Math 128 : Calculus 2 for the Sciences **Winter 2016**

Lecture 9: January 22, 2016

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9.1 The Comparison Test

if f and g are continous and $0 \leq g(x) \leq f(x)$ for $x \geq a$

- $\int_a^\infty f(x)dx$ converges then $\int_a^\infty g(x)dx$ converges
- $\int_a^\infty f(x)dx$ diverges then $\int_a^\infty g(x)dx$ diverges

Example 9.1 *Does* $\int_0^\infty \frac{x+5}{1+x^2}$ *converge or diverge ?*
Note : $\frac{x+5}{1+x^2} \geq \frac{x}{1+x^2}$

$$\int_0^\infty \frac{x}{1+x^2}dx = \lim_{b\to\infty}\int_0^b \frac{x}{1+x^2} = \lim_{b\to\infty}\left[\frac{\ln|1+x^2|}{2}\right]_0^b = \lim_{b\to\infty}\frac{\ln(1+b^2)}{2} - \frac{\ln(1)}{2} = \infty$$

$\int_0^\infty \frac{x}{1+x^2}dx$ *diverges so,* $\int_0^\infty \frac{x+5}{1+x^2}$ *diverges by the comparison test*

Example 9.2 *Does* $\int_1^\infty \frac{\sin^2 x}{x^2}dx$ *converge or diverge ?*

$$-1 \leq \sin x \leq 1$$
$$\implies 0 \leq \sin^2 x \leq 1$$
$$\implies 0 \leq \frac{\sin^2 x}{x} \leq \frac{1}{x^2}$$
$$\int_1^\infty \frac{1}{x^2}dx \text{ converges since } (P = 2 > 1)$$

$\therefore \int_1^\infty \frac{\sin^2 x}{x^2}$ *dx converges by the comparison test*

9.2 Review : Area Between Curves

Consider $f(x)g(x) on$ [a, b] with $f(x) \geq g(x)$. The area bounded by $f(x) \& g(x)$ between a and b can be repersented as.

$$A = \int_a^b f(x)dx - \int_a^b g(x)dx$$

End of Lecture Notes
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