

and 17

**Stat 230 - Probability**

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## 16.1 Negative Binomial Distribution

- The experiment consists of a sequence of independent trials
- Each trial in either a success or failure
- The probability of success  $p$  is constant from trial to trial
- The experiment continues until a total of  $k$  successes have been observed

### 16.1.1 Differences

In a binomial distribution we have  $n$  trials, but don't know the number of success that we will obtain.

In a negative binomial distribution, we know the number of successes,  $k$ , but we don't know the number of trials that we need to obtain  $k$  successes

### 16.1.2 Formula

$$f(x) = P(X = x) = \binom{x+k-1}{x} p^k (1-p)^x, x = 0, 1, 2, \dots$$

**Note :**  $x$  is the total number of failure before the  $k$ th success

## 16.2 Geometric Random Variable

Suppose that independent trials, each having probability  $p$ , of being success, are performed until a success occurs. If we let  $X$  equal the number of failures obtained before the first success then

$$f(x) = P(X = x) = p(1-p)^x$$

## 16.3 Poisson Distribution

$$f(x) = P(X = x) = \frac{e^{-\mu} \mu^x}{x!}$$

Where :

- $X$  = number of events of some type
- $\mu$  = expected (average) number of events
- $e$  = base of natural logarithm

### 16.3.1 Poisson Process

Another use of the poisson probability distribution arises in situations where events occur at certain points in time or in space.

For some positive  $\lambda$  the following assumptions hold

1. Independence : The number of occurrences in non-overlapping intervals are independent
2. Individuality : for sufficiently short time periods of length  $\delta t$ ; the probability that 2 or more events occurring in the interval is equal to zero
3. Uniformity : events occur at a uniform or homogeneous rate  $\lambda$  over time.

#### 16.3.1.1 Interpretation of $\mu$ and $\lambda$

- $\lambda$  : is the rate of occurrence parameter for the events. It represents the average rate of occurrences of events per unit of time
- $\lambda t = \mu$  : represents the average number of occurrences in units of time