Exercise 9.1

We're given the following functions and asked about some of their derivatives.

$$
\begin{aligned}
F(x,y) &= x^2y^3 \\
G(x,y) &= (F(x,y),y) = (x^2y^3,y) \\
H(x,y) &= F(F(x,y),y) = F(x^2y^3,y) = (x^2y^3)^2y^3 = x^4y^9
\end{aligned}
$$

In the calculus that follows, you'll notice that I map back to Leibniz's notation. While this feels like it goes against the spirit of the book, it's helpful until you're comfortable with functional notation.

9.1 (a)

$$
\begin{aligned}
\partial_o F(x,y) &= \frac{\partial}{\partial x}x^2y^3 = 2xy^3 \\
\partial_1 F(x,y) &= \frac{\partial}{\partial y}x^2y^3 = 3x^2y^2
\end{aligned}
$$

9.1 (b)

$$
\begin{aligned}
\partial_o F(F(x,y),y) = \partial_o H(x,y) &= \frac{\partial}{\partial x}x^4y^9 = 4x^3y^9 \\
\partial_1 F(F(x,y),y) = \partial_1 H(x,y) &= \frac{\partial}{\partial y}x^4y^9 = 9x^4y^8
\end{aligned}
$$

9.1 (c)

$$
\begin{aligned}
\partial_o G(x,y) &= \frac{\partial}{\partial x}(x^2y^3,y) = (2xy^3,0) \\
\partial_1 G(x,y) &= \frac{\partial}{\partial y}(x^2y^3,y) = (3x^2y^2,1)
\end{aligned}
$$

Notice that unlike F and H which return numbers, G returns a tuple containing F. So its partial derivatives are also tuples, with one set of derivatives matching those computed for F in 9.1 (a).

9.1 (d)

To arrive at the following, we group the partial derivatives determined previously into tuples, and then substitute the given inputs.

$$\begin{aligned} DF(a,b) &= [2ab^3, 3a^2b^2] \\ DG(3,5) &= [(2 \cdot 3 \cdot 5^3, 0), (3 \cdot 3^2 \cdot 5, 1)] \\ &= [(750, 0), (675, 1)] \\ DH(3a^2, 5b^3) &= [4(3a^2)^3(5b^3)^9, 9(3a^2)^4(5b^3)^8] \\ &= [210937500a^6b^{27}, 284765625a^8b^{24}] \end{aligned}$$