Math 239 - Introduction to Combinatorics **Spring 2017**

Lecture 21: June 16th, 2017

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Definition 21.1 *Let $X \subseteq V(G), (\emptyset \subset X \subset V(G))$ and let $y = V(G)\backslash X$*
The cut induced by X (denoted as $\delta(x)$)1 is the set of edges that have one end in X and the other end in y.

Theorem 21.2 *A graph is not connected if and only if there exists $X \subseteq V(G)$ with $\emptyset \subset X \subset V(G)$, such that $\delta(X)$ is not empty.*

Proof:

$\leftarrow$ Suppose G is connected. Let $X \subseteq V(G)$ and $\emptyset \subset X \subset V(G)$
Let $y = V\backslash X$. Pick $u \in X$ and $v \in y$. Since G is reconnected there is a path

$$P : ux_1, x_2, \ldots, x_{n-1}V$$

As $u \in X$ and $v \in y$, there is a first $i \in \langle 1, 2, \ldots, n\rangle$, such that

$$x_{i+1} \in y$$

$$x_i \in X \implies X_iX_{i+1} \in \delta(X)$$

$\rightarrow$ Suppose G is disconnected. Let C be a component of G and Let $X = V(C)$.
Since, G is not connected, it has atleast two components $\emptyset \subset X1 \subset V(G)$.
Now, let $y = V\backslash X$. Every edge with $x \in X$, has $y \in X$ has x and $j$are in the same component

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21.1 Bridges

Notation : If $e \in E(G)$ we denote $G - e$ (or $G\backslash e$) the graph whose vertex if V(G) and whose edge set is $E(G)\backslash\{e\}$. (So $G - e$ is the graph obtained from G by deleting the edge e.)

Definition 21.3 *A **Bridge** is an edge $e \in E(G)$ such that $G - e$ has more components than G.*

Lemma 21.4 *Let G be a connected graph and $e = xy \in E(G)$ a bride. Then $G - e$ has two components one including x and the others including y.*

Proof: Let $z \in V(G - e) = V(G)$ Since G is connected, there is a path $P : zv_1, v_2, \ldots v_{n-1}X$.

Case 1 : In this case, P is also a path in $G - e$, so z and x are in the same components of G, r

Case 2 : In this case $z - V_{n-1}X$, so$V_{n-1} = y$. As a result, $P' = Zv_1v_2 \ldots, V_{n-2}y$ is a path

In any case $z \in C_x$ or $z \in C_y$. Where C_x is the component including $x$and C_y is the component including y.
Since $G - e$ is disconnected $C_x \neq C_y$ ■