Math 239 - Introduction to Combinatorics **Spring 2017**

Lecture 28: July 5th, 2017

Lecturer: Alan Arroyo Guevara *Notes By: Harsh Mistry*

28.1 BFS Properties

Let (T,p) be a BFS tree with roots and let Level u be the vertices x such that the xr-path in T has length i.

1. G is connected $\iff$ V(T) = V(G)
2. P is a shortest xr-path in G
3. G has an odd cycle $\iff$ exists $xy \in E(G) \setminus E(T)$ with level(x)=level(y)

28.2 Planar Graphs

Definition 28.1 *A graph is **Planar** if it has a drawing in the plane in which every two edges intersect only at their ends.*

Any such drawings (without crossings) is a planar embedding

Definition 28.2 *A **face** is a region of a drawing*

Definition 28.3 *Given a face of a planar embedding, a **Boundary walk**.* $W_f = v_0, e_1 v_1, \ldots, v_{n-1} e_n v_n$ *is obtained by following the perimeter of a boundary of f.*

Note : The degree of f (denoted deg(f)) is the length of f.

Lemma 28.4 ***Faceshaking Lemma :*** *Let F be the set of faces in a planar embedding of a connected graph. Then,*

$$\sum_{f \in F} deg(f) = 2 \mid E(G) \mid$$

Proof: Each edge e has two sides (left and right side)

Fact

Both sides of an edge e belong to the same face $\iff$ e is a bridge

If both sides of e are on the same face, e contributes in 2 to the degree of such face
If the sides are on distinct faces, then e contributes in 1 to the degree of these faces.

Then, $\sum deg(f) = 2 \mid E(G) \mid$

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