Soru

10 Ocak 2022 Pazartesi 09:35

soru

$$y'' + 4y' + 4y = e^{-2t} \ln t$$

$$y'' - 2y' + y = t^{-1}e^t$$

$y'' - 2y' + y = 0,$

$r^2 - 2r + 1 = 0, \quad (r-1)^2 = 0, \quad r = 1, \quad y_h = (c_1 + tc_2)e^t = c_1 e^t + c_2 te^t$

$y_p = v_1 e^t + v_2 te^t$

$$v_1' e^t + v_2' te^t = 0$$
$$v_1' e^t + v_2'(e^t + te^t) = \frac{e^t}{t}$$

$$\Rightarrow W = \begin{vmatrix} e^t & te^t \\ e^t & e^t + te^t \end{vmatrix} = e^{2t} + te^{2t} - te^{2t} = e^{2t} \neq 0$$

$$v_1' = \frac{\begin{vmatrix} 0 & te^t \\ \frac{e^t}{t} & e^t + te^t \end{vmatrix}}{W} = \frac{-\frac{e^t}{\cancel{t}} \cdot \cancel{t}e^t}{e^{2t}} = -1, \qquad v_1' = -1$$

$$\boxed{v_1 = -\int dt = -t}$$

$$v_2' = \frac{\begin{vmatrix} e^t & 0 \\ e^t & \frac{e^t}{t} \end{vmatrix}}{W} = \frac{\frac{e^{2t}}{t}}{e^{2t}} = \frac{1}{t}, \qquad v_2' = \frac{1}{t}$$

$$\boxed{v_2 = \int \frac{1}{t}\, dt = \ln t}$$

$$\boxed{\begin{aligned} y_p &= v_1 e^t + v_2 te^t \\ y_p &= -te^t + \ln t \cdot te^t \end{aligned}}$$

$$\boxed{\begin{aligned} y &= y_h + y_p \\ y &= (c_1 + tc_2)e^t + te^t(\ln t - 1) \end{aligned}}$$

Given that $\underbrace{y_1(t) = t}$ is a solution to

$$y'' - \frac{1}{t}y' + \frac{1}{t^2}y = 0 \ , \qquad (17)$$

use the reduction of order procedure to determine a second linearly independent solution for $t > 0$.

$y_2 = y_1 \cdot v(t) = t \cdot v$

$y' = v + tv'$

$y'' = v' + v' + tv'' = tv'' + 2v'$

$tv'' + 2v' - \frac{1}{t}(v + tv') + \frac{1}{t^2}(t \cdot v) = 0$

$tv'' + 2v' - \cancel{\frac{v}{t}} - v' + \cancel{\frac{v}{t}} = 0$

$\boxed{tv'' + v' = 0}$

$v' = u$

$v'' = u'$

$\Rightarrow \quad t \cdot u' + u = 0$

$t\frac{du}{dt} = -u \Rightarrow \int \frac{du}{u} = \int -\frac{dt}{t}$

$\ln u = -\ln t \Rightarrow \ln u = \ln t^{-1}$

$u = t^{-1}$

$u = \frac{1}{t}$

$v' = \frac{1}{t}$

$v = \int \frac{1}{t} dt \Rightarrow \boxed{v = \ln t}$

$\boxed{y_2 = t \cdot v_2 = t \ln t}$ **

The following equation arises in the mathematical modeling of reverse osmosis.

(19) $$(\sin t)y'' - 2(\cos t)y' - (\sin t)y = 0\ , \qquad 0 < t < \pi\ .$$

Find a general solution.

$$y'' - 4y = 4t - 8e^{-2t} \ ;$$
$$y(0) = 0 \ , \qquad y'(0) = 5$$

In Problems 5–10, express the given function using window and step functions and compute its Laplace transform.

6. $g(t) = \begin{cases} 0\ , & 0 < t < 2\ , \\ t+1\ , & 2 < t \end{cases}$

$g(t) = 0 \cdot \cancel{\Pi_{0,2}(t)} + (t+1)\,u(t-2)$

$g(t) = (t+1)\,u(t-2)$

$\mathcal{L}\{g(t)\} = \mathcal{L}\{(t+1)\,u(t-2)\} = e^{-2s}\,\mathcal{L}\{(t+1)+2\}$

$= e^{-2s}\,\mathcal{L}\{t+3\}$

$\boxed{\mathcal{L}\{g(t)\} = e^{-2s}\left(\frac{1}{s^2} + \frac{3}{s}\right)}$

7.

$$\underset{=}{a}\,t+b$$

$$t+b$$

$$g(t)=\begin{cases}0, & 0<t<1\\ t, & 1<t<2\\ 1, & t>2\end{cases}$$

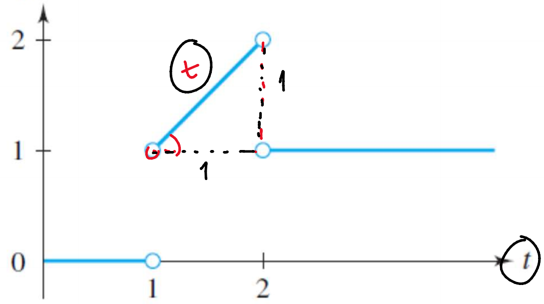

Figure 7.16 Function in Problem 7

$$g(t)=0\cdot\Pi_{0,1}(t)+t\,\Pi_{1,2}(t)+1\cdot u(t-2)$$

$$g(t)=t\big(u(t-1)-u(t-2)\big)+u(t-2)$$

$$g(t)=t\,u(t-1)+(1-t)\,u(t-2)$$

$$\mathcal{L}\{g(t)\}=\mathcal{L}\{t\,u(t-1)+(1-t)\,u(t-2)\}$$

$$=e^{-s}\mathcal{L}\{t+1\}+e^{-2s}\mathcal{L}\{3-t\}$$

$$\boxed{\mathcal{L}\{g(t)\}=e^{-s}\left(\frac{1}{s^2}+\frac{1}{s}\right)+e^{-2s}\left(\frac{3}{s}-\frac{1}{s^2}\right)}$$ **

$$\frac{y-y_0}{y_1-y_0} = \frac{x-x_0}{x_1-x_0} \quad , A(x_0,y_0),\ B(x_1,y_1)$$

9.

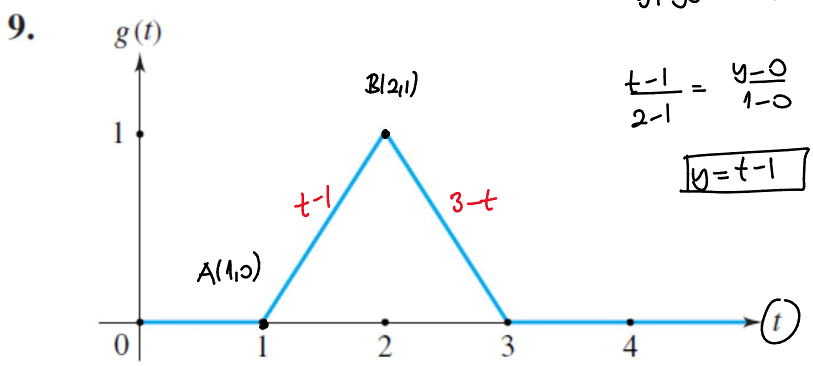

Figure 7.18 Function in Problem 9

$$\frac{t-1}{2-1} = \frac{y-0}{1-0}$$

$$\boxed{y = t-1}$$

$$g(t) = \cancel{0 \cdot \Pi_{0,1}(t)} + (t-1)\,\Pi_{1,2}(t) + (3-t)\,\Pi_{2,3}(t) + \cancel{0 \cdot u(t-3)}$$

$$g(t) = (t-1)\left[u(t-1) - u(t-2)\right] + (3-t)\cdot\left[u(t-2) - u(t-3)\right]$$

$$= (t-1)\,u(t-1) + (-t+1+3-t)\,u(t-2) + (t-3)\,u(t-3)$$

$$g(t) = (t-1)\,u(t-1) + (4-2t)\,u(t-2) + (t-3)\,u(t-3)$$

$$\mathcal{L}\{g(t)\} = e^{-s}\mathcal{L}\{t\} + e^{-2s}\mathcal{L}\{6-2t\} + e^{-3s}\mathcal{L}\{t\}$$

$$\boxed{\mathcal{L}\{g(t)\} = \frac{e^{-s}}{s^2} + e^{-2s}\left(\frac{6}{s} - \frac{2}{s^2}\right) + \frac{e^{-3s}}{s^2}}$$

In Problems 21–24, determine $\mathscr{L}\{f\}$, where $f(t)$ is periodic with the given period. Also graph $f(t)$.

24. $f(t) = \begin{cases} t\ , & 0 < t < 1\ , \\ 1 - t\ , & 1 < t < 2\ , \end{cases}$

and $f(t)$ has period 2.