Math 239 - Introduction to Combinatorics **Spring 2017**

Lecture 12: May 26th, 2017

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Remark

In general. an unambiguous expression is one for which every string can be constructed in a unique way by the rules in the expression.

Sometimes it is better to decompose after each occurrence or block of 1's

12.1 Tackling Binary problems

1. Decompose your set S using decomposition rules.
2. Find $[x^4]\phi_S(x)$

Problem 12.1 *Show that the number of binary strings of length n, where no block has length exactly two is equal to*

$$[x^n]\frac{1-x^2+x^3}{1-2x+x^2-x^3}$$

Solution:

1.

$$S = (\{\epsilon, 0\} \cup \{000\}\{0\}^*)(\{1, 111, 1111, \ldots\}\{0, 000, 0000\})^*(\{\epsilon, 1\} \cup \{111\}\{1\}^*)$$

2.

$$\begin{aligned}
\phi_{S_1}(x) &= 1 + x + x^3 + x^4 + \ldots \\
&= 1 + x + x^3(1 + x + x^2 + \ldots) \\
&= 1 + x + \frac{x^3}{1-x} \\
&= \frac{1-x^2+x^3}{1-x} \\
\phi_{S_3}(x) &= \frac{1-x^2+x^3}{1-x} \\
\phi_{S_2}(x) &= \phi_{\{1,111,\ldots\}}(x)\phi_{\{0,000,0000\}} \\
&= (x + x^3 + x^4 + \ldots)(x + x^3 + x^4 \ldots) \\
&= (x + x^3 + (1 + x + x^2 + \ldots))^2 \\
&= \left(x + \frac{x^3}{1-x}\right)^2 \\
&= \left(\frac{x - x^2 + x^3}{1-x}\right)^2
\end{aligned}$$

$$\begin{aligned}
\phi_S(x) &= \phi_{S_1}(x)\phi_{S_2}(x)\phi_{S_3}(x) \\
&= \frac{1-x^2+x^3}{1-x}\frac{1}{1-\left(\frac{x-x^2+x^3}{1-x}\right)^2}\frac{1-x^2+x^3}{1-x} \\
&= \dots \\
&= \frac{1-x^2+x^3}{1-2x+x^2-x^3}
\end{aligned}$$

Problem 12.2 *Find the number of binary strings of length n in which*

- *Every even block of 0's is followed by an odd number of 1's.*
- *Every odd block of 0's is followed by an even number of 1s (possibly followed by zero 1's)*

Solution :

1.

$$S = \{1\}^*(\{00\}\{00\}^*\{1\}\{11\}^* \cup \{0\}\{00\}^*\{11\}\{11\}^*)^*(\{\epsilon\} \cup \{0\}\{00\}^*)$$

2.

$$\phi_{S_1} = \frac{1}{1-\phi_{S_1}(x)} = \frac{1}{1-x}$$

$$\phi_{S_2}(x) = x^2\frac{1}{1-x^2}x\frac{1}{1-x^2}$$

$$\phi_{S_3}(x) = x\frac{1}{1-x^2}x^2\frac{1}{1-x^2}$$