Econ 301 - Microeconomic Theory 2	**Winter 2018**

Lecture 6: January 22, 2018

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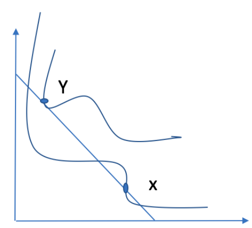
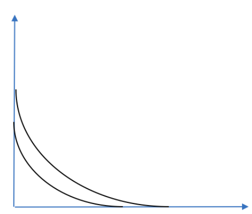
6.1 Consumer Choice Continued

6.1.1 Optimal consumer choice



- FOC $(L1) - (L\lambda)$ are only necessary for x^* to be optimal. (i.e any solution x^* to (PE) must be a solution to $(L1) - (L\lambda)$ but some solutions to $(L1) - (L\lambda)$ are not solutions to (PE))
- Need sufficient or second order conditions that guarantee that solutions to $(L1)-(L\lambda)$ are also solutions to (PE)
- Result if the consumers preferences are monotone and convex then any solution to $(L1) - (L\lambda)$ must be a solution to (PE)

Example 6.1 *Suppose* $U(x_1, x_2) = \sqrt{x_1} + x_2$ *and solve for* $\sqrt{x} + x_2 = c$



1. *Consumer UMP*
$$max_{x_1,x_2 \geq 0} \ \sqrt{x_1} + x_2 \ s.t \ \ p_1 x_1 + p_2 x_2 \leq m$$
2. *Preferences are monotone if* $x_1 > y_1$ *and* $x_2 > y_2$
$$\begin{aligned} U(x_1, x_2) &= \sqrt{x_1} + x_2 \\ &> \sqrt{y} + y_2 \\ &= U(y_1, y_2) \implies x \succ y \end{aligned}$$
Therefore, budget constraint holds as an equality at any solution to UMP

3. *Lagrangian*

$$L(x_1, x_2, \lambda) = \sqrt{x_1} + x_2 + \lambda[m - p_1 x_1 - p_2 x_2]$$

4. *Any solution to UMP such that* $x_1^*, x_2^* \neq 0$ *must solve FOC (First Order Conditions)*

$$\frac{d}{dx_1} L(x_1, x_2, \lambda) = \frac{1}{2\sqrt{x}} - \lambda p_1 = 0 \ \ (L1)$$

$$\frac{d}{dx_2} L(x_1, x_2, \lambda) = 1 - \lambda p_2 = 0 \ \ L(2)$$

$$\frac{d}{d\lambda} L(x_1, x_2, \lambda) = m - p_1 x_2^* - p_2 x_2^* = 0 \ \ (L\lambda)$$

Divide (L1) by (L2)

$$\frac{1}{2\sqrt{x_1}} = \frac{p_1}{p_2}$$

$$x_1^* = \frac{1}{4}(\frac{p_2}{p_1})^2 \ \ (\star)$$

Substitute $(\star)$ *into* $L(\lambda)$ *:*

$$x_2^* = \frac{m}{p_2} - \frac{1}{4}\frac{p_2}{p_1}$$

There is a condition though. $(L1) - (L\lambda)$ *only has a solution if* $\frac{m}{p_2} > \frac{1}{4}\frac{p_2}{p_1}$

5. *At a corner solution with* $x_1^* = 0$*, necessary condition*

$$\frac{\frac{d}{dx_1} U(0, m/p_2)}{\frac{d}{dx_2} U(0, m/p_2)} \leq \frac{p_1}{p2}$$

Unfortunately

$$\lim_{x \to \infty} \frac{\frac{1}{2\sqrt{x}}}{1} = \infty$$

This inequality is never satisfied, so the equation never holds.

Therefore, at $x_1 = 0$*, teh slope of the IC will never be greater than the budget line*

6. *At corner solution with* $x_2^* = 0$*, necessary condition*

$$\frac{\frac{d}{dx_1} U(m/p_1, 0)}{\frac{d}{dx_2} U(m/p_1, 0)} \geq \frac{p_1}{p2}$$

So,

$$\frac{\frac{1}{2\sqrt{m/p_1}}}{1} \geq \frac{p_1}{p_2} \implies \frac{m}{p_2} \leq \frac{1}{4}\frac{p_1}{p_2}]$$

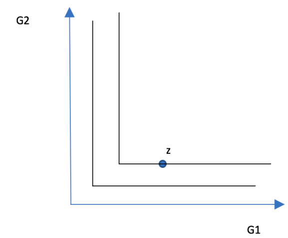
7. *Consumer preferences are convec so that any solution to necessary conditions must be solution to UMP*

8. *Demand function*

$$(x_1(p_1 m), x_2(p_2 m)) = \begin{cases} \left(\frac{1}{4}(\frac{m}{p_2})^2, \frac{m}{p_2} - \frac{1}{4}\frac{p_2}{p_1}\right) & \textit{if } \frac{m}{p_2} > \frac{1}{4}\frac{p_2}{p_1} \\ \left(\frac{m}{p_2}, 0\right) & \textit{if } \frac{m}{p_2} \leq \frac{1}{4}\frac{p_2}{p_1} \end{cases}$$

- If consumers utility function is not differentiable, then the Lagrangian does not apply

Example 6.2 *Consider perfect compliments where* $U(x_1, x_2) = min\{x_1, x_2\}$



1. *Consumers UMP*
$$max_{x_1,x_2 \geq 0} min\{x_1, x_2\} \quad \textit{such that } p_1x_1 + p_2x_2 \leq m$$

2. *Consumer preferences are monotone. Let* $x_1 > y_1$ *and* $x_2 > y_2$
$$\begin{aligned} U(x_1, x_2) &= min\{x_1, x_2\} \\ &> min\{y_1, y_2\} \\ &= U(y_1, y_2) \end{aligned}$$
Therefore, budget constraint holds as an equality at any solution to UMP

3. *Any optimal bundle* x^* *must be such that* $x_1^* = x_2^*$. *(I.e any bundle with* $x_1 > x_2$ *is worse than bundles* $\left(\frac{m}{p_1+p_2} + \frac{m}{p_1+p_2}\right)$

4. *Demand Function*
$$(x_1(p_1m), x_2(p_2m)) = \left(\frac{m}{p_1 + p_2}, \frac{m}{p_1 + p_2}\right)$$