Math 128 : Calculus 2 for the Sciences **Winter 2016**

Lecture 3: January 8, 2016

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3.1 IBP

Example 3.1 -

$$\begin{aligned}\int x^2 \cos 3x dx &= \frac{x^2 \sin 3x}{3} - \frac{2}{3}\int x \sin 3x dx \\ &= \frac{x^2 \sin 3x}{3} - \frac{2}{3}\left(\frac{-x^2 \cos 3x}{3} + \frac{1}{3}\int \cos 3x\right) \\ &= \frac{x^2 \sin 3x}{3} + \frac{2}{9}x \cos 3x - \frac{2}{27}\sin 3x + C\end{aligned}$$

$$u = x^2 \;\; dv = \cos 3x$$
$$du = 2xdx \;\; v = \frac{\sin 3x}{3}$$
$$u = x \;\; dv = \sin x$$
$$du = dx \;\; v = \frac{-\cos 3x}{3}$$

3.2 IBP for Definite Integrals

The IBP Formula can also be used for definite integrals

$$\int_a^b udv = uv \mid_a^b - \int_a^b vdu$$

Example 3.2 -

$$\begin{aligned}\int_0^{\frac{1}{2}} \cos^{-1} xdx &= x \cos^{-1} x \mid_0^{\frac{1}{2}} + \int_0^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}} \\ &= x \cos^{-1} x \mid_0^{\frac{1}{2}} + \int_1^{\frac{3}{4}} \frac{\frac{dt}{-2}}{\sqrt{2}} \\ &= \dots \\ &= \frac{\pi}{6} - \frac{\sqrt{3}}{2} + 1\end{aligned}$$

$$u = cos^{-1} \;\; du = \frac{-1}{\sqrt{-x^2}}$$
$$dv = 2xdx \;\; v = \frac{\sin 3x}{3}$$
$$Let\ t = 1 - x^2$$
$$dt = 2xdx$$

Practice : Sometimes IBP or U-sub will work

$$\int x\sqrt{x+1}$$

Evalute using :

1. IBP
2. U-Sub

Example 3.3 -

$$\int e^{-1} \sin 2x \; dx = \frac{-e^{-x}2x}{2} - \frac{1}{2}\int e^{-x}\cos 2x dx$$
$$= \frac{-e^{-x}2x}{2} - \frac{1}{2}\left(\frac{e^{-x}2x}{2} + \frac{1}{2}\int e^{-1}\sin 2x \; dx\right)$$
$$\frac{5}{4}\int e^{-1}\sin 2x \; dx = \frac{-e^{-x}2x}{2} - \frac{1}{4}e^{-x}\sin 2x$$
$$\therefore \int e^{-1}\sin 2x \; dx = \frac{-2}{5}e^{-x}\cos 2x - \frac{e^{-x}\sin 2x}{5} + C$$

$$u = e^{-x} \;\; du = -e^{-x}$$
$$dv = \sin 2x \; dx \;\; v = \frac{-\cos 2x}{2}$$
$$u = e^{-x} \;\; du = -e^{-x}$$
$$dv = \cos 2x \; dx \;\; v = \frac{\sin 2x}{2}$$

This is called Integration by reproduction

Practice :

$$\int \sin(\ln x) dx \;\; \text{Hint : let u = } \ln x \text{ then use IBP}$$

3.3 Trig Integrals

Recall : Basic Trig Derivatives

$$\frac{d}{x}\sin x \;\; \frac{d}{x}\cos x \;\; \frac{d}{x}tan \;\; \frac{d}{x}\sec$$

There is a pairing between sin/cos and tan/sec

3.3.1 Integrals Involving Sin and Cos

$$\int \sin^m x \cos^n x \; dx \text{ , m, n } \in \mathbb{Z} \text{ m, n} \geq 0$$

Example 3.4 -

$$\int \sin^5 x \cos x \; dx = \int u^5 \; dx$$
$$= \frac{u^6}{6} + C$$
$$= \frac{(\sin x)^6}{6} + C$$

$$\textit{Let } u = \sin x$$
$$du = \cos x \; dx$$

Example 3.5 -

$$\int \sin^5 x \ dx$$

This integral does not have a extra Cos like 3.4, which allowed for u-sub to work

$$\begin{aligned}
\int \sin^4 x \ \sin x \ dx &= \int (1-\cos^2 x)^2 \sin x \ dx \\
&= -\int (1-u)^2 du \\
&= -\int 1 - 2u^2 + u^4 \ du \\
&= u + \frac{2u^3}{3} - \frac{u}{5} + C \\
&= \cos x + \frac{2(\cos x)^3}{3} - \frac{\cos x}{5} + C
\end{aligned}$$

$$Let \quad u = \cos x$$
$$du = -sinxdx$$

The Method in 3.5 worked because the power was odd

Example 3.6 -

$$\int \sin^2 x \cos^5 x \ dx$$

The exponent of Cos is odd, so same method can be used

$$\begin{aligned}
\int \sin^2 x \ \cos^4 x \ \cos x \ dx &= \int \sin^2 x (1 - sin^2 x)^2 \cos x \ dx \\
&= \int u^2 (1-u^2) du \\
&= \int (u^2 - 2u^4 + u^6) du \\
&= \frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7} + C \\
&= \frac{\sin^3 x}{3} - \frac{2\sin^5 x}{5} + \frac{\sin^7 x}{7} + +C
\end{aligned}$$

$$Let \quad u = \sin x$$
$$du = cosx \ dx$$

End of Lecture Notes
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