Math 128 : Calculus 2 for the Sciences **Winter 2016**

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2.1 Integration by Subsitution Examples

Example 2.1

$$\begin{aligned}\int \frac{x^3}{(x+5)^2} &= \int \frac{(u-5)^3}{u}du \quad \textbf{\textit{Let } u = x + 5} \\ &= \int \frac{u^3 - 15u^2 + 75u - 125}{u^2} \\ &= \int u - 15 + \frac{75}{u} - \frac{125}{u^2} \\ &= \left[\frac{u^2}{2} - 15u + 75\ln u + \frac{125}{u} + c\right] \\ &= \left[\frac{(x+5)^2}{2} - 15(x+5) + 75\ln(x+5) + \frac{125}{x+5} + c\right]\end{aligned} \tag{2.1}$$

Example 2.2

$$\begin{aligned}\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}dx &= -1\int_1^{\frac{1}{2}} e^u du \quad \textbf{\textit{Let } u = } \frac{1}{x} \\ &= -\left[e^u\right]_1^{\frac{1}{2}} \\ &= -e^{\frac{1}{2}} + e\end{aligned} \tag{2.2}$$

Example 2.3 -

$$\begin{aligned}\int \tan x dx = \int \frac{\sin x}{\cos x} &= -\int \frac{1}{u}du \qquad \textit{Let } \ u = \cos x \\ &= -\ln|u| + c \qquad du = -sinxdx \\ &= -\ln|\cos x| + c \quad \textbf{\textit{or}} \ \ln|\sec| + c\end{aligned}$$

Practice Problem

$$\int \sec x \ dx \text{ Hint : Multiply by } \frac{\sec x + \tan x}{\sec x + \tan x}$$

Example 2.4

$$\int_{-1}^{1} \frac{\sin x}{1+x^2}dx = 0 \ \textbf{\textit{Integral is 0 because the function is odd}} \tag{2.3}$$

2.2 Integration by Parts (IBP)

IBP Formula

$$\int udv = uv - \int vdu$$

Why?
Reverse Engineering the product rule results in the IBP formula

Proof:

The product rule : $\frac{d}{dx}(f(x)g(x)) = f\prime(x)g(x) + f(x)g\prime(x)$

Integrate both sides with respect to x

$$\int \frac{d}{dx}(f(x)g(x))dx = \int f\prime(x)g(x) + f(x)g\prime(x)dx$$

$$\implies f(x)g(x) = \int f\prime(x)g(x)dx + \int f(x)g\prime(x)dx$$

$$\implies \int f(x)g\prime(x)dx = f(x)g(x) - \int f\prime(x)g(x)$$

let u = f(x) and v = g(x)

$$\therefore \int udv = uv - \int vdu$$

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Process :
So, divide the integrand into 2 parts u & dv. Then apply the IBP formula.

You'll find that calculating $\int udv$ reduces $\int vdu$

Example 2.5

$$\int xe^x$$

Suppose $u = x$ & $dv = e^x\ dx\ \rightarrow du = dx$ & $v = e^x$

$$\int xe^x\ dx = xe^x - \int e^x dx$$

$$= xe^x - e^x + c$$

Practice Problem

$$\int x^2 \cos x\ dx$$

End of Lecture Notes
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