Econ 301 - Microeconomic Theory 2	**Winter 2018**

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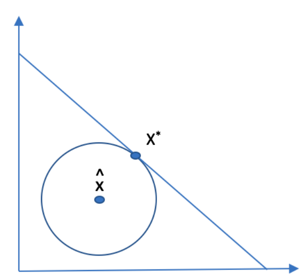
5.1 Consumer Choice Continued

5.1.1 Optimal consumer choice

In class numbering : 1.1.5

- Consumers choice problem has been reduced to constrained optimization problem (Utility Maximization Problem):
$$max_{x_1,x_2\geq 0}\ u(x_1,x_2) \text{ subject to } p_1x_1+p_2x_2\leq m \text{ (PI)}$$
- Solutions to this problem are <u>demand functions</u> $x_1(p_1m)$ and $x_2(p_2m)$
- Constraint in (PI) is an inequality constraint
- We want conditions under which solutions to (PI) are also solutions to the simpler problem
$$max_{x_1,x_2\geq 0}u(x_1,x_2) \text{ such that } p_1x_1+p2x_2=m \text{ (PE)}$$
- In other words, under what conditions do solutions to (PI) lie on the budget line.
- Result : If consumer preferences are monotone, then any solution to (PE) must be a solution to (PI)
- For some non-monotone preferences, consumers can find it optimal not to exhaust her budget.



  - $\hat{x}$ solution to (P.F.)
  - x^* solution to (P.E.), but not the solution to the original problem, as there is a higher indifference curve that intersects the budget line.
- If consumers preferences are monotone, (PE) still needs to be solved
- Use method of Lagrange if u is differentiable

1. Define <u>Lagrangian</u>
$$L(x_1, x_2, \lambda) = u(x_1, x_2) + \lambda(m - p_1x_1 - p_2x_2)$$
A function of bundles $x \in \mathbb{R}^2_+$ and <u>Lagrange multiplier</u> $\lambda \in \mathbb{R}$
2. If the solution x^* to (PE) is such that $x_1^*x_2^2 \neq 0$, then it must solve the system of first-order conditions:
$$\begin{cases} \frac{d}{dx}L(x_1^*, x_2^*, \lambda) = \frac{d}{dx}u(x_1^*, x_2^*) - \lambda p_i = 0 \text{ (Li)}, \; i = 1, 2, \ldots \\ \frac{d}{dx}L(x_1^*, x_2^*, \lambda) = m - p_1x_1^* - p_2x_2^* = 0 \text{ (L}\lambda\text{)}, \end{cases}$$

- The restriction that $x_1^*, x_2^* \neq 0$ implies that solution x^* is <u>interior</u>
- $L(1) - L(\lambda)$ is a system of three equations and three unknowns x_1^*, x_2^*, λ, and it can be solved in special cases.
- Substitute out λ by using (LI) and (L2)
$$\frac{\frac{d}{dx}u(x_1^*, x_2^*)}{\frac{d}{dx}u(x_1^*, x_2^*)} = \frac{p_1}{p_2} \text{ (MRS) - Marginal rate of exhance}$$
Consumers Marginal rate of substitution : rate at which consumers are wiling to exchange good 2 against additional units of good 1
- Let x^* denote consumers with level from x^*
$$u(x_1^*, x_2^*) - u^*$$
- Question : Starting from x^*, how much of good 2 would consumer be wiling to sacrifice in order to obtain an additional unit of good 1 whole keeping his/her utility atu^*
- Total derivative of $\star$ with x_11
$$\frac{d}{dx_1}u(x_1^*, x_2k)\frac{dx_1^*}{dx_1} + \frac{d}{dx2}u(x_1^*, x_20\frac{dx_1^*}{dx_1} = \frac{du^*}{dx_1} = 0$$
$$\frac{dx_2^*}{dx_1} = \frac{\frac{d}{dx_!1}u(x_1^*, x_2^*)}{\frac{d}{dx_2}u(x_1^*, x_2^*}$$
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- Suppose that solution to (PE) is such that $x_2^* = 0$. Then we must have that $x_1^* = \frac{m}{p_1}$
- A necessary condition for $(\frac{m}{p_1}, 0)$ to be optimal is that increasing consumption of good 2 while staying on budget line cannot increase consumers utility.
- i.e
$$\frac{d}{dx_1}u(\frac{m}{p_1}, 0)\frac{dx_1^*}{dx_1} + \frac{d}{dx_2}u(\frac{m}{p_1}, 0)\frac{dx_2^*}{dx_2} \leq 0$$
or
$$\frac{\frac{d}{dx_1}u(\frac{m}{p}, 0)}{\frac{d}{dx_2}u(\frac{m}{p}, 0)} \geq \frac{p_1}{p2}$$
In other words, the slope of the indifference curve is as steep or steeper than the slope of the budget line

- Similarly, necessary condition for $(0, \frac{m}{p_2})$ to be optimal is

$$\frac{\frac{d}{dx_1}u(\frac{m}{p_2}, 0)}{\frac{d}{dx_2}u(\frac{m}{p_2}, 0)} \leq \frac{p_1}{p2}$$

  In other words, the slope of the indifference curve is as steep or less steeper than the slope of the budget line