

Exercise 9.1

We're given the following functions and asked about some of their derivatives.

$$F(x, y) = x^2y^3$$

$$G(x, y) = (F(x, y), y) = (x^2y^3, y)$$

$$H(x, y) = F(F(x, y), y) = F(x^2y^3, y) = (x^2y^3)^2y^3 = x^4y^9$$

In the calculus that follows, you'll notice that I map back to Leibniz's notation. While this feels like it goes against the spirit of the book, it's helpful until you're comfortable with functional notation.

9.1 (a)

$$\partial_o F(x, y) = \frac{\partial}{\partial x} x^2y^3 = 2xy^3$$

$$\partial_1 F(x, y) = \frac{\partial}{\partial y} x^2y^3 = 3x^2y^2$$

9.1 (b)

$$\partial_o F(F(x, y), y) = \partial_o H(x, y) = \frac{\partial}{\partial x} x^4y^9 = 4x^3y^9$$

$$\partial_1 F(F(x, y), y) = \partial_1 H(x, y) = \frac{\partial}{\partial y} x^4y^9 = 9x^4y^8$$

9.1 (c)

$$\partial_o G(x, y) = \frac{\partial}{\partial x} (x^2y^3, y) = (2xy^3, 0)$$

$$\partial_1 G(x, y) = \frac{\partial}{\partial y} (x^2y^3, y) = (3x^2y^2, 1)$$

Notice that unlike F and H which return numbers, G returns a tuple containing F . So its partial derivatives are also tuples, with one set of derivatives matching those computed for F in 9.1 (a).

9.1 (d)

$$DF(a, b) = [2ab^3, 3a^2b^2]$$

$$DG(3, 5) = [(2 \cdot 3 \cdot 5^3, 0), (3 \cdot 3^2 \cdot 5, 1)] = [(750, 0), (675, 1)]$$

$$DH(3a^2, 5b^3) = [4(3a^2)^3(5b^3)^9, 9(3a^2)^4(5b^3)^8] = [210937500a^6b^{27}, 284765625a^8b^{24}]$$