Solving $\mathrm{SU}(3)$ Yang-Mills theory on the lattice: a calculation of selected gauge observables with gradient flow

Hans Mathias Mamen Vege
April 25, 2019

Supervisor: *Andrea Shindler*
Co-supervisor: *Morten Hjorth-Jensen*

University of Oslo

Introduction

Structure

- Quantum Chromodynamics(QCD).

- **QCD.** We will go through and explain what QCD as well as motivate its existence.

Structure

- Quantum Chromodynamics(QCD).
- Lattice QCD.

- **QCD.** We will go through and explain what QCD as well as motivate its existence.
- **LQCD.** We will briefly show how one discretise the lattice and perform calculations on it.

Structure

- Quantum Chromodynamics(QCD).
- Lattice QCD.
- Gradient flow.

- **QCD.** We will go through and explain what QCD as well as motivate its existence.
- **LQCD.** We will briefly show how one discretise the lattice and perform calculations on it.
- **Gradient flow.** We will quickly introduce gradient flow and explain its effects.

Structure

- Quantum Chromodynamics(QCD).
- Lattice QCD.
- Gradient flow.
- Developing a code for solving $\mathrm{SU}(3)$ Yang-Mills theory.

- **QCD.** We will go through and explain what QCD as well as motivate its existence.
- **LQCD.** We will briefly show how one discretise the lattice and perform calculations on it.
- **Gradient flow.** We will quickly introduce gradient flow and explain its effects.
- **GLAC.** Will briefly present the code which we developed as well as some benchmarks. We will also present the Metropolis algorithm.

Structure

- Quantum Chromodynamics(QCD).
- Lattice QCD.
- Gradient flow.
- Developing a code for solving $\mathrm{SU}(3)$ Yang-Mills theory.
- Results.

- **QCD.** We will go through and explain what QCD as well as motivate its existence.
- **LQCD.** We will briefly show how one discretise the lattice and perform calculations on it.
- **Gradient flow.** We will quickly introduce gradient flow and explain its effects.
- **GLAC.** Will briefly present the code which we developed as well as some benchmarks. We will also present the Metropolis algorithm.
- **Results.** We will present the results obtained from pure gauge calculations.

Structure

- Quantum Chromodynamics(QCD).
- Lattice QCD.
- Gradient flow.
- Developing a code for solving $\mathrm{SU}(3)$ Yang-Mills theory.
- Results.

Quantum Chromodynamics(QCD)

- The standard model: Six quarks and eight gluons

- The standard model.

- The standard model: Six quarks and eight gluons
- Asymptotic freedom

- The standard model.
- Asymptotic freedom.

- The standard model: Six quarks and eight gluons
- Asymptotic freedom
- Confinement

- The standard model.
- Asymptotic freedom.
- Confinement.

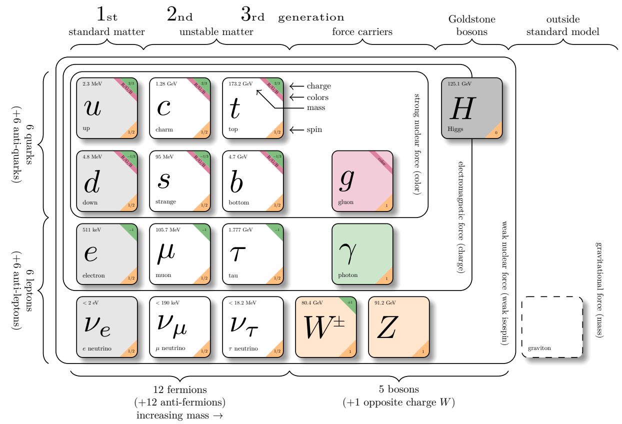
- The standard model: Six quarks and eight gluons
- Asymptotic freedom
- Confinement
- Highly nonlinear due to gluon self-interactions

- The standard model.
- Asymptotic freedom.
- Confinement.
- Nonlinearity.

QCD

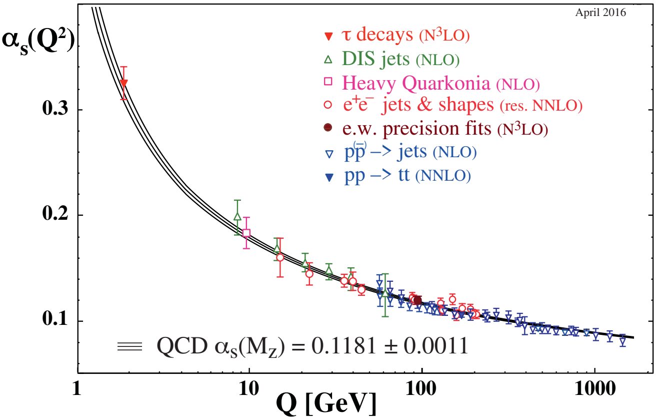
- The standard model: Six quarks and eight gluons
- Asymptotic freedom
- Confinement
- Highly nonlinear due to gluon self-interactions

The Standard Model

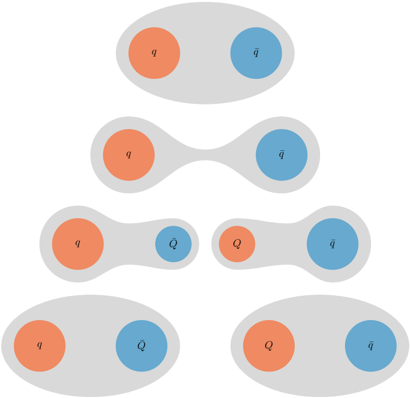


Consists of the innermost square of the six quarks and the gluons.

Asymptotic freedom



- The coupling constant **decreases** as we **increase** the energy
- One of the experimental proofs of QCD along with triple γ decay and muon cross section ration R.



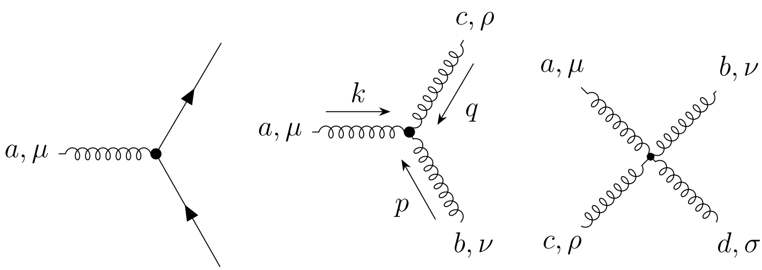
If we try to pull apart **two mesons**, more and more energy is required until we have enough energy to spontaneously create a **quark-antiquark** pair, forming thus **two new mesons.**

The QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{f=1}^{N_f} \bar{\psi}^{(f)} \left(i\not{D} - m^{(f)} \right) \psi^{(f)} - \frac{1}{4} G^{a}_{\mu\nu} G^{a\mu\nu},$$

with action

$$S = \int \mathrm{d}^4 x \mathcal{L}_{\mathrm{QCD}}. \tag{1}$$



- Instantons

- **Instantons** are local minimums to the Yang-Mills action in Euclidean space.

Topology in QCD

- Instantons
- Topological charge, Q

- **Instantons** are local minimums to the Yang-Mills action in Euclidean space.
- Measuring **topological charge** is a measure of the *Winding number* of the gauge field.

Topology in QCD

- Instantons
- Topological charge, Q
- Winding number

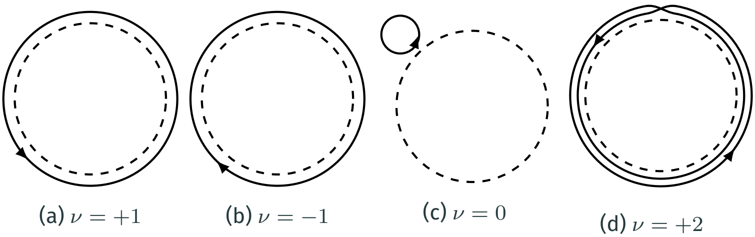
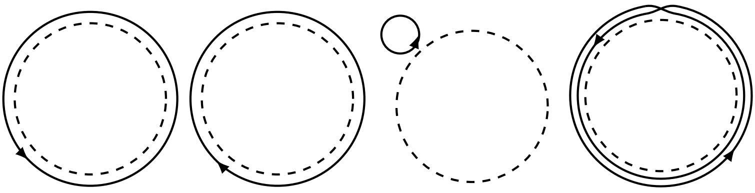


Figure 1: The figure is taken from Forkel [1, p. 32].

- **Instantons** are local minimums to the Yang-Mills action in Euclidean space.
- Measuring **topological charge** is a measure of the *Winding number* of the gauge field.
- An illustration of how one can view the winding number given a function f that parametrizes a path around a circle S^1. Given that it starts and ends at the same point, we have that the number of times it wraps around the circle gives us the winding number.

Topology in QCD

- Instantons
- Topological charge, Q
- Winding number



(a) $\nu = +1$ (b) $\nu = -1$ (c) $\nu = 0$ (d) $\nu = +2$

Figure 1: The figure is taken from Forkel [1, p. 32].

- **Instantons** are local minimums to the Yang-Mills action in Euclidean space.
- Measuring **topological charge** is a measure of the *Winding number* of the gauge field.
- An illustration of how one can view the winding number given a function f that parametrizes a path around a circle S^1. Given that it starts and ends at the same point, we have that the number of times it wraps around the circle gives us the winding number.

The Witten-Veneziano relation

A formula connecting pure gauge theory and full QCD.

$$m_{\eta'}^2 = \frac{2N_f}{f_\pi^2}\chi_{\text{top}} \tag{2}$$

- Pion decay constant $f_\pi = 0.130(5)/\sqrt{2}$ GeV.
- η' meson mass $m_{\eta'} = 0.95778(6)$ GeV.
- χ_{top} is the *topological susceptibility*.

- We use the experimental values for the pion decay constant and the η' mass.
- Allows us to estimate the number of flavors in our theory N_f.
- χ_{top} is the topological susceptibility, calculated from the expectation value of Q.

Lattice Quantum Chromodynamics(LQCD)

Discretizing spacetime

1. Divide spacetime into a cube of size $N^3 \times N_T$.

Discretizing spacetime

1. Divide spacetime into a cube of size $N^3 \times N_T$.
2. Fermions live on the each *point* in the cube.

Discretizing spacetime

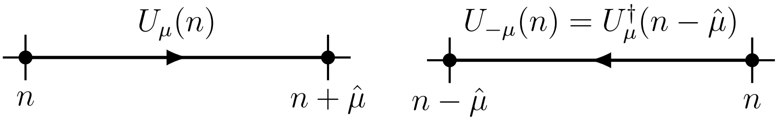
1. Divide spacetime into a cube of size $N^3 \times N_T$.
2. Fermions live on the each *point* in the cube.
3. The gauge fields live on the sites *in between* the points, and is called links.

Links

A link

$$U_\mu(n) = \exp\left[iaA_\mu(n)\right],$$

connects one lattice site to another.



where $U_{-\mu}(n) = U_\mu(n-\hat{\mu})^\dagger$.

- Defined from the gauge transporter.
- A link in the positive $\hat{\mu}$ direction is shown in the figure to the left.
- A link in the negative $\hat{\mu}$ direction is shown in the figure to the right.

Gauge invariance on the lattice

Links gauge transform as

$$
\begin{aligned}
U_\mu(n) &\to U'_\mu(n) = \Omega(n)\, U_\mu(n)\Omega(n+\hat{\mu})^\dagger, \\
U_{-\mu}(n) &\to U'_{-\mu}(n) = \Omega(n)\, U_\mu(n-\hat{\mu})^\dagger\Omega(n-\hat{\mu})^\dagger.
\end{aligned}
$$

Links gauge transform as

$$U_\mu(n) \to U'_\mu(n) = \Omega(n)\, U_\mu(n) \Omega(n+\hat{\mu})^\dagger,$$
$$U_{-\mu}(n) \to U'_{-\mu}(n) = \Omega(n)\, U_\mu(n-\hat{\mu})^\dagger \Omega(n-\hat{\mu})^\dagger.$$

Two main types of gauge invariant objects,

Gauge invariance on the lattice

Links gauge transform as

$$U_\mu(n) \to U'_\mu(n) = \Omega(n)\, U_\mu(n)\Omega(n+\hat{\mu})^\dagger,$$
$$U_{-\mu}(n) \to U'_{-\mu}(n) = \Omega(n)\, U_\mu(n-\hat{\mu})^\dagger\Omega(n-\hat{\mu})^\dagger.$$

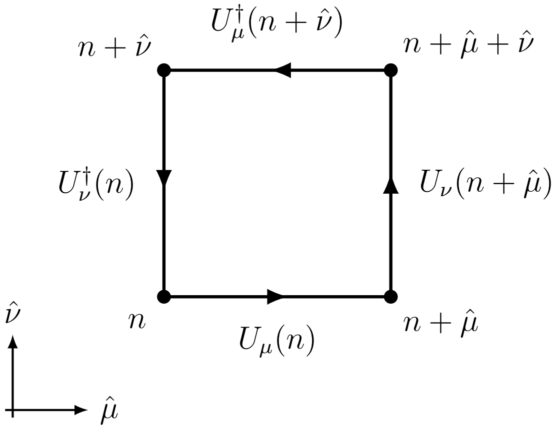
Two main types of gauge invariant objects,

- Fully connected gauge invariant objects.
- Objects with fermions ψ, $\bar{\psi}$ as end points.

The plaquette

The simplest gauge invariant object,

$$\begin{aligned} P_{\mu\nu}(n) &= U_\mu(n)\, U_\nu(n+\hat{\mu})\, U_{-\mu}(n+\hat{\mu}+\hat{\nu})\, U_{-\nu}(n+\hat{\nu}) \\ &= U_\mu(n)\, U_\nu(n+\hat{\mu})\, U_\mu(n+\hat{\nu})^\dagger\, U_\nu(n)^\dagger, \end{aligned}$$



The Wilson gauge action

The Wilson gauge action is given as

$$S_G[U] = \frac{\beta}{3} \sum_{n\in\Lambda} \sum_{\mu<\nu} \mathrm{Re}\,\mathrm{tr}\left[1 - P_{\mu\nu}(n)\right], \tag{3}$$

with $\beta = 6/g_S$.

- Using the definition of the link we saw earlier, we can reproduce the continuum action up to an discretization error of $\mathcal{O}(a^2)$.

Developing a code for solving $\mathrm{SU}(3)$ Yang-Mills theory on the lattice

The numerical challenge in lattice QCD

The path integral

The Metropolis algorithm

GLAC (GLuon ACtion)

Link sharing

Shifts

Scaling

Measuring observables on the lattice

How to measure

Energy

Gradient flow

The flow equation

Solving gradient flow on the lattice

Smearing the lattice

Results

Ensembles

Energy and the scale setting

Topological charge

Topological susceptibility

The fourth cumulant

The topological charge correlator

The effective glueball mass

Conclusion

Questions?

References

[1] Hilmar Forkel. A Primer on Instantons in QCD. *arXiv:hep-ph/0009136*, September 2000. URL `http://arxiv.org/abs/hep-ph/0009136`. arXiv: hep-ph/0009136.