Math 239 - Introduction to Combinatorics	**Spring 2017**

Lecture 29: July 7th, 2017

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Determining whether a graph G is Planar :

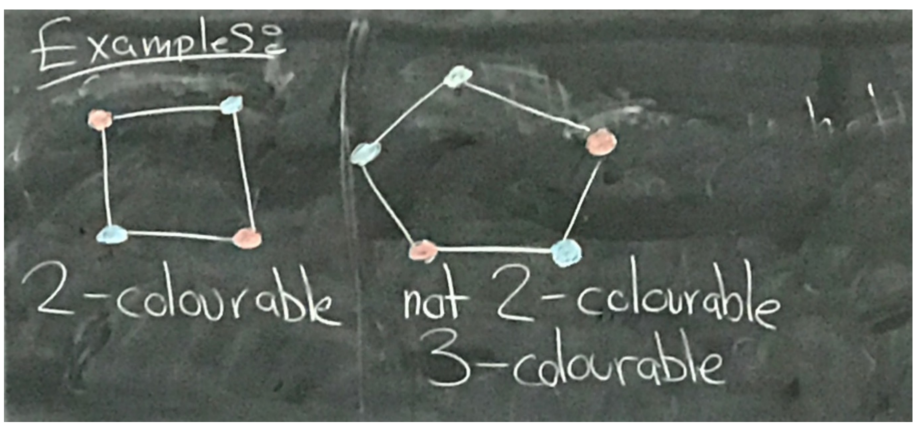
1. Try to redraw it without crossings
2. Find $K_{3,3}$ or K_5 sub division

29.1 Colouring Planar Graphs

Definition 29.1 *Given a graph G, a **k-colouring** is a function $f : V(G) \to \zeta$, where ζ is a set of size k (Known as the set of colours), such that every two adjacent vertices are assigned distinct graphs.*

*A graph that has a k-colouring is **k-colourable***

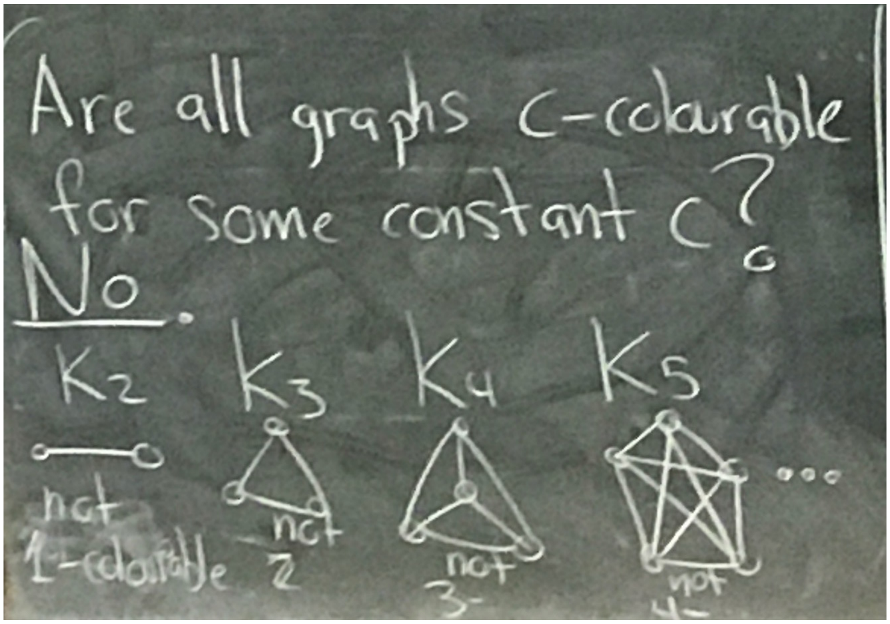
Example 29.2 *k-colourable examples*



Theorem 29.3 ***4-Colour Theorem :*** *All Planar graphs are 4-colourable*

Are all graphs c-colourable for some constant c? No.

Example 29.4 -



Theorem 29.5 ***6-Colour Theorem :*** *Every planar graph is 6-colourable*

Proof: Induction on $n = \mid V(G) \mid$

Base : n = 1, a graph with one vertedx is planar an also 6-colourable

I.H : Suppose that every planar graph with at most n-1 vertices has a 6 colour ring using colours in

$$\zeta = \langle 1, 2, 3, 4, 5, 6 \rangle$$

I.S : Recall that since G is planar, it has a vertex v with $deg(v) \in S$
So, Let G be a planar graph with n vertices. Since G is planar, it has a vertex $v \in V(G)$ with $deg(v) \leq 5$. Since $G - v$ is planar, By our inductive hypothesis, $G - v$ has a 6-colouring using colours in 6.
Note that as v has at most 5 neighbours, atleast one of the colours in 6 is unused. We can then use this colour to colour in v. Thus, we obtain a 6-colouring of G

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