Math 136 - Linear Algebra	**Winter 2016**

Lecture 24: March 4, 2016

Lecturer: Yongqiang Zhao — *Notes By: Harsh Mistry*

24.1 Matrix Inverses

Theorem 24.1 *If β and ζ are bases for an n-dimensional vector space $\mathbb{V}$, then the change of coordinate matrices ${}_{\zeta}P_{\beta}$ and ${}_{\beta}P_{\zeta}$ satisfy*

$$ {}_{\zeta}P_{\beta\beta}P_{\zeta} = I = {}_{\beta}P_{\zeta\zeta}P_{\beta} $$

This shows that ${}_{\zeta}P_{\beta}$ and ${}_{\beta}P_{\zeta}$ are multiplicative inverses of each other

24.2 Left and Right Inverse

Definition 24.2 *Let A be a $m \times n$ matrix.*
If B is an $m \times n$ matrix such that $AB = I_m$, then B is a right inverse of A
If C is an $n \times m$ matrix such that $CA = I_n$, then C is called a left inverse of A

Note (For Right Inverses) : $[\vec{e_1} \dots \vec{e_m}] = AB = [A\vec{b_1} \dots A\vec{b_m}]$

Theorem 24.3 *If A is an $m \times n$ matrix with $m > n$, then A cannot have a right inverse*

Corollary 24.4 *If A is an $m \times n$ matrix with $m < n$, then A cannot have a left inverse*

24.3 Matrix Inverse

Definition 24.5 *An $n \times n$ matrix is called a square matrix*

Definition 24.6 *Let A be a $n \times n$ matrix. IF B is a matrix such that $AB = I = BA$, then B is called a inverse of A.*
We write $B = A^{-1}$ and we A is said to be invertible

Remark : If $B = A^{-1}$ then $A = B^{-1}$

Theorem 24.7 *The inverse of a matrix is unique*

Proof: B = BI = B(AC) = (BA)C = IC = C ■

Theorem 24.8 *If A and B are $n \times n$ matrices such that $AB = I$, then A and B are invertible and $rankA = rankB = n$*

End of Lecture Notes
Notes by : Harsh Mistry