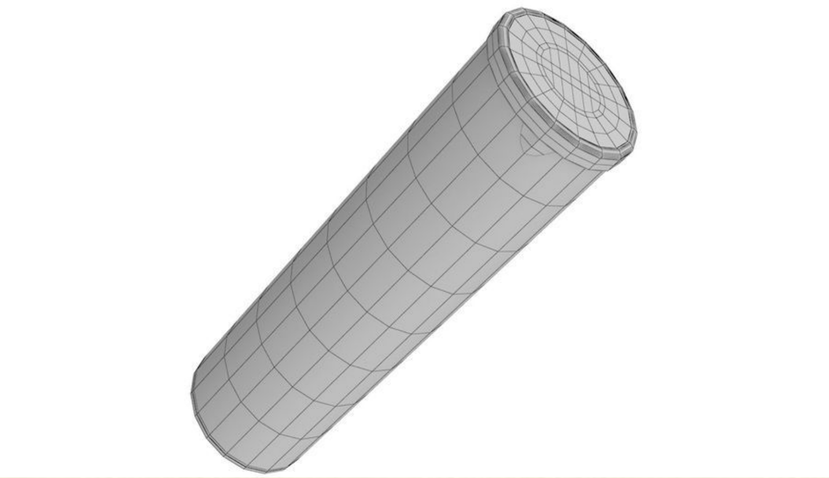
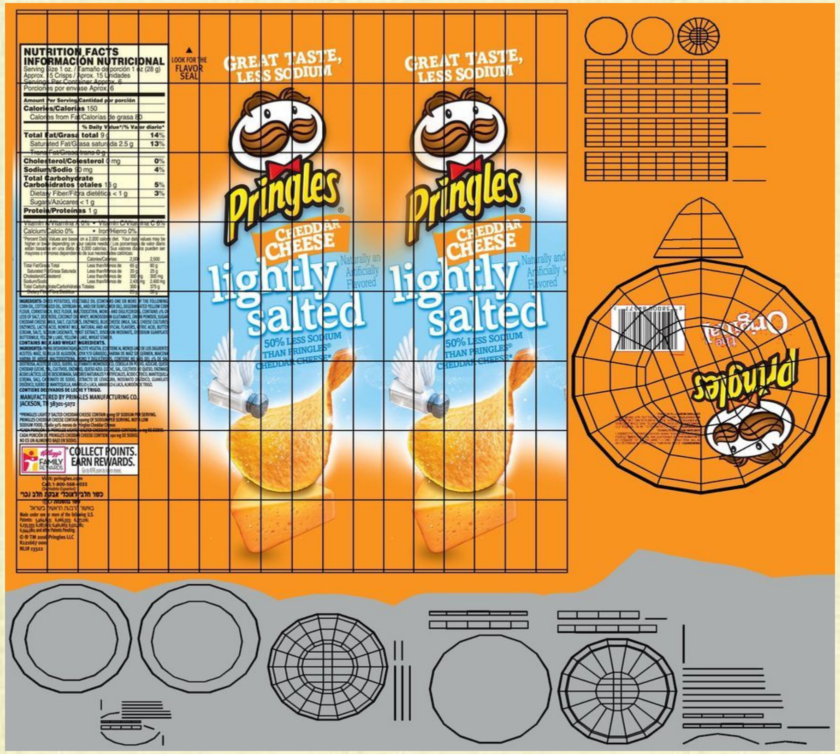
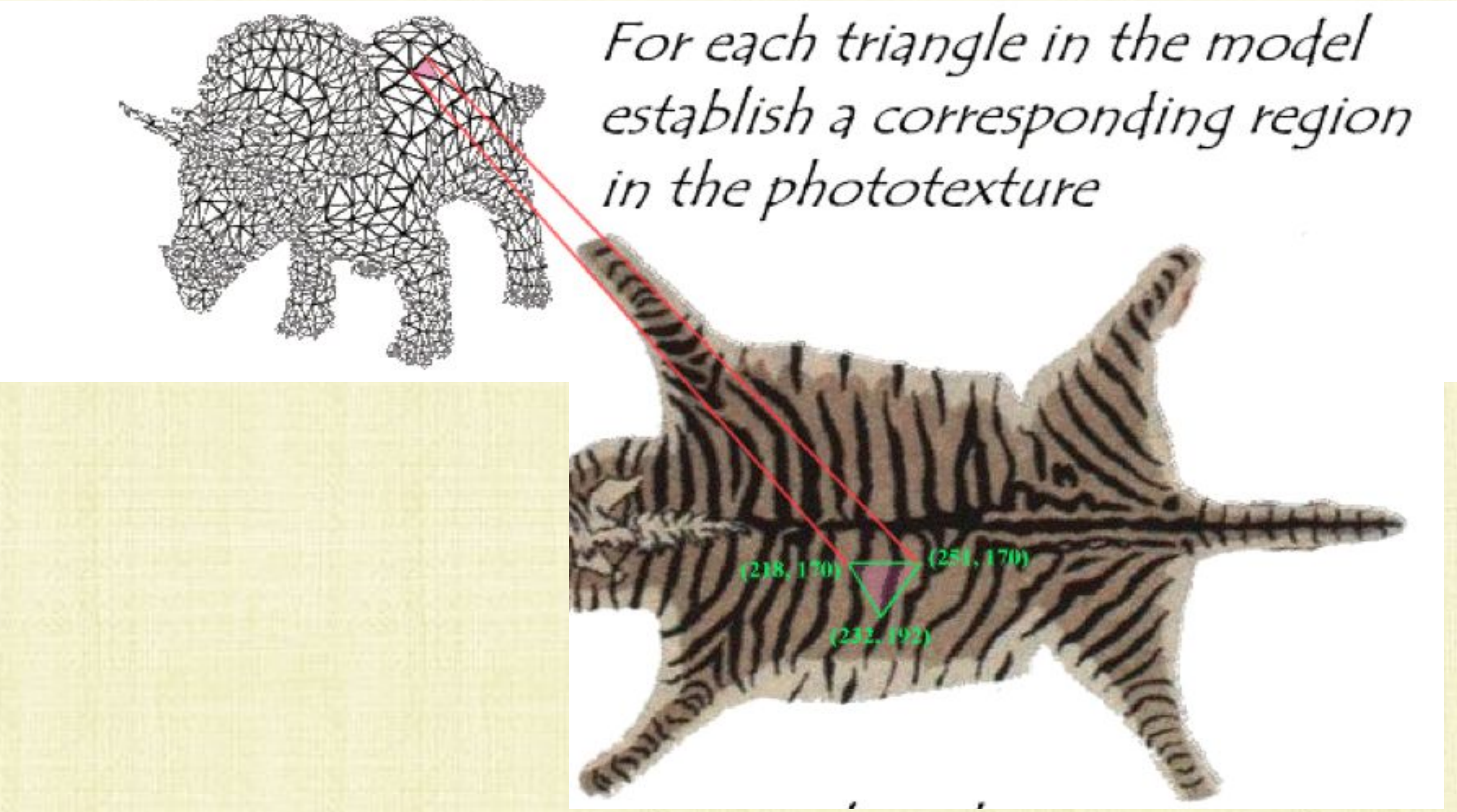
Texture Mapping



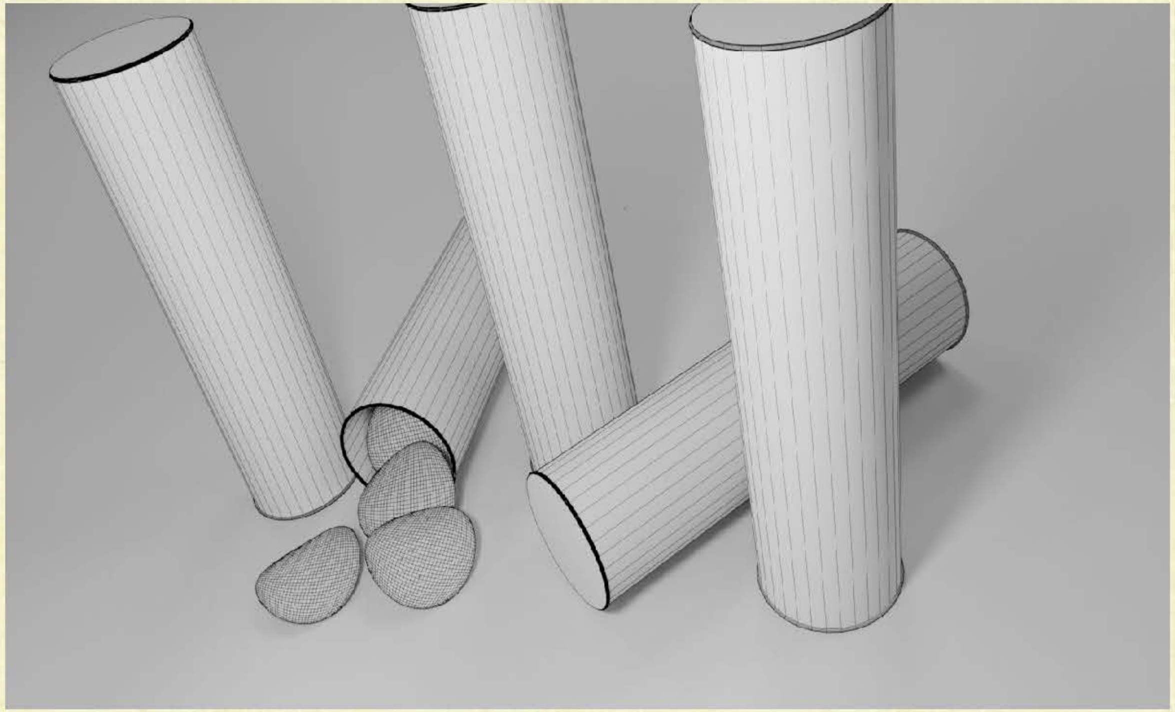


Texture Mapping

- Adds back the details lost by assuming that the BRDF doesn't change along an object's surface
- RGB reflectance is stored as an image (called a texture)
- The image colors are mapped to the object's surface (one triangle at a time)

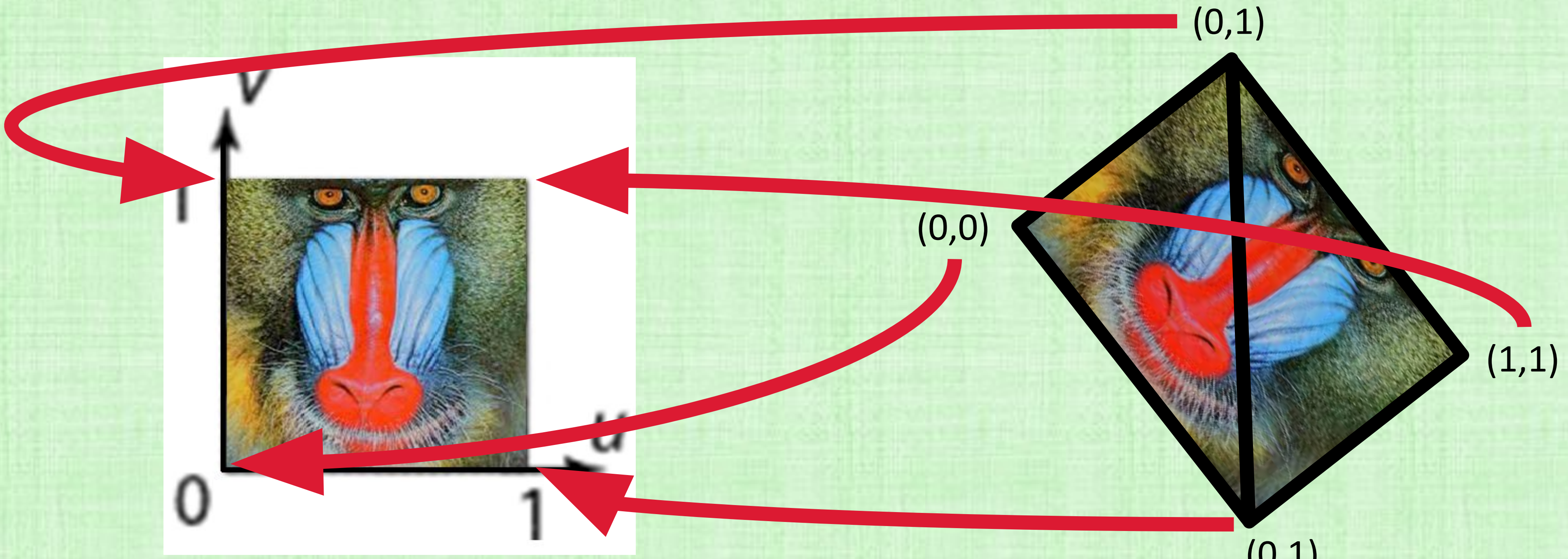


Similar to Wrapping Gifts



Texture Coordinates

- The texture is defined in a 2D coordinate system: (u, v)
- Texture mapping assigns a (u, v) coordinate to each triangle vertex
- Then, the texture is “stuck” onto the triangle:
  - Let p be a point inside the triangle, with barycentric weights $\alpha_0, \alpha_1, \alpha_2$
  - The <u>color</u> assigned to p is the <u>texture color</u> at $\big(u(p), v(p)\big) = \alpha_0(u_0, v_0) + \alpha_1(u_1, v_1) + \alpha_2(u_2, v_2)$
  - That is, texture coordinates are barycentrically interpolated



texture image

2 triangles

Recall: Screen Space vs. World Space Barycentric Weights

- Express the pixel p' terms of its screen space barycentric weights: $\alpha'_0, \alpha'_1, \alpha'_2$
- Express the point p that projects to p' in terms of unknown world space barycentric weights: $\alpha_o, \alpha_1, \alpha_2$
- Project p into screen space and set the result equal to p'
- Solve for $\alpha_o, \alpha_1, \alpha_2$ to obtain:

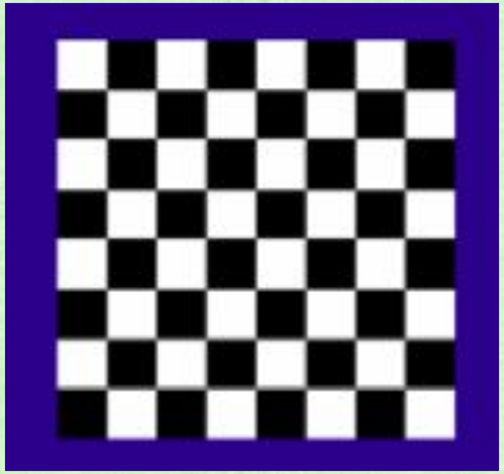
$$\alpha_0 = \frac{z_1 z_2 \alpha'_0}{z_1 z_2 \alpha'_0 + z_0 z_2 \alpha'_1 + z_0 z_1 \alpha'_2}$$

$$\alpha_1 = \frac{z_0 z_2 \alpha'_1}{z_1 z_2 \alpha'_0 + z_0 z_2 \alpha'_1 + z_0 z_1 \alpha'_2}$$

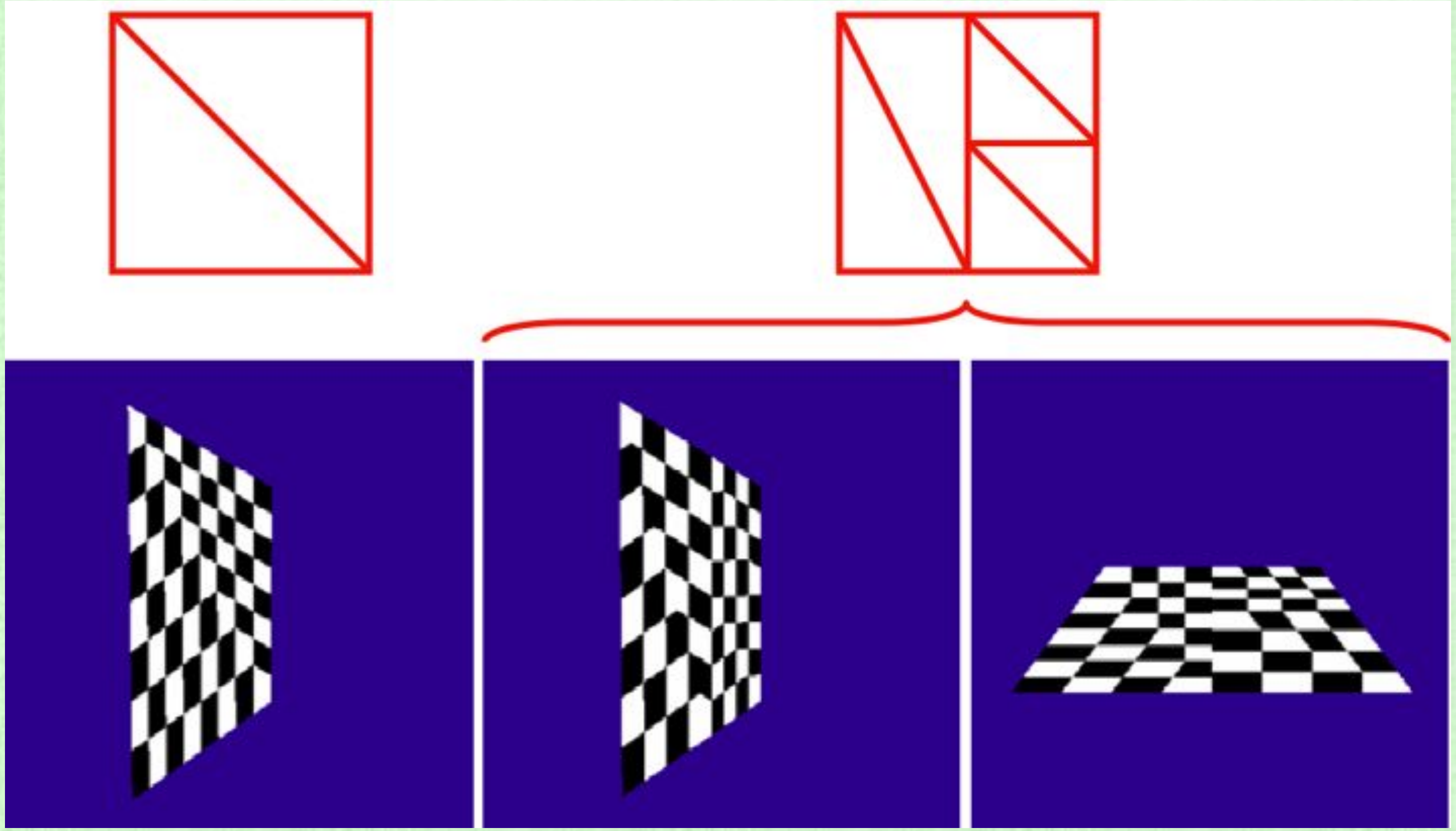
$$\alpha_2 = \frac{z_0 z_1 \alpha'_2}{z_1 z_2 \alpha'_0 + z_0 z_2 \alpha'_1 + z_0 z_1 \alpha'_2}$$

Screen Space vs. World Space Barycentric Weights

- Perspective transformation nonlinearly changes a triangle's shape
- Interpolating texture coordinates in screen space results in texture distortion

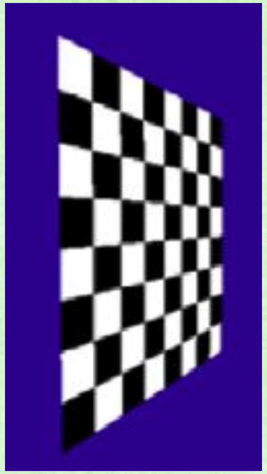


texture



screen space

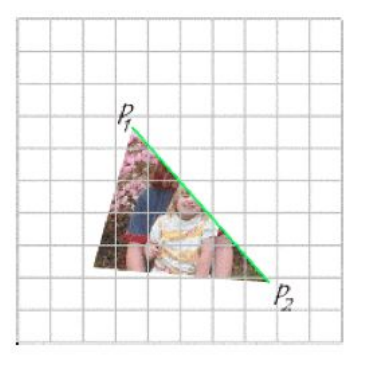
mesh refinement helps (less z variance per triangle)

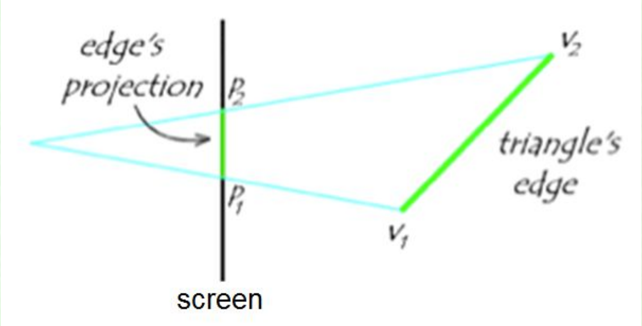


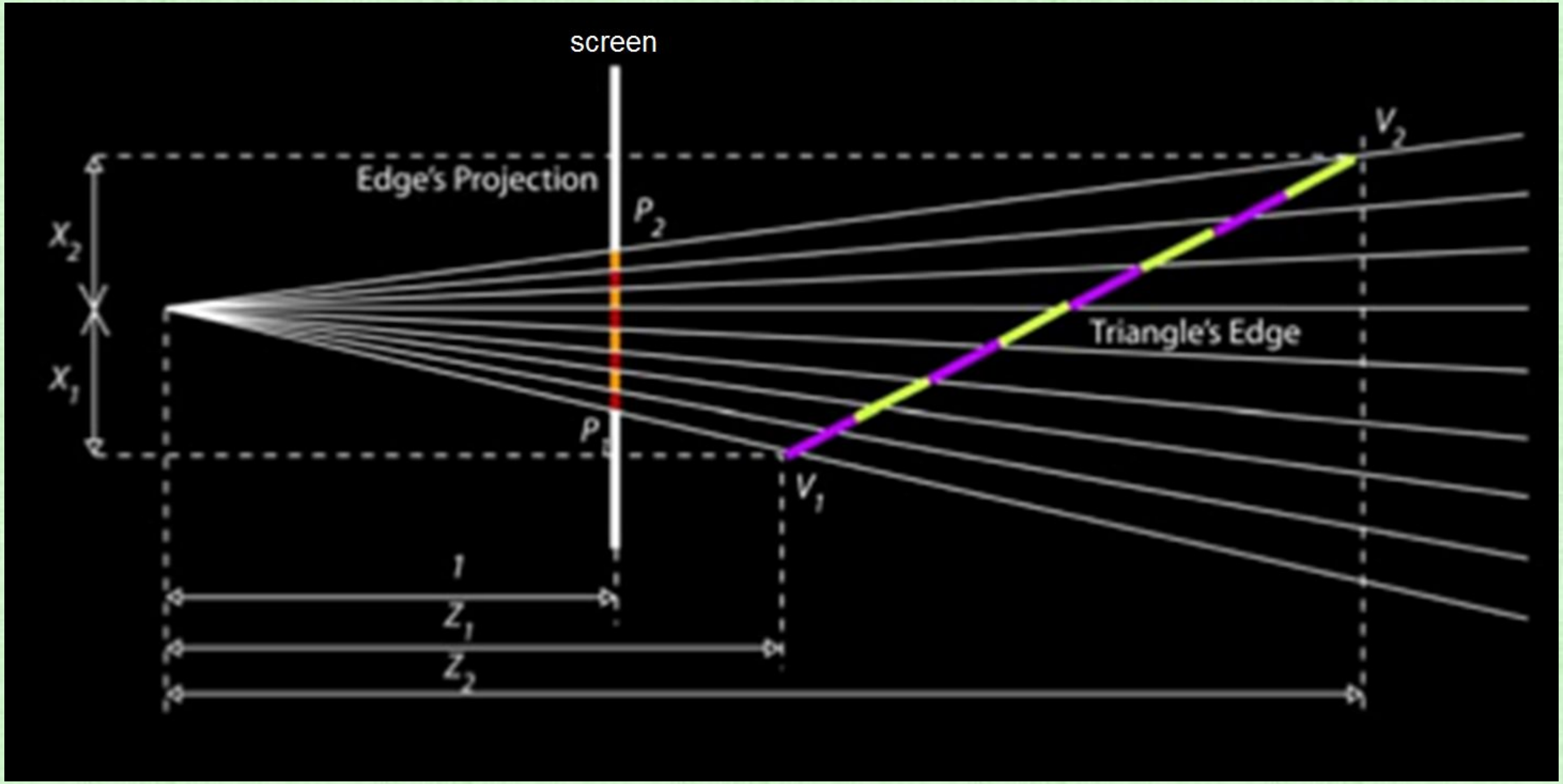
world space

Texture Distortion

- Consider one triangle edge
- Uniform increments along the edge in screen space world do not correspond to uniform increments in world space



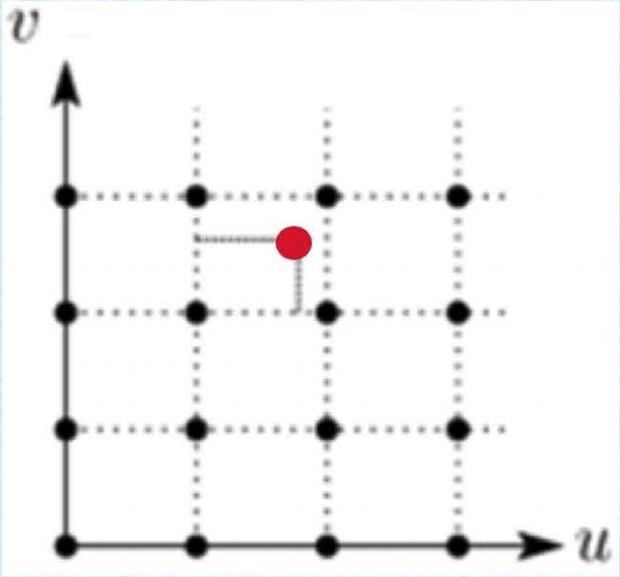




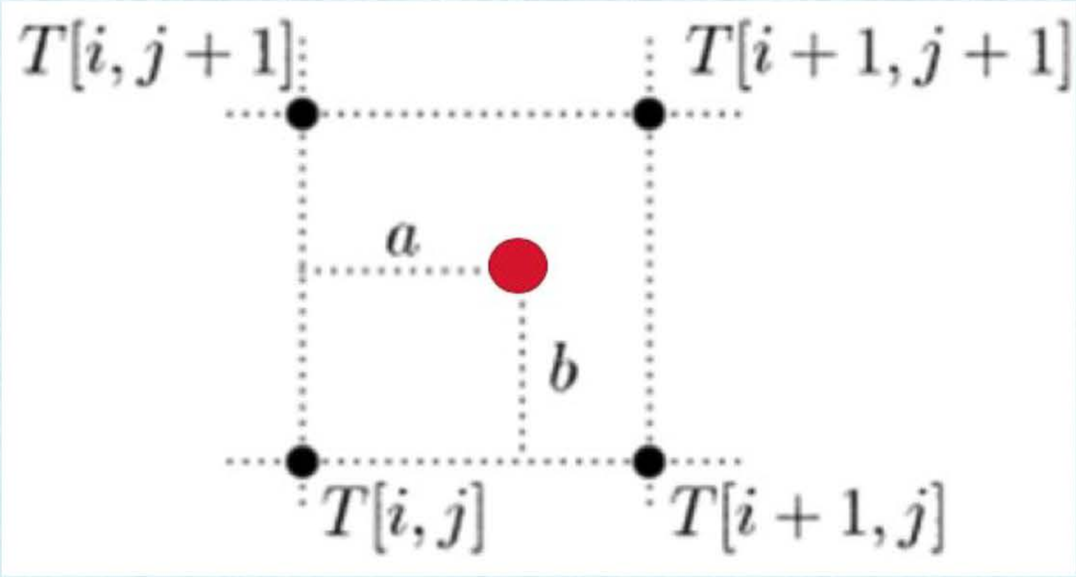
Interpolating from the Texture Image

- $(u(p), v(p))$ is surrounded by 4 pixels in the texture image
- Use bilinear interpolation to interpolate values for: T = R, G, B, α, etc.
  - First, linearly interpolate in the u direction; then, in the v direction (or vice versa)

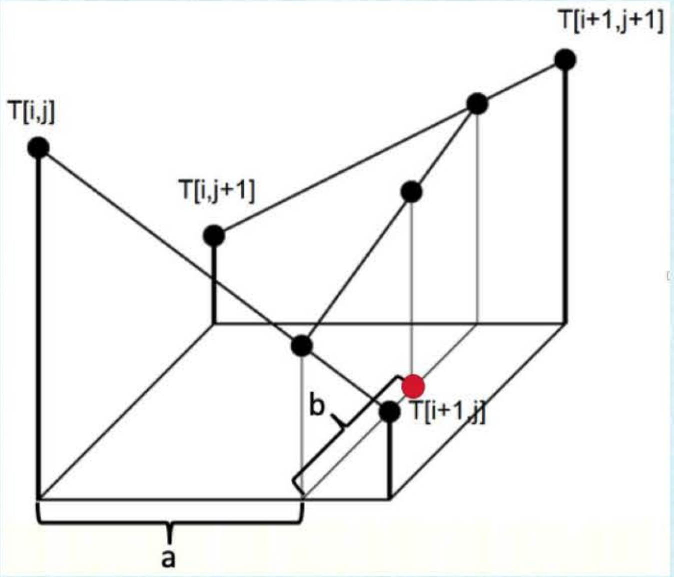
$$T(u,v) = (1-a)(1-b)T_{i,j} + a(1-b)T_{i+1,j} + (1-a)bT_{i,j+1} + abT_{i+1,j+1}$$



texture image



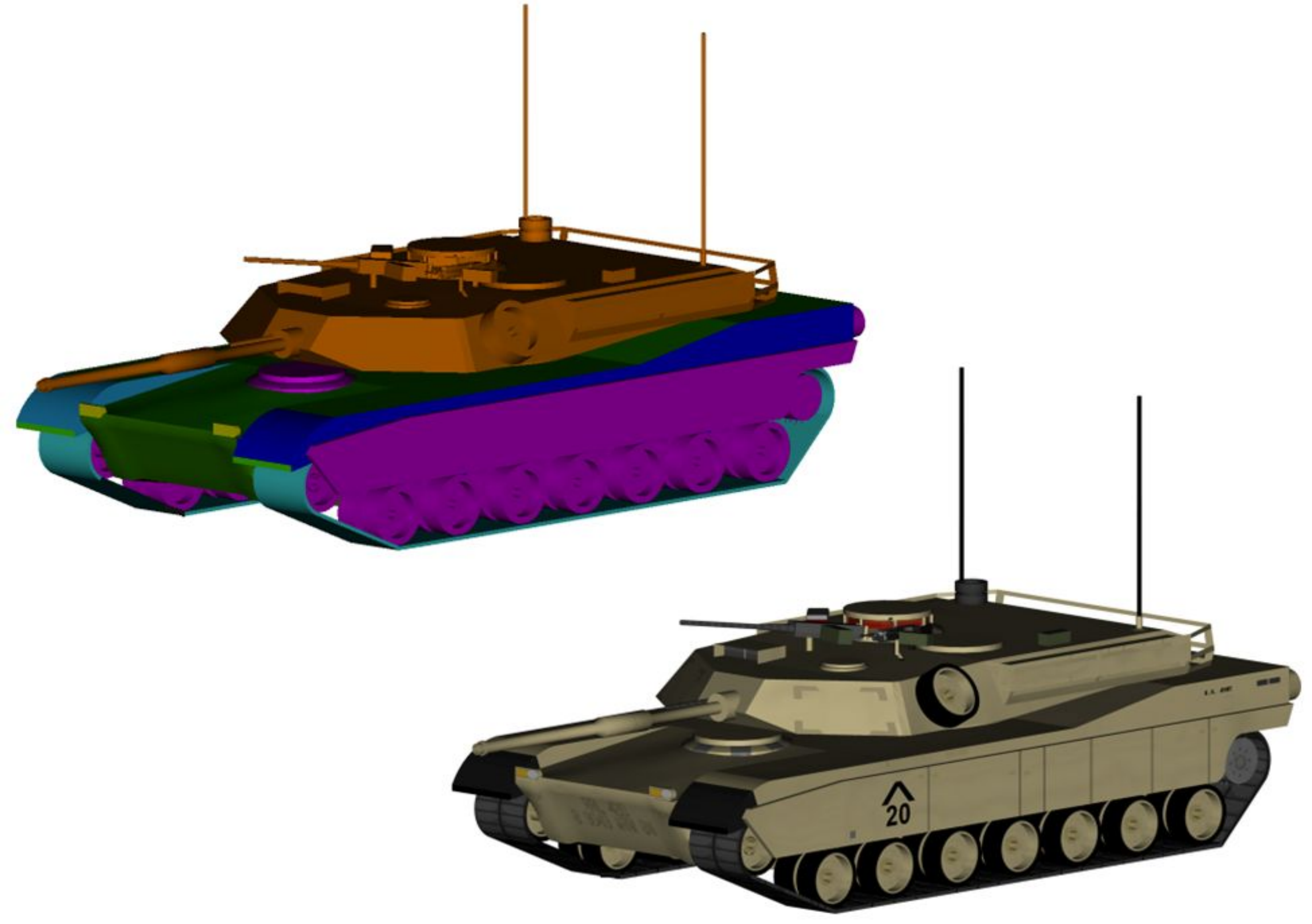
close-up view (of 4 surrounding pixels)



bilinear interpolation

Assigning Texture Coordinates

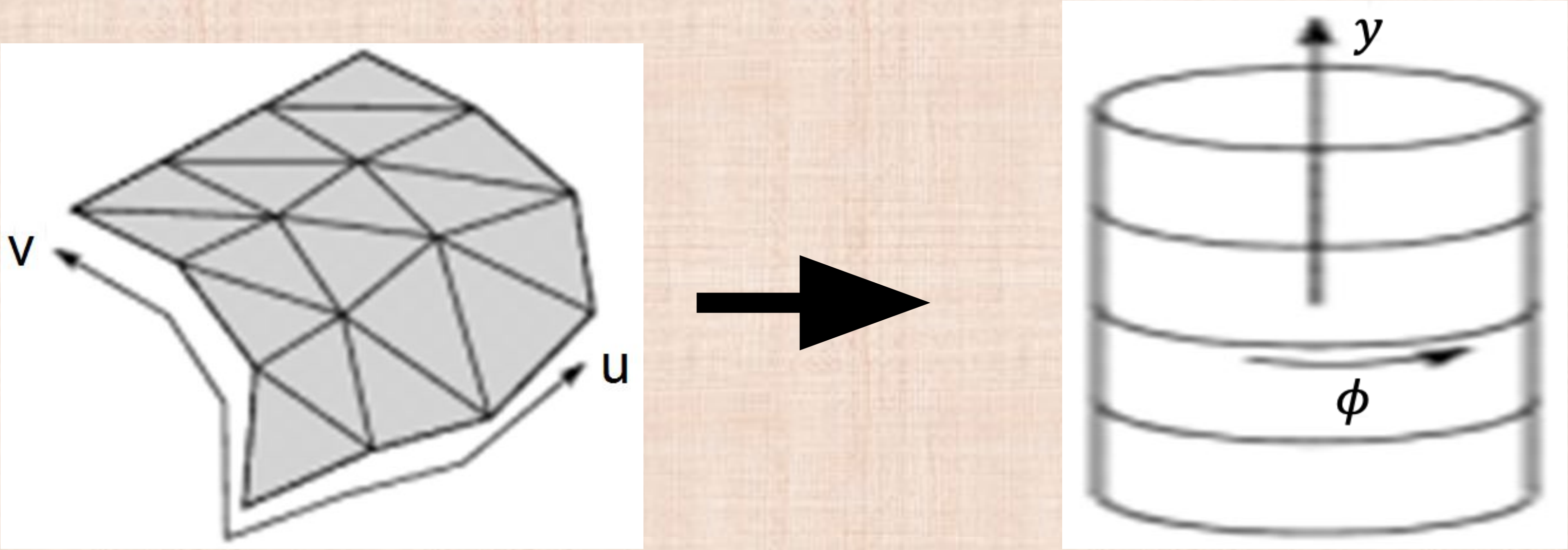
- Assign texture coordinates on complex objects one part/component at a time



Assigning Texture Coordinates

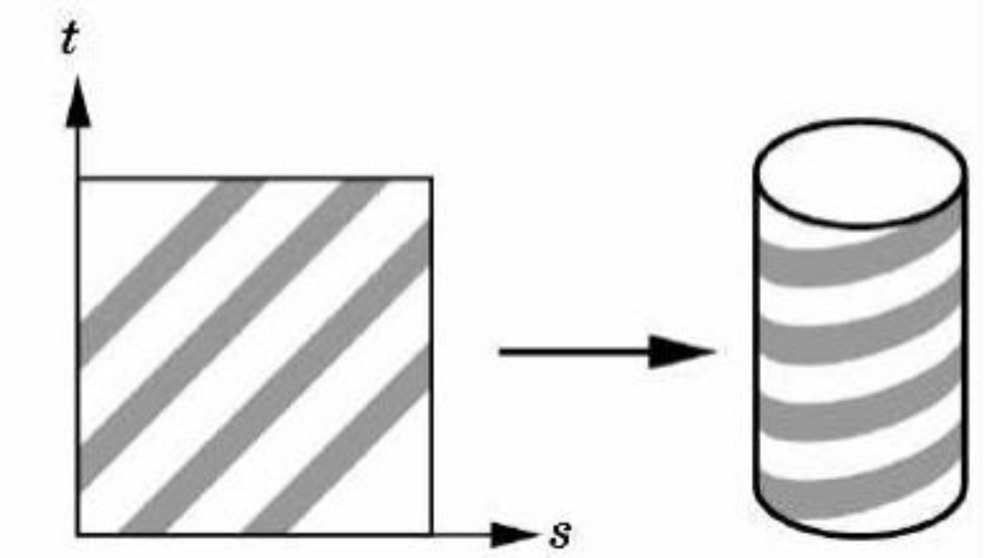
- For complex surfaces, manually assigning (u, v) one vertex at a time can be tedious

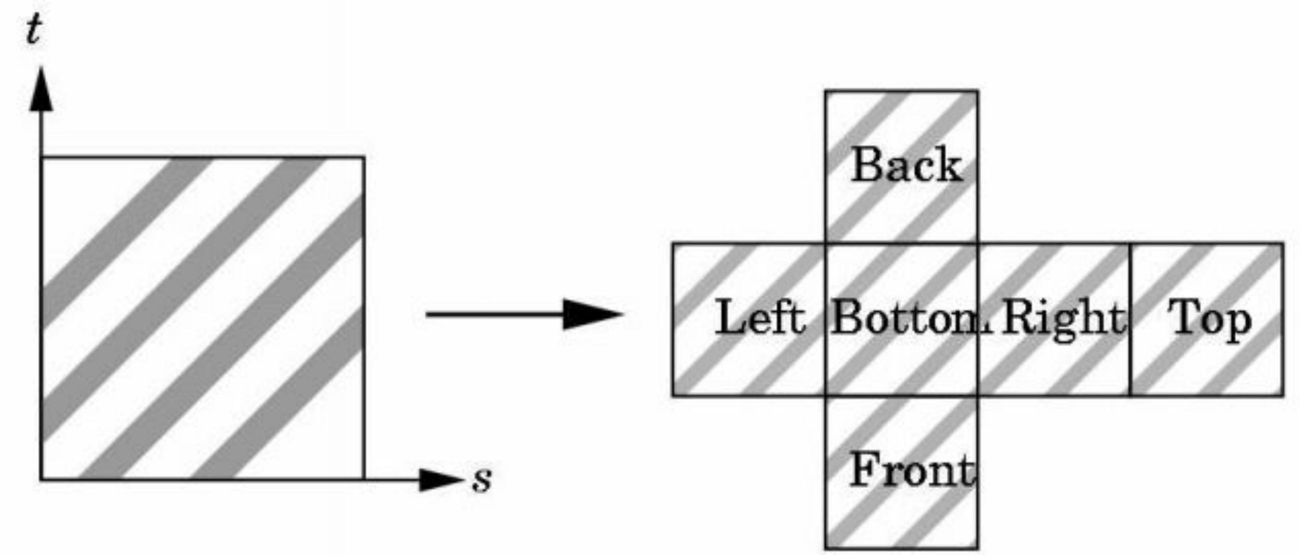
- For some surfaces, the (u, v) texture coordinates can be generated procedurally
- E.g. Cylinder (wrap the image around the outside)
  - map the $[0,1]$ values of the u coordinate to $[0,2\pi]$ for ϕ
  - map the $[0,1]$ values of the v coordinate to $[0, h]$ for y



Proxy Objects – Step 1

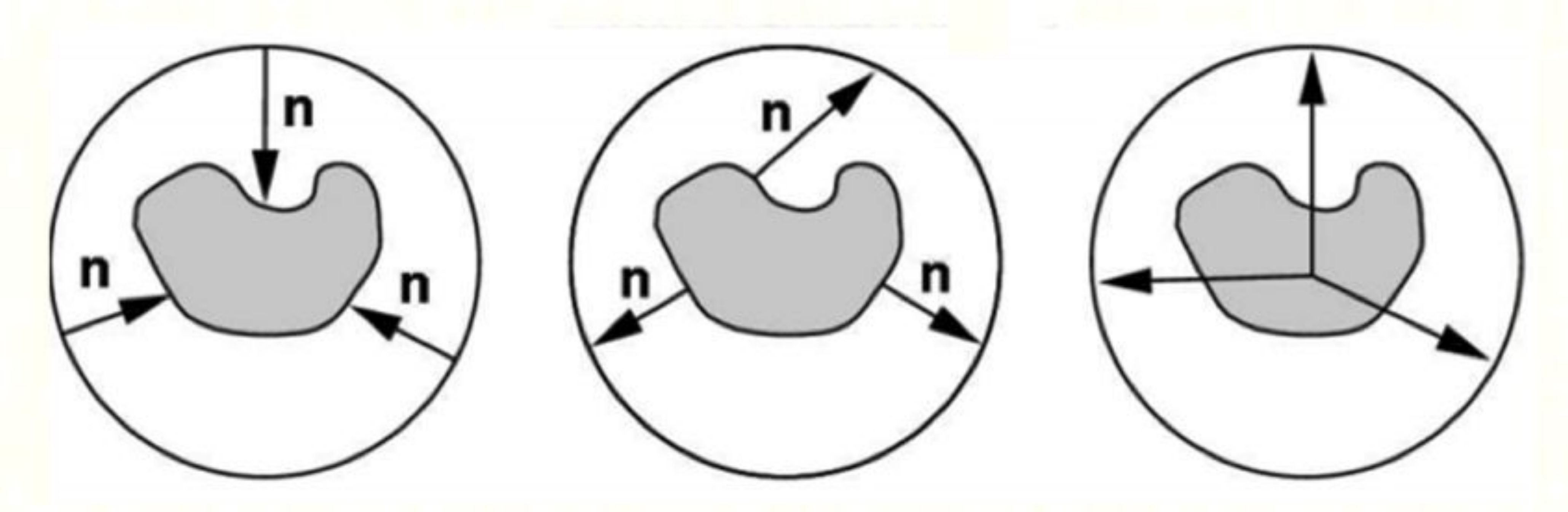
- Assign texture coordinates to intermediate/proxy objects:
  - Example: Cylinder
    - wrap texture coordinates around the outside of the cylinder
    - not the top or bottom (to avoid distorting the texture)
  - Example: Cube
    - unwrap cube, and map texture coordinates over the unwrapped cube
    - texture is seamless across some of the edges, but not necessarily other edges





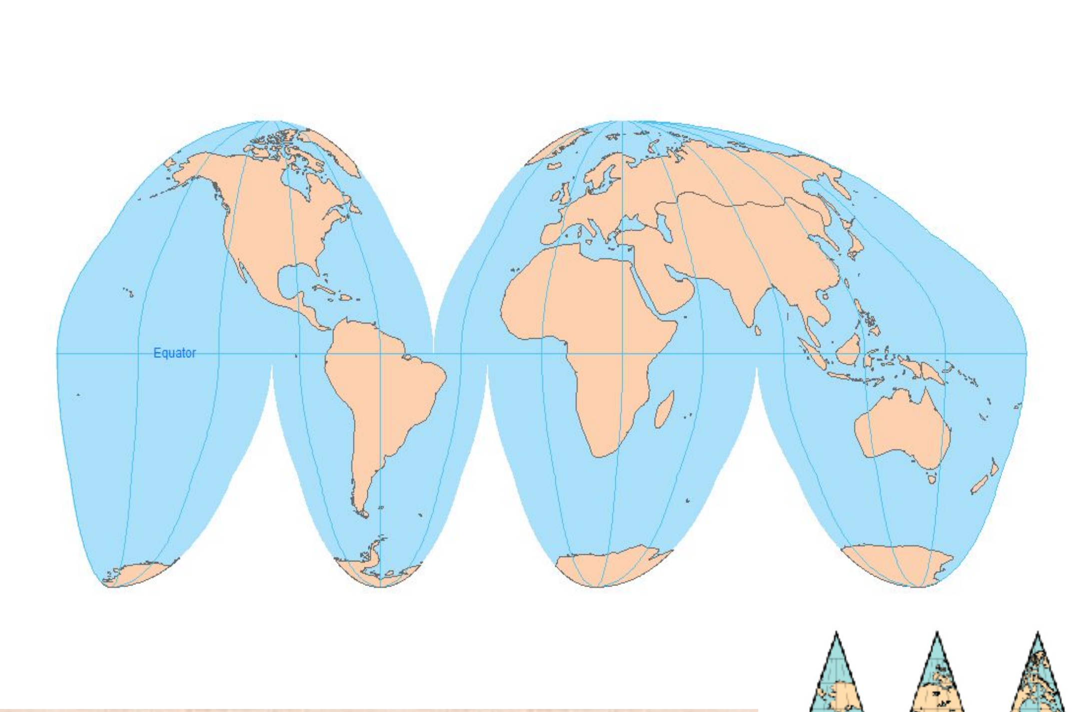
Proxy Objects – Step 2

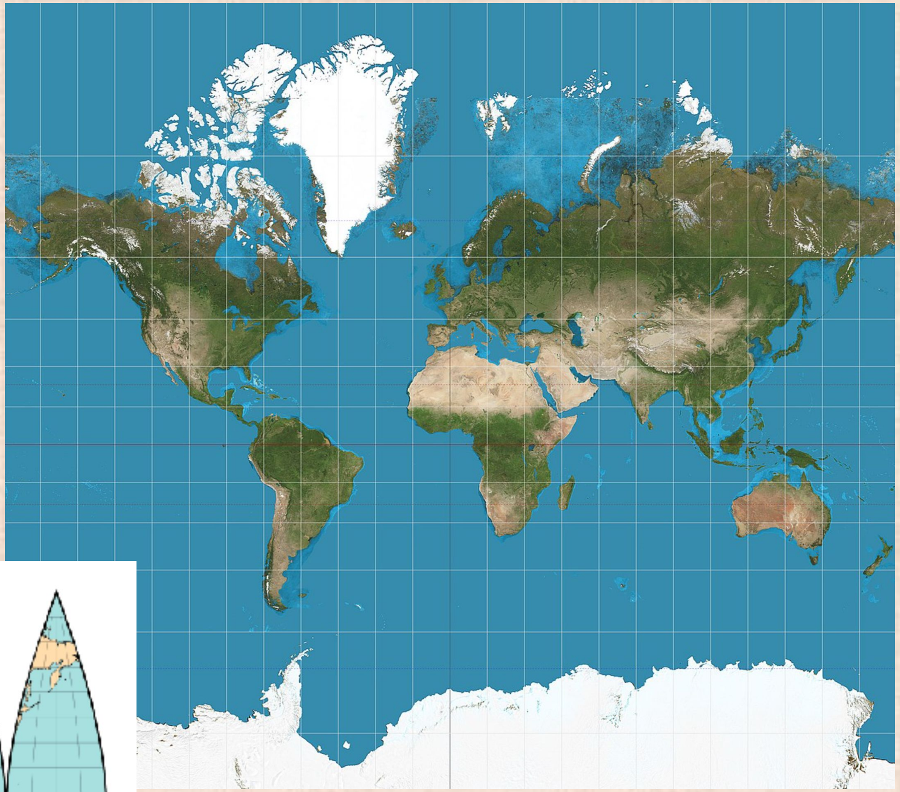
- Next, map the texture coordinates from the intermediate/proxy object to the final object
- Three ways of doing this:
  - Use the intermediate/proxy object's surface normal
  - Use the target object's surface normal
  - Use rays emanating from a "center"-point or "center"-line of the target object

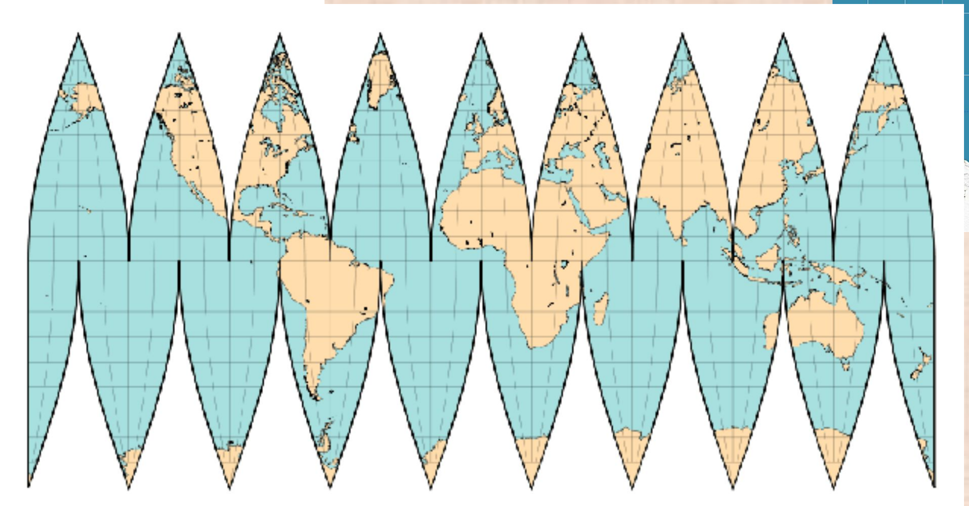


Distortion

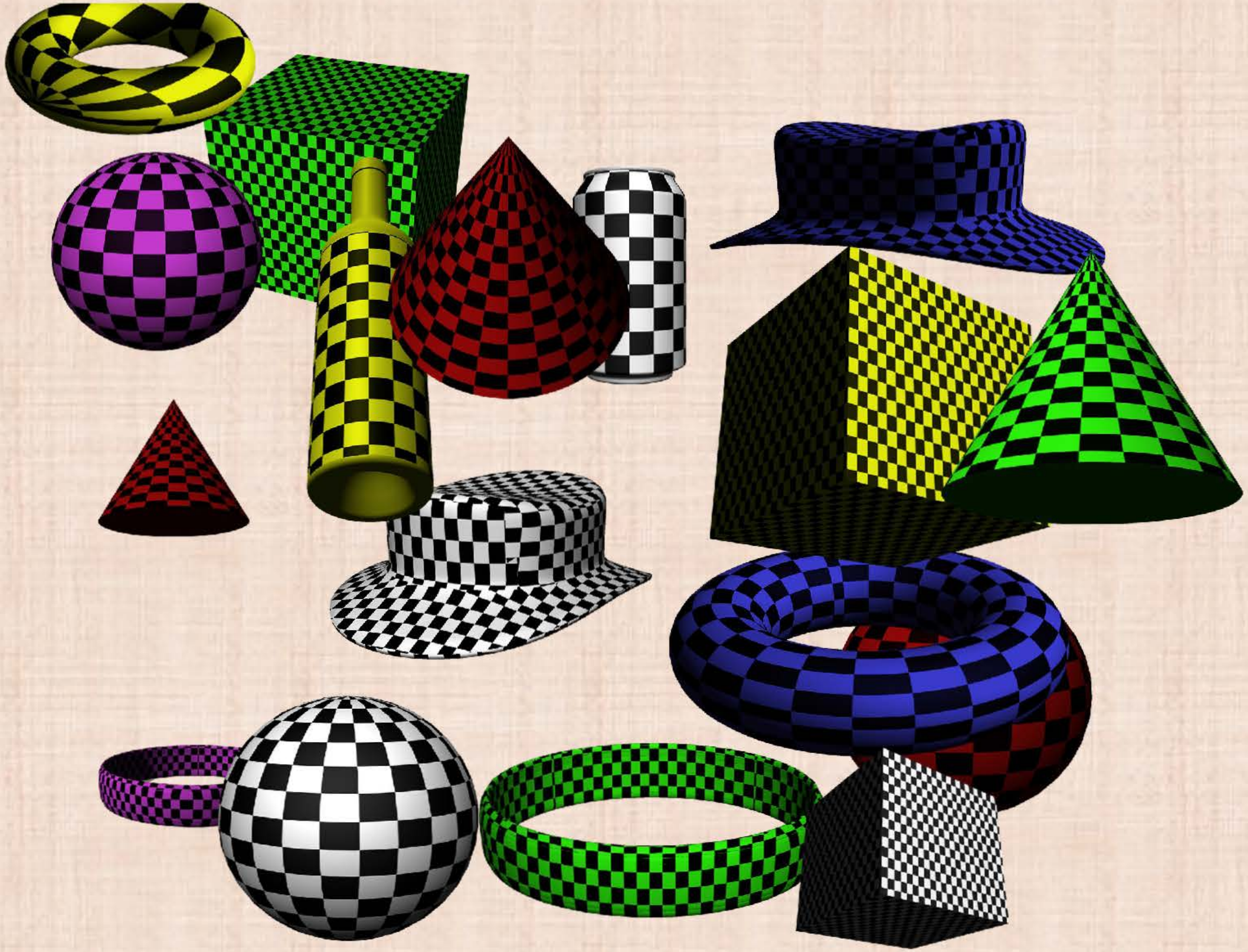
- It’s difficult to find low-distortion mappings (back and forth) from a 2D plane to 3D surfaces





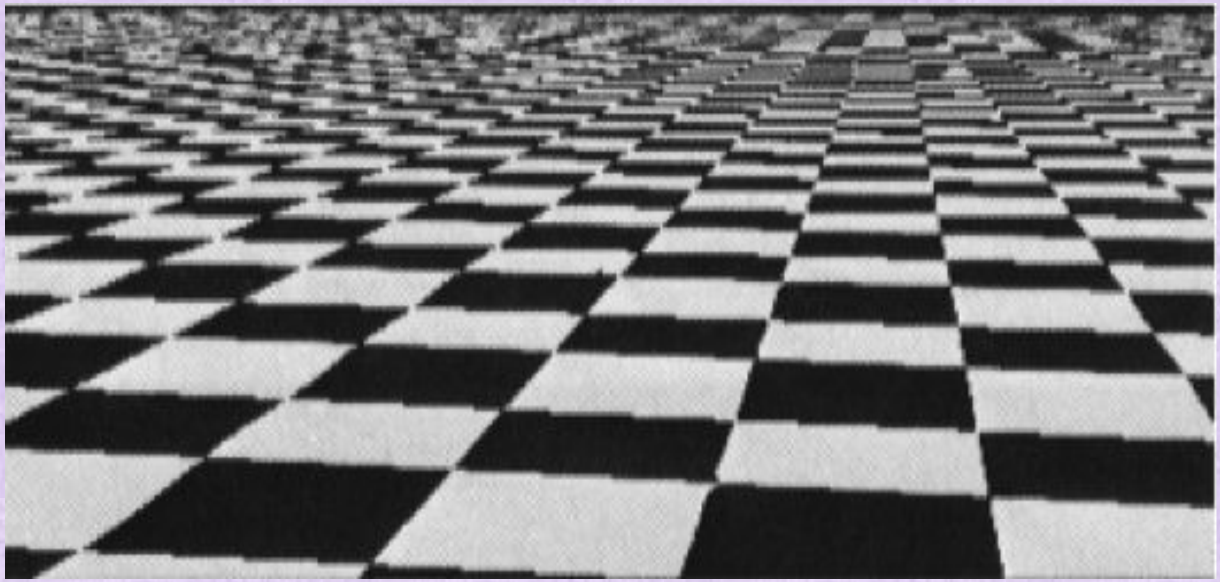


DEBUG with checkerboard textures

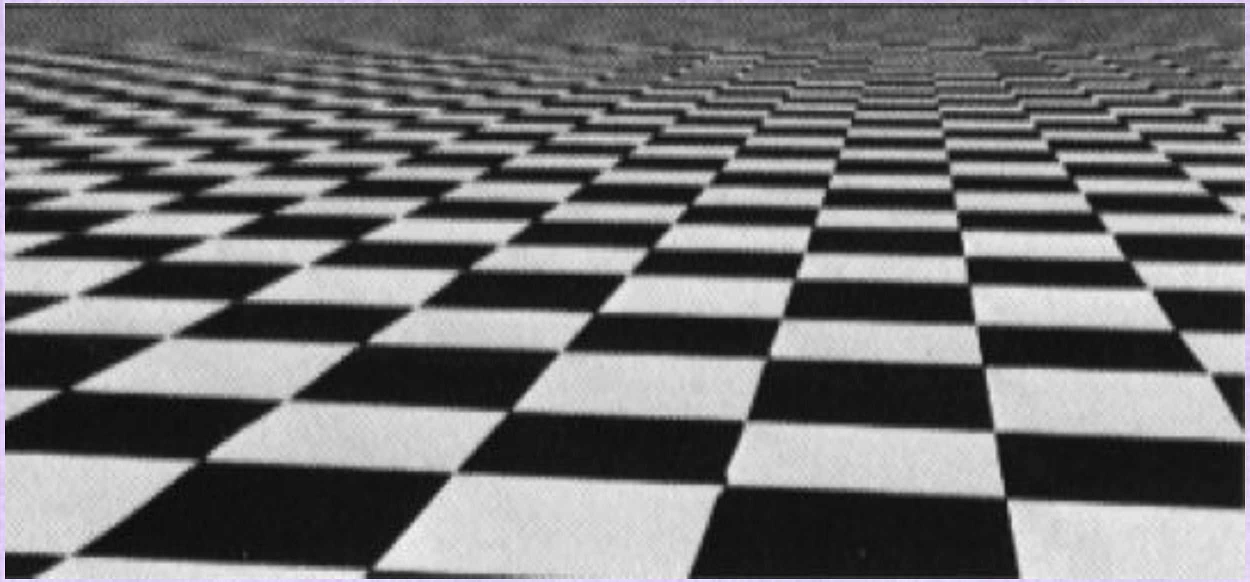


Aliasing

- Textures often alias when viewed from a distance



incorrect

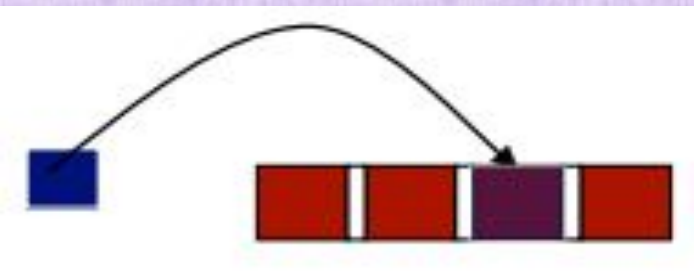


correct

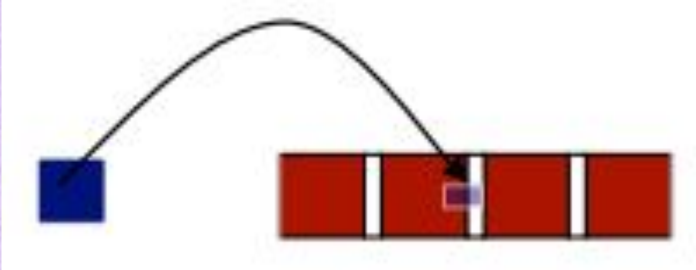
Aliasing

- Aliasing occurs when the sampling frequency is too low compared to the texture resolution (which is the signal frequency)

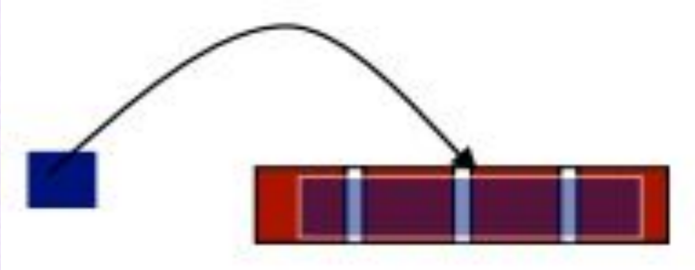
- At an optimal distance, there is a 1 to 1 mapping from triangle pixels to texture pixels (texels)
- At closer distances, triangle pixels (correctly) interpolate from texture pixels
- At far distances, a triangle pixel <u>should</u> use several texture pixels
    - But, interpolation ignores all but the nearest texture pixels (resulting in aliasing)



1 to 1 (optimal)



pixel interpolates from texels



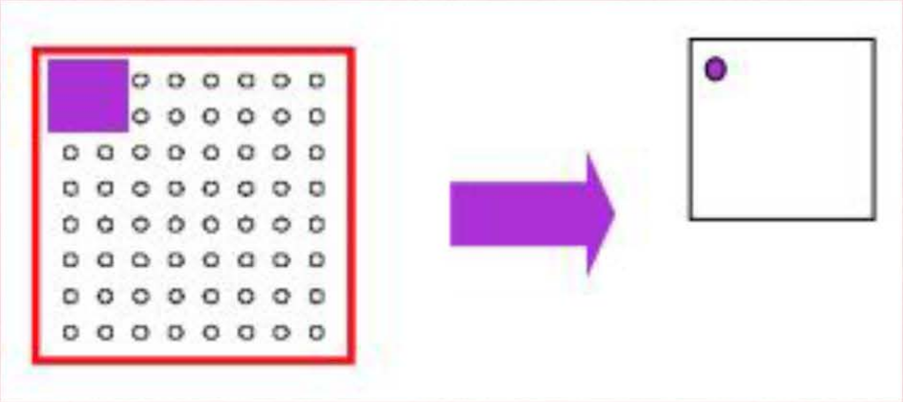
pixel should use multiple texels

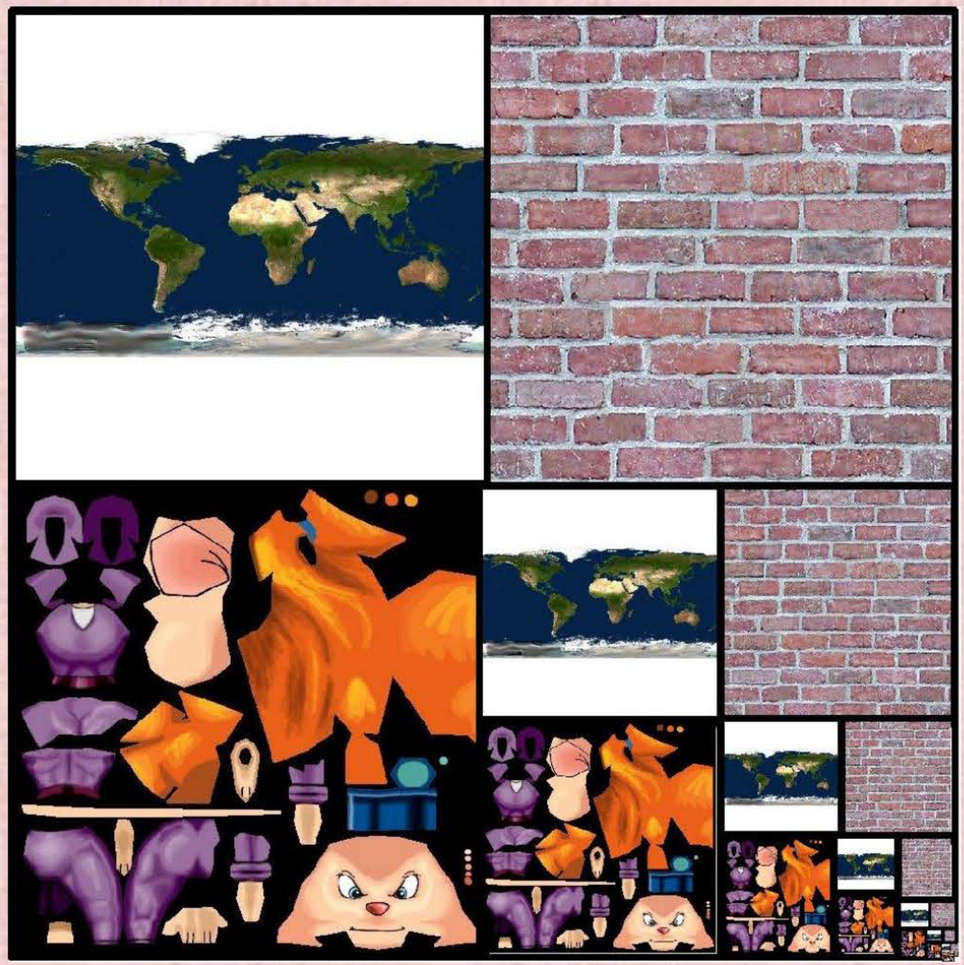
MIP Maps

- Multum in Parvo (much in little)
- Precompute texture maps at multiple resolutions, using <u>averaging</u> as a low pass filter
- When texture mapping, choose the image size that approximately gives a 1 to 1 pixel to texel correspondence
- The averaging “bakes-in” all the nearby pixels that otherwise would not be sampled correctly

MIP Maps

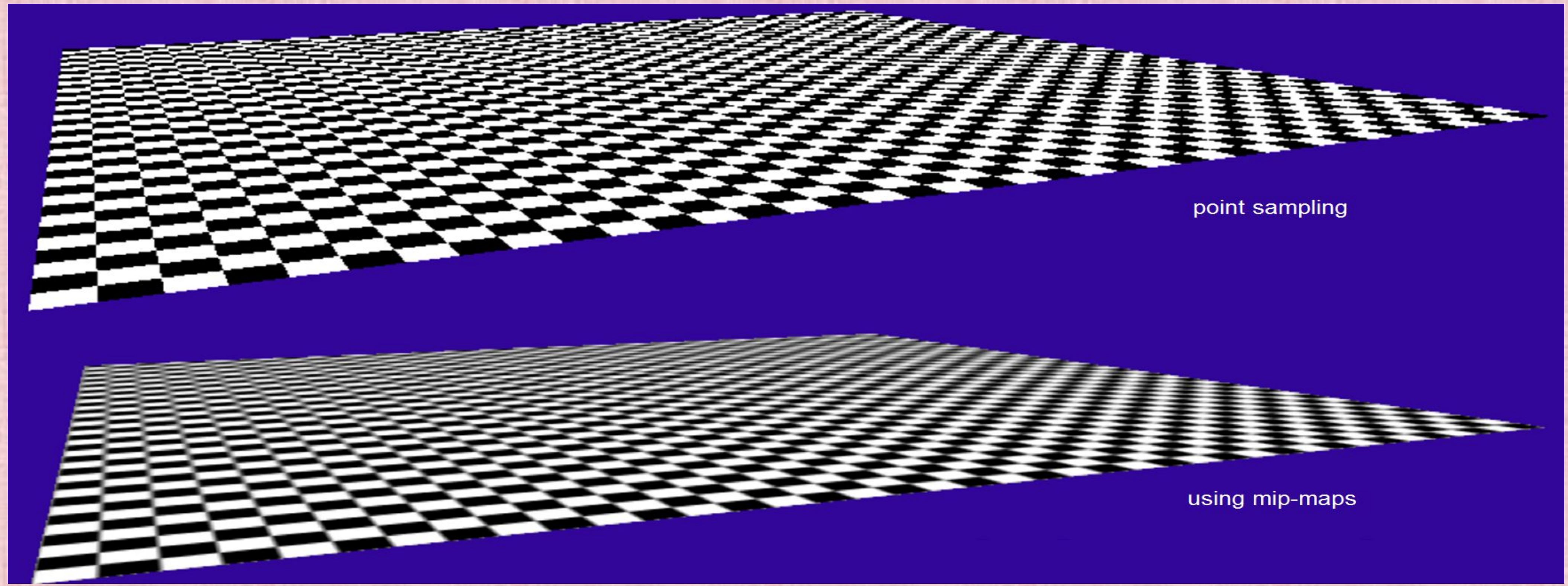
- 4 neighboring pixels of one level are averaged to form a single pixel at the next lower level
- Since $1 + \frac{1}{4} + \frac{1}{16} + \cdots = \frac{4}{3}$, can store EVERY coarser resolution using only 1/3 additional space





Using MIP Maps

- Find the MIP map image just above and just below the screen space pixel resolution
- Use bilinear interpolation on both the higher/lower resolution MIP map images
- Linearly interpolate between the results (with weights based on comparing the screen space resolution to that of the two MIP maps)



RIP Maps

- A triangle tilted away from the camera has a different texel sampling rates in the horizontal and vertical than directions
- A MIP map can only match one of the two sampling rates
- RIP maps are anisotropic in order to account for this

- **RIP maps require 4 times the storage:**

$$\left(1+\frac{1}{4}+\frac{1}{16}+\cdots\right)\left[1+2\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\right)\right]=4$$

RIP map

MIP map