Lecture 32: July 14th, 2017

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Lemma 32.1 *If a matching M has an augmenting path, then M is not maximum*

Note : Given M and an augmenting path P, the notation for swapping edges of P is

$$M' = M \triangle E(P)$$

32.1   Covers

Definition 32.2 *A **cover** is a set C of vertices such that every edge is incident with atleast one vertex in C.*

Definition 32.3 *A cover C is **minimum** if for every other cover C', $\mid C \mid \leq \mid C' \mid$*

Lemma 32.4 *If M is a matching and C is a cover of G, then*

$$\mid M \mid \leq \mid C \mid$$

Proof:

- Since C is a cover, every edge $m \in M$ has at least one end $Vm \in C$.
- Since M is a matching $Vm neq Vm'$ for $m, m' \in M (m \neq m'$

Combining these two observations we find that

$$\mid M \mid \leq \mid C \mid$$

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32.2   Konig's Theorem

Theorem 32.5 *In a bipartite graph, if M is a max matching and C is a min cover then*

$$\mid M \mid = \mid C \mid$$

We will prove this theorem, using an algorithm that terminates with a max matching M and a min cover C.