Math 239 - Introduction to Combinatorics | Spring 2017

Lecture 18: June 9th, 2017

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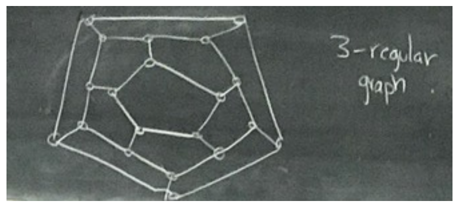
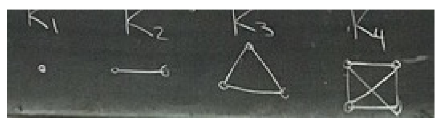
Definition 18.1 *Given an integer $k \geq 0$. a k regular graph is a graph in which every vertex has degree k.*

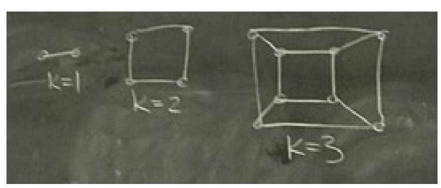
Example 18.2 *3-Regular Graph*



Example 18.3 *Let $n \in \mathbb{N}$. The complete graph K_n is the graph with n vertices such that the graph with n vertices such that every two vertices are adjacent*



Problem 18.4 *Let $Q_k = (V, E)$, $V = \{0,1\}^k$ and $E = \{S_1S_2 : S_1S_2 \in \{0,1\}^k, S_1$ and S_2 differ in exactly one digit$\}$ Show Q_k is regular and find $\mid E(Q_k) \mid$*



Solution : *Let $s \in \{0,1\}^k$. Every neighbour s' of s is obtained by changing one digit S, do s has k neighbours $\implies Q_k$ is k-regular*

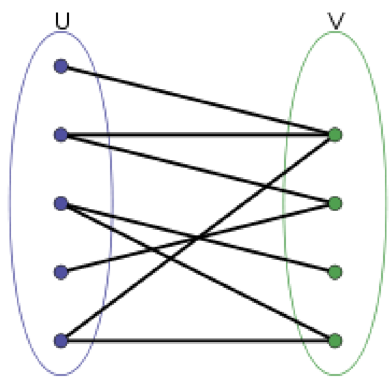
$$\begin{aligned}
2\cdot \mid E(Q) \mid &= \sum_{s \in V(G_k)} deg(s) \\
&= \sum_{s \in V(G_k)} k \\
&= k\cdot \mid V(Q_k) \mid \\
&= k \cdot 2^k \\
\implies \mid E(Q_k) \mid &= k \cdot 2^{k-1}
\end{aligned}$$

Definition 18.5 *The **degree sequence** of a graph is the sequence of degree in decreasing order*

Exercise

- Prove that if two graphs have distinct degree sequences $\implies$ they are not isomorphic
- Find two non-isomorphic graphs with the same degree sequence.

18.1 Bipartite Graphs



Definition 18.6 *A **bipartite** graph is a graph where the set of vertices can be partitioned into two sets A,B such that every edge joins a vertex in A to a vertex in B*

Problem 18.7 *Show the k-cube Q_k is bipartite*
Solution :
Set $A = \{s \in \{0,1\}^k : s \text{ has an even number of 1's}\}$
and $b = \{s \in \{0,1\}^k : s \text{ has an odd number of 1's}\}$
Since for every $s_1 s_2 \in E(Q_k)$, s_1 and s_2 differ in one digit. So either s_1 has an even number of 1's and s_0 has an odd number of 1's or viceversa