CS 245 - Logic and Computation	**Fall 2016**

Lecture 3: September 15, 2016

Lecturer: Kevin Lanctot — *Notes By: Harsh Mistry*

3.1 Review

- Propositions : True / False
- Syntax : Format
- Symbols :
  - p, q, r
  - $\wedge, \vee, \neg, \implies, \iff$
  - ()
- Expression : $((P \wedge Q) \implies R)$

3.2 Unique Readability of Formulas

Theorem 3.1 *Every well-formed formula has a unique derivation as a well-formed formula. So, each well-formed formula has exactly one the following forms:*

1. *An atom*
2. $(\neg\alpha)$
3. $(\alpha \wedge \beta)$
4. $(\alpha \vee \beta)$
5. $(\alpha \implies \beta)$
6. $(\alpha \iff \beta)$

In each case, it is of that form in exactly one way. In essence, there is only one way of reading the formula and breaking it down.

Simple and Strong Induction Review

	Simple Induction	Strong Induction (Course of value)
Basis	Show P(0)	Show P(0)
Ind. Hypothesis	P(k) holds	**P(m) holds for every M $\leq$ k**
Ind. Step	Show P(k+1) holds	Show P(k+1) Holds
Conclusion	P(k) holds for every k	P(k) holds for every k

3.2.1 Structural Induction

A formula is not a natural number, but it suffices to prove any one of the following

- For every natural number n, every formula with n or fewer symbols has property P.
- For every natural number n, every formula with n or fewer connectives has property P.
- For every natural number n, every formula whose parse tree has height n or less has property P.
- or , For every natural number n, every formula whose parse tree has height n or less has property P.

Induction applied tp any of the formulations above, requires steps that show:
If $P(\alpha)$ and $P(\beta)$ then $P((\neg\alpha))$ and $P(\alpha * \beta)$.

Formulas α and β have smaller n values that $\neg\alpha$ and $\alpha * \beta$ do. Where * represents a connective operator ,

3.2.2 Principal of Structural Induction

Theorem 3.2 *Let R be a property. Suppose that*

1. *for each atomic formula p, we have R(p); and*
2. *for each formula α, if $R(\alpha)$ then $R((\neg\alpha))$; and*
3. *for each pair of formulas α and β and each binary connective *, if $R(\alpha)$ and $R(\beta)$ then $R(\alpha * \beta)$*

Then $R(\alpha)$ for every formula α

Use of this principle is called ***Structural Induction****, which is a special case of mathematical induction*

3.2.3 Proof of Unique Readability

Proof: Property P(n) :

Every Formula φ contain at most n connectives satisfies all there of the following.

A: The first symbol of φ is either '(' or variable

B: φ has an equal number of '(' and ')' and each proper prefix of φ has more '(' than ')'.

C: φ has a unique construction as a formula

Note : A **Proper Prefix** of φ is a non-empty expression x such that φ is xy for some non-empty expression y.

We can prove property P for all n by induction

Inductive Hypothesis : P(k) holds for some natural number k

We Must Prove : P(k+1) holds, to do this we let formula α have k+1 connectives

A key case : α is $\beta * \gamma$. For property C, we must show that α is $\beta' *' \gamma'$ for **Formulas** β' and γ', then $\beta = \beta'$, $* = *'$, and $\gamma = \gamma'$,

If β' has the same length as β, then they must be the same string

Otherwise, either β' is a proper prefix of β or β is a proper prefix of β' . But since β and β' are formulas with at most k connectives, the inductive hypothesis applies to them. In particular, each has property B, and thus neither can be a proper prefix of the other.

Thus β has a unique derivation, as required by property C.

■

3.2.4 Options for a Proof

1. Prove A, B and C simultaneously, as a single compound property. (Section 3.2.2)
2. Prove them separately: first A, then B, and finally C.

3.3 Truth Tables

Columns: List all the propositional variables on left and all the subformulas (in increasing order of the number of connectives) on the right.

Rows: Create a row for every possible combination of truth valuations for the propositional variables.