Lecture 20: February 24, 2016

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20.1 Vector Spaces Continued

$$C(\mathbb{R}) = \{f : \mathbb{R} \to \mathbb{R} \mid \text{ f is continuous } \}$$

Definition 20.1 *Let V and W be 2 vector spaces we define the cartesian product of V and W as :*

$$V \times W = \{(\vec{v}, \vec{w}) \mid \vec{v} \in V, \vec{w} \in W\}$$

If we define addition and scaler multiplication as following

$$(\vec{v_1}, \vec{w_1}) + (\vec{v_2}, \vec{w_2}) = (\vec{v_1} + \vec{v_2}, \vec{w_1} + \vec{w_2})$$
$$c(\vec{v_1}, \vec{w_1}) = (c\vec{v_1}, c\vec{w_1})$$

then V x W is a vector space

Theorem 20.2 *In any vector in Span V,* $0\vec{v} = \vec{0}, \forall \vec{v} \in V$*, Also,* -$\vec{v} = (-1)\vec{v}$

Proof:

$$0\vec{v} = (0 + 0)\vec{v} = 0\vec{v} + 0\vec{v}$$
$$0\vec{v} + (-0\vec{v}) = 0\vec{v} + 0\vec{v} + (-0\vec{v})$$
$$\vec{0} = 0\vec{v} + \vec{0} = 0\vec{v}$$
$$\therefore \vec{0} = 0\vec{v}$$

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Definition 20.3 *A non=empty subset of a vector space V is called a subaspce if :*

- $\forall \vec{x}, \vec{y} \in S, \vec{x} + \vec{y} \in S$
- $\forall \vec{x} \in S, \forall c \in \mathbb{R}, c\vec{x} \in S$

Trivial Examples : V and $\{\vec{0}\}$ are subspaces

Example 20.4 *Prove S is subsapce of $M_{m\times n}$*

$$S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a+b-c-d=0 \right\}$$

Clearly S is non-empty as $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in S$

$$\begin{aligned} A_1 + A_2 &= \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix} \\ &= (a_1+a_2)+(b_1+b_2)-(c_1+c_2)-(d_1+d_2) \\ &= (a_1+b_1-c_2-d_1)+(a_2+b_2-c_2-d_2) = 0 \\ &\implies A_1+A_2 \in S \\ tA &= \begin{bmatrix} ta & tb \\ tc & td \end{bmatrix} \\ &= ta+tb-tc-td = 0 \\ &\implies tA \in S \end{aligned}$$

$\therefore$ *S is a subspace of $M_{m\times n}$*

Definition 20.5 *A square matrix A is called symmetric if $A = A^T$*

$$Sym_n(\mathbb{R}) = \{A \in M_{m\times n}(\mathbb{R}) \mid A = A^T\}$$

Check to see if its a subspace

- $0 \in Sym_n(\mathbb{R}) \to 0T = 0$
- $\forall AB \in Sym_n(\mathbb{R}), (A+B)^T = A^T + B^T = A + B \implies A+B \in Sym_n(\mathbb{R})$
- $tA \in Sym_n(\mathbb{R})$

Note : $Sym_n(\mathbb{R}) = \{((a_{ij}) \mid a_{ij} = a_{ji}\}$

Definition 20.6 *A square matrix M is called Skew-symmetric if $A = -A^T$*

$$S_n = \{A \in M_{m\times n}(\mathbb{R}) \mid A = -A^T\}$$

End of Lecture Notes
Notes by : Harsh Mistry