

## Lecture 29: March 16, 2016

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## 29.1 Ratio Test Con't

Notes:

- Works well when  $a_n$  contains  $n!$  or  $(\text{constant})^n$
- Does not work well if  $a_n$  is rational
- Does not require all positive terms
- The term that goes in the numerator matters

## 29.2 Power Series

A power series is an infinite series of the form

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots$$

where  $x$  is a variable and  $c$ 's are constants, called the coefficients of the series.  
More generally, a power series centered at  $a$  or about  $a$  is

$$\sum_{n=0}^{\infty} c_n (x - a)^n = c_0 c_1 (x - a) + c_2 (x - a)^2 + c_3 (x - a)^3 + \dots$$

Also, we know the geometric series  $\sum x^{n-1}$  is a power series which diverges when  $|x| \leq 1$   
So this tells that  $\sum x^{n-1} = \frac{1}{1-x}$  on the interval  $(-1,1)$

We say that  $(-1,1)$  is the interval of convergence and  $R = 1$  is the radius of convergence for the series

For any power series we are often interested in determining for which values of  $x$  the power series converges.  
We can use the ratio test to do so.

**Example 29.1** For what values of  $x$  does  $\sum_{n=0}^{\infty} \frac{1}{n+1} (x-2)^n$  converge?

- Using the ratio test we get  $\lim_{n \rightarrow \infty} = |x - 2|$
- It converges absolutely if  $|x - 2| < 2 \iff 1 < x < 3$
- It diverges if  $|x - 2| > 1 \iff x > 3 \text{ or } x < 1$

- At  $x = 1$ , the series converges and At  $x = 3$ , the series diverges
- So, the power series converges for  $1 \leq x < 3$

**Thus,**

The Interval of convergence is  $[1,3)$

The Radius of convergence is 1

The center of convergence is 2

**End of Lecture Notes**  
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