Math 239 - Introduction to Combinatorics **Spring 2017**

Lecture 20: June 14th, 2017

Lecturer: Alan Arroyo Guevara *Notes By: Harsh Mistry*

Definition 20.1 *A graph is **Connected**, id for every two vertices $x, y \in V(G)$ there is a path from x to y.*

Theorem 20.2 *Let G be a graph and $v \in V(G)$. G is connected if and only if for every $w \in V(G)$ there is a path from v to w in G.*

Proof: Prove as an exercise.
Hint : Suppose that for very $w \in V(G)$, there is a path from v to w ■

Problem 20.3 *Show that the k-cube Q_k is connected.*
Solution : *By previous theorem, we only need to show that every $s \in V(Q_k)$ is connected (by using a path) to $00\dots0$, So suppose s has ℓ 1s.*

- $S_\ell = S$
- $S_{\ell-1}$ = *the string obtained from S_ℓ by replacing the first 1 in S_ℓ with a 0*

$\vdots$

- $S_0 = 00\dots0$

$P : S_\ell S_{\ell-1} \dots S_0$ *connects S to $00\dots0$*

Definition 20.4 *Given a $G = (V, E)$, a **Subgraph** $H = (V', E')$ is a graph such that $V' \subseteq V$ and $E' \subseteq E$*

Definition 20.5 *A **Component** of G is a sub-graph H such that*

(a) H is connected

(b) H is not a proper subgraph of a connected subgraph.

Given $u, v \in V(G)$, if we define $u \sim v$ if u and v are in the same component then

- $a \sim a$
- If $a \sim b \implies b \sim a$
- $a \sim b$ and $b \sim c \implies a \sim c$