Math 136 - Linear Algebra **Winter 2016**

Lecture 9: January 22, 2016

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9.1 Linear Systems Continued

Theorem 9.1 *Given a linear system*

$$(A)\begin{cases} a_{11}x_1 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + \ldots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + \ldots + a_{mn}x_n = b_m \end{cases}$$

with the solution set S_1 *and corresponding homogeneous system*

$$(B)\begin{cases} a_{11}x_1 + \ldots + a_{1n}x_n = 0 \\ a_{21}x_1 + \ldots + a_{2n}x_n = 0 \\ \vdots \\ a_{m1}x_1 + \ldots + a_{mn}x_n = 0 \end{cases}$$

with the solution set S_2 *then,*

- $\forall \vec{a}, \vec{c} \in S_1, \vec{a} - \vec{c} \in S_2$
- $\forall \vec{x} \in S_1, \vec{y} \in S_2, \vec{x} + \vec{y} \in S_1$
- *For a fixed* $\vec{x_0} \in S_1, \forall \vec{x} \in S_1, \vec{x} = \vec{y} + \vec{x_0}$ *with* $\vec{y} \in S_2$

Proof: $\forall \vec{a}, \vec{c} \in S_1$, Let $\vec{a} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$ and $\vec{c} = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}$

(1) $a_{i1}a_1 + \ldots + a_{in}a_n = b_i \ \ 1 \le i \le m$

(2) $a_{i1}c_1 + \ldots + a_{in}c_n = b_i \ \ 1 \le i \le m$

(3) $a_{i1}(a_1 - c_1) + \ldots + a_{in}(a_n - c_n) = 0 \ \ 1 \le i \le m \ \big((1) - (2)\big)$

$\implies \vec{a} - \vec{c} \in S_1$

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Proof: $\forall \vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \vec{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \in S_2$

(1) $a_{i1}a_1 + \ldots + a_{in}a_n = b_i \ \ 1 \le i \le m$

(2) $a_{i1}x_1 + \ldots + a_{in}y_n = 0 \ \ 1 \le i \le m$

(3) $a_{i1}(x_1 + y_1) + \ldots + a_{in}(x_n + y_n) = b_i \ \ 1 \le i \le m \ \big((1) + (2)\big)$

$\implies \vec{x} + \vec{y} \in S_1$

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Proof: $\forall \vec{x} \in S_1, \vec{x} - \vec{x_0} \in S_2, if \vec{y} = \vec{x} - \vec{x_0} \implies \vec{x} = \vec{x_0} + \vec{y}, \vec{y} \in S_2$ ■

Solution Sets

Let $\alpha_1 = \begin{bmatrix} a_{11} \\ \vdots \\ a_{1n} \end{bmatrix} \dots \alpha_m = \begin{bmatrix} a_{m1} \\ \vdots \\ a_{mn} \end{bmatrix}$

$$\begin{cases} a_{11}x_1 + \ldots + a_{1n}x_n = 0 \\ a_{21}x_1 + \ldots + a_{2n}x_n = 0 \\ \vdots \\ a_{m1}x_1 + \ldots + a_{mn}x_n = 0 \end{cases} \implies \begin{cases} \alpha_1 \vec{x} = 0 \\ \alpha_2 \vec{x} = 0 \\ \vdots \\ \alpha_n \vec{x} = 0 \end{cases} \implies \text{Solution Set} = \{\vec{x} \in \mathbb{R} \mid \vec{x} \perp d_1 1 \leq i \leq m\}$$

solution set can also be written as $\{\alpha_1 \dots \alpha_n\}$

9.2 Solving Linear Systems

Example 9.2 -

$$\begin{cases} 2x + y = 2 \ (1) \\ x - 2y = 4 \ (2) \end{cases} \rightarrow \begin{cases} 2x + y = 2 \ (1) \\ 5y = 10 \ (3) \end{cases} \rightarrow \begin{cases} 2x + y = 2 \ (1) \\ y = 2 \ (4) \end{cases} \rightarrow \begin{cases} 2x = 0 \ (5) \\ y = 2 \ (6) \end{cases} \rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$$

Definition 9.3 -

We Call $\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ & \dots & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$ *the coefficient Matrix*

and $\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \mid b_1 \\ a_{21} & a_{22} & \dots & a_{2n} \mid b_2 \\ & \dots & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \mid b_m \end{bmatrix}$ *A augmented matrix* $(A \mid \vec{b})$

Elementary Row Operations (ERO) : As a previous example we perform teh following 3 operations

1. Multiply a row with a constant (cR_i)
2. Add a multiple of one row to another $R_i + cR_j$
3. Swap one row for another ($R_i \longleftrightarrow R_j$)

Theorem 9.4 *If the augmented matrix* $(A_2 \mid \vec{b_2})$ *can be obtained from* $(A_1 \mid \vec{b_1})$ *by performing ERO's, then the linear systems are equivalent*

End of Lecture Notes
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