Math 136 - Linear Algebra **Winter 2016**

Lecture 29: March 16, 2016

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29.1 More Determinants!

Corollary 29.1 *If A is an $n \times n$ matrix and E is an $n \times n$ elementary matrix, then $detEA = detE\ detA$*

Theorem 29.2 *Addition to the Invertible Matrix Theorem*
An $n \times n$ matrix A is invertible if and only if $detA \neq 0$

Proof: Let R be the RREF of A, then there exists k elementary matrices $E_1 \dots E_k$ such that $A = E_1E_2 \dots E_kR$ Then,

$$A = det(E_1E_2 \dots E_kR) = det(E_1)det(E_2) \dots det(E_k)det(R)$$

Thus,

$$detA \neq 0 \iff detR \neq 0 \text{ since the determinant of an elementary marix is non zero}$$

$$\therefore detR \neq 0 \iff rankR = n \iff \text{ A is invertible}$$

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Theorem 29.3 *If A and B are $n \times n$, then det(AB) = det(A) det (B)*

Proof: Write A as $A = E_1E_2 \dots E_kR$ such that R is the RREF of A. If A is invertible, then

$$R = I_n$$

$$detA = det(E_1)det(E_2) \dots det(E_k)det(R)$$

$$det(AB) = E_1E_2 \dots E_kB = det(E_1)det(E_2) \dots det(E_k)det(B) = detAdetB$$

If AB is non-invertible, then E has at least one row of zeros and RB also contains one row of zeros which implies $det(RB) = 0$

$$det(AB) = E_1E_2 \dots E_kRB = det(E_1)det(E_2) \dots det(E_k)det(RB) = 0$$

While,

$$detA = det(E_1)det(E_2) \dots det(E_k)det(R) = det(E_1)det(E_2) \dots det(E_k)det(R) = 0 \text{since det R} = 0$$

$$\implies det(AB) = 0 = detAdetB$$

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Corollary 29.4 *If A is an invertible matrix, then $detA^{-1} = \frac{1}{detA}$*

Theorem 29.5 *False Expansion Theorem*
If A is an $n \times n$ matrix with cofactors C_{ij}, then

$$\sum_{k=1}^{n} (A)_{ik}(C)_{jk} = 0, \text{ whenever } i \neq j$$

Theorem 29.6 *If A is invertible, then $(A^{-1})_{ij} = \frac{1}{detA} C_{ij}$*

Quick Fact : for any two matricies $A_{m\times n}$ $B_{n\times s}$, If A contains a row of zeros, then AB also contains a row of zeros. (This is very useful in alot of proofs)

End of Lecture Notes
Notes by : Harsh Mistry