Math 239 - Introduction to Combinatorics **Spring 2017**

Lecture 31: July 12th, 2017

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31.1 Matchings

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Definition 31.1 *Given a graph G a matching is set M of edges of G such that every vertex in G is incident to at most one edge in M*

Definition 31.2 *Let M be a matching of G*

1. *M is **Maximum** if every other matching M' has $\mid M' \mid \in \mid M \mid$*
   (Red edges form a maximum matching in G_1 and G_2 but not in G_3)
2. *A matching is **perfect** if $\mid M \mid = \frac{1}{2} \mid V(G) \mid$*
   (Red edges form a perfect matching just for G_1)
3. *A vertex $v \in V(G)$ is **saturated** by M if it is incident to an edge in M.*

31.2 Augmenting paths

Goal: Given a graph, find a maximum matching.

- How to tell that a matching is not maximum? Use Augmenting paths
- How to tell a graph is not connected? There exists a cut
- How to tell a graph is bipartite? Check for Odd cycles
- How to tell that a matching is not planar Check for $K_{3,3}$ or K_5 subdivisions

Definition 31.3 *Given a graph G and a matching M a path P is **alternating** if when we traverse P, the edges alternate between M and $E(G) \setminus M$*

*If in addition the ends of P are unsaturated, then P is **Augmenting***