

and 15

Stat 230 - Probability

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Lecturer: Nagham Mohammad

Notes By: Harsh Mistry

14.1 Discrete Probability Distributions

14.1.1 Examples of Discrete Probability Distributions:

- Discrete Uniform Distribution
- The Hypergeometric Distribution.
- The Binomial Distributions
- The Negative Binomial Distribution
- Poisson Distribution

14.1.2 Discrete Uniform Distribution

A random variable X has a discrete uniform distribution if each of the n values in its range, say, $a, a+1, a+2, \dots, b$ has equal probability. Then,

$$f(x) = P(X = x) = \begin{cases} \frac{1}{b-a+1} & \text{for } x = a, a+1, \dots, b \text{ and } n = b - a + 1 \\ 0 & \text{otherwise} \end{cases}$$

14.1.3 The Hypergeometric Distribution

- n objects in a sample taken from a finite population of size N .
- Sample taken without replacement.
- Objects can be classified into two distinct types: success(S) or failure(F).
- There are r successes in the population
- Let X be the number of successes obtained

r	The number of S in the population
$N-r$	The number of F in the population
n	The number of items in the sample
x	The number of S in the sample
$n-x$	The number of F in the sample

$$f(x) = P(X = x) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}}$$

$$x = 0, 1, \dots, \min(n, r) \text{ Where } \max(0, n - N + r) \leq x \leq \min(n, r)$$

14.1.4 Binomial Distribution

There are many experiments for which the results of each trial can be reduced to two outcomes: success and failure.

14.1.4.1 Physical Setup

1. The experiment is repeated for a fixed number of trials, where n is fixed in advance of the experiment.
2. There are only two possible outcomes of interest for each trial. The outcomes can be classified as a success S or as a failure F .
3. The trials are independent of the other trials.
4. The probability of successful $P(S)$ is constant from trial to trial denoted by p .

14.1.4.2 Notation

$p = P(S)$	The probability of success in a single trial.
$q = P(F)$	The probability of failure in a single trial ($q = 1 - p$)
X	The number of successes in n trials: $x = 0, 1, 2, 3, \dots, n$.

14.1.4.3 Function

$$f(x) = P(X = x) = b(x; n, p) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & , x = 0, 1, 2, \dots, n \\ 0 & , \text{otherwise} \end{cases}$$

14.1.5 Binomial vs. Hyper geometric Distribution

The main difference is that the binomial distribution requires INDEPENDENT trials, and the probability of success(p) is the same in each trial. Hypergeometric, the draws are made from a finite number of objects(N) without replacement and the trials are not independent.