Math 128 : Calculus 2 for the Sciences **Winter 2016**

Lecture 17-20: February 10 - 24 , 2016

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17.1 Parametric Equations

Up to now we have defined curves by

- y = f(x), x = f(y)
- F(x,y) = 0

For many applications this repersentation is not ideal (or even posssible)
Another way to define a curve :
Suppose x and y are given as functions of a third variable

$$x = f(t)$$
$$y = g(t)$$

t us the parameter that links x and y

17.2 Tangents

Given a curve with parametric equations x = f(t) , y = g(t), can we find $\frac{dy}{dx}$

$$\frac{dy}{dt} = \frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \text{ provided } \frac{dx}{dt} \neq 0$$

So the curve will have:

- Horizontal tangents where $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$
- vertical tangents where $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$

If both $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} = 0$ at some t^*, then you need to look at $\lim_{t \to t^*} \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

17.3 Area

Given $y = g(t), x = f(t), \alpha \leq t \leq \beta$

$$A = \int_a^b F(x)dx = \int_a^b ydx = \int_\alpha^\beta g(t)f'(t)dt$$

17.4 Arc Length

If C is a curve describe by parametric equations $y = g(t), x = f(t), \alpha \leq t \leq \beta$, and C is travered exactly once as t increases from $\alpha \rightarrow \beta$, then the length of the curve is given by

$$L = \int_\alpha^\beta \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt$$

17.5 Polar Coordinates

Given point (x,y), (r, θ) is a polar coordinante

- r is the distance from O (origin) to P (point)
- θ is the angle OP makes with the polar axis

17.6 Polar Coordinates $\longleftrightarrow$ Cartesian coodinates

In general, if P has polar coordinates (r, θ), its cartiesian coordinates are given by,

$$x = r\cos\theta, \quad y = r\sin\theta$$

In general, if P has Cartiesian coordinates (x,y), its polar coordinates (r, θ) are given by,

$$r^2 = x^2 + y^2, \quad \tan\theta = \frac{y}{x}$$

17.7 Symmetry

if the polar equation is unchanged when

- θ is replaced by $-\theta$: Then the equation is symmetric about polar axis
- θ is replaced by $\pi - \theta$: Then the equation is symmetric about vertical line $\theta = \frac{\pi}{2}$

End of Lecture Notes
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