

Nonlinear Reconstruction

New paradigm¹

¹*in the Large-scale structure*

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We present a new method to reconstruct the primordial (linear) density field using the estimated nonlinear displacement field. The divergence of the displacement field gives the reconstructed density field. We solve the nonlinear displacement field

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Introduction.—The observed large-scale structure provides ... In this Letter, we solve the nonlinear displacement field from the nonlinear density field and present a new method to reconstruct the primordial density field and hence the linear BAO information.

The analysis usually uses the density field directly. measure the power spectrum. However, modeling the small-scale inhomogeneities limits to $k < 0.1 h/\text{Mpc}$. We find the nonlinearities in the displacement field is much more linear than the density field. The correlation of the divergence of the displacement field with the primordial (linear) density field is much better than that of the nonlinear density field. The divergence of the reconstructed nonlinear displacement gives the reconstructed density field, which is much more linear than the nonlinear density field.

Displacement decomposition.—In the Lagrangian picture of structure formation, the displacement field $\mathbf{s}(\mathbf{q}, \tau)$ fully describes the motion of each mass element. The Eulerian position \mathbf{x} of a mass element is given by

$$\mathbf{x}(\mathbf{q}, \tau) = \mathbf{q} + \mathbf{s}(\mathbf{q}, \tau), \quad (1)$$

where \mathbf{q} is the initial Lagrangian position of this mass element. The displacement field $\mathbf{s}(\mathbf{q})$ can be decomposed into a gradient part and a curl part,

$$\mathbf{s}(\mathbf{q}) = \mathbf{s}_E(\mathbf{q}) + \mathbf{s}_B(\mathbf{q}), \quad (2)$$

where $\nabla \times \mathbf{s}_E = 0$ and $\nabla \cdot \mathbf{s}_B = 0$. The gradient part can be completely described by a scalar potential, while the curl part has two independent components.

In the 1D cosmology, the displacement field has only one component though it is a vector field [1]. This allows us to determine the displacement field, which is a vector field, from the density field, which is a scalar field [2]. However, the motion has three degrees of freedom in 3D instead of one. Since from cosmological observations we only have the density field, we expect to reconstruct the scalar part of the displacement field.

Reconstruction algorithm.—The basic idea is to build a curvilinear coordinate system $\boldsymbol{\xi} \equiv (\xi_1, \xi_2, \xi_3)$, where the mass per volume element is constant [3, 4]. In order to determine the physical position of each grid point, we need to specify the Cartesian coordinate $\mathbf{x}(\boldsymbol{\xi}, t)$ of each curvilinear coordinate. As in Refs.[3, 4], we define a coordinate transformation that is a pure gradient,

$$x^i = \xi^\mu \delta_\mu^i + \Delta x^i, \quad (3)$$

where

$$\Delta x^i \equiv \frac{\partial \phi}{\partial \xi^\nu} \delta^{i\nu}. \quad (4)$$

Here, $\Delta \mathbf{x}$ is called the *lattice displacement* and ϕ the *deformation potential* [3, 4]. Here, Latin indices denote Cartesian coordinate labels x^i , Greek indices imply curvilinear coordinates ξ^α .

One efficient algorithm to solve the deformation potential is the moving mesh method [3, 4]. The moving mesh method is originally introduced for

$$\partial_\mu (\rho \sqrt{g} e_i^\mu \delta^{i\nu} \partial_\nu \phi) = \Delta \rho, \quad (5)$$

where $\sqrt{g} = \det(\partial x^i / \partial \xi^\alpha)$ and $\Delta \rho = \bar{\rho} - \rho \sqrt{g}$

Thus, $\boldsymbol{\xi}$ is the estimated initial Lagrangian coordinate \mathbf{q} and $\Delta \mathbf{x}(\boldsymbol{\xi}) = \mathbf{x} - \boldsymbol{\xi}$ is the estimated nonlinear Lagrangian displacement field $\mathbf{s}_E(\mathbf{q})$, which describes the motion beyond the Zel'dovich approximation. In the case that particles follow an irrotational potential flow and no shell crossing happens, the reconstructed displacement is exact up to a global spatial translation. However, shell crossing happens in the nonlinear regime and limits the reconstruction.

Implementation and results.—To test the performance of the new reconstruction algorithm, we run N -body simulations with the CUBEP³M code [5]. The simulation involves 2048^3 dark matter particles in a box of side length $600 \text{ Mpc}/h$. In the analysis, we use the output at $z = 0$. Mass densities are computed on 512^3 grids. The reconstructed ... 1500 times. We also scale the initial density field at $z = 100$ by the linear growth factor to get the linear density field at $z = 0$.

Figure 1 shows a slice of the nonlinear density field. We overplot the deformed grid on the density field.

Figure 2 shows the power spectra of the nonlinear and reconstructed density fields.

Figure 3 shows the cross-correlation coefficients of the nonlinear and reconstructed density field with the linear density field.

Figure shows the PDFs of the density fields and displacement fields.

Discussions.— The reconstructed nonlinear displacement can be further ... before apply to real data, consider RSD, halo fields, SDSS main samples ... Peculiar velocity cosmology, predicting the displacement, robust measurement of volume weighted velocity field, free of

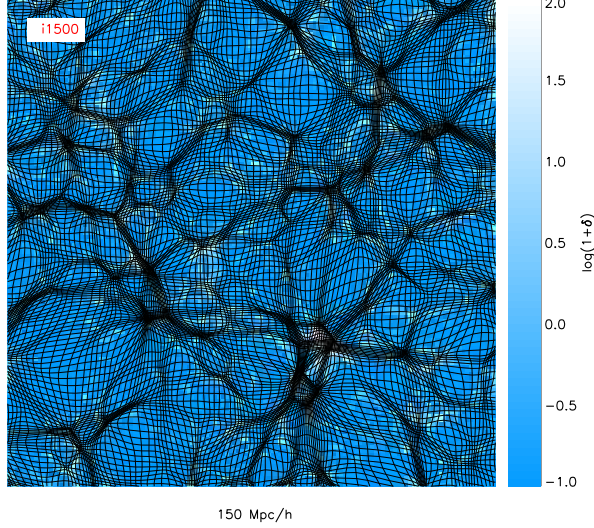


FIG. 1: The density field and the deformed grid.

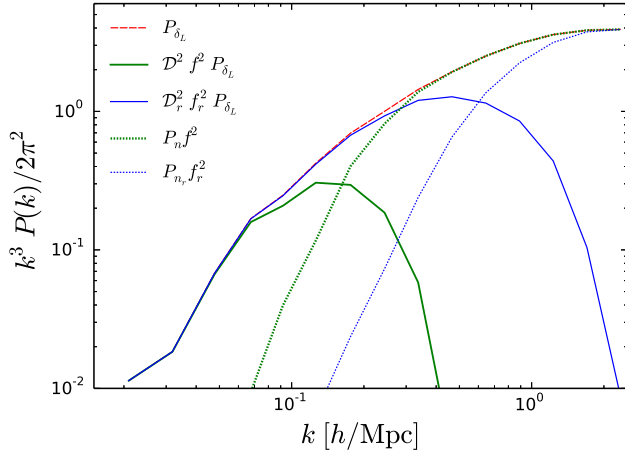


FIG. 2: The power spectra of the nonlinear and reconstructed density fields.

sampling artifact, and good at predicting the velocity field from the density field. also important for the kSZ reconstruction

neutrino mass measurement, nonlinearity is fully exploited. also important for the velocity field reconstruction for measuring the dipole

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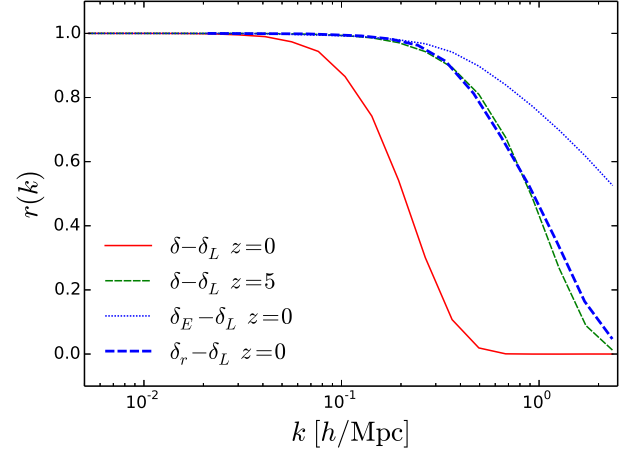


FIG. 3: The cross-correlation coefficients of the nonlinear and reconstructed density fields with the linear density field.

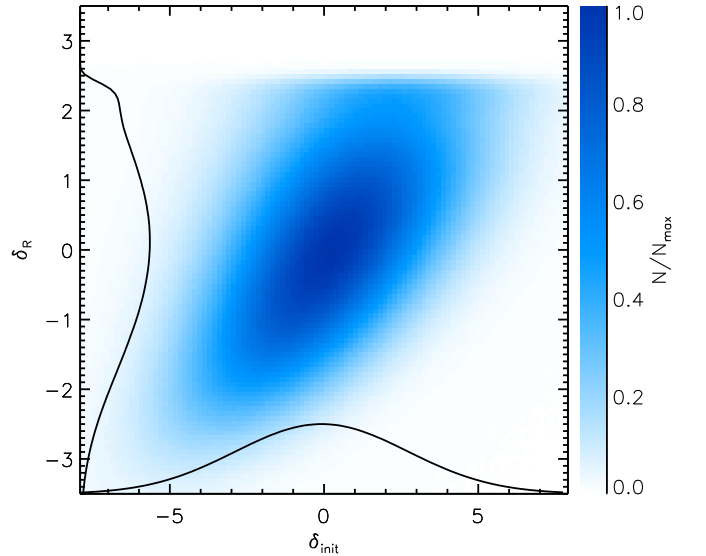


FIG. 4: The PDFs of the reconstructed and linear density fields.

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