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Stat 231 - Statistics **Spring 2017**

Lecture 11, 12: May 24th - 26th, 2017

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Exponential Model

$$Y_i \sim Exp(\gamma)$$

Density Function :

$$f(y) = \frac{1}{\gamma}e^{-y/\gamma}$$

Likelihood Function :

$$L(\gamma) = \frac{1}{\gamma}e^{-y_1/\gamma} \cdot \ldots \cdot \frac{1}{y}e^{-y_n/\gamma} = \frac{1}{\gamma^n}e^{-\frac{1}{\gamma}\sum y_i}$$

Log-Likelihood Function

$$l(\gamma) = -n\ln\gamma - \frac{1}{\gamma}\sum y_i$$

Gaussian Distribution

$$Y_i \sim G(\gamma, \sigma)$$

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2\sigma^2}(y-\gamma)^2}$$

Likelihood Function :

$$L(\gamma, \sigma) = \frac{1}{(2\pi)^{n/2}\sigma^n}e^{-\frac{1}{2\sigma^2}(y_i-\gamma)^2}$$

Log-Likelihood Function

$$l(\gamma, \sigma) = \frac{-n}{2} \cdot \ln(2\pi) - n\ln\sigma - \frac{1}{2\sigma^2}\sum(y_i - \sigma)^2$$

Invariance Property Of The MLE

If $\hat{\theta}$ is the MLE for θ, then $g(\hat{\theta})$ in the MLE for $g(\theta)$ if g is continuous.

Uniform Distribution

$Y_i \sim U[0, \theta]$

Density Function :

$$f(y) = \begin{cases} \frac{1}{\theta} & \text{if } 0 \leq y \leq \theta \\ 0 & \text{elsewhere} \end{cases}$$

Likelihood Function :

$$L(\theta) = \begin{cases} \frac{1}{\theta^n} y & \text{if } 0 \leq y_i \leq \theta, \forall i \\ 0 & \text{if } \theta < max\{y_1, \ldots, y_n\} \end{cases}$$

11.1 Model Selection

Model : "Identify" the random variable from which $\{y_1, \ldots, y_n\}$ is drawn

Subjective Tests

We run numerical and graphical tests on the data to select the "right" model.

Numerical Tests

- Check whether the data set satisfies the theoretical properties of the distributions assumed in your model

11.1.0.1 Graphical Tools

- Super impose the relative frequency histogram of your data set to the theoretical distirbution function assumed and see whether the shapes match

The Q-Q plot

Typically used to check whether Gaussian is the "right mode"