Econ 301 - Microeconomic Theory 2 **Winter 2018**

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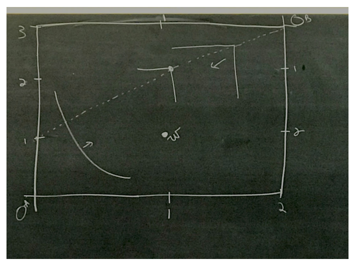
10.1 Competitive Equilibrium Continued

Example 10.1 *Suppose* $\omega^A = (1,1)$, $\omega^B = (1,2)$, $u^A(x_1^A, x_2^A) = x_1^{A1/2} x_2^{A1/2}$ $u^B(x_1^B, x_2^B) = min\{x_1^B, x_2^B\}$



Consumer Demands :

$$(x_1^A(p_1 m), x_2^A(p_1 m)) = \left(\frac{m^A}{2p_1}, \frac{m^A}{2p_2}\right) = \left(\frac{p_1+p_2}{2p_1}, \frac{p_1+p_2}{2p_2}\right)$$

$$(x_1^B(p_1 m), x_2^B(p_1 m)) = \left(\frac{m^B}{p_1+p_2}, \frac{m^B}{p_1+p_2}\right) = \left(\frac{p_1+2p_2}{p_1+p_2}, \frac{p_1+2p_2}{p_1+p_2}\right)$$

Would P(1, 1) be a good prediction of the prices in this economy?

$$x_1^A((1,1),\omega^A) = 1, \;\; x_1^B((1,1),\omega^B) = \frac{3}{2}$$

No, because $1 + \frac{3}{2} > 2$ *which aggregate demand for good 1 to exceed aggregate endowment*

What if $p = (1, \sqrt{2} - 1)$

$$(x_1^A(p_1 m), x_2^A(p_1 m)) = \left(\frac{1}{\sqrt{2}}, \frac{\sqrt{2}}{2\sqrt{2}\cdot 2}\right)$$

$$(x_1^B(p_1 m), x_2^B(p_1 m)) = \left(\frac{2\sqrt{2}-1}{\sqrt{2}}, \frac{2\sqrt{2}-1}{\sqrt{2}}\right)$$

Definition 10.2 *A Competitive equilibrium* (x^{A*}, x^{B*}, p^*) *consists of an allocation of goods* $X^{J*} = (X_1^{J*}, X_2^{J*})$ *for each consumer* $p^* = (p_1^*, p_2^*)$ *which satisfy*

1. *Given prices* $p^* = (p_1^*, p_2^*)$, *the allocation* X^{J*} *for consumer* $J = 1, 2$ *is a solution to UMP*

$$\max_{x^J \in \mathbb{R}_+^2} u^J(x_1^J, x_2^J) \;\; s.t. \;\; p_1 x_1^J + p_2 x_2^J \leq p_1 \omega_1^J + p_2 \omega_2^J$$

2. *For each good $i = 1, 2$, the aggregate allocations exhaust aggregate endowments :*

$$x_i^{A*} + x_i^{B*} = \omega_i^A + \omega_i^B \quad (MCi)$$

- (MC1) and (MC2) are market clearing conditions
  - Through consumers' demand functions, (MC1) and (MC2) is a system of 2 equations in 2 unknowns p_1^* and p_2^*
  - If (p_1^*, p_2^*) one equilibrium prices, then given any $\alpha > 0$, $(\alpha p_1^*, \alpha p_2^*)$ are equilibrium prices that support the same allocation and hence demands are the same under (p_1^*, p_2^*) and $(\alpha p_1^*, \alpha p_2^*)$
  - A common approach is to normalize $p_1^* = 1$
- Now we have 2 conditions (MC1) and (MC2) to determine p_2^*?
- Result ; If consumer' preferences are monotone and if (MC1) holds, then (MC2) also holds
  - If consumers' preferences are monotone, then optimal bundles exhaust their budget :

$$x_1^{J*} + p_2^* x_2^{J*} = \omega_1^J + p_2^* \omega_2^J \text{ for } J = 1, 2$$

- Then aggregate spending must equal value of aggregate endowment :

$$x_1^{A*} + x_1^{B*} + p_2^*[x_2^{A*} + x_2^{B*}] = \omega_1^A = \omega_1^B - p_2^*[\omega_2^A + \omega_2^B]$$

- Rewrite : (LHS = 0, if (MC1) holds)

$$x_1^{A*} - x_1^{B*} - [\omega_1^A + \omega_1^B] = p_2^*[\omega_2^A + \omega_2^B - [x_2^{A*} + x_2^{B*}]]$$

Example 10.3 *Continued from 9.1*
Any equilibrium prices $p^* = (1, p_2^*)$ must satisfy (MC1) :

$$x_1^A((1, p_1^*), \omega^A) + x_1^B((1, p_2^*), \omega^B = 2$$

$$\frac{1 + P_2^*}{2} + \frac{1 + 2p_2^*}{1 + p_2^*} = 2 \implies (p_2^*)^2 + 2p_2^* - 1 = 0$$

- *Two Roots are $-\sqrt{2} - 1 < 0$ and $\sqrt{2} + 1 > 0$. Therefore prices $p^*(1, \sqrt{2} - 1)$ and allocations*

$$X^{A*} = \left(\frac{1}{\sqrt{2}}, \frac{\sqrt{2}}{2\sqrt{2} - 2}\right) \text{ and } X^{B*} = \left(\frac{2\sqrt{2} - 1}{\sqrt{2}}, \frac{2\sqrt{2} - 1}{\sqrt{2}}\right)$$

  are a competitive equilibrium