Math 128 : Calculus 2 for the Sciences **Winter 2016**

Lecture 2: January 6, 2016

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2.1 Integration by Subsitution Examples

Example 2.1

$$\begin{aligned}\int \frac{x^3}{(x+5)^2} &= \int \frac{(u-5)^3}{u} du \quad \textbf{\textit{Let u = x + 5}} \\ &= \int \frac{u^3 - 15u^2 + 75u - 125}{u^2} \\ &= \int u - 15 + \frac{75}{u} - \frac{125}{u^2} \\ &= \left[\tfrac{u^2}{2} - 15u + 75\ln u + \tfrac{125}{u} + c\right] \\ &= \left[\tfrac{(x+5)^2}{2} - 15(x+5) + 75\ln(x+5) + \tfrac{125}{x+5} + c\right]\end{aligned} \tag{2.1}$$

Example 2.2

$$\begin{aligned}\int_1^2 \frac{e^{\frac{1}{x}}}{x^2} dx &= -1\int_1^{\frac{1}{2}} e^u du \quad \textbf{\textit{Let u =}} \ \frac{1}{x} \\ &= -\left[e^u\right]_1^{\frac{1}{2}} \\ &= -e^{\frac{1}{2}} + e\end{aligned} \tag{2.2}$$

Example 2.3

$$\begin{aligned}\int \tan x dx = \int \frac{\sin x}{\cos x} &= -\int \frac{1}{u} du \quad \textbf{\textit{Let u =}} \ \cos x \\ &= -\ln \mid u \mid \ + c \\ &= -\ln \mid \cos x \mid \ + c \quad \textbf{\textit{or}} \ \ln \mid \sec \mid \ + c\end{aligned} \tag{2.3}$$

Practice Problem

$$\int \sec x \ dx \ \text{Hint : Multiply by} \ \frac{\sec x + \tan x}{\sec x + \tan x}$$

Example 2.4

$$\int_{-1}^1 \frac{\sin x}{1+x^2} dx = 0 \ \textbf{\textit{Integral is 0 because the function is odd}} \tag{2.4}$$

2.2 Integration by Parts (IBP)

IBP Formula

$$\int udv = uv - \int vdu$$

Why?
Reverse Engineering the product rule results in the IBP formula

Proof:

The product rule : $\frac{d}{dx}(f(x)g(x)) = f\prime(x)g(x) + f(x)g\prime(x)$

Integrate both sides with respect to x

$$\int \frac{d}{dx}(f(x)g(x))dx = \int f\prime(x)g(x) + f(x)g\prime(x)dx$$

$$\implies f(x)g(x) = \int f\prime(x)g(x)dx + \int f(x)g\prime(x)dx$$

$$\implies \int f(x)g\prime(x)dx = f(x)g(x) - \int f\prime(x)g(x)$$

let u = f(x) and v = g(x)

$$\therefore \int udv = uv - \int vdu$$

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Process :
So, divide the integrand into 2 parts u & dv. Then apply the IBP formula.

You'll find that calculating $\int udv$ reduces $\int vdu$

Example 2.5

$$\int xe^x$$

Suppose $u = x$ & $dv = e^x\ dx \ \rightarrow du = dx$ & $v = e^x$

$$\int xe^x\ dx = xe^x - \int e^x dx$$

$$= xe^x - e^x + c$$

Practice Problem

$$\int x^2 \cos x\ dx$$

End of Lecture Notes
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