Math 136 - Linear Algebra **Winter 2016**

Lecture 3: January 8, 2016

Lecturer: Yongqiang Zhao *Notes By: Harsh Mistry*

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3.1 Geometry of Spanning

Example 3.1

$$Span\left\{\begin{bmatrix}1\\1\end{bmatrix}\right\} \subseteq \mathbb{R}^2$$

By Definition,

$$\vec{x} \in Span\left\{\begin{bmatrix}1\\1\end{bmatrix}\right\}\ ,\ \vec{x} = t\ \begin{bmatrix}1\\1\end{bmatrix}\ ,\ t \in \mathbb{R}$$

$$\therefore \quad Span\left\{\begin{bmatrix}1\\1\end{bmatrix}\right\}\ is\ a\ line\ with\ the\ direction\ vector\ \begin{bmatrix}1\\1\end{bmatrix}$$

Example 3.2

$$S = Span\left\{\begin{bmatrix}1\\0\\0\end{bmatrix} \begin{bmatrix}0\\1\\0\end{bmatrix}\right\}$$

No vectors in the set are scaler multiples, therfore the spanning setting can not be further simplified

$$\vec{x} = x\begin{bmatrix}1\\0\\0\end{bmatrix} + y\begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$\therefore \quad S\ is\ the\ xy\ plane\ in\ \mathbb{R}^3$$

3.2 Simplify Spanning Sets

Theorem 3.3

If $\vec{v}_{k+1}$ *is a linear combination of* $\{\vec{v}_1 \dots \vec{v}_k\}$ *then,* $Span\{\vec{v}_1 \dots \vec{v}_{k+1}\} = \{\vec{v}_1 \dots \vec{v}_k\}$

Example 3.4 ***Simplify***

a)

$$S = Span\left\{\begin{bmatrix}1\\0\\1\end{bmatrix}\begin{bmatrix}2\\0\\2\end{bmatrix}\begin{bmatrix}0\\0\\0\end{bmatrix}\right\}$$
$$= Span\left\{\begin{bmatrix}1\\0\\1\end{bmatrix}\begin{bmatrix}2\\0\\2\end{bmatrix}\right\}$$
$$= x\begin{bmatrix}1\\0\\1\end{bmatrix} + 2y\begin{bmatrix}1\\0\\1\end{bmatrix}$$
$$= (x+2y)\begin{bmatrix}1\\0\\1\end{bmatrix} \text{ , } Let\ c = x + 2y$$
$$= c\begin{bmatrix}1\\0\\1\end{bmatrix}$$

b)

$$S = Span\left\{\begin{bmatrix}1\\0\\1\end{bmatrix}\begin{bmatrix}2\\1\\0\end{bmatrix}\begin{bmatrix}-1\\-1\\1\end{bmatrix}\right\}$$
$$= Span\left\{\begin{bmatrix}1\\0\\1\end{bmatrix}\begin{bmatrix}2\\1\\0\end{bmatrix}\right\}$$
$$note:\ \begin{bmatrix}1\\0\\1\end{bmatrix} - \begin{bmatrix}2\\1\\0\end{bmatrix} = \begin{bmatrix}-1\\-1\\1\end{bmatrix}$$

3.3 Linear Independance

Definition 3.5

$\{\vec{v_1} \ldots \vec{v_k}\} \subseteq \mathbb{R}$ is called linear dependant if $\exists$ n (non-zero) solution to $t_1 v_1 + \ldots + t_k v_k = \vec{0}$

If the only solution to $t_1 v_1 + \ldots + t_k v_k = \vec{0}$ is $t_1 = t_2 = t_k$, we say it is linearly independent

Theorem 3.6

$\{\vec{v_1} \ldots \vec{v_k}\}$ is linearly dependant iff $\exists\ c_i$, $1 \le i \le k$ such that,

$v_1 \in\ Span\{\vec{v_1} \ldots \vec{v_{i-1}}, \vec{v_{i+1}} \ldots \vec{v_k}\}$

Idea :

$\vec{v} = t\vec{v_1} + t_{i-1}\vec{v_{i-1}} + t_{i+1}\vec{v_{i+1}} + \ldots + t_k\vec{v_k}$

Corollary 3.7

If $\{\vec{v_1} \ldots \vec{k}\}$ contains the 0 vector, then it is linearly dependant

Example 3.8 ***Determine whether the following sets are linear dependant or independant***

a)

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \right\}$$

$$\begin{cases} t_1 = 0 \\ 3t_2 - t_3 = 0 \\ t_1 + 2t_2 + 2t_3 = 0 \end{cases}$$

$t_1 = t_2 = t_3 = 0$

$\therefore$ *set is linear independant*

b)

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$$= \begin{cases} t_1 - t_2 + t_3 = 0 \text{ (1)} \\ t_2 + t_3 = 0 \text{ (2)} \\ -t_1 + 2t_2 = 0 \text{ (3)} \end{cases} \qquad \text{(2) - (1) = (3)}$$

$$= \begin{cases} t_1 - t_2 + t_3 = 0 \\ t_2 + t_3 = 0 \end{cases}$$

$t_3 = -1$, $t_1 = 2$, $t_2 = 1$

$$2 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \vec{0} \implies \textit{Linear dependant}$$

Definition 3.9

If $S = Span\{\vec{v_1} \dots \vec{v_k}\}$ *and* $\{\vec{v_1} \dots \vec{v_k}\}$ *are linearly independant*

Then, $\{\vec{v_1} \dots \vec{v_k}\}$ *is called a basis of* S

Practice : Prove

$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^2$

Check :

1.
$$\mathbb{R}^2 = span \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$$

2. Set is linear independant

End of Lecture Notes
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