### Experiment design

The general experiment design and diagram were provided in (Nguyen and Liebman 2022), but an updated diagram is provided below to cover tillage regimes. Contrasting herbicide regimes have been used for both corn and soybean at the experiment site from 2008. Soybean plots were managed by conventional herbicide starting in 2017.

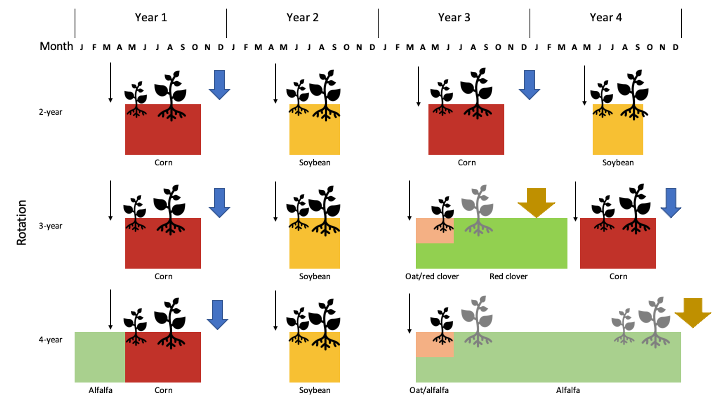


Figure 1: cap

*Population transition matrices were calculated using Wood’s quadratic programming method (Section 6.2, Caswell 2001).*

The waterhemp population is considered size-structured with overlapping cohorts. Cohorts in this experiment reflected the time stamps when the waterhemp sub-populations were surveyed. Two consecutive surveys were approximately two to three weeks apart, as the weather permitted.

### Matrix form

*The periodic matrices are sub-divisions of an annual projection matrix: each is constructed based on critical points in a weed species’ life cycle, which will provide a means to track when and where major changes in population dynamics happen.* Details on periodic matrices construction and justification are provided in the next section.

Each transition matrix of eight rows by eight columns (8 x 8) consists of four blocks using the format of matrix 4.8 in (Chapter 4, Caswell 2001). Each rotational transition matrix, , is the product of two, three or four annual projection matrices, (c = 2, 3, or 4, corresponding to the number of crop phases in each rotation). Each annual projection matrices, corresponding to a crop environment, , is the product of six sub-annual matrices, with .s

The general form of any matrix (periodic sub-annual (), annual (), or rotational ()) is

where,  
, 2 x 2, is the transition within the seedbank population,  
, 6 x 6, is the transition within the plant population,  
, 6 x 2, is the transition from the seedbank to the plant population (emergence), and  
, 2 x 6, is the transition from the plant to the seedbank population (distribution of newly produced seeds to the soil seedbank)

Within a year, waterhemp population projection from time to was reflected by the changes in number of seeds and plants and calculated as follow (Chapters 2, 3 and 4, Caswell 2001).

where,  
 is the square transition matrix from time to time , and  
 is the population vector (in column matrix form) of eight rows and one column.  
Waterhemp population transition in one crop phase, from pre-planting tillage to winter dormancy is calculated with

where,  
 is the index for the crop phase in a particular rotation,  
 is the Leslie’s population matrix in crop phase , and (**leslieUseMatricesCertain1945?**)  
 is the population transition matrix during summer

The main factors that contribute to the success of the control practices are tillage regime, chemical herbicide, cultivation practice and crops’ competitiveness. Among those four factors, crop competitiveness will not be factorized because this factor is dependent on other factors.

The following parameters were collected and plugged into a general population model to examine when and where large changes in population dynamics might occur. The list of parameters and the corresponding formula is presented in Table 4.1 below.

Table 4.1 - Model parameters

| Parameter | Formula | Notation | Definition | Reference |
| --- | --- | --- | --- | --- |
| Common indexes |  |  | Cohort number, 1 to 6 |  |
|  |  |  | Top (1) and bottom (2) soil strata |  |
|  |  |  | The time between two consecutive measurements |  |
|  |  |  | Crop: corn, soybean, oat, alfalfa |  |
|  |  |  | Rotation: 2-,3-,4-year |  |
|  |  |  | Corn weed management: low or conventional |  |
|  |  |  | Seedbank density at stratum |  |
|  |  |  | Cohort plant density |  |
| ——— | ————————- | ——————- | —————————– | ————– |
| Seedling establishment |  |  | The proportion of the seedbank’s top stratum to germinate to cohort |  |
| Surface seedbank density |  |  | in Crop , rotation , herbicide |  |
| Seedling density |  |  | in Crop , rotation , herbicide |  |
| ——— | ————————- | ——————- | —————————– | ————– |
| Plant survival |  |  | The proportion seedling in each cohort that survives until the end of the crop season |  |
| Mature plant density |  |  | in Crop , rotation , herbicide |  |
| ——— | ————————- | ——————- | —————————– | ————– |
|  |  |  | fecundity of a surviving plant (presumably resistant) relative to a susceptible one |  |

### 0.0.1 Model assumptions

Table 1: Female-only population dynamics model assumptions for common waterhemp (*Amaranthus tuberculatus*)

| Stage | Assumption | References |
| --- | --- | --- |
| All | Sex is stable throughout the life cycle. | (schaffnerObservationsExperimentsSex1935?); (murrayGeneticsSexDetermination1940?) |
|  | Equal growth rate across individuals of the same size and shape under the same treatments (crop x rotation x herbicide) | Chapter 8, Caswell, 2001 |
| Seed | Equal germination probability across sexes |  |
|  | The sex ratio is 1:1. | Costea, 2005 |
|  | Sex is determined at seed formation | Montgomery et al., 2019; 2021 |
|  | Seeds only emerge from the 0-2cm soil stratum | Deduced collectively from the recommended sowing depth of 1-3 times the seed diameter (Chapter 3, foodandagricultureorganizationoftheunitednationsAridZoneForestry1989?) and AMATA seed size of 1 mm or less in diameter Costea, 2005. |
|  | Germination is fatal from the 2-20 cm soil stratum |  |
|  | Equal decay rate in the soil across sexes |  |
|  | Equal decay rate across all burial depths | (buhlerEmergencePersistenceSeed2001a?), (steckelTillageCroppingSystem2007?) |
|  | Equal palatability to granivores across sexes |  |
|  | Granivore activities are an important threat. | van der Laat et al., 2015 |
|  |  |  |
| Young plant | Female plants are more likely to survive under stressful conditions than male plants. | Deduced collectively from the general 1:1 sex ratio (costeaBiologyInvasiveAlien2005?) and differentiated sex ratio at maturity across weed management systems at the experiment site in 2018. |
|  | Crops are the main competitors for resources. | Deduced from weeds being effectively suppressed under contrasting cropping systems at the experiment site (huntReducingFreshwaterToxicity2017?). |
|  | Competition with other weeds is negligible. |  |
|  | Intraspecific competition is negligible. |  |
|  | The weed control program catered to the specific crop is the main cause of mortality. | Ryan et al., 2010 |
| Mature plant | Pollen source is unlimited | pollens can stay viable for five days after dispersion (Liu et al., 2012) and substantial population density and sex ratio at the experiment site (Nguyen and Liebman, 2022a) |
|  | 50% of the seeds produced by each female plant are female. | Costea, 2005 |
|  | Male : female ratio can deviate from 1:1 ratio under different conditions | more males than females observed at the experiment site when composted swine manure was applied(menalledImpactCompostedSwine2004?); 5 to 10 females per 1 male in a low-density population in Ohio (prattAmaranthusRudisTuberculatus2001?) |
|  | Genetic material exchange is more likely to occur within 0 to 50 meters around the pollen source. | (sarangiPollenmediatedGeneFlow2017?); (liuPollenBiologyDispersal2012?) |
|  | Fitness cost is negligible in populations with resistance to atrazine, protoporphyrinogen oxidase inhibitors, 4?hydroxyphenylpryuvate dioxygenase inhibitors or glyphosate. | (wuLimitedFitnessCosts2018?); Sosnoskie et al., 2013 |

### Parameterization

The empirical data was organized by corn weed management regimes, i.e., conventional (conv) versus low herbicide, because the herbicide regime change was recent while the legacy effect of weed control program on each subplot may last for a few years (**rasmussenLegacyPesticidePollution2015?**; **gibsonBenchmarkStudyGlyphosateresistant2016?**). *Details on the empirical and literature data collection and justification are listed below.*

From spring to winter, the sequences of sub-annual matrices are: pre-planting tillage, emergence, summer survival, fecundity, post-harvest tillage, and overwinter survival. Each sub-annual period is an 8x8 matrix consists of four blocks using the format of matrix 4.8 in (Chapter 4, Caswell 2001).

#### Vertical redistribution of seeds, in pre-planting and post-harvest tillage

In fall 2019, 36 soil cores were collected for seedbank densities estimation at two soil strata, 0-2cm as the top (1), and 2-20 cm depth as the bottom (2).

The redistribution of female seeds that were produced at the end of the crop season is calculated per each crop phase under each rotation using the transition matrices corresponding to each tillage regime by Spokas et al. (2007). No-till is represented by an identical matrix after (**cousensModelEffectsCultivation1990?**).

#### Seedling recruitment

Seedling recruitment was calculated as the proportion of emerged seedlings with regards to the top 0-2 cm top soil stratum: , where

#### In-season survival

#### Plant fecundity

#### Overwinter survival

what assumptions, list in previous table

### Population growth rate and its variance

The variance of population growth rate is calculated with where is the variance-covariance matrix of each periodic matrix.

Variance of annualized population growth rate is calculated with Taylor series expansion using the general formula: , in which is the nth-root function used to annualize the rotation-wise growth rates. For example, the variance of annualized population growth rate in the 2-year rotation was , where .

### Elasticity of population growth rate on each lower-level parameter

Matrix calculation in this study was performed in R version 4.2.0 (**rdevelopmentcoreteamLanguageEnvironmentStatistical2022?**). The population growth rate, , sensitivity and elasticity were calculated with eigen.analysis function (**stubbenPopbioConstructionAnalyse2020?**).

Caswell, Hal. 2001. *Matrix Population Models: Construction, Analysis, and Interpretation*. Second. Sunderland, Mass.: Sunderland, Mass. : Sinauer Associates.

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Spokas, K., F. Forcella, D. Archer, and D. Reicosky. 2007. “SeedChaser: Vertical Soil Tillage Distribution Model.” *Computers and Electronics in Agriculture* 57 (1): 62–73. <https://doi.org/dzh845>.