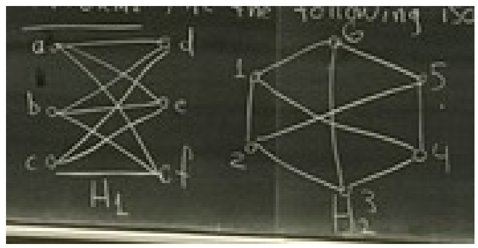
Math 239 - Introduction to Combinatorics **Spring 2017**

Lecture 17: June 7th, 2017

Lecturer: Alan Arroyo Guevara *Notes By: Harsh Mistry*

Problem 17.1 *Are the following Isomorphic?*



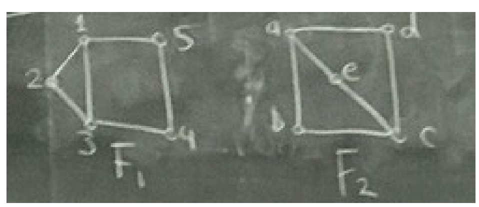
v	a	b	c	d	e	f
$f(v)$	1	3	5	2	4	6

So, yes they are isomorphic.

Tip

Isomorphism between G_1 and G_2 is the same as "drag and move" vertices of G_1 to get G_2
To display this, download **yEd graph editor**.

Problem 17.2 *Are the following isomorphic?*



Solution : *No, vertices 1, 2, 3, are pairwise adjacent in F_1. If there is an isomorphism $f : v(F_1) \to V(F_2)$ then f(1), f(2), f(3) would also be pairwise adjacent in F_2, but no such triple exists in F_2. So they are not isomorphic.*

Tip

To show two graphs are not isomorphic, find a property in one of them, that you can't find in the other

17.1 Degrees of Vertices

Definition 17.3 *Given $v \in V(G)$, the* ***degree*** *of v, denoted as $deg(v)$, is the number of neighbours of v in G.*

Lemma 17.4 *Handshaking Lemma: For any graph G*

$$\sum_{v \in V(G)} deg(v) = 2\cdot \mid E(G) \mid$$

Proof: Let $S = \{(v, e) : v \in V(G), e \in E(G) \text{ e is incident to v}\}$
Every $v \in V(G)$ is in deg(v) pairs in S.

$$\mid S \mid = \sum_{v \in V(G)} deg(v)$$

Every $e \in E(G)$ is in 2 pairs, then

$$\mid S \mid = 2\cdot \mid E(G) \mid$$

■

Corollary 17.5 *In a graph, there is an even number of vertices with an odd degree.*

Proof: Let G be a graph and Let $V(G) = V_e \cup V_o$ where

$$v_e = \text{ even degree vertices}$$

$$v_o = \text{ odd degree vertices}$$

By the Handshake Lemma,

$$\begin{aligned}
2\cdot \mid E(G) \mid &= \sum_{v \in V(G)} deg(v) \\
&= \sum_{v \in V_e} deg(v) + \sum_{v \in V_o} deg(v) \\
&\implies \sum_{v \in V_0} deg(v) = 2\cdot \mid E(G) \mid \sum_{v \in V_e} deg(v) \\
&\implies \sum_{v \in V_o} \text{deg(v) is even} \\
&\implies \mid V_o \mid \text{ is even because the sum of odd numbers is even only when } \mid V_o \mid \text{ is even}
\end{aligned}$$

■

Definition 17.6 *Given an integer $k \geq 0$. a k regular graph is a graph in which every vertex has degree k.*