Math 136 - Linear Algebra **Winter 2016**

Lecture 4: January 11, 2016

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4.1 Basis Examples

Standard Basis

$$\mathbb{R}^n \left\{ \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \cdots \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \right\}$$

So, a standard basis is when all numbers in the vector except the corresponding component are 0

Example 4.1 ***Determine if the set is a basis***

$$\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\} \textbf{\textit{ For }} \mathbb{R}^2$$

$$\forall \vec{x} \in \mathbb{R}^2 \text{ , } \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ , } \mathit{If} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + t_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{cases} t_1 + 2t_2 = x_1 \\ 2t_1 + t_2 = x_2 \end{cases} \implies \begin{cases} t_1 = \frac{2x_2 - x_1}{3} \\ t_2 = \frac{2x_1 - x_2}{3} \end{cases} \implies \vec{x} \in \mathit{Span} \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\} \implies \mathbb{R}^2 = \mathit{Span} \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$$

Suppose $\vec{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ *, then we get* $t_1 = t_2 = 0$ *, so* $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$ *is linearly independant*

Definition 4.2 -

1. *Let* $\vec{v}\ \vec{b} \in \mathbb{R}^n$, $\vec{v} \neq \vec{0}$
   The set $\{\vec{x} \mid \vec{x} = t\vec{x} + t\vec{b} \text{ , } t \in \mathbb{R}\}$ *is called a line in* $\mathbb{R}^n$
2. *Let* $\vec{v}\ \vec{v_2} \in \mathbb{R}^n$, $\{\vec{v_1}\vec{v_2}\}$ *being linear independant and* $\vec{b} \in \mathbb{R}^n$
   then set $\{\vec{x} \mid \vec{x} = t_1\vec{v_1} + t_2\vec{v_2} + \vec{b} \text{ , } t_1\ t_2 \in \mathbb{R}\}$ *is called a plane*
3. *Let* $\vec{v_1} \ldots \vec{v_{n-1}} \in$ *and* $\mathbb{R}^n\{\vec{v_1} \ldots \vec{v_{n-1}}\}$ *be linearly independant*
   $\vec{b} \in \mathbb{R}^n$ *the set* $\{\vec{x} \mid \vec{x} = t_1\vec{v_1} \ldots t_{n-1}\vec{vn - 1} + \vec{b_1}, t_n \ldots t_{n-1} \in \mathbb{R}\}$

4.2 Subspaces

Definition 4.3 -
A non-empty set S or $\mathbb{R}^n$ is called a subaspace if S is closed under adddion and scaler multiplication

- $\forall \vec{x}\ \vec{y} \in S, \vec{x} + \vec{y} \in S$
- $\forall \vec{x} \in S, \forall C \in \mathbb{R}, C\vec{x} \in S$

Remarks

- For any subspace S $\vec{0} \in S$
- $\{\vec{0}\}$ and $\mathbb{R}^n$ are subspaces of $\mathbb{R}^n$

Example 4.4 *Determine if the given set is a subspace*
a)

$$S_1 \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2 \mid x_1 - x_2 = 0 \right\}$$

1. $\vec{0} \in S$ *, So* S_1 *is non-empty*
2. $\forall \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \in S_1 \implies x_1 - x_2 = 0$ *and* $y_1 - y_2 = 0 \implies (x_1 + y_1) - (x_2 + y_2) = 0 \implies \vec{x} + \vec{y} \in S_1$
3. $\forall c \in \mathbb{R}, cx_1 - cx_2 = c(x_1 - x_2) = 0 \implies c\vec{x} \in S_1$

$\therefore S_1$ *is a subspace*

b)

$$S_1 \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2 \mid x_1 - x_2 = 1 \right\}$$

$\vec{0} \notin\ S_2\ \therefore S_2$ *is not a subspace*

Theorem 4.5 *Given* $S = \{\vec{v_1}, \vec{v_2}, \vec{v_k}\}, Span_b$ *is a subspace*

4.3 Basis of a subspace

Questions for Next Lecture : Find the basis of the following subspaces

- $S_1 = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R} \mid x_1 + x_2 - x_3 = 0 \right\}$
- $S_2 = \left\{ \begin{bmatrix} a - b \\ b - c \\ c - a \end{bmatrix} \in \mathbb{R} \mid a, b, c \in \mathbb{R} \right\}$
- $\forall \vec{x} \in S_1, \vec{x} \begin{bmatrix} x_1 \\ x_2 \\ x_1 + x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

End of Lecture Notes
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