Quantifying the Return on Investment (ROI) of Campaign Contributions

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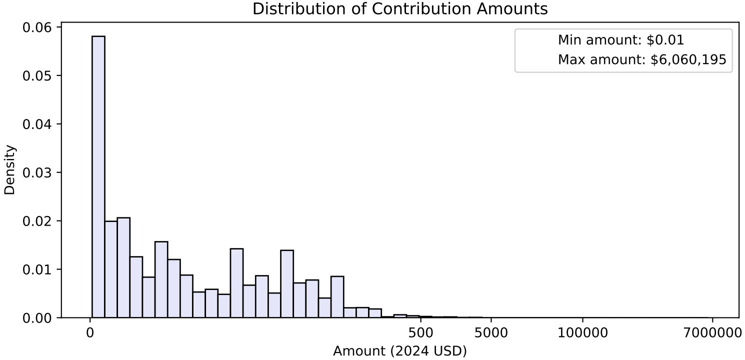
Update: Contribution Amount Distribution



Update: Class Imbalance

Days	W Count	W %	L Count	L %
360	4713	80%	1188	20%
240	4924	72%	1910	28%
120	5129	59%	3567	41%
60	5157	55%	4227	45%
30	5165	54%	4458	46%
14	5167	53%	4526	47%
7	5168	53%	4557	47%
1	5169	53%	4567	47%

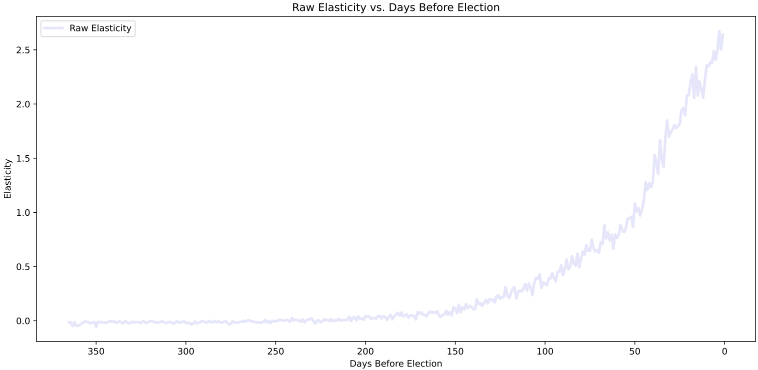
Recap

- Timing of contributions matters.
- Individual contributions deliver highest marginal returns, whereas committee money is weaker or mixed.
- Generalized additive models uncover non-monotonic saturation points.
- Money explains only a portion of outcome variation.

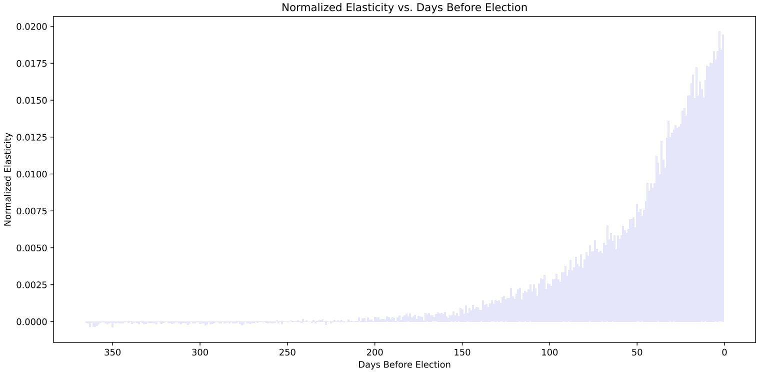
MLE Logistic Elasticities

Days	Elasticity
360	-0.011
240	-0.007
120	0.176
60	0.579
30	1.130
14	1.744
7	1.677
1	1.706

Logistic Raw Elasticities



Logistic Normalized Elasticities



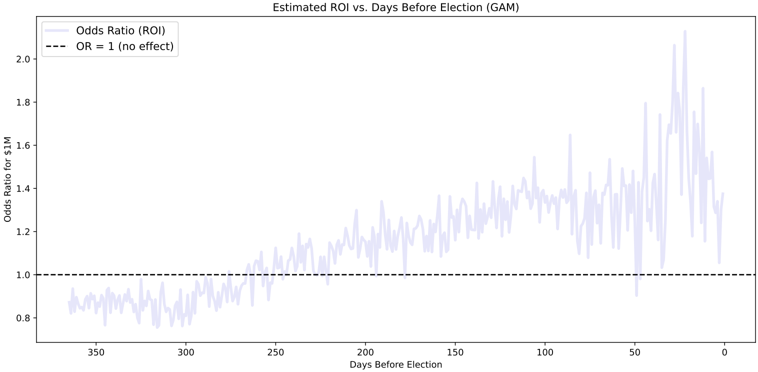
GAM Logistic Elasticities

Days	Elasticity
360	-0.029
240	0.012
120	0.084
60	-0.069
30	0.370
14	0.040
7	0.405
1	0.690

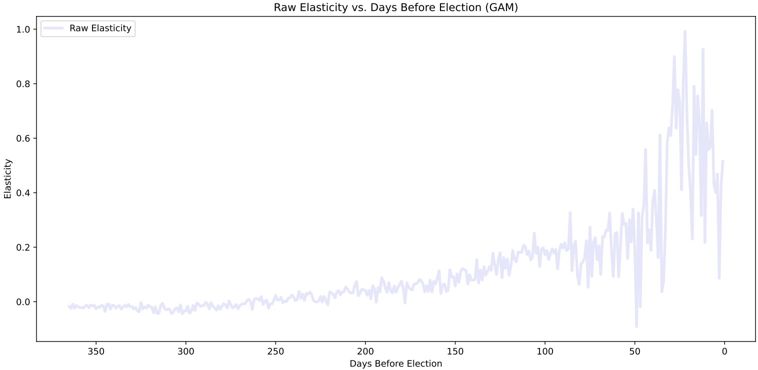
Elasticity Points

- ☆ Elasticity — Unit-free measure of responsiveness.
- ☆ Raw elasticities measure percent change in probability of winning from a one percent increase in spending at a particular cutoff.
- ☆ A one percent reallocation of total spending among time periods changes total spending by zero, so it can't change the outcome in first order.

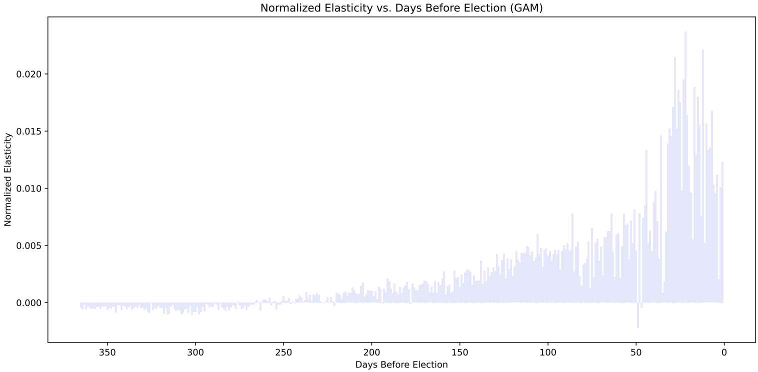
GAM Logistic Odds-Ratio Trend



GAM Logistic Raw Elasticities



GAM Logistic Normalized Elasticities



Linear Elasticities

Days	Elasticity
360	-0.059
240	-0.053
120	-0.018
60	0.018
30	0.094
14	0.135
7	0.136
1	0.171

GAM Linear Elasticities

Days	Elasticity
360	-0.002
240	-0.001
120	0.010
60	0.020
30	0.015
14	-0.004
7	-0.007
1	-0.006

Win Probability Maximization

- $\max_{\{b_t \geq 0\}} \sigma\left(\alpha + \sum_t \beta_t \log(1 + b_t)\right)$
- Maximize candidate's probability of winning a single race in single district.
- Total spending cannot exceed budget B; daily spend b_t is non-negative.
- α reflects factors unrelated to spending; adjusts final win probability, but not optimal timing of spending.

Win Probability Maximization (Continued)

- Solver converged to solution where first-order conditions and budget constraint are satisfied.
- Code minimizes $-\Pr(\text{win})$.

Win Probability Maximization (Continued)

✿ Budget of $100 thousand:

Days	Spend
22	$10,101.13
28	$928.32
23	$672.75
12	$657.99
26	$625.58
29	$559.54
44	$557.22
25	$501.78
17	$496.87
36	$477.98

Win Probability Maximization (Continued)

- ✿ Focus largest spends about $3 - 6$ weeks before election.
- ✿ Earlier and much later spending is less efficient per these ROI estimates.

Alternative Decision Problems (1)

- Assume D districts.
- Maximize Expected Wins:

$$\max_{\{b_{it} \geq 0\}} \sum_{i=1}^{D} \sigma\Big(\alpha_i + \sum_{t=1}^{T} \beta_t \log(1 + b_{it})\Big)$$

- b_{it} is dollars in district i at time t; β_t is the ROI curve; a_i is baseline log-odds for district i; σ is logistic link.

Alternative Decision Problems (2)

- Assume D districts.
- Maximize Threshold Wins:

$$\max_{\{b_{it}\}} \Pr\left(\sum_{i=1}^{D} W_i \geq M \right)$$

- M is the number of seats.
- $W_i \sim \text{Bernoulli}(p_i(b_i))$

Alternative Decision Problems

- In either case, when district-specific α_i and β_t are available, form each district's win probability term just as done in the one-race GAM.

Contrast

- Expected Wins: FOCs imply equalizing every district-time's marginal gain per dollar across all i, t.
- Threshold Wins: Moving a district's p_i from, say, 0.45 to 0.55 has a bigger effect than moving 0.8 to 0.9; one "overinvests" in mid-range p_i.
- Thus, under threshold objective, resources flow disproportionately to swing districts/times; under expected wins they spread more evenly.

Theory and Uncertainty

- Treat each estimated β_t as a random variable $B_t \sim N(\hat{\beta}_t, \sigma_t^2)$
- Problem:
  $\max \mathbb{E}[\sum_i \sigma(\alpha_i + \sum_t B_t \log(1 + b_{it}))]$
- Worst-case Problem:
  $\max_{\{b_{it}\}} \min_{\beta_t \in [\hat{\beta}_t \pm \Delta_t]} \sum_i \sigma\Big(\alpha_i + \sum_t \beta_t \log(1 + b_{it})\Big)$

Key Findings

- ✔ Back-loading: Majority of first-order effect comes in last 30 days.
- ✔ Diminishing returns fit the data well; spend beyond a certain window adds little.
- ✔ Elasticity framework translates β_t curves into unit-free shares (normalized elasticities sum to 1 by construction).

Practical Implications

- Campaign manager should concentrate resources in $14 - 30$ day window to maximize win probability under fixed budget.
- Under uncertainty, a robust allocation (using lower-bound β_t) further favors that same window, but more conservatively.
- Limitations: Current work is for a single race; district-level data would let us optimize across multiple seats.

Conclusion

By quantifying time-varying ROI and embedding it in a clear decision framework, we are able to let campaigns know not just what matters, but when and how much to spend for maximal impact.

References I

- Bonica, Adam. *Database on Ideology, Money in Politics, and Elections: Public version 4.0 [Computer file]*. `http://data.stanford.edu/dime`. Stanford University Libraries, Stanford, CA, 2024.