Math 239 - Introduction to Combinatorics **Spring 2017**

Lecture 22: June 19th, 2017

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22.1 Eulerian Circuits

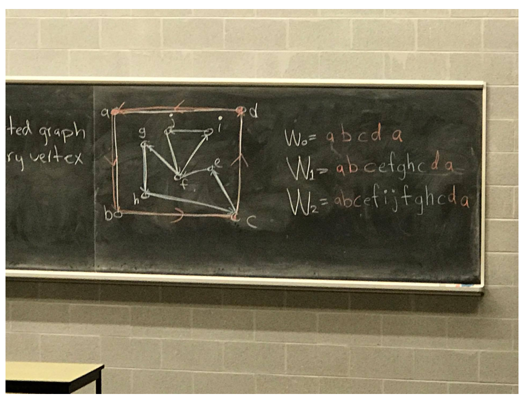
Eulers Question: What graphs have a closed walk that contains all the edges of G exactly once?

Such walks are known as **Eulerian tours**

Theorem 22.1 *(Euler) A connected graph has an Euler Tour $\iff$ every vertex has an even degree.*

Proof: In course notes ■

Example 22.2 -



22.2 Trees

Definition 22.3 *A **tree** is a connected graph with no cycles*

Lemma 22.4 *There is a unique path between vertices u and v in a tree T.*

Proof: Suppose there are two paths, $P : x_0x_1 \ldots x_n$ and $Q : y_0y_1 \ldots y_m$ where $u = x_0 = y_0$ and $v = x_n = y_m$

Let $i + 1$ be the smallest index for which $x_{i+1} \neq y_{i+1}$.
There is a closed walk connecting the ends of $e := x_ix_{i+1}$that does not go through $e : W$ is defined by following Q from y_i to y_m and then returning back from x_m to x_{i+1}.

Since x_i and x_{i+1} are connected in $T - e$, then e is not a bridge *implies* there is a cycles in T containing e, thus contradicting that T is a tree ∎

Observation : Every edge of a tree is a bridge

Theorem 22.5 *A tree T with* $\mid V(G) \mid \geq 2$ *has atleast two vertices of degree 1*

Theorem 22.6 *If T is a tree* $\implies \mid E(T) \mid = \mid V(T) \mid -1$

Proof:

Base : n= 1 and and n=2 holds

I.H : Suppose that every tree with $n-1$ vertices has $n-2$ edges.

I.S : Let T be a tree with n vertices. By the previous theorem, T has vertex c such that $deg(v) = 1$

Next Lecture

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