

Lecture 25: June 26th, 2017

Lecturer: Alan Arroyo Guevara

Notes By: Harsh Mistry

25.1 BFS Properties

Let (T, p) be a BFS tree with roots and let $\text{Level } u$ be the vertices x such that the xr -path in T has length u .

1. G is connected $\iff V(T) = V(G)$
2. P is a shortest xr -path in G
3. G has an odd cycle \iff exists $xy \in E(G) \setminus E(T)$ with $\text{level}(x) = \text{level}(y)$

25.2 Planar Graphs

Definition 25.1 A graph is **Planar** if it has a drawing in the plane in which every two edges intersect only at their ends.

Any such drawings (without crossings) is a planar embedding

Definition 25.2 A **face** is a region of a drawing

Definition 25.3 Given a face of a planar embedding, a **Boundary walk**. $W_f = v_0, e_1 v_1, \dots, v_{n-1} e_n v_n$ is obtained by following the perimeter of a boundary of f .

Note : The degree of f (denoted $\deg(f)$) is the length of f .

Lemma 25.4 Faceshaking Lemma : Let F be the set of faces in a planar embedding of a connected graph. Then,

$$\sum_{f \in F} \deg(f) = 2 |E(G)|$$

Proof: Each edge e has two sides (left and right side)

Fact

Both sides of an edge e belong to the same face $\iff e$ is a bridge

If both sides of e are on the same face, e contributes in 2 to the degree of such face

If the sides are on distinct faces, then e contributes in 1 to the degree of these faces.

Then, $\sum \deg(f) = 2 |E(G)|$

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