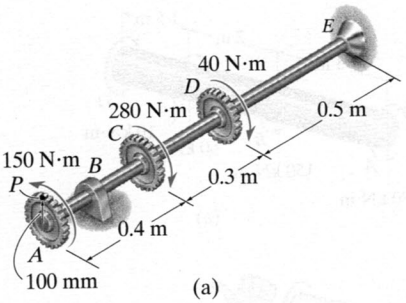
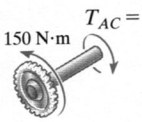
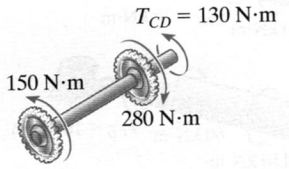
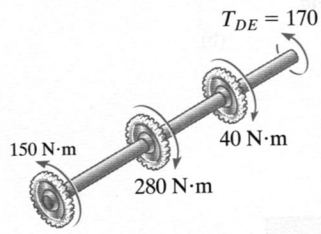
EXAMPLE 5.6



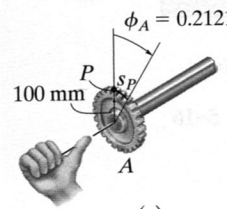
(a)







(b)



(c)

Fig. 5–17

The gears attached to the fixed-end steel shaft are subjected to the torques shown in Fig. 5–17*a*. If the shaft has a diameter of 14 mm, determine the displacement of the tooth P on gear A. $G = 80$ GPa.

SOLUTION

Internal Torque. By inspection, the torques in segments AC, CD, and DE are different yet *constant* throughout each segment. Free-body diagrams of these segments along with the calculated internal torques are shown in Fig. 5–17*b*. Using the right-hand rule and the established sign convention that positive torque is directed away from the sectioned end of the shaft, we have

$$T_{AC} = +150\,\text{N}\cdot\text{m} \qquad T_{CD} = -130\,\text{N}\cdot\text{m} \qquad T_{DE} = -170\,\text{N}\cdot\text{m}$$

Angle of Twist. The polar moment of inertia for the shaft is

$$J = \frac{\pi}{2}(0.007\,\text{m})^4 = 3.771\,(10^{-9})\,\text{m}^4$$

Applying Eq. 5–16 to each segment and adding the results algebraically, we have

$$\phi_A = \sum\frac{TL}{JG} = \frac{(+150\,\text{N}\cdot\text{m})\,(0.4\,\text{m})}{3.771\,(10^{-9})\text{m}^4\,[80\,(10^9)\text{N/m}^2]}$$

$$+\frac{(-130\,\text{N}\cdot\text{m})\,(0.3\,\text{m})}{3.771\,(10^{-9})\text{m}^4\,[80\,(10^9)\text{N/m}^2]} + \frac{(-170\,\text{N}\cdot\text{m})\,(0.5\,\text{m})}{3.771\,(10^{-9})\text{m}^4\,[80\,(10^9)\,\text{N/m}^2]}$$

$$\phi_A = -0.2121\,\text{rad}$$

Since the answer is negative, by the right-hand rule the thumb is directed *toward* the support E of the shaft, and therefore gear A will rotate as shown in Fig. 5–17*c*.

The displacement of tooth P on gear A is

$$s_P = \phi_A r = (0.2121\,\text{rad})\,(100\,\text{mm}) = 21.2\,\text{mm} \qquad \textit{Ans.}$$