Math 136 - Linear Algebra	**Winter 2016**

Lecture 28: March 14, 2016

Lecturer: Yongqiang Zhao — *Notes By: Harsh Mistry*

28.1 Determinants Continued

Definition 28.1 *An $m \times n$ matrix U is said to be upper triangular if $u_{ij} = 0$ whenever $i > j$. An $m \times n$ matrix L is said to be lower triangular if $l_{ij} = 0$ whenever $i < j$*

Example 28.2 -
Upper Matrix Example

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 1 \\ 0 & 0 & 5 \end{bmatrix}$$

Lower Matrix Example

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Theorem 28.3 *If an $n \times m$ matrix A is upper triangular or lower triangular, then*

$$detA = a_{11}a_{22}\dots a_{nn}$$

Theorem 28.4 *If B is an $n \times n$ matrix obtained from A by swapping two rows of A, then $detB = -detA$*

Corollary 28.5 *If $n \times n$ matrix A has two identical rows, then $detA = 0$*

Theorem 28.6 *If B is the matrix obtained from S by multiplying one row of A by a non-zero constant c, then $detB = cderA$*

Theorem 28.7 *If B is the matrix obtained from A by adding r times the k-th row of a to the j-th row, then $detB = detA$*

Theorem 28.8 *If A is an $n \times n$ matrix, then $detA = detA^T$*

Combining Theorms 28.8, 28.7, and 28.6, we find that we can do column operations

- Adding a multiple of one column to another does not change the determinant
- Multiplying a column by a non-zero scaler c multiplies the determinant by c

- Swapping two columns multiplies the determinant by (-1)

Note : Column Operations can be only used when simplifying a determinant

End of Lecture Notes
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