Quiz 2 **CSCE 421**
Name:
UIN (or TAMU Email):

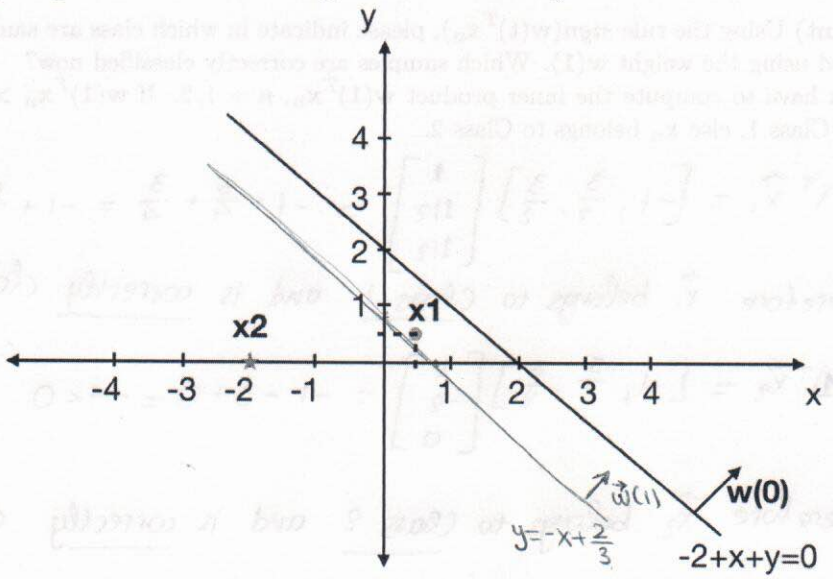
Total 5 points.

Question 1 (3 points)

The goal of this problem is to run one step of a linear perceptron algorithm. Assume that you have two training samples:

1. Sample $\mathbf{x_1} = [1, \frac{1}{2}, \frac{1}{2}]^T$ lies at point $(\frac{1}{2}, \frac{1}{2})$ of the x-y space and belongs to Class 1 ($y_1 = 1$)
2. Sample $\mathbf{x_2} = [1, -2, 0]^T$ lies at point $(-2, 1)$ of the x-y space and belongs to Class 2 ($y_2 = -1$)

The linear perceptron is initialized using the line $-2 + x + y = 0$.



(a) (1 point) Please find the weight $\mathbf{w(0)}$ corresponding to line $-2 + x + y = 0$.
Hint: The weight corresponding to line $w_0 + w_1 x + w_2 y = 0$ can be written as $\mathbf{w} = [w_0, w_1, w_2]^T$.

$\vec{w}(0) = [-2, 1, 1]^T$

(b) (1 point) Based on the weight update rule from the linear perceptron algorithm, please find the value of the new weight $\mathbf{w(1)}$ using the training sample $\mathbf{x_1}$. Find and plot the new line

corresponding to weight $\mathbf{w(1)}$ in the 2D space.
Hint: The update rule is $\mathbf{w(1)} = \mathbf{w(0)} + y_1\mathbf{x_1}$, where $\mathbf{x_1}$ and y_1 are the feature and class label of misclassified sample 1.

$$\vec{\omega}(1) = \vec{\omega}(0) + \vec{x}_1 = [-2,1,1]^T + \left[1, \frac{1}{2}, \frac{1}{2}\right]^T = \left[-1, \frac{3}{2}, \frac{3}{2}\right]$$

The line corresponding to $\vec{\omega}(1)$ is:

$$-1 + \frac{3}{2}x + \frac{3}{2}y = 0 \Rightarrow \quad y = -x + \frac{2}{3}$$

(c) (1 point) Using the rule $sign(\mathbf{w(t)}^T\mathbf{x_n})$, please indicate in which class are samples $\mathbf{x_1}$ and $\mathbf{x_2}$ classified using the weight $\mathbf{w(1)}$. Which samples are correctly classified now?
Hint: You have to compute the inner product $\mathbf{w(1)}^T\mathbf{x_n}$, $n = 1, 2$. If $\mathbf{w(1)}^T\mathbf{x_n} > 0$, then $\mathbf{x_n}$ belongs to Class 1, else $\mathbf{x_n}$ belongs to Class 2.

$$\vec{\omega}(1)^T \vec{x}_1 = \left[-1, \frac{3}{2}, \frac{3}{2}\right] \begin{bmatrix} 1 \\ 1/2 \\ 1/2 \end{bmatrix} = -1 + \frac{3}{4} + \frac{3}{4} = -1 + \frac{6}{4} = \frac{1}{2} > 0$$

therefore $\vec{x}_1$ belongs to Class 1 and is correctly classified

$$\vec{\omega}(1)^T \vec{x}_2 = \left[-1, \frac{3}{2}, \frac{3}{2}\right] \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} = -1 - 3 + 0 = -4 < 0$$

therefore $\vec{x}_2$ belongs to Class 2 and is correctly classified

Question 2 (2 points)

Please describe the difference between the linear regression and the linear perceptron classifier (2-3 sentences). What problem does each machine learning algorithm try to solve and how are these different?

The linear regression algorithm will predict a continuous outcome based on the input features.

The linear perceptron performs a classification task, therefore will predict a categorical outcome (or a class, or a binary outcome) based on the input features.