Note on Reconstruction beyond the Zel'dovich approximation

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In this paper we introduce a new way to reconstruct BAO peaks in real space.

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I. INTRODUCTION

The imprint of baryon acoustic oscillations (BAO) in the large-scale structure provides a standard ruler to measure the expansion rate of the Universe. Precise measurements of the BAO feature are crucial for probing the dynamics of dark energy. Nonlinear evolution in the density field damps the oscillations in the linear power specturm, reducing the BAO signal can be extracted from observations. The lost linear BAO information can be partially recovered by the reconstruction technique []. Precision modelling of the reconstructed density field and further improvements of the reconstruction method are of great importance for measureing the BAO peak at the subpercent level in the current and future dark energy experiments.

In the standard BAO reconstruction algorithm, the negative Zel'dovich (linear) displacement is used to reverse the large-scale bulk flows. The small-scale inhomogeneities in the nonlinear density field is ususally suppressed using a Gaussian smoothing of scale R so that the smoothed density field provides a reliable estimation on the linear displacement. The smoothing scale R should be comparable to the nonlinear scale ~ 10 Mpc$/h$, where linear approximation breaks down. Thus, the nonlinear information is not used in the reconstruction. However, using smaller smoothing scales in order to include the small-scale modes would make the Zel'dovich approximation unvalid and induce nonlinearities which are difficult to model. The gain would be huge if these small-scale modes can be exploited, providing the information about the higher-order displacement field. The estimated negative Zel'dovich displacement is evaluated at the Eulerian position $\boldsymbol{x}$ rather than the Lagrangian position $\boldsymbol{q}$. The difference between these two positions is often neglected in the reconstruction, which is a reasonable first order approximation (See Section 3.1 in Ref. [] for a discussion about this). Recently it is suggested that a new correction term due to this should be included in modelling the displacement (See Appendix A in Ref. for more detailed discussions). In this Letter, we point out that the nonlinear displecement can be solved from the nonlinear density field and present a new method to reconstruct the linear BAO information from the nonlinear density field.

The basic idea is to build a curvilinear coordinate system, where the mass per volume element is constant [3,4]. Consider a numerical grid of coordinates $\boldsymbol{\xi} \equiv (\xi_1,\ \xi_2,\ \xi_3)$. In order to determine the physical position of each lattice point, one needs to specify the Cartesian coordinate $\boldsymbol{x}(\boldsymbol{\xi},\ t)$ of each curvilinear coordinate. In the Cartesian coordinate system, the metric is just the Kronecker delta function δ_{ij}, while the curvilinear metric is

$$g_{\alpha\beta} = \frac{\partial x^i}{\partial \xi^\alpha}\frac{\partial x^j}{\partial \xi^\beta}\delta_{ij}, \tag{1}$$

where Latin indices denote Cartesian coordinate labels x^i, while Greek indices imply curvilinear coordinates ξ^α. As in Refs. [3,4], we define a coordinate transformation that is a pure gradient,

$$x^i = \xi^\mu \delta^i_\mu + \Delta x^i, \tag{2}$$

where

$$\Delta x^i \equiv \frac{\partial \phi}{\partial \xi^\nu}\delta^{i\nu}. \tag{3}$$

The lattice displacement $\boldsymbol{\Delta x}$ is completely determined by the potential ϕ. The mass density at the curvilinear coordinate $\boldsymbol{\xi}$ is

$$\rho(\boldsymbol{\xi}) = \sqrt{g}\rho(\boldsymbol{x}), \tag{4}$$

where $\sqrt{g} \equiv \det(\partial x^i/\partial \xi^\alpha)$ is the volume element. The potential field $\phi(\boldsymbol{\xi})$ which gives $\rho(\boldsymbol{\xi}) = 0$ can be solved using the multigrid alogirthm described in Ref. []. Since we only have the density field, which is a scalar field, it only allows us to reconstruct the scaler potential ϕ rather than the vector. The reconstructed density field $\delta_r(\boldsymbol{\xi})$ is given by

$$\delta_r(\boldsymbol{\xi}) = \nabla_{\boldsymbol{\xi}} \cdot \Delta\boldsymbol{x}(\boldsymbol{\xi}). \tag{5}$$

In the Lagrangian approach to the nonlinear structure formation, the displacement field $\boldsymbol{\Psi}(\boldsymbol{q},\tau)$ fully describes the motion of the mass elements. The Eulerian position $\boldsymbol{x}$ of a mass element is

$$\boldsymbol{x} = \boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q},\tau), \tag{6}$$

where $\boldsymbol{q}$ is the initial Lagrangian position of this mass element. The displacement field $\boldsymbol{\Psi}$ can be decomposed into an irrotational part and a curl part,

$$\boldsymbol{\Psi} = \boldsymbol{\Psi}_E + \boldsymbol{\Psi}_B, \tag{7}$$

where $\nabla\times\boldsymbol{\Psi}_E = 0$ and $\nabla\cdot\boldsymbol{\Psi}_B = 0$. In the case that particles describe an irrotational potential flow, $\boldsymbol{\Psi}_E$ fully describes the motion of the mass elements. In reality, shell crossing happens in the nonlinear regime, and the emergence of vorticity leads to the growth of $\boldsymbol{\Psi}_B$. The reconstructed lattice displacement $\Delta\boldsymbol{x}$ describes the nonlinear motion beyond the Zel'dovich displacement.

The real displacement field is available in N-body simulations. It can be decomposed and compared with the reconstructed lattice displacement to see how much information is reconstructed.

II. DISPLACEMENT DECOMPOSITION

The displacement field $\boldsymbol{\Psi}$ can be decomposed into an irrotational part and a curl part,

$$\boldsymbol{\Psi} = \boldsymbol{\Psi}_E + \boldsymbol{\Psi}_B, \tag{8}$$

where $\nabla\times\boldsymbol{\Psi}_E = 0$ and $\nabla\cdot\boldsymbol{\Psi}_B = 0$. We further decompose $\boldsymbol{\Psi}_E$ as

$$\boldsymbol{\Psi}_E = \boldsymbol{\Psi}_E^{(l)} + \boldsymbol{s}_E^{(nl)}. \tag{9}$$

The density field is given by

$$\delta_E^{(l)} = -\nabla\cdot\boldsymbol{\Psi}_E^{(l)} \tag{10}$$

The density field is given by

$$\delta_E^{(nl)} = -\nabla\cdot\boldsymbol{\Psi}_E^{(nl)} \tag{11}$$

The displacement can be written as

$$\boldsymbol{\Psi} = \boldsymbol{\Psi}_E^{(l)} + \boldsymbol{\Psi}_E^{(nl)} + \boldsymbol{\Psi}_B, \tag{12}$$

A. Power spectra of displacement fields

Show the power spectra of different displacement componenets ...

B. PDFs

Show the pdf of the displacement fields, maybe the density field?

C. Cross correlations

Show the cross correlation coefficients of the nonlinear density field and the linear density field ...

III. RECONSTRUCTION ALGORITHM

Discuss how to solve the deformation potential.

The Lagrangian displacement can be solved easily in 1+1 dimensions by ordering of the mass elements.

(1) Solve the displacement $\Psi(q)$ field by ordering of the mass elements.

(2) Take the differential derivative of $\Psi(q)$ to get the reconstructed density field $\delta_r(\boldsymbol{q}) = -\nabla_{\boldsymbol{q}}\Psi(\boldsymbol{q})$. $\delta_r(\boldsymbol{q}) = -\partial\Psi(q)/\partial q$.

IV. SIMULATIONS AND PERFORMANCE

We adopt the 1D N-body simualtions in Ref. [1] and use outputs at $z=0$. The simulation box is 10^8 Mpc with 3×10^8 grids and 3×10^8 PM elements. We scale the intial density field by the linear growth factor to get the linear density field δ_L at $z=0$. Note that δ_L has been rescaled using the linear growth function to the same redshift as δ.

A. Deformation potential

Show the visual grid deformation.

B. Statistics

Show resonctruction from linear density fields and $z=0$ density field. Different configurations.

To convenienty quantify the linear information δ_L in the nonlinear density field δ, we decompose the nonlinear density field δ as

$$\delta(\boldsymbol{k}) = b(\boldsymbol{k})\delta_L(\boldsymbol{k}) + \delta_N(\boldsymbol{k}). \tag{13}$$

Here, $b\delta_L$ is completely correlated with the linear density field δ_L and $b = P_{\delta\delta_L}/P_{\delta_L}$. Nonlinear evolution drives b to drop from unity, reducing the linear signal. δ_N is generated in the nonlinear evolution and thus uncorrelated with the linear density field δ_L, further reducing $b\delta_L$ with respect to δ. This part induces noises in the measurement of BAO. Hence we denote it with a subscript "N". Such decomposition helps to write the nonlinear power spectrum as

$$P_\delta(k) = \mathcal{D}(k) P_{\delta_L}(k) + P_{\delta_N}(k), \tag{14}$$

where $\mathcal{D} = b^2$ describes the damping of the linear power spectrum. The reconstructed power spectrum P_{δ_r} can be describe in the same way. Here, $b(\boldsymbol{k})$ is often referred as the "propagator" and $n(\boldsymbol{k})$ is usually called the mode-coupling term [2–4]. still to be modified

Figure **??** shows the damping functions for the raw and reconstructed fields. The nonlinear damping of the linear power spectrum is significantly reduced after reconstruction. We also overplot the best-fitting Gaussian BAO damping model,

$$\mathcal{D}(k) = \mathrm{e}^{-k^2\Sigma^2/2}, \tag{15}$$

with $\Sigma =$? Mpc and ? Mpc for the nonlinear and reconstructed fields. The new BAO reconstruction algorithm reduces the the nonlinear damping scale Σ by ?? per cent, i.e., a of ??. The damping factor is above 0.9 for $k \lesssim$? Mpc^{-1} indicating (almost) perfect reconstruction. how to best quantify the reduction of damping? However, the 100 per cent reconstruction, cancelling any nonlinear effects, is still unachievable, as some information has been irreversibly lost. (more discissions)

Reconstruction also reduces the noise term P_{δ_N}. To demonstrate this, in Fig. **??** we plot the cross-correltion coefficient

$$r(k) = \frac{P_{\delta\delta_L}(k)}{\sqrt{P_\delta(k) P_{\delta_L}(k)}} = \frac{1}{\sqrt{1+\eta(k)}}, \tag{16}$$

where $\eta = P_n/(\mathcal{D}P_{\delta_L})$ quantifies the relative amplitude of δ_N with respect to $b\delta_L$. The correlation of δ_r with δ_L is as good as that of δ at $z = 3$. how to quantify this better?

V. DISCUSSIONS AND CONCLUSIONS

The new method significantly improves the expansion rate measurement from BAO. (more discussions?)

This method can be generalized to the 3D case. We leave this to future work.

Comparision with and Implications for the standard BAO reconstruction: exact Lagrangian displacement, nonlinear displacement, which is easier to model.

If use the displacement solved in this paper for the standard BAO rec, we expect the performance will become much better but still not as good as our results.

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