

Lecture 12: February 14, 2018

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12.1 Competitive Equilibrium Continued

Example 12.1 *Competitive equilibria need not exist. Let $\omega^A = (1, 1) = \omega^B$, $u^A(x_1^A, x_2^A) = \max\{x_1^A, x_2^A\}$ and $u^B(x_1^B, x_2^B) = x_1^{B\frac{1}{2}}x_2^{B\frac{1}{2}}$*



- Demand Function for A

$$(x_1^A(p_1\omega^A), x_2^A(p_1\omega^A)) = \begin{cases} \left(\frac{p_1+p_2}{p_1}, 0\right) & \text{if } \frac{p_1}{p_2} < 1 \\ \left(\frac{p_1+p_2}{p_1}\right) \text{ or } \left(0, \frac{p_1+p_2}{p_2}\right) & \text{if } \frac{p_1}{p_2} = 1 \\ \left(0, \frac{p_1+p_2}{p_2}\right) & \text{if } \frac{p_1}{p_2} > 1 \end{cases}$$

- Demand Function for B

$$(x_1^B(p_1\omega^B), x_2^B(p_1\omega^B)) = \left(\frac{p_1+p_2}{2p_1}, \frac{p_1+p_2}{2p_2}\right)$$

- Case 1 : Can a competitive equilibrium have $p_1^* = 0$? No, $x_1^B(p_1^*\omega^B)$ is undefined if $p_1^* = 0$
- Case 2 : Can we have a competitive equilibrium with $\frac{p_1^*}{p_2} < 1$? No, (MC2) fails

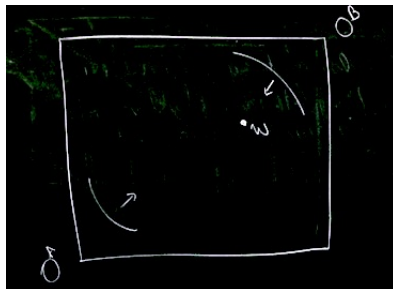
$$\begin{aligned} x^B(p_1^*\omega^B) &= \frac{p_1^* + p_2^*}{2p_2} = \frac{1}{2} \frac{p_1^*}{p_2^*} + \frac{1}{2} < 1 \\ &= \omega_2^A + \omega_2^B \end{aligned}$$

- Case 3 : A similar argument shows that no competitive equilibrium with $\frac{p_1^*}{p_2} > 1$ exists
- Case 4 : Can we have a competitive equilibrium with $\frac{p_1^*}{p_2} = 1$? No, A demands 2 units if some good B demands 1 unit of each good
- Therefore, some consumers don't meet equilibrium

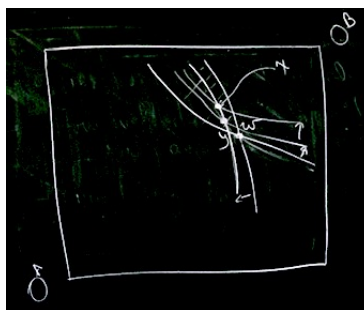
12.2 Welfare

In-Class Numbering : 3.0

- So far, we have studied positive, or descriptive properties of competitive equilibriums (i.e do they exists, how do prices vary with endowments, preferences, etc)
- Given this, our goal is to study normative properties of competitive equilibrium (i.e which depend on value judgement)
 - How well do markets perform in allocating goods across consumers?
 - How do they compare against alternative exchange mechanisms?
 - We need to develop normative criteria to evaluate the performance of markets and/or other mechanisms.
- What would bargaining between consumers look like?
 - Consider a two consumer economy



- Bargaining protocols can be varied and complex, as a result we focus on describing allocations that are consistent with some bargaining process.



- Any bargained agreement must deliver allocation x^J to consumer $J = AB$ such that

$$u^J(x_1^J, x_2^J) \geq u^J(\omega_1^J, \omega_2^J)$$

- Any bargained agreement must be feasible

$$x_i^A + x_i^B \leq \omega_1^A + \omega_2^B \text{ for } i = 1, 2 \text{ (with equality if preferences are monotone)}$$

- Are all feasible and individually rational allocations candidate bargained agreements? No, such allocations can be c to mutually beneficial counterproposal

To be continued next lecture with examples