Math 128 : Calculus 2 for the Sciences **Winter 2016**

Lecture 27: March 11, 2016

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27.1 Alternating Series Test (AST)

Theorem 27.1 *If the alternating series*

$$\sum_{n=1}^{\infty}(-1)^{n-1}b_n = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + \dots$$

where $b_n > 0$ satisfies

1. $b_{n+1} \leq b_n$
2. $\lim_{n\to\infty} b_n = 0$

Then the series is convergent

27.2 Alternating Series Estimation

Let S be the sum of a series we know converges

$$S = \sum_{n=1}^{\infty} a_n = a_1 + a_2 + \dots + a_N + \sum_{n=N+1}^{\infty} a_n = S_N + \sum_{n=N+1}^{\infty} a_n,$$

Where $S_N = a_1 + a_2 + \dots + a_N$ is the Nth partial sum
S_N can be used as an approximation for the sum of the series

Theorem 27.2 *If the alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}b_n$, where each $b_i > 0$, converges by AST, then the error in $\sum_{n=1}^{\infty}(-1)^{n-1}b_n \approx S_N$ satisfies*

$$[R_n] = [S - S_N] \leq b_{N+1}$$

In order words, the max error is the absolute value of the next term

End of Lecture Notes
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