Math 128 : Calculus 2 for the Sciences **Winter 2016**

Lecture 24: March 4, 2016

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24.1 Series Continued

Definition 24.1 *We say that a series $\sum_{n=1}^{\infty} a_n$ converges if the sequence of partial sums $\{S_n\}$ converges The limits of $\{S_n\}$ is called the sum of the series.*

$$\lim_{n\to\infty} S_n = S = \sum_{n=1}^{\infty} a_n$$

Theorem 24.2 *The Geometric series*

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + \ldots + ar^(n-1) + \ldots$$

is convergent if $\mid r \mid \leq 1$ and its sum is

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$$

if $\mid r \mid \geq 1$, the geometric series is divergent

$$\begin{aligned} s_n = &\quad a + ar + \ldots + ar^{n-1} \\ rs_n = &\quad\quad ar + ar^2 + ar^{n-1} + ar^n \end{aligned}$$

$$s_n - rs_n = a - ar^n \implies s_n = \frac{a(1-r^n)}{1-r}, \text{ provided } r \neq 1$$

If $-1 \leq r \leq 1$, then $\lim_{n\to\infty} s_n = \frac{a}{1-r}$
If $r > 1$ or $r \leq -1$, the limits do not exist, so the series diverges
If $r = 1$, then $s_n = a + \ldots + a = na$ and $\lim_{n\to\infty} s_n = \lim_{n\to\infty} na = \infty$, so the series diverges

Note: Its better to think of the formula as $\frac{\text{"First term"}}{\text{1 - common factor}}$

24.2 Telescoping Series

For some series it will not be possible to find a closed formula for $\{S_n\}$
We can however, determine whether or not the series converges. We rely on a number of tests to achieve this.

Theorem 24.3 *If* $\sum_{n=1}^{\infty} a_n$ *converges then* $\lim_{n\to\infty} a_n = 0$

Corollary 24.4 *The* n^{th}*-Term Test/Test for Divergence*
If $\lim_{n\to\infty} \neq 0$ *or DNE, the series* $\sum_{n=1}^{\infty} a_n$ *diverges*

End of Lecture Notes
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