HW3

Hannah Zmuda

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Question 1

Problem

Use Monte Carlo simulation to investigate whether the empirical Type I error rate of the t-test is approximately equal to the nominal significance level α, when the sampled population is non-normal. The t-test is robust to mild departures from normality. Discuss the simulation results for the cases where the sampled population is (i) $\chi^2(1)$, (ii) Uniform(0,2), and (iii) Exponential(rate=1). In each case, test H0 : $\mu = \mu_0$ vs H0 : $\mu \neq \mu_0$, where μ_0 is the mean of $\chi^2(1)$, Uniform(0,2), and Exponential(1), respectively.

Solution

```
#Part a: looking at a type I error
#all follow a similar algorithm as shown on page 193 in SCRR
n <- 100 #number of replicates
a <- 0.05 #significance level alpha
muA <- mean(rchisq(n, df = 1)) #mu0 in part a
muB <- mean(runif(n, 0, 2)) #mu0 in part b
muC <- mean(rexp(n, rate = 1)) #mu0 in part c
#become alternatives in the estimate power of a test (b).

m <- 1000 #number of replicates
pA <- numeric(m)
pB <- numeric(m)
pC <- numeric(m)

for(i in 1:m){
  xA <- rchisq(n, df = 1) #sample dist part a
  xB <- runif(n, 0, 2) #sample dist part b
  xC <- rexp(n, rate = 1) #sample dist part c

  ttestA <- t.test(xA, alternative = "two.sided",mu = muA)
  ttestB <- t.test(xB, alternative = "two.sided",mu = muB)
  ttestC <- t.test(xC, alternative = "two.sided",mu = muC)

  pA[i] <- ttestA$p.value
  pB[i] <- ttestB$p.value
  pC[i] <- ttestC$p.value
}
```

```r
pHatA <- mean(pA < a)
pHatB <- mean(pB < a)
pHatC <- mean(pC < a)

seHatA <- sqrt(pHatA * (1- pHatA)/m)
seHatB <- sqrt(pHatB * (1- pHatB)/m)
seHatC <- sqrt(pHatC * (1- pHatC)/m)
#Means:
cat("The means are: ", c(pHatA, pHatB, pHatC))
```

```
## The means are:  0.108 0.136 0.062
```

```r
#SE:
cat("The standard errors are: ", c(seHatA, seHatB, seHatC))
```

```
## The standard errors are:  0.00981509 0.01083993 0.007626008
```

```r
#Part b: Estimating power of a test and outputting empirical power curves of the t-test from the three
mu <- mean(rnorm(n))

powA <- power.t.test(m, delta = pHatA, sig.level = 0.05, power = NULL, alternative = "two.sided")
powB <- power.t.test(m, delta = pHatB, sig.level = 0.05, power = NULL, alternative = "two.sided")
powC <- power.t.test(m, delta = pHatC, sig.level = 0.05, power = NULL, alternative = "two.sided")

for(j in 1:m){
  #how do I get multiple means and powers when I am using the one pHat calculated in part a?

}
```

Question 2

Question 3

Question 4

Question 5