Exercise 1.1

For each of the mechanical systems described below, give the number of degrees of freedom of the configuration space.

1.1 (a)

Three juggling pins.

Rigid bodies generally have 6 degrees of freedom: three parameters to specify their position in space and three to specify their orientation.

Since we have three pins, the number of degrees of freedom is $6 \times 3 = \boxed{18}$.

1.1 (b)

A spherical pendulum, consisting of a point mass (the pendulum bob) hanging from a rigid massless rod attached to a fixed support point. The pendulum bob may move in any direction subject to the constraint imposed by the rigid rod. The point mass is subject to the uniform force of gravity.

A point mass generally has three degrees of freedom, corresponding to its position in space. The rod acts as a constraint, forcing it to move on a spherical surface. Therefore, the number of degrees of freedom are $3 - 1 = \boxed{2}$.

These correspond to the two angles you need to know (θ (polar) and ϕ (azimuth)) to locate the pendulum bob. Wikipedia has a good diagram of these angles.

1.1 (c)

A spherical double pendulum, consisting of one point mass hanging from a rigid massless rod attached to a second point mass hanging from a second massless rod attached to a fixed support point. The point masses are subject to the uniform force of gravity.

This is just the double of the previous answer, as we have two point masses and two constraints. $2 \times (3 - 1) = \boxed{4}$.

1.1 (d)

A point mass sliding without friction on a rigid curved wire.

Since we know about the shape of the curved wire, the point mass is simply moving in a 1D curve in 3D space. Therefore all we we have is $\boxed{1}$ degree of freedom, which is how far along the wire it is.

1.1 (e)

A top consisting of a rigid axisymmetric body with one point on the symmetry axis of the body attached to a fixed support, subject to a uniform gravitational force.

Since one point of the top is fixed, its translational freedom goes away leaving us with only three degrees of freedom for its orientation. Further, because it's axisymmetric, it loses one of these degrees of freedom as you cannot discern rotations about its axis of symmetry.

Therefore, this has $3 - 1 = \boxed{2}$ degrees of freedom. These correspond to the two angles needed to describe the orientation of its axis of symmetry.

1.1 (f)

The same as (e), but not axisymmetric.

Because it's not axisymmetric, we don't lose that one rotational degree of freedom (rotation about its axis of symmetry) we subtracted above. So this case has $\boxed{3}$ degrees of freedom. These correspond to two angles needed to specify the orientation of its axis of rotation, and one to specify its rotation about that axis.