Econ 301 - Microeconomic Theory 2 **Winter 2018**

Lecture 11: February 12, 2018

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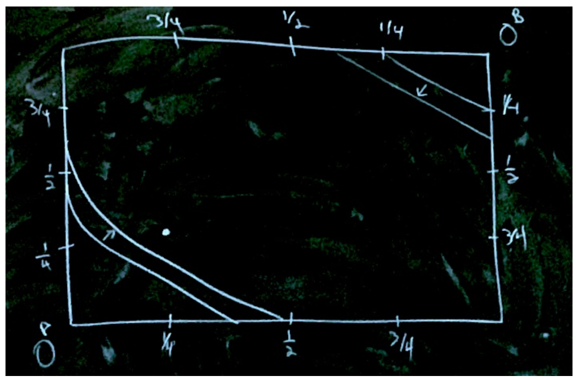
11.1 Competitive Equilibrium Continued

Example 11.1 *Say* $\omega^A = \left(\frac{1}{4}\right)$, $\omega^B = \left(\frac{3}{4}, \frac{3}{4}\right)$ *and* $u^A(x_1^A, x_2^A) = \ln x_1^A + x_2^A$, $u^B(x_1^B, x_2^B) = x_1^B + x_2^B$



- *Given prices* (p_1, p_2). *UMP of consumer A.*

$$\max_{x_1, x_2 \geq 0} \ln x_1^A + x_2^B \quad s.t. \;\; p_1 x_1^A + p_2 x_2^A \leq \frac{1}{4}p_1 + \frac{1}{4}p_2$$

- *Demand functions of A*

$$(x_1^A, (p\omega^A), x_2^A(p\omega^A)) = \begin{cases} \left(\frac{p_1+p_2}{4p_1}, 0\right) & if \; \frac{p_1}{p_2} \leq 3 \\ \left(\frac{p_2}{p_1}, \frac{p_1 - 3p_1}{4p_2}\right) & if \; \frac{p_1}{p_2} > 3 \end{cases}$$

- *UMP of Consumer B*

$$\max_{x_1, x_2 \geq 0} x_1^B + x_2^B \quad s.t. \;\; p_1 x_1^A + p_2 x_2^A \leq \frac{3}{4}p_1 + \frac{3}{4}p_2$$

- *Demand functions of B*

$$(x_1^B, (p\omega^B), x_2^B(p\omega^B)) = \begin{cases} \left(\frac{3p_1+3p_2}{4p_1}, 0\right) & if \; \frac{p_1}{p_2} < 1 \\ \left(0, \frac{3p_1+3p_2}{4p_2}\right) & if \; \frac{p_1}{p_2} > 3 \end{cases}$$

- *Normalize* $p_1^* = 1$, *find prices* $(1, p_1^*)$ *that clear one of the goods market*
- *Case 1: Can we have* $\frac{1}{p_1^*} > 3$ *? (MC2) is not satisfied*

$$x_2^A(p_1^* \omega^A) + x_2^A(p_1^*, \omega^B) = \frac{1 - 3p_2^*}{4p_2^*} + \frac{3 + 3p_2^*}{3p_2^*} = \frac{1}{p_2^*}$$

- *Case 2: Can we have $1 < \frac{1}{p_1^*} \leq 3$? (MC2) is not satisfied*

$$x_2 \frac{1-3p_2^*}{4p_2^*} + \frac{3+3p_2^*}{3p_2^*} = \frac{1}{p_2^*}^A (p_1^* \omega^A) + x_2^A(p_1^*, \omega^B) = \frac{3+3p_2^*}{4p_2^*} = \frac{3}{4} + \frac{3}{4}(\frac{1}{p_2^*})$$
$$> \frac{3}{2} > 1 = \omega_2^A + \omega_2^B$$

- *Case 3: Can we have $\frac{1}{p_1^*} > 1$? (MC2) is not satisfied*

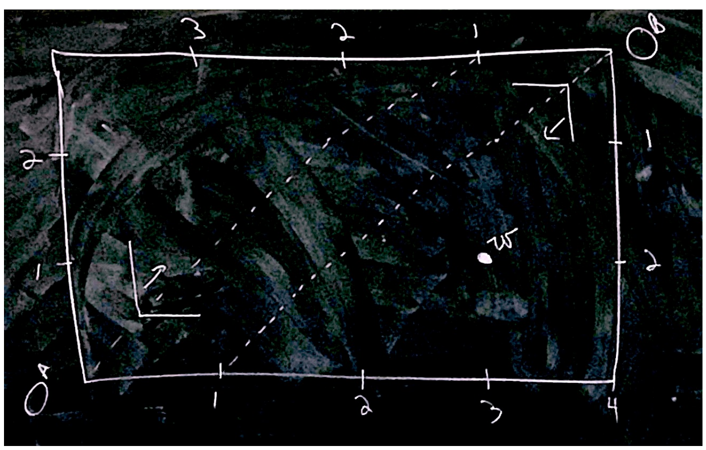
$$x_2 \frac{1-3p_2^*}{4p_2^*} + \frac{3+3p_2^*}{3p_2^*} = \frac{1}{p_2^*}^A (p_1^* \omega^A) + x_2^A(p_1^*, \omega^B) = 0 < 1 = \omega_2^A + \omega_2^B$$

- *Case 4: Can we have $\frac{1}{p_1^*} = 1$? (MC1) is satisfied*

$$x_2 \frac{1-3p_2^*}{4p_2^*} + \frac{3+3p_2^*}{3p_2^*} = \frac{1}{p_2^*}^A (p_1^* \omega^A) + x_2^A(p_1^*, \omega^B) = \frac{1}{2} + x_1^B(p_1^* \omega^B) = 1$$
$$\implies x_1^B(p_1^* \omega^B) = \frac{1}{2}$$
$$\implies \text{ Given budget constraint } x_2^B(p_1^* \omega^B) = 1$$

- *Prices (1,1) and allocations $(x_1^{A*}, x_2^{A*}) = \left(\frac{1}{2}, 0\right)$ and $(x_1^{B*}, x_2^{B*}) = \left(\frac{1}{2}, 1\right)$ form a competitive equilibrium*

Example 11.2 *Say $\omega^A = (3,1)$, $\omega^B = (1,2)$, $u^A(x_1^A, x_2^A) = \min\{x_1^A, x_2^B\}$, and $u^B(x_1^B, x_2^B) = \min\{x_1^B, x_2^B\}$*



- *Can we have $p_1^*, p_2^* \neq 0$? (MC1) fails*

$$\begin{aligned} x_1^A(p_1^* \omega^A) + x_1^B(p_1^* \omega^B) &= \frac{3p_1^* + p_2^*}{p_1^* + p_2^*} + \frac{p_1^* + 2p_2^*}{p_1^* + p_2^*} \\ &= \frac{4p_1^* + 3p_2^*}{p_1^* + p_2^*} \\ &= 3 + \frac{p_1^*}{p_1^* + p_2^*} < 4 \\ &= \omega_1^B + \omega_1^B \end{aligned}$$

- *Can we have* $p_1^* = 0$ *?*
  - *Demand functions for* $J = A, B$ *:*

$$(x_1^J(p_1\omega^J), x_2^J(p_1\omega^J)) = (\text{any } x_1^J \geq \omega_2^J, \omega_2^J)$$

$\implies$ *Given any* $1 \leq x_1^{A*} \leq 2$, $p^* = (0, 1)$ *and allocations* $x_1^{A*} = (x^A, 1)$ *and* $x_{B*} = (4 - x_1^A, 2)$ *form a competitive equilibrium*

  - *Additional case : Can we have* $p_2^* = 0$*? No, (MC2) fails*

$$\begin{aligned} x_2^A(p_1^*\omega^A) + x_2^B(p_1^*, \omega^A) &\geq \omega_1^A + \omega_1^B \\ &= 4 \\ &> 3 = \omega_2^A + \omega_2^B \end{aligned}$$