Lattice Quantum Chromo Dynamics

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Introduction

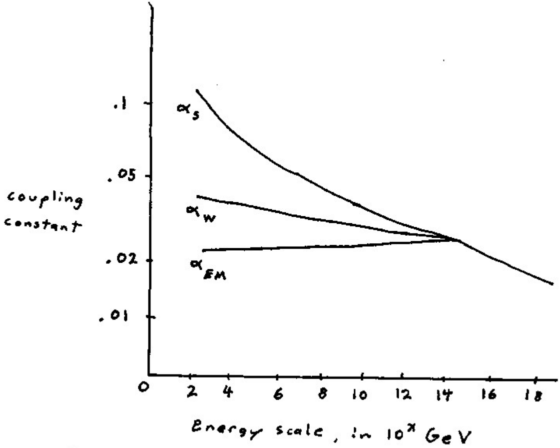
The fundamental forces

There are four fundamental forces in nature

- Gravity
- Electromagnetism
- Weak force
- Strong force

Coupling of the forces

Hope that at some energy these couplings will be equal...



The Standard Model

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + i\bar{\psi}\not{D}\psi + h.c.$$
$$+ \bar{\psi}_i y_{ij}\psi_j\phi + h.c. + |D_\mu\phi|^2 - V(\phi)$$

But we are only interested in the strong force…

QCD

The full QCD Lagrangian,

$$\mathcal{L} = -\frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu\, a} + \bar{\psi}_i (i\gamma^{\mu} D_{\mu ij} - m\delta_{ij}) \psi_j$$

a is color indices, i, j is flavour indices.

$$\begin{aligned} D_{\mu} &= \partial_{\mu} - igt^{a} A^{a}_{\mu} \\ G^{a}_{\mu\nu} &= \frac{i}{g}[D_{\mu}, D_{\nu}] = \partial_{\mu} A^{a}_{\nu} - \partial_{\nu} A^{a}_{\mu} + gf^{abc} A^{b}_{\mu} A^{c}_{\nu} \end{aligned}$$

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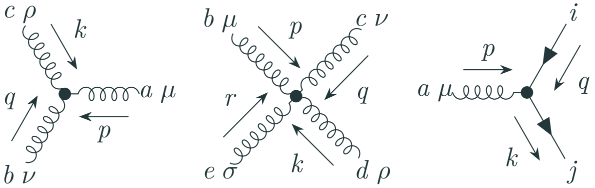
Why QCD is weird

- In the low energy limit, QCD is non-perturbative!
- Self-interaction



- huge mass difference in hadrons:

$$m_u + m_u + m_d \; ? = m_p$$

$$(2.3 + 2.3 + 4.8) \text{ MeV } ? = 938 \text{ MeV}$$

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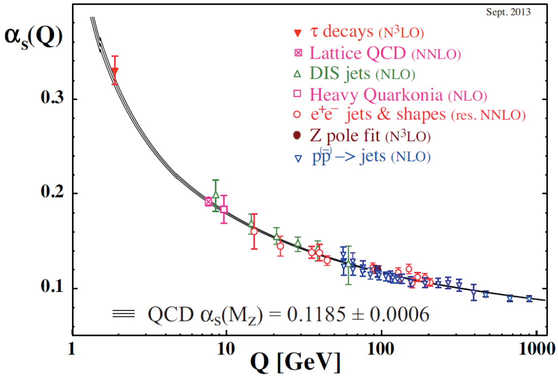
A closer look at the coupling...

This one is VERY different from the others



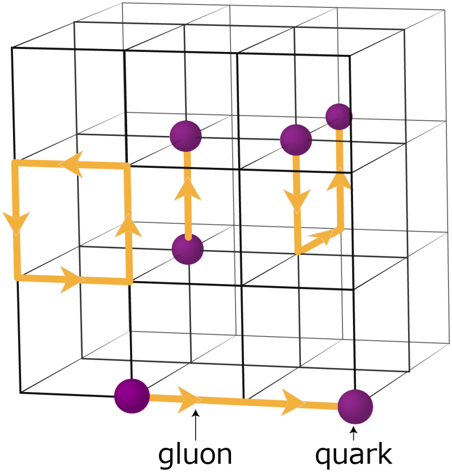
Low-energy regime is **non-perturbative**.

Why is is a problem?

- Understand confinement
- Derive nuclear forces from ”first principles”
- Can be used to investigate Dark Matter phenomena (in a far future...).

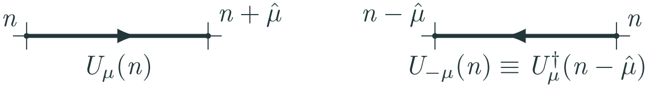
Lattice QCD

Solution: Lattice QCD



Discretizing the Gluon field in spacetime by introducing *link variables*, $U_\mu(x)$.

A bit more formally...



Schematic representation of the *link variables* $U_\mu(n)$ and $U_{-\mu}(n)$ on the lattice.

- These are 3×3 unitary matrices.
- Requirement of *gauge invariance* is strongly maintained (throws Lorentz invariance out of the window ect.). $U' = \Omega^\dagger(x)\, U \Omega(x)$
- A closed-path product, when traced, is *gauge invariant*

What do we do with it?

Path Integrals!

$$\int \mathcal{D}F[x(t)] \rightarrow \frac{1}{A}\int dx_1\, dx_2 \ldots dx_{N-1} F[x(t)]$$

These objects are hard to work with though...

Analogy with QM

In Quantum Mechanics an observable is given by:

$$\frac{\langle\psi|\,\hat{O}\,|\psi\rangle}{\langle\psi|\psi\rangle} = \frac{\int dx\ \psi^*(x)\hat{O}\psi(x)}{\int dx\ \psi^*(x)\psi(x)}$$

In Field Theories:

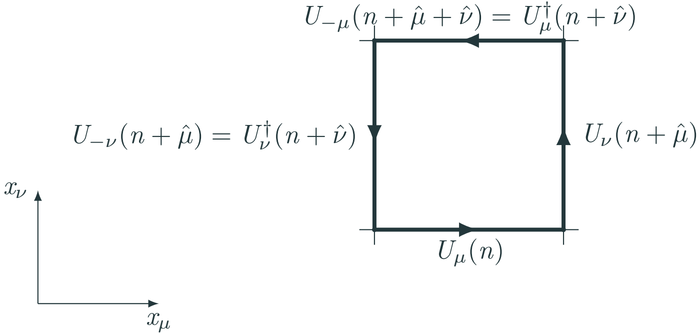
$$\langle O[\phi]\rangle = \frac{1}{Z[\phi]}\int \mathcal{D}[\phi]\ O[\phi]e^{-S_E[\phi]}$$

$$Z = \int \mathcal{D}[\phi]\ e^{-S_E[\phi]}$$

The Euclidean Action

Split in Gluonic and Fermionic part:
$S_E[\psi, \bar{\psi}, U] = S_F[\psi, \bar{\psi}, U] + S_G[U]$

$$S_G[U] = \frac{a^4}{2g^2} \sum_{n \in \Lambda} \sum_{\mu\nu} \text{Re Tr}(\mathbb{1} - P_{\mu\nu}) \xrightarrow{a \to 0} \frac{1}{4g^2} \int d^4x G_{\mu\nu}(x)^2$$



A Fermion Trick

Integrate the fermions analytically (Grassmann Numbers!)

$$Z = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}U e^{-S[\psi,\bar{\psi},U]} = \int \mathcal{D}U e^{-S_G[U]} \det M[U]$$

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}U \; O[\psi, \bar{\psi}, U] e^{-S_G[U]} \det M[U].$$

A better trick: $\det M[U] \to const$ (no dynamical fermions)

Analogy with SM

As in Statistical Mechanics:

$$P[U] = \frac{e^{-S_G[U]} \det M[U]}{Z}.$$

so that observables become:

$$\langle O \rangle = \int \mathcal{D}U \; P[U] O[\psi, \bar{\psi}, U] \approx \frac{1}{N} \sum_{i=1}^{N} O(\psi, \bar{\psi}, U_i),$$

Strategy of LQCD

Create a program that...

- Generates ensembles of field configurations U_i with the given PDF.
- Stores them to memory!
- Later compute the desired observables as averages over the ensemble.

Fun technicalities

Data

The size is HUGE

$$\underbrace{N^3}_{\text{spatial dimension}} \times \underbrace{N_t}_{\text{time dimension}} \times \underbrace{4}_{\text{links per site}} \times \underbrace{9}_{\text{SU(3) matrix size}} \times \underbrace{2}_{\text{real and imaginary part}}$$

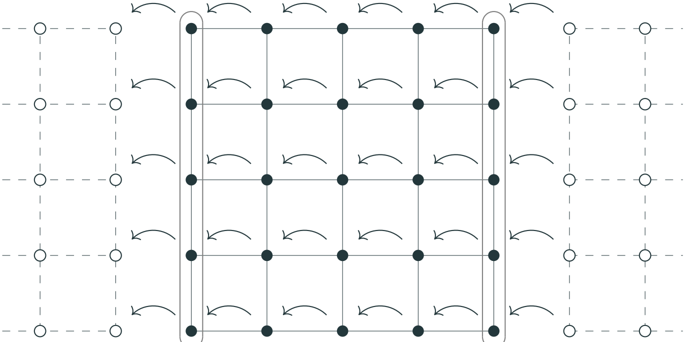
Table 1: Every set is generated with 600 correlation updates between each configuration, and 30 updates per link, except for 6.45, which has 1600 correlation updates. The the side of the cube is then around $L = 2.2$ fermi.

β	Size	N_{cfgs}	Size
6.0	$24^{\times}48$	1000	356 GB
6.1	$28^{\times}56$	500	330 GB
6.2	$32^{\times}64$	500	562 GB
6.45	$48^{\times}96$	250	1.4 TB

Parallelization

Too big systems to handle on one processor $\rightarrow$ split into sublattices
How do you share a face in 4D?

- 3 different approaches
  - **Halo-exchange**. Edge cases, not possible in the Metropolis algorithm
  - **Single link-sharing**. Relatively simple to implement, but slow if network is slow.
  - **Shifts**. Large non-blocking data packets at a time.



What we did

Gradient Flow

A way of renormalizing the field...
Basically, a diffusion-like equation in a 5th (!) dimension.

$$\partial_{t_f} B_\mu = D_\nu G_{\nu\mu}$$

with $B_\mu|_{t_f=0} = A_\mu$ The other terms in the equation are defined as:

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu],$$
$$D_\mu B_\nu = \partial_\mu B_\nu + [B_\mu, B_\nu].$$

A VERY bad approximation is:

$$\partial_{t_f} B_\mu \approx \partial_\nu \partial_\mu B_\nu \rightarrow \text{diffusion equation in 5D}$$

Numerical integration of the Gradient Flow

$$
\begin{aligned}
W_0 &= V_t \\
W_1 &= \exp\left[\frac{1}{4}Z_0\right] W_0 \\
W_2 &= \exp\left[\frac{8}{9}Z_1 - \frac{17}{36}Z_0\right] W_1 \\
V_{t+\epsilon} &= \exp\left[\frac{3}{4}Z_2 - \frac{8}{9}Z_1 + \frac{17}{36}Z_0\right] W_2,
\end{aligned}
$$

Z is the derivative of the Action S_G. Exponential of $\mathfrak{su}(3)$ elements...
The goal is to see the evolution of physical observables as the field is smeared by the flow equation.

But Why?

But Why?

- Get nice animations!
- Eventually write a thesis...

References

- G.P. Lepage, *Lattice QCD for Novices*, arXiv:hep-lat/0506036 (2005)
- C. Gattringer & C.P. Lang, *Quantum Chromodynamics on the Lattice*, Springer (2010)