Math 136 - Linear Algebra **Winter 2016**

Lecture 10: January 25, 2016

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10.1 Reduced Row Echelon Form (RREF)

1. All zero rows are "downstairs"
2. The first non-zero entry in each non-zero row is 1 and is called the leading one
3. The leading one in non-zero row is to the right of the leading one is the row above
4. A leading one is the only non-zero entry in its column

Theorem 10.1 *Every matrix is equivalent to a unique RREF*

Example 10.2 *Solve the following system*

$$\begin{cases} -x_2 + x_3 + x_4 = 4 \\ x_1 + x_2 + x_4 = 1 \\ 2x_1 + x_2 + x_3 + x_4 = -2 \end{cases} \to \begin{bmatrix} 0 & -1 & 1 & 1 & 4 \\ 1 & 1 & 0 & 1 & 1 \\ 2 & 1 & 1 & 1 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & -1 & 1 & 1 & 4 \\ 2 & 1 & 1 & 1 & -2 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & -1 & 1 & 1 & 4 \\ 0 & -1 & 1 & -1 & -4 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & -1 & 1 & 1 & 4 \\ 0 & 0 & 0 & -2 & 8 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 & -4 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 1 & 0 & 1 & -2 \\ 0 & 1 & -1 & -1 & -4 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix} \implies \begin{cases} x_1 + x_3 = -3 \\ x_2 - x_3 = 0 \\ x_4 = 4 \end{cases}$$

$$\text{Let } t = x_3 \quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3-t \\ t \\ t \\ 4 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 0 \\ 4 \end{bmatrix} + t \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, t \in \mathbb{R}$$

Definition 10.3 *Any variable whose column does not contain a leading one in the RREF of the coefficient matrix is called a free variable*

Definition 10.4 *The rank of a matrix A is the number of leading one's in its RREF and is denoted as $rankA$, $rank(A)$, or $r(A)$*

Example 10.5

$$r \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 2 \qquad r \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 3$$

Theorem 10.6 $A_{m \times n}, r(A) \leq min\{m, n\}$

Theorem 10.7 *Given a linear system* $(A \mid \vec{b}\ )$

1. $(A \mid \vec{b}\ )$ *is consistent iff* $r(A \mid \vec{b}) = r(A)$
2. *if* $(A \mid \vec{b}\ )$ *is consistent, then it contains* $n - r(A)$ *free variables*
3. $r(A) = m$ *iff* $(A \mid \vec{b}\ )$ *is consistent for for every* $\vec{b} \in \mathbb{R}^m$

End of Lecture Notes
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