Math 136 - Linear Algebra **Winter 2016**

Lecture 7: January 18, 2016

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7.1 Scaler Equations Examples

Example 7.1 *Find a normal vector of the plane* $x_1 + x_2 + 3x_3 = 2$

$$\vec{n} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

Example 7.2 *Find a scaler equivalent of the of the plane with ethe normal vector* $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$ *and passes through the point (2, 1, 0)*

$\vec{b} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \quad (\vec{x} - \vec{b}) \cdot \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} = 0 \implies -x_1 + x_2 + 2x_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} = 1 \quad \therefore x_1 - x_2 - 2x_3 = 1$

Example 7.3 *Find a scaler equation of the plan that passes through P(1, 2 , 0) ,* P_2*(2, 1, 1),* P_3*(-1, 0, 2)*

Let $\vec{u} = P_1\vec{P}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ *and* $\vec{v} = P_1\vec{P}_3 = \begin{bmatrix} -2 \\ -2 \\ 2 \end{bmatrix}$

$\{\vec{x}, \vec{v}\}$ *is linear independant* $\implies \vec{x} = s\vec{u} + t\vec{v} + \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \leftarrow$ *Vector equation*

Scaler : $\vec{n} = \vec{u} \times \vec{v} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \times \begin{bmatrix} -2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 4 \end{bmatrix} \rightarrow \vec{x} \cdot \vec{n} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \cdot \vec{n} \implies x_2 + x_3 = 2$

7.2 Projections

Given $\vec{u} \in \mathbb{R}^n$, a Line L with $\vec{x} = t\vec{v} + \vec{b}, t \in \mathbb{R}$
$\vec{u} \cdot \vec{v} = (c\vec{u} + w)\vec{v} = c\|\vec{v}\|^2$

Definition 7.4 - *Given* $\vec{v} \in \mathbb{R}, \vec{u} \in \mathbb{R}^n, \vec{v} \neq \vec{0}$
We define the projection of $\vec{u}$ *on* $\vec{v}$ *as*

$$Proj_{\vec{v}}(\vec{u}) = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2}\vec{v}$$

Also the perpendicular vector of $\vec{u}$ *onto* $\vec{v}$ *is defined as*

$$Perp_{\vec{v}}(\vec{u}) = \vec{u} - Proj_{\vec{v}}(\vec{u})$$

Proposition 7.5 *Given $\vec{v} \in \mathbb{R}^n, \vec{v} \neq \vec{0}$, and $c \in \mathbb{R}, c \neq 0$ we have*

1. $Proj_{c\vec{v}}(\vec{u}) = Proj_{\vec{v}}(\vec{u})$
2. $Proj_{\vec{v}}(\cdot)$ *is the "Linear Map"*
3. $Perp_{\vec{v}}(\cdot)$ *is also a linearmap*

7.3 Projection onto Planes

For a Plane P with $\vec{n}$, we define the projection to P as $Proj_P(\vec{u}) = Perp_{\vec{n}}(\vec{u})$

To Be Continued Next Lecture

End of Lecture Notes
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