Operatör Metodu

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$y'' - 6y' + 9y = t^{-3}e^{3t}$

$$y'' + 5y' + 6y = 18t^2 \longrightarrow \frac{1}{D^2+5D+6} = \frac{1}{(D+2)(D+3)}$$

$$y'' + 4y' + 4y = e^{-2t}\ln t$$

$$y''+2y'+3y = e^x \longrightarrow D^2y + 2Dy + 3y = e^x$$

$$(D^2+2D+3)y = e^x$$

$\frac{d}{dx} = D$ ✓ , $\frac{dy}{dx} = Dy$

$\frac{d^2}{dx^2} = D^2$ ✓ $\frac{d^2y}{dx^2} = D^2y$

$\frac{d^3}{dx^3} = D^3$ ✓ $\frac{d^3y}{dx^3} = D^3y$

ÖR $y'' - 6y' + 9y = x^{-3} \cdot e^{3x}$ d.d.ç. ?

$y'' - 6y' + 9y = 0$

$r^2 - 6r + 9 = 0$

$(r-3)^2 = 0$, $r_{1,2} = 3$ (çift katlı) , $\boxed{y_h = (c_1 + x c_2)e^{3x}}$

$y_p = ?$, Operatör metodunu kullanırsak

$\frac{d}{dx} = D$, $y'' = D^2y$, $D \to$ türev

$\frac{1}{D} \to$ integral

$$D^2y - 6Dy + 9y = x^{-3}e^{3x}$$

$$(D^2 - 6D + 9)y = x^{-3}e^{3x}$$

$$y_p = \frac{1}{D^2-6D+9} x^{-3} e^{3x}$$

$$y_p = \frac{1}{(D-3)(D-3)} x^{-3} e^{3x}$$ $u(x)$

$$u(x) = \frac{1}{D-3} x^{-3} e^{3x}$$

$(D-3)u = x^{-3} e^{3x}$

$D-3$

$(D-3)u = x^{-3} e^{3x}$

$\frac{du}{dx} - 3u = x^{-3} e^{3x}$ (Linear d.d)

$\underbrace{e^{-\int 3dx}}_{e^{-3x}} . u = \int x^{-3} \underbrace{e^{3x} \underbrace{e^{-\int 3dx}}_{e^{-3x}}}_{1} dx$

$e^{-3x} . u = \int x^{-3} dx$

$e^{-3x} . u = -\frac{1}{2x^2} \Rightarrow \boxed{u(x) = -\frac{e^{3x}}{2x^2}}$

$y_p = \frac{1}{D-3} . u(x) = \frac{1}{D-3} \left[-\frac{e^{3x}}{2x^2} \right]$

$\frac{dy}{dx} - 3y = -\frac{e^{3x}}{2x^2}$ (Linear d.d)

$\underbrace{e^{-\int 3dx}}_{} . y = -\int \frac{e^{3x}}{2x^2} \underbrace{e^{-\int 3dx}}_{e^{-3x}} dx$

$e^{-3x} . y = -\int \frac{dx}{2x^2}$

$e^{-3x} . y = \frac{1}{2x} \Rightarrow \boxed{y_p = \frac{e^{3x}}{2x}}$

$\boxed{y = y_h + y_p = \left[c_1 + x c_2 + \frac{1}{2x} \right] e^{3x}}$ ✓

SR $y'' + 4y' + 4y = e^{-2t} \ln t$

$y''+4y'+4y=0$

$r^2+4r+4=0$, $(r+2)^2=0$, $r_{1,2}=-2$, $y_h=(c_1+tc_2)e^{-2t}$

$y_p=?$

$\frac{d}{dt}=D$

$D^2y+4Dy+4y=e^{-2t}\ln t$

$(D^2+4D+4)y=e^{-2t}\ln t$

$y_p=\frac{1}{D^2+4D+4}e^{-2t}\ln t$

$y_p=\frac{1}{(D+2)(D+2)}e^{-2t}\ln t$ $u(t)$

$u(t)=\frac{1}{D+2}e^{-2t}\ln t$

$\frac{du}{dt}+2u=e^{-2t}\ln t$ (Lineer. dd)

$e^{\int 2dt}\cdot u=\int e^{-2t}\ln t\, \underbrace{e^{\int 2dt}}_{e^{2t}}\, dt$

$e^{2t}\cdot u=\int \ln t\, dt$

$\int \ln t\, dt$, $\ln t=u$, $dt=dv$

$\frac{dt}{t}=du$, $t=v$

$=t\ln t-\int t\frac{dt}{t}$

$=t(\ln t-1)$

$e^{2t}\cdot u=t(\ln t-1)$

$\boxed{u=e^{-2t}\cdot t(\ln t-1)}$

$y_p=\frac{1}{D+2}u(t)=\frac{1}{D+2}\left[e^{-2t}\cdot t(\ln t-1)\right]$

$$\frac{dy}{dt} + 2y = e^{-2t}\, t\,(\ln t - 1) \qquad (\text{Lineer dd})$$

$$e^{\int 2\,dt} \cdot y = \int \cancel{e^{-2t}} \cdot t\,(\ln t - 1)\, e^{\int 2\,dt}\, dt$$

$$\cancel{e^{2t}}$$

$$e^{2t} \cdot y = \int t\,(\ln t - 1)\, dt$$

$$\int t \ln t\, dt \;, \quad \ln t = u\,, \quad t\,dt = dV$$

$$\frac{dt}{t} = du\,, \quad \frac{t^2}{2} = V$$

$$= \frac{t^2}{2}\ln t - \int \frac{t^2}{2} \cdot \frac{1}{t}\, dt$$

$$= \frac{t^2}{2}\ln t - \frac{t^2}{4}$$

$$e^{2t} \cdot y = \frac{t^2}{2}\ln t - \frac{t^2}{4} - \frac{t^2}{2}$$

$$\boxed{y_p = e^{-2t}\left[\frac{t^2}{2}(\ln t - 1) - \frac{t^2}{4}\right]}$$

$$y = y_h + y_p$$

$$\boxed{y = \left[c_1 + t c_2 + \frac{t^2}{2}(\ln t - 1) - \frac{t^2}{4}\right] e^{-2t}}$$

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