Math 128 : Calculus 2 for the Sciences **Winter 2016**

Lecture 6: January 15, 2016

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6.1 Trig Substitution

Interested in integrals containing $\sqrt{a^2 - b^2x^2}$, $\sqrt{a^2 + b^2x^2}$ or $\sqrt{b^2x^2 - a^2}$ (also powers like $(a^2 - b^2x^2)^{\frac{5}{2}}$)

Example 6.1 -
we can use subsitution to solve if we complete the sqauare

$$\begin{aligned}
\int \frac{1}{\sqrt{x^2+4x+5}}dx &= \int \frac{1}{\sqrt{(x+2)^2-4+5}}dx \\
&= \int \frac{1}{\sqrt{(x+2)^2+1}}dx \\
&= \int \frac{1}{\sqrt{(\tan^2\theta+1}}\sec^2\theta d\theta \qquad\qquad x+2=\tan\theta \implies dx = \sec^2\theta \\
&= \int \frac{1}{\sqrt{(\sec^2\theta}}\sec^2\theta d\theta \\
&= \int \sec\theta d\theta \\
&= \ln \mid \tan\theta + \sec\theta \mid + c \\
&= \ln \mid (x+2)\sqrt{x^2+4x+5} \mid + c
\end{aligned}$$

6.2 Partial Fractions

Used for integrating rational functions which are in the forn $f(x) = \frac{p(x)}{q(x)}$ where p(x) and c(x) are polynomials

Example 6.2 -
$\int \frac{6x+8}{x^2+3x+2}dx$ *can be evaulated with regular subsitution, but takes to long.*
Using partial fractions we find that $\int \frac{6x+8}{x^2+3x+2}dx = \int \frac{2}{x+1} + \frac{4}{x+2}$ *which is easier to integrate and results in* $2\ln \mid x+1 \mid 4\ln \mid x+1 \mid + c$

Steps

1. If degree p(x) is greater that degree q(x), then divide q(x) into p(x) using long divison
2. Factor q(x)
3. For every linear factor $(ax+b)^n$ in q(x) include the following terms $\frac{A_1}{ax+B} + \ldots + \frac{A_n}{(ax+B)^n}$
4. Multiply by all factors to get rid of the denominators

5. Compare coefficients
6. Solve for A , B , C , etc
7. Sub Terms back in and integrate

End of Lecture Notes
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