Math 136 - Linear Algebra **Winter 2016**

Lecture 14: February 3, 2016

Lecturer: Yongqiang Zhao *Notes By: Harsh Mistry*

14.1 Linear Mappings

Definition 14.1 *If A is a $m \times n$ matrix, then we can define the function $f : \mathbb{R}^n \to \mathbb{R}^m$ as $f(\vec{x}) = A\vec{x}$ this is called a matrix mapping*

Matrix Mapping Notation :

$$f(\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}) = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

$$f(x_1 \dots x_n) = (y_1 \dots y_n)$$

Theorem 14.2 *If A is a $m \times n$ matrix and $f : \mathbb{R}^n \to \mathbb{R}^m$ is defined by $f(\vec{x}) = A\vec{x}$, then for all $\vec{x}, \vec{y} \in \mathbb{R}^n and b, c \in \mathbb{R}$ we have :*

$$f(b\vec{x} + c\vec{y}) = bf(\vec{x}) + cf(\vec{y})$$

Definition 14.3 *A function $A : \mathbb{R}^n \to \mathbb{R}^m$ is said to be a **Linear mapping** $is for every \vec{x}, \vec{y} \in \mathbb{R}^n$ and $b, c \in \mathbb{R}$ we have :*

$$L(b\vec{x} + c\vec{y}) = bL(\vec{x}) + cL(\vec{y})$$

Remarks

- If two linear mappings (L & M) are said to be equal for all $\vec{x} \in \mathbb{R}^n$, we write L = M
- A linear mapping L : $\mathbb{R}^n \to \mathbb{R}^n$ is called a linear operator

Theorem 14.4 *Every linear mapping L : $\mathbb{R}^n \to \mathbb{R}^m$ can be repersented as a matrix mapping with a matrix whose i-th column is the image of the i-th standard basis vector of $\mathbb{R}^n$ under L for all $1 \leq i \leq n$. That is, $L(\vec{x})$ = $[L]\vec{x}$ where,*

$$\big[L\big] = \big[L(\vec{e_1}) \dots L(\vec{e_2})\big]$$

End of Lecture Notes
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