Probing Neutrino Hierarchy and Chirality via Wakes

Hong-Ming Zhu,[1] Ue-Li Pen,[2,3] Xuelei Chen,[1,4] and Derek Inman[2]

[1] *Key Laboratory for Computational Astrophysics, National Astronomical Observatories, Chinese Academy of Sciences, 20A Datun Road, Beijing 100012, China*

[2] *Canadian Institute for Theoretical Astrophysics, 60 St. George Street, Toronto, Ontario M5S 3H8, Canada*

[3] *Canadian Institute for Advanced Research, CIFAR Program in Gravitation and Cosmology, Toronto, Ontario M5G 1Z8, Canada*

[4] *Center of High Energy Physics, Peking University, Beijing 100871, China*

The relic neutrinos are expected to acquire a bulk relative velocity with respect to the dark matter at low redshifts, and neutrino wakes are expected to develop downstream of the dark matter halos. We propose a method of measuring the neutrino mass based on this mechanism. This neutrino wake will cause a dipole distortion of the galaxy-galaxy lensing pattern. This effect could be detected by combining upcoming lensing surveys, e.g. the LSST and Euclid surveys with a low redshift galaxy survey or a 21cm intensity mapping survey which can map the neutrino flow field. The data obtained with LSST and Euclid should enable us to make positive detection if the three neutrino masses are Quasi-Degenerate, and a future high precision 21cm lensing survey would allow the normal hierarchy and inverted hierarchy cases to be distinguished, and even the right handed Dirac neutrinos may be detectable.

PACS numbers:

Introduction.—The squared mass differences of the three neutrino species have been measured from neutrino oscillation experiments, but the individual masses are still unknown. Based on the measured mass square differences, the neutrino masses may form the so called *normal hierarchy* ($m_1 \sim m_2 \ll m_3$), or the *inverted hierarchy* ($m_3 \ll m_1 \sim m_2$), or they may be *quasi-degenerate* ($m_1 \sim m_2 \sim m_3$)(c.f. §14 of [1]). Determining the neutrino mass hierarchy is a very important problem in modern physics [2]. The suppression of matter power spectrum on small scale provides a way to measure the sum of the neutrino masses[3, 4], complementary to particle physics experiments, and in some cases the combination of these two approach may allow the determination of the mass hierarchy [5].

Recently, it was shown that a bulk relative velocity field between neutrinos and cold dark matter (CDM) exists, with coherent flows over several Mpc [6]. This produces a cross-correlation dipole between CDM and neutrinos on large scales. While on nonlinear scales, as neutrinos flow over dark matter halos, they are gravitationally focused into a *wake*. This downstream excess could then be observed through gravitational lensing. This wake is unique to neutrinos as the CDM-baryon relative velocity is far too small to mimic the effect. The full three-dimensional relative velocity field can be computed from the galaxy density field using linear perturbation theory with good accuracy. By exploiting the asymmetry of relative velocity, the neutrino wakes can be isolated and used to determine the neutrino masses.

Besides the neutrino masses, the very nature of the neutrinos, i.e. whether they are Majorana or Dirac particles, are still unknown. If neutrinos are Majorana particles, the lepton number conservation law is broken, and this may induce neutrinoless β-decay, but so far such decays have not been detected despite of many search efforts [7]. This question may be answered if the Big Bang relic neutrinos can be detected with tritium capture experiment [8], for the total capture rate in the Majorana case could be twice as large as that of the Dirac case [9]. However, detecting the relic neutrinos is very difficult due to their small kinetic energy, and furthermore, even if the relic neutrinos are detected, this effect is confounded by the fact that the local neutrino overdensity due to gravitational clustering is unknown. Here we note that for Dirac neutrinos there are right-handed neutrinos (ν_R) distinct from the left handed ones (ν_L), and they could be produced during the Big Bang. In many beyond-the-Standard-Model theories, e.g. the models with a heavier right-handed coupling gauge boson Z' [10], the left-right symmetry is restored at high energy, then in the very early universe there would be a thermalized ν_R background in addition to the ν_L one. However, the left-right symmetry must be broken at low energy, and as the Z' is heavier than the Z-boson, the primordial ν_R would decouple before the ν_L , and evolve as a separate relic background. At later time, these neutrinos would propagate in mass eigenstates, with both right and left (re-generated from Yukawa coupling) components, but their temperature and number density would be distinct from the primordial ν_L background which decoupled much later. Since the relative velocity with respect to CDM depends on the initial velocity dispersion[6], the relative velocity field of the ν_R and the resulting wakes would also be different, thus enabling a new way to answer this important question[1].

In this Letter we discuss how these neutrino wakes are produced by non-linear CDM halos and compute the expected size and signal of the wakes, and investigate the observability with upcoming surveys. Moreover, we also delineate the evolution of the ν_R for the case of Dirac neutrinos, and consider its detection with this method.

Neutrino wakes and lensing signal.—At high redshift individual neutrinos move at large velocities, but the bulk flow of neutrinos with respect to the CDM is small. Because of their large velocities, neutrinos traverse large distances and experience the inhomogeneous gravitational potentials at different places, as a result a bulk relative velocity field between the neutrinos and dark matter particles is induced. The relative velocity field can be computed from the primordial density perturbations and its variance is given by

$$\langle v_{\nu c}^2 \rangle(z) \;=\; \int \frac{dk}{k} \Delta_\zeta^2(k) \left[\frac{\theta_\nu(k,z) - \theta_c(k,z)}{k} \right]^2, \quad (1)$$

where $\theta \equiv \nabla \cdot \boldsymbol{v}$ is the velocity divergence transfer function, and $\Delta_\zeta^2(k)$ is the primordial curvature perturbation spectrum [6]. This variance was computed for ν_L in [6] where it was found to be comparable with the thermal velocity at late times. The ν_L relative velocity power spectrum is shown in Fig. 1 for the case of a single 0.05 eV neutrino. The relative velocity fields are well correlated at large scales [6]. The coherency scale, which is defined here as the distance where the correlation function drops to half its maximum value, is $R = 14.5$ Mpc$/h$ for ν_L at $z = 0.3$. In the following calculations, we take $\sqrt{\langle v_{\nu_L c}^2 \rangle} = 418$km/s at $z = 0.3$ as the typical relative velocity. The one dimensional neutrino velocity dispersion is $\sigma(z) = 2.077 T_\nu(z)/m_\nu \simeq 2716$km/s for ν_L at $z = 0.3$.

We now compute the neutrino wake induced by a dark matter halo with a given mass M via linear response theory. We approximate the neutrinos' distribution as Maxwell-Boltzmann, which is sufficiently accurate for the relevant densities and temperatures. Assuming neutrinos flow over the dark matter halo coherently with velocity $v_{\nu c}(z) = \sqrt{\langle v_{\nu_L c}^2 \rangle}(z)$, the density contrast at point $\boldsymbol{r}$ is given by [12]:

$$\begin{aligned} \delta_\nu(\boldsymbol{r}) \;&=\; \frac{r_B}{r} \exp\left(- \frac{v_{\nu c}^2 \sin^2\vartheta}{2\sigma^2} \right) \left[1 + \mathrm{erf}\left(\frac{v_{\nu c} \cos\vartheta}{\sqrt{2}\sigma} \right) \right] \\ &\approx\; \frac{r_B}{r} \left(1 + \frac{2}{\sqrt{\pi}} X \cos\vartheta - X^2 \sin^2\vartheta \right), \quad (2) \end{aligned}$$

where $r_B(z) = GM/\sigma^2(z)$ is the Bondi radius, ϑ is the angle between the relative velocity and $\boldsymbol{r}$, and $X(z) = v_{\nu c}(z)/(\sqrt{2}\sigma(z))$. The accuracy of the approximate form in the second line is better than 95% for the cases we consider.

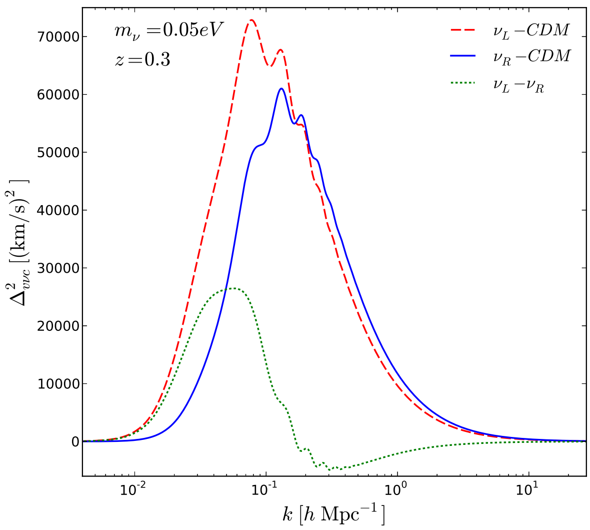

FIG. 1: Relative velocity power spectra between ν_L , ν_R and CDM at $z = 0.3$.

With Eq. (2) in hand we can now compute the expected lensing signal from this wake. The geometry is shown in Fig. 2. Taking the z-axis as the direction of our line of sight (l.o.s.) and the relative velocity $\boldsymbol{v}_{\nu c}$ to lie in the $x - z$ plane at angle θ from the z-axis, then the induced density contrast at any point $\boldsymbol{r} \equiv (x, y, z)$ can be determined from Eq. (2) via coordinate transformations. Here we also define the polar coordinates (ϖ, ϕ) on the lens $(x - y)$ plane, $x = \varpi \cos\phi, y = \varpi \sin\phi$, for later use.

The perturbed surface neutrino mass-density is obtained by integration along the l.o.s., $\Sigma_\nu(x, y) = \rho_0 \int_{-aL/2}^{aL/2} dz\, \delta_\nu(x, y, z)$, where ρ_0 is the unperturbed neutrino mass density, a is the scale factor and L is the effective coherent scale of the neutrino-CDM relative flow field, defined as $(4/3)\pi R^3 = L^3$. We find $L = 23.4$ Mpc$/h$ for ν_L at $z = 0.3$ for $m_\nu = 0.05$eV. We first consider the contribution from a single halo at $z = 0.3$. Using Eq. (2), we obtain

$$\begin{aligned} \frac{\Sigma_\nu}{\rho_0 r_B} &= (1 - X^2 \sin^2\theta) \ln\left[\frac{1 + \sin\eta}{1 - \sin\eta} \right] + \frac{4\eta X}{\sqrt{\pi}} \sin\theta \cos\phi \\ &+ X^2 (3 \sin^2\theta - 2) \sin\eta - X^2 (\cos^2\theta - 1) \sin\eta \cos 2\phi, \end{aligned}$$

where $\eta = \arctan(aL/2\varpi)$. The contribution to the weak lensing convergence of neutrinos in a redshift slice is given by $\kappa(\varpi, \phi) = \Sigma_\nu(\varpi, \phi)/\Sigma_{\rm cr}$, where $\Sigma_{\rm cr} = \frac{c^2}{4\pi G} \frac{(1+z_d) D_s}{D_d D_{ds}}$ is the critical surface density, z_d is the redshift of the halo, and D_s, D_d, D_{ds} are the comoving distances between the observer and source, the observer and lens, and the lens and source respectively.

[1] In the see-saw models of neutrino mass [11], the light ν_L are primarily Majorana particles. There are also right handed neutrinos in such models, but they are very heavy and should have decayed in the early Universe.

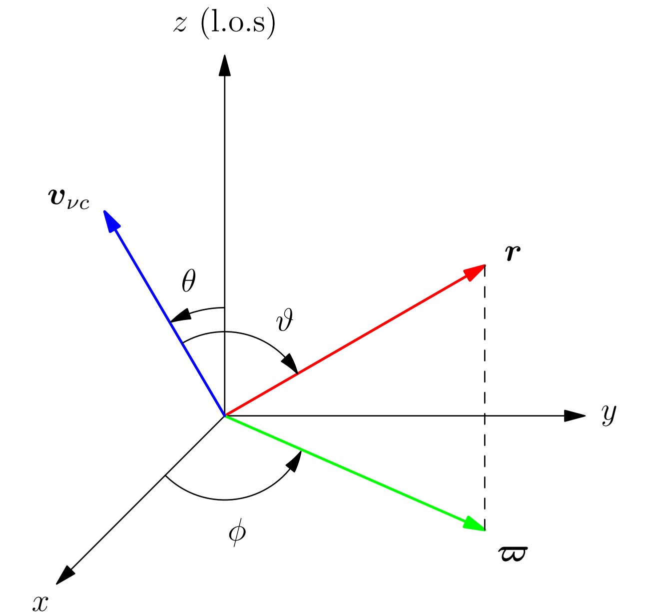

FIG. 2: The coordinate system employed in calculating the lensing signal. $\boldsymbol{v}_{\nu c}$ is the relative velocity, $\boldsymbol{r}$ is the point at which we compute the density contrast and (ϖ, ϕ) are polar coordinates in the lensing plane.

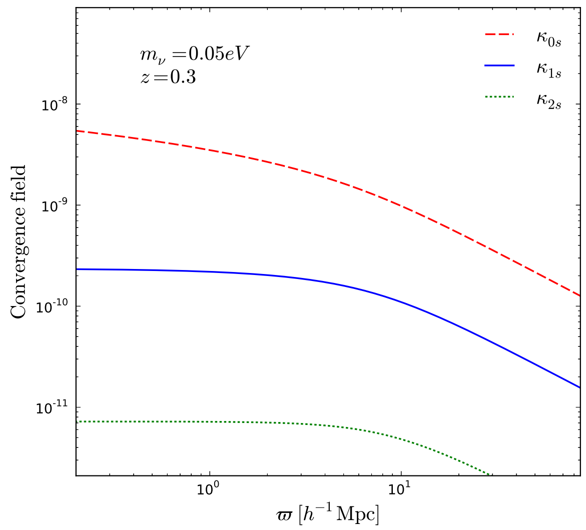

FIG. 3: Different components of the convergence field for a single $10^{13} M_\odot$ halo at z=0.3, with $m_\nu = 0.05$ eV when $\theta = \pi/2$.

We can make a multipole expansion for κ:

$$\kappa(\varpi, \phi) = \kappa_0(\varpi) + \kappa_1(\varpi)\cos\phi + \kappa_2(\varpi)\cos 2\phi.$$

As a simple order-of-magnitude estimate, we shall assume that the background galaxies are located at $z = 0.8$, which is about the median redshift of galaxies in the Large Synoptic Survey Telescope (LSST) survey[13] and the Euclid survey[14, 15]. The middle point between the source and observer in comoving distance is then $z = 0.3$. In Fig. 3 we plot the contribution to the different components of convergence field from a single halo κ_{0s}, κ_{1s} and κ_{2s} at $z = 0.3$ for $m_\nu = 0.05$ eV in the case that the relative velocity is perpendicular to the l.o.s., i.e. $\theta = \pi/2$. Here we take $M = 10^{13} M_\odot$, which is the typical mass of a halo.

Although the effect is fairly small, no other effect is known to produce the dipole term κ_{1s} except for the neutrino wake lensing. The halo density field itself may have a dipole; however, its contribution can be removed by correlating with the relative velocity field direction: $\xi(\boldsymbol{r}) = \langle \delta_h(\boldsymbol{x})\delta_h(\boldsymbol{x}+\boldsymbol{r})\sin\theta\rangle$, as it is antisymmetric. We have verified this is the case by N-body simulations, without the neutrinos the measured lensing signal is indeed consistent with zero within the numerical accuracy. The clustering of neutrinos around a growing dark matter halo moving with a bulk velocity to neutrinos can be calculated similarily as in [16].

Principle of Observation.—We can reconstruct the relative velocity field from the large scale structure survey data δ_g, in Fourier space the velocity is given by

$$\boldsymbol{v}_{\nu c}(\boldsymbol{k}) = \frac{i\hat{\boldsymbol{k}}}{k}\frac{\theta_c(k) - \theta_\nu(k)}{T_c(k)} s \frac{\delta_g(\boldsymbol{k})}{b_g}, \qquad (3)$$

where θ_c and θ_ν are the velocity divergence transfer functions, T_c is the density transfer function and b_g is a linear galaxy bias as the galaxy density field is different from that of dark matter. The influence of different values of the bias is marginal as we only need the direction of the relative velocity, which can be reconstructed accurately [17]. The lensing dipole can then be calculated with respect to the direction of velocity, so that the neutrino wakes add coherently. However, as the neutrino mass is unknown, we would have to generate the relative velocity fields for many neutrino masses and determine the neutrino mass as the best fit for κ_1.

In a velocity coherent volume $V = L^3$, each halo produces a wake in the same direction and add constructively to the dipole term ($\cos\phi$ has the same sign). For simple estimate, the number of halos of typical mass in such a volume is $\bar{n}V$, where $\bar{n} = \rho_m/M$ and ρ_m is the matter density at the corresponding redshift. Thus we can multiply the response density by $\bar{n}V$, or in more sophisticated treatment calculate the integrated response using the Press-Schechter model.

This procedure is complicated by the fact that along each l.o.s., in the lensing effective length, there are on average $D_{\rm eff}/L$ coherent relative velocity volumes, where $D_{\rm eff}$ is the effective comoving length over which the lensing effect is significant. Each of these contributes to the dipole term, but the dipole directions are randomly oriented with respect to one another. If the contribution to

κ_1 from each of these coherent volumes can be measured individually, one may construct a stacked κ_1, which will total as $D_{\rm eff}/L$ as discussed previously. However, it is perhaps too difficult to measure κ_1 for individual velocity coherent volumes. Instead, the total κ_1 along each l.o.s. may be observed and the dipoles from each velocity coherent volume will now add incoherently, resulting that κ_1 grows as $\kappa_1 \approx \bar{n}V\sqrt{D_{\rm eff}/L}\kappa_{1s}$, where κ_{1s} was the single halo contribution shown in Fig.3. For our simple estimate, we define $D_{\rm eff}$ as the comoving length along our l.o.s. where the weight $D_{\rm d}D_{\rm ds}/D_{\rm s}/(1+z_d)$ is larger than half of its maximum value. For the fiducial case here, $D_{\rm eff} = 1357{\rm Mpc}/h$. The result of a full numerical integration gives result about 15% smaller than this simple estimate.

Right handed neutrinos.—If the neutrinos are Dirac type, the primordial right handed neutrinos have the same mass as the left handed ones, but with a different temperature, so that its distribution is also different. The decoupling of the ν_Rs depend on the model of their interactions, and one has to consider the rich phenomenology and experimental constraints on such models [1, 10]. Here, as a simple example, we assume that the ν_Rs were fully thermalized during the early universe but decoupled before any other SM particle. This is a plausible case, for the ν_Rs do not participate in the SM interactions, and their earlier thermalization must be due to a beyond-standard-model interaction which freeze out at energies above 1 TeV. The temperature of these neutrinos can be computed by ensuring conservation of entropy density $g_{*S}^{today}T_\gamma^3 = g_{*S}^{\nu_R}T_{\nu_R}^3$, where $g_{*S}^{\nu_R}$ is the effective degree of freedom when ν_R decoupled. If the Standard Model is applicable at such energy scales except for the addition of right hand neutrinos, we have $g_{*S}^{SM} = 106.75$, and $g_{*S}^{\nu_R} = g_{*S}^{SM} + \frac{7}{8}$ (fermions) $\times 3$ (number of species) $\times 2$ (particle and anti-particle). Currently, $g_{*S} = 3.91$ but again a factor of $\frac{7}{8}\times 6 \times (\frac{T_{\nu_R}}{T_\gamma})^3$ needs to be added. This leaves

$$\left(\frac{T_{\nu_R}}{T_\gamma}\right)^3 = \frac{106.75 + \frac{7}{8}\times 6}{3.91 + \frac{7}{8}\times 6 \times \left(\frac{T_{\nu_R}}{T_\gamma}\right)^3} \tag{4}$$

so the extrapolated right hand neutrino temperature is $T_{\nu_R} = \left(\frac{43}{427}\right)^{1/3} T_{\nu_L} \simeq 0.905\,{\rm K}$. With ν_R at this temperature, the number of relativistic degrees of freedom during the nucleosynthesis era is $N_{\rm eff} = 3.04\times(1+(\frac{T_{\nu_R}}{T_{\nu_L}})^4) \simeq 3.18$. This is consistent with the current big bang nucleosynthesis (BBN) constraints [18]. Note that the ν_R density might be greater if its production mechanism is non-thermal, but the BBN constraints its density to be not much higher than the thermal value. To model ν_R , we use the CLASS code [19] with neutrinos of the same mass but a lowered temperature. The results presented here only depend on the ν_R being thermalized and their current temperature.

We can compute the variance of ν_R , the procedure is similar to the ν_L , except that ν_R decoupled earlier, have lower temperature, i.e. velocity dispersion, and so their relative velocities are different from that of ν_L . The ν_L and ν_R relative velocity spectrum is shown in Fig. 1. The coherency scale is $R = 10.0$ Mpc$/h$ for ν_R at $z = 0.3$. In this model we take $\sqrt{\langle v_{\nu_R c}^2\rangle} = 373$ km/s as typical relative velocities. The one dimensional neutrino velocity dispersions are $\sigma(z) = 2.077T_\nu(z)/m_\nu \simeq 1263$ km/s for ν_R at $z = 0.3$. The direction of the flow deviates from that of ν_L , with the angle $\langle\cos\theta\rangle = \int \Delta_\zeta^2\theta_{\nu c}^L\theta_{\nu c}^R/k^3dk/\sqrt{\int\Delta_\zeta^2(\theta_{\nu c}^L/k)^2{\rm d}{\rm ln}k\int\Delta_\zeta^2(\theta_{\nu c}^R/k)^2{\rm d}{\rm ln}k}$, and we find $\theta \sim 20^o$.

The right handed neutrino wake direction differs from the left handed wake by an angle of typically 20 degrees. This direction is computable, and differs from patch to patch. The signal is a small effect on the amplitude of the wake, which is degenerate with measurement systematics and small variations in the left handed neutrino mass. Instead, we use the difference in direction, and only consider the right handed wake component orthogonal to the left handed wake to be observable, and adjust sensitivity estimates correspondingly.

The Observability.—For forecasting purposes, we consider a combined LSST+Euclid dataset as well as a future 21cm lensing survey. The LSST will survey about 3 billion galaxies and the expected error for κ is about $0.28/\sqrt{N} \simeq 5.2\times10^{-6}$, where $N = 2.88\times10^9$ [13]. The Euclid survey gives a similar expected error [14][15]. Here for simple estimates we will neglect the overlap between these two surveys and assume that they provide independent data set. By combining these data, we can reach a precision of $\sigma_{\kappa_1} = 5.2\times10^{-6}/\sqrt{2} = 3.68\times10^{-6}$.

Ultimately, extremely high precision measurements can be achieved with 21cm lensing observations such as the one proposed in [20]. For such a survey, the error on κ is $(k_{\rm max}/k_{\rm min})^{-1.5}$. For a survey with $k_{\rm max} = 1.47\times10^2h/{\rm Mpc}$ and $k_{\rm min} = 1.47\times10^{-3}h/{\rm Mpc}$, the expected error is about 3.16×10^{-8}, which is far smaller than the signals. With such future 21cm lensing surveys, we can measure neutrino masses with unprecedented precision, and may even determine whether the neutrinos are Majorana or Dirac particles.

Based on the measurement of neutrino mass differences from the neutrino oscillation experiments, three possible hierarchies are being considered(c.f. §14.8 of [1]): (i) the Normal Hierarchy (NH) case, with $m_1 \sim m_2 \approx 0, m_3 \approx 0.05\,{\rm eV}$; (ii) the Inverted Hierarchy case (IH), with $0 \sim m_3 \ll m_1 \sim m_2 \approx 0.05\,{\rm eV}$; (iii) the Quasi-Degenerate case (QD), with $m_1 \approx m_2 \approx m_3 \approx 0.1\,{\rm eV}$. In the IH and NH cases, we can neglect the contribution of the lighter neutrinos, so that the NH is equivalent to a single neutrino with mass of 0.05 eV, while IH is equivalent to two 0.05 eV neutrinos. The QD case is almost equivalent to three 0.1 eV neutrinos.

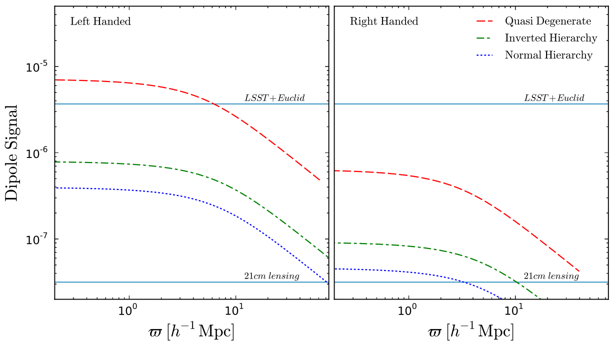

FIG. 4: Dipole components $\bar{\kappa}_1$ of the convergence field with consideration of neutrino hierarchy. The noise floor for LSST+Euclid and for 21 cm surveys are also shown. For the right panel, only the signal orthogonal to the left handed neutrinos is shown.

The dipoles on the sky depend on the angle θ between the relative velocity direction and the l.o.s., the signal reaches its maximum when the relative velocity is perpendicular to the l.o.s. ($\theta = \pi/2$) and vanishes when $\theta = 0$. Thus the final observed signal strength should an average over different angles. In Figure 4, we plot the expected dipole term $\bar{\kappa}_1 = \int \kappa_1(\theta)d\theta / \int d\theta$ for ν_L (left panel), and ν_R (right panel) for these three cases. The dipole signal for ν_R is suppressed by a $\cos 20^o$ since we only consider the wake component geometrically orthogonal to ν_L. We also plot the measurement errors for the LSST+Euclid lensing survey and the future 21cm lensing survey described above. As can be seen, the LSST+Euclid data should be able to positively detect the QD case. With the future 21cm lensing survey, the NH and IH cases can be distinguished, and even the ν_R may be detectable.

Conclusion.—We have computed the density contrast of neutrino wakes produced by the relative motions of neutrinos and dark matter. We have estimated the observability of these wakes via gravitational lensing and shown that it may be possible to observe both the mass hierarchy as well as right handed neutrinos (should they exist). The wake directions differ by a distinctive angular signature.

We acknowledge the support of the Chinese MoST 863 program under Grant No. 2012AA121701, the CAS Science Strategic Priority Research Program XDB09000000, the NSFC under Grant No. 11373030, Tsinghua University, CHEP at Peking University, and NSERC.

[1] K. A. Olive and Particle Data Group, Chinese Physics C **38**, 090001 (2014), 1412.1408.
[2] R. N. Cahn, D. A. Dwyer, S. J. Freedman, W. C. Haxton, R. W. Kadel, Y. G. Kolomensky, K. B. Luk, P. McDonald, G. D. Orebi Gann, and A. W. P. Poon, ArXiv e-prints (2013), 1307.5487.
[3] J. R. Bond, G. Efstathiou, and J. Silk, Physical Review Letters **45**, 1980 (1980).
[4] W. Hu, D. J. Eisenstein, and M. Tegmark, Physical Review Letters **80**, 5255 (1998), astro-ph/9712057.
[5] S. Dodelson and J. Lykken, ArXiv e-prints (2014), 1403.5173.
[6] H.-M. Zhu, U.-L. Pen, X. Chen, D. Inman, and Y. Yu, Physical Review Letters **113**, 131301 (2014), 1311.3422.
[7] A. de Gouvea, K. Pitts, K. Scholberg, G. P. Zeller, J. Alonso, A. Bernstein, M. Bishai, S. Elliott, K. Heeger, K. Hoffman, et al., ArXiv e-prints (2013), 1310.4340.
[8] S. Betts, W. R. Blanchard, R. H. Carnevale, C. Chang, C. Chen, S. Chidzik, L. Ciebiera, P. Cloessner, A. Cocco, A. Cohen, et al., ArXiv e-prints (2013), 1307.4738.
[9] A. J. Long, C. Lunardini, and E. Sabancilar, J. Cosmology Astropart. Phys. **8**, 038 (2014), 1405.7654.
[10] P. Langacker, Reviews of Modern Physics **81**, 1199 (2009), 0801.1345.
[11] Z.-Z. Xing, International Journal of Modern Physics A **23**, 4255 (2008), 0810.1421.
[12] J. Binney and S. Tremaine, *Galactic Dynamics: Second Edition* (Princeton University Press, 2008).
[13] LSST Science Collaboration, P. A. Abell, J. Allison, S. F. Anderson, J. R. Andrew, J. R. P. Angel, L. Armus, D. Arnett, S. J. Asztalos, T. S. Axelrod, et al., ArXiv e-prints (2009), 0912.0201.
[14] R. Laureijs, J. Amiaux, S. Arduini, J. . Augueères, J. Brinchmann, R. Cole, M. Cropper, C. Dabin, L. Duvet, A. Ealet, et al., ArXiv e-prints (2011), 1110.3193.
[15] L. Amendola, S. Appleby, D. Bacon, T. Baker, M. Baldi, N. Bartolo, A. Blanchard, C. Bonvin, S. Borgani, E. Branchini, et al., Living Reviews in Relativity **16**, 6 (2013), 1206.1225.
[16] M. LoVerde and M. Zaldarriaga, Phys. Rev. D **89**, 063502 (2014), 1310.6459.
[17] D. Inman, J. D. Emberson, U.-L. Pen, A. Farchi, H.-R. Yu, and J. Harnois-Déraps, Phys. Rev. D **92**, 023502 (2015), 1503.07480.
[18] L. A. Anchordoqui, H. Goldberg, and G. Steigman, Physics Letters B **718**, 1162 (2013), 1211.0186.
[19] D. Blas, J. Lesgourgues, and T. Tram, J. Cosmology Astropart. Phys. **7**, 034 (2011), 1104.2933.
[20] K. W. Masui and U.-L. Pen, Physical Review Letters **105**, 161302 (2010), 1006.4181.