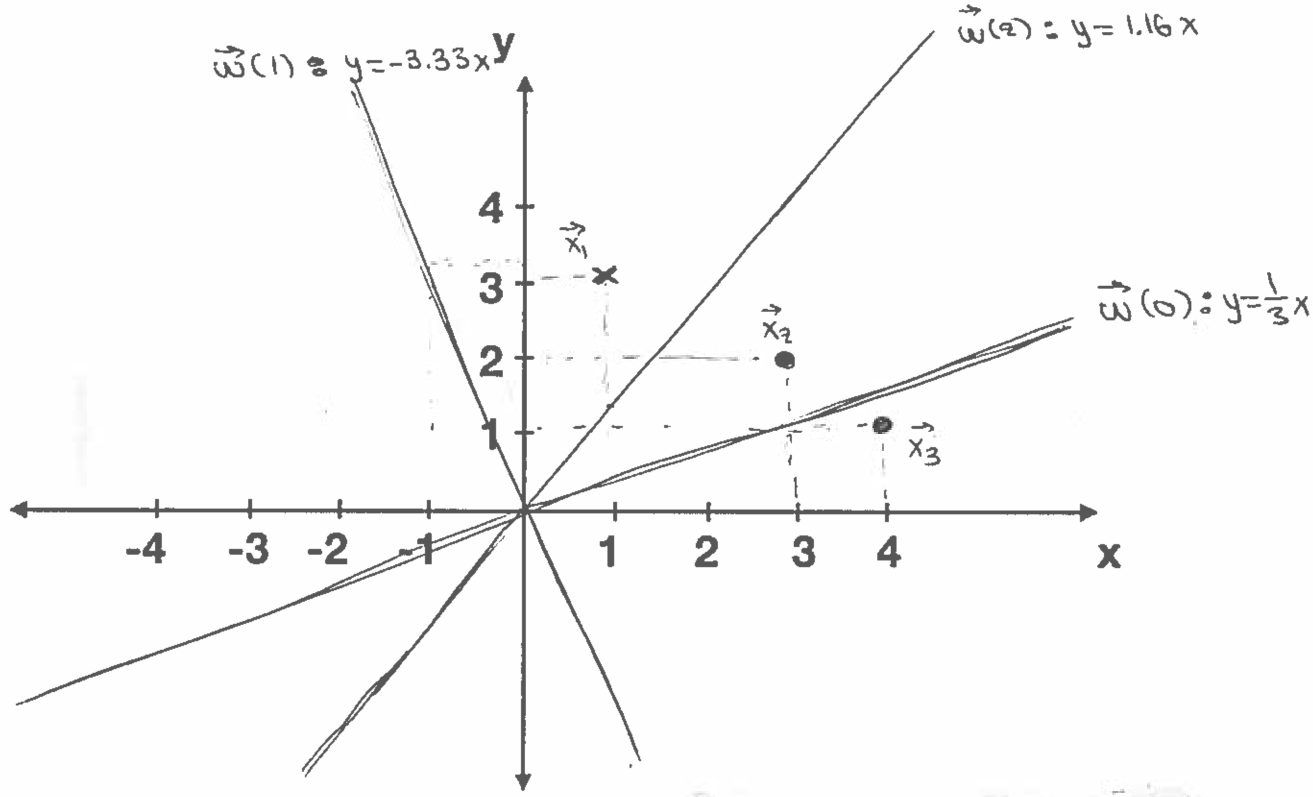
Linear Perceptron Practice Problem

~~Linear Algebra Review Questions~~ CSCE 421

The goal of this problem is to run a linear perceptron algorithm. Assume that you have three training samples:

1. Sample $\mathbf{x}_1 = [1, 3]^T$ from Class 1 ($y_1 = 1$)
2. Sample $\mathbf{x}_2 = [3, 2]^T$ from Class 2 ($y_2 = -11$)
3. Sample $\mathbf{x}_3 = [4, 1]^T$ from Class 2 ($y_3 = -1$)

The linear perceptron is initialized with a line with corresponding weight $\mathbf{w}(0) = [-\frac{1}{3}, 1]^T$. In the following, for the sake of simplicity, you will assume that all lines of the perceptron intersect point $(0, 0)$, therefore **you do not have to include any intercept w_0 or x_0 in the following calculations.**



For $\vec{w}(0) = [-\frac{1}{3}, 1]^T$: $-\frac{1}{3}x + y = 0 \Rightarrow y = \frac{1}{3}x$

(1) Plot $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ in the given 2D space. Plot the line corresponding to weight $\mathbf{w}(0)$.

(2) Using the rule $sign(\mathbf{w}(\mathbf{t})^T\mathbf{x}_n)$, please indicate in which class are samples $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ classified using the weight $\mathbf{w}(0)$. Which samples are not correctly classified based on this rule? **Note:** You have to compute the inner product $\mathbf{w}(0)^T\mathbf{x}_n$, $n = 1, 2, 3$, and see if it is greater or less than 0.

$\vec{w}(0)^T \vec{x}_1 = -\frac{1}{3} + 3 = \frac{8}{3} > 0$ ✓

$\vec{w}(0)^T \vec{x}_2 = -1 + 2 = 1 > 0$ X

$\vec{w}(0)^T \vec{x}_3 = -\frac{4}{3} + 1 = -\frac{1}{3} < 0$ ✓

Sample $\vec{x}_2$ is not classified correctly using $\vec{w}(0)$.

(3) Using the weight update rule from the linear perceptron algorithm, please find the value of the new weight $\mathbf{w}(1)$. Find and plot the new line corresponding to weight $\mathbf{w}(1)$ in the 2D space.
Note: The update rule is $\mathbf{w}(t+1) = \mathbf{w}(t) + y_s\mathbf{x_s}$, where $\mathbf{x_s}$ and $y_s \in \{-1, 1\}$ is the feature and class label of missclassified sample s.

$$\vec{\omega}(1) = \vec{\omega}(0) - \vec{x}_2 = \left[-\tfrac{1}{3}, 1\right]^T - [3, 2]^T = \left[-\tfrac{10}{3}, -1\right]^T$$

the "−" sign is
because $\vec{x}_2$ is from Class 2 ($y_2 = -1$)

The corresponding line is $-\frac{10}{3}x - y = 0 \Rightarrow y = -\frac{10}{3}x \Rightarrow y = -3.33x$

(4) Using the rule $sign(\mathbf{w}(t)^T\mathbf{x_n})$, please indicate in which class are samples $\mathbf{x_1}$, $\mathbf{x_2}$, and $\mathbf{x_3}$ classified using the weight $\mathbf{w}(1)$. Which samples are not correctly classified based on this rule?
Note: You have to compute the inner product $\mathbf{w}(1)^T\mathbf{x_n}$, $n = 1, 2, 3$, and see if it is greater or less than 0.

$$\vec{\omega}(1)^T \vec{x}_1 = -\frac{10}{3} - 3 = -\frac{19}{3} < 0 \quad \times$$

$$\vec{\omega}(1)^T \vec{x}_2 = -10 - 2 = -12 < 0 \quad \checkmark$$

$$\vec{\omega}(1)^T \vec{x}_3 = -\frac{40}{3} - 1 = -\frac{43}{3} < 0 \quad \checkmark$$

sample $\vec{x}_1$ is missclassified based on $\vec{\omega}(1)$.

(5) Using the weight update rule from the linear perceptron algorithm, please find the value of the new weight $\mathbf{w}(2)$. Find and plot the new line corresponding to weight $\mathbf{w}(2)$ in the 2D space. How many samples are correctly classified now?
Note: The update rule is $\mathbf{w}(t+1) = \mathbf{w}(t) + y_s\mathbf{x_s}$, where $\mathbf{x_s}$ and $y_s \in \{-1, 1\}$ is the feature and class label of missclassified sample s.

$$\vec{\omega}(2) = \vec{\omega}(1) + \vec{x}_1 = \left[-\tfrac{10}{3}, -1\right]^T + [1, 3]^T = \left[-\tfrac{7}{3}, 2\right]^T$$

↳ the "+" sign is because $\vec{x}_1$ belongs to Class 1 ($y_1 = 1$)

$-\frac{7}{3}x + 2y = 0 \Rightarrow y = \frac{7}{6}x \Rightarrow y = 1.16x$

$$\vec{\omega}(2)^T \vec{x}_1 = -\frac{7}{3} + 6 = \frac{11}{3} > 0 \checkmark$$

$$\vec{\omega}(2)^T \vec{x}_2 = -7 + 4 = -3 < 0 \checkmark$$

$$\vec{\omega}(2)^T \vec{x}_3 = -\frac{28}{3} + 2 = -\frac{22}{3} < 0 \checkmark$$

All samples are classified correctly based on $\vec{\omega}(2)$.