Lecture 13: February 1, 2016

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13.1 Differential equations

A differential equations (DE) is an equation involving an unknown function and one or more of its derivatives

Example 13.1

$$\frac{dp}{dt} = kp \leftarrow \textit{ Law of Natural Growth}$$

$$\frac{dp}{dt} = kp(1 - \frac{p}{m}) \leftarrow \textit{ Logistic Model for Growth}$$

$$\frac{md^2x}{dt^2} = -kx \leftarrow \textit{ Spring Motion}$$

$$\frac{d^2\theta}{dt^2} + \frac{g}{r}\sin\theta = 0 \leftarrow \textit{ Motion of a Pendulum}$$

- The order of a DE is the order of the highest derivative
- By solving a DE we mean finding the function which makes the equation

Example 13.2 *Verify that* $y = \frac{x^2}{2} + 4x$ *is a solution to the DE* $\frac{dy}{dx} = x^2 + \frac{y}{x}$
To verify, simply just check the left and right side.

Example 13.3 *Solve* $\frac{dy}{dx} = 6x^2 + 2x$

$$y = 2x^3 + x^2 + C$$

Example 13.4 *Solve* $\frac{dy}{dx} = y$

$$y = e^x$$

But there can be can multiple solutions to this DE. So we reprsent the solution using a general solution. These general solutions repersent a family of solutions. The general solution for this DE is :

$$y = ce^x$$

End of Lecture Notes
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