

We are going to prove that the vector perpendicular to line $w_1x_1 + w_2x_2 + w_0 = 0$ can be written as $\mathbf{n} = [w_1, w_2]^T$.

Let's assume the line $w_1x_1 + w_2x_2 + w_0 = 0$. Its intersection with x_1 and x_2 axes can be represented by the vectors $\mathbf{v} = \left[-\frac{w_0}{w_1}, 0\right]^T$ and $\mathbf{u} = \left[0, -\frac{w_0}{w_2}\right]^T$, respectively.

The vector starting from $\left[0, -\frac{w_0}{w_2}\right]^T$ and ending at $\left[-\frac{w_0}{w_1}, 0\right]^T$ can be written as the difference between \mathbf{v} and \mathbf{u} , as $\mathbf{a} = \left[-\frac{w_0}{w_1}, \frac{w_0}{w_2}\right]^T$. Vector \mathbf{a} lies on the above line.

Let $\mathbf{n} = [n_1, n_2]$ be the perpendicular vector to the line, which is also perpendicular to \mathbf{a} , therefore their inner product will be zero, i.e., $\mathbf{a} \cdot \mathbf{n} = 0$.

Based on the above equation we get $-n_1 \frac{w_0}{w_1} + n_2 \frac{w_0}{w_2} = 0 \implies \frac{n_1}{w_1} = \frac{n_2}{w_2}$. If we assume that $n_2 = w_2$, then the perpendicular vector can be written as $\mathbf{n} = [w_1, w_2]^T$.

\mathbf{n} could be any vector as long as its elements verify the association $\frac{n_1}{w_1} = \frac{n_2}{w_2}$.

