, 12, 13

Stat 230 - Probability **Fall 2016**

Lecture 11, 12, 13: October 3, 5, 7, 2016

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11.1 Conditional Independence

Two events A and B are **Conditionally independent** given a third event Y if the occurrence or non-occurrence of A and the occurrence or non-occurrence of B are independent in their conditional probability given Y.

$$P(A \cap B \mid Y) = P(A \mid Y)P(B \mid Y)$$

11.2 Discrete Random Variables and Probability Models

11.2.1 Random Variable

- When an experiment is performed we are interested mainly in some function of the outcome as opposed to the actual outcome itself
- Random Variable : is a function that assigns a real number to each point in a sample space S.
- In essence, a random variable is a function whose domain is the sample space and whose range is the set of possible values of the variable.

Two Types of Random Variables

- Discrete random variable is a r.v whose possible values either integer or countable set om which there is a first element, second element, and so on.
- Continuous random variable is a r.v whose set of possible values consists of an entire interval on the number line.

11.2.2 Probability Function

- Probability function of a random variable, X, is a function

$$f(x) = P(X = x) \text{ Defined for all } x \in A$$

11.2.3 Probability Distributions for Discrete Random Variables

- A probability distributions of a random variable X is a description of the probabilities associated with the possible values of X
- The set of pairs $\{(x, f(x)) : x \in A\}$ is called the probability distribution of X
- A probability distribution says how the total probability of 1 is distributed among the various value of X

Properties

- $f(x) \geq 0$ for all $x \in A$
- $\sum_{x \in A} f(x) = 1$
- The p.f can be presented nicely in tabular form
- The p.f can also be displayed in a line graph
- The p.f can also be displayed using histogram which is called a probability histogram
  - The height of each rectangle is proportional to $f(x)$
  - The base is the same for all rectangle
- A p.f may be specified as a formula

Note : Its not always possible to find a simple formula

11.2.4 The Cumulative Distributive Function.

F(x) for discrete r.v for variable X with p.f. f(x) is defined for every number $x \in R$ by

$$F(x) = P(X \leq x) = \sum_{u:u \leq x} f(u)$$

Properties : Let F(X) denote the probability that the random variable X takes on a value that is less than or equal to x

- F is a non decreasing function; that is, if $a < b$, then $F(a) \leq F(b)$.
- $0 \leq F(X) \leq 1$, for all $x \in R$
- $\lim_{x \to \infty} F(x) = 1, \lim_{x \to -\infty} = F(x) = 0$

Proposition: For any numbers a and b with $a \leq b$

$$P(a \leq X \leq b) = F(b) = F(a-)$$

Where $(a-)$ is the largest x value that is strictly less than a