Math 136 - Linear Algebra **Winter 2016**

Lecture 2: January 6, 2016

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2.1 Proofs for Fundemental Operations

Commutativity

$$\text{Let } \vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ xn \end{bmatrix} \text{ and } \vec{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \text{ be 2 vectors}$$

$$\vec{x} + \vec{y} = \begin{bmatrix} x_1 + y_1 \\ \vdots \\ x_n + y_n \end{bmatrix} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \vec{y} + \vec{x}$$

$$\therefore \vec{x} + \vec{y} = \vec{y} + \vec{x}$$

Vector Distributivity

$$c(\vec{x} + \vec{y}) = c\begin{bmatrix} x_1 + y_1 \\ \vdots \\ x_n + y_n \end{bmatrix} = \begin{bmatrix} c(x_1 + y_1) \\ \vdots \\ c(x_n + y_n) \end{bmatrix} = \begin{bmatrix} cx_1 + cy_1 \\ \vdots \\ cx_n + cy_n \end{bmatrix} = \begin{bmatrix} cx_1 \\ \vdots \\ cx_n \end{bmatrix} + \begin{bmatrix} cy_1 \\ \vdots \\ cy_n \end{bmatrix} = c\vec{x} + c\vec{y}$$

$$\therefore c(\vec{x} + \vec{y}) = c\vec{x} + c\vec{y}$$

2.2 Span

Definition 2.1 ***Linear Span***
For a given set of vectors

$$B = \{\vec{u}, \vec{u_2} \dots \vec{u_n}\} \subset \mathbb{R}^n$$

we define the span set as

$$Span_b = \{t_1\vec{v_1} + t_2\vec{v_2} + t_n\vec{v_n} \mid t_1 \dots t_n \in \mathbb{R}\} \quad B \text{ is a spanning set of } Span_b$$

So, a span is just the set of all linear combinations of the vectors in the set, which can also be written as

$$Span\vec{v_1}, \vec{v_2} \dots \vec{v_n}$$

Example 2.2

$$\textbf{\textit{Show that }} S = Span \left\{ \begin{bmatrix}1\\0\\0\end{bmatrix} \begin{bmatrix}0\\1\\0\end{bmatrix} \begin{bmatrix}0\\0\\1\end{bmatrix} \right\} = \mathbb{R}^3$$

Clearly $S \subseteq \mathbb{R}^3$, *we need to show* $\mathbb{R}^3 \subseteq S$

$$\implies \forall \vec{x} = \begin{bmatrix}x\\y\\z\end{bmatrix} \in \mathbb{R}^3$$

$$\implies \vec{x} = \begin{bmatrix}x\\y\\z\end{bmatrix} = \vec{x} \begin{bmatrix}1\\0\\0\end{bmatrix} \vec{y} \begin{bmatrix}0\\1\\0\end{bmatrix} \vec{z} \begin{bmatrix}0\\0\\1\end{bmatrix} \in S$$

$$\implies \mathbb{R}^3 \subseteq S \text{ thus } S = \mathbb{R}^3$$

Example 2.3

Does the spanning set $S = Span \left\{ \begin{bmatrix}1\\0\\2\end{bmatrix} \begin{bmatrix}2\\0\\1\end{bmatrix} \right\}$ *contain the vector* $\begin{bmatrix}0\\1\\0\end{bmatrix}$

Solution : *Suppose that* $\begin{bmatrix}0\\1\\0\end{bmatrix} \in S$

$$\begin{aligned}\begin{bmatrix}0\\1\\0\end{bmatrix} &= t_1 \begin{bmatrix}1\\0\\2\end{bmatrix} + t_2 \begin{bmatrix}2\\0\\1\end{bmatrix} \\ &= \begin{bmatrix}t_1 + 2t_2\\0\\2t_1 + t_1\end{bmatrix} \\ 1 &= 0\textbf{\textit{, Contradiction, 1 can't equal 0}}\end{aligned} \tag{2.1}$$

$$\therefore \begin{bmatrix}0\\1\\0\end{bmatrix} \text{ is not in the span set } S$$

Example 2.4

$$S = Span \left\{ \begin{bmatrix}1\\2\end{bmatrix} \begin{bmatrix}2\\4\end{bmatrix} \right\} \subseteq \mathbb{R}^2 \ \textbf{\textit{Show }} S = Span \left\{ \begin{bmatrix}1\\2\end{bmatrix} \right\}$$

$$\begin{aligned}\text{for any } \vec{x} \in S \text{ , } \vec{x} = t_1 \begin{bmatrix}1\\2\end{bmatrix} + \begin{bmatrix}2\\4\end{bmatrix} &= t_1 \begin{bmatrix}1\\2\end{bmatrix} + 2t_2 \begin{bmatrix}1\\2\end{bmatrix} \\ &= (t_1 + t_2) \begin{bmatrix}1\\2\end{bmatrix} \text{ Let } t = (t_1 + t_2) \\ &= t \begin{bmatrix}1\\2\end{bmatrix} \in Span \begin{bmatrix}1\\2\end{bmatrix}\end{aligned} \tag{2.2}$$

End of Lecture Notes
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