Solving $\mathrm{SU}(3)$ Yang-Mills theory on the lattice: a calculation of selected gauge observables with gradient flow

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Introduction

Structure

- Quantum Chromodynamics(QCD).

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- Developing a code for solving $\mathrm{SU}(3)$ Yang-Mills theory.

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- **GLAC.** Will briefly present the code which we developed as well as some benchmarks. We will also present the Metropolis algorithm.

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- **Results.** We will present the results obtained from pure gauge calculations.

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Quantum Chromodynamics(QCD)

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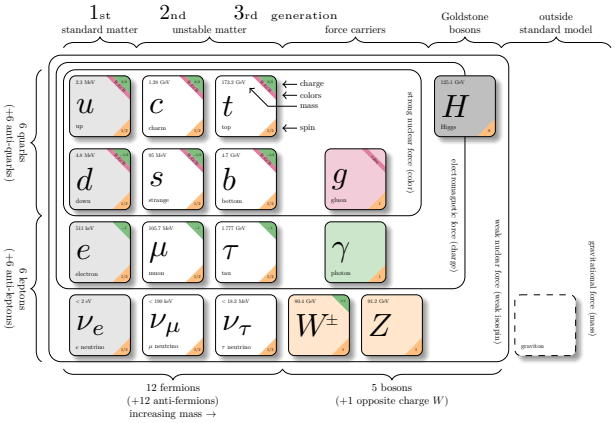
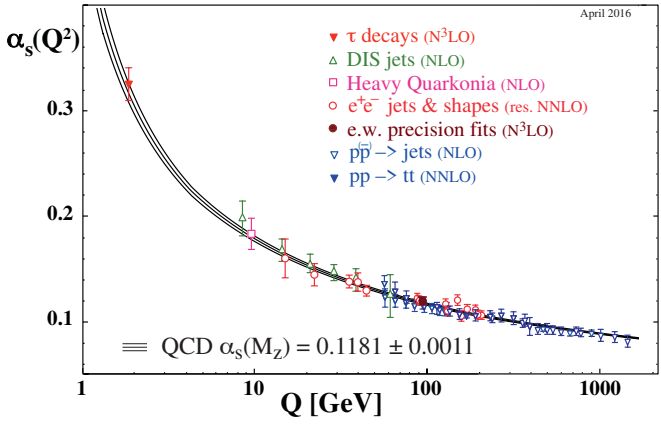
The Standard Model



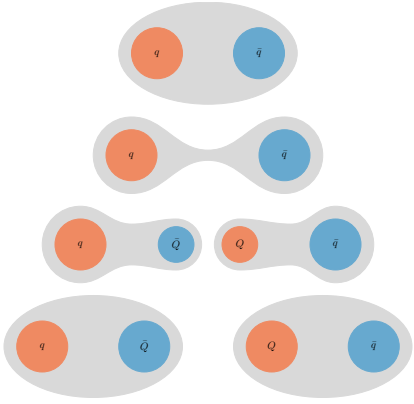
Consists of the innermost square of the six quarks and the gluons.

Asymptotic freedom



- The coupling constant **decreases** as we **increase** the energy
- One of the experimental proofs of QCD along with triple γ decay and muon cross section ration R.

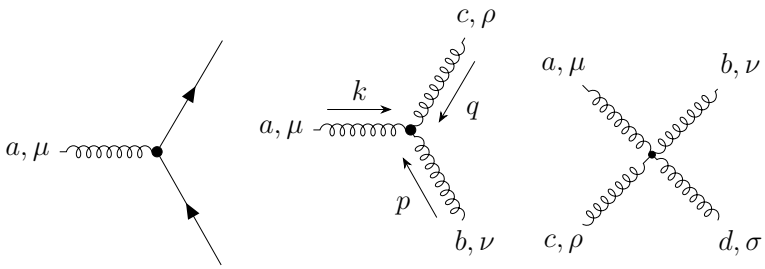
Confinement



If we try to pull apart **two mesons**, more and more energy is required until we have enough energy to spontaneously create a **quark-antiquark** pair, forming thus **two new mesons.**

The non-linearity of QCD

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{f=1}^{N_f} \bar{\psi}^{(f)} \left(i \not{D} - m^{(f)} \right) \psi^{(f)} - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu},$$



Measuring topological charge is a measure of the *Winding number* of the gauge field.

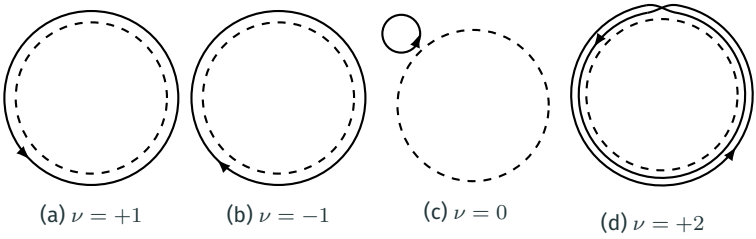


Figure 1: The figure is taken from [1, p. 32].

An illustration of how one can view the winding number given a function f that parametrizes a path around a circle S^1. Given that it starts and ends at the same point, we have that the number of times it wraps around the circle gives us the winding number.

The Witten-Veneziano relation

- A

Lattice Quantum Chromodynamics(LQCD)

Discretizing spacetime

- A

Links

Pure gauge action

Recovering the continuum action action

Developing a code for solving $\mathrm{SU}(3)$ Yang-Mills theory on the lattice

The numerical challenge in lattice QCD

The path integral

The Metropolis algorithm

Link sharing

Shifts

Scaling

Measuring observables on the lattice

How to measure

Topological charge

Energy

Gradient flow

The flow equation

Solving gradient flow on the lattice

Smearing the lattice

Results

Ensembles

Energy and the scale setting

Topological charge

Topological susceptibility

The fourth cumulant

The topological charge correlator

The effective glueball mass

Conclusion

Questions?

References

[1] Hilmar Forkel. A Primer on Instantons in QCD. *arXiv:hep-ph/0009136*, September 2000. URL http://arxiv.org/abs/hep-ph/0009136. arXiv: hep-ph/0009136.