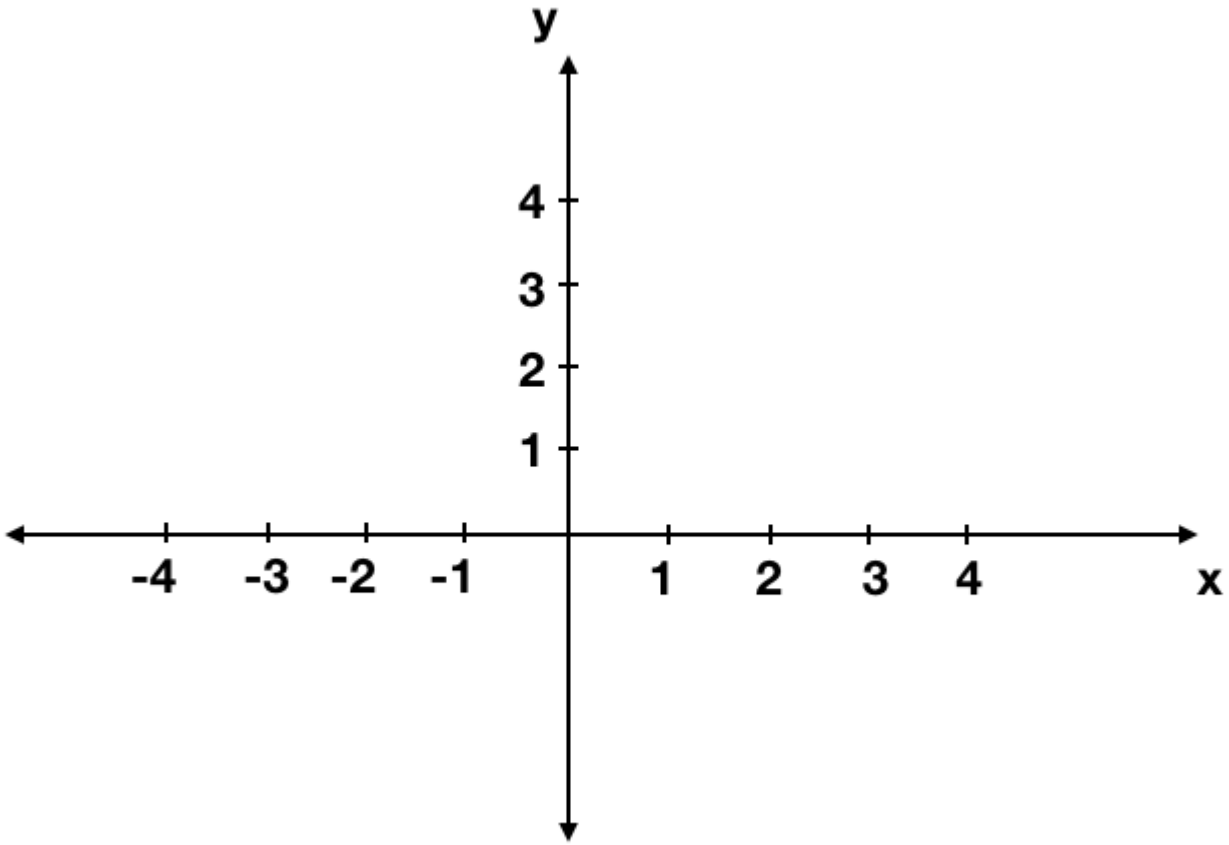
Linear Perceptron Practice Problem **CSCE 421**

The goal of this problem is to run a linear perceptron algorithm. Assume that you have three training samples:

1. Sample $\mathbf{x_1} = [1, 3]^T$ from Class 1 ($y_1 = 1$)
2. Sample $\mathbf{x_2} = [3, 2]^T$ from Class 2 ($y_2 = -1$)
3. Sample $\mathbf{x_3} = [4, 1]^T$ from Class 2 ($y_3 = -1$)

The linear perceptron is initialized with a line with corresponding weight $\mathbf{w(0)} = [-\frac{1}{3}, 1]^T$. In the following, for the sake of simplicity, you will assume that all lines of the perceptron intersect point $(0, 0)$, therefore **you do not have to include any intercept w_0 or x_0 in the following calculations.**



(1) Plot $\mathbf{x_1}$, $\mathbf{x_2}$, and $\mathbf{x_3}$ in the given 2D space. Find and plot the line corresponding to weight $\mathbf{w(0)}$.

(2) Using the rule $sign(\mathbf{w(t)}^T\mathbf{x_n})$, please indicate in which class are samples $\mathbf{x_1}$, $\mathbf{x_2}$, and $\mathbf{x_3}$ classified using the weight $\mathbf{w(0)}$. Which samples are not correctly classified based on this rule?
Note: You have to compute the inner product $\mathbf{w(0)}^T\mathbf{x_n}$, $n = 1, 2, 3$, and see if it is greater or less than 0.

(3) Using the weight update rule from the linear perceptron algorithm, please find the value of the new weight $\mathbf{w(1)}$. Find and plot the new line corresponding to weight $\mathbf{w(1)}$ in the 2D space.
Note: The update rule is $\mathbf{w(t+1)} = \mathbf{w(t)} + y_s\mathbf{x_s}$, where $\mathbf{x_s}$ and $y_s \in \{-1, 1\}$ is the feature and class label of missclassified sample s.

(4) Using the rule $sign(\mathbf{w(t)}^T\mathbf{x_n})$, please indicate in which class are samples $\mathbf{x_1}$, $\mathbf{x_2}$, and $\mathbf{x_3}$ classified using the weight $\mathbf{w(1)}$. Which samples are not correctly classified based on this rule?
Note: You have to compute the inner product $\mathbf{w(1)}^T\mathbf{x_n}$, $n = 1, 2, 3$, and see if it is greater or less than 0.

(5) Using the weight update rule from the linear perceptron algorithm, please find the value of the new weight $\mathbf{w(2)}$. Find and plot the new line corresponding to weight $\mathbf{w(2)}$ in the 2D space. How many samples are correctly classified now?
Note: The update rule is $\mathbf{w(t+1)} = \mathbf{w(t)} + y_s\mathbf{x_s}$, where $\mathbf{x_s}$ and $y_s \in \{-1, 1\}$ is the feature and class label of missclassified sample s.