Math 136 - Linear Algebra **Winter 2016**

Lecture 27: March 11, 2016

Lecturer: Yongqiang Zhao *Notes By: Harsh Mistry*

27.1 Determinants

Definition 27.1 *Let* $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. *We define the determinant of A to be* $ad - bc$ *and write*

$$detA = ad - bc = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Definition 27.2 *Let A bs an* $n \times n$ *matrix with* $n > 1$. *Let A(i,j) be the* $(n-1) \times (n-1)$ *matrix obtained from A by deleting the i-th row and the j-th column. the cofactor* a_{ij} *is*

$$C_{ij} = (-1)^{i+j} detA(i,j)$$

Definition 27.3 *If A is the* 1×1 *matrix* $A = [a]$, *then* $detA = a$. *If A is an* $n \times n$ *matrix with* $n \geqslant 2$, *then the* ***determinant*** *of A is defined to be*

$$detA = \sum_{j=1}^{n} a_{1j} C_{1j}$$

Remarks :

1. The Determinant of am $n \times n$ matrix is defined in terms of cofactors which are determinants of $(n-1) \times (n-1)$
2. We often repersent the determinant of a matrix with vertical straight lines.

$$det \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a22 \end{bmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

Theorem 27.4 *Let A be an* $n \times n$ *matrix, For any i with* $1 \leqslant i \leqslant n$

$$detA = \sum_{k=1}^{n} a_{ik} C_{ik}$$

is called ***the cofactor expansion across the i-th row****, Or for any j with* $1 \leqslant j \leqslant n$

$$detA = \sum_{k=1}^{n} a_{kj} C_{kj}$$

is called ***the cofactor expansion across the j-th column***

End of Lecture Notes
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