



We will prove that the derivative of the cost function with respect to the weight  $w_{kl}^{(l)}$  of the  $l^{th}$  hidden layer is  $\frac{\partial J(\mathbf{W}, \mathbf{b})}{\partial w_{kl}^{(l)}} = \alpha_j^{l-1} f'(z_k^l) \sum_m \delta_m^{(l+1)} w_{mk}^{(l+1)}$ , where  $f$  is the activation function and  $\delta_m^{(l+1)}$  is the error propagated from layer  $l+1$ .

In the following, we will assume zero bias term for the sake of simplicity.

$$z_k^l = \sum_j w_{kl}^{(l)} \alpha_j^{l-1}$$

$$\alpha_k^l = f(z_k^l)$$

$$z_m^{l+1} = \sum_k w_{mk}^{(l+1)} \alpha_k^l$$

$$\begin{aligned} \frac{\partial J(\mathbf{W}, \mathbf{b})}{\partial w_{kl}^{(l)}} &= \underbrace{\frac{\partial J(\mathbf{W}, \mathbf{b})}{\partial z_k^l}}_{\delta_k^l} \cdot \frac{\partial z_k^l}{\partial w_{kl}^{(l)}} = \frac{\partial J(\mathbf{W}, \mathbf{b})}{\partial \alpha_k^l} \cdot \underbrace{\frac{\partial \alpha_k^l}{\partial z_k^l}}_{f'(z_k^l)} \cdot \underbrace{\frac{\partial z_k^l}{\partial w_{kl}^{(l)}}}_{\alpha_j^{l-1}} = \left( \sum_m \underbrace{\frac{\partial J(\mathbf{W}, \mathbf{b})}{\partial z_m^{l+1}}}_{\delta_m^{l+1}} \frac{\partial z_m^{l+1}}{\partial \alpha_k^l} \right) \cdot f'(z_k^l) \cdot \alpha_j^{l-1} = \\ & \left( \sum_m \delta_m^{l+1} w_{mk}^{(l+1)} \right) f'(z_k^l) \cdot \alpha_j^{l-1} \end{aligned}$$