| **Econ 301 - Microeconomic Theory 2** | **Winter 2018** |
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## 8.1 Intertemporal Choice Continued

- Suppose consumers can borrow against period-2 income at interest rate $r$
- Maximum loan b that consumers can take out on period -1 is such that $(1+r)b = m_1$ or $b = \frac{m_2}{1+r}$
- Budget set with saving and borrowing

$$\beta = \{(c_1, c_2) \in \mathbb{R}^2 \mid pc_2 \leq m_2 + (1+r)[m_1 \cdot pc_1]$$

  - If $m_1 \cdot pc_1 > 0$ consumers is a <u>saver</u>
  - If $m_1 \cdot pc_2 < 0$ consumer is a <u>borrower</u>
  - Price of consumption in both periods is $p_1$ and does not change. This **does not** mean that the market rate of exchange of consumption at periods 1 and 2 is 1.
  - Market rate of exchange is $1+r$
  - Consumer can exchange consumption in period 2 against consumption in period 1 only through financial markets and cost of this is $1+r$
  - With this we can rewrite the budget set

$$\beta = \{(c_1, c_2) \in \mathbb{R}^2_+ \mid p_1 c_1 + \frac{p_2 c_2}{1+r} \leq m_1 + \frac{m_2}{1+r}\}$$

    basically present value of lifetime consumption $\leq$ present value of lifetime income

- We assume that consumers preferences over consumer path $(c_1, c_2)$ represented by utility function

$$u(c_1) + bu(c_2) \text{ where } u : \mathbb{R} \to \mathbb{R} \text{ and } 0 \leq b \leq 1$$

  - Given consumption $c_1$ in period $i = 1, 2$ consumers utility is $u(c_i)$
  - Form perspective of period 1, period 2 utility is discounted by $b$

- Consumers dynamic UMP

$$\max_{c_1, c_2 \geq 0} u(c_1) + bu(c_2) \text{ such that } p_1 c_1 + \frac{p}{1+r} c_2 \leq m_1 + \frac{m_2}{1+r}$$

- If $u'(c) > 0$ for all $c > 0$ then consumers preferences over consumption paths are monotone and budget constant holds as equality at any solution to UMP.
  - If $u$ is differentiable that Lagrangean

$$L(c_1, c_2, \lambda) = u(c_1) + bu(c_2) + \lambda \left[ m_1 + \frac{m_2}{1+r} - pc_1 - \frac{pc_2}{1_r} \right]$$

- At any optimal consumption path such that $c_1^*, c_2^* \neq 0$ have FOC.

$$\frac{d}{dc_1}L(c_1^*, c_2^*, \lambda) = u'(c_1^*) - \lambda p = 0 \text{ (L1)}$$

$$\frac{d}{dc_2}L(c_1^*, c_2^*, \lambda) = bu'(c_2^*) - \lambda\frac{p}{1+r} = 0 \text{ (L2)}$$

$$\frac{d}{d\lambda}L(c_1^*, c_2^*, \lambda) = m_1 + \frac{m_2}{1+r} - [pc_1^* + \frac{pc_2^*}{1+r}] = 0 \text{ (L}\lambda\text{)}$$

- Substitute for $\lambda$ with (L1) and (L2)

$$\frac{u'(c_1^*)}{bu'(c_2^*)} = 1 + r$$

  marginal rate of inter temporal substitution = market rate of exchange of period 1 and 2 consumption.

- If $u'(0) = \infty$, then optimal consumption paths such that $c_1, c_2 \neq 0$
  Then necessary condition is $\frac{u'(c_1)}{bu'(c_2^*)} \geq 1 + r$
- If $u''^{(c)} \leq 0$ for all $c \geq 0$, then consumers preferences over consumption paths are convex, so that necessary conditions are also sufficient.
- This model can be used to study consumption dynamics.
- rewrite (MRS) :

$$\frac{u'(c_1^*)}{u'(c_2^*)} = \frac{b}{1/1+r}, \quad \text{if } u'' \leq 0, \text{ then } c_1^* \geq c_2^* \implies u^prime(c_1^*) \leq u'(c_2^*)$$

- $b$ is rate at which consumers accepts period-1 utility against period-2 utility
- $\frac{1}{1+r}$ is rate at which market accepts period-1 consumption against period-2 consumption.
- If $b > \frac{1}{1+r}$, then $c_1^* < c_2^*$, backload consumption
- If $b < \frac{1}{1+r}$, then $c_1^* > c_2^*$, frontload consumption
- If $b = \frac{1}{1+r}$, then $c_1^* = c_2^*$, consumption smoothing across period