Math 136 - Linear Algebra **Winter 2016**

Lecture 25: March 7, 2016

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25.1 Matrix Inverse Theorems

Theorem 25.1 *If A and B are invertible matrices and $c \in \mathbb{R}$ with $c \neq 0$, then*

1. $(cA)^{-1} = \frac{1}{c}A^{-1}$
2. $(A^T)^{-1} = (A^{-1})^T$
3. $(AB)^{-1} = B^{-1}A^{-1}$

Theorem 25.2 *If A is a $n \times m$ matrix such that $RankA = n$, then A is invertible*

Theorem 25.3 ***Invertible Matrix Theorem***
For any $n \times m$ matrix A, the following are equivalent:

1. *A is invertible*
2. *The RREF of A is I*
3. *$rankA = n$*
4. *The system of equation $A\vec{x} = \vec{b}$ is consistent with a unique solution for all $\vec{b} \in \mathbb{R}^n$*
5. *The nullspace of A is $\{0\}$*
6. *The columns of A form a basis for $\mathbb{R}^n$*
7. *The rows of A form a basis for $\mathbb{R}^n$*
8. *A^T is invertible*

Note :
A is invertible $\iff$ The system of equation $A\vec{x} = \vec{b}$ is consistent with a unique solution for all $\vec{b} \in \mathbb{R}^n$

$$\begin{aligned} A\vec{x} &= \vec{b} \\ A^{-1}(A\vec{x}) &= A^{-1}\vec{b} \\ (A^{-1}A)\vec{x} &= A^{-1}\vec{b} \\ \vec{x} &= A^{-1}\vec{b} \end{aligned}$$

End of Lecture Notes
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