Math 239 - Introduction to Combinatorics	**Spring 2017**

Lecture 19: June 12th, 2017

Lecturer: Alan Arroyo Guevara — *Notes By: Harsh Mistry*

Definition 19.1 *A **Closed Walk** is one which $v_0 = v_n$.*
*A **Cycle** is a walk where all $v_0, v_1, \ldots, v_{n-1}$ are distinct and $v_0 = v_n$*

Theorem 19.2 *Let $x, y \in V(G)$. If there is a walk from x to y, then there is a path from x to y*

Proof: Consider a walk $W : v_n c_1, v_1 c_2, \ldots, v_{n-1} c_n$ where $v_0 = x$, $v_n = u$ and suppose length(W) is as small as possible.

- If W is a path, we are done.
- Suppose W is not a path, then there exists $0 \leq i \leq j \leq n$ such that $v_i = v_j$. Then, $W' = v_0 v_1 \ldots, v_i v_{j+1} v_{j+2} \ldots v_n$ has a shorter length than W, which is a contradiction.

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Theorem 19.3 *If every vertex in G has degree at least 2, then G has a cycle.*

Proof: Take a path $P = v_0 v_1 \ldots v_1$ of maximal length, v_0 has a neighbour $Z \neq v_1$.
Such Z must be in P otherwise $P^prime : z v_0 v_1 \ldots v_n$ would be larger than P.
Thus v_0 has a neighbour v_i with $i = 2, \ldots, n$. Then $v_0 v_1 \ldots v_i v_0$ is a a cycle ■

Notions of Graph Theory:

- Hamilton Cycle : A cycle that goes through all the vertices of the graph