Math 136 - Linear Algebra **Winter 2016**

Lecture 23: March 2, 2016

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23.1 Coordinates continued

Recall : Let $\beta = \{\vec{v_1} \dots \vec{v_n}\}$ be a basis for a vector space $\mathbb{V}$
if $\vec{v} = b_1\vec{v_1} + \dots + b_n\vec{v_n}$, then $b_1 \dots b_1$ are called β-coordinates of $\vec{v}$, and we define the β-coordinate vector by

$$| \vec{v} |_\beta = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

Theorem 23.1 *if $\mathbb{V}$ is s vector space with basis $\beta = \{\vec{v_1}, \dots, \vec{v_n}\}$, then for any $\vec{v}, \vec{w} \in \mathbb{V}$ and $s, t \in \mathbb{R}$ we have*

$$[s\vec{v} + t\vec{w}]_\beta = s[\vec{v}]_\beta + t[\vec{w}]_\beta$$

Corollary 23.2 *if $\mathbb{S}$ is a subspace of a finite dimensional vector space $\mathbb{V}$, then $dim\mathbb{S} \neq dim\mathbb{V}$*

23.2 Change of Coordinates

We might want to change which basis we are using for a vector space. To acheive this, we start by converting to and from the standard basis in $\mathbb{R}^3$
Let β be a basis for $\mathbb{R}^3$ and $\vec{x} \in \mathbb{R}^3$. Since

$$\vec{x} = x_1\vec{e_1} + x_2\vec{e_2} + x_3\vec{e_3}$$

we find the coordinates of the standard basis vectors respect to the basis β, to make calculating $[\vec{x}]_\beta$ easier

$$\begin{aligned} \left[\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right]_\beta &= x_1[\vec{e_1}]_\beta + x_2[\vec{e_2}]_\beta + x_3[\vec{e_3}]_\beta \\ &= \left[[\vec{e}_1]_\beta [\vec{e_2}]_\beta [\vec{e_3}]_\beta\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \end{aligned}$$

$\beta P_s = \left[[\vec{e}_1]_\beta [\vec{e_2}]_\beta [\vec{e_3}]_\beta\right]$ is called the change of coordinates matrix from the standard basis S to the basis β

In general : Let $\beta = \{\vec{v_1}, \ldots, \vec{v_n}\}$ and ζ both be basis of a vector space $\mathbb{V}$. We want to change coordinates from β-coordinates to ζ-coordinates.
$\vec{x} = b_1\vec{v_1} + \ldots + b_n\vec{v_n}$, we want to find $[\vec{x}]_\zeta$

$$[\vec{x}]_\zeta = \big[[\vec{v_1}]_\zeta \ldots [\vec{v_n}]_\zeta\big] \, [\vec{x}]_\beta$$

Definition 23.3 *Let $\beta = \{\vec{v_1} \ldots \vec{v_n}\}$ and ζ be bases for a vector space $\mathbb{V}$*
The Change of Coordinates Matrix from β-coordinates to ζ-coordinates is defined by

$$\zeta P_\beta = \big[[\vec{v_1}]_\zeta \ldots [\vec{v_n}]_\zeta\big]$$

and for any $\vec{x}$ in $\mathbb{V}$ we have

$$[\vec{x}]_\zeta = \zeta P_\beta [\vec{x}]_\beta$$

End of Lecture Notes
Notes by : Harsh Mistry