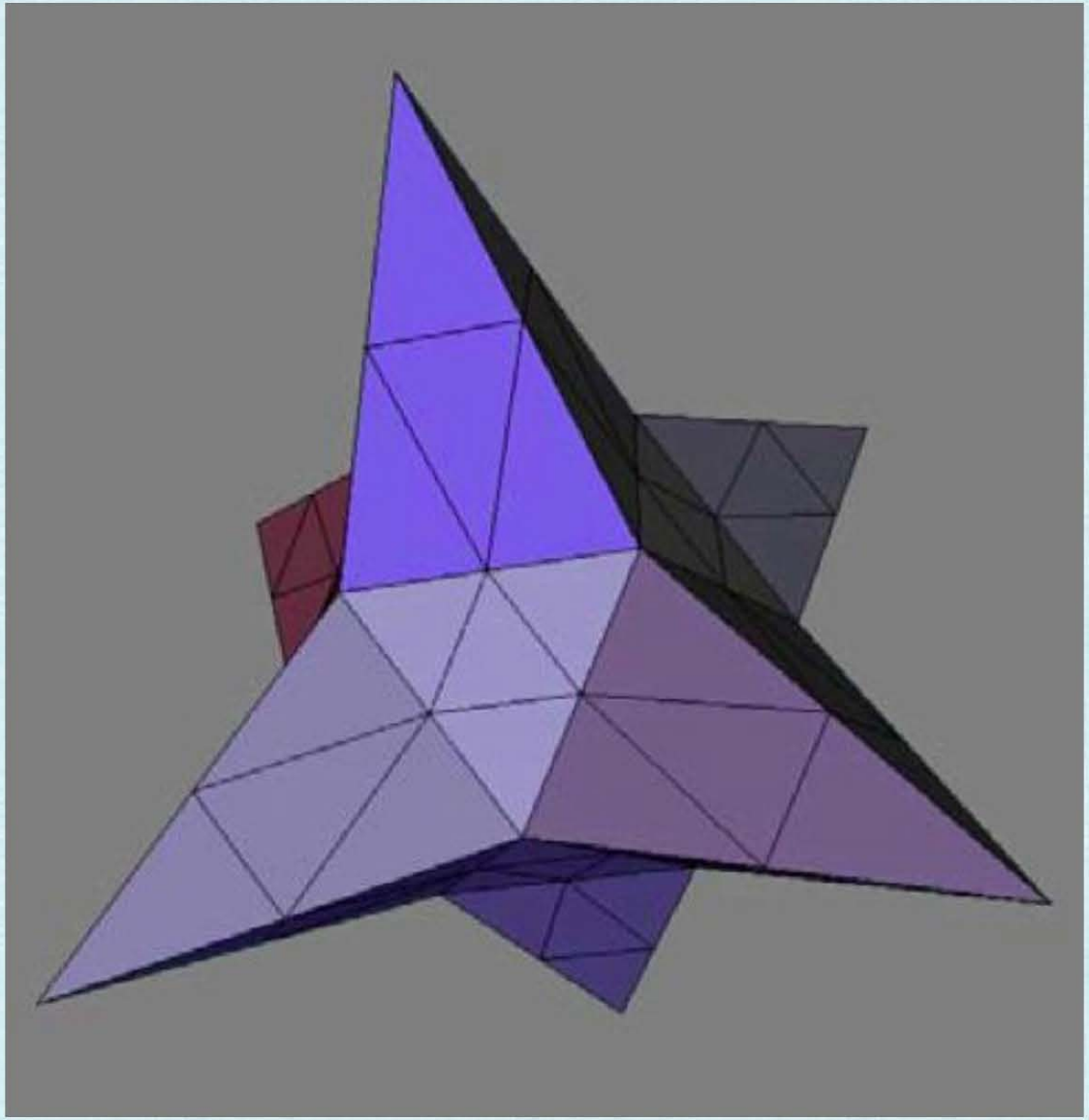




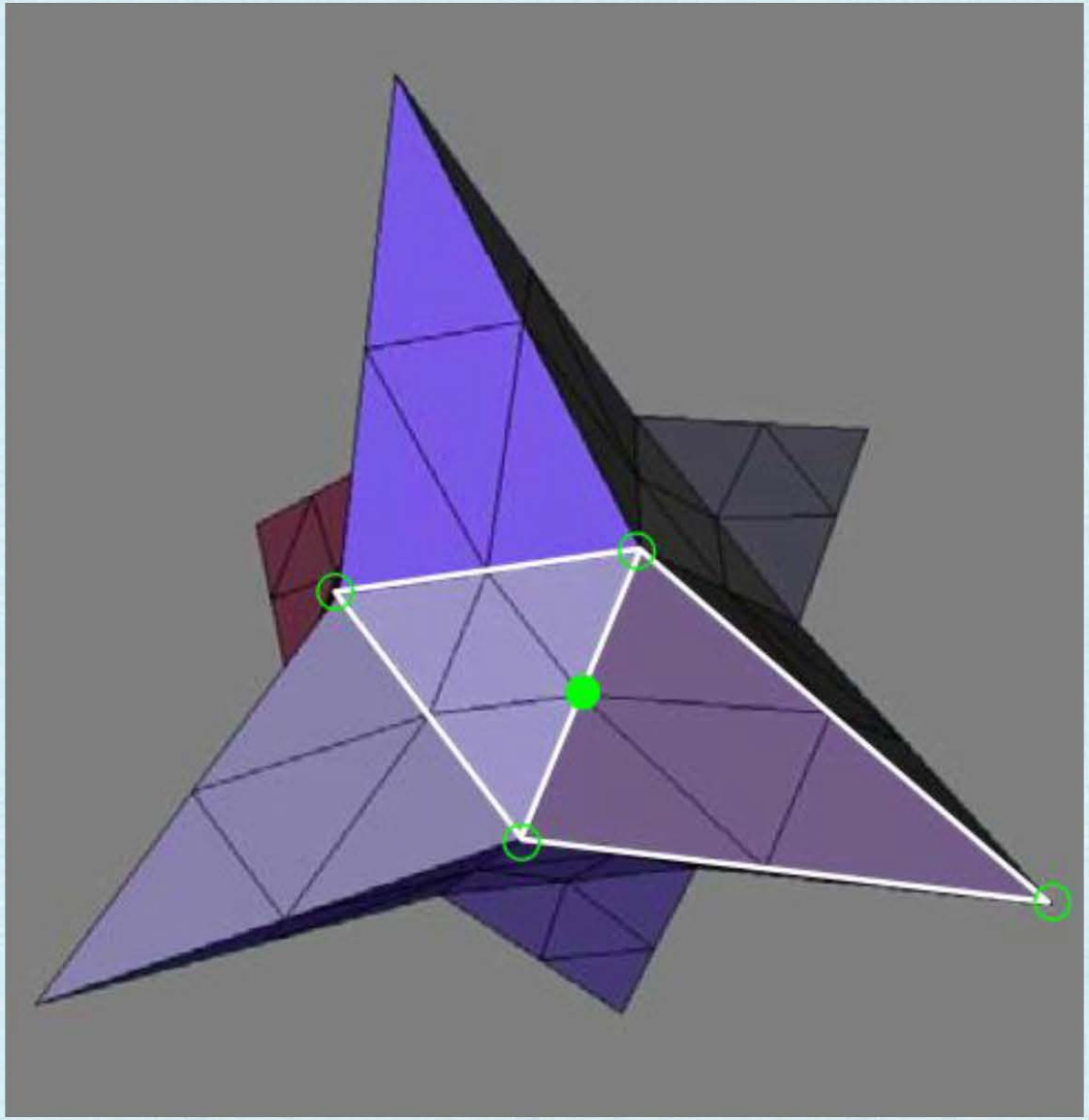
Initial Mesh



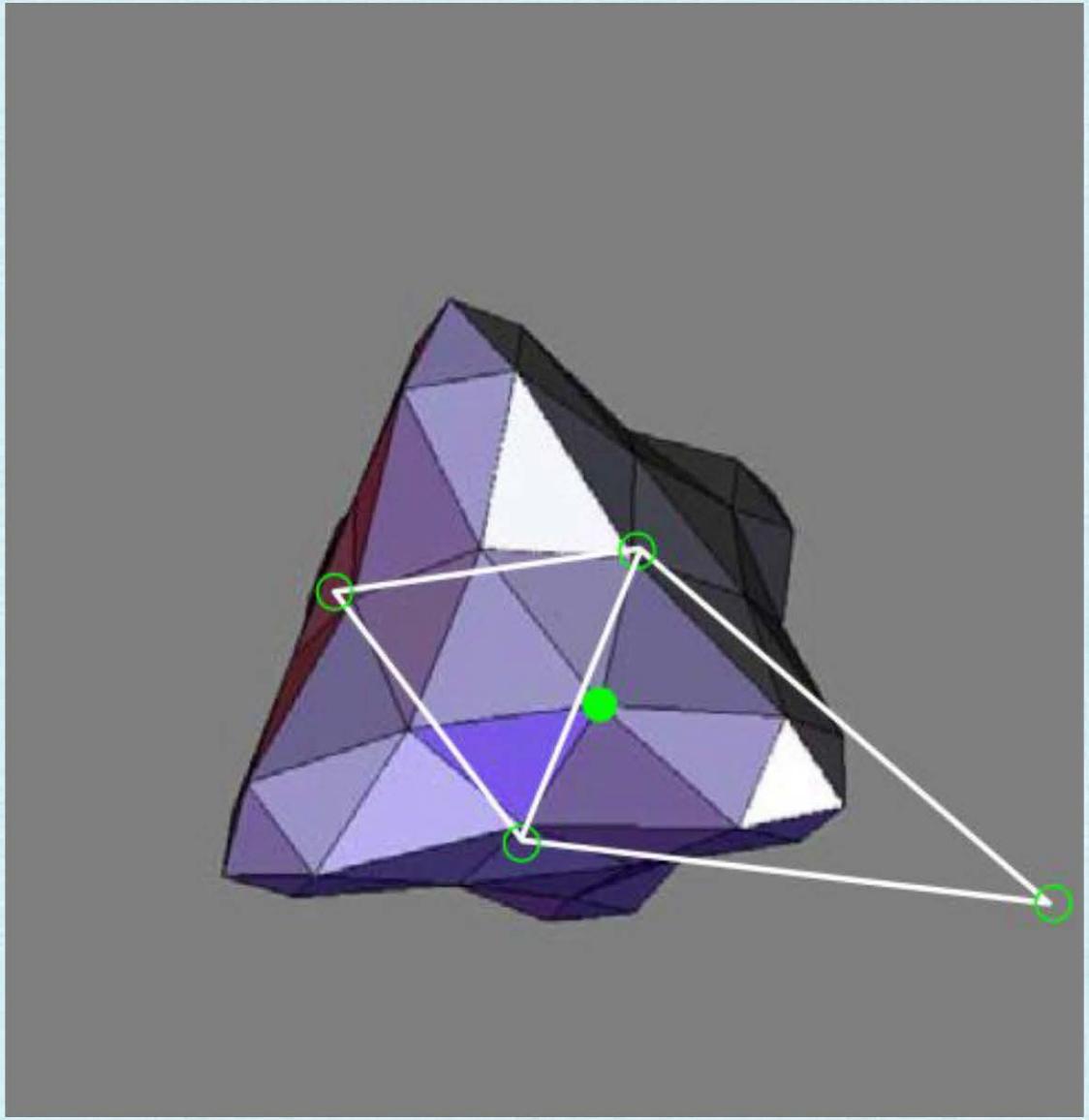
Add New Vertices



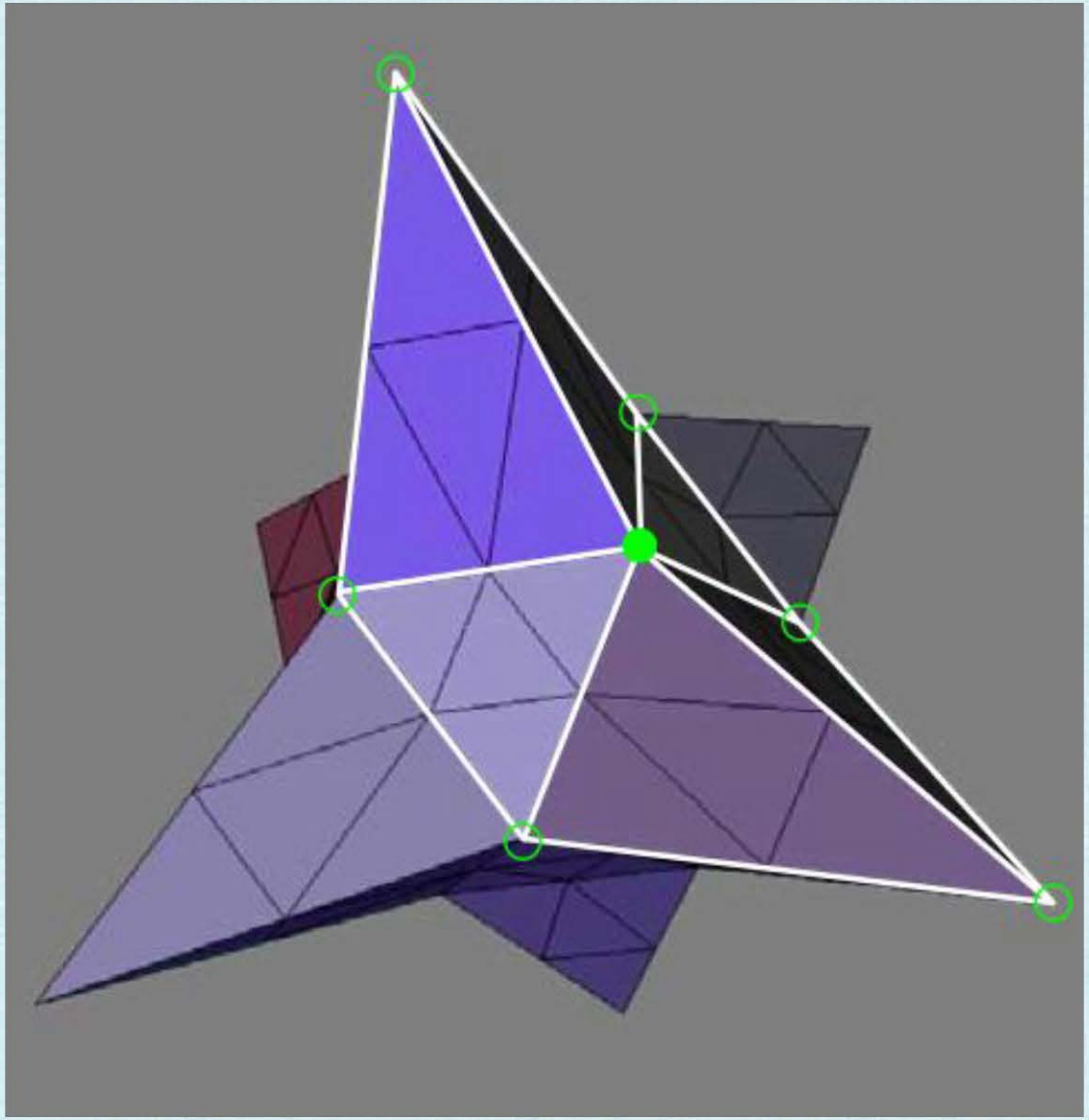
New Vertex and Stencil



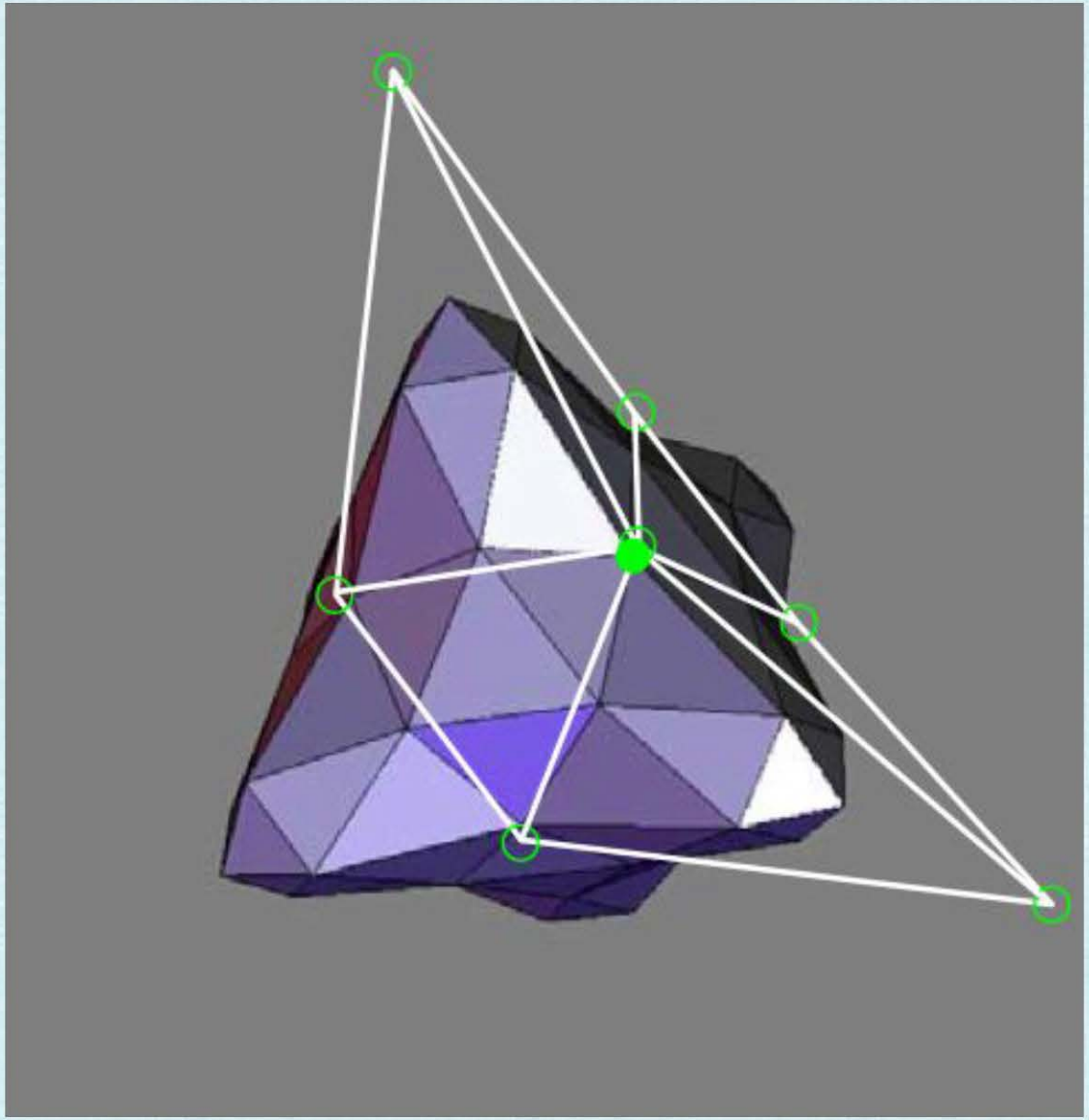
Move New Vertex



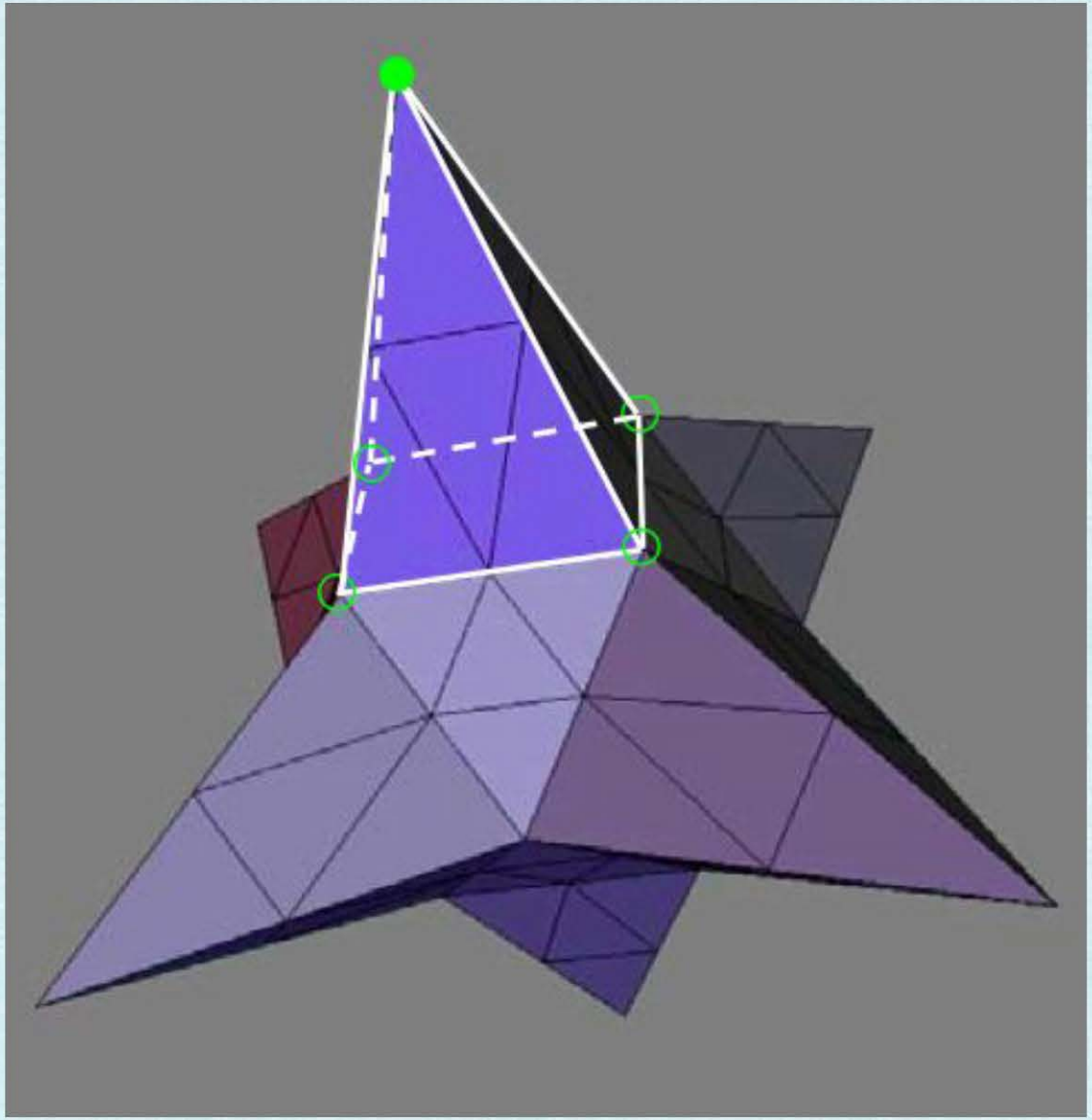
Original (Regular) Vertex and Stencil



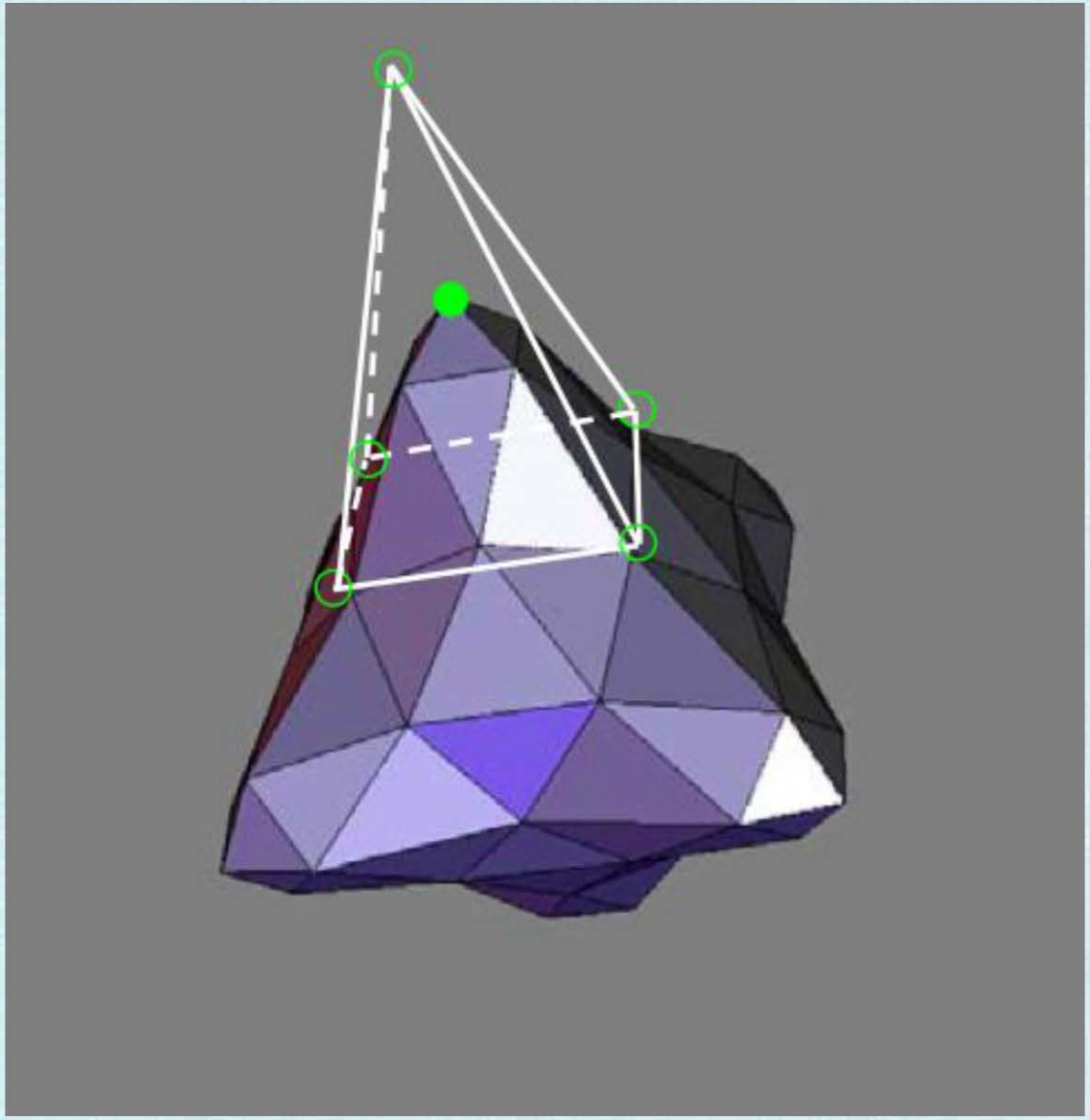
Move Original (Regular) Vertex



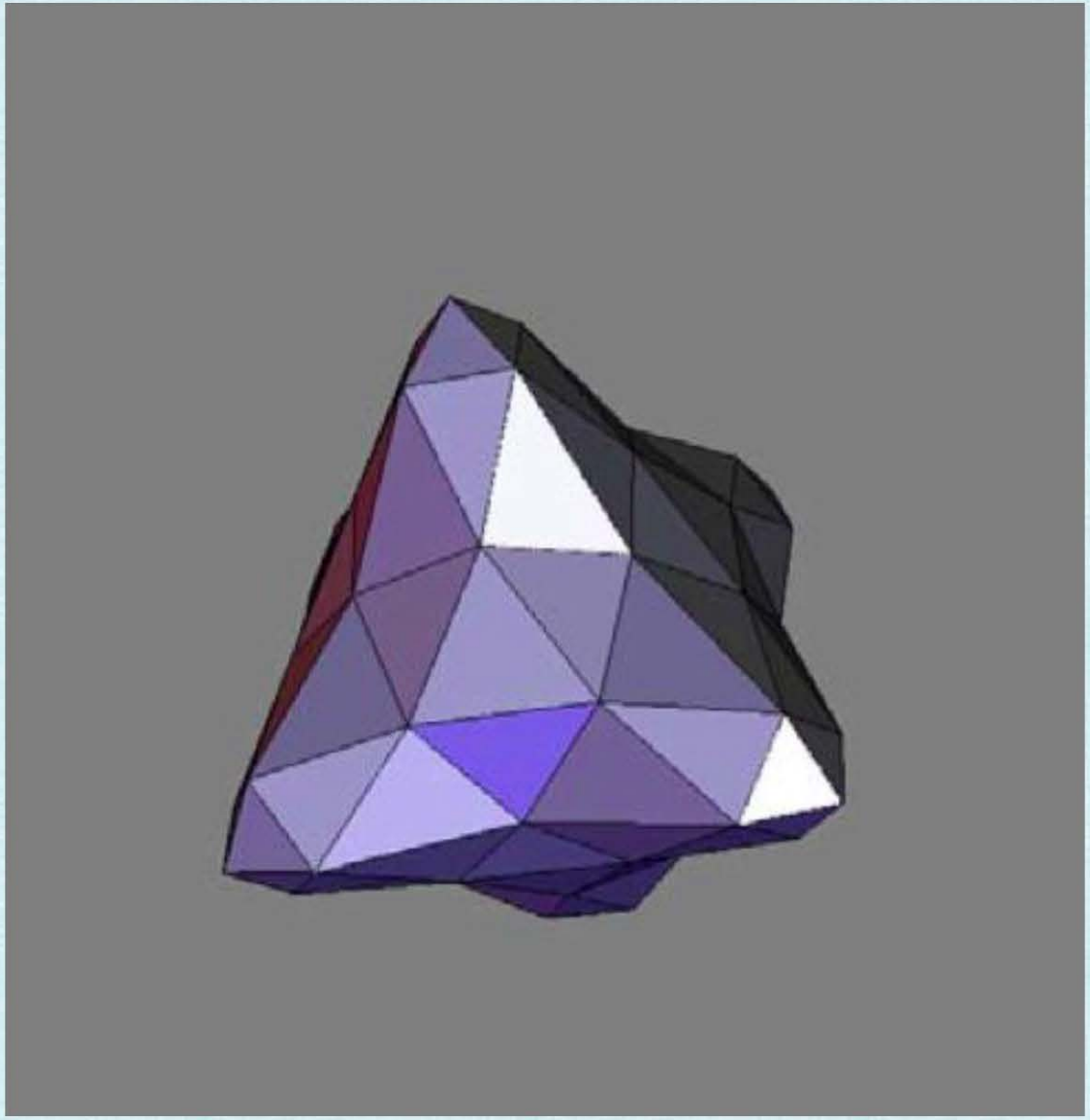
Original (Extraordinary) Vertex and Stencil



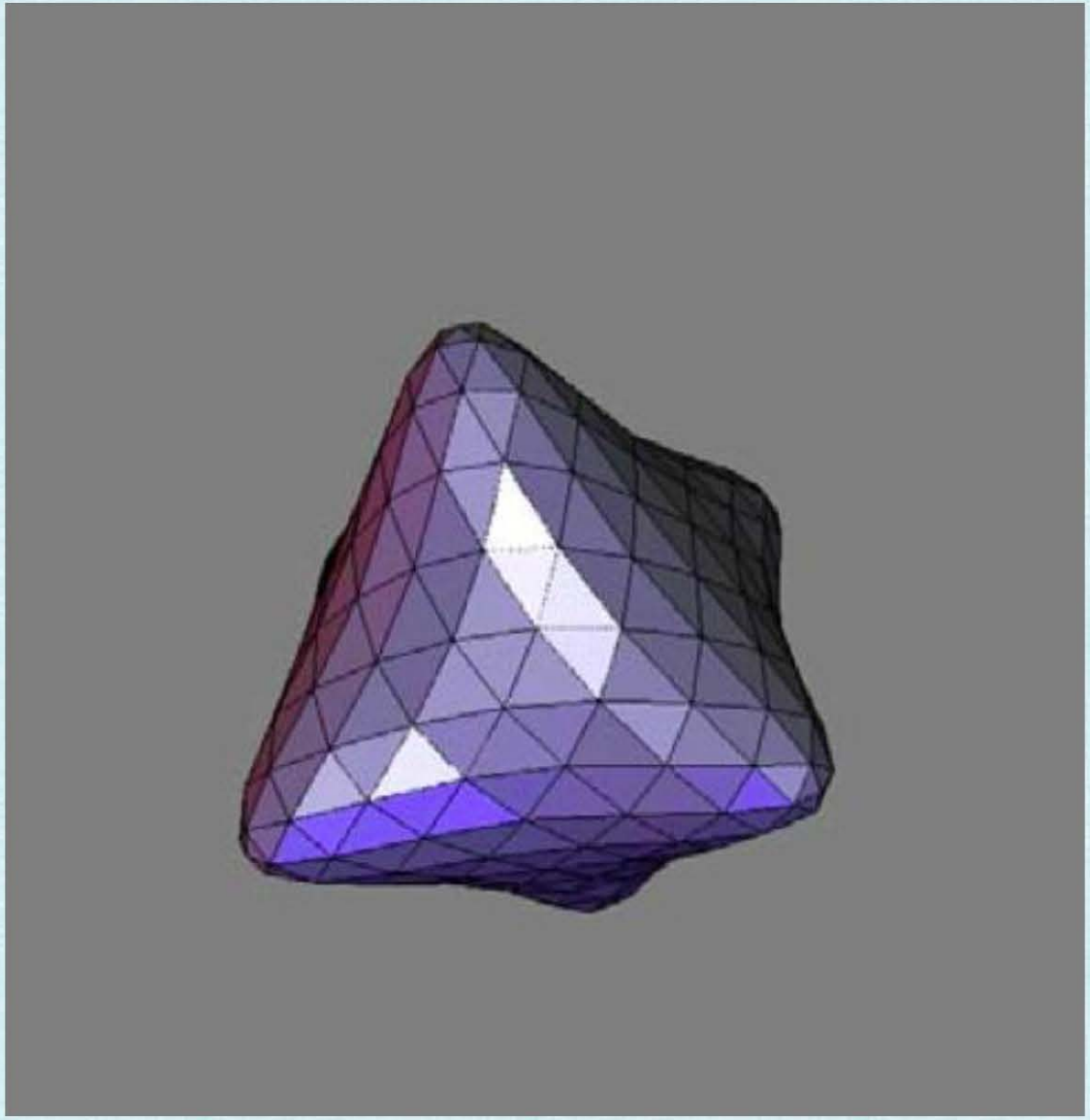
Move Original (Extraordinary) Vertex



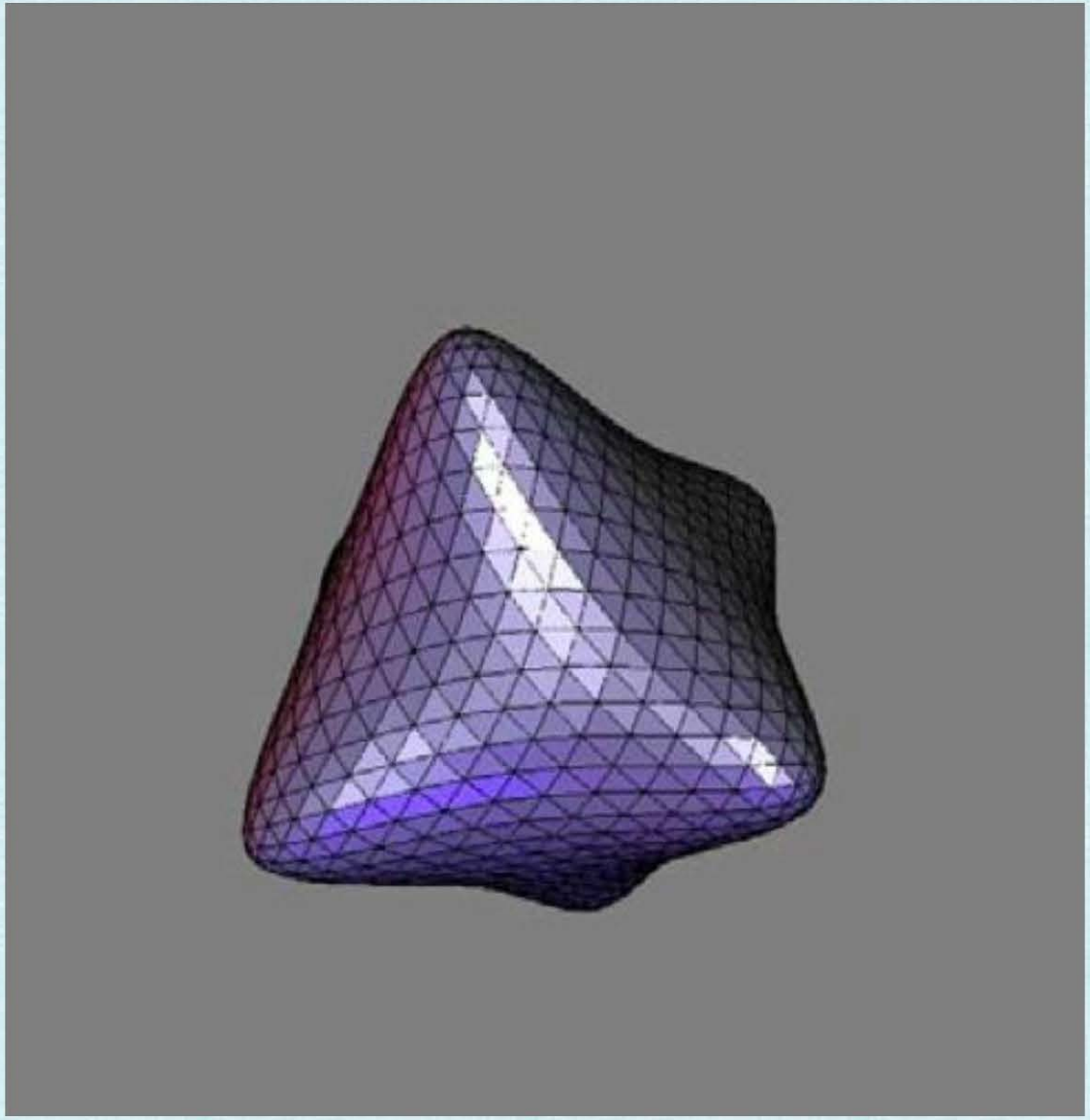
Subdivided Surface



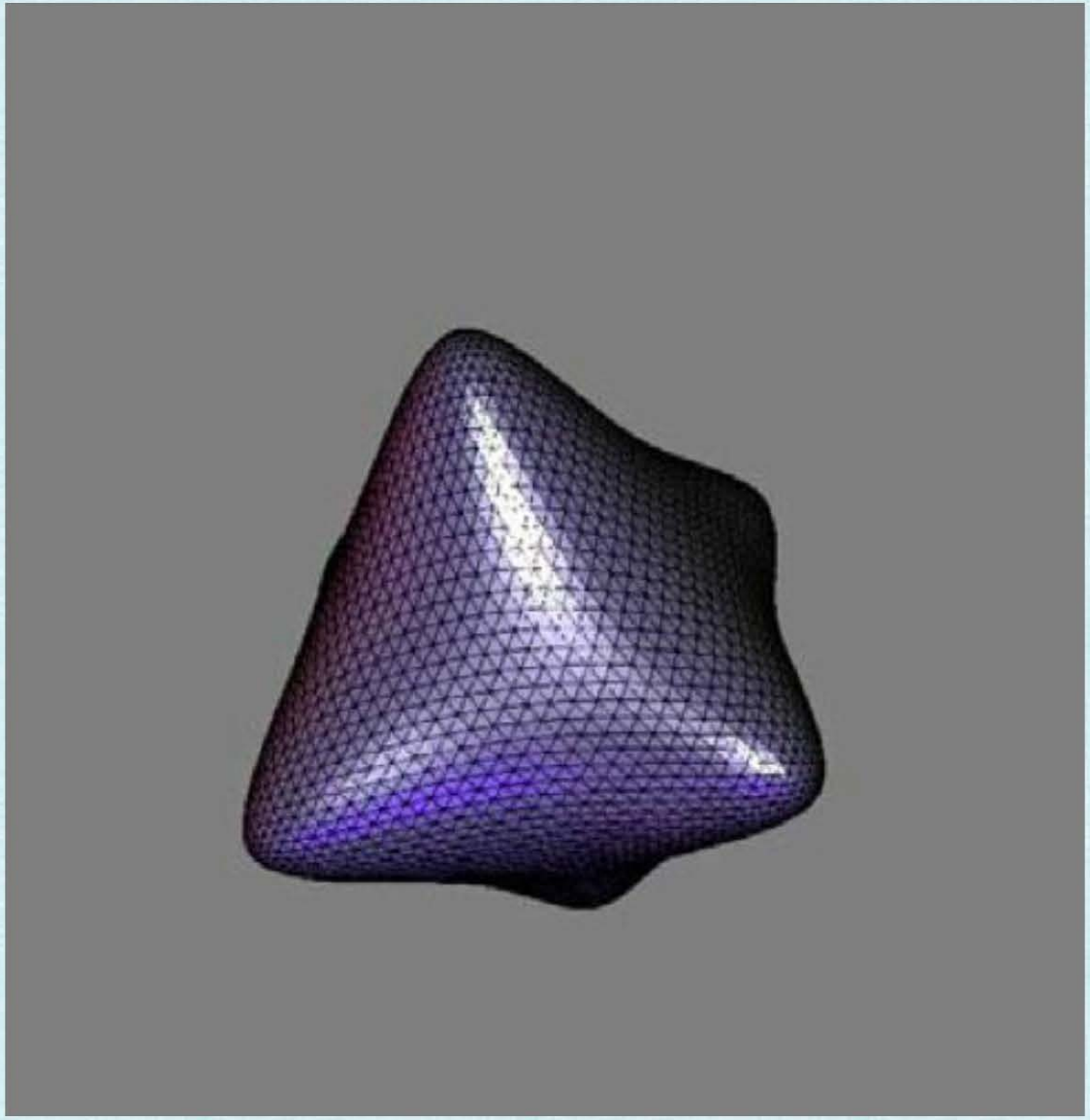
Subdivide Again



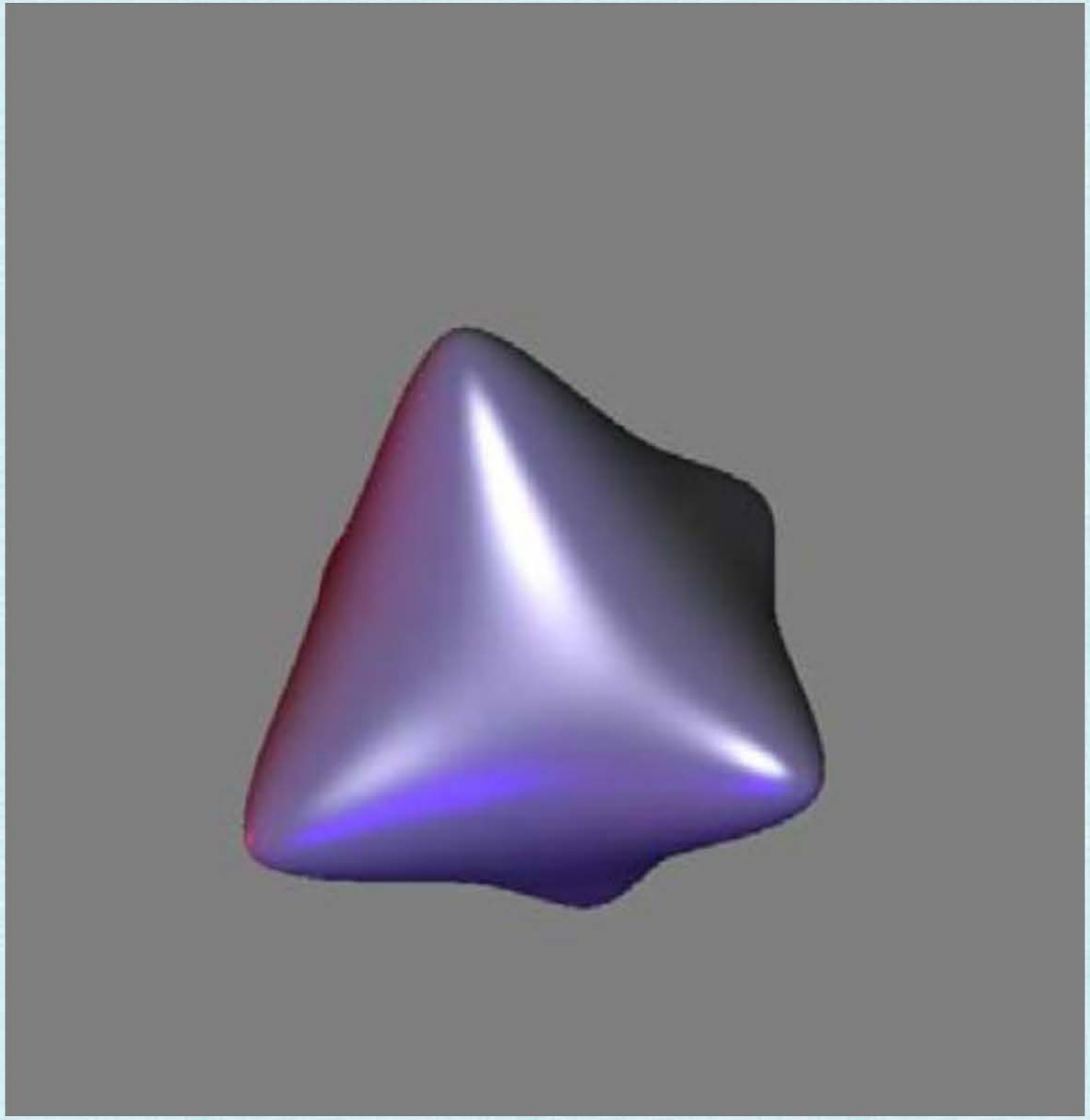
And Again



And Again



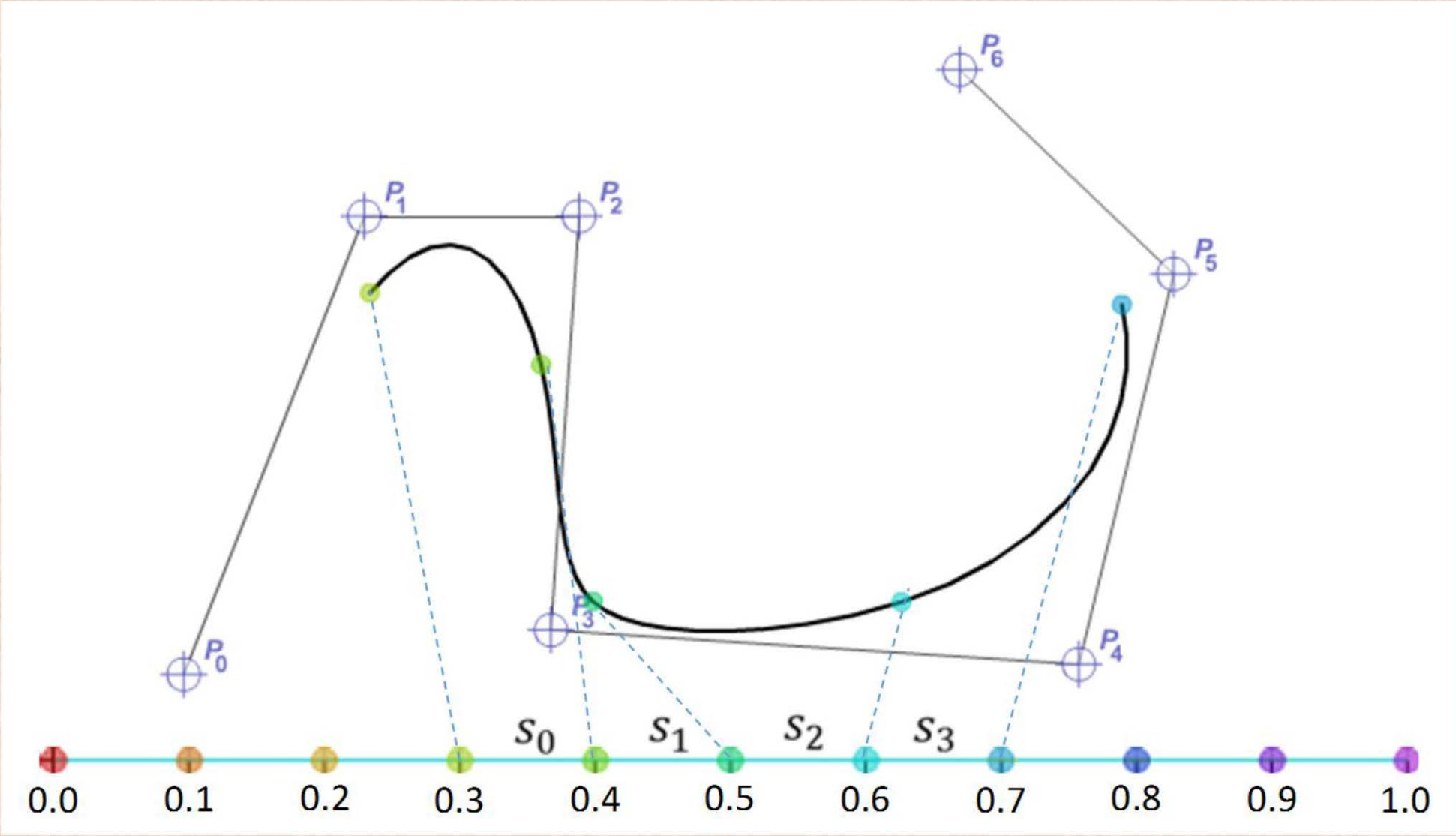
Final (Smooth) Limit Surface



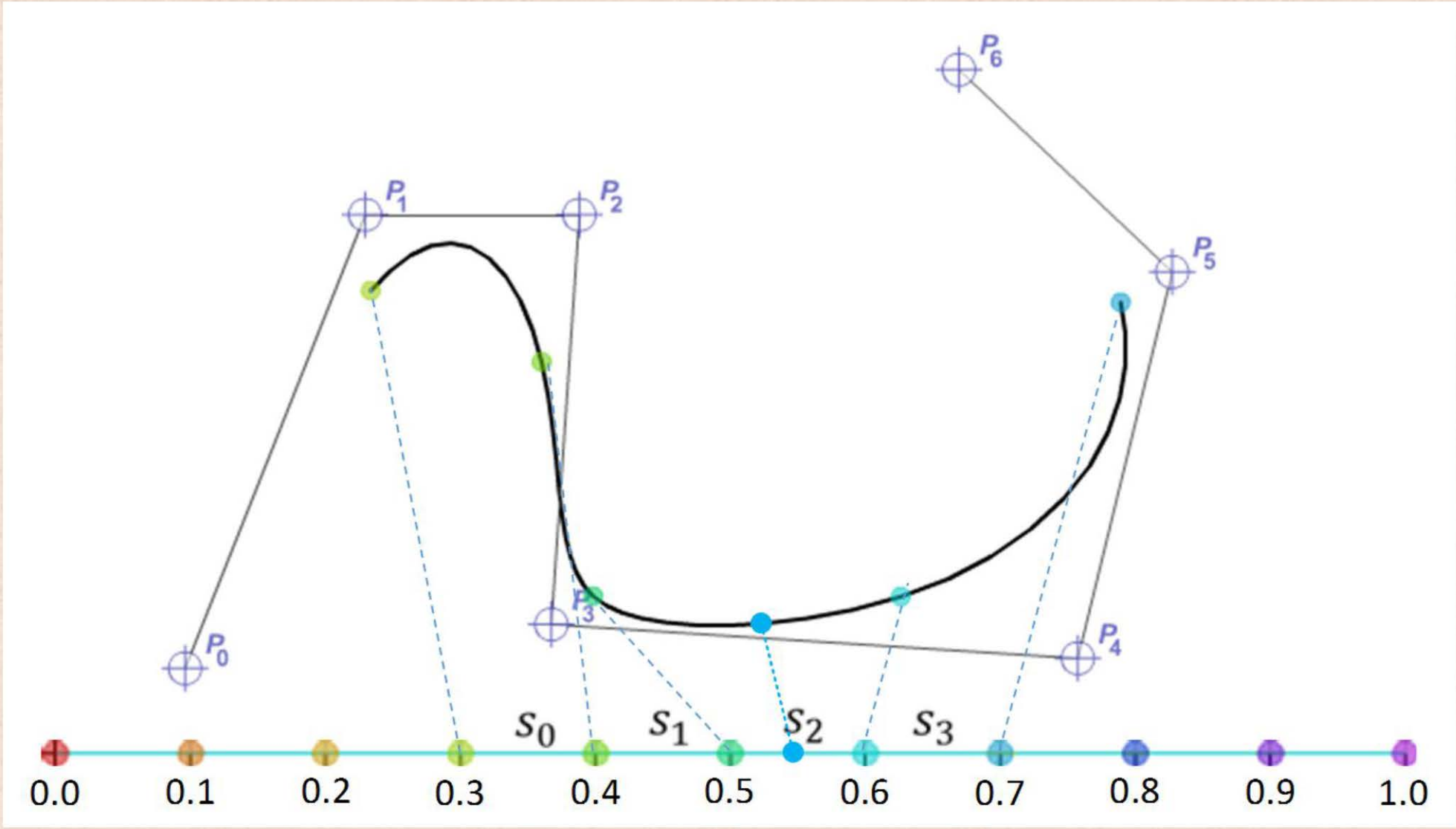
Subdividing Spline Curves

- First, insert new knots (and control points) without changing the shape of the curve
- Then, move around control points to modify the shape of the curve

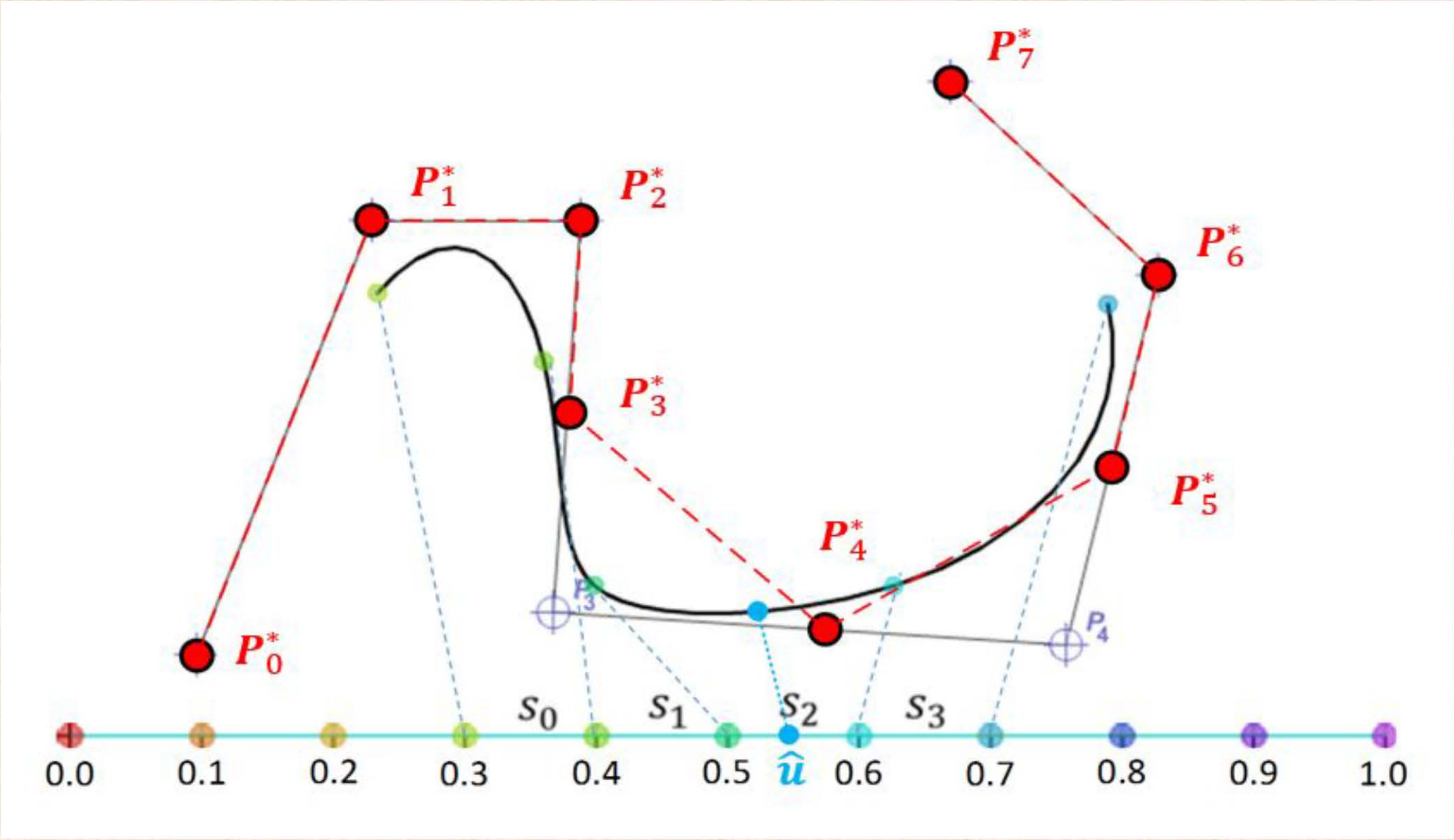
Original Curve



Insert a Knot

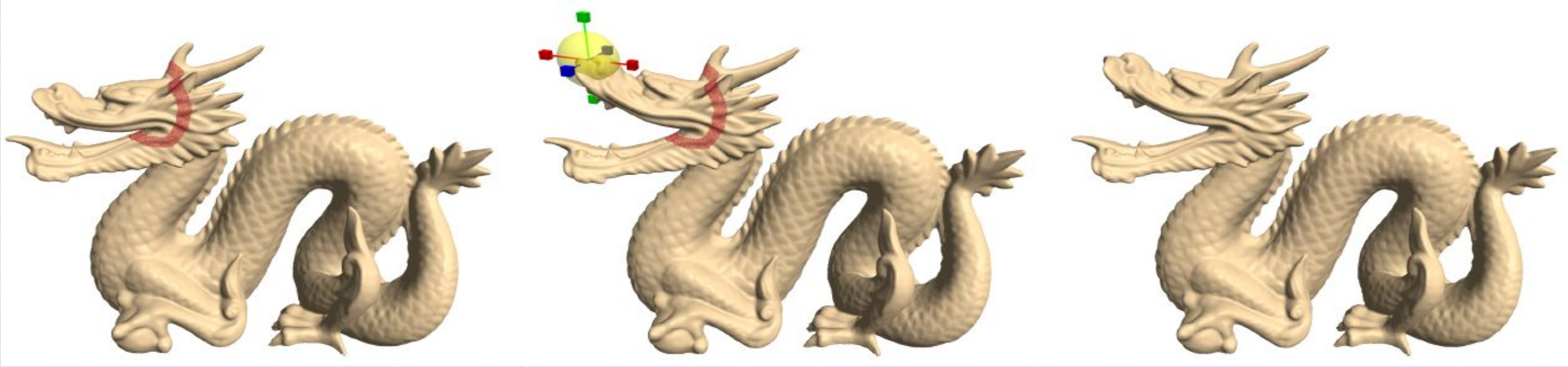


Add/Adjust Control Points



Laplacian Mesh Editing

- Spline mesh editing (e.g. with NURBS) uses control points
- Laplacian mesh editing allows one to directly move mesh points
  - Select a subset of the mesh to move (and target positions)
  - Select a region of interest containing the subset (so the deformation is smooth)



Differential Coordinates

- The differential coordinate is the difference between a vertex's position x_i and the average position of its neighbors:

$$d_i = L(x_i) = x_i - \frac{1}{n_i}\sum_{j \in N_i} x_j$$

- Let X be the vector of mesh vertices (one entry for each vertex)
- Let D be the vector of differential coordinates (one entry for each vertex)
- Since each d_i is linear in x_i and all x_j, can write $D = MX$ where M is a constant coefficient (sparse) matrix

- Differential coordinates approximate the local shape:
  - The direction of d_i approximates the direction of the normal vector
  - The magnitude of d_i approximates the mean curvature

Laplacian Mesh Editing

- Select mesh points x_c and target positions p_c
- Select a region of interest Ω containing (at least) all the x_c

- Solve (the sparse linear system) $M\hat{X} = D$ for all $\hat{x}_i \in \Omega$, with soft constraints $\hat{x}_c = p_c$
- $M\hat{X} = D$ attempts to preserve the shape, while $\hat{x}_c = p_c$ aims for the deformation

- Minimize $E(\hat{X}) = \sum_{i\in\Omega}\|L(\hat{x}_i) - d_i\|_2^2 + \sum_c\|\hat{x}_c - p_c\|_2^2$

- Better results are obtained by approximating part of the deformation via a transform $T_i(\hat{X})$, computed for each vertex via: $\min_{T_i}\left(\|T_i x_i - \hat{x}_i\|_2^2 + \sum_{j\in N_i}\left\|T_i x_j - \hat{x}_j\right\|_2^2\right)$
- Then $E(\hat{X}) = \sum_{i\in\Omega}\left\|L(\hat{x}_i) - T_i(\hat{X})d_i\right\|_2^2 + \sum_c\|\hat{x}_c - p_c\|_2^2$

Blender, Clip Studio Paint, TurboSquid

- Pear is downloaded from TurboSquid, all other geometry created from scratch
- Cat character is created through modeling from geometric primitives and sculpting
- Cup is broken through cell fracture and rigid body collisions
- Liquid simulations to model fluid inside cup
- Leaves/trees created through particle system
- Textures are created/modified through Clip Studio Paint or taken from online

- Set train at different positions on separate keyframes to create motion blur.
- Used loop cuts to make fine-grained adjustments to the geometry of the mesh.
- Applied noise textures, bump map, and color ramp to create plastic material.
- Marked edges as sharp and used Auto Smooth to ensure that intentional creases are not smoothed out by Shade Smooth or Subdivision Surface Modifier.
- Downloaded the "UV Squares" add-on that helps streamline UV unwrapping by converting a UV map to a square grid.

Edwin Pua, CS148 2021

- Tree created from scratch using a skin modifier on a skeleton of vertices and edges forming the shape of a tree, and subdivided later to apply weight painting for the particle system.
- Bridge, stone, ground, and mountains created from scratch using extrusion and various low-poly modeling techniques.
- Lotus flower, lily pads, cherry blossoms downloaded from online sources.
- Generated cherry blossoms onto the tree randomly using weight painting and hair particle system onto the tree.

Amy Flo, Vivian Xiao

Group Members: Nicholas Negrete (individual project)

- Scaled a sphere to model the basic body shape, then applied decimate to get the “crystal” look
- Applied spin to a cylinder to model the rounded swan neck
- Used the bevel function to round the wood surface edges
- Used the “inset faces” method on the face of a cube and moved the face back to create the case
- Used boolean modifiers to make the different parts look like one piece of glass

Crystal Explorer by Kent Vainio (2020)

METHODS:

- Cloth and rigid body simulations for the cape and treasure chest items
- Blender’s hair particle system to create the fluffy fur, cape fur and antenna
- Multiple procedural textures (mostly Voronoi and Musgrave) along with various shaders (glass, emission, etc.) to create a vibrant scene
- Moogle (the fluffy creature - a character from the Final Fantasy video game series) was modeled and sculpted from scratch using reference images

Used loop cut, subdivision, solidify, bezel tool, bisect tool and displacement to create the whiskey, moon rock award, photo frames, etc
Used cloth simulation to model the newspaper. Took pictures of the book covers and created UV textures with a photo editing tool for books.
Combined LightPath node, Transparent, Glossy and Emission BSDF to create a reflective window and its imperfect surface
Downloaded the rocket ship, lunar lander and NASA medal in SolidWorks assembly format and converted with AutoDesk Inventor

Yanjia Li, Lingjie Kong, CS148 2020

Open Shading Language (OSL) Node; Displacement map; Dynamic Sky; Particle System; Motion Blur

- Geometrical low-poly modeling from scratch for all the geometries except for the grass (downloaded from TurboSquid)
- OSL node for procedural stone road texture (idea from Erindale YouTube Channel)
- Random displacement mapping to generate water wrinkles (from a subdivided plane) and tree crown (from an icosphere)
- Particle system with wind force for drifting cherry blossom petals; motion blur applied while rendering
- Blender official add-on “Dynamic Sky” for the basic light source and background; additional lighting for the reflection on the water wrinkles

Yifan Wang, CS148 2020

Example Slides

- Extra Credit: 5 HW points (i.e. worth one full HW assignment) for contributing:
  - a slide with a nice image
  - a few bullet points below the image on how the geometry was modeled
  - names of both partners (if applicable)

- The CAs will assign anywhere from 0-5 HW points on a submission (guaranteed 5 points if we use it for future classes)
- Recall: there is no overflow past the 50 percentile HW cap

- Deadline: end of the quarter