1. Dynamic parameters:

   K_b , K_t , R_a , J_m , D_m , T_{max} at input voltage , $\omega_{no\ load}$ at specific input voltage.

J_m: to find J we need T at time=0 and the angular acceleration at t=0.

$$To(0) = Tmax$$

We test T_{max} at V_{in} =6v using torque meter from Mechatronics Lab :

$$Tmax = 15\ kg.cm$$

$$To(0) = Tmax = \frac{15 * 9.81}{100} = 1.4715\ N.m$$

$\alpha_{no\ load}$ testing using the encoder at t=0 and V_{in}=6v

$\alpha_{no\ load}$=40131 rad/s^2

$$T=\alpha*J_m \rightarrow J_m = \frac{T}{\alpha} = \frac{1.4715}{40131} = 3.67857 * 10^{-5}$$

Jm from gemomitry $=\frac{1}{2} * m * r^2$

Dencity =8050 kg/m^3 for most commen steel

V=(pi*r^2)*L

V=3.14*(0.003)^2*0.23

V=0.0000068

M= V*dencity=8050*0.0000068= 0.054503 kg

Jm from gemomitry $=\frac{1}{2} * (0.054503) * (0.003)^2$

Jm from gemomitry=2.45263* 10^{-5}

Now we take the averge

$$Ja=\frac{3.67857*10^{-5}+2.45356*10^{-5}}{2} = 3.066 * 10^{-5}$$

K_t/R_a , K_b ,and D_m :

D_m assumed to be zero since the frection is neglected .

We test $T_{full\ load}$ (T_{max}) ,and $\omega_{no\ load}$,at input voltage =12v

$$Tmax = 30\ kg.cm = \frac{30 * 9.81}{100} = 2.943\ N.m$$

$$\omega_{no\ load} = 31.4286\ rad/s$$

2.

$$\frac{\theta(s)}{T(s)} = \frac{{Kt}/{Ra * Ja}}{s(s + \frac{1}{Ja}(Dm + \frac{Kt}{Ra} * Kb))}$$

$$\frac{Kt}{Ra} = \frac{T_{no\ load}}{e_{in}} = \frac{2.943}{12} = 0.2452$$

$$Kb = \frac{e_{in}}{\omega_{\text{no load}}} = \frac{12}{31.4286} = 0.3818$$

$$\frac{\theta(s)}{T(s)} = \frac{8000}{S^2 + 3055 * S}$$

$$(S^2 + 3055 * S) * \theta(s) = 8000 * T(s)$$

$$\ddot{\theta}(t) + 3055 * \dot{\theta}(t) - 8000 * T(t) = 0$$

3. $$G(s) = \frac{\dot{\theta}_m(s)}{V(s)}$$

$$S * \dot{\theta}(s) + 3055 * \dot{\theta}(s) = 8000 * T(s)$$

$$\frac{\dot{\theta}(s)}{T(s)} = \frac{8000}{S + 3055}$$

4. Design using sisotool (root locus) for settling time T_S =1s , %over shot =0% , and error (e=0).

In real motor we input 6 V, the output was 15 rad/s, after finding the transfer function we input 6 V to transfer function in Simulink the output was 15 rad/s, which means our model is acceptable, see figures (1,2)

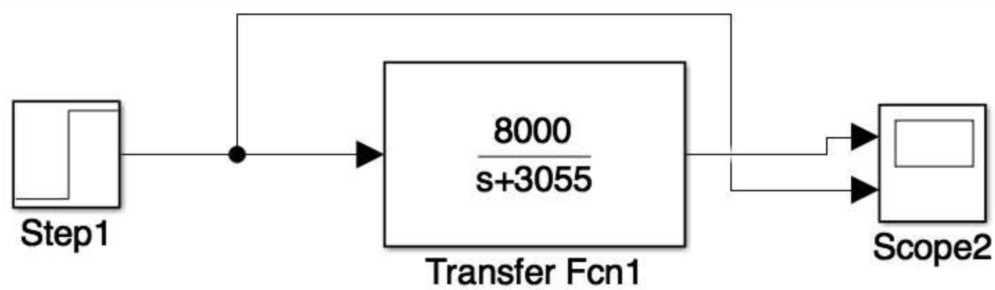


Figure 1 forwarded transfer function

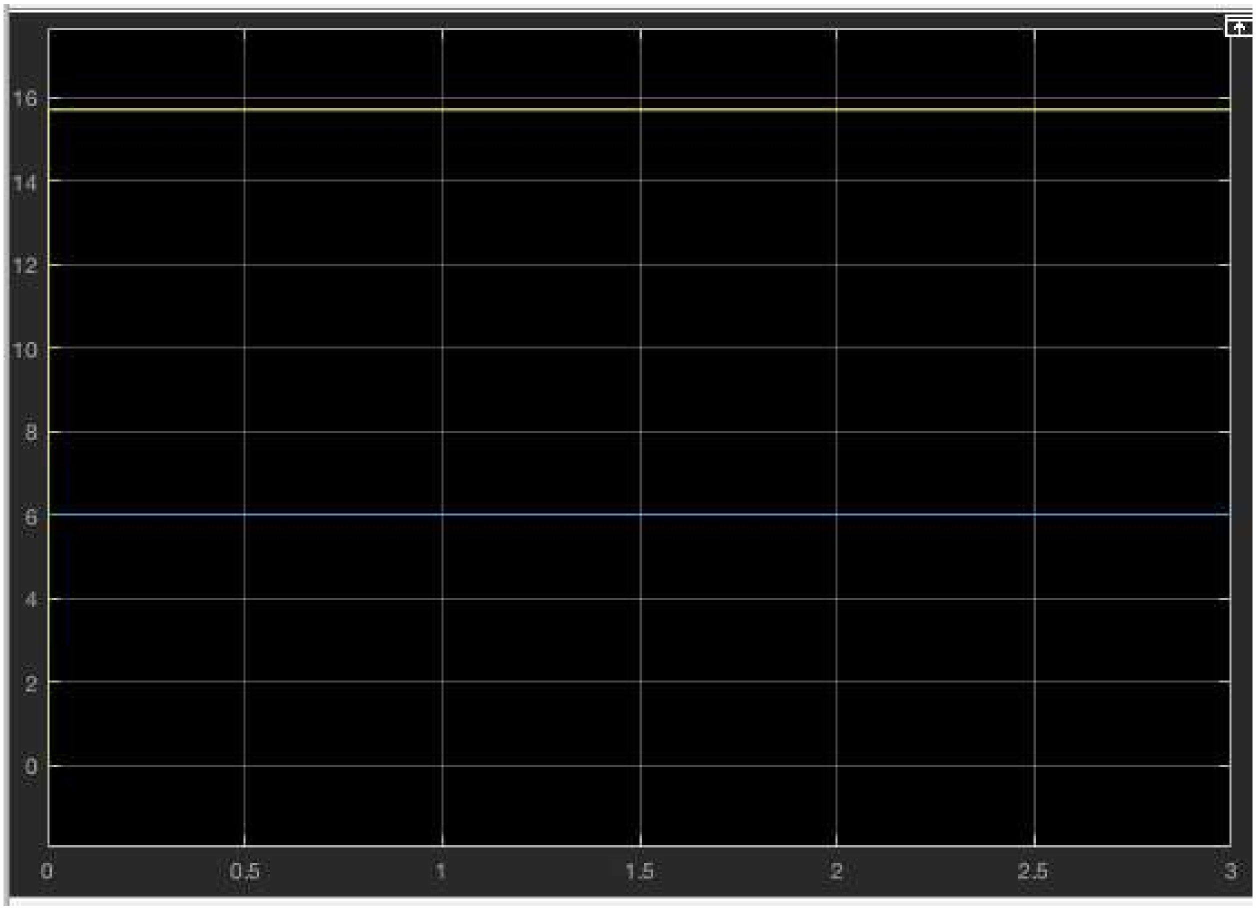


Figure 2 input 6V and output 15 rad /s

Close loop system

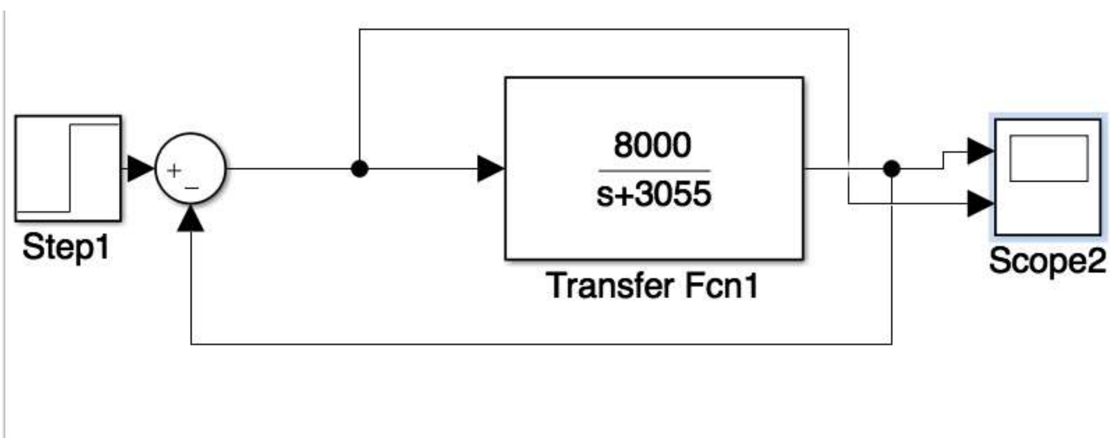


Figure 3 close loop transfer function

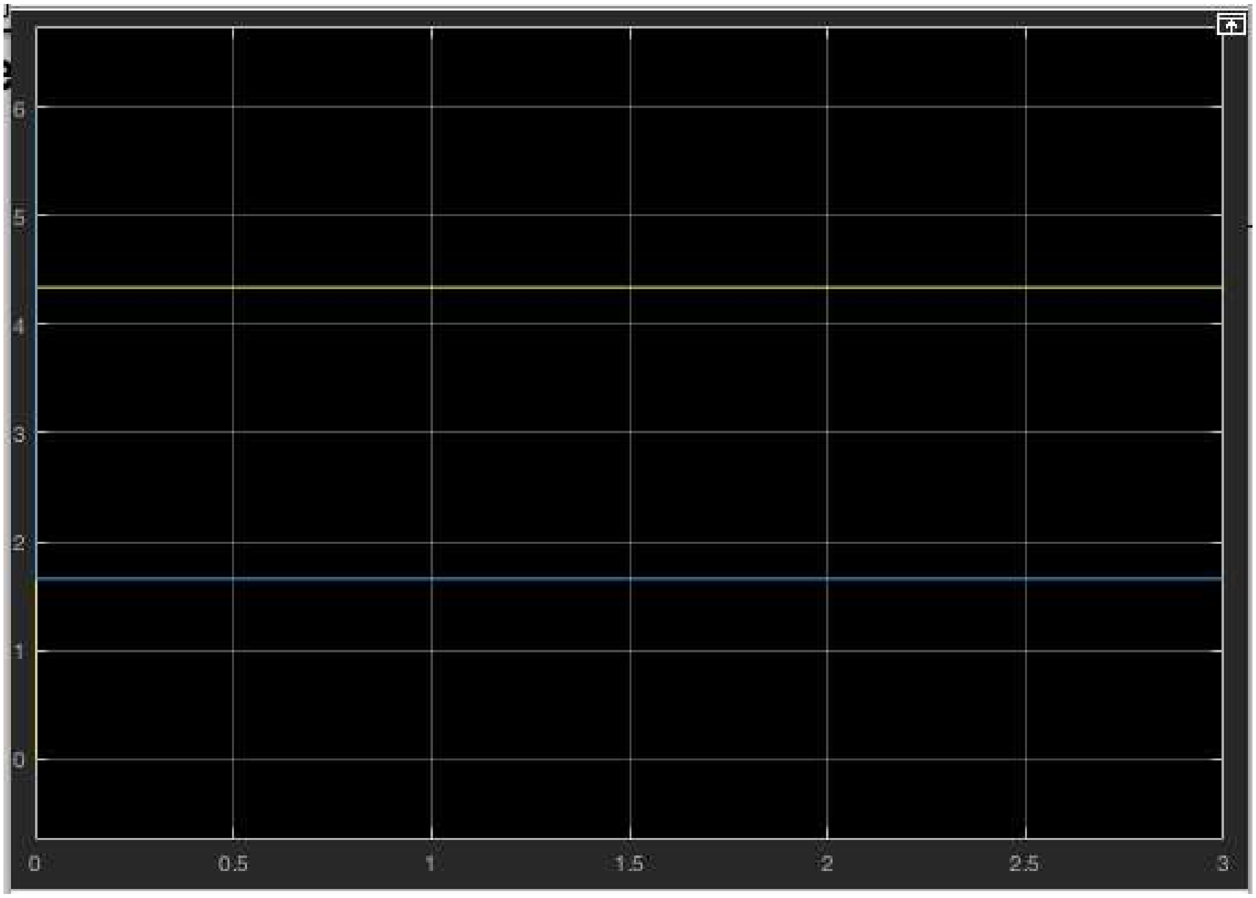


Figure 4 close loop system response

In figure (4), you see the step response and its obvious that the system has a large steady state error so we need a controller.

- Controller belt

COED:
D=[8000];
N=[1 3055];
Sisotool(tf(D,N));

Rout loucas :

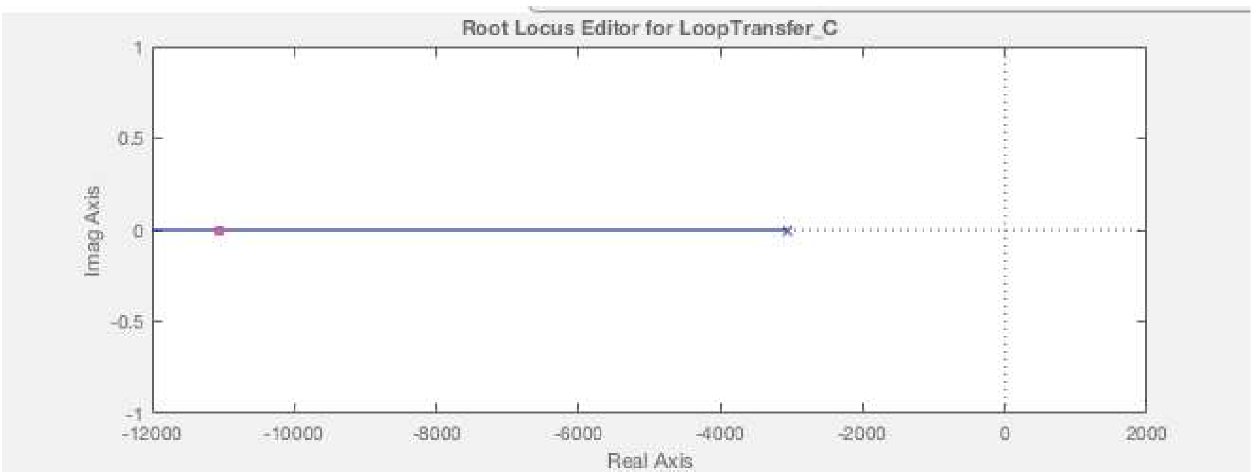
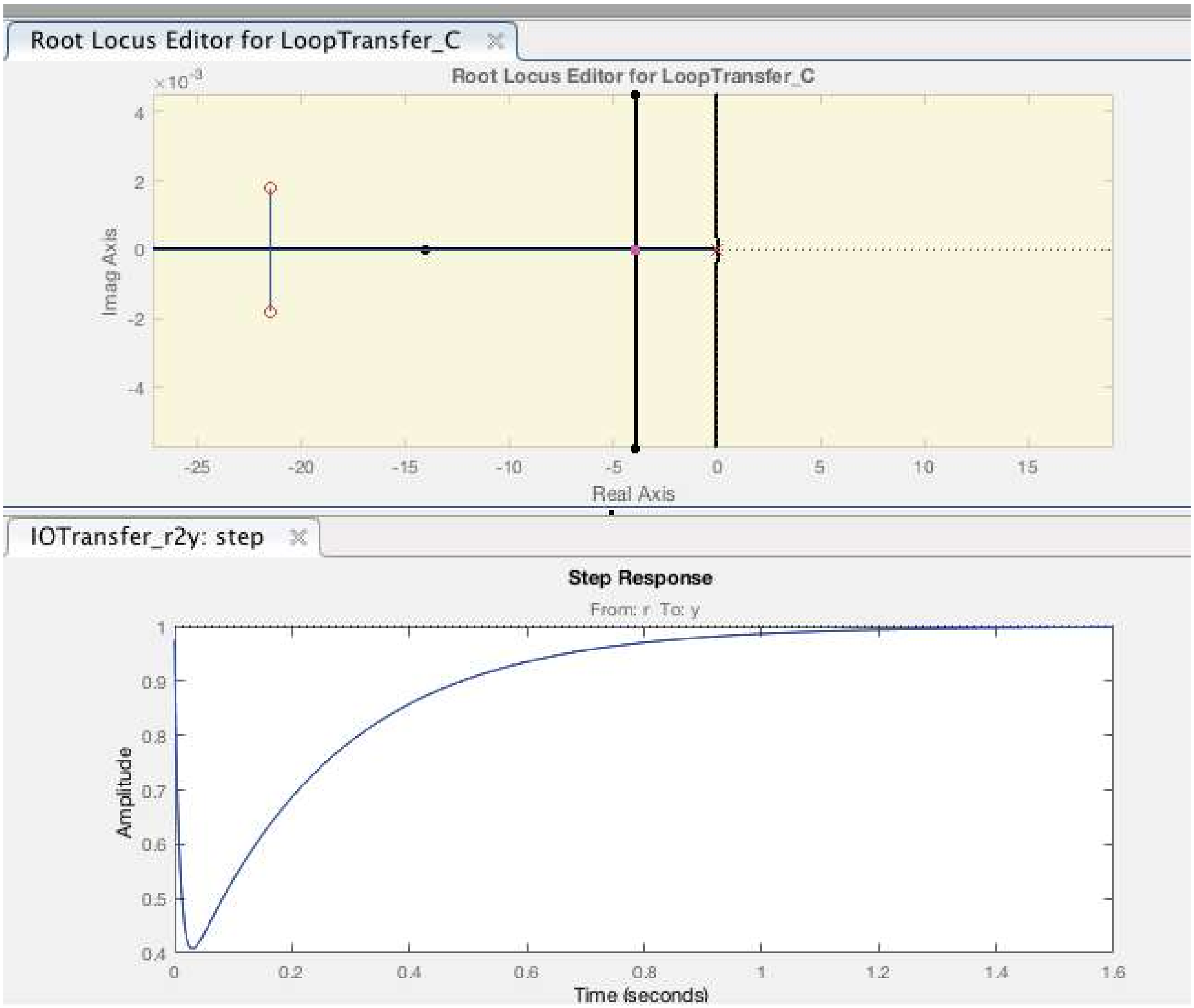


Figure 5 Root locus

Pid controller:



PI:

If we use pi controller the output will be zero with zero over shot and 1 s settling time

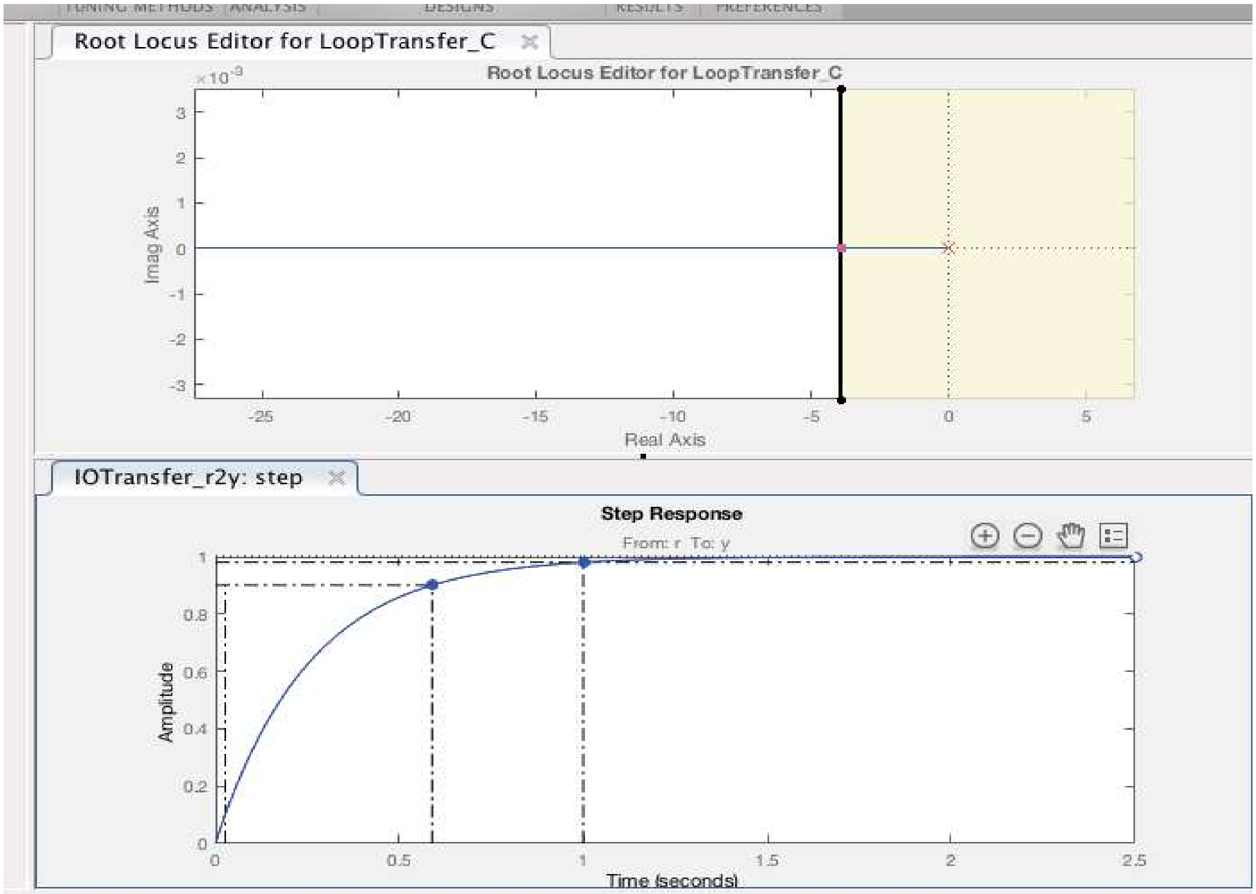


Figure 6 pi controller design for using sisotool

$K_p=0.004$, $K_i=2.6928$

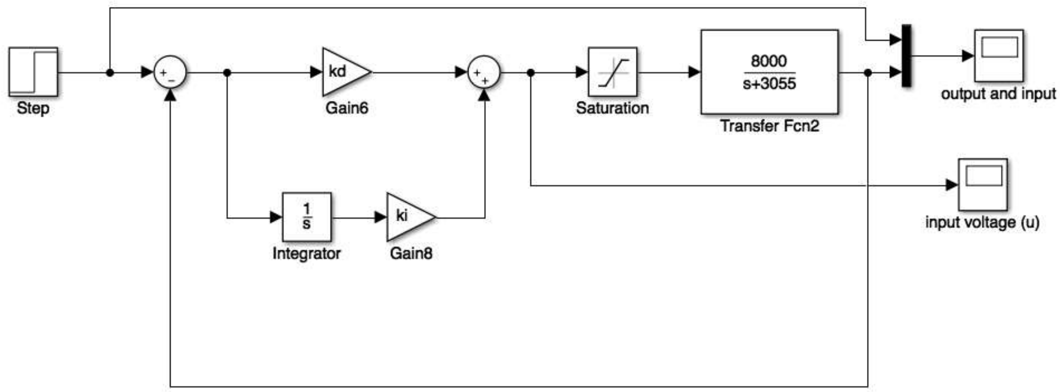


Figure 7 Simulink for pi controller

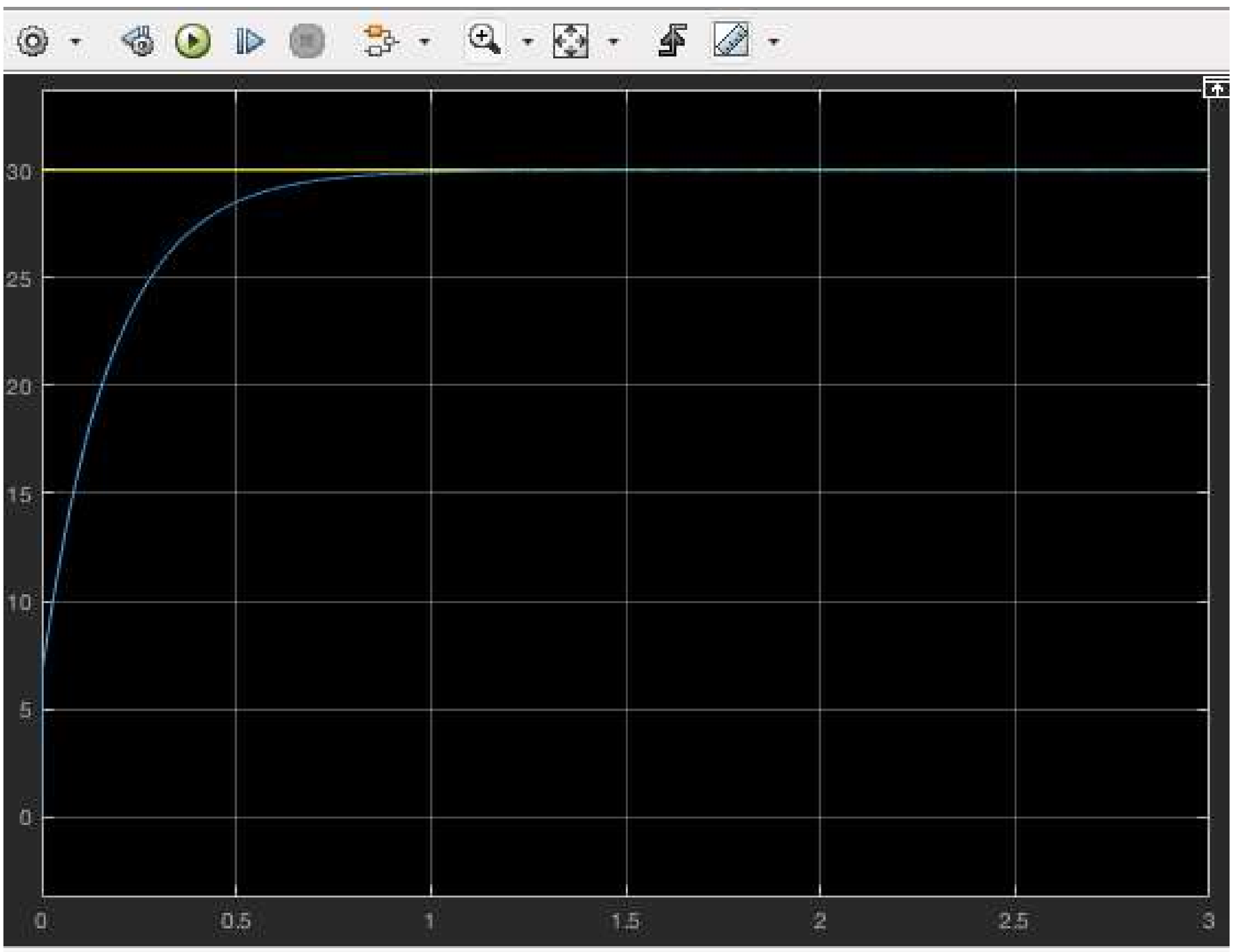


Figure 8 output for pi controller using sumulink

Since our motor works at 12 v max the input must not pass this value when we use pid controller this value reach to 34 v and this make sum problems but in pi we have better output with max input voltage (u)=12 V, see figure (11).

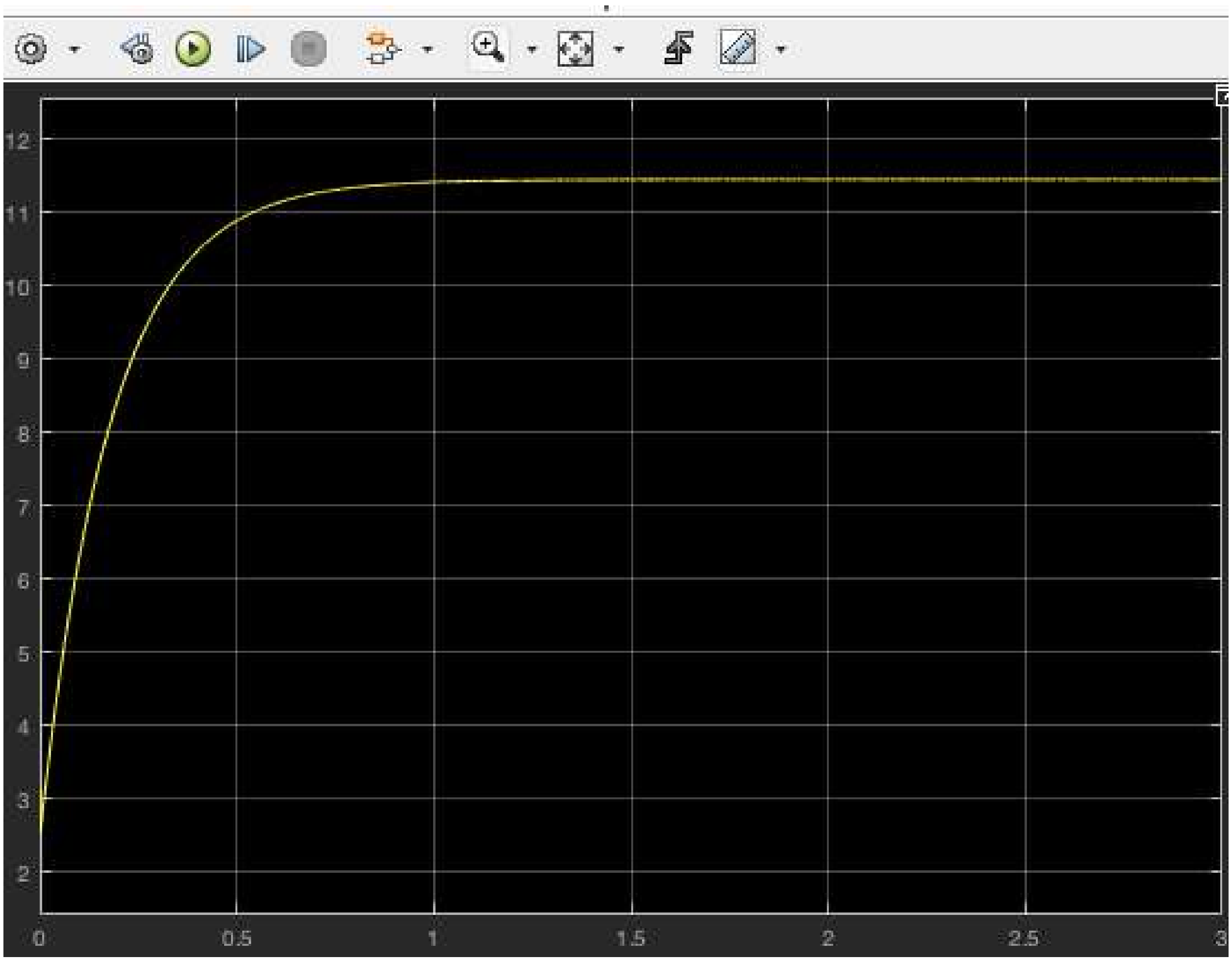


Figure 9 the value of input voltage when we input max speed (30 rad/s)

Pd:

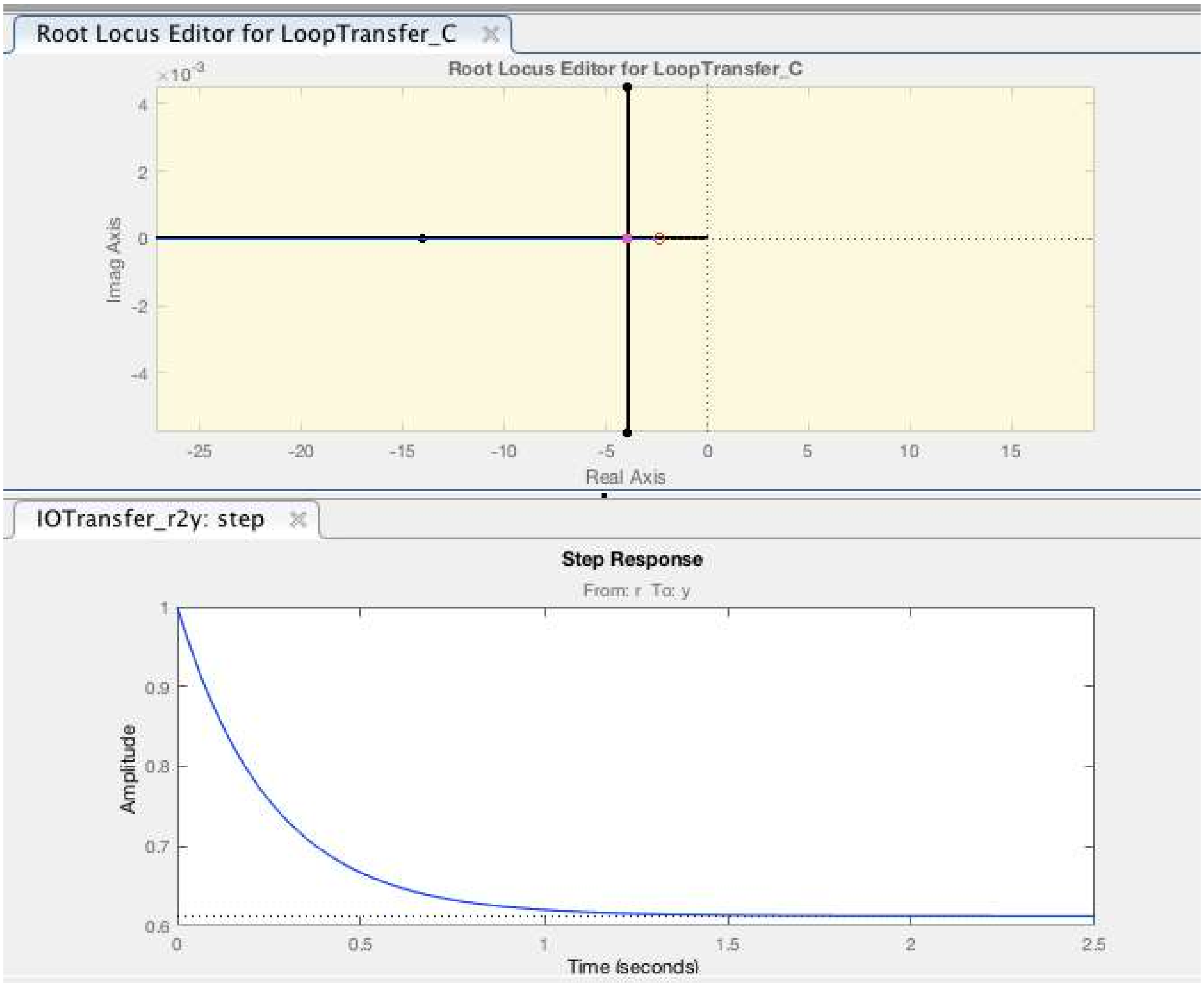


Figure 10 pd response

In PD controller the we still have the steady state error.

I controller:

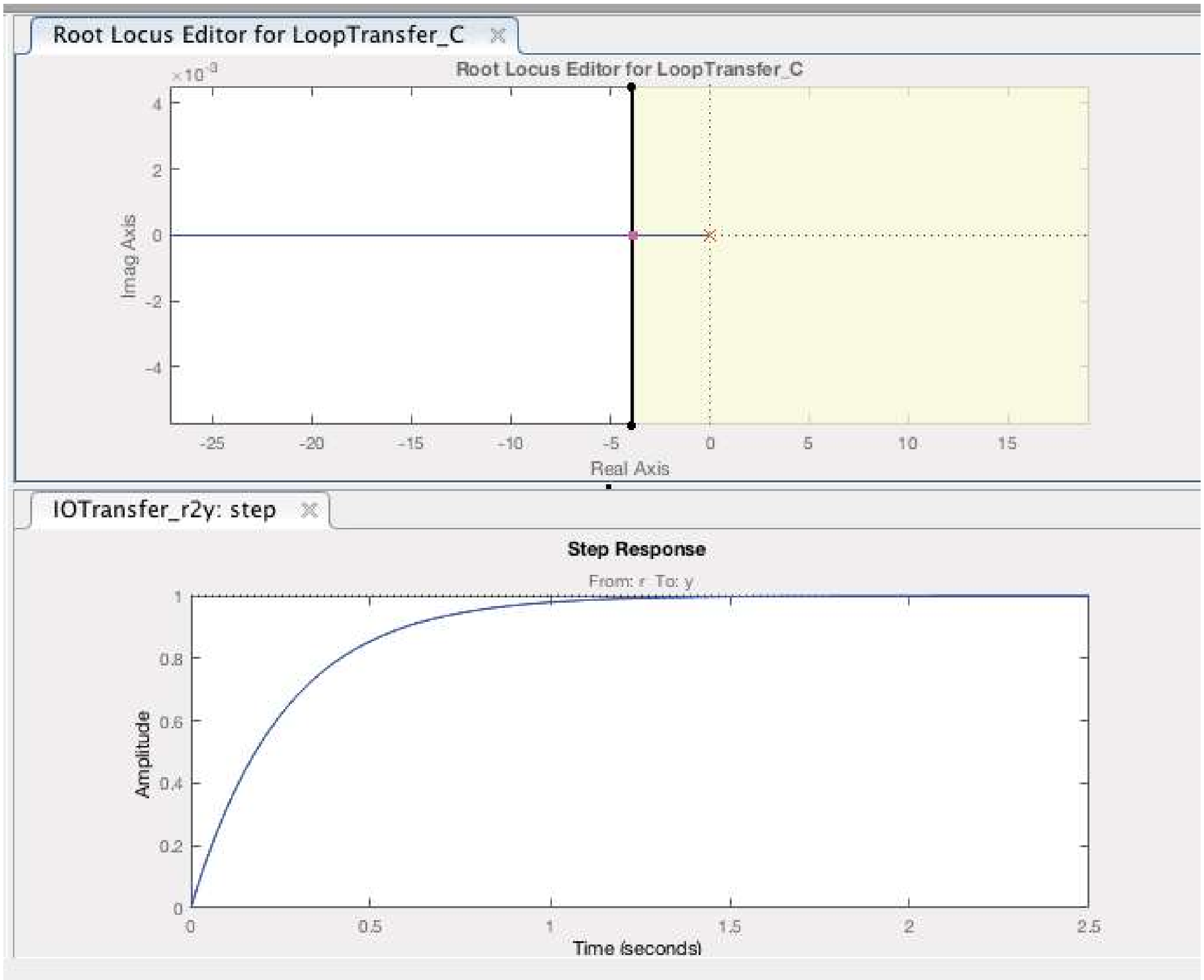


Figure 11 I controller design

Figure 12

In I controller the system has all design requirement, when KI=1.4727

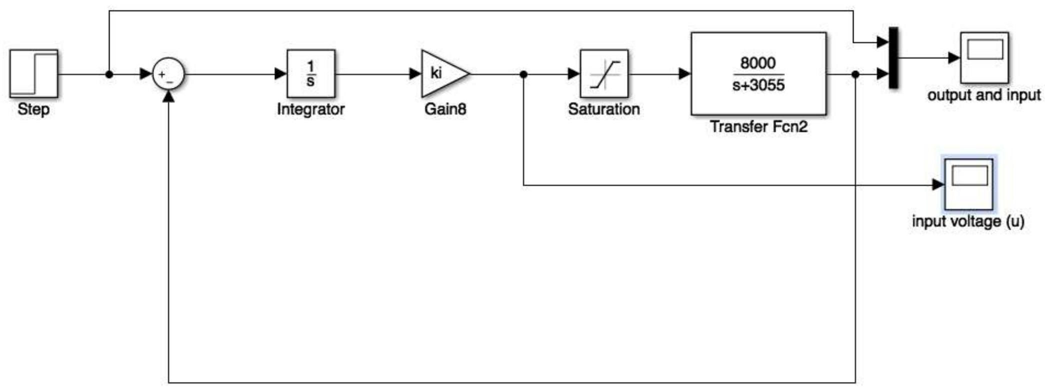


Figure 13 I controller Simulink

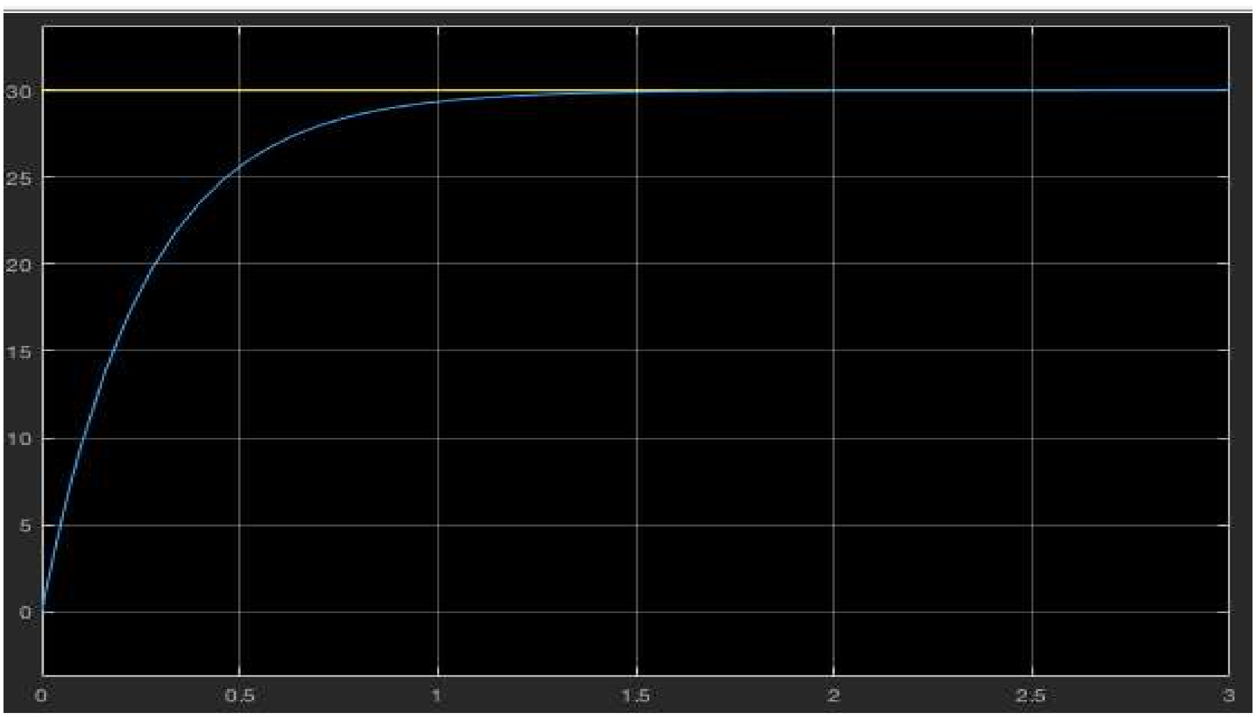


Figure 14 response for I controller (speed if motor)

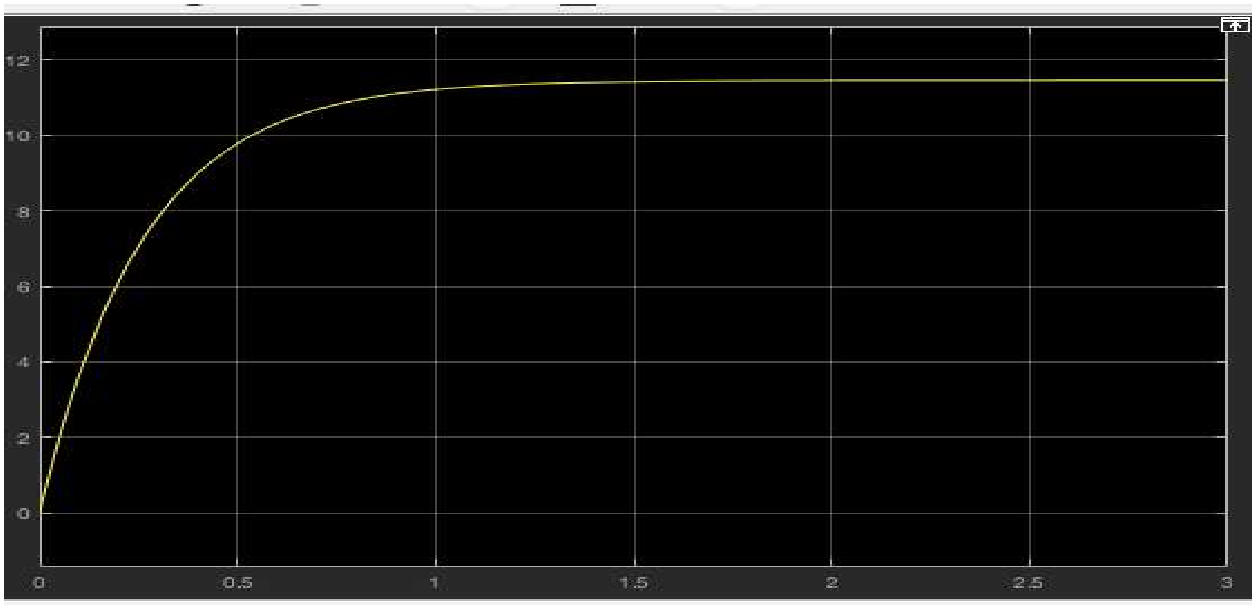


Figure 15 max input voltage for I controller

5.Sampling time (Ts):

We design for 0% over shot and 1 s settling time

So zeta =1

And Tsetling=$\frac{4}{z*wn}$

So $wn = 4\frac{rad}{s}$

$$f = \frac{4}{2*3.14} = 0.6369\ Hz$$

$$Tpieriod = \frac{1}{f} = \frac{1}{0.6369} = 1.57\ s$$

$$Tallw = \frac{1}{2}*1.57 = 0.785s$$

$$TS = \frac{1}{10}*0.785 = 0.0785\ \text{s}$$

6_discreet controller

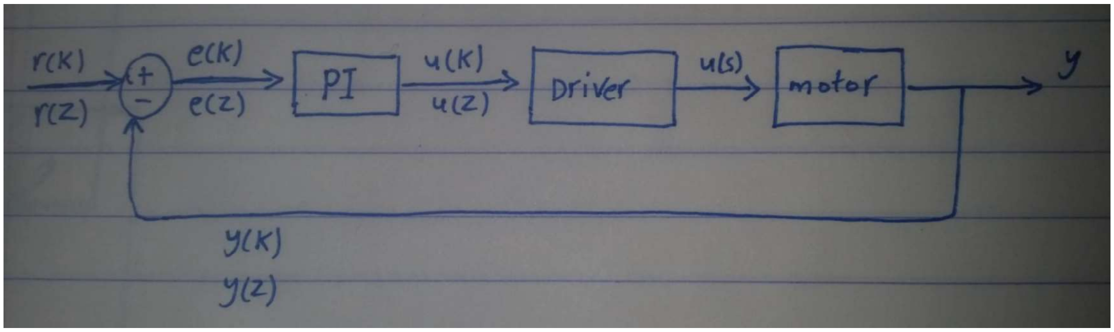


Figure 16

- PI controller:

$$PI(s) = \mathrm{Kp} + \frac{Ki}{S}$$

$$PI(z) = \frac{u(z)}{e(z)}$$

$$PI(z) = \frac{u(z)}{e(z)} = \mathrm{Kp} + \frac{Ki * Ts}{2} * (\frac{1 + z^{-1}}{1 - z^{-1}})$$

$$u(z) * [2 * (1 - z^{-1})] = e(z) * [2 * Kp + Ki * (Ts) * (1 + z^{-1})]$$

$$u(z) = u(z-1) + \left[Kp * \big(e(z) - e(z-1)\big) + \frac{Ki}{2} * (Ts) * (e(z) + e(z-1))\right]$$

Substitute Kp=0.004 , Ki=2.6928 , Ts=0.0785 :

$$u(z) = u(z-1) + \left[0.004 * \big(e(z) - e(z-1)\big) + 0.1057 * (e(z) + e(z-1))\right]$$

$$u(k) = \sum_{k=1}^{\infty} \{u(k-1) + \left[0.004 * \big(e(k) - e(k-1)\big) + 0.1057 * \big(e(k) + e(k-1)\big)\right]\}$$

- I controller:

$$I(z) = \frac{u(z)}{e(z)} = \frac{Ki * Ts}{2} * (\frac{1 + z^{-1}}{1 - z^{-1}})$$

$$u(z) * [2 * (1 - z^{-1})] = e(z) * [Ki * (Ts) * (1 + z^{-1})]$$

$$u(z) = u(z - 1) + \left[\frac{Ki}{2} * (Ts) * (e(z) + e(z - 1))\right]$$

ki=1.4727

$$u(z) = u(z - 1) + [0.0578 * (e(z) + e(z - 1))]$$

$$u(k) = \sum_{k=1}^{\infty} \{u(k - 1) + \left[0.0578 * \left(e(k) + e(k - 1)\right)\right]\}$$

OUTPUT IN REAL TEST USING EXCEL

We connect the motor and use the serial monitor to read the error, and the speed of motor and plot this values using excel, in figure (17), you see the error, and in figure (18), the speed of motor when input is 3 rad/s.

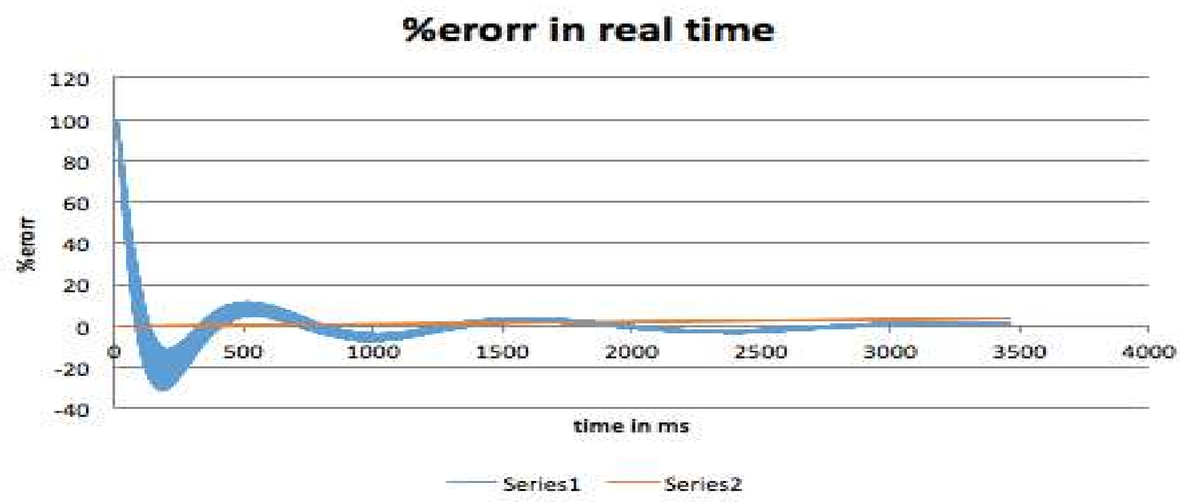


Figure 17 %error

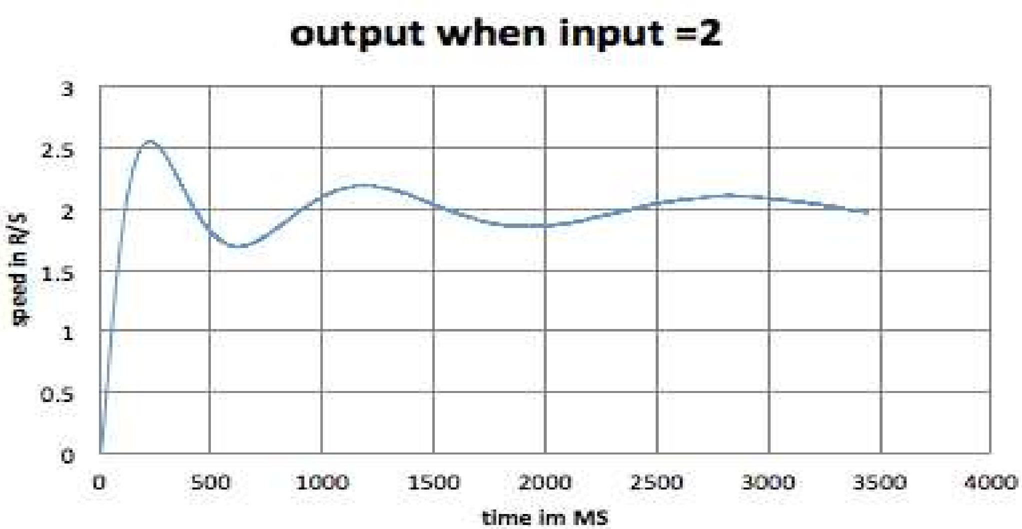


Figure 18 speed of motor when input 3 rad/s