, 9, 10

Stat 230 - Probability	**Fall 2016**

Lecture 8, 9, 10: September 26, 28, 30, 2016

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8.1 Conditional Probability

The importance of this concept is:

We are often interested in calculating probabilities when some partial information concerning the result of an experiment is available.

For any two events A and B with $P(B) > 0$, the conditional probability of A given B has occurred is defined by

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Note that the conditional probability given $P(A) > 0$ is

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

Note

$$P(A \mid B) + P(A^c \mid B) = 1$$
$$P(B \mid A) + P(B^c \mid A) = 1$$

8.2 The Multiplication Rule for $P(A \cap B)$

$$P(A \cap B) = P(A \mid B) \cdot P(B) \text{ or } P(A \cap B) = P(A) \cdot P(B \mid A)$$

Let A, B, C, D, ... Be arbitrary events in a sample space. Assume that $P(A) > 0, P(AB) > 0, P(ABC) > 0$. Then

$$P(ABC) = P(A)P(B \mid A)P(C \mid AB)$$
$$P(ABCD) = P(A)P(B \mid A)P(C \mid AB)P(D \mid ABC)$$

8.3 Independence

- For any two events A and B defined on S with $P(B) > 0, P(A) > 0$

- Then A and B are independent if and only if either of the statements is true.

$$P(A) = P(A \mid B)$$
$$P(B) = P(B \mid A)$$

8.4 Tree Diagrams

A tree diagram is useful for displaying all outcomes for a multistage experiment and determining their probabilities.

8.5 The Law of Total Probability

- Let $A_1, A_2, \ldots, A_k$ be mutually exclusive and exhaustive events. Then for any other event B

$$P(B) = P(B \mid A_1)P(A_1) + \ldots + P(B \mid A_k)P(A_k) = \sum_{i=1}^{k} P(B \mid A_i)P(A_i)$$

- The events are exhaustive if one A_i must occur, so that $A_i \cup \ldots \cup A_k = S$
- A set of event is said to be exhaustive when at least one of the events compulsorily occurs.

8.5.1 Terminology

- A false positive results when a test indicates a positive status when the true status is negative $(T \mid D^c)$
- A false negative results when a test indicates a negative status when the true status is positive $(T^C \mid D)$
- The Sensitivity (true positive rate) of a test is a probability of a positive test result given the presence of the disease $P(T \mid D)$.
- The Specificity(true negative rate)of a test is a probability of a negative test result given the absence of the disease $P(T^c \mid D^c)$

Note

- Sensitivity is complementary to the false negative rate.

$$P(T \mid D) + (T^c \mid D) = 1$$

- Specificity is complementary to the false positive rate.

$$P(T^c \mid D^c) + (T \mid D^c) = 1$$

8.6 Bayes' Theorem

Let $A_1, A_2, \ldots, A_k$ be mutually exclusive and exhaustive events with prior probabilities $P(A_i)$ i = 1, 2 ..., k. then for any other event B for which P(B) ¿ 0 the posterior probability of A_j given that B has occurred is

$$\begin{aligned} P(A_j \mid B) &= \frac{P(A_j \cap B)}{P(B)} \\ &= \frac{P(B \mid A_j)P(A_j)}{\sum_{i=1}^{k} P(B \mid A_i)P(A_i)} j = 1,\ 2,\ ...,\ k \end{aligned}$$