**Math 128 : Calculus 2 for the Sciences** **Winter 2016**

# Lecture 5: January 13, 2016

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## 5.1 Trig Substitution

Interested in integrals containing $\sqrt{a^2 - b^2x^2}$, $\sqrt{a^2 + b^2x^2}$ or $\sqrt{b^2x^2 - a^2}$ (also powers like $(a^2 - b^2x^2)^{\frac{5}{2}}$ )

**Example 5.1** -

$$\begin{aligned}\int \frac{1}{\sqrt{1-x^2}}dx &= \int \frac{1}{\sqrt{1-\sin^2\theta}}\cos\theta d\theta \\ &= \int \frac{\cos\theta}{\sqrt{\cos^2\theta}}d\theta \\ &= \int \frac{\cos\theta}{|\cos\theta|}d\theta \\ &= \int 1 d\theta \\ &= \theta + c \\ &= \arcsin x + c\end{aligned}$$

$$x = \sin\theta \implies dx = \cos x \implies \theta = \arcsin x$$

**Example 5.2** -

$$\begin{aligned}\int \frac{1}{\sqrt{4-9x^2}}dx &= \int \frac{1}{\sqrt{4-9(\frac{4}{9}\sin^2\theta)}}\frac{2}{3}\cos\theta d\theta \\ &= \frac{2}{3}\int \frac{\cos\theta}{2\sqrt{\cos^2\theta}}d\theta \\ &= \frac{1}{3}\int \frac{\cos\theta}{|\cos\theta|}d\theta \\ &= \frac{1}{3}\int 1 d\theta \\ &= \frac{1}{2}\theta + c \\ &= \frac{\arcsin\frac{3x}{2}}{3} + c\end{aligned}$$

$$x = \frac{2}{3}\sin\theta \implies dx = \frac{2}{3}\cos x$$

**Example 5.3** -

$$\int \frac{\sqrt{25x^2-4}}{x}dx = \frac{\sqrt{25(\frac{4}{25}\sec\theta)^2-4}}{\frac{2}{5}\sec\theta}\frac{2}{5}\sec\theta\tan\theta d\theta$$
$$= \int 2\sqrt{\sec^2\theta - 1}\tan\theta d\theta$$
$$= 2\int \sqrt{\tan^2\theta}\tan\theta d\theta$$
$$= 2\int \mid \tan\theta \mid \tan\theta d\theta$$
$$= 2\int \tan^2\theta d\theta \ ( \ \textit{since}\ \pi \leq \theta \leq \frac{3\pi}{2})$$
$$= 2\int \sec^2\theta - 1 d\theta$$
$$= 2\tan^2(\cos^{-1}\frac{5x}{2}) - 2\sec^{-1}\frac{5x}{2} + C$$
$$= 2(\frac{\sqrt{25x^2-4}}{2}) - 2\sec^{-1}\frac{5x}{2} + C$$

$$x = \frac{2}{5}\sec\theta \implies dx = \frac{2}{5}\sec\theta\tan\theta d\theta$$
$$\theta = \sec^{-1}\frac{5x}{2}$$

**In General**

$\sqrt{a^2-b^2x^2}$ sub in $x = \frac{a}{b}\sin\theta$ and use the identity $\cos^2\theta = 1-\sin^2\theta$
$\sqrt{a^2+b^2x^2}$ sub in $x = \frac{a}{b}\tan\theta$ and use the identity $\sec^2\theta = 1+\tan^2\theta$
$\sqrt{b^2x^2-a^2}$ sub in $c = \frac{a}{b}\sec\theta$ and use the identity $\tan^2\theta = \sec^2 - 1\theta$

**End of Lecture Notes**
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