

CSCE 222: Discrete Structures for Computing  
Section 501,502  
Spring 2018

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**Homework 1**

**Due: 28 January 2017 (Sunday) before 11:59 p.m.** on gradescope ([gradescope.com](https://www.gradescope.com)).  
You must show your work in order to receive credit.

**Aggie Honor Statement:** On my honor as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment.

**Checklist:** Did you...

1. abide by the Aggie Honor Code?
2. solve all problems?
3. start a new page for each problem?
4. show your work clearly?
5. type your solution?
6. submit a PDF to gradescope?

**Problem 1.**

Let  $p, q, r$  be the propositions

$p$  : Grizzly bears have been seen in the area.

$q$  : Hiking is safe on the trail.

$r$  : Berries are ripe along the trail.

Write these propositions using  $p, q, r$  and logical connectives:

1. Berries are ripe along the trail, but grizzly bears have not been seen in the area.
2. Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.
3. If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.
4. It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.
5. For hiking on the trail to be safe, it is necessary but not sufficient that the berries not be ripe along the trail and for grizzly bears not to have been seen in the area.
6. Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

Construct a truth table for any 2 of the propositions (your choice).

**Solution.**

1.  $r \wedge \neg p$
2.  $\neg p \wedge q \wedge r$
3.  $r \rightarrow (q \leftrightarrow \neg p)$
4.  $\neg q \wedge \neg p \wedge r$
5.  $(q \rightarrow (\neg r \wedge \neg p))$
6.  $(p \wedge r) \rightarrow \neg q$

Table 1:

| $r$ | $p$ | $\neg p$ | $r \wedge \neg p$ |
|-----|-----|----------|-------------------|
| T   | F   | F        | F                 |
| T   | F   | T        | T                 |
| F   | T   | F        | F                 |
| F   | F   | T        | F                 |

Table 2:

| $p$ | $q$ | $r$ | $\neg p$ | $\neg p \wedge q \wedge r$ |
|-----|-----|-----|----------|----------------------------|
| F   | F   | F   | T        | F                          |
| F   | F   | T   | T        | F                          |
| F   | T   | F   | T        | F                          |
| F   | T   | T   | T        | T                          |
| T   | F   | F   | F        | F                          |
| T   | F   | T   | F        | F                          |
| T   | T   | F   | F        | F                          |
| T   | T   | T   | F        | F                          |

**Problem 2.**

Four friends have been identified as suspects for unauthorized access into a computer system. They have made statements to the investigating authorities. Alice said “Carlos did it.” Bob said “I did not do it.” Carlos said “Diana did it.” Diana said “Carlos is a liar”.

1. If exactly one of the four is telling the truth, who did it? Explain your reasoning.
2. If exactly one of the four is lying, who did it? Explain your reasoning.

**Solution.**

There are 4 claims.

p: Carlos did it.

q: I did not do it.

r: Diana did it.

s: Carlos is a liar.

$p \rightarrow \neg r$

$p \rightarrow q$

$r \oplus s$

*Scenario 1*

”Exactly one is telling the truth”

Carlos and Diana contradict, therefore one must be truthful. If exactly one is truthful, then Diana is telling the truth. That means Bob is lying and he did it.

*Scenario 2*

”Exactly one is lying”

Carlos and Diana contradict, therefore one must be lying. If Diana was lying then Alice would be too. Because of this Carlos must be lying and Alice is truthful. Carlos did it.

**Problem 3.**

The following specification is taken from the specification of a telephone system:

If the directory database is opened, then the monitor is put in a closed state, if the system is not in its initial state.

Find an equivalent, easier to understand, specification that involves only disjunctions and negations.

**Solution.**

p: directory database is opened

q: monitor is in a closed state

r: system is in its initial state

$$\begin{aligned} & p \rightarrow (\neg r \rightarrow q) && . \\ \equiv & p \rightarrow (r \vee q) && \text{definition of implication} \\ \equiv & \neg p \vee (r \vee q) && \text{definition of implication} \\ \equiv & \neg p \vee r \vee q \end{aligned}$$

**Problem 4.**

Use logical equivalences to determine the correct equivalence for each expression:

Answer Bank:

|     |          |          |                      |                   |                   |                        |                       |
|-----|----------|----------|----------------------|-------------------|-------------------|------------------------|-----------------------|
| $T$ | $p$      | $q$      | $p \wedge q$         | $p \wedge \neg q$ | $\neg p \wedge q$ | $\neg p \wedge \neg q$ | $p \oplus q$          |
| $F$ | $\neg p$ | $\neg q$ | $\neg p \vee \neg q$ | $\neg p \vee q$   | $p \vee \neg q$   | $p \vee q$             | $p \leftrightarrow q$ |

- $p \vee (\neg(p \rightarrow p) \vee (\neg p \wedge (\neg p \vee (p \wedge (p \vee p))))) \equiv \underline{\hspace{2cm}}$
- $((p \vee p) \vee p) \rightarrow \neg p) \rightarrow (\neg(\neg(p \wedge \neg p) \vee \neg(p \wedge \neg(p \rightarrow p))) \wedge \neg(p \rightarrow (p \wedge \neg p))) \equiv \underline{\hspace{2cm}}$
- $p \rightarrow \neg(((p \wedge p) \leftrightarrow \neg p) \oplus ((p \vee (p \leftrightarrow p)) \leftrightarrow p)) \equiv \underline{\hspace{2cm}}$
- $\neg(\neg(p \vee \neg q) \wedge \neg q) \rightarrow \neg(\neg(q \wedge \neg(q \rightarrow q)) \wedge \neg q) \wedge \neg p) \equiv \underline{\hspace{2cm}}$
- $\neg q \wedge ((q \rightarrow \neg p) \wedge \neg(p \rightarrow ((q \rightarrow \neg q) \wedge p)) \rightarrow \neg p) \equiv \underline{\hspace{2cm}}$
- $(\neg(\neg(p \rightarrow (q \wedge \neg q)) \wedge \neg(p \leftrightarrow (\neg q \vee q))) \vee (\neg(\neg p \vee \neg q) \vee \neg(\neg q \vee \neg q))) \equiv \underline{\hspace{2cm}}$
- $(\neg q \wedge (\neg p \wedge \neg p)) \wedge \neg(\neg q \vee \neg p) \wedge ((p \rightarrow \neg p) \vee \neg p) \equiv \underline{\hspace{2cm}}$

**Solution.**

Example

$$\begin{aligned}
 & (\neg((p \vee p) \rightarrow p) \vee \neg \neg \neg p) \\
 \equiv & (\neg((p \vee p) \rightarrow p) \vee \neg \neg p) && \text{double negation} \\
 \equiv & (\neg((p \vee p) \rightarrow p) \vee p) && \text{double negation} \\
 \equiv & (\neg(p \rightarrow p) \vee p) && \text{idempotent} \\
 \equiv & (\neg(\neg p \vee p) \vee p) && \text{definition of implication} \\
 \equiv & (\neg T \vee p) && \text{negation (tautology)} \\
 \equiv & (F \vee p) \\
 \equiv & p && \text{identity}
 \end{aligned}$$

*Question 1*

$$\begin{aligned}
 & p \vee (\neg(p \rightarrow p) \vee (\neg p \wedge (\neg p \vee (p \wedge (p \vee p))))) \\
 \equiv & p \vee (\neg(p \rightarrow p) \vee (\neg p \wedge (\neg p \vee (p \wedge p)))) && \text{idempotent} \\
 \equiv & p \vee (\neg(p \rightarrow p) \vee (\neg p \wedge (\neg p \vee p))) && \text{idempotent} \\
 \equiv & p \vee (\neg(p \rightarrow p) \vee (\neg p \wedge T)) && \text{negation laws} \\
 \equiv & p \vee (\neg(\neg p \vee p) \vee (\neg p \wedge T)) && \text{definition of implication} \\
 \equiv & p \vee (\neg T \vee (\neg p \wedge T)) && \text{negation} \\
 \equiv & p \vee (F \vee (\neg p \wedge T)) && \text{identity} \\
 \equiv & p \vee (F \vee \neg p) && \text{identity} \\
 \equiv & p \vee \neg p && \text{negation} \\
 \equiv & T
 \end{aligned}$$

## Question 2

|   |                           |
|---|---------------------------|
| $((p \vee p) \vee p) \rightarrow \neg p \rightarrow (\neg(\neg(p \wedge \neg p) \vee \neg(p \wedge \neg(p \rightarrow p))) \wedge \neg(p \rightarrow (p \wedge \neg p)))$ |                           |
| $\equiv(((p \vee p) \rightarrow \neg p) \rightarrow (\neg(\neg(p \wedge \neg p) \vee \neg(p \wedge \neg(p \rightarrow p))) \wedge \neg(p \rightarrow (p \wedge \neg p)))$ | commutative               |
| $\equiv((p \rightarrow \neg p) \rightarrow (\neg(\neg(p \wedge \neg p) \vee \neg(p \wedge \neg(p \rightarrow p))) \wedge \neg(p \rightarrow (p \wedge \neg p)))$          | commutative               |
| $\equiv((\neg p \vee \neg p) \rightarrow (\neg(\neg(p \wedge \neg p) \vee \neg(p \wedge \neg(p \rightarrow p))) \wedge \neg(p \rightarrow (p \wedge \neg p)))$            | definition of implication |
| $\equiv(p \rightarrow (\neg(\neg(p \wedge \neg p) \vee \neg(p \wedge \neg(p \rightarrow p))) \wedge \neg(p \rightarrow (p \wedge \neg p)))$                               | commutative               |
| $\equiv(p \rightarrow (\neg(\neg F) \vee \neg(p \wedge \neg(p \rightarrow p))) \wedge \neg(p \rightarrow (p \wedge \neg p)))$   | negation                  |
| $\equiv(p \rightarrow F \vee \neg(p \wedge \neg(p \rightarrow p))) \wedge \neg(p \rightarrow (p \wedge \neg p))$  | double negation           |
| $\equiv(p \rightarrow F \vee \neg(p \wedge \neg(p \rightarrow p))) \wedge \neg(p \rightarrow F)$  | negation laws             |
| $\equiv(p \rightarrow F \vee \neg(p \wedge \neg(\neg p \vee p))) \wedge \neg(p \rightarrow F)$  | definition of implication |
| $\equiv(p \rightarrow F \vee \neg(p \wedge \neg(\neg p \vee p))) \wedge \neg(\neg p \vee F)$  | definition of implication |
| $\equiv(p \rightarrow F \vee \neg(p \wedge \neg T)) \wedge \neg(\neg p \vee F)$   | negation                  |
| $\equiv(p \rightarrow F \vee \neg(p \wedge F)) \wedge \neg \neg p$  | negation                  |
| $\equiv(p \rightarrow F \vee \neg(p \wedge F)) \wedge p$  | double negation           |
| $\equiv(p \rightarrow F \vee \neg F) \wedge p$  | domination                |
| $\equiv(p \rightarrow F \vee T) \wedge p$   | negative false            |
| $\equiv p \rightarrow T \wedge p$   | identity                  |
| $\equiv p \rightarrow p$  | identity                  |
| $\equiv \neg p \vee p$  | definition of implication |
| $\equiv T$  | negation                  |

## Question 3

|   |                           |
|---|---------------------------|
| $p \rightarrow \neg(((p \wedge p) \leftrightarrow \neg p) \oplus ((p \vee (p \leftrightarrow p)) \leftrightarrow p))$ | .                         |
| $\equiv p \rightarrow \neg((p \leftrightarrow \neg p) \oplus ((p \vee (p \leftrightarrow p)) \leftrightarrow p))$     | idempotent                |
| $\equiv p \rightarrow \neg((p \leftrightarrow \neg p) \oplus ((p \vee T) \leftrightarrow p))$                         | bi conditional definition |
| $\equiv p \rightarrow \neg(F \oplus ((p \vee T) \leftrightarrow p))$  | bi conditional definition |
| $\equiv p \rightarrow \neg(F \oplus (T \leftrightarrow p))$   | bi conditional definition |
| $\equiv p \rightarrow \neg(F \oplus p)$   | domination                |
| $\equiv p \rightarrow \neg((F \wedge \neg p) \vee (T \wedge p))$  | xor definition            |
| $\equiv p \rightarrow \neg(F \vee (T \wedge p))$  | domination                |
| $\equiv p \rightarrow \neg(F \vee p)$   | identity                  |
| $\equiv p \rightarrow \neg p$   | identity                  |
| $\equiv F$  | bi conditional definition |

## Question 4

|   |                           |
|---|---------------------------|
| $\neg(\neg(p \vee \neg\neg q) \wedge \neg q) \rightarrow \neg\neg\neg\neg((\neg(q \wedge \neg(q \rightarrow q)) \wedge \neg\neg q) \wedge \neg p)$  |                           |
| $\equiv \neg(p \vee \neg\neg q) \wedge \neg q \rightarrow \neg\neg\neg\neg((\neg(q \wedge \neg(q \rightarrow q)) \wedge \neg\neg q) \wedge \neg p)$ | double negation           |
| $\equiv ((p \vee \neg\neg q \wedge \neg q) \rightarrow \neg\neg\neg\neg((\neg(q \wedge \neg(q \rightarrow q)) \wedge \neg\neg q) \wedge \neg p)$    | double negation           |
| $\equiv ((p \vee \neg\neg q \wedge \neg q) \rightarrow \neg\neg((\neg(q \wedge \neg(q \rightarrow q)) \wedge \neg\neg q) \wedge \neg p)$            | double negation           |
| $\equiv ((p \vee \neg\neg q) \wedge \neg q) \rightarrow \neg((\neg(q \wedge \neg(q \rightarrow q)) \wedge \neg\neg q) \wedge \neg p)$               | double negation           |
| $\equiv ((p \vee \neg q) \wedge \neg q) \rightarrow \neg((\neg(q \wedge \neg(q \rightarrow q)) \wedge \neg\neg q) \wedge \neg p)$                   | double negation           |
| $\equiv ((p \vee \neg q) \wedge \neg q) \rightarrow \neg((\neg(q \wedge \neg(q \rightarrow q)) \wedge q) \wedge \neg p)$                            | double negation           |
| $\equiv ((p \vee \neg q) \wedge \neg q) \rightarrow \neg((\neg(q \wedge \neg T) \wedge q) \wedge p)$  | definition of implication |
| $\equiv ((p \vee \neg q) \wedge \neg q) \rightarrow \neg(q \vee p)$   | domination                |
| $\equiv (\neg q) \rightarrow \neg(q \vee \neg p)$   | de morgans                |
| $\equiv q \vee \neg(\neg q \vee \neg p)$  | implication               |
| $\equiv (q \vee \neg q) \vee p$   | associative               |
| $\equiv T \vee p$   | negation                  |
| $\equiv T$  | domination                |

## Question 5

|   |                 |
|---|-----------------|
| $\neg\neg q \wedge ((q \rightarrow \neg p) \wedge \neg(p \rightarrow ((q \rightarrow \neg q) \wedge p)) \rightarrow \neg p))$ | .               |
| $\equiv q \wedge ((q \rightarrow \neg p) \wedge \neg(p \rightarrow ((q \rightarrow \neg q) \wedge p)) \rightarrow \neg p))$   | double negation |
| $\equiv q \wedge ((q \rightarrow \neg p) \wedge \neg(p \rightarrow ((q \rightarrow \neg q) \wedge p)) \rightarrow \neg p))$   | double negation |
| $\equiv q \wedge ((q \rightarrow \neg p) \wedge \neg(p \rightarrow ((F \wedge p)) \rightarrow \neg p))$                       | implication     |
| $\equiv q \wedge ((q \rightarrow \neg p) \wedge \neg(p \rightarrow F) \rightarrow \neg p))$                                   | domination      |
| $\equiv q \wedge ((\neg q \vee \neg p) \wedge \neg(p \rightarrow F) \rightarrow \neg p))$                                     | bi conditional  |
| $\equiv q \wedge ((\neg q \vee \neg p) \wedge \neg(\neg p \vee F) \rightarrow \neg p))$                                       | implication     |
| $\equiv q \wedge ((\neg q \vee \neg p) \wedge \neg(\neg p \rightarrow \neg p))$   | identity        |
| $\equiv q \wedge ((\neg q \vee \neg p) \wedge F)$   | implication     |
| $\equiv ((q \wedge \neg q) \vee (q \wedge \neg p)) \wedge F$  | distributive    |
| $\equiv (F \vee (q \wedge \neg p)) \wedge F$  | negation        |
| $\equiv (q \wedge \neg p) \wedge F$   | identity        |
| $\equiv F$  | domination      |

## Question 6

|   |                 |
|---|-----------------|
| $\neg\neg\neg((p \rightarrow (q \wedge \neg q)) \wedge \neg(p \leftrightarrow (\neg q \vee q))) \vee (\neg(\neg p \vee \neg q) \vee \neg(\neg q \vee \neg\neg q)))$ | .               |
| $\equiv \neg((p \rightarrow (q \wedge \neg q)) \wedge \neg(p \leftrightarrow (\neg q \vee q))) \vee (\neg(\neg p \vee \neg q) \vee \neg(\neg q \vee \neg\neg q))$   | double negation |
| $\equiv \neg((p \rightarrow (q \wedge \neg q)) \wedge \neg(p \leftrightarrow (\neg q \vee q))) \vee (\neg(\neg p \vee \neg q) \vee \neg(\neg q \vee \neg q))$       | double negation |
| $\equiv \neg((p \rightarrow F) \wedge \neg(p \leftrightarrow (\neg q \vee q))) \vee (\neg(\neg p \vee \neg q) \vee \neg(\neg q \vee \neg q))$                       | negation        |
| $\equiv \neg((p \rightarrow F) \wedge \neg(p \leftrightarrow T)) \vee (\neg(\neg p \vee \neg q) \vee \neg(\neg q \vee \neg q))$                                     | negation        |
| $\equiv \neg(\neg p \wedge \neg p) \vee (\neg(\neg p \vee \neg q) \vee \neg(\neg q \vee \neg q))$   | identity        |
| $\equiv \neg(\neg p) \vee (\neg(\neg p \vee \neg q) \vee \neg(\neg q \vee \neg q))$   | idempotent      |
| $\equiv p \vee (\neg(\neg p \vee \neg q) \vee \neg(\neg q \vee \neg q))$  | double negation |
| $\equiv p \vee (p \wedge q \vee \neg(\neg q \vee \neg q))$  | de morgan's     |
| $\equiv p \vee (p \wedge q) \vee \neg\neg q$  | idempotent      |
| $\equiv p \vee (p \wedge q) \vee q$   | double negation |
| $\equiv p \vee q$   | absorption      |

## Question 7

|  |                 |
|--|-----------------|
| $(\neg\neg q \wedge (\neg\neg p \wedge \neg p)) \wedge \neg\neg\neg(\neg(\neg q \vee \neg p) \wedge ((p \rightarrow \neg p) \vee \neg p))$ | .               |
| $\equiv (\neg\neg q \wedge (\neg\neg p \wedge \neg p)) \wedge \neg(\neg(\neg q \vee \neg p) \wedge ((p \rightarrow \neg p) \vee \neg p))$  | double negation |
| $\equiv (q \wedge (\neg\neg p \wedge \neg p)) \wedge \neg(\neg(\neg q \vee \neg p) \wedge ((p \rightarrow \neg p) \vee \neg p))$           | double negation |
| $\equiv (q \wedge (\neg p \wedge \neg p)) \wedge \neg(\neg(\neg q \vee \neg p) \wedge ((p \rightarrow \neg p) \vee \neg p))$               | double negation |
| $\equiv (q \wedge (\neg p \wedge \neg p)) \wedge \neg(\neg(\neg q \vee \neg p) \wedge ((p \rightarrow p) \vee \neg p))$                    | double negation |
| $\equiv (q \wedge (\neg p \wedge p)) \wedge \neg(\neg(\neg q \vee \neg p) \wedge ((p \rightarrow p) \vee \neg p))$                         | double negation |
| $\equiv (q \wedge (\neg p \wedge p)) \wedge \neg(\neg(q \vee \neg p) \wedge ((p \rightarrow p) \vee p))$                                   | double negation |
| $\equiv (q \wedge (\neg p \wedge p)) \wedge \neg(\neg(q \vee \neg p) \wedge ((p \rightarrow p) \vee p))$                                   | double negation |
| $\equiv (q \wedge F) \wedge \neg(\neg(q \vee \neg p) \wedge ((p \rightarrow p) \vee p))$   | negation        |
| $\equiv F \wedge \neg(\neg(q \vee \neg p) \wedge ((p \rightarrow p) \vee p))$  | domination      |
| $\equiv F \wedge \neg(\neg(q \vee \neg p) \wedge (T \vee p))$  | implication     |
| $\equiv F \wedge \neg(\neg(q \vee \neg p) \wedge T)$   | domination      |
| $\equiv F \wedge \neg((\neg q \wedge p) \wedge T)$   | de morgan's     |
| $\equiv F \wedge (\neg(\neg q \wedge p) \vee F)$   | de morgan's     |
| $\equiv F \wedge ((q \vee \neg p) \vee F)$   | de morgan's     |
| $\equiv F \wedge (q \vee \neg p)$  | identity        |
| $\equiv F$   | domination      |



**Problem 5.**

A set of logical operators is called **functionally complete** if every compound proposition is logically equivalent to a compound proposition involving only these logical operators. For example, the set  $\{\neg, \wedge, \vee\}$  is functionally complete. Furthermore, due to De Morgan's laws, each of the sets  $\{\neg, \wedge\}$  and  $\{\neg, \vee\}$  are also functionally complete. The singleton set  $\{\downarrow\}$  (NOR) is functionally complete, and  $\downarrow$  is therefore called a universal operator since all statements in propositional logic can be formed using only  $\downarrow$ . The same is true of  $\{\uparrow\}$  (NAND).  $\{\downarrow\}$  and  $\{\uparrow\}$  are the only 1-element functionally complete operator sets, but there are several 2-element functionally complete operator sets, of which  $\{\neg, \wedge\}$  and  $\{\neg, \vee\}$  are just two examples.

1. Show that  $\{\neg, \wedge, \vee\}$  is a functionally complete set of logical operators.

*Hint: every truth table over  $n$  variables can be expressed in **disjunctive normal form** (a disjunction of conjunctions that specifies when the compound proposition is true).*

2. Show that  $\{\neg, \wedge\}$  is a functionally complete set of logical operators.

*(Give a logical equivalence for disjunction that involves only negation and conjunction, then construct a truth table to show that the equivalence is valid.)*

3. Show that  $\{\uparrow\}$  is a functionally complete set of logical operators.

*(Give logical equivalences for negation and conjunction that involve only NAND, then construct a truth table to show that the equivalences are valid.)*

4. Express the following propositions using only NAND.

*You must show your work. If you cannot show your work, find another way to solve it.*

- (a)  $p \vee q$
- (b)  $p \rightarrow q$
- (c)  $p \oplus q$

**Solution.***Question 1*

Table 3: Add caption

| p | q | $\neg p$ | $\neg q$ | $p \wedge q$ | $\neg p \wedge \neg q$ | $p \vee q$ | $\neg p \vee \neg q$ |
|---|---|----------|----------|--------------|------------------------|------------|----------------------|
| T | T | F        | F        | T            | F                      | T          | F                    |
| F | T | F        | F        | F            | F                      | T          | T                    |
| T | F | F        | T        | F            | F                      | T          | T                    |
| F | F | T        | T        | F            | T                      | F          | T                    |

*Question 2*

Table 4: Add caption

| p | q | $\neg p$ | $\neg q$ | $p \wedge q$ | $\neg p \wedge \neg q$ |
|---|---|----------|----------|--------------|------------------------|
| T | T | F        | F        | T            | F                      |
| F | T | F        | F        | F            | F                      |
| T | F | F        | T        | F            | F                      |
| F | F | T        | T        | F            | T                      |

*Question 3*

$$p \wedge q \equiv (p|q)|(p|q)$$

$$| \equiv \neg(p \wedge q)$$

$$\neg p \equiv p|p$$

