Assume a 1-dimensional linear regression model $y = w_0 + w_1 x$. The residual sum of squares (RSS) of the training data $\mathcal{D}^{train} = \{(x_1, y_1), \ldots, (x_N, y_N)\}$ can be written as:

$$RSS(w_0, w_1) = \sum_{n=1}^{N} (y_n - w_0 - w_1 x_n)^2$$

We estimate the weights w_0, w_1 by minimizing the above error.

(1) Show that minimizing RSS results in the following closed-form expression:

$$w_1^* = \frac{\sum_{n=1}^{N} x_n y_n - N\left(\frac{1}{N}\sum_{n=1}^{N} x_n\right)\left(\frac{1}{N}\sum_{n=1}^{N} y_n\right)}{\sum_{n=1}^{N} x_n^2 - N\left(\frac{1}{N}\sum_{n=1}^{N} x_n\right)^2}$$

$$w_0^* = \left(\frac{1}{N}\sum_{n=1}^{N} y_n\right) - w_1\left(\frac{1}{N}\sum_{n=1}^{N} x_n\right)$$

Tip: Set the partial derivatives $\frac{\vartheta RSS(w_0,w_1)}{\vartheta w_0}$ and $\frac{\vartheta RSS(w_0,w_1)}{\vartheta w_1}$ equal to 0. Then solve a 2×2 system of linear equations with respect to w_0 and w_1.

(2) Show that the above expressions for w_0^* and w_1^* are equivalent to the following:

$$w_1^* = \frac{\sum\limits_{n=1}^{N} (x_n - \bar{x})(y_n - \bar{y})}{\sum\limits_{i=1}^{N} (x_n - \bar{x})^2}$$

$$w_0^* = \bar{y} - w_1 \bar{x}$$

where $\bar{x} = \frac{1}{N} \sum_{n=1}^{N} x_n$ and $\bar{y} = \frac{1}{N} \sum_{n=1}^{N} y_n$ are the sample means of input features and outcome values, respectively.

(3) How would you interpret w_0^*?