Math 239 - Introduction to Combinatorics **Spring 2017**

Lecture 13: May 29th, 2017

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13.1 Recurrences

We reduced our counting problems about finding coefficients as

$$[x^n]\frac{2x^2-6x+8}{x^3-x^2-x+1}$$

Problem 13.1 ***Find :***

$$[x^n]\frac{2x^3-6x+8}{x^3-x^2-x+1}$$

Solution using partial fractions:

$$\begin{aligned}
\frac{2x^3-6x+8}{x^3-x^2-x+1} &= [x^n]\left(2+\frac{3}{x+1}-\frac{1}{(x-1)}+\frac{2}{(x-1)^2}\right)\\
&= [x^n]\left(2+\frac{3}{x-(-1)}-\frac{1}{(x-1)}+\frac{2}{(x-1)^2}\right)\\
&= [x^n]\left(2+3\sum_{m\geq 0}(-x)^m+\sum_{m\geq 0}x^m+2\sum_{m\geq 0}\binom{m+2-1}{2-1}x^m\right)\\
&= [x^n]\left(2+3\sum_{m\geq 0}(-1)x^m+\sum_{m\geq 0}x^m+2\sum_{m\geq 0}(m+1)x^m\right)\\
&= \begin{cases} 2+3(-1)^0+1+2(0+1) & ,n=0\\ 3(-1)^n+1+2(n+1) & ,n\geq 1\end{cases}
\end{aligned}$$

Problem 13.2 *Use the recurrence method to find :*

$$[x^n]\frac{2x^3-6x+8}{x^3-x^2-x+1}$$

Solution using recurrence method :

$$\begin{aligned}
\frac{2x^3-6x+8}{x^3-x^2-x+1} &= \sum_{n\geq 0}a_nx^n = a_0+a_1x+a_2x^2+\dots\\
&\iff 8-6x+2x^3=(1-x+x^2+x^3)(a_0+a_1x+a_2x^2+\dots)\\
&\implies a_0+(a_1-a_0)x+(a_2-a_1-a_0)x^2+(a_3-a_2-a_1+a_0)x^3+(a_1-a_3-a_2+a_1)x^4
\end{aligned}$$