Math 136 - Linear Algebra **Winter 2016**

Lecture 1: January 4, 2016

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1.1 Vector Addition and Scaler Multiplication

Recall :
2-dimensional Euclidian Space

$$\mathbb{R}^2 = \{(x, y)|x, y \in \mathbb{R}\}$$

3-dimensional Euclidian Space

$$\mathbb{R}^3 = \{(x, y, z)|x, y, z \in \mathbb{R}\}$$

Example 1.1

$$\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix} \textit{ corresponds to the point}(x, y)$$

Definition 1.2 *We represent vectors as column vectors (matrices of size n 1) to distinguish them from points*

$$\vec{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}, v_i \in \mathbb{R}, 1 \leq i \leq n$$

This is called a n-dimensional Euclidean Space

1.2 Operations

Equality

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\vec{x} = \vec{y} \text{ iff } x_i = y_i, 1 \leq i \leq n$$

Addition/Subtraction

$$\text{Let } \vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \text{ and } \vec{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \text{ be 2 vectors}$$

$$\vec{x} \pm \vec{y} = \begin{bmatrix} x_1 \pm y_1 \\ \vdots \\ x_n \pm y_n \end{bmatrix}$$

Scaler Multiplication

$$c\vec{x} = \begin{bmatrix} cx_1 \\ \vdots \\ cx_n \end{bmatrix}$$

1.3 Linear Combination

Definition 1.3

$$\textit{For } \vec{V_1} \ldots \vec{V_2} \ldots \vec{V_n} \in \mathbb{R}$$

$$\textit{We call the sum } V_1C_1 + V_2C_2 + \ldots V_nC_i \quad \textit{the linear combination of } \vec{V_1} \ldots \vec{V_2} \ldots \vec{V_n}$$

1.4 Fundemental Properties of Vector Operations

Theorem 1.4

$$\textit{If } \ \vec{x}, \vec{y}, \vec{w} \ \in \mathbb{R}^n \ , c, d \in \mathbb{R}$$

- *Closure Under Addition:*
$$\vec{x} + \vec{y} \in \mathbb{R}^n$$
- *Associativity :*
$$(\vec{x} + \vec{y}) + \vec{w} = \vec{x} + (\vec{y} + \vec{w})$$
- *Commutativity :*
$$\vec{x} + \vec{y} = \vec{y} + \vec{x}$$
- *Additive identity :*
$$\exists \vec{0} \in \mathbb{R}, \forall \vec{x} \in \mathbb{R}^n, \vec{x} + \vec{0} = \vec{x}$$
- *Additive inverse :*
$$\forall \vec{x} \in \mathbb{R}^n, \exists - \vec{x} \in \mathbb{R}^n, \vec{x} + (-\vec{x}) = \vec{0}$$
- *Closure under scalar multiplication:*
$$c\vec{x} \in \mathbb{R}^n$$
- $$c(d\vec{x}) = (cd)\vec{x}$$
- *Scalar distributivity:*
$$(c + d)\vec{x} = c\vec{x} + d\vec{x}$$

- *Vector distributivity:*
$$c(\vec{x} + \vec{y}) = c\vec{x} + c\vec{y}$$
- *Multiplicative identity:*
$$1\vec{x} = \vec{x}$$

End of Lecture Notes
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