Math 136 - Linear Algebra **Winter 2016**

Lecture 8: January 20, 2016

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8.1 Projection Examples

Example 8.1 *Find projection of* $\vec{u} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$ *onto the line* $\vec{x} = t\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 2016 \\ 2017 \\ 2018 \end{bmatrix}$

$$Proj_{\vec{v}}(\vec{u}) = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2}\vec{v} = \frac{-3}{9}\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{-1}{3} \\ \frac{-2}{3} \\ \frac{-2}{3} \end{bmatrix}$$

Example 8.2 *Find the projection of* $\vec{u}$ *onto the plane* $2x_1 - x_2 + 2x_3 = 2016$

$$\vec{n} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

$$Proj_{\vec{v}}(\vec{u}) = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2}\vec{v} = \frac{1}{9}\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

$$Proj_{Plane}(\vec{u}) = \vec{u} - Proj_{\vec{n}}(\vec{u})$$

$$= \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} - \begin{bmatrix} \frac{2}{9} \\ \frac{3}{9} \\ \frac{2}{9} \end{bmatrix} = \begin{bmatrix} \frac{7}{9} \\ \frac{-8}{9} \\ \frac{-11}{9} \end{bmatrix}$$

8.2 Chapter 2 : System of Linear Equations

Definition 8.3 *A set of m linear equations with n variables* $x_1 \ldots x_n$ *is called a system of m linear equations*

$$(*) \begin{cases} a_{11}x_1 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + \ldots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + \ldots + a_{mn}x_n = b_m \end{cases}$$

A solution for $(*)$ *will be written as* $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ *solution vector*

If $(*)$ *has atleast 1 solution, then it is consistent, otherwise it is inconsistent*

Example 8.4

$$\begin{cases} x + y = 1 \\ x - y = 1 \end{cases}$$

$$\begin{cases} x = 1 \\ y = 0 \end{cases} \quad \textit{A unique solution}$$

Example 8.5

$$\begin{cases} 2x - y = 2 \\ -x - \frac{1}{2}y = 1 \end{cases} \iff \begin{cases} 2x - y = 2 \\ 2x - y = 2 \end{cases} \iff 2x - y = 2 \textit{ Let } y = 2t \text{ - } 2 \textit{ and } x = t$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} t \\ 2t - 2 \end{bmatrix} = t \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

Solution Set : $\{\vec{x} \mid \vec{x} = t \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}, t \in \mathbb{R}\}$, *There is a infinite number of solutions*

Example 8.6

$$\begin{cases} x + 3y = 6 \\ 3x + 9y = 10 \end{cases} \iff \begin{cases} x + 3y = 1 \\ x + 3y = \frac{10}{3} \end{cases}$$

Not consistent! , So there is no solution

Geometry : Solving linear system $\iff$ finding the intersection of set of hypothesis in $\mathbb{R}^n$

Definition 8.7 -
A linear system that has the form

$$(**) \begin{cases} a_{11}x_1 + \ldots + a_{1n}x_n = 0 \\ a_{21}x_1 + \ldots + a_{2n}x_n = 0 \\ \vdots \\ a_{m1}x_1 + \ldots + a_{mn}x_n = 0 \end{cases}$$

is called a homogeneous system

Remark : A homogeneous systenm is alwasy consistent as $\vec{0} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

Theorem 8.8 *The solution set of* $(**)$ *is a subspace of* $\mathbb{R}^n$

Theorem 8.9 *Given a linear system that is consistent*

$$(A)\begin{cases} a_{11}x_1 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + \ldots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + \ldots + a_{mn}x_n = b_m \end{cases}$$

$$(B)\begin{cases} a_{11}x_1 + \ldots + a_{1n}x_n = 0 \\ a_{21}x_1 + \ldots + a_{2n}x_n = 0 \\ \vdots \\ a_{m1}x_1 + \ldots + a_{mn}x_n = 0 \end{cases}$$

To be Continued Next Lecture

End of Lecture Notes
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