Math 136 - Linear Algebra **Winter 2016**

Lecture 18: February 12, 2016

Lecturer: Yongqiang Zhao *Notes By: Harsh Mistry*

18.1 Operations on Linear Mappings

Definition 18.1 *Let* $L : \mathbb{R}^n \to \mathbb{R}^m$ *and* $M : \mathbb{R}^n \to \mathbb{R}^m$ *be linear mappings and let* $c \in \mathbb{R}$*, we define* $L + M : \mathbb{R}^n \to \mathbb{R}^m$ *and* $cL : \mathbb{R}^n \to \mathbb{R}^m$ *by*

$$(L + M)(\vec{x}) - L(\vec{x}) + M(\vec{x})$$

$$(cL)(\vec{x}) = cL(\vec{x})$$

Theorem 18.2 *If* $L,M : \mathbb{R}^n \to \mathbb{R}^m$ *are liner mappings and* $c \in \mathbb{R}$*, then* $L + M : \mathbb{R}^n \to \mathbb{R}^m$ *and* $cL : \mathbb{R}^n \to \mathbb{R}^m$ *are linear mappings.*
Furthermore,

$$\left[L + M\right] = \left[L\right] + \left[M\right]$$

$$\left[cL\right] = c\left[L\right]$$

Definition 18.3 *We denote the set of all possible linear mappings with domain* $\mathbb{R}^n$ *and co-domain* $\mathbb{R}^m$ *as* $\mathbb{L}$

Theorem 18.4 *If* $L,\ M \in \mathbb{L}$ *and* $c,\ d \in \mathbb{R}$ *then,*

1. $L + M \in \mathbb{L}$
2. $(L + M) + N = L + (M + N)$
3. $L + M = M + L$
4. *There exists a linear mapping* $O : \mathbb{R}^n \to \mathbb{R}^m$*, such that* $L + O = L$ *for all* L
5. *There exists a linear mapping (-L):* $\mathbb{R}^n \to \mathbb{R}^m$ *with the property that L + (-L) = O*
6. $cL \in \mathbb{L}$
7. $c(dL) = (cd)L$
8. $(c + d)L = cL + cM$
9. $c(L + M) = cL + cM$
10. $1L = L$

Definition 18.5 *Let $L : \mathbb{R}^n \to \mathbb{R}^m$ and $M : \mathbb{R}^n \to \mathbb{R}^m$ be linear mappings.*
The composition of M and L is the function $M \circ L : \mathbb{R}^n \to \mathbb{R}^m$ defined by

$$(M \circ L)(\vec{x}) = M(L(\vec{x}))$$

Remarks : The range of L must be a subset of the domain of M for M $\circ$ L to be defined

Theorem 18.6 *If $L : \mathbb{R}^n \to \mathbb{R}^m$ and $M : \mathbb{R}^m \to \mathbb{R}^p$ are linear mappings, then $M \circ L : \mathbb{R}^n \to \mathbb{R}^m$ is a linear mapping and*

$$(M \circ L) = (M)(L)$$

Definition 18.7 *The Linear Mapping $ld : \mathbb{R}^n \to \mathbb{R}^m$ defined by $ld(\vec{x}) = \vec{x}$ is called the identity mapping*

18.2 Vector Spaces

Definition 18.8 *A set $\mathbb{V}$ with an operation of addition, denoted $\vec{x} + \vec{y}$ and an operation of scaler multiplication denoted $c\vec{x}$ is called a vector space over $\mathbb{R}$ if for every $\vec{v}, \vec{x}, \vec{y} \in \mathbb{V}$and $c,\ d \in \mathbb{R}$ we have :*

1. $\vec{x} + \vec{y} \in \mathbb{V}$
2. $(\vec{x} + \vec{y}) + \vec{v} = \vec{x} + (\vec{y} + \vec{v})$
3. $\vec{x} + \vec{y} = \vec{y} + \vec{x}$
4. *There is a vector $\vec{0} \in \mathbb{V}$ called the zero vector, such that $\vec{x} + \vec{0} = \vec{x}\ \forall \vec{x} \in \mathbb{V}$*
5. *There exists an element $-\vec{x} \in \mathbb{V}$ called the additive inverse of $\vec{x}$, such that $\vec{x} + \vec{-x} = \vec{0}$*
6. $c\vec{x} \in \mathbb{V}$
7. $c(d\vec{x}) = (cd)\vec{x}$
8. $(c + d)\vec{x} = c\vec{x} + d\vec{x}$
9. $c(\vec{x} + \vec{y}) = c\vec{x} + c\vec{y}$
10. $1x = x$

Elements of $\mathbb{V}$ are called vectors

Remarks : some times $\oplus$ and $\odot$ are used to differentiate these from normal scaler multiplication and scaler addition

End of Lecture Notes
Notes By : Harsh Mistry