EXAMPLE F.1-1A W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, CONTINUOUSLY BRACED

Given:

Select a W-shape beam for span and uniform dead and live loads as shown in Figure F.1-1A. Limit the member to a maximum nominal depth of 18 in. Limit the live load deflection to *L*/360. The beam is simply supported and continuously braced. The beam is ASTM A992 material.

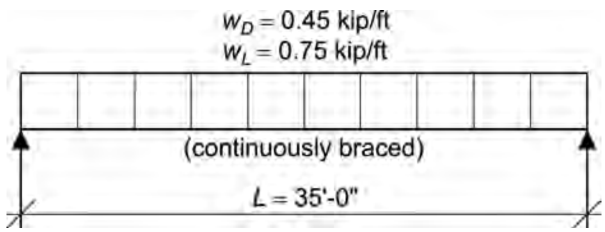

Fig. F.1-1A. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$\begin{aligned} w_u &= 1.2(0.45\text{ kip/ft}) + 1.6(0.75\text{ kip/ft}) \\ &= 1.74\text{ kip/ft} \end{aligned}$ From AISC *Manual* Table 3-23, Case 1: $\begin{aligned} M_u &= \frac{w_u L^2}{8} \\ &= \frac{(1.74\text{ kip/ft})(35\text{ ft})^2}{8} \\ &= 266\text{ kip-ft} \end{aligned}$	$\begin{aligned} w_a &= 0.45\text{ kip/ft} + 0.75\text{ kip/ft} \\ &= 1.20\text{ kip/ft} \end{aligned}$ From AISC *Manual* Table 3-23, Case 1: $\begin{aligned} M_a &= \frac{w_a L^2}{8} \\ &= \frac{(1.20\text{ kip/ft})(35\text{ ft})^2}{8} \\ &= 184\text{ kip-ft} \end{aligned}$

Required Moment of Inertia for Live-Load Deflection Criterion of L/360

$$\begin{aligned} \Delta_{max} &= \frac{L}{360} \\ &= \frac{(35\text{ ft})(12\text{ in./ft})}{360} \\ &= 1.17\text{ in.} \end{aligned}$$

$$I_{x(reqd)} = \frac{5w_L L^4}{384E\Delta_{max}}$$ (from AISC *Manual* Table 3-23, Case 1)

$$= \frac{5(0.75\text{ kip/ft})(35\text{ ft})^4(12\text{ in./ft})^3}{384(29{,}000\text{ ksi})(1.17\text{ in.})}$$

$$= 746\text{ in.}^4$$

Beam Selection

Select a W18×50 from AISC *Manual* Table 3-3.

$I_x = 800\text{ in.}^4 > 746\text{ in.}^4$ **o.k.**

Per the User Note in AISC *Specification* Section F2, the section is compact. Because the beam is continuously braced and compact, only the yielding limit state applies.

From AISC *Manual* Table 3-2, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = \phi_b M_{px}$ $= 379\text{ kip-ft} > 266\text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = \dfrac{M_{px}}{\Omega_b}$ $= 252\text{ kip-ft} > 184\text{ kip-ft}$ **o.k.**

EXAMPLE F.1-1B W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, CONTINUOUSLY BRACED

Given:

Verify the available flexural strength of the ASTM A992 W18×50 beam selected in Example F.1-1A by directly applying the requirements of the AISC *Specification*.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
$Z_x = 101$ in.^{3}

The required flexural strength from Example F.1-1A is:

LRFD	ASD
$M_u = 266$ kip-ft	$M_a = 184$ kip-ft

Nominal Flexural Strength

Per the User Note in AISC *Specification* Section F2, the section is compact. Because the beam is continuously braced and compact, only the yielding limit state applies.

$$M_n = M_p = F_y Z_x \qquad \text{(\textit{Spec}. Eq. F2-1)}$$
$$= (50 \text{ ksi})(101 \text{ in.}^3)$$
$$= 5{,}050 \text{ kip-in. or } 421 \text{ kip-ft}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(421 \text{ kip-ft})$ $= 379 \text{ kip-ft} > 266 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{421 \text{ kip-ft}}{1.67}$ $= 252 \text{ kip-ft} > 184 \text{ kip-ft}$ **o.k.**

EXAMPLE F.1-2A W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, BRACED AT THIRD POINTS

Given:

Use the AISC *Manual* tables to verify the available flexural strength of the W18×50 beam size selected in Example F.1-1A for span and uniform dead and live loads as shown in Figure F.1-2A. The beam is simply supported and braced at the ends and third points. The beam is ASTM A992 material.

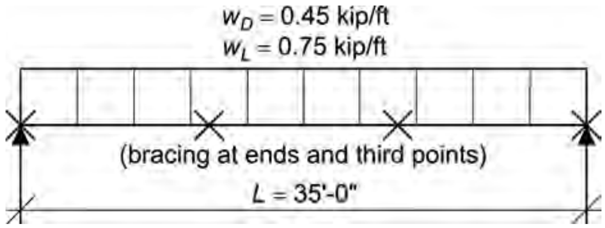

Fig. F.1-2A. Beam loading and bracing diagram.

Solution:

The required flexural strength at midspan from Example F.1-1A is:

LRFD	ASD
$M_u = 266$ kip-ft	$M_a = 184$ kip-ft

Unbraced Length

$$L_b = \frac{35 \text{ ft}}{3} = 11.7 \text{ ft}$$

By inspection, the middle segment will govern. From AISC *Manual* Table 3-1, for a uniformly loaded beam braced at the ends and third points, C_b = 1.01 in the middle segment. Conservatively neglect this small adjustment in this case.

Available Flexural Strength

Enter AISC *Manual* Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 11.7 ft. Obtain the available strength from the appropriate vertical scale to the left.

From AISC *Manual* Table 3-10, the available flexural strength is:

LRFD	ASD
$\phi_b M_n \approx 302 \text{ kip-ft} > 266 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} \approx 201 \text{ kip-ft} > 184 \text{ kip-ft}$ **o.k.**

EXAMPLE F.1-2B W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, BRACED AT THIRD POINTS

Given:

Verify the available flexural strength of the W18×50 beam selected in Example F.1-1A with the beam braced at the ends and third points by directly applying the requirements of the AISC *Specification*. The beam is ASTM A992 material.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
r_y = 1.65 in.
S_x = 88.9 in.^{3}
J = 1.24 in.^{4}
r_{ts} = 1.98 in.
h_o = 17.4 in.

The required flexural strength from Example F.1-1A is:

LRFD	ASD
$M_u = 266$ kip-ft	$M_a = 184$ kip-ft

Nominal Flexural Strength

Calculate C_b. For the lateral-torsional buckling limit state, the nonuniform moment modification factor can be calculated using AISC *Specification* Equation F1-1. For the center segment of the beam, the required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: M_{max} = 1.00, M_A = 0.972, M_B = 1.00, and M_C = 0.972.

$$\begin{aligned} C_b &= \frac{12.5M_{max}}{2.5M_{max}+3M_A+4M_B+3M_C} \\ &= \frac{12.5(1.00)}{2.5(1.00)+3(0.972)+4(1.00)+3(0.972)} \\ &= 1.01 \end{aligned}$$ (*Spec*. Eq. F1-1)

For the end-span beam segments, the required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: M_{max} = 0.889, M_A = 0.306, M_B = 0.556, and M_C = 0.750.

$$\begin{aligned} C_b &= \frac{12.5M_{max}}{2.5M_{max}+3M_A+4M_B+3M_C} \\ &= \frac{12.5(0.889)}{2.5(0.889)+3(0.306)+4(0.556)+3(0.750)} \\ &= 1.46 \end{aligned}$$ (*Spec*. Eq. F1-1)

Thus, the center span, with the higher required strength and lower C_b, will govern.

The limiting laterally unbraced length for the limit state of yielding is:

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} \qquad (Spec.\text{ Eq. F2-5})$$

$$= 1.76(1.65\text{ in.})\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$

$$= 69.9\text{ in. or } 5.83\text{ ft}$$

The limiting unbraced length for the limit state of inelastic lateral-torsional buckling, with c = 1 from AISC *Specification* Equation F2-8a for doubly symmetric I-shaped members, is:

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_o}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}} \qquad (Spec.\text{ Eq. F2-6})$$

$$= 1.95(1.98\text{ in.})\left[\frac{29{,}000\text{ ksi}}{0.7(50\text{ ksi})}\right]\sqrt{\frac{(1.24\text{ in.}^4)(1.0)}{(88.9\text{ in.}^3)(17.4\text{ in.})} + \sqrt{\left[\frac{(1.24\text{ in.}^4)(1.0)}{(88.9\text{ in.}^3)(17.4\text{ in.})}\right]^2 + 6.76\left[\frac{0.7(50\text{ ksi})}{29{,}000\text{ ksi}}\right]^2}}$$

$$= 203\text{ in. or } 16.9\text{ ft}$$

For a compact beam with an unbraced length of $L_p < L_b \le L_r$, the lesser of either the flexural yielding limit state or the inelastic lateral-torsional buckling limit state controls the nominal strength.

M_p = 5,050 kip-in. (from Example F.1-1B)

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad (Spec.\text{ Eq. F2-2})$$

$$= 1.01\left\{5{,}050\text{ kip-in.} - \left[5{,}050\text{ kip-in.} - 0.7(50\text{ ksi})(88.9\text{ in.}^3)\right]\left(\frac{11.7\text{ ft} - 5.83\text{ ft}}{16.9\text{ ft} - 5.83\text{ ft}}\right)\right\} \le 5{,}050\text{ kip-in.}$$

$$= 4{,}060\text{ kip-in.} < 5{,}050\text{ kip-in.}$$

$$= 4{,}060\text{ kip-in. or } 339\text{ kip-ft}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(339\text{ kip-ft})$ $= 305\text{ kip-ft} > 266\text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{339\text{ kip-ft}}{1.67}$ $= 203\text{ kip-ft} > 184\text{ kip-ft}$ **o.k.**

EXAMPLE F.1-3A W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, BRACED AT MIDSPAN

Given:

Use the AISC *Manual* tables to verify the available flexural strength of the W18×50 beam size selected in Example F.1-1A for span and uniform dead and live loads as shown in Figure F.1-3A. The beam is simply supported and braced at the ends and midpoint. The beam is ASTM A992 material.

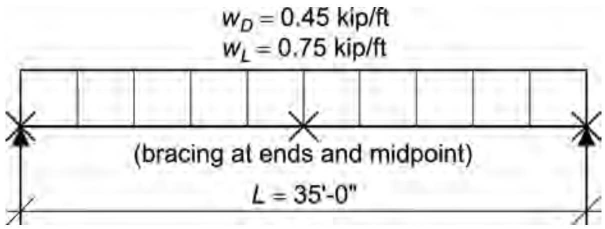

Fig. F.1-3A. Beam loading and bracing diagram.

Solution:

The required flexural strength at midspan from Example F.1-1A is:

LRFD	ASD
$M_u = 266$ kip-ft	$M_a = 184$ kip-ft

Unbraced Length

$$L_b = \frac{35 \text{ ft}}{2} = 17.5 \text{ ft}$$

From AISC *Manual* Table 3-1, for a uniformly loaded beam braced at the ends and at the center point, C_b = 1.30. There are several ways to make adjustments to AISC *Manual* Table 3-10 to account for C_b greater than 1.0.

Procedure A

Available moments from the sloped and curved portions of the plots from AISC *Manual* Table 3-10 may be multiplied by C_b, but may not exceed the value of the horizontal portion (ϕM_p for LRFD, M_p/Ω for ASD).

Obtain the available strength of a W18×50 with an unbraced length of 17.5 ft from AISC *Manual* Table 3-10.

Enter AISC *Manual* Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 17.5 ft. Obtain the available strength from the appropriate vertical scale to the left.

LRFD	ASD
$\phi_b M_n \approx 222$ kip-ft From AISC *Manual* Table 3-2: $\phi_b M_p = 379$ kip-ft (upper limit on $C_b \phi_b M_n$)	$\dfrac{M_n}{\Omega_b} \approx 148$ kip-ft From AISC *Manual* Table 3-2: $\dfrac{M_p}{\Omega_b} = 252$ kip-ft (upper limit on $C_b \dfrac{M_n}{\Omega_b}$)

LRFD	ASD
Adjust for C_b.	Adjust for C_b.
$1.30(222\text{ kip-ft}) = 289\text{ kip-ft}$	$1.30(148\text{ kip-ft}) = 192\text{ kip-ft}$
Check limit.	Check limit.
$289\text{ kip-ft} < \phi_b M_p = 379\text{ kip-ft}$ **o.k.**	$192\text{ kip-ft} < \dfrac{M_p}{\Omega_b} = 252\text{ kip-ft}$ **o.k.**
Check available versus required strength.	Check available versus required strength.
$289\text{ kip-ft} > 266\text{ kip-ft}$ **o.k.**	$192\text{ kip-ft} > 184\text{ kip-ft}$ **o.k.**

Procedure B

For preliminary selection, the required strength can be divided by C_b and directly compared to the strengths in AISC *Manual* Table 3-10. Members selected in this way must be checked to ensure that the required strength does not exceed the available plastic moment strength of the section.

Calculate the adjusted required strength.

LRFD	ASD
$M_u' = \dfrac{266\text{ kip-ft}}{1.30}$ $= 205\text{ kip-ft}$	$M_a' = \dfrac{184\text{ kip-ft}}{1.30}$ $= 142\text{ kip-ft}$

Obtain the available strength for a W18×50 with an unbraced length of 17.5 ft from AISC *Manual* Table 3-10.

LRFD	ASD
$\phi_b M_n \approx 222\text{ kip-ft} > 205\text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} \approx 148\text{ kip-ft} > 142\text{ kip-ft}$ **o.k.**
$\phi_b M_p = 379\text{ kip-ft} > 266\text{ kip-ft}$ **o.k.**	$\dfrac{M_p}{\Omega_b} = 252\text{ kip-ft} > 184\text{ kip-ft}$ **o.k.**

EXAMPLE F.1-3B W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR-AXIS BENDING, BRACED AT MIDSPAN

Given:

Verify the available flexural strength of the W18×50 beam selected in Example F.1-1A with the beam braced at the ends and center point by directly applying the requirements of the AISC *Specification*. The beam is ASTM A992 material.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
r_{ts} = 1.98 in.
S_x = 88.9 in.^{3}
J = 1.24 in.^{4}
h_o = 17.4 in.

The required flexural strength from Example F.1-1A is:

LRFD	ASD
$M_u = 266$ kip-ft	$M_a = 184$ kip-ft

Nominal Flexural Strength

Calculate C_b. The required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: M_{max} = 1.00, M_A = 0.438, M_B = 0.750, and M_C = 0.938.

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \quad \text{(}Spec\text{. Eq. F1-1)}$$

$$= \frac{12.5(1.00)}{2.5(1.00) + 3(0.438) + 4(0.750) + 3(0.938)}$$

$$= 1.30$$

From AISC *Manual* Table 3-2:

L_p = 5.83 ft
L_r = 16.9 ft

From Example F.1-3A:

L_b = 17.5 ft

For a compact beam with an unbraced length $L_b > L_r$, the limit state of elastic lateral-torsional buckling applies.

Calculate F_{cr}, where c = 1.0 for doubly symmetric I-shapes.

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2} \qquad \text{(\textit{Spec}. Eq. F2-4)}$$

$$= \frac{1.30\pi^2 (29{,}000 \text{ ksi})}{\left[\frac{(17.5 \text{ ft})(12 \text{ in./ft})}{1.98 \text{ in.}}\right]^2} \sqrt{1 + 0.078 \frac{(1.24 \text{ in.}^4)(1.0)}{(88.9 \text{ in.}^3)(17.4 \text{ in.})} \left[\frac{(17.5 \text{ ft})(12 \text{ in./ft})}{1.98 \text{ in.}}\right]^2}$$

$$= 43.2 \text{ ksi}$$

$$M_p = 5{,}050 \text{ kip-in. (from Example F.1-1B)}$$

$$M_n = F_{cr} S_x \le M_p \qquad \text{(\textit{Spec}. Eq. F2-3)}$$

$$= (43.2 \text{ ksi})(88.9 \text{ in.}^3) \le 5{,}050 \text{ kip-in.}$$

$$= 3{,}840 \text{ kip-in.} < 5{,}050 \text{ kip-in.}$$

$$= 3{,}840 \text{ kip-in. or } 320 \text{ kip-ft}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(320 \text{ kip-ft})$ $= 288 \text{ kip-ft} > 266 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{320 \text{ kip-ft}}{1.67}$ $= 192 \text{ kip-ft} > 184 \text{ kip-ft}$ **o.k.**