Math 128 : Calculus 2 for the Sciences	**Winter 2016**

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Lecturer: Jen Nelson — *Notes By: Harsh Mistry*

7.1 Partial Fraction Examples

Example 7.1 -

$$\int \frac{4x}{3x^2 - 5x^2 + x + 1} dx$$
$$\frac{4x}{3x^2 - 5x + 4} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{3x+1}$$
$$\implies 4x = A(x+1)(3x+1) + B(3x+1) + c(x-1)^2$$

- $x = 1 : 4(1) = B(3(1)+1) \implies B = 1$
- $x = \frac{1}{3} : \frac{4}{3} = c(\frac{-4}{3})^2 \implies C = \frac{-3}{4}$
- $x = 0 : 0 = -A + B + C \implies A = \frac{1}{4}$

Example 7.2

$$\int \frac{x^3 - 2x^2 + 8x - 4}{x(x^2+2)^2} dx = \int \frac{A}{x} + \frac{Bx + C}{x^2+2} + \frac{Dx + E}{(x^2+2)^2} dx$$

Expand, Multiply, and then compare coefficients (Do as practice)
A = -1 B = -1 C = 1, D = 0, E = 6

$$\begin{aligned}
\int \frac{x^3 - 2x^2 + 8x - 4}{x(x^2+2)^2} dx7 &= \int \frac{-1}{x} + \frac{x+1}{x^2+2} + \frac{6}{(x^2+2)^2} dx \\
&= \int \frac{-1}{x} dx + \int \frac{x+1}{x^2+2} dx + \int \frac{6}{(x^2+2)^2} dx \\
&\vdots \\
&= -\ln | x | + \frac{1}{2} \ln | x^2 + 2 | + \frac{5\sqrt{2}}{4} \arctan(\frac{x}{\sqrt{2}}) + \frac{3x}{4(x^2+2)}
\end{aligned}$$

Example 7.3 *Determine the form of the partial fraction decomposition of :*

$$\frac{x+1}{(x)(x-1)^2(x+2)(x^2-2x+4)(x^2+1)^2}$$
$$= \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-1} + \frac{D}{(x-1)^2} + \frac{Ex+F}{x^2-2x+4} + \frac{Gx+H}{x^2+1} + \frac{Jx+K}{(x^2+1)^2}$$

7.2 Improper Integral (Type 1)

Infinite Interval ($a = \infty$ and/or $b = \infty$)
Consider $\int_1^\infty \frac{1}{x^2} dx$ This integeral is improper since the upper limit is not finite. Think of the integral in terms of area.

Consider the area under $\frac{1}{x^2} on$ [1, t] where t is finite.
We can then use FTC II: $\int_1^t \frac{1}{x^2} dx = \frac{1}{x} |_1^t = 1 - \frac{1}{t}$
We can determine the area on $[1, \infty]$ by letting $t \to \infty$:

$$\int_1^\infty \frac{1}{x^2} dx = \lim t \to \infty \int_1^t \frac{1}{x^2} dx = \lim_{t\to\infty} (1 - \frac{1}{t}) = 1$$

End of Lecture Notes
Notes By : Harsh Mistry