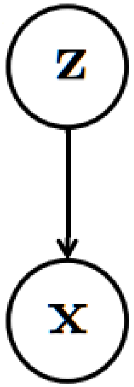
Variational Auto encoder (VEA)

Variational Inference

- Problem Definition
  - Observable Data: $x = \{x_1, x_2, ..., x_n\}$
  - Hidden Variable: $z = \{z_1, z_2, ..., z_n\}$



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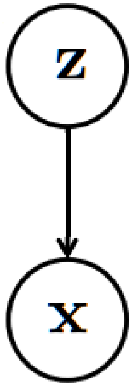


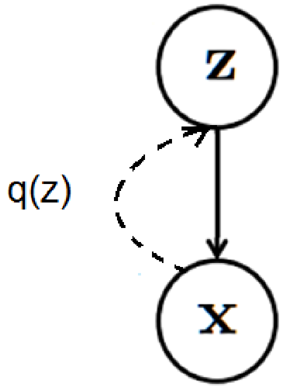
$$p(z|x) = \frac{p(z,x)}{p(x)} = \frac{p(x|z)p(z)}{\int p(x|z)p(z)dz}$$

Variational Inference

- Solutions
  - Monte Carlo Sampling
    - Metropolis Hasting
    - Gibbs Sampling
  - Variational Inference

Variational Inference

- Approximate $p(z|x)$ by $q(z)$



- Minimize the KL Divergence:

$$D_{KL}\left[q(z)||p(z|x)\right] = -\int q(z)log\frac{p(z|x)}{q(z)}dz$$

Variational Lower Bound

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$$= -\int q(z)log\frac{p(z,x)}{q(z)p(x)}dz$$

Variational Lower Bound

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$$= -\int q(z)log\frac{p(z,x)}{q(z)p(x)}dz$$

$$= -\int q(z)log\frac{p(z,x)}{q(z)}dz + \int q(z)log\big(p(x)\big)dz$$

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$$= -\int q(z)\Big(log\big(p(z,x)\big) - log\big(q(z)\big)\Big)dz + log\big(p(x)\big)$$

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$$= -\int q(z)\Big(log\big(p(z,x)\big) - log\big(q(z)\big)\Big)dz + log\big(p(x)\big)$$

$$= -\underbrace{(E_{q(z)}\left[log\big(p(z,x)\big)\right] - E_{q(z)}\left[log\big(q(z)\big)\right])}_{\text{Evidence Lower Bound (ELBO)}} + log\big(p(x)\big)$$

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$$D_{KL}\Big[q(z)||p(z|x)\Big] = -L\Big[q(z)\Big] + log\Big(p(x)\Big)$$

Variational Lower Bound

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$$D_{KL}\left[q(z)||p(z|x)\right] = -L\left[q(z)\right] + log\left(p(x)\right)$$

$$log\left(p(x)\right) = D_{KL}\left[q(z)||p(z|x)\right] + L\left[q(z)\right]$$

Variational Lower Bound

$$D_{KL}\Big[q(z)||p(z|x)\Big] = -\underbrace{\left(E_{q(z)}\Big[log\big(p(z,x)\big)\Big] - E_{q(z)}\Big[log\big(q(z)\big)\Big]\right)}_{\text{Evidence Lower Bound (ELBO)}} + log\big(p(x)\big)$$

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Minimizing $D_{KL}\Big[q(z)||p(z|x)\Big]$

is equal to Maximizing $L\Big[q(z)\Big]$