Math 136 - Linear Algebra **Winter 2016**

Lecture 6: January 15, 2016

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6.1 Dot Product Continued

Definition 6.1 - *For any two vectors* $\vec{x}\vec{y}$ *, we define the angle between* $\vec{x}\vec{y}$ *to be* $\cos\theta = \frac{\vec{x}\cdot\vec{y}}{\|\vec{x}\|\|\vec{y}\|}$

- *If* $\vec{x}\cdot\vec{y} = 0$ *, we say* $\vec{x}$ *and* $\vec{y}$ *are orthogonal*
- *Any any two vectors of ,* $\{e_1, e_2 \dots e_n\}$ *standard basis for* $\mathbb{R}^n$ *, are orthogonal*

6.2 Cross product

Given two vectors $\vec{x}$ and $\vec{y}$ in $\mathbb{R}^3$, find a third vectors which is orthongonal to $\vec{x}+\vec{y}$

Definition 6.2 - *Given* $\vec{x}$ *and* $\vec{y}$ $\in \mathbb{R}^3, \vec{x}\times\vec{y} = \begin{bmatrix} x_2y_3 - x_3y_2 \\ x_3y_1 - x_1y_3 \\ x_1y_1 - x_2y_1 \end{bmatrix}$

geometrically *:* $\vec{x}\times\vec{y} = \|\vec{x}\|\|\vec{y}\|\sin\theta$

Proposition 6.3 *Suppose that* $\vec{v}, \vec{w}, \vec{x} \in \mathbb{R}^3$ *and* $c \in \mathbb{R}$

1. *If* $\vec{n} = \vec{v}\times\vec{w}$ *then for any* $\vec{y} \in Span\{\vec{v}, \vec{w}\}$ *we have* $\vec{y}\cdot\vec{n} = 0$
2. $\vec{v}\times\vec{w} = -\vec{w}\times\vec{v}$
3. $\vec{v}\times\vec{v} = 0$
4. $\vec{v}\times\vec{w} = \vec{0}$ *iff either* $\vec{v} = \vec{0}$ *or* $\vec{w}$ *is a scaler multiple of* $\vec{w}$
5. $\vec{v}\times(\vec{x}+\vec{w}) = \vec{v}\times\vec{w} + \vec{v}\times\vec{x}$
6. $(c\vec{v})\times\vec{w} = c(\vec{x}\times\vec{w})$

Example 6.4 *Let* $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\vec{z} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\vec{x}\times\vec{z} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3-2 \\ 1-3 \\ 2-1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ $\therefore \vec{x}$ *is not orthogonal*

6.3 Scaler Equations

Recall : A plane in $\mathbb{R}^3$ is given by $\vec{x} = s\vec{v} + t\vec{w} + \vec{b}$ and $\{\vec{v}, \vec{w}\}$ is linear independant

Definition 6.5 - *Given* $\vec{n} = \vec{v} \times \vec{w}$ *, the scaler equation for a plan is* $(\vec{x} - \vec{b}) \cdot \vec{n} = 0$

Example 6.6 - *Finding a scaler equation*

$$\vec{x} = s \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\vec{n} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$0 = (\vec{x} - \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}) \cdot \vec{n}$$

$$\vec{x} \cdot \vec{n} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \vec{n}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \implies (x_3 - x_1 = z)$$

End of Lecture Notes
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