Math 136 - Linear Algebra **Winter 2016**

Lecture 12: January 29, 2016

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12.1 Matrix Operations

$(A)_{ij} = a_{ij}$ The ij-th entry

Special Cases

- $M_{m\times 1}(\mathbb{R}) = \mathbb{R}^m$ $\begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \in M_{nx1}(R)$
- $M_{1\times n}(\mathbb{R}) \to (a_1, a_2, \ldots, a_n)$ Row vector
- $M_{1\times 1}(\mathbb{R}) = \mathbb{R}$

Definition 12.1 *$\forall A, B \in M_{m\times n}(\mathbb{R}), C \in \mathbb{R}$ we define $A + B = (a_{ij} + b_{ij}$*
If $A = (a_{ij})$ $B = (b_{ij})$ $CA = (Ca_{ij})$

The Set $M_{m\times n}(\mathbb{R})$ is a "vector space"

Theorem 12.2 *$\forall A, B \in M_{m\times n}(\mathbb{R}), c, d \in \mathbb{R}$*

1. *$A + B \in M_{m\times n}(\mathbb{R})$*
2. *(A + B) + C = A + (B + C)*
3. *A + B = B + A*
4. *$\exists O_{m\times n} = \begin{bmatrix} 0 \ldots 0 \\ 0 \ldots 0 \end{bmatrix} \in M_{m\times n}(\mathbb{R})$ Such That $\forall A, A + O_{m\times n} = A$*
5. *$\forall A$, $-A = (-1)(A) \implies A + (-A) = 0_{m\times n}$*
6. *$cA \in M_{m\times n}(\mathbb{R})$*
7. *c(dA) = (cd)A = d(cA)*
8. *(c + d)A = cA + dA*
9. *c(A + B) = cA + cB*

Definition 12.3 *The Transpose of $A_{m\times n}$ is the the matrix A^T with $(A^T)_{ij} = a_{ji}$ then $A^T \in M_{n\times n}(\mathbb{R})$*

Example 12.4 -
$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$
$\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad \vec{v}^T = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$

Properties of transpose $\forall A, B \in M_{m\times n}(\mathbb{R}), \forall x \in \mathbb{R}$

1. $(A^T)^T = A$
2. $(A+B)^T = A^T + B^T$
3. $(cA)^T = cA^T$

12.2 Matrix Multiplication

Given $A = (a_{ij}) \in M_{m\times n}(\mathbb{R})$ and $B = (b_{ij}) \in M_{n\times s}(\mathbb{R})$, we define matrix multiplication $AB \in M_{m\times s}(\mathbb{R})$ as $(AB)_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$

$$\begin{bmatrix} a_{11} & \dots & a_{1n} \\ \dots & \dots & \dots \\ a_{m1} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} b_{11} & \dots & b_{1s} \\ \dots & \dots & \dots \\ b_{n1} & \dots & a_{ns} \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{n} a_{k1}b_{k1} & \dots & \sum_{k=1}^{n} a_k b_{ks} \\ \dots & \dots & \dots \\ \sum_{k=1}^{n} a_{km}b_{k1} & \dots & \sum_{k=1}^{n} a_{mk}b_{ks} \end{bmatrix}$$

Example 12.5

$$\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1+6 & 2+8 \\ 0-3 & 0=4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ -3 & -4 \end{bmatrix}$$

Note : $AB \neq BA$

Example 12.6

$$\begin{cases} a_{11}x_1 + \ldots + a_{xn}x_n = b_1 \\ \vdots \\ a_m x_1 + \ldots + a_{mn}x_n = b_m \end{cases} \iff A\vec{x} = \vec{b}$$

End of Lecture Notes
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