EXAMPLE 14.1

Determine the structure stiffness matrix for the two-member truss shown in Fig. 14–7*a*. AE is constant.

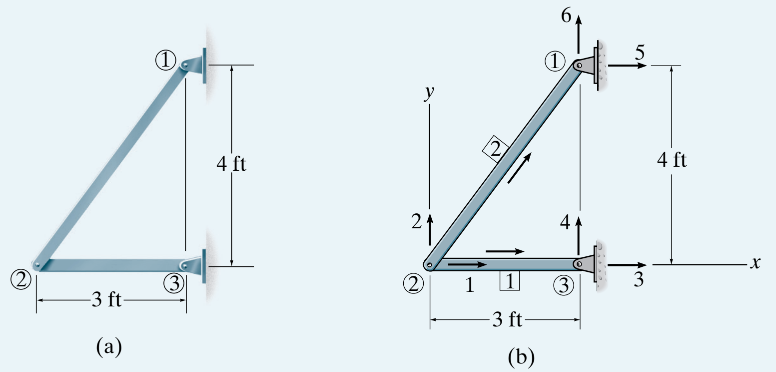

Fig. 14–7

SOLUTION

By inspection, ② will have two unknown displacement components, whereas joints ① and ③ are constrained from displacement. Consequently, the displacement components at joint ② are code numbered first, followed by those at joints ③ and ①, Fig. 14–7*b*. The origin of the global coordinate system can be located at any point. For convenience, we will choose joint ② as shown. The members are identified arbitrarily and arrows are written along the two members to identify the near and far ends of each member. The direction cosines and the stiffness matrix for each member can now be determined.

Member 1. Since ② is the near end and ③ is the far end, then by Eqs. 14–5 and 14–6, we have

$$\lambda_x = \frac{3-0}{3} = 1 \qquad \lambda_y = \frac{0-0}{3} = 0$$

Using Eq. 14–16, dividing each element by $L = 3$ ft, we have

$$\mathbf{k}_1 = AE \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.333 & 0 & -0.333 & 0 \\ 0 & 0 & 0 & 0 \\ -0.333 & 0 & 0.333 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$

The calculations can be checked in part by noting that $\mathbf{k}_1$ is *symmetric.* Note that the rows and columns in $\mathbf{k}_1$ are identified by the x, y degrees of freedom at the near end, followed by the far end, that is, 1, 2, 3, 4, respectively, for member 1, Fig. 14–7*b*. This is done in order to identify the elements for later assembly into the $\mathbf{K}$ matrix.

Member 2. Since ② is the near end and ① is the far end, we have

$$\lambda_x = \frac{3-0}{5} = 0.6 \qquad \lambda_y = \frac{4-0}{5} = 0.8$$

Thus Eq. 14–16 with $L = 5$ ft becomes

$$\mathbf{k}_2 = AE \begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \left[\begin{array}{cccc} 0.072 & 0.096 & -0.072 & -0.096 \\ 0.096 & 0.128 & -0.096 & -0.128 \\ -0.072 & -0.096 & 0.072 & 0.096 \\ -0.096 & -0.128 & 0.096 & 0.128 \end{array}\right] \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 6 \end{array}$$

Here the rows and columns are identified as 1, 2, 5, 6, since these numbers represent, respectively, the x, y degrees of freedom at the near and far ends of member 2.

Structure Stiffness Matrix. This matrix has an order of 6×6 since there are six designated degrees of freedom for the truss, Fig. 14–7b. Corresponding elements of the above two matrices are added algebraically to form the structure stiffness matrix. Perhaps the assembly process is easier to see if the missing numerical columns and rows in $\mathbf{k}_1$ and $\mathbf{k}_2$ are expanded with zeros to form two 6×6 matrices. Then

$$\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2$$

$$\mathbf{K} = AE \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{cccccc} 0.333 & 0 & -0.333 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.333 & 0 & 0.333 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} + AE \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{cccccc} 0.072 & 0.096 & 0 & 0 & -0.072 & -0.096 \\ 0.096 & 0.128 & 0 & 0 & -0.096 & -0.128 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.072 & -0.096 & 0 & 0 & 0.072 & 0.096 \\ -0.096 & -0.128 & 0 & 0 & 0.096 & 0.128 \end{array}\right] \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

$$\mathbf{K} = AE \left[\begin{array}{cccccc} 0.405 & 0.096 & -0.333 & 0 & -0.072 & -0.096 \\ 0.096 & 0.128 & 0 & 0 & -0.096 & -0.128 \\ -0.333 & 0 & 0.333 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.072 & -0.096 & 0 & 0 & 0.072 & 0.096 \\ -0.096 & -0.128 & 0 & 0 & 0.096 & 0.128 \end{array}\right]$$

If a computer is used for this operation, generally one starts with $\mathbf{K}$ having all zero elements; then as the member global stiffness matrices are generated, they are placed directly into their respective element positions in the $\mathbf{K}$ matrix, rather than developing the member stiffness matrices, storing them, then assembling them.