Math 239 - Introduction to Combinatorics	**Spring 2017**

Lecture 16: June 5th, 2017

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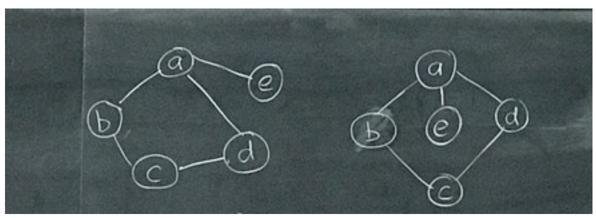
16.1 Graph Theory

Definition 16.1 *A graph G is a pair G=(V,E), where B is an finite set, called the vertices of G, and E is a set of unordered pairs (2-sets) of elements in V, called the edges of G.*

Example 16.2

$$V = \langle a, b, c, d, e \rangle$$

$$E = \langle \{a, b\}, \{b, c\}, \{c, d\}, \{d, a\}, \{a, e\} \rangle$$



In drawings of graphs, vertices are represented as points in the plan an edges are arcs connecting them.

Remarks

- Instead of saying "edge $\{u, v\}$", we say "edge uv"
- Also $V(G) = V$ and $E(G) = E$
- We also don't study graphs with multiple edges or loops

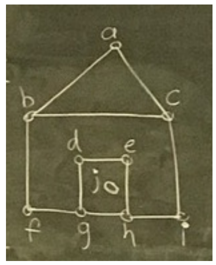
Terminology

Let G be a graph an $e = uv \in E(G)$, we say

- u and v are adjacent
- v is a neighbour of u
- e is incident to u
- e is joins u and v

Example 16.3 -

- *a and b are adjacent*
- *a and d are not adjacent*
- *j has no neighbours*
- *bf is incident with f.*

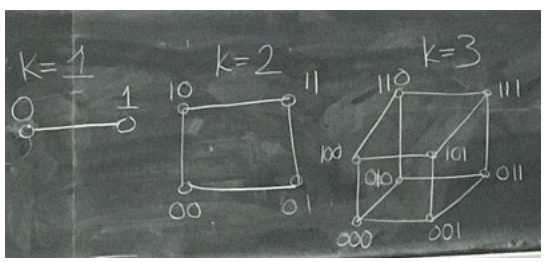


Graphs can also be used to represent interesting objects

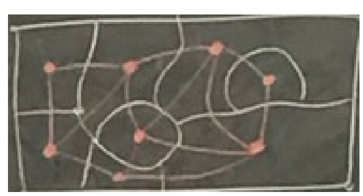
Example 16.4 *(Cubes)*
Let k be a positive integer and define $Q_k = (V, E)$

$$V = \{0, 1\} stringsoflengthk$$

$E = S_1S_2 : S_1S_2 \in V, s_1$ *and* s_2 *differ in at most one digit*



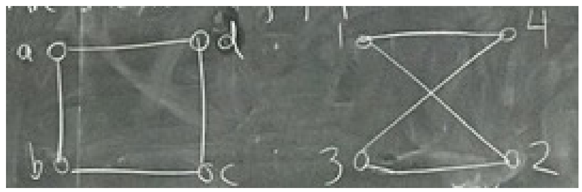
Example 16.5 *(Maps)* ***Four Colour Conjecture*** *: Every map can be coloured using 4-colours, such that no two regions share a "real boundary" having the same colour.*



A Plane graph *is a graph drawn with no crossing between the edges. From every map you can get a plane graph*

16.1.1 Isomorphisms

Are the following graphs the same :



Definition 16.6 *Two graphs G_1 and G_2 are isomorphic iff there exists a bijection*

$$f : V(G_1) \longrightarrow V(G_2)$$

such that

$$uv \in E(G_1) \iff f(u)(v) \in E(G_2)$$