

## Lecture 11: January 27, 2016

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## 11.1 Volumes Continued

**Example 11.1** Find the volume of the solid obtained by rotating the area bounded by  $x = y^2$  and  $x + y = 2$  about  $y = -3$

$y = 1, -2$

volume of one shell  $= 2\pi(y + 3)(2 - y - y^2)dy$

$$\begin{aligned} \text{Total Volume} &= \int_{-2}^1 2\pi(y + 3)(2 - y - y^2)dy \\ &= \frac{45\pi}{2} \end{aligned}$$

**Practice :** Find the volume enclosed by  $x = y^3, y = \sqrt{2 - x}, y = 0$  rotated about  $y = 1$ . Try to use both vertical rectangles and horizontal rectangles.

**Tips :**

- Draw a diagram
- Check rotating both vertical and horizontal rectangles to determine which orientation to use
- Be careful with radius of cylinder/disk/washer if not rotating about x
- Limits of integration must be for same variable that is in the integral

## 11.2 Arc Length

Consider  $y = f(x)$

If  $f(x)$  is continuous on  $[a, b]$ , then the length of the curve  $y = f(x)$  on  $a \leq x \leq b$  is

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

If instead we have  $x = g(y)$  where  $c \leq y \leq d$ , then we use

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}$$

**End of Lecture Notes**  
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