Math 239 - Introduction to Combinatorics **Spring 2017**

Lecture 33: July 17th, 2017

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Definition 33.1 *Consider a matching M. Given a unsaturated vertex $a \in A$ and $x \in A \cup B$, then x is **M-Reachable** or just **Reachable** from a, if there is a alternating path connecting a to x*

Notation:

- x_o = vertices in A that are unsaturated
- x = vertices in A that are M-Reachable for some $a \in x_o$
- y = vertices in B that are M-Reachable for some $a \in x_o$

33.1 Algorithm For Finding M-Reachable Vertices

Input : Bipartite g and matching M **Output** : X, Y and for every $v \in X \cup Y$ an alternating path P_x

Initialize: $x_0 \leftarrow \emptyset$ and $P_v \leftarrow \emptyset$ **Input** : Bipartite g and matching M
Output : X, Y and for every $v \in X \cup Y$ an alternating path P_x

Initialize: $x_0 \leftarrow \emptyset$ and $P_v \leftarrow \emptyset$

- For each unsaturated $v_0 \in A$
  - $x_0 \leftarrow x_0 \cup \{V_0\}$
  - $P_x \leftarrow V_0$
- For $i = 1, 2, 3, \ldots$
  - For each vertex $v_i \in \cup X_j$ for which $\exists V_{i-1} \in X_{i-1}$
    - $X_i \leftarrow X_i \cup \{V_i\}$
    - $P_{vi} \leftarrow P_{vi-1}, V_i$
- $X \leftarrow X_0 \cup X_2 \cup X_4 \cup \ldots$
- $Y \leftarrow X_1 \cup X_3 \cup X_5 \cup \ldots$