, 17, 18

Stat 231 - Statistics **Spring 2017**

Lecture 16, 17, 18: May 29th - June 2nd, 2017

Lecturer: Suryapratim Banerjee *Notes By: Harsh Mistry*

16.1 Interval Estimation

Interpretation

- The confidence Interval is an Estimate of the r.v.s L, U where $[L, U]$ contains θ with 95% confidence
- If the experiment was repeated many times, approximately 95% of the intervals constructed would contain θ

Distribution

Sampling Distribution of the Sample Mean

$$\bar{Y} \sim G(\gamma, \frac{\sigma}{\sqrt{n}})$$

1. Find the estimate for the unknown parameter.
   MLE for $\gamma = \bar{y} = \frac{1}{n}\sum y_i$
   $\hat{\gamma} = \bar{y}$
2. Identify the estimator and its distribution
   $\bar{y}$ =r.v. from which $\bar{y}$ is an outcome
   $\bar{y}$ = Estimator
3. Construct the pivotal quantity from the sampling distribution
   $$\frac{\bar{y} - \gamma}{\frac{\sigma}{\sqrt{n}}} = z = G(0, 1)$$
4. Find the extreme points of your pivotal distribution
5. Use step 4 to construct the coverage Interval
6. Use the coverage interval to construct your confidence interval.
   Confidence Interval = $[\bar{y} \pm z^* \frac{\sigma}{\sqrt{n}}]$

16.2 Distribution Theory

16.2.1 The Chi-Squared Distribution

Definition 16.1 *Let W be a random variable such that*

$$W = Z_1^2 + Z_2^2 + \ldots + Z_n^2$$

Where $Z_i \sim G(0, 1)$ and Z_i's are independent, Then W is said to follow a chi-squared distribution with n degrees of freedom

$$W \sim X_n^2$$

Properties of Chi-Squared Distibutions

- n - degrees of freedom parameter of the chi-squared distribution
- As n changes the shape changes
- W can take values between 0 and ∞
- E(W) = n and Var(W)= 2n
- Suppose $W_1 \sim X_n^2$ and $W_2 \sim X_n^2$, where W_1 and W_2 are independent

$$W_1 + W_2 \sim X_{n_1+n_2}^2$$

16.2.2 Students T-distribution

Definition 16.2 *A random variable T is said to follow a student's T distribution with n degrees of freedom if*

$$T = \frac{z}{W}$$

Where $Z \sim G(0, 1)$, $W = \sqrt{x_n^2/n}$, and Z/W are independent

Properties

- T can take all values $(-\infty, \infty)$
- T is symmetric around zero for any n
- For "small" n, T looks like the z distribution, but fatter tails $K > 3$
- As n becomes larger $T \longrightarrow Z$ and the *pdf* converges for $n \longrightarrow \infty$