Math 239 - Introduction to Combinatorics	**Spring 2017**

Lecture 15: June 2nd, 2017

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15.1 Non Homogeneous Linear Recurrence

Definition 15.1 *A non-homogeneous linear recurrence is a sequence that satisfies*

$$a_n + b_1 a_{n-1} + b_2 a_{n-2} + \ldots + b_k a_{n-k}$$

and initial conditions $a_0, a_1, \ldots, a_{k-1}$

Problem 15.2 *Find the explicit* $a_n, a_n - 3a_{n-1} + 2a_{n-2} = 10 \cdot 3^{n-1}$, $(1n \geq 2)$

Solution : *Guess Method*

1. *Goal : Find* $\{b_n\}_{n\geq 0}$ *satisfying* $b_n - 3b_{n-1} + 2b_{n-2} = 10 \cdot 3^{n-1}$
2. *Subtract hypothetical* b_n *from* a_n *because* $\{a_n - b_n\}_{n\geq 0}$

   $$0 = (a_n - b_n) - 3(a_{n-1} - b_{n-1}) + 2(a_{n-2} - b_{n-2}) = 10 \cdot 3^{n-1} - 10 \cdot 3^{n-1}$$

   $$\begin{aligned}\{a_n - b_n\}_{n\geq 0} \text{ has a } C(x) &= (1)x^2 + (-3)x + 2 \\ &= (x-2)(x-1)\end{aligned}$$

   $$\begin{aligned}a_n - b_n &= 2^n + B1^n \\ &= a_n = b_n + A2^n + B\end{aligned}$$

3. *Guess to find* $\{b_n\}_{n\geq 0}$
   Guess that b_n *is similar to* $f(n) \implies$ *in this case it would be like* $10 \cdot 3^{n-1}$

   $$\begin{aligned}10 \cdot 3^{n-1} &= b_n - 3b_{n-1} + 2b_{n-2} \\ &= \alpha 3^n - 3(\alpha 3^{n=1}) + 2(\alpha 3^{n-2}) \\ &= \alpha 3^{n-2}(3^2 - 3_2) \\ &= \alpha \cdot 2 \cdot 3^{n-2} \\ \alpha &= \frac{10 \cdot 3^{n-1}}{2 \cdot 3^{n-2}} = 15\end{aligned}$$

   So we let $b_n = 15 \cdot 3^n$

4. *Use initial condition*

   $$a_0 = 11 = b_0 + A \cdot 2^0 + B = 15 \cdot 3^0 + A \cdot 2^0 + B = 15 + A + B$$

   $$a_1 = 42 = b_1 + A \cdot 2^1 + B = 15 \cdot 3^1 + A \cdot 2^1 + B = 46 + 2A + B$$

   Solve the Syetem to Get : $A = 1, B = -5$

   $a_n = 15 \cdot 3^n + 2^n - 5$

How to Guess $\{b_n\}_{n\geq 0}$

- If $f(n) = \beta \cdot C^n$
  Try $b_n = \alpha C^n$
- If $f(n)$ is a polynomial in n Try $b_n =$ polynomial of degree at most n