and 7

CS 245 - Logic and Computation **Fall 2016**

Lecture 6 and 7: September 25, 29, 2016

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6.1 Proofs and Entailment

We want show :

1. Soundness : $\sum \vdash_{Res} \varphi$ implies that $\sum \models \varphi$, that is, if we prove something using Resolution then it is true
2. Completeness : $Sum \models \varphi$ implies that $\sum \vdash \varphi$, that is, every entailment has a proof

6.1.1 Resolution is Sound

For resolution to be meaningful, we need the following.

Theorem 6.1 *Suppose that $\{\alpha_1, \ldots, \alpha_2\} \vdash_{Res} \perp$; that is, there is a resolution refutation with premises $\alpha_1, \ldots, \alpha_n$ ans conclusion $\perp$. Then the set $\{\alpha 1, \ldots, \alpha_n\}$ is unsatisfiable (Contradictory)*

That is, $\sum \cup \{\neg\varphi\} \vdash_{Res} \perp$, then $\sum \cup \{\neg\varphi\}$is a contradiction. Therefore, $\sum \models \varphi$.

In essence, the solution proof system is sound.

6.1.2 Resolution Can Fail

Definition 6.2 *A proof system S is complete if every entailment has a proof; that is, if every entailment has a proof; that is, if*

$$\sum \models \alpha \textit{ \textbf{Implies} } \sum \vdash_S \alpha$$

Theorem 6.3 *Resolution is complete refutation system for CNF formulas. That is, if there is no proof of $\perp$ from a set of premises in CNF, then $\sum$ satisfiable.*

Claim 6.4 *Suppose that a resolution proof reaches a dead endthat is, no new clause can be obtained, and yet $\perp$ has not been derived. Then the entire set of formulas (including the premises!) is satisfiable.*

The resolution method yields an algorithm to determine whether a given formula, or set of formulas, is satisfiable or contradictory.

- Convert to CNF

- Form resolvents (the result of applying resolutions inference rule) until either $\perp$ is derived, or no more derivations are possible.

If $\perp$ is derived, the original formula/set is contradictory. Otherwise, the preceding proof describes how to find a satisfying valuation.

6.2 Proofs in Propositional Logic : Natural Deduction

- $\wedge$-Introduction Rule : If $\sum \vdash \varphi$ and $\sum \vdash \alpha$, then $\sum \vdash \varphi \wedge \alpha$
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  $$\frac{\varphi \ \alpha}{\varphi \wedge \alpha}$$
- $\wedge$- Elimination Rule : If $\sum \vdash \varphi \wedge \alpha$. then $\sum \vdash \varphi$ and $\sum \vdash \alpha$
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  $$\frac{\varphi \wedge \alpha}{\varphi} \frac{\varphi \wedge \alpha}{\alpha}$$
- $\rightarrow$-elimination rule : If $sum \vdash \varphi \rightarrow \alpha$ and $\sum \vdash \varphi$,then $\sum \vdash a$
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  $$\frac{\varphi \rightarrow \alpha \ \varphi}{\alpha}$$
- $\rightarrow$-Introduction rule : If $\sum \cup \{\varphi\} \vdash \alpha$, then $\sum \vdash \varphi \rightarrow \alpha$
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  $$\begin{array}{c} \varphi \\ \vdots \\ \alpha \\ \hline \varphi \rightarrow \alpha \end{array}$$