Math 239 - Introduction to Combinatorics **Spring 2017**

Lecture 10: May 23rd, 2017

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10.1 Binary Strings

Definition 10.1 *The empty string ϵ is a string of length 0 with the property that for every other string a*

$$\epsilon a = a = a\epsilon$$

Definition 10.2 *(String Product) Let A,B be sets of string, then*

$$AB = \{ab \mid a \in A, b \in B\}$$

Example 10.3 $A = \{1, 01\}$, $B = \{1, 10\}$

$$AB = \{11, 110, 011, 0110\}$$

$$BA = \{11, 101, 101, 1001\} = \{11, 101, 1001\}$$

Definition 10.4 *We say AB is unambiguous if for every string $S \in AB$ there is a unique $a \in A$ and unique $b \in B$ such that $S = ab$*

Definition 10.5 $A \cup B$ *is unambiguous* $\iff$ $A \cap B = \emptyset$

Definition 10.6 *(String Power)*

$$A^k := AA \dots A$$
$$A^0 := \{\epsilon\}^k$$

Example 10.7 $\{0, 1\}^k = all\{0, 1\}oflengthk$
A^k *is* <u>*Unambiguous*</u> *if for every $s \in A^k$, there are unique $a_1, a_2, a_3, \dots, a_k \in A$ such that $s = a_1 a_2 \dots a_k$*

Example 10.8 $A = \{\epsilon, 1\}$, *is A^3 ambiguous?*
Yes, *because* $1 = (1)(\epsilon)(\epsilon) = (\epsilon)(1)(\epsilon) = (\epsilon)(\epsilon)(1)$

Definition 10.9 *(Star)*

$$A^* := \{\epsilon\} \cup A \cup A^2 \cup \dots$$

A^k *is unambiguous if*

- *Each A^k is unambiguous* $(k = 0, 1, \dots)$
- $A^k \cap A^j = \emptyset,\ k \neq j$

Definition 10.10 *Given a string s, a block is a maximal non-empty substring of 0's and 1's*

Problem 10.11 *Describe in words the following set of strings*

- $\{0,1\}^*$ = *All binary strings*
- $\{\epsilon, 0, 1\}^*$ = *All binary strings, but this is ambiguous*
- $\{11\}$ = *all even length binary strings with only 1's*
- $\{0\}\{0\}^*\{11\}^*$ = *strings starting with a block of zeros, and followed by an even number of 1's*
- $\left(\{0\}\{0\}^0\{11\}^*\right)^*$ = *strings starting with a block of 0's in which every 0 is followed by an even number of 1s*