Econ 301 - Microeconomic Theory 2 **Winter 2018**

Lecture 18: March 19, 2018

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18.1 Externalities Continued

Note : This lecture builds upon the example in Lecture 17

- A competitive equilibrium consists of prices $p^* = (p_1^*, p_2^*, p_R^*)$ and allocations $x^{A*} = (x_1^{A*}, x_2^{A*}, x_R^{A*})$and $x^{B*} = (x_1^{B*}, x_2^{B*}, x_R^{B*})$ which satisfy
  1. Given price p^* and allocation X^{A*} solves
  $$\max_{x_1^{A*}, x_2^{A*}, x_R^{A*} \geq 0} x_1^{A\frac{1}{2}} x_2^{A\frac{1}{2}} \text{ s.t } \quad p_1^* x_1^A + p_2^* x_2^A + p_R^* x_R^A \leq 2p_1^* + p_2^* + p_R^* \omega_R^A$$
  allocation x^{B*} solves
  $$\max_{x_1^{B*}, x_2^{B*}, x_R^{B*} \geq 0} x_1^{B\frac{1}{2}} x_2^{B\frac{1}{2}} \text{ s.t } \quad p_1^* x_1^B + p_2 x_2^B + p_R^* x_R^B \leq p_1^* + p_2^* + p_R^* \omega_R^B$$
  2. Allocations x^{A*} and x^{B*} clear al markets $x_i^{A*} + x_i^{B*} = \omega_i^A + \omega_i^B$ for all $i = 1, 3, R$
- Derive a competitive equilibrium by first simplifying the consumers UMP.
- In any equilibrium, $p_R^* > 0$
  - If $p_R^* = 0$, consumers B's demand for rights is undefined.
  - Additionally, $p_1^* > 0$
- In any equilibrium, $p_2^* = 0$
  - If $p_2^* > 0$,then we must have $x_2^{B*} = 0$
  - By (MC2), $x_2^{A*} = 2$
  - Due to that fact thet $x_2^{A*} \leq x_R^{A*}$, we have $x_R^{A*} \geq 2$
  - By (MCR), $x_R^{A*} = 2$ and $x_R^{B*} = 0$
  - Due to the fact that $p_1^*, p_2^* > 0, \quad x_R^{A*}$ is never optimal
- In any equilibrium $x_2^{A*} = x_R^{A*}$
  - $x_2^{A*} < x_R^{A*}$ can not be optimal because u^A is increasing in x_2^A and $p_2^* = 0$
- With these findings we can now rewrite the consumers UMP from the previous example
$$\max_{x_1^J, x_R^J \geq 0} x_1^{J\frac{1}{2}} x_2^{J\frac{1}{2}} \text{ s.t } \quad p_1^* x_1^J p_2^* x_R^J \leq p_1^* \omega_1^J + p_R^* \omega_R^J$$
- This gives a standard problem with the demand function :
$$(x_1^{J*}(p_1^* \omega^J), x_R^{J*}(p_1^* \omega^J)) = \left(\frac{p_1^* \omega_1^J + p_2^* \omega_R^J}{2p_1^*}, \frac{p_1^* \omega_1^J + p_2^* \omega_R^J}{2p_R^*} \right)$$

- Normalize $p_1^* = 1$, (MCR) yields

$$\frac{2 + p_R^* \omega_R^A}{2p_R^*} + \frac{1 + p_2^* \omega_R^B}{2p_R^*} = 2$$
$$\implies p_R^* = \frac{3}{2}$$

- Prices $p^* = (1, 0, \frac{3}{2})$ and allocations

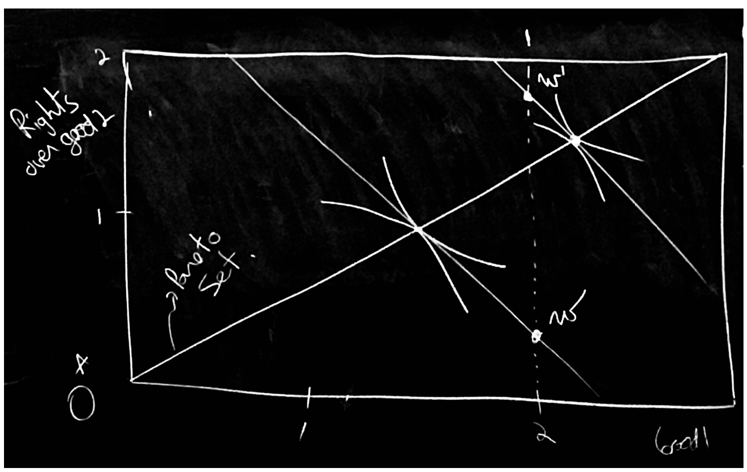
$$x^{A*} = \left(1 + \frac{3}{4}\omega_R^A,\ \ \frac{2}{3} + \frac{1}{2}\omega_R^A,\ \ \frac{2}{3} + \frac{1}{2}\omega_R^A\right)$$

$$x^{B*} = \left(1 + \frac{3}{4}\omega_R^B,\ \ \frac{1}{3} + \frac{1}{2}\omega_R^B,\ \ \frac{1}{3} + \frac{1}{2}\omega_R^B\right)$$

  form a competitive equilibrium

- Given any initial endowments of rights x^{A*} and x^{B*} are Pareto-efficient.

$$\frac{\frac{d}{dx_1^A} u^A(x_1^{A*}, x_R^{A*})}{\frac{d}{dx_R^A} u^A(x_1^{A*}, x_R^{A*})} = \frac{p_1^*}{p_2^*} = \frac{2}{3} = \frac{\frac{d}{dx_1^B} u^B(x_1^{B*}, x_R^{B*})}{\frac{d}{dx_R^B} u^A(x_1^{B*}, x_R^{B*})}$$



Thus, the Pareto Set is

$$\begin{pmatrix} \frac{x_R^A}{x_1^A} \end{pmatrix} = \begin{pmatrix} \frac{2 - x_R^A}{3 - x_1^A} \end{pmatrix} \implies x_R^* = \frac{2}{3} x_1^A$$

- Externalities create missing-market problem, but if property rights are established over externalities, then welfare theorems apply.

- In practice, completing markets entails the creation of new institutions and distribution of property rights which can be difficult. Additionally, initial allocations have large effects on equilibrium welfare.