Math 136 - Linear Algebra **Winter 2016**

Lecture 26: March 9, 2016

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26.1 Elementary Matrices

Definition 26.1 *An $n \times m$ matrix E is called an elementary matrix if it can be obtained from the $n \times n$ indentiy matrix by performing exactly one matrix operation.*

Example 26.2 *The following are elementary matricies*

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Theorem 26.3 *If A is an $m \times n$ matrix and E is the $m \times m$ elementary matrix corresponding to the row operation $R_i + cR_j$, for $i \neq j$, then EA is the matrix obtained from A by performing the row operation $R_i + cR_j$ on A.*

Theorem 26.4 *If A is an $m \times n$ matrix and E is the $m \times m$ matrix corresponding to the row operation cR_i, then EA is the matrix obtained from A by performing the row operation cR_i on A*

Theorem 26.5 *If A is an $m \times n$ matrix and E is the $m \times m$ matrix corresponding to the row operation $cR_i \longleftrightarrow r_j$ for $i \neq j$, then EA is the matrix obtained from A by performing the row operation $cR_i \longleftrightarrow r_j$ on A*

Note : Multiplying a matrix on the left by an elemtary matrix is the same as performing the corresponding elementary row operation on A

Corollary 26.6 *If A is an $m \times n$ matrix and E is an $m \times m$ elementary matrix, then*

$$rank(EA) = rank A$$

Theorem 26.7 *If A is an $m \times n$ matrix with reduced row ecelon form R, then there exists a sequence $E_1, \ldots, E_k$ of $m \times m$ elementary matrices such that $E_k \ldots E_2 E_1 A = E$, particularly*

$$A = E_1^{-1} E_2^{-1} \ldots\ E_k^{-1} R$$

Proof: The first conclusion follows from the Gauss Elimination. The second part holds since

$$A = (E_1 \ldots E_k)^{-1} R = E_k^{-1} \ldots E_1^{-1} R$$

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Corollary 26.8 *If A is a $n \times n$ invertible matrix, then A and A^{-1} can be written as a product of elemntary matricies*

End of Lecture Notes
Notes by : Harsh Mistry