

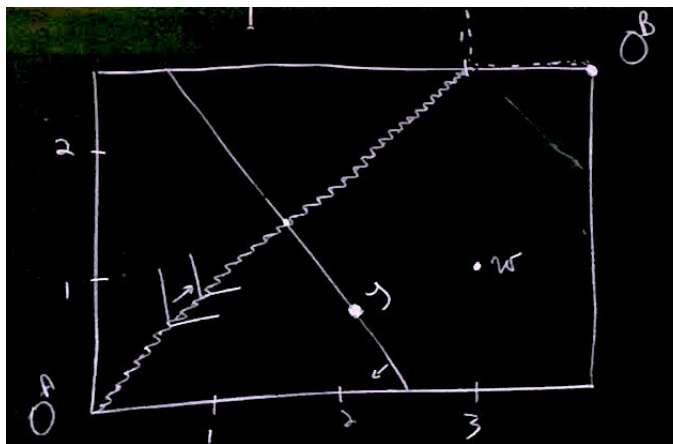
## Lecture 14: February 28, 2018

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## 14.1 Welfare Continued

**Example 14.1** Say  $\omega^A = (3, 1)$ ,  $\omega^B = (1, 2)$ ,  $u^A(x_1^A, x_2^A) = \min\{x_1^A, x_2^A\}$ , and  $u^B(x_1^B, x_2^B) = x_1^B + x_2^B$



- Fix any allocations  $y^A$  and  $y^B$
- If  $y_1^A > y_2^A$ , allocations  $y^A$  and  $y^B$  are pareto-dominated by allocations  $x^A = \left(\frac{y_1^A + y_2^A}{2}, \frac{y_1^A + y_2^A}{2}\right)$  and  $x^B = (4 - x_1^A, 3 - x_2^A)$
- Pareto set is the allocations  $x^A$  and  $x^B$  such that  $x_1^A = x_2^A$

## 14.1.1 First Welfare Theorem

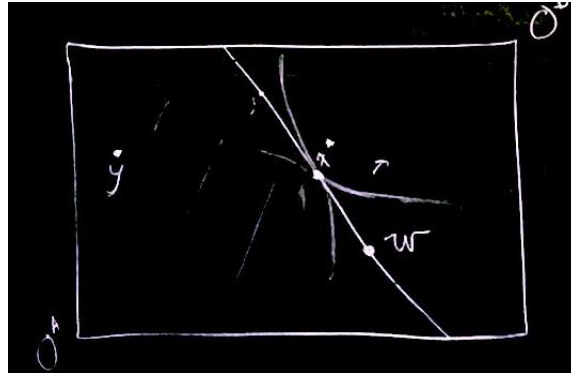
## In-Class Numbering : 3.1

- **Question:** what is the relationship between Pareto-efficient allocations and competitive equilibrium allocations?

**Theorem 14.2 FWT :** Suppose that consumers' preferences are monotone and that price  $p^*$  and allocations  $x^{A*}$  and  $x^{B*}$  form a competitive equilibrium. Then  $x^{A*}$  and  $x^{B*}$  are Pareto-efficient.

- Competitive Equilibria must exhaust gains from trade

**Proof:** By Contradiction. Suppose  $p^*$ ,  $x^{A*}$  and  $x^{B*}$  are a competitive equilibrium where  $x^{A*}$  and  $x^{B*}$  are not Pareto-efficient. Then there exists feasible allocations  $y^A$  and  $y^B$  such that  $u^J(y_1^J, y_2^J) \geq u^J(x_1^{J*}, x_2^{J*})$  for all  $J = 1, 2$ , with one strict inequality. Say  $u^A(y_1^A, y_2^A) > u^A(x_1^{A*}, x_2^{A*})$



- Since  $y^A \succ x^{A*}$ , then we must have  $p_1^* y_1^A + p_2^* y_2^A > p_1^* x_1^{A*} + p_2^* x_2^{A*}$
- We have  $y^A \succeq x^{B*}$ 
  - If  $y^B \succ x^{B*}$ , then  $p_1 y_1^B + p_2 y_2^B > p_1^* x_1^{B*} + p_2^* x_2^{B*}$
  - If  $y^B \sim x^{B*}$ , then we must have  $p_1^* x_1^{B*} + p_2^* x_2^{B*} [ < \text{contradicts optimality of monotonicity}]$
  - Then,

$$p_1^* y_1^A + p_2^* y_2^A + p_1^* y_1^B + p_2^* y_2^B > p_1^* x_1^{A*} + p_2^* x_2^{A*} + p_1^* x_1^{B*} + p_2^* x_2^{B*} \\ = p_1^* \omega_1^A + p_2^* \omega_2^A + p_1^* \omega_2^B + p_2^* \omega_2^B$$

- So we have,

$$p_1^* [y_1^A + y_1^B - \omega_1^A - \omega_2^B] + p_2^* [y_2^A + y_2^B - \omega_2^A - \omega_2^B] > 0$$

**Contradiction**, feasibility of  $y^A, y^B$  is violated. Basically, this statement implies we have more of a good than what actually exists.

**Note :** Proof is included to supplement the First Welfare Theorem to make understanding it easier. The proof itself, despite the awesomeness of mathematical proofs, is not a testable topic. It can essentially be ignored.

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- First Welfare Theorem says that competitive equilibrium allocations reproduce the outcomes of some bargaining protocol
- In bargaining, computing outcomes (P-E Allocations) requires a lot of information about consumers preferences and aggregate endowments.
- Markets only require consumers to know their own preferences and endowments.
- Prices aggregate economy-wide information.