# The Reliability Analysis of Concrete Gravity Dam

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# Abstract

The stability of a gravity dam against sliding along deep-seated weak planes is a universal and important problem encountered in the construction of dams. Taking Nongling dam as an example, the research in this paper is focused on the stability analysis of the most dangerous dam section by the traditional rigid body limit state equilibrium method and form analysis. By calculating the stability index of the dam, it can be concluded that the safety of the dam against sliding along deep-seated weak planes can be evaluated by both two methods leading to the same results. A comparison is also indicated during the analysis of the reliability index.


**Keywords:** stability; gravity dam; traditional rigid body limit state equilibrium method; form analysis;

# Contents

# 1 Introduction

A gravity dam is an engineering structure that by its own weight resists the forces imposed with a desired factor of safety. Gravity dams are designed so that every dam section is stable, independent of any other dam section. Concrete gravity dams are an important part of the nation's infrastructure, serving as part of inland waterway transportation systems and conservation and recreational projects. The stability of the gravity dam is among one of the most serious problems in the design and construction procedure.

The general way to maintain a concept of stability is the method called rigid body limit equilibrium method which adopts the safety factor K as the index. There are three general methods in the simplification of the model. Here, we adopted the Equal K Method. In the report, a formula of reliability for calculating the stability against sliding of gravity dam is also deduced based on reliability theory. The formula is based on the limit state function which contains multiple random variables. The reliability index from the FORM has been calculated in Matlab to make a comparison with the traditional safety factor K and discuss the feasibility of the reliability theory that can be widely used in the gravity dam stability analysis.

As China has the most advantageous techniques in the region of dam design, the report has based on the design provision of concrete gravity dam of China and the example is excerpted from the Nongling hydraulic station.

# 2 Methodology

## 2.1 Reliability-based method

When there are n random variables Xi (1, 2…. n) affects the reliability of structure, the structural limit state function is

$$Z=g(X1, X2,\ldots.Xn)=0 \qquad (1)$$

Typically, (1) equation contains multiple basic random variables with complicated distribution. In order to calculate component failure probability, we can get the probability theoretically by the multidimensional integral:

$$Pf= P\{Z<0\}=\iiint_{Z<0} f(x1, x2\ldots\ldots, xn)dx1\ dx2\ldots..\ dxn$$

Obviously, only when the number of random variables is countable and it's easy to conduct the direct integral, the formula is feasible. Generally, we use probability design method such as first order second moment method, response surface method, Monte Carlo method, stochastic finite element method, etc. In this report, FORM will be used for the reliability-based analysis.

### 2.1.1 Limit State function

For the slide plane of double-taper wedge of the gravity dam, the limit state function under the load combination of water pressure, self-weight, uplift pressure can be conducted as:

$$Z = f_1'[(G1+\sum W)\cos\alpha-\sum P\sin\alpha-Q\sin(\varphi-\alpha)+U_3\sin\alpha-U_1]+c'_1\ A_1$$

$$-[(G1+\sum W)\sin\alpha+\sum P\cos\alpha-Q\cos(\varphi-\alpha)-U_3\cos\alpha]$$

Where Q is the resistance force $aQ^2+bQ+c=0$

$a= fc_1'\sin(\varphi-\alpha)\cos(\beta+\varphi)- fc_2'\cos(\varphi-\alpha)\sin(\beta+\varphi);$

$b= fc_2'\sin(\beta+\varphi)\ [(G1+\sum W)\sin\alpha+\sum P\cos\alpha-U_3\cos\alpha]-\cos(\varphi-\alpha)\ [fc_2'(G2\cos\beta+U3\sin\beta$

$-U2)+c_2'A_2]+ fc_1'\sin(\varphi-\alpha)(U3\cos\beta-G2\sin\beta)+\cos(\beta+\varphi)\{ fc_1'[(G1+\sum W)\cos\alpha-\sum P\sin\alpha$

$+U_3\sin\alpha-U_1]+c_1'A_1\}$;

$c= [fc_2'(G2\cos\beta+U3\sin\beta-U2)+c_2'A_2]*[(G1+\sum W)\sin\alpha+\sum P\cos\alpha-U_3\cos\alpha]-\{ fc_1'[(G1+\sum W)$

$\cos\alpha-\sum P\sin\alpha+U_3\sin\alpha-U_1]+c_1'A_1\}*(U3\cos\beta-G2\sin\beta)$;

### 2.1.2 FORM analysis index

$$\beta_{FORM=\alpha}{}^{T}u^*$$

where α is the unit normal vector and u* is the design point. Because the coordinates of the design point is unknown, we need to calculate it by iteration.

From the analysis before, fc' and c' are the most sensitive factors with respect to the stability of a gravity dam. Here we consider $fc'_1$, $fc'_2$, $c'_1$, $c'_2$ as the random variables which as statistically independent. Their distributions are shown in the Table. 1

**Table 1 Design parameters for the random variables**

| Parameters | Value or mean | Distribution | COV (%) |
|---|---|---|---|
| $fc_1'$ | 0.32 | normal | 26 |
| $fc_2'$ | 0.9 | normal | 20 |
| $c_1'$ | 0.09 | normal | 45 |
| $c_2'$ | 0.65 | normal | 36 |

Note: fc' and c' are the shear resistance parameters of the soil in the left and right block showed in Fig.1 respectively. The unit of c' is MPa.

The principal procedure is shown as follows.

- Transform the g from original space to the standard space G. $\alpha=\frac{-\nabla G}{||G||}$
- Assume the initial value of the design point u*. That is, using the mean value of the random variables $\mu_{fc1'}$, $\mu_{fc2'}$, $\mu_{c1'}$, $\mu_{c2'}$ for $fc_1'$ $fc_2'$ $c_1'$ $c_2'$
- Determine the partial derivative of the limit state function with respect to every random variable, namely, $\frac{\partial Z}{\partial fc1}, \frac{\partial Z}{\partial fc2}, \frac{\partial Z}{\partial c1}, \frac{\partial Z}{\partial c2}$

  $\frac{\partial Z}{\partial fc1}=(G1+\sum W)\cos\alpha-\sum P\sin\alpha-Q\sin(\varphi-\alpha)+U_3\sin\alpha-U_1+[\cos(\varphi-\alpha)-fc_1'\sin(\varphi-\alpha)]\frac{\partial Q}{\partial fc1}$;

  $\frac{\partial Z}{\partial fc2}=[\cos(\varphi-\alpha)-fc_1'\sin(\varphi-\alpha)]\frac{\partial Q}{\partial fc2}$;

  $\frac{\partial Z}{\partial c1}=[\cos(\varphi-\alpha)-fc_1'\sin(\varphi-\alpha)]\frac{\partial Q}{\partial c1}+c_1$;

  $\frac{\partial Z}{\partial c2}=[\cos(\varphi-\alpha)-fc_1'\sin(\varphi-\alpha)]\frac{\partial Q}{\partial c2}$;

  where

  $\frac{\partial Q}{\partial fc1}=\frac{1}{2aQ+b}[-Q^2\sin(\varphi-\alpha)\cos(\beta+\varphi)-Q\sin(\varphi-\alpha)(U3\cos\beta-G2\sin\beta)+[(G1+\sum W)\cos\alpha-\sum P\sin\alpha+U_3\sin\alpha-U_1]*[U3\cos\beta-G2\sin\beta+Q\cos(\beta+\varphi)]$

  $\frac{\partial Q}{\partial fc2}=\frac{1}{2aQ+b}[Q^2\cos(\varphi-\alpha)\sin(\beta+\varphi)+Q(G2\cos\beta+U3\sin\beta-U2)-[(G1+\sum W)\sin\alpha+\sum P\cos\alpha-U_3\cos\alpha]*[G2\cos\beta+U3\sin\beta+Q\sin(\beta+\varphi)]$;

  $\frac{\partial Q}{\partial c1}=\frac{1}{2aQ+b}[Q\cos(\beta+\varphi)+U3\cos\beta-G2\sin\beta]A_1$;

  $\frac{\partial Q}{\partial c1}=\frac{1}{2aQ+b}[Q\cos(\varphi-\alpha)-\sum P\cos\alpha-(G1+\sum W)\sin\alpha+U_3\sin\alpha]A_2$;

- Substitute the mean value of the four random variables to calculate $\frac{\partial Z}{\partial fc1}, \frac{\partial Z}{\partial fc2}, \frac{\partial Z}{\partial c1}, \frac{\partial Z}{\partial c2}$

- Calculate $u_{i+1}$ to see that whether the design point is OK. $u_{i+1}=\frac{G(ui)-\nabla GT(ui)ui}{||\nabla G(ui)||} * \frac{-\nabla G(ui)}{||G(ui)||}$.

  If the difference between $u_{i+1}$ and $u_i$ is within the permissible range, $u_i$ is really close to the real design point which can be utilized for calculate $\beta$. If the error goes far beyond the range, we should do more iteration to achieve the convergence. This step is done by Matlab and the code is attached in the Appendix.

- Calculate $\beta$ and $P_f$ as a byproduct. $P_f = \emptyset(-\beta)$

## 2.2 Traditional rigid body limit equilibrium method

Generally, there are three ways to calculate the safety factor in the rigid body limit equilibrium method: (1) Residual Thrust Method (2) Passive Resistance Method (3) Equal K Method. Here we adopt the Equal K method.

Meanwhile, as the formulation of slide plane of double-taper wedge indicates the most dangerous failure mechanism of the gravity dam, we consider gravity dam cross section as Fig.1.

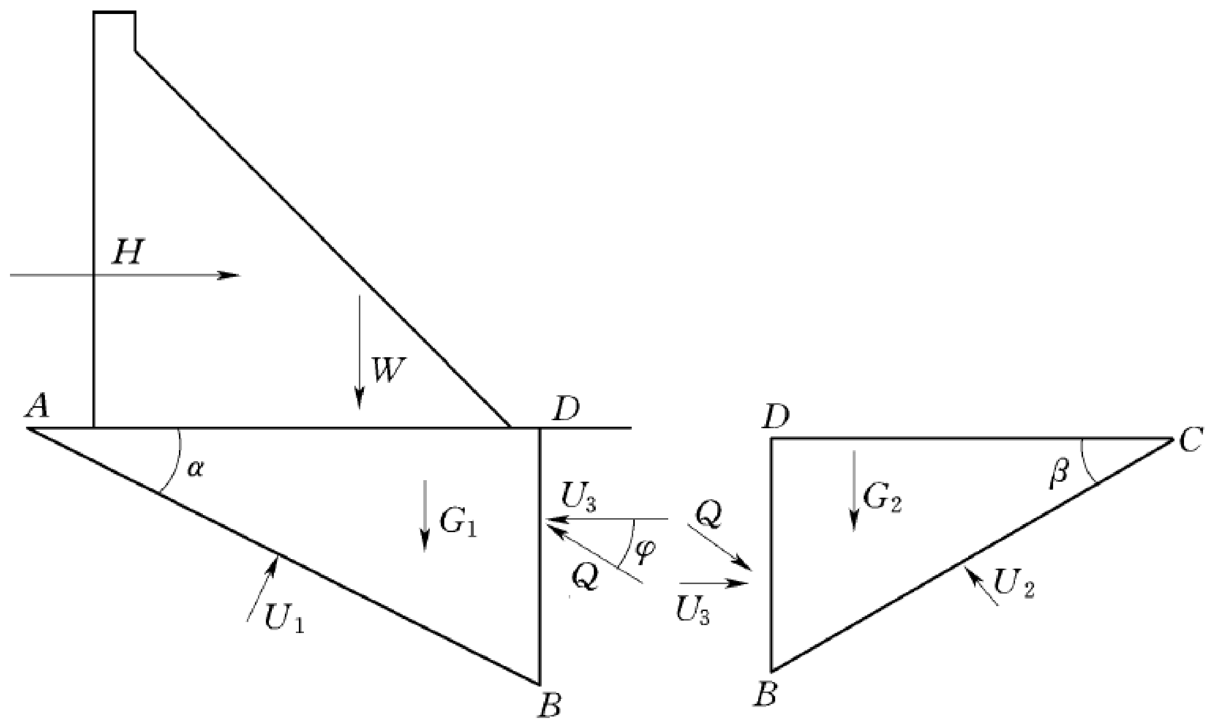


Figure 1: the gravity dam cross section view

When using the Equal-K method, the double-taper wedge can be separated as Figure.2

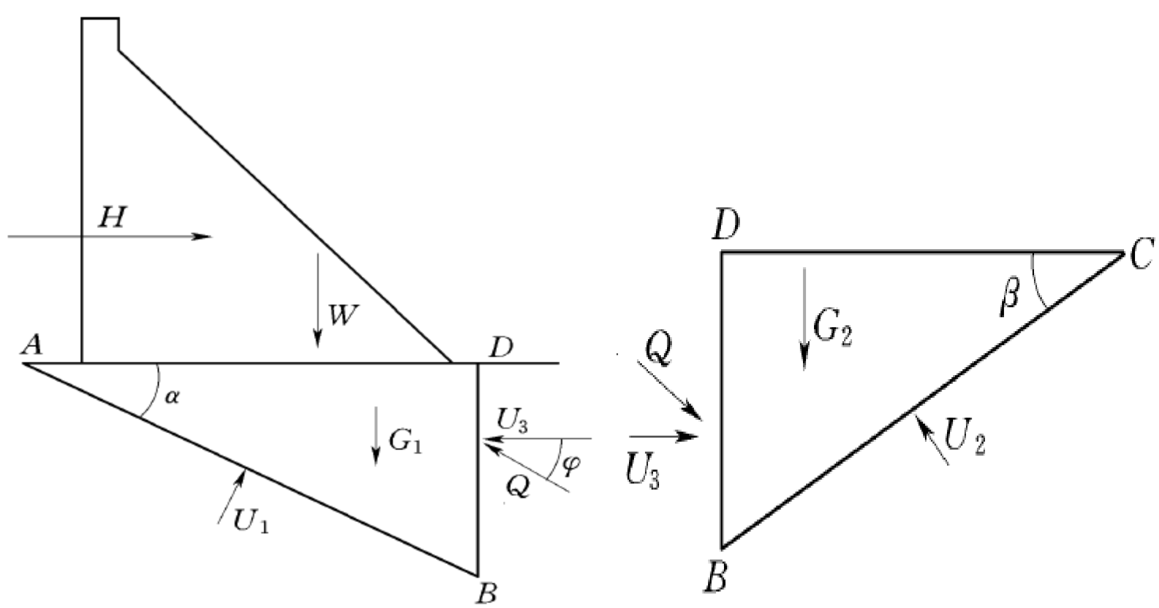

Figure 2 Two separate parts of the gravity dam for K calculation

Where G1.G2---- the self-weight of the rocks under the surface for the left and right rocks.

W------- the self-weight of the concrete gravity dam.

H-------- the total horizontal water pressure. It equals to the difference between the upstream and the downstream water force.

U1/U2/U3----the uplift pressure which acted on the upstream and downstream as well as the interface of the two rocks.

Q--------- the interaction force on the interface, which needs to be calculated by iteration.

## 2.2.1 Calculation of K

Safety factor $K_1 = \dfrac{f1[(W+G1)cos\alpha-\sum Psin\alpha-Rsin(\varphi-\alpha)-U1+U3sin\alpha]+c1A1}{\sum Pcos\alpha+(W+G1)sin\alpha-U3cos\alpha-Rcos(\varphi-\alpha)}$

$$K_2 = \frac{f2[G2cos\beta+Rsin(\varphi+\beta)+U3sin\beta-U2]+c2A2}{Rcos(\varphi+\beta)-G2sin\beta+U3cos\beta}$$

We assume $K_1=K_2$ to calculate Resistance force R. We need the iteration to decide the R so that K1=K2.Then we can plug back to calculate K.

Note: $K_1$,$K_2$ refer to the safety factor of the left block and right block respectively.

For simplification, the design provision suggested that U3 equals to zero which indicates that there's no uplift pressure at the interface of the two separate rocks. What's more, φ is also defined as zero for the sake of computational time. If φ is nonzero, then complicated iteration should be utilized to solve the problem.

# 3 Design example

The report took the Nongling Hydropower Station in China as an example. We focused on the most dangerous dam section 0+247 whose cross section is shown in Figure 3.

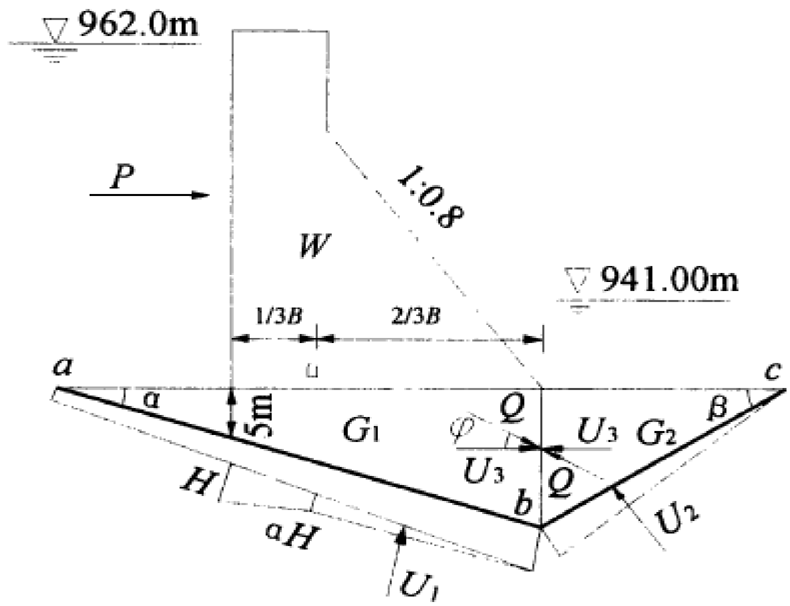


Figure 3: 0+247 cross section view

The dam foundation surface elevation is 936.50 m and the dam crest elevation is 964.50 m with width of 8.0m. Based on qualitative data, the main and the auxiliary critical slip angel is 22°and 28°.That is, $\alpha = 22° \ and \ \beta = 28°$ respectively. The concrete density is 24 kN/m3 while the rock density is 26 kN/m3. Assume φ=0° and $U_3$ =0 for simplification.

The upstream water level is 962.00m and the downstream water level is 941.00m. The load combination is adopted as the basic combination under the normal storage level. Loads includes the self-weight of the rocks and dam itself above slip surface, the water pressure and uplift pressure both upstream and downstream. The osmotic pressure reduction factor of draining curtain is α= 0. 35 while the permissible error for the reliability index is 0.005.

The calculation of the report is divided into two part: (1) reliability-based analysis; (2) rigid body limit state equilibrium method.

# 4 Results

## 4.1 Reliability-based analysis procedure

The reliability-based analysis is realized by ferum. Here we adopt the most general form analysis. The essential part is to calculate the derivatives of the limit state function with respect to $f_1$' $f_2$' $c_1$' $c_2$'. Once the calculation of $\frac{\partial Z}{\partial fc1}, \frac{\partial Z}{\partial fc2}, \frac{\partial Z}{\partial c1}, \frac{\partial Z}{\partial c2}$ are done, we can utilize Matlab to achieve the reliability index and the failure probability.

$\alpha = [-\ 0.2110626; -\ 0.6325519;\ \ -\ 0.2830141; -\ 0.6893720]^T$

In the FORM, the normal vector is $\alpha$=

$[-\ 0.2110626; -\ 0.6325519;\ \ -\ 0.2830141; -\ 0.6893720]^T$ $\beta_{FOSM}$= 4.84>4 for the code-based project, which indicates the dam section is safety against slipping as a double-taper wedge. The failure probability *Pf*=6.4920e-07, thus dictates that the most dangerous section is nearly impossible to fail based on the infinitesimal failure probability.

Note: The criteria of β>4 is based on the design provision of concrete gravity dam of China.

## 4.2 Traditional rigid body limit equilibrium method

Given that all the properties of the rocks and concrete as well as upstream and downstream water level, we can calculate that the basic forces we need for iteration of R, which had been shown in Table 2.

**Table 2 Forces that contributes to the calculations of K**

| Forces | Unit( KN) |
|---|---|
| G1 | 6952.22 |
| G2 | 5823.28 |
| W | 9216 |
| H | 3150 |
| U1 | 2151 |
| U3 | 5923 |

Thus by iteration using Matlab, we can calculate that R= 7282 KN. Plugging back to the K equation, $K$ =3.108>3.0 which also shows the stability of this dam section.

# 5 Conclusion

1. By comparing the traditional rigid body limit equilibrium method and reliability-based analysis, we can safely draw the conclusion that they lead to the same results, i.e. they are both valid measures of the stability of the gravity dam.
2. The rigid body limit equilibrium method is reasonable. It's an easier way to check the stability of the gravity dam under different load combinations (here use the normal load combination, which includes the self-weight of the concrete and rock, the water pressure as well as the uplift force).

3. However, the traditional rigid body limit equilibrium method can only shows the final results of the reliability index which means that it cannot tell us about the exact probability of failure under specific circumstance. In contrast, the reliability-based analysis can not only indicate the same conclusion but also dictate the failure risk in a mathematical way by the failure probability. If the reliability is small (just as in this case), it means that the design of gravity dam is really safe.
4. The probability-based analysis is more reliable compared with the traditional rigid body limit equilibrium method. The latter method use some simplifications such as decreasing the random variables to $fc_1$' $fc_2$' $c_1$' and $c_2$' only. However, the reality is much more complicated than the simplification. The influential random variables may be different under various situations. The ferum analysis allows us to discuss with more than 4 random variables as well as using different reliability analysis method at the same time such as form, sorm, etc. Thus, the mathematical probability analysis is much more reliable than the traditional limit state equilibrium method.

# 6 Deficiency

1. The paper only considers about four mostly influential random variables, which is suggested by experimental papers and reports. The experience shows that for the stability of gravity dam is most influential by the shear resistance factor such as $fc_1$' $fc_2$' $c_1$' and $c_2$'. However, the influential factors may be different with each other along with the change of the location and the water level, etc. Hence, in order to improve the quality of prediction, most influential factors should be taken into consideration and sensitivity analysis among all the parameters should be done to generalize the problem.

2. The distribution of the random variables are regarded as normal distribution and statistically independent. But actually they may subject to lognormal distributions based on the various properties of soil and rocks. Correlations between the random variables may also lead to some errors in the failure probability.
3. The project has adopted many simplifications and assumptions in calculating of the value of K. The uplift force at the interface and φ are treated as zero in the project. However, the interaction force between the two blocks doesn’t necessarily to be horizontal and the uplift force may be nonzero due to the influence of small openings in the rocks. This will lead to the more computational time and complicated code for iteration.
4. The limit state functions may not be so accurate to some extent. There are other influential components which cannot be measured such as the initial imperfections of the rocks and dynamic load such as earthquakes and wave pressure.
5. The report took the basic load combination for the stability analysis which may not represent the specific case. However, during the empirical design of the dam section, specific load combinations need to be considered for the uncountable damage they might cause.

# 7 Appendix

Appendix is the code for the ferum part of the reliability-based analysis and the code for the iteration of K. Here attached the inputfile-template for the limit state function and the iteration code.

1. Ferum form part

```
clear probdata femodel analysisopt gfundata randomfield systems results output_filename
```

```
output_filename = 'outputfile_project.txt';

probdata.marg(1,:) =  [ 1 0.32  0.0832  0.32  0 0 0 0 0];
probdata.marg(2,:) =  [ 1 0.9  0.18   0.9  0 0 0 0 0];
probdata.marg(3,:) =  [ 1 90   40.5  90    0 0 0 0 0];
probdata.marg(4,:) =  [ 1 650  234  650   0 0 0 0 0];

probdata.correlation = [ 1.0    0    0    0 ;
                         0.0   1.0   0    0 ;
                         0.0   0.0  1.0   0 ;
                         0.0   0.0  0.0  1.0];

probdata.parameter = distribution_parameter(probdata.marg);

analysisopt.ig_max     = 10000;
analysisopt.il_max     = 5;
analysisopt.e1         = 0.001;
analysisopt.e2         = 0.001;
analysisopt.step_code = 0;
analysisopt.grad_flag = 'ddm';
analysisopt.sim_point = 'dspt';
analysisopt.stdv_sim  = 1;
analysisopt.num_sim   = 1000;
analysisopt.target_cov = 0.05;

gfundata(1).evaluator = 'basic';
gfundata(1).type = 'expression';
gfundata(1).parameter = 'no';


gfundata(1).expression =
'11660*x(1)+39.24*x(3)+0.5*(0.3746*x(1)+0.9272)*((((5384*x(2)+29.03*x(4)-9363*x(1)+3
4.64*x(3))^2+4*(0.33*x(1)+0.44*x(2))*(2893676*x(1)-11302105*x(2)+281092*x(4)+97385
*x(3))))^(0.5)-5384*x(2)-29.03*x(4)+9363*x(1)-34.64*x(3))/(-0.33*x(1)-0.44*x(2))-8878';
gfundata(1).dgdq = {
 '(1873*(5384*x(2)-9363*x(1)+(866*x(3))/25+(2903*x(4))/100-((5384*x(2)-9363*x(1)+(86
 6*x(3))/25+(2903*x(4))/100)^2+((33*x(1))/25+(44*x(2))/25)*(2893676*x(1)-11302105*x(
 2)+97385*x(3)+281092*x(4)))^(1/2)))/(10000*((33*x(1))/100+(11*x(2))/25))-(33*((1873*
 x(1))/10000+1159/2500)*(5384*x(2)-9363*x(1)+(866*x(3))/25+(2903*x(4))/100-((5384*x(
 2)-9363*x(1)+(866*x(3))/25+(2903*x(4))/100)^2+((33*x(1))/25+(44*x(2))/25)*(2893676*
 x(1)-11302105*x(2))+97385*x(3)+281092*x(4)))^(1/2))/(100*((33*x(1))/100+(11*x(2))/25
 )^2)+((((1873*x(1))/10000+1159/2500)*(((2766167321*x(2))/25-(4574271066*x(1))/25+(1
 3003011*x(3))/25+(8628717*x(4))/50)/(2*((5384*x(2)-9363*x(1)+(866*x(3))/25+(2903*x(
 4))/100)^2+((33*x(1))/25+(44*x(2))/25)*(2893676*x(1)-11302105*x(2)+97385*x(3)+2810
 92*x(4)))^(1/2))-9363))/((33*x(1))/100+(11*x(2))/25)+11660';
```

```
'-(11*((1873*x(1))/10000+1159/2500)*(5384*x(2)-9363*x(1)+(866*x(3))/25+(2903*x(4))/
100-((5384*x(2)-9363*x(1)+(866*x(3))/25+(2903*x(4))/100)^2+((33*x(1))/25+(44*x(2))/2
5)*(2893676*x(1)-11302105*x(2)+97385*x(3)+281092*x(4)))^(1/2)))/(25*((33*x(1))/100
+(11*x(2))/25)^2)-((((90957512*x(2))/5-(2766167321*x(1))/25+(13610028*x(3))/25+(201
82924*x(4))/25)/(2*((5384*x(2)-9363*x(1)+(866*x(3))/25+(2903*x(4))/100)^2
+((33*x(1))/25+(44*x(2))/25)*(2893676*x(1)-11302105*x(2)+97385*x(3)+281092*x(4)))
^(1/2))-5384)*((1873*x(1))/10000+1159/2500))/((33*x(1))/100+(11*x(2))/25)';

'981/25-(((1873*x(1))/10000+1159/2500)*(((13610028*x(2))/25-(13003011*x(1))/25+(149
9912*x(3))/625+(1256999*x(4))/625)/(2*((5384*x(2)-9363*x(1)+(866*x(3))/25+(2903*x(
4))/100)^2+((33*x(1))/25+(44*x(2))/25)*(2893676*x(1)-11302105*x(2)+97385*x(3)+281
092*x(4)))^(1/2))-866/25))/((33*x(1))/100+(11*x(2))/25)';

'-(((1873*x(1))/10000+1159/2500)*(((20182924*x(2))/25-(8628717*x(1))/50+(1256999*x(
3))/625+(8427409*x(4))/5000)/(2*((5384*x(2)-9363*x(1)+(866*x(3))/25+(2903*x(4))/100
)^2+((33*x(1))/25+(44*x(2))/25)*(2893676*x(1)-11302105*x(2)+97385*x(3)+281092*x(4
)))^(1/2))-2903/100))/((33*x(1))/100+(11*x(2))/25)'};

femodel = 0;
randomfield.mesh = 0;
```

2. Iteration for K

```
function Sav=af2

global f1 c1 f2 c2 A1 A2 alfa fai beita G1 G2 W H U1 F1 F2;

pars=[0.32 90000  0.9 650000 39.24 31.31 22 0 28  6952220 5823280 9216000 3150000
      2151000 5923000 0];

N=1;

Sav=zeros(N,3);

for i=1:N

  paras=pars(i,:);

 f1=paras(1); c1=paras(2); f2=paras(3); c2=paras(4); A1=paras(5); A2=paras(6);

 alfa=paras(7)*3.1416/180; fai=paras(8)*3.1416/180; beita=paras(9)*3.1416/180;

G1=paras(10); G2=paras(11); W=paras(12); H=paras(13); U1=paras(14);

F1=paras(15); F2=paras(16);
```

```
    Q=4000000;
    Q=fzero(@func,Q);
    kq=K(Q);

   Sav(i,:)=[kq(2),Q,func(Q)];
  end
  end

    function k=K(Q)
    global f1 c1 f2 c2 A1 A2 alfa fai beita G1 G2 W H U1 F1 F2;
    k(1)=f1*(G1*cos(beita)+Q*sin(fai+beita)-F1+F2*sin(beita))+c1*A1;
   k(1)=k(1)/(Q*cos(fai+beita)-G1*sin(beita)+F2*cos(beita));
   k(2)=f2*((W+G2)*cos(alfa)-H*sin(alfa)-Q*sin(fai-alfa)-U1+F2*sin(alfa))+c2*A2;
   k(2)=k(2)/((W+G2)*sin(alfa)+H*cos(alfa)-Q*cos(fai-alfa)-F2*cos(alfa));
end

   function y=func(Q)
   k=K(Q);
   y=k(1)-k(2);
end
```

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