

Lecture 21: February 26, 2016

Lecturer: Yongqiang Zhao

Notes By: Harsh Mistry

21.1 Bases

Definition 21.1 Let \mathbb{V} be a vector space. The set β is called a basis of \mathbb{V} if β is a linearly independent spanning set for \mathbb{V}

We define a basis for $\{\vec{0}_{\mathbb{V}}\}$ to be the empty set

Theorem 21.2 Let $\beta = \{\vec{v}_1 \dots \vec{v}_n\}$ be a basis for a vector space \mathbb{V} and let $\zeta = \{\vec{v}_1 \dots \vec{v}_k\}$ be a set in \mathbb{V} . If $k > n$, then ζ is linear

Theorem 21.3 if $\beta = \{\vec{v}_1 \dots \vec{v}_n\}$ and $\zeta = \{\vec{v}_1 \dots \vec{v}_k\}$ are bases for a vector space \mathbb{V} , then $k = n$

21.2 Dimension

Definition 21.4 If $\beta = \{\vec{v}_1 \dots \vec{v}_n\}$ is a basis for a vector space \mathbb{V} , then we say the dimension of \mathbb{V} is n and write

$$\dim \mathbb{V} = n$$

If \mathbb{V} is the trivial vector space, then $\dim \mathbb{V} = 0$. If \mathbb{V} does not have a basis with a finite number of vectors in it, then \mathbb{V} is said to be **Infinite Dimensional**

Theorem 21.5 If \mathbb{V} is an n -dimensional vector space $n > 0$ then

1. a set of more than n vectors in \mathbb{V} must be linear dependent
2. a set of fewer than n vectors in \mathbb{V} cannot span \mathbb{V}
3. a set of n vectors in \mathbb{V} is linear independent if and only if it spans \mathbb{V}

End of Lecture Notes
Notes by : Harsh Mistry