Math 136 - Linear Algebra	**Winter 2016**

Lecture 16: February 8, 2016

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16.1 Linear Mapping Example

Example 16.1 *Write down the standard matrix for $Proj_{\vec{a}}$ with $\vec{a} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \in \mathbb{R}^n$*

$Proj_{\vec{a}}(\vec{e_n}) = \frac{\vec{e_n} \cdot \vec{a}}{\|\vec{a}\|^2}\vec{a} = \frac{1}{n}\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$

16.2 Rotation

Definition 16.2 *Let $\mathbb{R}_\theta : \mathbb{R}^2 \to \mathbb{R}^2$ denote the function of the map that rortates a vector $\vec{x} \in \mathbb{R}^2$ about the orgin counterclockwise through a angle θ, then we have*

$$R_\theta(x_1, x_2) = (x_1 \cos\theta - x_2 \sin\theta, x_1 \sin\theta + x_2 \cos\theta)$$

Example 16.3 *Let $R(\vec{e_1}) = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$ $\quad R(\vec{e_2}) = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$ then,*

$$\left[\mathbb{R}_\theta\right] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Theorem 16.4 *For $\mathbb{R}_\theta : \mathbb{R}^2 \to \mathbb{R}^2$ $\forall \vec{x}, \vec{y} \in \mathbb{R}^2$ we have.*

1. $\|\mathbb{R}_\theta(\vec{x})\| = \|\vec{x}\|$
2. $\mathbb{R}_\theta(\vec{x}) \cdot \mathbb{R}_\theta(\vec{y}) = \vec{x} \cdot \vec{y}$

16.3 Reflection

Definition 16.5 *Let $Refl_p : \mathbb{R}^n \to \mathbb{R}^n$ denote the mapping that sends $\vec{x}$ to its mirror image in hyperplane P with normal vector $\vec{n}$, then we define a reflection as :*

$$Refl_P(\vec{x}) = \vec{x} - 2Proj_{\vec{n}}(\vec{x})$$

16.4 Special Subspaces

Definition 16.6 *Let $L : \mathbb{R}^n \to \mathbb{R}^m$ be a linear mapping, then the range of L is defined by :*

$$Range(L) = \{L(\vec{x}) \mid \vec{x} \in \mathbb{R}^m\}$$

Theorem 16.7 *The range of a linear mapping $L : \mathbb{R}^n \to \mathbb{R}^m$ is a subspace of $\mathbb{R}^m$*

Proof:

$L(\vec{0}) = \vec{0} \in Range(L)$, non- empty

$\forall L(\vec{x}), L(\vec{y}) \in Range(L) \quad L(\vec{x}) + L(\vec{y}) = L(\vec{x} + \vec{x}) \in Range(L)\ \forall L(\vec{x}) \in Range(L)$

$\forall L(\vec{x}) \in Range(L), \forall c \in \mathbb{R} \quad cL(\vec{x}) = L(c\vec{x}) \in Range(L)$

$\implies Range(L)$ is a subspace of $\mathbb{R}^m$

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End of Lecture Notes
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