CS 370 - Numerical Computation **Fall 2018**

Parametric Curves And Intro to ODEs

Lecturer: Christopher Batty *Notes By: Harsh Mistry*

- Our interpolants so far only handled functions $y = p(x)$ or one coordinate is a function of the other. This prevents us from modelling more complicated curves such as ones that fold back over itself
- Parametric curves enable us to model more general curves

3.1 Parametric Curves

- Let x and y each be seperate functions of a new parameter, t. Then a points position is given by the vector $\vec{P(t)} = (x(t), y(t))$
- Parameter t increases monotonically along the curve, but x and y may increase and decrease as needed to describe any shape.
- An example would be parameter t might represent time, so an object's coordinates, (x, y) change as time passes

Note

The notion of parametric curves is not specific to any type of curve or interpolating function (like piecewise linear, Hermite, cubic splines, or even fancier Bezier/B-spline curves, etc.)

It's a more powerful/flexible way of describing curves in **general**

- We say that the curve is "parametrized" by t. For example, the (x, y) position on the curve is dictated by parameter t

Example 3.1 *Line Example*
The simple line $y = 3x + 2$ can equivalently be described by the two coordinate functions

$$x(t) = t$$

$$y(t) = 3t + 2$$

Example 3.2 *Semi-Circle Example*
Consider a curve along a semi-circle in the upper half plane, oriented from $(1, 0)$ to $(-1, 0)$.
The usual implicit equation for a unit circle is $x^2 + y^2 = 1$

The Parametric for is

$$x(t) = \cos(\pi t) \quad y(t) = \sin(\pi t) \text{ for } 0 \le t \le 1$$

3.1.1 Non-Uniqueness

A given curve can be "parametrized" in different ways, while yielding the exact same shape.

Example 3.3 -

1. $x(t) = \cos(\pi t)$, $y(t) = \sin(\pi t)$ *for* $0 \leq t \leq 1$
2. $x(t) = \cos(\pi(1-t)), y(t) = \sin(\pi(1-t))$ *for* $0 \leq t \leq 1$

Parametrization traverses the curve in the opposite direction (left to right) as t *goes from 0 to 1*

3.1.2 Speeds

2 parametrizations can also traverse the curve in the same direction, but at different speeds/rates./

Example 3.4 -

1. $x(t) = \cos(\pi t)$, $y(t) = \sin(\pi t)$ *for* $0 \leq t \leq 1$
2. $x(t) = \cos(\pi t^2)$, $y(t) = \sin(\pi t^2)$ *for* $0 \leq t \leq 1$

Both Curves cover the same semi-circle curve, in the same direction, but return different poitns for any given value of t