Econ 301 - Microeconomic Theory 2 **Winter 2018**

Lecture 13: February 26, 2018

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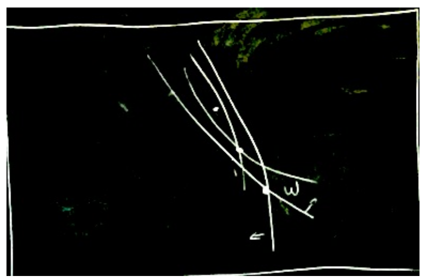
13.1 Welfare Continued



Definition 13.1 *Allocations x^A and x^B Pareto dominante allocations y^A and y^B if*

$$u^J(x_1^J, x_2^J) \geq u^J(y_1^J, y_1^J) \text{ for all } J = 1, 2,$$

with (at least) one inequality strict.

Definition 13.2 *Pareto-efficient allocations x^A and x^B are not Pareto-dominated by any feasible allocations y^A and y^B*

Result : Allocations consistent with bargaining between consumers are Pareto efficient allocations x^A and x^B such that $u^J(x_1^J, x_2^J) \geq u^J(\omega_1^J, \omega_2^J)$ for all $J = 1, 2$

Pareto efficiency is fundamental normative criterion in economic. Basically, its not a strong argument, but its an criteria that can support your statement.

- As a value judgement, Pareto is weak. Pareto-dominated allocations can be ruled out by unanimous votes
- Pareto-efficient allocations are often unsatisfactory relative to additional normative criteria : (e.g no notion of fairness)
- To find Pareto-efficient allocations :
  - Start with allocations y^A and y^B
  - Find optimal allocations x^A and x^B for consumer A such that consumer B is indifferent between x^B and y^B

- i.e. solution to UMP with equality constraint

$$\max_{\{0\leq x_i^A \leq \omega_i^A+\omega_i^B\}_{i=1,2}} u^A(x_1^A, x_2^A) \text{ s.t. } u^B(\omega_1^A+\omega_1^B - x_1^A, \omega_2^A+\omega_2^B - x_2^B) = u^B(y_1^B, y_2^B)$$

- Lagrangian

$$L(x_1^A, x_2^A, \lambda) = u^A(x_1^A, x_2^A) + \lambda[u^B(y^B, y_2^B) - u^B(\omega_1^A - \omega_1^B - x_1^A, \omega_2^A + \omega_2^B - x_2^A)]$$

- At an interior solution, we have the FOC

$$\frac{dL}{dx_i^A}(x_1^{A*}, x_2^{A*}, \lambda) = \frac{d}{dx_i^A}u^A(x_1^A, x_2^A) + \lambda\frac{d}{dx_i^A}u^B(\omega_1^A - \omega_1^B - x_1^A, \omega_2^A + \omega_2^B - x_2^A) = 0 \quad (Li), i = 1, 2$$

$$\frac{dL}{d\lambda}(x_1^{A*}, x_2^{A*}, \lambda) = 0 \quad (L\lambda)$$

- Let $x_i^B = \omega_i^A + \omega_i^B - x_i^{A*}$
- Using (L1) - (L2) :

$$\frac{\frac{d}{dx_1^A}u^A(x_1^A, x_2^A)}{\frac{d}{dx_2^A}u^A(x_1^A, x_2^A)} = \frac{\frac{d}{dx_1^B}u^B(x_1^B, x_2^B)}{\frac{d}{dx_2^B}u^B(x_1^B, x_2^B}$$

- Result : If both consumers' preferences are monotone and convex, then the solutions to FOC are Pareto-efficient allocations.

- Allocations y^A and y^B are arbitrary, can find more Pareto-efficient allocations by considering different initial allocations

Definition 13.3 *The set of all Pareto-efficient allocations is the* <u>*Pareto Set*</u>*, or* <u>*contract curve*</u>

Example 13.4 $\omega^A = (1,1), \omega^B = (1,2), \omega^A(x_1^A, x_2^A) = x_1^{A\frac{1}{2}} x_2^{A\frac{1}{2}}, u^B(x_1^B, x_2^B) = x_1^{B\frac{1}{2}} x_2^{B\frac{1}{2}}$

- *Given allocations* x^A *and* x^B

$$\frac{\frac{d}{dx_1^A} u^A(x_1^A, x_2^A)}{\frac{d}{dx_2^A} u^A(x_1^A, x_2^A)} = \frac{\frac{1}{2}\left(\frac{x_2^A}{x_1^A}\right)^{\frac{1}{2}}}{\frac{1}{2}\left(\frac{x_1^A}{x_2^A}\right)^{\frac{1}{2}}} = \frac{x_2^A}{x_1^A}$$

$$\frac{\frac{d}{dx_1^B} u^B(x_1^B, x_2^B)}{\frac{d}{dx_2^B} u^B(x_1^B, x_2^B} = \frac{1}{3}\frac{x_2^B}{x_1^B}$$

- *Both consumers preferences are monotone and convex, so the Pareto set consists of all solutions to*

$$\frac{\frac{d}{dx_1^A} u^A(x_1^A, x_2^A)}{\frac{d}{dx_2^A} u^A(x_1^A, x_2^A)} = \frac{\frac{d}{dx_1^B} u^B(2 - x_1^A, 2 - x)2^A)}{\frac{d}{dx_2^B} u^B(2 - x_1^A, 2 - x)2^A)}$$

Or

$$\frac{x_2^A}{x_1^A} = \frac{1}{3}\frac{3 - x_2^A}{2 - x_1^A}$$

$$x_2^A = \frac{3x_1^A}{2[3 - x_1^A]}$$