Linear programming applications to power system economics, planning and operations

Article *in* IEEE Transactions on Power Systems · September 1992

DOI: 10.1109/59.207329 · Source: IEEE Xplore

M. Shahidehpour
Illinois Institute of Technology

LINEAR PROGRAMMING APPLICATIONS TO POWER SYSTEM ECONOMICS, PLANNING AND OPERATIONS

Jerome K. Delson
Applied Analytics Associates
Palo Alto, California

S. M. Shahidehpour
Illinois Institute of Technology
Chicago, Illinois

Abstract: Linear programming is a tool that has yet to reach its full potential in power system engineering. To illustrate in a tutorial style how it is currently being applied and how its use evolved, applications are outlined in three areas: generation scheduling, loss minimization through allocation of reactive power supply, and planning of capital investments in generation equipment. The applications include not only linear programming but also its extensions to integer and quadratic programming and to the use of Benders and Dantzig-Wolfe decomposition techniques. The planning issues discussed show the limitations of traditional engineering economics to power system planning. This occurs when there is a spread between the interest rates for lending and for borrowing funds and also when investment funds have limits and thus are rationed. The result of this review is the recommendation that power system planning models should incorporate financial flows with the linear programming approach to capital budgeting originally formulated in 1963 by H. M. Weingartner. The need for such an approach is illustrated in the appendix with examples of how capital market conditions can upset the type of engineering economic decision making currently used in planning models. The Lagrangian relaxation method, which can extend computational feasibility for linear and integer programming, is also described in the appendix.

Keywords: Optimization, Linear Programming, Lagrangian Relaxation, Capital Budgeting, Engineering Economics, Power System Planning, Power System Operations, Reactive Power Supply.

I. INTRODUCTION

Linear programming is used in a variety of power system business and engineering applications, but it is still unfamiliar to many practicing engineers and its properties have not yet been fully exploited for power system planning. The paper, therefore, has two aims: to display power system engineering applications of linear programming and to indicate the potential for its future use.

Power system planning falters on the interrelation of capital budgeting and financing. A text on corporate finance claims linear programming models are "tailor-made" for solving such capital budgeting problems and asks "Why then are they not universally accepted in either theory or practice?" [1 p.103]. The reasons they give are:

- Cost of computation and for gathering data.
- Limitation of the models in dealing with uncertainty.
- Lack of need in companies where constraints are "soft" not "hard".

These factors may apply, but there is a special explanation for power system engineers. Linear programming has yet to find its way into the power engineering curriculum. (The range of its application should be sufficient to spur interest). Also, unfortunately, the engineering economics that power engineers do receive explicitly excludes the interaction of financing and capital budgeting decisions.

For some applications engineers rely or at least have available approaches based on linear programming. Three will be reviewed. The first application is the unit commitment or generation scheduling problem. It can be solved using integer programming, which is an extension of linear programming. Practical application to large scale problems has been made possible by a computational technique, Lagrangian relaxation, which is reviewed in Appendix A.

The second application is reactive power allocation, which aims at loss minimization and adjustment of voltage profile through judicious use of capacitors. This application uses decomposition to separate the operations and investment subproblems.

The third application is power system planning. Linear programming was not adequate for most applications until Benders decomposition was used. Although this has proved successful in theory and practice, the methods now in use have not fulfilled their potential. The present inadequacies stem from engineering economics, not from programming techniques. Shortcomings occur in the discounting method used to formulate the objective function. This is illustrated in Appendix B and leads to a recommendation for revising power system planning models.

II. OPERATIONS ANALYSIS

Generation Scheduling Problem

The economic importance of generation scheduling and the potential for significant savings have been recognized for decades, but perhaps current conditions make it seem even more important. Utilities that face financial difficulties may operate with slim margins and aging generators. They would be concerned with cost and with reducing stress on these machines.

Prior to the 1960's and to the application of linear programming to generator scheduling, optimization was confined to ***economic dispatch***, not the "on/off" question of unit commitment. The method used marginal generating costs, and the Lagrangian multiplier "lambda" entered the dispatch vernacular [2]. With the advent of digital computers, iteration and gradient methods were used to solve the dispatch equations. Fuel-constrained units and hydroelectric units, if present, were included in the gradient search loop, but this increased the computation burden, particularly for large-scale systems.

In the early 1960's, analytical models were implemented for unit commitment in order to replace the heuristic priority list method then in use. These models incorporated economic dispatch as a subroutine. A mixed integer optimization model was proposed [3]. This model utilized the branch-and-bound algorithm. More advanced models followed [4,5], and unit commitment for hydrothermal systems was studied.

It was in this period that dynamic programming took root as a unit commitment technique. Dynamic programming is not related to linear programming but provided a combinatorial procedure to enumerate and evaluate solutions, rejecting immediately many inferior combinations. The computation burden still expanded exponentially with the size of the problem, and for large systems the basic algorithm was impractical. To limit the computation, many heuristic strategies were used. This limited the dynamic range for the search, and in some

91 WM 176-8 PWRS A paper recommended and approved by the IEEE Power System Engineering Committee of the IEEE Power Engineering Society for presentation at the IEEE/PES 1991 Winter Meeting, New York, New York, February 3-7, 1991. Manuscript submitted August 31, 1990; made available for printing January 16, 1991.

cases priority lists were reintroduced [6-11].

More recently, linear programming was again applied to the unit commitment problem, this time using the Benders decomposition technique [12,13]. Here, the on/off portion of the unit commitment problem is handled as a master integer program and economic dispatch as a separate subproblem [14]. This framework is readily extended to scheduling of hydrothermal systems [15-17].

To handle the calculation burden that would otherwise be impractical in large scale linear systems, Lagrangian relaxation was applied to the unit commitment problem in the late 1970's. The technique is motivated by duality properties of linear programming and is outlined in Appendix A. In the problem formulated to minimize generation costs, Lagrangian relaxation provided a sharp, efficient lower bound for use in the branch-and-bound technique, but the upper bound still required repeated, time consuming solutions of the economic dispatch problem. As a compromise, Lagrangian relaxation algorithms usually terminate near an optimal solution, when the relative difference between the primal and dual objective functions is less than a prescribed tolerance [18-24].

An extension to linear programming called quadratic programming keeps constraints linear but allows quadratic terms in the objective function. This is a relatively minor complication if the objective function is convex [59]. Quadratic objective functions have been used in lieu of piece-wise linear generator cost functions and to represent costs associated with *changes* in generator output level. The programs also include ramp rate limits on generator output and are called dynamic unit commitment. Ramp rate limits link one time step to the next and can significantly change dispatch schedules [68, 69].

Linear programming also offers the potential opportunity to replace deterministic spinning reserve constraints with a probabilistic approach. This may lead to a more accurate index of system reliability but the procedure is not readily implemented [25-27].

The initial impetus for use of linear or integer programming in unit commitment was to replace less accurate heuristic methods. There is interest now to return to heuristic methods both for speed and even accuracy. The attempts include application of simulated annealing and artificial intelligence, and the use of expert systems and artificial neural networks [17,28,29]

Loss Minimization Criterion in Power Systems

Another example that illustrates this mathematical approach to analyzing power system operation is loss minimization. The analysis is subject to constraints: the combined generation and transmission system must satisfy load demands by providing a reliable supply of power, maintaining load bus voltages within their permissible limits.

The decisions regarding reactive power generation for minimizing real power losses with existing equipment are referred to as the operation subproblem. The decision concerning the cost of adding capacitor banks to the system is referred to as the investment subproblem. The combination of these two subproblems forms the reactive power allocation problem.

Within the past decade, the linear programming approach has been recognized as an effective and reliable optimization tool for this problem as it can directly enforce limits on variables and on linear functions of variables. The enforcement of these constraints had presented difficulties to more classical techniques based on the non-linear gradient or Newton's method. The drawbacks of linear programming lie in the requirement that all relations must be linear or approximated by linear functions [30-36].

Linearization of the objective function is carried out in the neighborhood of the current operating state of the system. To linearize system loss, the variation of Jacobian coefficients is limited to a small range by introducing restricted step sizes for voltage magnitude and transformer tap setting changes. In more recent approaches, the solution of the operation subproblem is based on the Dantzig-Wolfe decomposition technique, in which the primal and dual solutions of the revised simplex method are utilized. Without the decomposition technique, this application of linear programming would be impractical [36-38].

Investment planning for new reactive power sources (shunt capacitors and inductors) has been implemented by optimization methods that treat reactive sources either as divisible [39,40] or discrete [41-43] variables. Ref. [40] presented a method for large scale reactive power planning, which included contingency conditions based on a Newton-Raphson algorithm. However, unnecessary installations of new reactive power sources were the result of an inadequate criterion for the proper selection of candidate buses in the system.

Ref. [39] extended reactive power allocation to include the subproblem for investment in reactive sources. All buses with static capacitors were initially candidates for reactive power expansion, but the method of selection of a subset of buses based on this approach did not lead to the optimum solution.

Refs. [41-43] formulated reactive power planning as a mixed-integer programming problem using binary variables to determine the location of new reactive power sources. The generalized Benders decomposition method was employed to decompose the planning problem into two independent subproblems.

Ref. [42] divided the problem into an integer program with binary variables and a linear program. The solution of the integer program requires iteration on the Lagrange multipliers of dual problems, which makes the application of the algorithm rather complex.

In [43], reactive power planning is treated as a mixed-integer linear program. The optimization is approximate, but the final solution is improved further before testing its convergence. This approach overcame difficulties faced by other methods in selecting candidate buses for installation of new reactive power sources, but it treats only normal operations and did not consider contingent cases.

Refs. [44-46] emphasize system operation characteristics in planning new reactive power sources. The method treats all variables as real and not as discrete integer variables. Benders decomposition method is implemented to solve this complex problem, which is decomposed into two linear subproblems.

With this formulation, the authors have managed to bypass the difficulties related to integer programming. The approach reduces computation time and secures the convergence of the equations. It employs the Dantzig-Wolfe decomposition approach supplemented by the Benders decomposition method. The proposed method would allow an increased number of buses in the initial candidate set, so that it can include all buses that either have reactive power sources or are connected to tap changing transformer terminals.

III. CAPITAL INVESTMENT

The Time Factor in Cash Flow

Investments in capital equipment are distinguished by longevity. The consequences of a purchase remain important for several years. Benefits, for example from installing a generator, may extend for several decades. To measure benefits against costs, the common approach is to place both on a commensurate basis by discounting future cash flows to their present value. This involves use of an interest rate that represents the time value of money.

The applications reviewed in Section II were labeled "operations" because their time period was either brief or the pattern of cash flow was assumed to be static. Capital investment in capacitors and inductors was treated as a static problem, one in which cost relationships do not change over time. If, however, generation costs and hence cost of losses were expected to change in such a way that it would alter the balance between investment cost and benefit, the time factor would require more explicit treatment.

Capital investment in generation is a multi-period problem. For example, the issue may be to find the best years in which to add generating capacity, and the analysis would have to take the time value of money into consideration.

Models Based on Discounted Cash Flow

Optimization techniques for planning investment in generating capacity started with semi-static models, which were later replaced with dynamic multi-period models. This evolution was matched by increasing complexity in the decisions faced by planners.

When the first optimizing models were developed in the 50's and 60's, United States utilities were experiencing a more or less steady rate of load growth, which was accompanied by advances in technology and progressive economies of scale. Units were being installed in sizes that matched the increasing increment in load. In spite of larger unit sizes, reliability was maintained through improvement in design and inter-utility coordination of operations. Generator scheduling also exhibited a smooth progression in which new generators would enter the loading order at base load and displace existing thermal units to higher positions in the loading order.

Planning procedures became less predictable with introduction of nuclear power and the increased severity of environmental regulations for fossil and particularly coal-fired units. Less reliance could be placed on cost calculations to settle choices between nuclear and coal. Later, with escalation in fuel price and more uncertainty in load growth, mid-range and peaking units with short construction lead times gained favor. Options such as pumped hydroelectric plants for energy storage, more inter-system coordination, and use of load management became attractive alternatives to the installation of capacity, and the planning process became more challenging.

The first widely used generation expansion optimization program that accompanied the advent of enhanced digital computer capability in the 1960's was semi-static [47]. The program developed its plan as it advanced through the data one year at a time. Consequences of generator selection would carry forward in time, but conditions such as future rate or pattern of load growth would not affect the decisions. At each stage the objective was to minimize the discounted present worth of capital investment and operating costs using only generalized assumptions about the future.

Dynamic Multi-Period Models

Probably the first dynamic multi-period optimizing models for power system capital investment were linear programming models. Linear programming was conceived in 1947, prompted in part by the Leontief inter-industry economic model and in part by the advent of electronic computers. News of the technique spread quickly and it was applied with consider success by economists. Impressive results were presented at a major conference on the subject in 1949. Linear programming texts were written, and they were soon adopted in business management courses [48-50].

A text, first published in France in 1959 and then in English translation in 1962 showed how to apply linear programming to power system planning and illustrated the connection with electricity pricing [51]. In 1972, a World Bank tutorial paper on the use of linear programming for generation expansion promoted its application, particularly for use in less industrialized countries that did not have a well established pattern for installation of capacity [52]. Linear programming was also applied to coordinated generation and transmission planning. An early study analyzed the Pacific Northwest-Southwest Interconnection and displayed the duality pricing relationships [53].

Two basic limitations faced these models: the production costing subproblem did not adequately account for the reliability consequences of unplanned generator outages, and the program did not take into account differences in economy of scale among types of generating units. The result was a bias in results, for example not recognizing adequately the contribution of peaking units to system reliability. Also an attempt to approximate probabilistic production costing in the basic linear programming format proved ineffective [54].

Meanwhile, dynamic programming overcame both of these basic limitations of linear programming and received wide acceptance [55]. Later, a sophisticated approximation to the probability calculations, based on cumulants, was incorporated and eased the dynamic programming computational burden considerably [26,27].

A more recent breakthrough in application of linear programming brought it to a comparable footing with dynamic programming in regards production costing [56]. A generalized Benders decomposition technique permitted inclusion of the probabilistic effect of generator outages on production cost as realistically as had been achieved by dynamic programming. Furthermore, the cumulants approximation was incorporated into a commercially available package that gave the user the choice of optimizing generation expansion using either dynamic programming or linear programming with Benders decomposition [57]. The package could but has not yet been generalized to handle economy of scale.

In the application of the Benders method, capital investment decisions are computed in a master program and production costing is computed in subprograms. The subprograms communicate to the master program with pricing signals. These subprograms can be linear but any algorithm can be used that produces the price signals, provided a convexity condition is maintained.

Other developments can be mentioned briefly. Using decomposition, a planning framework was developed for complex simultaneous planning of generation and transmission [58]. Quadratic programming had been applied to planning problems outside the power industry, incorporating the public policy objective of maximizing social welfare as its objective function [59].

Limitations of Discounted Cash Flow

Although all the above models use discounted cash flow to form their objective function, the method breaks down if the capital market is not ideal. Three problems are illustrated with numerical examples in the Appendix B. For the first two cases, the difficulty arises because the borrowing and lending interest rates are unequal. The third problem arises when only a limited amount of capital is available.

The first example shows that results depend on whether decisions are based on present, future or annual worth. Such a confused situation can be rectified if a uniform type of annual payout is selected to represent the investors' time preference for consumption. Then project selection could be based on finding the combination that would support the greatest real rate of growth in dividends, given the firm's initial rate of dividends per share. Some firms opt for a low initial dividend with a higher rate of growth while others choose a higher initial dividend but lower rate of growth.

The second example shows that when there is a spread between borrowing and lending rates the amount of internal financing (net funds from owners and net funds generated by ongoing projects) can become decisive.

Conventional engineering economics assumes no spread between borrowing and lending rates, in which case the choice between internal or external sources of funding do not affect project selection. Under this assumption, an increase in internal funds could reduce the amount borrowed to finance one option and increase the amount lent to the capital market for the other option, but the two actions would have the same "cost of funds" impact on the firm. Thus, in conventional engineering economics project selection depends only the discount rate and the magnitude and timing of cash flows. The example shows that this does not hold if there is a spread between borrowing and lending interest rates.

The third situation is capital rationing. In private finance this is what home buyers face when they apply for a loan and are given a limit depending on their income. In power system planning, capital rationing can apply whether or not there is a spread between the borrowing and lending interest rates, but if it does apply only the borrowing rate will be a critical factor. Again, an example in the appendix shows how a borrowing limit can affect investment decisions.

Limitations of Corporate Financial Models

Awareness of the importance of financial decisions led to the development of the corporate financial model. This model is based on an analysis of historical cash flows and interrelationships. It can include detail such as an observed elapsed time between sales of power and receipt of customer payment for it. This type of model also can include physical interrelationships for calculation of production costs, a reliability model for triggering the addition of generating capacity, and a model of regulatory action. Such detail is particularly useful for short-term budgeting, but this requires extensive data to be collected from both the commercial and engineering sections of the firm [60].

Corporate financial models have been used to show how a proposed expansion plan can affect dividends to shareholders. One study shows that if dividends were used as a yardstick, managers might reach an opposite decision from that based on discounted cash flow [61]. This divergence of results is consistent with the points illustrated in Appendix B, as the study assumed a spread between borrowing and lending interest rates.

Corporate financial models, however, lack optimization. Under the heading, "There is no finance in corporate financial models" Ref. [1] states:

> The first reason is that most such models incorporate an accountant's view of the world. They are designed to forecast accounting statements, and their equations naturally embody the accounting conventions employed by the firm. Consequently the models do not emphasize the tools of financial analysis: incremental cash flow, present value, market risk, and so on. Second, corporate financial models produce no signposts pointing toward optimal financial decisions. They do not even tell which alternatives are worth examining. All this is left up to their users.

As a step toward optimization, corporate financial models can be incorporated as part of an interconnected set of simulation-type modules. Here the module for load forecasting includes the effect of price on load growth. The module for rate regulation links income (prices) with the flow of capital investment. The module for capital investment utilizes projected load data. Finally, the financial module computes company returns based on revenues and financial flows. These modules can be interconnected in a circular feed-forward, feed-back manner, the output of one feeding the input of the next, but this arrangement does not have an underlying structure that assures an optimum solution.

Incorporation of Financial Analyses

In contrast, an underlying structure is provided if financial transactions are integrated into the guts of the program that guides the capital investment decisions. Thus, Ref. [1] continues,

> However, it is possible to build linear programming models that help search for the best financial strategy subject to specified assumptions and constraints. These "intelligent" financial planning models should prove more flexible tools for sensitivity analysis and more effective in screening alternative financial strategies. Ideally they will suggest strategies that would never occur to the unaided financial manager.

Such linear programming models were first developed in Ref. [62] and are referred to as capital budgeting models. They use separate borrowing and lending interest rates and incorporate capital rationing within the model framework and thus overcome the type of problems illustrated in Appendix B.

Power system planning models based on linear or integer programming (with or without enhancements achieved through use of Benders decomposition) readily fit this mold. Financial flows explicitly enter the constraint set. However, because the planning period is explicitly defined in linear programming models, practical applications need to consider a relatively long period of time, use nonuniform time steps, or some other artifice to reduce end effects [63].

IV. CONCLUSIONS

A review of some linear programming studies has shown that this technique has been applied effectively to power system engineering problems. An extension, integer programming, is used to model the on/off decision of unit commitment. Decomposition allows separation of reactive power subproblems in power system loss minimization. Lagrangian relaxation can be applied as an efficient computation algorithm for the large scale models that result.

Similar developments occurred in optimizing models for generation planning. In its original form, linear programming was limited by its linear treatment of production costing and its inability to deal with economy of scale. When such limitations applied, the models were useful only in a screening role or for general assessment of environmental impact. Dynamic programming proved more versatile. Application of Benders decomposition and integer programming have, however, restored the balance, and with these enhancements linear programming is suitably versatile for power system planning.

At present dynamic programming does not have an advantage over linear programming in optimizing generation expansion. Linear programming is not only suitable but has the advantage that it fits a basic formulation, the Weingartner model, that addresses directly the interaction of financing on capital budgeting and overcomes the limitations of discounted cash flow. The model displays a complete set of financial variables for each time period, facilitating formulation of financial relationships, including corporate income tax. Such capital budgeting relationships may be more difficult to incorporate into dynamic programming because in that analysis values for time-related variables only unfold sequentially during the course of the solution.

Linear and dynamic programming can be used in conjunction, but with decomposition the practical scale of linear programming models is greater than that of dynamic programming. This permits very large problems to be considered such as the combined planning of transmission and generation investments.

APPENDIX A: THEORY AND TECHNIQUES

Duality

Many texts on linear programming discuss duality, each with its own approach to the subject. Ref. [64] introduces the concept with a well-chosen numerical example that shows how the coefficients of the linear constraints naturally form a transposed dual problem with the direction of the inequalities reversed ("greater than" becoming "less than") and the direction of the optimization reversed ("minimization" for "maximization"). The example also motivates understanding of the central theorem on the relationship of the primal and dual solutions (if a solution exists and is optimal, the dual problem will also have an optimal solution and magnitude of the two objective functions will be equal) and the role that feasible solutions play in establishing bounds for the solution for both primal and dual. Finally, it brings in the concepts of complementary slackness and shadow prices.

From the time linear programming was conceived (before it was born), economists were receptive and intrigued by the role and interpretation of duality. Dualism in general had proved useful in other fields: in Chinese religion, Yin and Yang; in philosophy of dialectics, thesis and antithesis; and in electrical circuit theory, the relation of the nodal equations of one circuit to the mesh equations of its dual. In all these fields, duality brings an element of completeness to the models. Duality in linear programming unites resource allocation and pricing.

Thus,

> ...no matter how technocratic the bias of the planner and how abhorrent to him are the unplanned workings of the free market, every optimal planning decision which he makes must have implicit in it the rationale of the pricing mechanism and the allocation of resources produced by the profit system. [50 p.116]

For example, the act of installing a base-load coal generator and postponing the purchase of peaking capacity (or vice versa) influences a pricing relationship revealed in the dual that places a value on the resources used.

The duality principal in linear programming also leads to efficient computational techniques. It forms the basis for decomposition and for Lagrangian relaxation.

Decomposition has been applied to managerial theory. In a departmentalized firm, prices become the coordinating mechanism. Using Dantzig-Wolfe decomposition, subproblems analogous to subdivisions of the firm are linked to represent the common resources they share. Prices attributed to these resources are factors in terms added to the objective function of each subproblem. These prices are the dual variables of the master program. The master program, analogous to the company manager, calculates the final decisions and sends out price signals to the subdivisions.

In another type of model, under conditions of mathematical separability, the problem becomes further decentralized as the central authority only sends out price signals. The decisions are made by solution of the subproblems [65].

The Basis for Lagrangian Relaxation

A tutorial paper, Ref.[66], shows how to apply Lagrangian relaxation to an integer program, i.e., a linear program where some of the variables are restricted to integral values. The following is based primarily on that source. Assume the goal is to maximize an objective function and the constraint inequalities are in the form "less than or equal" their right-hand side. [If the goal were minimization, the words upper bound and lower bound would be reversed in what follows.]

The solution procedure reduces the dimensions of the problem by removing some constraints and inserting them into a new objective function. The way this is done is to assign a variable, called a generalized Lagrange multiplier, to each constraint that will be removed. First, the constants on the right-hand side of the constraint are shifted to the left hand side. Then each revised left-hand side is multiplied by a newly defined variable, called its associated Lagrange multiplier, and the resulting functional products are added to the original objective function. This revised objective function and the remaining unaltered constraints formulate the "relaxed" problem.

The original program has indeed been relaxed. The new problem has fewer constraints. Therefore, some of its feasible solutions may not be feasible for the original problem. It is also "Lagrangian" since, instead of ignoring the constraints that were removed, they appear as terms in the new problem's objective function. The process is similar to the use of regular Lagrange Multipliers in the optimization of continuous, constrained functions.

Ref. [66] indicates that the art of Lagrangian relaxation is to pick which constraints to remove from the original problem, keeping in mind that each constraint removed adds an additional variable (Lagrange multiplier) into the new problem's objective function. Ideally, the reduced problem will be in some recognizable form, such as the knapsack or shortest-route problem.

To summarize, the original problem contains:

- Constraints that will be unaltered
- Constraints that will be relaxed
- The objective function

The dualized or relaxed problem contains:

- The constraints that were not altered
- A modified objective function

Although the number of constraints is reduced, none of the dual variables (shadow prices) have been lost.

First, there is a dual variable for each of the constraints that were not altered. The values of these dual variables have been modified because in the original problem each dual variable measures the changes with respect to the original objective function when the right-hand side of an associated constraint is increased. The dual variables corresponding to the constraints in the relaxed problem have a similar role, but this is in regards to the modified objective function.

Secondly, although the relaxed constraints are not part of the constraint set of the relaxed version of the problem, the Lagrange multipliers that appear in its objective function serve a similar role because they measure how much the objective function of the relaxed version increases if any of the relaxed constraints are not satisfied.

The feasible solutions of the relaxed problem form an upper bound to the solution of the original problem under certain conditions. First, relaxation (removal) of some of the constraints means the solution of the relaxed problem would at least equal that of the original problem if no terms were added to the objective function. The question, therefore, is the effect of the terms added to the objective function– whether they increase or decrease its value.

The terms added to the objective function are the vector product of Lagrangian multipliers times the slack in the constraints to which they are matched. As regular linear programming variables, the Lagrange multipliers are assigned only non-negative values. Thus, the terms added to the objective function will be non-negative if the dualized constraints are not violated, i.e., if the slacks are positive.

Although this reasoning appears circular (Why bother removing constraints that are automatically satisfied?), it does have a practical computational advantage.

The author of Ref. [66] says, "In my experience, it is rare in practice that the Lagrangian solution will be feasible in the original problem. However, it is not uncommon that the Lagrangian solution will be nearly feasible and can be made feasible with some minor modifications." If the modifications are made, the constraints will not be violated, and the solution to the modified problem will be an upper bound for the original problem.

For the lower bound, any feasible, non-optimal solution of the original problem can be used. This is useful information. Generally the computation may be stopped when the lower and upper bounds are close enough to meet the accuracy requirements. Thus, an overall goal is to make the upper bound as tight or as close to the lower bound as possible.

The Lagrangian Relaxation Algorithm

The Lagrangian terms tend to widen the gap between the upper and lower bounds since they add to the value of the relaxed problem's objective function. They may therefore be called "penalty" terms, and the Lagrangian relaxation process may be characterized as one in which some constraints are "dualized" by replacing them with penalty terms in the objective function.

It is a useful thought process is to consider the constraints in the original problem as resource constraints, "with the right side representing the available supply of some resource and the left side the amount of resource demanded in a particular solution. We can then interpret the dual variable as a "price" charged for the resource. It turns out that if we can discover a price for which the supply and demand for the resource are equal, then this value will also give a tight upper bound. However, such a price might not exist." [66]

In his example, the author starts with a zero price and finds that the variables that solve the relaxed problem violate the constraints in the original problem. At that price "demand exceeds supply." Next, he tries a higher price and finds that he has overshot the solution as all the variables have become zero and none of the resource is used. This suggests a logic for an iterative process, which is formalized in the subgradient formula that is used to solve the relaxed problem.

The success of Lagrangian relaxation depends on the existence of a saddle point in the relaxed problem. This is related to a convexity requirement [66]. To reduce the gap between the lower and upper bounds, the objective function of the relaxed problem is *minimized* with respect to its Lagrange multiplier variables and *maximized* with respect to the other variables, i.e., the variables that appear in both the original and relaxed problems. The objective function of the relaxed problem is maximized with respect to the other variables because this also increases the value of the original problem's objective function and thus raises the lower bound. Minimizing with respect to the Lagrangian variables reduces the value of the relaxed problem's objective function without changing the value of the original problem's objective function and thus reduces its upper bound and the gap between the bounds. If the upper and lower bounds should meet, the procedure reaches an exact solution, not an approximation.

A saddle-shaped solution surface for the relaxed problem can be visualized as a function of the Lagrangian variables. It is an envelope formed by maximizing the relaxed problem's objective function with respect to the original variables for each set of Lagrangian values, while at the same time satisfying just the set of constraints that were retained in both problems. The subgradient method moves along this surface to find the saddle point where the objective function is minimized with respect to the Lagrangian multipliers.

The overall procedure starts with any feasible solution for the original problem, such as setting all variables to zero. Then, in the relaxed problem, an initial, arbitrary set of Lagrange multiplier values is selected and held constant while that objective function is maximized with respect to the original variables so that the function increases and rises to the saddle-shaped surface. This obtains a new set of variables and corresponds to a new value of the objective function for the original problem. A subgradient formula is then used to produce a new set of Lagrange multipliers that will reduce the value of the objective function of the relaxed problem and bring it closer to its saddle point. The formula is based on the current gap between the bounds and on the slack in the constraints that were relaxed. The process is iterated a number of times. When the gap between the bounds is small enough, the last solution of the original problem is made feasible with a branch-and-bound algorithm if this is necessary.

APPENDIX B: IMPERFECT CAPITAL MARKET

Effect of Time Preference on Project Selection

Under the usual assumptions of engineering economics, it does not matter whether project selection is based on present, future, or annual worth. This is not true, however, when there is a spread between borrowing and lending interest rates, i.e., when funds such as cash recovered from projects cannot be invested financially and receive the same interest rate that must be paid for borrowing funds.

Consider a farming project. Alfalfa requires $1000 for seeds and chemicals and yields $1200 a year later. An alternative is to grow cotton, which has a higher percentage return but is more land intensive. With cotton, only $600 can be invested, but this will yield $750 for the crop. If the farmer has $1,000 in hand and if the borrowing rate is 15% and the lending rate is 10% the comparison becomes:

Option	Present Worth	Future Worth
Alfalfa	1200/1.15=1043	<u>1200</u>
Cotton	(750/1.15)+400=<u>1052</u>	750+(400x1.1)=1190
Neither	1000	1000x1.1=1100

Between alfalfa and cotton, present worth favors cotton; future worth favors alfalfa. The present worth calculation is based on what the farmer could obtain by borrowing (at 15%) against the future crop. Future worth is based on the value of the crop plus the return (at 10%) for the funds that could not be used in the project. The choice becomes a consumption preference: is $1052 now worth more to the farmer than $1200 next year?

Effect of Internal Financing on Project Selection

In the following examples, the investor's time preference for receiving returns is not the issue because the farmer decides to pick whichever project maximizes end-of-year value. The choice, however, will be shown to depend on the amount of internal financing, i.e., the funds on hand to start with.

Case 1 - Full Internal Financing: Same facts as the previous farm example. Hence, alfalfa with the higher future worth is selected.

Case 2 - Partial Internal Financing: Same facts except that only $600, not $1000 is on hand at the start of the year. For the alfalfa option $400 is borrowed at 15% interest. The cash flows are:

	Start of Year	End of Year
Alfalfa		
Project Flow	-1000	+1200
Borrowed Funds	+400	-460
Owner's Net Flow	-600	+740
Cotton		
Owner's Net Flow	-600	+750

Here, cotton, which has the higher future worth, is selected.

Between Cases 1 and 2, the difference in project selection is due to the amount of available internal financing, $1000 in Case 1, $600 in Case 2. Without knowledge of the firm's internal financing, the correct choice could not be made; the project cash flows and financial market interest rates were not enough.

Effect of Capital Rationing

Capital rationing can also affect project choice. For example, suppose the farmer has only $400 but can borrow whatever funds are necessary at 12% interest. For alfalfa, the farmer borrows $600 and for cotton, $200. At the 12% interest rate, the farmer's end-of-year net return is $528 for alfalfa, which is greater than $526 for cotton.

If, however, the farmer can borrow no more than $500, then only part of the alfalfa crop can be planted. The net return for alfalfa is $520, and cotton becomes the crop with greater return.

	Start of Year	End of Year
Alfalfa (Unlimited Borrowing)		
Project Flow	-1000	+1200
Borrowed Funds	+600	-672
Owner's Net Flow	-400	+528
Alfalfa (Limited Borrowing)		
Project Flow	-900	+1080
Borrowed Funds	+500	-560
Owner's Net Flow	-400	+520
Cotton (Borrows $200)		
Project Flow	-600	+750
Borrowed Funds	+200	-224
Owner's Net Flow	-400	+526

Since capital rationing reduces the size of the alfalfa crop, alfalfa loses its advantage. If all the land can be planted in alfalfa, its return (profit) would be higher than that of cotton even though its rate of return is lower. If the acreage is limited because of capital rationing, cotton, with its greater rate of return, has the advantage.

Rationing of capital also violates an assumption underlying the calculation of present worth. Note that with the limit on borrowing, the future net return of $520 for alfalfa *has no present value*! The farmer's credit has been exhausted, and there is no financial market willing to translate the future crop into a present value.

REFERENCES

1. Brealey, Richard; Myers, Stewart. *Principles of Corporate Finance* New York: McGraw-Hill, 1981.
2. Kirchmeyer, Leon K. *Characteristics and Economic Operation of Steam Plants*. New York: Wiley, 1956. Chapter 2 is reprinted in *EPRI Monographs on Simulation of Electric Power Production. Book 5: Classic Articles.* (Preliminary publication) Palo Alto: EPRI, 1987.
3. Garver, L.L., "Power Generation Scheduling by Integer Programming: Development of Theory", AIEE Transactions, Vol. 81, pp. 730-735, 1962.
4. Dillion, T.S.; Edwin, K.W.; Kochs, H.D.; Taud, R.J., "Integer Programming Approach to the Problem of Optimal Unit Commitment with Probabilistic Reserve Determination", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-97, No. 6, pp. 2154-2166, November/December 1978.
5. Muchkstadt, J.A.; Wilson, R.C., "An Application of Mixed-Integer Programming Duality to Scheduling Thermal Generating System", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-87, No. 12, pp. 1968-1977, 1968.
6. Lee, F., "Short-Term Thermal Unit Commitment", IEEE Trans. on Power Systems, Vol. 3, No., pp. 421-428, May 1988.
7. Pang, C.K.; Chen, H.C., "Optimal Short-Term Thermal Unit Commitment", IEEE Transactions on Power Apparatus and System, Vol. PAS-95, No. 4, pp. 1336-1346, July/August 1976.
8. Pang, C.K.; Sheble, G.B.; Albuyeh, F., "Evaluation of Dynamic Programming Based Methods and Multiple Area Representation for Thermal Unit Commitments", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-100, No. 3, pp. 1212-1218, March 1981.
9. Snyder Jr., W.L.; Powell Jr., H.D.; Rayburn, J.C., "Dynamic Programming Approach to Unit Commitment", IEEE Trans. on Power Systems, Vol PWRS-2, No. 2, pp. 339-350, May 1987.
10. Van Den Bosch, P.P.J.; Honderd, G., "A Solution of the Unit Commitment Problem via Decomposition and Dynamic Programming", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-104, No. 7, pp. 1684-1690, July 1985.
11. Van Meeteren, H.P.,"Scheduling of Generation Allocation of Fuel, Using Dynamic and Linear Programming", IEEE Transactions, on Power Apparatus and Systems, Vol. PAS-103, No. 7, pp. 1562-1568, July 1984. 309-314, 1985.
12. Geoffrion, A.M., "Generalized Benders Decomposition", Journal of Optimization Theory and Application, 10, pp. 237-260, 1972.
13. Geoffrion, A.M., "Lagrangian Relaxation for Integer Programming", Mathematical Programming Study, Vol. 2, pp. 82-114, 1974.
14. Turgeon, A., "Optimal Scheduling of Thermal Generating Units", IEEE Transactions on Automatic Control, Vol. AC-23, No. 6, pp. 1000-1005, December 1978.
15. Baptistella, L.F. B.; Geromel, J.C., "Decomposition Approach to Problem of Unit Commitment Schedule for Hydrothermal Systems", IEE Proceedings, Vol. 127, Part D, No. 6, pp. 250-258, November 1980.
16. Habibollahzadeh, H.; Bubenko, J.A., "Application of Decomposition Techniques to Short-Term Operation Planning of Hydrothermal Power System", IEEE Transactions on Power Systems, Vol. PWRS-1, No. 1, pp. 41-47, February 1986.
17. Tong, S.K.; Shahidehpour, S.M., "An Innovative Approach to Generation Scheduling in Large-Scale Hydrothermal Power System with Fuel Constrained Units", IEEE Transactions on Power Systems, Vol. 5, No. 2, pp. 665-673, May 1990.
18. Fisher, M.L., "The Lagrangian Relaxation for Solving Integer Programming Problems", Management Science, Vol. 27, No. 1, pp. 1-18, January 1981.
19. Aoki, A.; Satoh, T.; Itoh, M.;Ichimori, T.; Masegi, K., "Unit Commitment in a Large-Scale Power System including Fuel Constrained Thermal and Pumped-Storage Hydro", IEEE Transactions on Power Systems, Vol. PWRS-2, No. 4, pp. 1077-1084, November 1987.
20. Cohen, I.; Wan, S.H., "A Method for Solving the Fuel Constrained Unit Commitment Problem", IEEE Transactions on Power Systems, Vol. PWRS-2, No. 3, pp 608-614, August 1987.
21. Ouyang, Z.; Shahidehpour, S. M.; Divakaruni, M., "A Generation Scheduling Decision Supported by Neural Networks", Proceedings of EPRI Conference on Decision Support Methods for the Electric Power Industry, Cambridge, Mass., May 1990.
21. Merlin, A.; Sandrin, P., "A New Method for Unit Commitment at Electricite de France", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-102, No. 5, pp. 1218-1225, May 1983.
22. Tong, S.K.; Shahidehpour, S.M., "A Combination of Lagrangian Relaxation and Linear Programming Approaches for Fuel Constrained Unit Commitment Problem", IEE Proceedings, Part C, Vol. 136, No. 3, pp. 162-174, May 1989.
23. Virmani, S.; Adrian, E.; Imof, K.; Mukherjee, S., "Implementation of a Lagrangian Relaxation Based Unit Commitment Problem", IEEE Transactions on Power Systems, Vol. 4, No. 4, pp. 1373-1380, November 1989
24. Zhuang, F.; Galiana, F.D., "Towards a More Rigorous and Practical Unit Commitment by Lagrangian Relaxation", Proceedings of IEEE Conference PICA 1987, pp. 385-392, 1987.
25. Tong, S.K.; Shahidehpour, S.M., "Hydrothermal Unit Commitment with Probabilistic Constraints Using Segmentation Method", IEEE Trans. on Power Systems, Vol. 5, No. 1, pp. 276-282, Feb. 1990.
26. Booth, R.R., "Power System Simulation Model Based on Probability Analysis", IEEE Transactions of Power Apparatus and Systems, Vol. PAS-91, No. 1, pp 62-69, January/February 1972.
27. Schenk, K.F.; Misra, R.B.; Vassos, S.; Wen, W., "A New Method for the Evaluation of Expected Energy Generation and Loss of Load Probability", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-103, No. 2, pp. 294-299, February 1984.
29. Zhuang, F.; Galiana, F.D., "Unit Commitment by Simulated Annealing", IEEE Transactions on Power Systems, Vol. 5, No. 1, pp. 311-317, February 1990.
30. Hobson, Eric, "Network Constrained Reactive Power Control Using Linear Programming", IEEE Trans. Power Apparatus & Systems, Vol. PAS-99, May/June 1980, pp. 868-877.
31. Mamandur, K. R. C.; Chenoweth, R. D., "Optimal Control of Reactive Power Flow for Improvements in Voltage Profiles and for Real Power Losses Minimization", IEEE Trans. Power Apparatus & Systems, Vol. PAS-100, July 1981, pp. 3185-3193.
32. Stadlin, W. O.; Fletcher, P. L., "Voltage Versus Reactive Current Model for Dispatch and Control", IEEE Trans. Power Apparatus & Systems, Vol. PAS-101, Oct. 1982, pp. 3751-3758.
33. Iyer, S. Rama, *et al.*, "New Technique for Optimal Reactive-Power Allocation for Loss Minimization in Power System", IEE Proceedings, 1983, pp. 178-182.
34. Elangovan, S., "New Approach for Real Power Loss Minimization", IEE Proceedings, Nov. 1983. pp. 295-299.
35. Lee, K. Y. *et al.*, "A United Approach to Optimal Real Reactive Power Dispatch", IEEE Trans. Power Apparatus & Systems, Vol. PAS-104, May 1985, pp. 1147-1153.
36. Shahidehpour, S. M.; Qiu, J., "A New Approach for Minimizing Power Losses and Improving Voltage Profile", IEEE Trans. Power Apparatus & Systems, Vol. PAS-106, May 1987, 287-295.

37. Deeb, N.; Shahidehpour, S. M., "An Efficient Technique for Reactive Power Dispatch Using a Revised Linear Programming Approach", Electric Power Systems Research Journal. Vol. 15, No. 1, 1988, pp. 121-134.

38. Deeb, N.; Shahidehpour, S. M., "Linear Reactive Power Optimization in a Large Power Network Using the Decomposition Approach", IEEE Transactions on Power Systems, Vol. 5, No. 2, May 1990, pp. 428-438.

39. Iba, K. *et al.*, "Practical Reactive Power Allocation/Operation Planning Using Successive Linear Programming", IEEE Trans. on Power Systems, Vol. 3, No. 2, pp. 558-566, 1988.

40. Fernandes, R. *et al.*, "Large Scale Reactive Power Planning", IEEE Trans. Power Apparatus & Systems, Vol. PAS-102, May 1983, pp. 1083-1088.

41. Granville, S. *et al.*, "An Integrated Methodology for VAR Sources Planning", IEEE Trans. on Power Systems, Vol. 3, No. 2, pp. 549-557, 1988.

42. Iyer, S. Rama; Ramachandran, K.; Hariharan, S., "Optimal Reactive Power Allocation for Improved System Performance", IEEE Trans. Power Apparatus & Systems, Vol. PAS-103, June 1984, pp. 1509-1515.

43. Aoki, K.; Fan, W.; Nishikori, A., "Optimal VAR Planning by Approximation Method for Recursive Mixed-Integer Linear Programming", IEEE Trans. on Power Systems, Vol. 3, No. 4, pp. 1741-1747, 1988.

44. Deeb, N.; Shahidehpour, S. M., "Economic Allocation of Reactive Power Supply in an Electric Power Network", Proc. of the IEEE International Symposium on Circuits and Systems, pp. 1859-1862, May 1989.

45. Deeb, N.; Shahidehpour, S. M., "A Decomposition Approach for Minimizing Real Power Losses in Power Systems", Proceedings of IEE, Part C, 1990.

46. Deeb, N.; Shahidehpour, S. M., "Cost Minimization in Allocation of Reactive Power Sources", Electrical Power and Energy Systems Journal, 1990.

47. General Electric Company, "Optimized Generation Planning". Discussed in papers by W. D. Marsh, L. L. Garver, R. P. Felak and others in the *Proceedings of the American Power Conference*, 1975 to 1978.

48. Dantzig, George B., *Linear Programming and Extensions*. Revised edition. Princeton, NJ: Princeton University Press, 1966.

49. Dorfman, Robert; Samuelson, Paul A.; Solow, Robert M., *Linear Programming and Economic Analysis*. New York: McGraw-Hill, 1958.

50. Baumol, William J., *Economic Theory and Operations Analysis* 4th edition. Englewood Cliffs NJ: Prentice-Hall, 1977.

51. Massé, Pierre. *Optimal Investment Decisions* (English translation) Englewood Cliffs, NJ: Prentice-Hall, 1962.

52. Anderson, D., "Models for Determining Least-Cost Investment in Electricity Supply," Bell Journal of Economics and Management Science, Vol. 3, No. 1, pp. 267-299, Spring, 1972.

53. Sautter, Edouard A. "Studies in the Long Range Planning of Interties between Electric Power systems." Ph.D. Thesis, Institute in Engineering Economic Systems, Stanford University, 1964.

54. Gordian Associates, "A Generation Planning System: Methodology and Case Study". EPRI Report EA-1807, 1981.

55. Jenkins, R. T.; Joy, D. S., "Wien Automatic System Planning Package (WASP) - An Electric Utility Optimal Generation Expansion Planning Computer Code," Oak Ridge National Laboratory, Oak Ridge, TN, ORNL 4945, 1974.

56. Bloom, Jeremy. "Mathematical Generation Planning Models Using Decomposition and Probabilistic Simulation". EPRI Report EA-2566-SR, 1982.

57. Stone & Webster Corporation, "Electric Generation Expansion Analysis System" (6 volumes) EPRI Report EL-2561.

58. Stanford University, "Mathematical Decomposition Techniques for Power System Expansion Planning" (5 volumes) EPRI Report EL-5299, 1988.

59. Boot, C. G..*Quadratic Programming*. Amsterdam: North Holland Publishing Company, 1964.

60. Kohlmeier, J.M.; Schlick, F. J.; Phillips, J. P., "Utility Planning Model (UPM)". EPRI Report EA-4807M, 1986.

61. Carlin, J. F., *et al.*, "Corporate Model of an Electric Utility", IEEE Spectrum, P. 75-84, June, 1969.

62. Weingartner, H. Martin. *Mathematical Programming and the Analysis of* Capital Budgeting Problems. Englewood Cliffs, NJ: Prentice-Hall, 1963. Reprinted by Markham Publishing Co. 1967.

63. Grinold, R. C., "Correction of End Effects in Energy Planning Models". EPRI Report EA-1231, 1979.

64. Chvatal, Vasek. *Linear Programming* New York: W. H. Freeman, 1983.

65. Laskon, Leon S. *Optimization Theory for Large Systems*. New York: Macmillan, 1970.

66. Fisher, M. L., "An Application Oriented Guide to Lagrangian Relaxation". INTERFACES, Vol. 15:2, March-April 1985, pp. 10-21.

67. Shapiro, Jeremy F.. *Mathematical Programming: Structures and Algorithms*. New York: Wiley, 1979.

68. Patton, A. D., "Dynamic Optimal Dispatch of Real Power for Thermal Generating Units." Proceedings IEEE Power Industry Computer Applications Conference, June 3-6, 1973, pp 403-411.

69. Innorta, M. *et al.*, "Security Constrained Dynamic Dispatch of Real Power for Thermal Units", IEEE Transactions on Power Systems, Vol.3, No. 2, May 1988, pp. 774-781.

BIOGRAPHIES

JEROME K. DELSON is a senior IEEE member, registered professional engineer, and an electrical engineering graduate of the Illinois Institute of Technology (B.S. 1949) and the California Institute of Technology (M.S., Ph.D 1953). He has taught engineering at Ohio University (Stocker Visiting Professor, 1988), Ben Gurion and Tel Aviv Universities. He worked at the General Electric Company in Analytical Engineering and other departments from 1953 to 1958, served in the planning departments of electrical utilities in Denmark and Israel, and was a supervisory engineer for the 1964 National Power Survey. He was a Project Manager at the Electric Power Research Institute (1979-1987) and presently operates an analytical engineering service for electric power utilities.

S. M. SHAHIDEHPOUR received his Ph.D. in Electrical Engineering from University of Missouri-Columbia in 1981, and that year joined the University of Michigan-Columbia where he received the Distinguished Faculty Award in 1983. He has been with the Illinois Institute of Technology since 1983, where he is currently an Associate Professor and the Associate Chairman of the Electrical and Computer Engineering Department. He has 63 publications on power system planning and operation, is a senior member of IEEE and Associate Director of the American Power Conference. He received the 1990 Excellence in Teaching Award at IIT and the C. Holmes MacDonald Outstanding Young Electrical Engineering Professor Award.

Discussion

J. A. Bloom, (GPU Service Corp., Parsippany, NJ): I want to congratulate the authors for a timely and relevant survey. I believe this paper will serve as a valuable guide to power system engineers and educators.

The authors comment that "Linear programming... has not yet been fully exploited for power system planning." I would like to add my own observations on this situation and to suggest several avenues to remedy it. As the authors have demonstrated with numerous citations, linear programming is an enormously powerful and useful technique. Nevertheless, it has not been widely accepted as a standard tool by power system engineers, primarily, I believe, for two reasons:

— linear programming is perceived to be highly complex mathematically; and
— useful tools for formulating realistic problems have not been widely available.

The perception of complexity, I believe, results from textbooks and teaching methods which focus on algorithms for solving linear programs rather than on applications of the technique. Fortunately, good textbooks are available which include substantial discussions and examples of realistic applications [1]. However, there is still a need for good case studies of applications to power system planning problems and for course materials and teaching methods which focus on formulation rather than solution methods.

Tools for formulating linear programs of realistic size and complexity are now becoming widely and conveniently available [2, 3]. These tools have significantly reduced the effort needed to set up a linear program, which previously required laboriously programming a matrix generator for the problem. These tools use natural, algebraic notation and have interfaces to reliable solvers.

More sophisticated methods of mathematical programming (such as Benders' decomposition and Lagrangian relaxation) are still not easily accessible to general-purpose users. They will need to be incorporated into easy-to-use formulation tools to achieve wider use. Integer programming remains difficult, primarily because, unlike linear programs, success in solving them is intimately linked formulation. A poorly formulated integer program may be computationally impossible to solve while an equivalent reformulation may solve relatively easily. New solution methods based upon "strong cutting planes" [4, 5] have elucidated how formulation facilitates solution and lead to hope that general-purpose formulation and solution tools for integer programs may become available.

Could the authors please comment on their experience and suggestions for incorporating linear programming and related topics in the power system engineering curriculum?

References

1. Bradley, S. P., Hax, A. C., and Magnanti, T. L. *Applied Mathematical Programming.* Addison-Wesley, 1977.
2. Brooke, A., Kendrick, D., and Meeraus, A. *GAMS: A User's Guide.* Scientific Press, 1988.
3. Schrage, L. *Linear, Integer, and Quadratic Programming with LINDO.* Scientific Press, 1986.
4. Nemhauser, G. L., and Wolsey, L. A. *Integer and Combinatorial Optimization.* Wiley, 1988.
5. Crowder, H. P., Johnson, E. L., and Padberg, M. W. Solving Large-Scale Zero-One Linear Programming Problems. *Operations Research,* vol. 31 (1983), pp. 804–834.

Manuscript received March 1, 1991.

B. Manhire and **W. C. Smith,** (Ohio University, Athens, Ohio): The authors have written a very thought provoking paper on some applications of linear programming in electric power engineering. We would like to ask the following questions.

Discounted cash flow is based on present worth principles routinely used in planning power projects. Does the example given on capital rationing imply that this technique should be abandoned in all circumstances?

The discussion of corporate financial models indicates that they can be linked through feedback loops to a variety of other models. The paper indicates that there is a drawback in the optimization process but the arrangement appears flexible and could probably accommodate load management options. Could load management also be included in the Weingartner capital budgeting model?

If the Weingartner capital budgeting model includes financial flows, are financing details such as debt/equity split or dividend policy entered by the user or developed by the model?

Next, must the objective be to maximize profit or dividends? Would this be appropriate, for example, for a cooperatively owned power system?

Finally we note with interest the authors' comments regarding linear programming and the power engineering curriculum. Being academics, we are naturally interested in learning more about how the authors would recommend that linear programming be incorporated into this curriculum.

Manuscript received August 26, 1991.

(J.K. Delson and S.M. Shahidehpour)-- Although Dr. Bloom, Prof. Manhire and Dr. Smith Focus on different issues in their discussions, they all pick up the point that the usefulness of linear programming could be more widely appreciated in power engineering if it became a part of power education. Dr. Bloom suggests development of case studies and application-oriented material. His suggestion may help provide our answer to Prof. Manhire and Dr. Smith's question on fitting linear programming into the curriculum. Since LP models are particularly suited to the economic issues that arise in planning, operations and finances, a course in these areas might be the place to introduce the technique.

On the questions raised, the issue is not abandonment of discounted cash flow, but rather the recognition of separate borrowing and lending interest rates. The problem is knowing which of the two rates to use in each of the various time intervals. The optimization process in linear programming provides the answer, but the problem may also be solved iteratively by trial and error, and sometimes this is the most efficient approach.

A formal programming model requires an explicit statement of purpose. This sometimes is lacking in load management studies. If a social welfare objective is chosen, price and quantity both vary and an objective based on their product forms a quadratic function and leads naturally to the use of quadratic programming.

In terms of incorporating financial detail into a planning model, a compromise can be reached in which the general pattern of corporate financing is accepted just as in the "weighted cost of capital", but separate financial flows remain explicitly recognized. This would improve understanding of the interaction of financing, inflation and taxation as they affect project choice.

In closing, we thank Dr. Bloom, Prof. Manhire and Dr. Smith for their thoughtful discussion.