Solution Atcoder - arc066 Problem B

classic

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1 Link Problem

2 Solution

Let $u = a \oplus b$
$v = a + b$

$a + b = (a \oplus b) + 2 \cdot (a \& b)$

or $v = a + 2 \cdot (a \& b)$

Let $d = a \& b$:

- $v = u + 2d$
- $u \& d = 0$
- $u + 2d \leq N$ and $u, d \geq 0$

Change problem: Find number of pair (u, d) such that:

- $u \& d = 0$
- $u + 2d \leq N$

We will have a valid pair (u, v) with $v = u + 2d$.

Let $f(x)$ is the number of pairs (u, d) that satisfy $u \& d = 0$ and $u + 2d \leq x$.

Consider the lowest bit of x:

Case 1: x odd

Suppose $x = 2k + 1$

- u even, d even:

  Let $u = 2u'$, $d = 2d'$

  Condition $u \& d = 0 \Leftrightarrow (2u') \& (2d') = 0 \Leftrightarrow u' \& d' = 0$

  Condition $u + 2d \leq 2k + 1 \Leftrightarrow 2u' + 4d' \leq 2k + 1 \Leftrightarrow u' + 2d' \leq k$

The number of pairs in this case is $f(k)$

- u odd, d even:

  Let $u = 2u' + 1$, $d = 2d'$

  Condition $u + 2d \leq 2k + 1 \Leftrightarrow (2u' + 1) + 4d' \leq 2k + 1 \Leftrightarrow u' + 2d' \leq k$

  The number of pairs in this case is $f(k)$

- u even, d odd:

  Let $u = 2u'$, $d = 2d' + 1$

  Condition $u + 2d = 2u' + 4d' + 2 = 2(u' + 2d' + 1) \leq 2k + 1 \leq 2k$

  $\Leftrightarrow u' + 2d' + 1 \leq k \Leftrightarrow u' + 2d' \leq k - 1$

  The number of pairs in this case is $f(k - 1)$

Summary: $f(2k + 1) = f(k) + f(k) + f(k - 1) = 2 \cdot f(k) + f(k - 1)$

Case 2: x even

- u even, d even:

  Let $u = 2u'$, $d = 2d'$

  Condition: $u' \& d' = 0$ and $u' + 2d' \leq k$

  Number of pairs: $f(k)$

- u odd, d even:

  Let $u = 2u' + 1$, $d = 2d'$

  Condition: $u' + 2d' \leq k - 1$ (because $2u' + 1 + 4d' \leq 2k$)

  Number of pairs: $f(k - 1)$

- u even, d odd:

  Let $u = 2u'$, $d = 2d' + 1$

  Condition: $u' + 2d' + 1 \leq k \Leftrightarrow u' + 2d' \leq k - 1$

  Number of pairs: $f(k - 1)$

Summary: $f(2k) = f(k) + f(k - 1) + f(k - 1) = f(k) + 2 \cdot f(k - 1)$