Lattice Quantum Chromo Dynamics

Giovanni Pederiva, Mathias Vege

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University of Oslo

Introduction

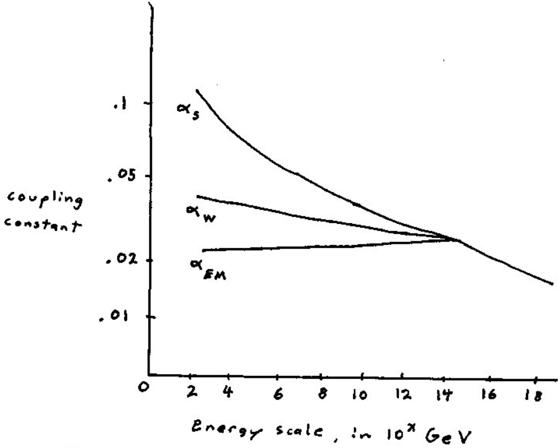
The fundamental forces

There are four fundamental forces in nature

- Gravity
- Electromagnetism
- Weak force
- Strong force

Coupling of the forces

Hope that at some energy these couplings will be equal...



$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + i\bar{\psi}\not{D}\psi + h.c.$$
$$+ \bar{\psi}_i y_{ij}\psi_j\phi + h.c. + |D_\mu\phi|^2 - V(\phi)$$

But we are only interested in the strong force...

The full QCD Lagrangian,

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\, a} + \bar{\psi}_i (i\gamma^\mu D_{\mu ij} - m\delta_{ij})\psi_j$$

a is color indices, i, j is flavour indices.

$$\begin{aligned} D_\mu &= \partial_\mu - igt^a A^a_\mu \\ G^a_{\mu\nu} &= \frac{i}{g}[D_\mu, D_\nu] = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc} A^b_\mu A^c_\nu \end{aligned}$$

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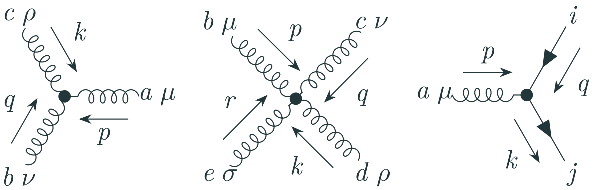
Why QCD is weird

- In the low energy limit, QCD is non-perturbative!
- Self-interaction



- huge mass difference in hadrons:

$$m_u + m_u + m_d \ ?= m_p$$

$$(2.3 + 2.3 + 4.8)\ \text{MeV} \ ?= 938\ \text{MeV}$$

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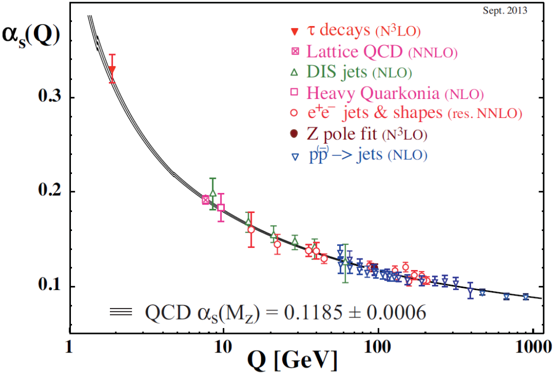
A closer look at the coupling...

This one is VERY different from the others



Lattice QCD

Solution: Lattice QCD

- Low-energy regime is non-perturbative.
- To understand nuclear forces, one needs a first principles approach.
- Understanding nuclear physics from LQCD.
- Can be used to investigate Dark Matter phenomena (in a far future...).

Challenges with LQCD

- Create a program that...
  - generates field configurations.
  - flows generated field configurations.
  - Samples the field for P, E, Q, Q_{t_e}.
- Take care of technicalities such as parallelization, $SU(3)$ exponentiation, ect.
- Perform analysis with bootstrap, jackknife, autocorrelation.
- Side projects: animating the field energy density and topological charge.

Lattice QCD

- Requirement of *Gauge invariance* is strongly maintained(throws Lorentz invariance out of the window ect.). $U' = \Omega^\dagger(x) U \Omega(x)$
- Discretizing the Gluon field in spacetime by introducing *link variables*, $U_\mu(x)$.

Path integrals

For field theories we discretize the field into points in space time.
For a scalar field, this becomes

$$\int \mathcal{D}\phi \rightarrow \int \prod_{x_i \in lattice} d\phi(x_i)$$

From this we can extract observables $\Gamma[\phi]$ dependent on this field as

$$\langle \Gamma[\phi] \rangle = \frac{1}{Z} \int \mathcal{D}\phi \Gamma[\phi] e^{-S[\phi]}$$

where the normalization Z is given again by

$$Z = \int \mathcal{D}\phi e^{-S[\phi]}$$

Fun technicalities

Parallelization

- How do you share a face in 4D?
- 3 different approaches
  - Face-sharing. Edge cases, not possible in the Metropolis algorithm
  - Single link-sharing. Relatively simple to implement, but slow if network is slow.
  - Shifts. Large non-blocking data packets at a time.

Sampling

Data is generated with the Metropolis algorithm, and following setup,

- Thermalize lattice. Roughly 10000 updates.
  1. Perform N_{corr} updates on the *whole* Lattice.
  2. For each N_{corr} update, we also perform N_{up} updates on every *single* link.
  3. Measure the change in ΔS, the action at each update. Accept config if $\exp(-\Delta S) > r$, where r is a number drawn from a uniform distribution.
  4. Flow the lattice and calculate observable at each step or save it to file.

Flow

A way of renormalizing the field.

Data

Table 1: Every set is generated with 600 correlation updates between each configuration, and 30 updates per link, except for 6.45, which has 1600 correlation updates. The the side of the cube is then around $L = 2.2$ fermi.

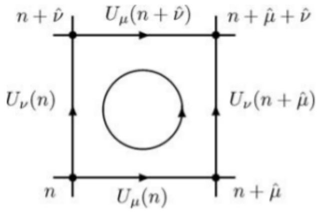
β	Size	N_{cfgs}	Size
6.0	$24^{\times}48$	1000	356 GB
6.1	$28^{\times}56$	500	330 GB
6.2	$32^{\times}64$	500	562 GB
6.45	$48^{\times}96$	250	1.4 TB

Observables

Plaquette

The *plaquette* is the simplest possible gauge invariant object on the lattice. It is a square made of 4 link variables:

$$P_{\mu\nu} = U_\mu(n)U_\nu(n+\hat{\mu})U_{-\mu}(n+\hat{\mu}+\hat{\nu})U_{-\nu}(n+\hat{\nu}) \quad = U_\mu(n)U_\nu(n+\hat{\mu})U_\mu(n \tag{1}$$



Energy

$$\langle E \rangle = -\frac{1}{64\,V} F^{a}_{\mu\nu} F^{a\mu\nu} \tag{2}$$

Tological charge

Can be thought of as the curl of the gluon field. Analogous to $u \times v = \varepsilon^{ijk} u_j v_k$

$$Q = -\sum_x \frac{1}{64 \cdot 32\pi^2} \epsilon_{\mu\nu\rho\sigma} Tr\{G^{clov}_{\mu\nu} G^{clov}_{\rho\sigma}\} \tag{3}$$

Topological Susceptibility

$$\chi^{1/4} = \frac{\hbar c}{a V^{1/4}} \langle Q^2 \rangle^{1/4} \tag{4}$$

N_f - checking the flavor

Using Witten-Veneziano formula to look at how different χ behave in the continuum limit and which that gives the best error estimate.

$$\frac{F_\pi^2 m_{\eta'}^2}{2N_f} = \chi$$

where F_π is the decay rate constant of pion given as $F_\pi = 130.0 \pm 5.0$MeV. The mass of the meson η' is given as $m_{\eta'} = 957.78 \pm 0.06$MeV. N_f is the number of flavors.

References

- G.P. Lepage, *Lattice QCD for Novices*, arXiv:hep-lat/0506036 (2005)
- C. Gattringer & C.P. Lang, *Quantum Chromodynamics on the Lattice*, Springer (2010)