# Transformation with matrix exercises

Takuma Yamamiya

2018//04/23

## 1 Exercises 1-1

Point(p,q) (p × q ≠ 0) is transformed on itself by matrix M.

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$M \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} pa + qb \\ pc + qd \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}$$

If pq ≠ 0,

$$b = \frac{(1-a)p}{q}$$

$$d = \frac{q - cp}{q}$$

If p=0, q ≠ 0, b=0,d=1.
If q=0, p ≠ 0, a=0,c=1.

## 2 Exercises 1-2

Point(2,3) is transformed to (4,6) by matrix M.
Where is point(4,6) transformed to by M?

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$M \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2a + 3b \\ 2c + 3d \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

$$M \begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 4a + 6b \\ 4c + 6d \end{pmatrix} = 2 \begin{pmatrix} 2a + 3b \\ 2c + 3d \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \end{pmatrix}$$

## 3 Exercise 1-3

All points except (0,0) is not transformed on itself by matrix M.

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

When $p = 0, q = q_1$ $(q_1 \neq 0)$

$$M \begin{pmatrix} 0 \\ q_1 \end{pmatrix} = \begin{pmatrix} bq_1 \\ dq_1 \end{pmatrix}$$

$$b \neq 0, d \neq 1 \tag{1}$$

In the same way, when $p = p_1(p_1 \neq 0), q = 0$,

$$M \begin{pmatrix} p_1 \\ 0 \end{pmatrix} = \begin{pmatrix} ap_1 \\ cp_1 \end{pmatrix}$$

$$a \neq 1, c \neq 0 \tag{2}$$

p*q $\neq$ 0, two formula,

$$b = \frac{(1-a)p}{q}$$
$$d = \frac{q-cp}{q}$$

is not satisfied at the same time. (Exercise1-1)
When$(p,q) = (p_0, \frac{1-a}{b}p_0)$ $(satisfying(1))$,

$$d \neq \frac{q-cp}{q} = 1 - \frac{bc}{1-a}$$

$$ad - bc - a - d + 1 \neq 0 \tag{3}$$

M satisfy (1),(2),(3).

## 4 Exercise 2-1 to 2-4

Answered on R file

## 5 Exercise 2-5

$$M = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Eigenvector $\mathbf{v}$ satisfy

$$M\mathbf{v} = \lambda\mathbf{v}$$

When $\mathbf{v} = (v_1, v_2)$,

$$M \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \lambda \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} v_1 + 2v_2 \\ 3v_1 + 4v_2 \end{pmatrix}$$

Solve two equation

$$\lambda = \frac{5 \pm \sqrt{33}}{2}$$

$$v_2/v_1 = \frac{\lambda - 1}{2} = \frac{3 \pm \sqrt{33}}{4}$$

# 6 Exercise 2-6

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Eigenvector $\mathbf{v}$ satisfy

$$M\mathbf{v} = \lambda\mathbf{v}$$

When $\mathbf{v} = (v_1, v_2)$,

$$M \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \lambda \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} av_1 + bv_2 \\ cv_1 + dv_2 \end{pmatrix}$$

Solve two equation

$$\lambda = \frac{(a+d) \pm \sqrt{(a+d)^2 - 4(ad - bc)}}{2}$$

# 7 Exercise 2-7

When $\lambda$ is complex number, a=b=1

$$(1+d)^2 - 4(d-c) < 0$$

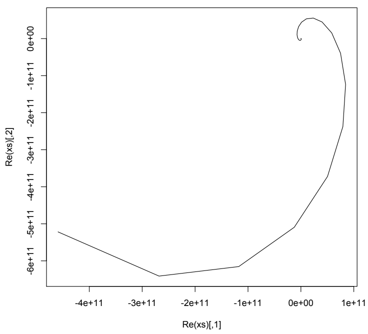
$$(d-1)^2 + 4c < 0$$

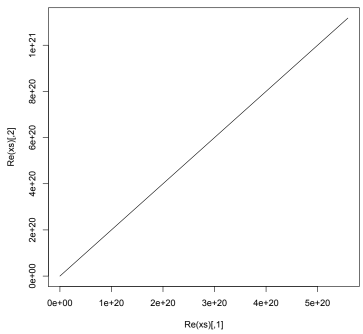
For example, when c is -5,d is 1, the equation above is satisfied.

$$\lambda = 1 \pm \sqrt{5}i$$

# 8 Exercise 2-8

I plotted $M^t x$ with M which has 2 complex eigen numbers(former figure below) , and whose eigen numbers are 0 and one real number(latter figure below), but I can't plot by M which has 2 real eigen numbers.
Why? I also submit my R code. I'm happy to tell me why.




# 9 Exercise 3-1

$$M = \begin{pmatrix} 2 & 3 & -7 \\ 3 & 4 & -9 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$M \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix}$$

$$M \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix}$$