

## Lecture 8: January 20, 2016

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## 8.1 Improper Integral Type 1

Infinite Interval  $a = -\infty$  and/or  $b = \infty$ 

In General :

- $\int_a^\infty f(x)dx = \lim_{b \rightarrow \infty} \int_a^b f(x)dx$ , provided  $f$  is continuous on  $[a, \infty)$  and the limit exists
- $\int_{-\infty}^b f(x)dx = \lim_{a \rightarrow -\infty} \int_a^b f(x)dx$ , provided  $f$  is continuous on  $(-\infty, b]$  and the limit exists
- $\int_{-\infty}^\infty f(x)dx = \lim_{a \rightarrow -\infty} \int_a^c f(x)dx + \lim_{b \rightarrow \infty} \int_c^b f(x)dx$  provided that both limits exist

If the limit exists we say the improper integral is equal to the limit or is Convergent and converges. If the limit does not exist, the improper integral has no value and is divergent

## 8.2 Improper Integral Type 2

Discontinuous Integrand ( $f$  discontinuous on  $[a, b]$ )

In General :

- If  $f$  is continuous on  $[a, b)$  but not at  $b$  then  
 $\int_a^b f(x)dx = \lim_{c \rightarrow b^-} \int_a^c f(x)dx$ , provided the limit exists
- If  $f$  is continuous on  $(a, b]$  but not at  $a$  then  
 $\int_a^b f(x)dx = \lim_{c \rightarrow a^+} \int_c^b f(x)dx$ , provided the limit exists
- If  $f$  is continuous on  $(a, b]$  but not at  $d$ , where  $a < d < b$ , then  
 $\int_a^b f(x)dx = \lim_{c \rightarrow d^-} \int_a^c f(x)dx + \lim_{c \rightarrow d^+} \int_c^b f(x)dx$  provided both limits exist.

**End of Lecture Notes**  
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