Math 136 - Linear Algebra **Winter 2016**

Lecture 13: February 1, 2016

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13.1 Matrix Multiplication Continued

Matrix Multiplication can be used to denote dot product $\vec{x}^T\vec{y} = \vec{x} \cdot \vec{y}$

Identity Matrix :

$$I_1 = \begin{bmatrix}1\end{bmatrix}\ I_2 = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\ \dots\ I_n - \begin{bmatrix}1 & 0 & \dots & 0\\0 & 1 & \dots & 0\\0 & 0 & \dots & 1\end{bmatrix}$$

$\forall A = (a_{ij}) \in M_{m\times n}(\mathbb{R}), AI_n = A\ I_mA = A$

Proposition 13.1 *For materices A, B, C (assuming the required product makes sense), $t \in \mathbb{R}$ we have,*

1. *A(B+C) = AB + AC*
2. *t(AB) = (tA)B = A(tB)*
3. *A(BC) = (AB) C*
4. $(AB)^T) = B^TA^T$

Example 13.2 *Given $A_{m\times n}, B_{n\times s}, C_{n\times s}$ Does the following statement hold : $AB = AC \implies B = C$?*
No, To show this, Let $A = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$ $B = \begin{bmatrix}1 & 2\\0 & 0\end{bmatrix}$ $c = \begin{bmatrix}0 & 2016\\0 & 0\end{bmatrix}$

$$AB = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix} \text{ and } AC = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix} \implies AB = AC,\ \text{but } B \neq C$$

Theorem 13.3 $\forall A, B \in M_{m\times n}(\mathbb{R})$ *Such That* $A\vec{x} = B\vec{x}$ *for any* $\vec{x} \in \mathbb{R}^n, then A = B$

Proof: Given A = $(a_1, a_2, \dots, a_n)$ and B = $(b_1, b_2, \dots, b_n)$

$$\text{Set } \vec{a_i} = \begin{bmatrix}a_{1i}\\\vdots\\a_{mi}\end{bmatrix}, 1 \leq i \leq n \text{ and } \vec{b_i} = \begin{bmatrix}b_{1i}\\\vdots\\b_{mi}\end{bmatrix}, 1 \leq i \leq n$$

$$A\vec{e_1} = \vec{a_1}\ \dots\ A\vec{e_n} = \vec{a_n} \text{ and } B\vec{e_1} = \vec{b_1}\ \dots\ B\vec{e_n} = \vec{b_n} \implies A = B$$

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Note :

$$AB = A(b_1, b_2, \dots, b_n) = (Ab_1, Ab_2, \dots, Ab_n)$$

End of Lecture Notes
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