Shock wave in Traffic Flow Model

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Abstract

The purpose of the article is to combine the METANET traffic model with the model predictive control (MPC) to predict the optimal speed for vehicles in a high density traffic. We introduce and study the METANET model and add extensions to the model with the Rankine-Hugoniot Condition on the Greenshield's Model. We argue that the chart comparing traffic at certain time intervals with and without MPC proves more advantageous towards the traffic with MPC. The concept of using time prediction method is to predict when the traffic will happen and reduce the congestion in traffic.

Introduction

Stuck in a intersection, you always watch to unfold the Fundamental Problem of Traffic. On the green light, the first car accelerates, and then the next and then keep going until you are stopped by the red light. You instinctively think that had all the cars accelerate simultaneously, you would have made it through. Human has slow reaction time and short attention spans. This incoordination limits how many cars can get through an intersection and one backs up to the next; that's when city-sized gridlock cascades happen, taking forever to damp out. In general analysis, the more intersections, equal mores incoordination which equals more traffic congestion. This is the motive behind freeway, big highways: "No intersections".

In application, we see these traffic problems in Linear Algebra. Looking at the traffic flow network in a four one-way streets in Kumasi, Ghana [1] or the traffic in downtown Jacksonville, Florida [12], we convert the traffic flowing in and flowing out of the system into a matrix and determine the density of each junction of traffic. Theoretically, even though there are splits and merges, there is no stopping, no coordination problems, and no traffic. However, intersection outside of a highway will back up onto it. Human reaction times limit how many cars can escape off-ramp when the light changes. Moreover, even without intersections, there would still be traffic on the highway. For example, take a one lane highway with a constant flow and moderate density, then all of a sudden an object cross the road. The driver sees it brakes a little, the following car brakes a little harder and the chain reaction keeps on going until someone reach a complete stop. In a multi-lane highway, if a driver decided to change lanes rapidly with cars too close behind would be enough to birth one. Though the object is no longer be there, it left a phantom intersection on the highway, or we can understand at a shock wave point. "Shock waves are waves that originate from a sudden, substantial change in the state of the traffic flow. That is, a shock wave is defined by a discontinuity in the flow-density conditions in the time-space domain." [3]

We think of the shock waves as a snake slithering down the road eating on coming cars and damp out the other end. We set up an experiment where cars with a moderate density and initial velocity driving in a circle. As soon as one car has a different velocity than the initial velocity. This will start a shock wave of traffic that will last forever, even though there is no accident nor problem with the road, and cars. The solution for the problem is to coordinate all the drivers and have them accelerate simultaneously, then the shock wave will be eventually cease.

Since shock waves are a global problem, it has long since needed a remedy. However, since the root cause seems to stem from human interaction the applied solution can only go so far. The Department of Transportation published a report on solutions to several traffic grid issues and their perceived solution and results. A success story for the Colorado Department of Transportation (CDOT), in 2006 a construction project went underway to create a grade separation by lowering Wadsworth Boulevard by 25 feet, creating a new passage under Grandview Avenue and the Northern Railroad. The $32 million project ran from October 2006 to December 2008 [8]. The goal of the project was a success in providing a grade separated intersection to eliminate all grade crossing conflicts and to accommodate other intersecting maneuvers by merging. The reduced the shock wave effect of the initial conflict because now vehicles are forced to merge at relatively low speeds, allowing for a consistent flow of traffic.

Cambridge Systematics was hired by Michigan's Department of Transportation to perform a study of the before and after effects of removing ramp meters from local highways and interstates. They gathered information from 5 weeks for each dataset. They found that without meters, there was a 9 percent reduction in freeway volume; a 22 percent increase in travel times; a 26 percent increase in crashes (even after adjusting for prior seasonal rates). Most survey respondents believed traffic had worsened. After the study, however, 20 percent wanted meters left off; 10 percent want them "returned"; 70 percent want modifications. MnDot took into consideration the wants of the communities and decided on a middle ground that the meters would wait no more than 4 minutes on local ramps or 2 minutes on freeway-to-freeway ramps. Vehicles queued back to city streets will be "released" (meters temporarily shut off) and meter operation will better-respond to congestion-only times via improved use of detectors.

In the city of Atlanta, the shoulder lane on 85-N are converted into a temporary lane during high traffic hours. The change to this was done to postpone the onset of congestion. The lane creates the use of uniform speeds and increases the capacity of vehicles allowed on 85N. It was a low cost plan that stemmed direct results. A similar conversion is seen with the use of toll road "Peach Pass Lane" by not limiting the prior HOV lane to only vehicles that are supporting an effort to carpool, congestion is also alleviated for those traveling to and from Metro Atlanta.

Driver's in Pennsylvania faced a different issue but the same solution was warranted to resolve it. Drivers did not have adequate acceleration lanes causing vehicles to often come to a complete stop to wait for an acceptable gap in traffic. This caused an unsafe and inefficient merge condition. In 2007 the Pennsylvania Department of Transportation (PennDOT) and the Federal Highway Administration (FHWA) devised a low-cost solution which involves converting the existing right shoulder into a third freeway lane and re-striping the existing ramp to allow oncoming traffic to continue onto the freeway without stopping [9]. With only an estimated cost of $250,000. The project will completely remove the merge condition and bottleneck at the freeway entrance and eliminate ramp and mainline queues. This project demonstrates that low-cost, low-impact bottleneck improvements can make significant improvements in traffic flow [9].

Climate change is approaching a doomsday event according to the Boris Johnson, the British Prime minister. The United Nations summit for climate change prompts every nation to do its part in creating a lasting environment for communities worldwide. More than 50% of the world population lives in cities. Additionally, cities are responsible for more than 75% of the global energy consumption and the global greenhouse gas (GHG) emissions [7]. As previous discussed traffic patterns can exponentially increase the negative effects of outdated city plans.

In 2000 De Vlieger, D. De Keukeleere, J.G. Kretzschmar of the Flemish Institute for Technological Research published a research journal about the environmental effects of driving behavior and congestion related to passenger cars. Their findings concluded that "traffic conditions have a major effect on fuel consumption and emissions during rush hours, ... with an increase by 10 to 200%" [4]. The variation of increase is proposed to be dependent on traffic patterns and driver behavior, meaning that if a driver is more aggressive, they will create larger amount of emissions and fuel. This brings us back to the notion of applied solutions for our shockwave problem. The use of alternative traffic patterns is necessary on a climate scale as well as for the reduction of traffic.

In 2018 Christian Hofer, Georg Jäger, and Manfred Füllsack published Large scale simulation of CO2 emissions caused by urban car traffic: An agent-based network approach. In the journal they presented a flexible structure model, which can be used to evaluate policies beyond what is presented in their original study. It can easily be adapted to other conditions and geographical regions [7].

The METANET model was created as optimization formulation for macroscopic traffic flow problem and solved using a genetic algorithm. The road networks are represented as directed graphs with nodes and links; the links are homogeneous road sections with constant number of lanes and no changes to the curvature or gradients of the roads. The links connect the nodes, and in turn, the nodes are used at places where the geometry of the motorway changes or at the on-ramp or off-ramp junctions [11]. On the other hand, the model predictive control are able to solve the problem of the optimal coordination of speed limits [2]. We use the METANET model to make predictions of the traffic and apply the model predictive control (MPC) to find the optimal control inputs.

Therefore, as a driver participating in traffic, you have a high responsibility of causing traffic by your driving behavior. Always keeps the distance two cars from the car in front and behind you. Tailgating instigates trouble since it is more likely to cause accident, or you as the tailgater can cause shock wave point if the driver ahead breaks. However, this will never be the solution for traffic congestion. In traffic, the congestion will grow if the shock wave travels faster than the damping point and vice versa. We as a group are seeking an optimization solution for this every day problem, a structural systematization solution.

Methods

Our goal is to find an optimization solution for traffic conditions that produce shock wave. As previously discussed, shock waves are "waves that originate from a sudden, substantial change in the state of the traffic flow. A shock wave is defined by a discontinuity in the flow-density conditions in the time space domain. As a group we applied the model predictive control to optimally coordinate variable speed limits. The purpose of the control is to find the control signals that minimize the total time the vehicles spend in the network.

We look back to the Fundamental Problem of Traffic. Every vehicles will enter the road from a pit with an initial velocity. At the connection point between the pit and the ring road, you set up a meter with a camera installed Model Predictive Control, and a speed limit prediction. We indicate that if each car cross the meter reach $v_1 < v_0$, then the meter turn on the yellow signal. If the camera detects a complete stop on sight, then the meter turns on traffic control mode. Depends on the distance between the stopped car and the camera, the camera will determine the stop-and-go time for each vehicle such that the car exits out the shock wave point and will follow the speed v_2, until the last car has reach v_2. If the camera no longer detects congestion, the meter stops and v_2 change back to v_1 until the next shock wave point.

We want to apply the model predictive control to optimally coordinate variable speed limits. The purpose of the control is to find the control signals that minimize the total time the vehicles spend in the network. We want to predict the time that shock wave will occur with given certain density and traffic flow velocity. We first start with traffic flow equation

$$\rho_t + C(\rho)\rho_x = 0$$

and use Greenshield's Model to predict the ideal velocity that vehicle can go in a high density roadway. Finally, we have a piece-wise function

$$\begin{cases} \rho_L & x \leq -\epsilon \\ \rho_L + \frac{\rho_R - \rho_L}{2\epsilon}(x+\epsilon) & -\epsilon \leq x \leq \epsilon \\ \rho_R & x \geq \epsilon \end{cases}$$

predicts the density of traffic. The condition to form a shockwave is $\rho_L < \rho_R$. As we can predict the t_0 value, we can combine with the MPC to optimize vehicle speed limit.

We want to use the Rankine-Hugoniot condition on Greenshield's Model to find the time value that vehicle will approach to shock wave point. We then use the time value to combine with model predictive control to optimally coordinate variable speed limit.

Results

Observing the camera to indicate the density at a congestion point at one point interval. We use the MPC on the off-ramp meter and on-ramp traffic lights. In the off-ramp meter, we use green light for normal density, yellow light when there are higher density to indicate the drivers to be cautious of the cars around them. During rush hour, we apply the MPC algorithm to determine the stop time for exit ramp vehicles. While in the on-ramp traffic lights, during normal density, the MPC will prioritize vehicles ready to exit the roads and enter the freeway. When there are higher density of cars, the MPC will use traffic sensor to control traffic and moderate timing for traffic lights to avoid congestion at choke points. Finally, during rush hours, the MPC will prioritize vehicles moving in local roads to reduce the amounts of vehicle entering the freeway.

We found that the optimal prediction horizon is approximately $N_p = 10$ min. A control horizon $N_c = 9$ min was necessary. When the difference $N_p - N_c$ is constant, an increase of N_p causes a small decrease of the total time spent (TTS). In the controlled case, the shock wave disappears after approximately 1.5 hours, while in the no control case, the shock wave travels through the whole link. The active speed limits start to limit the flow at $t = 10$ min and create a low density wave traveling downstream. The low density wave meets the upstream traveling shock wave and reduces its density and stops it [2]. The upstream "end" of the shock wave has a fixed location while the downstream "end" dissolves into free flow traffic in the uncontrolled situation. The shock wave eventually dissolves. In high density areas, the speed limits persist until the shock wave dissolves. The outflow after the shock wave entered the link is restored to earlier capacity in controlled case.

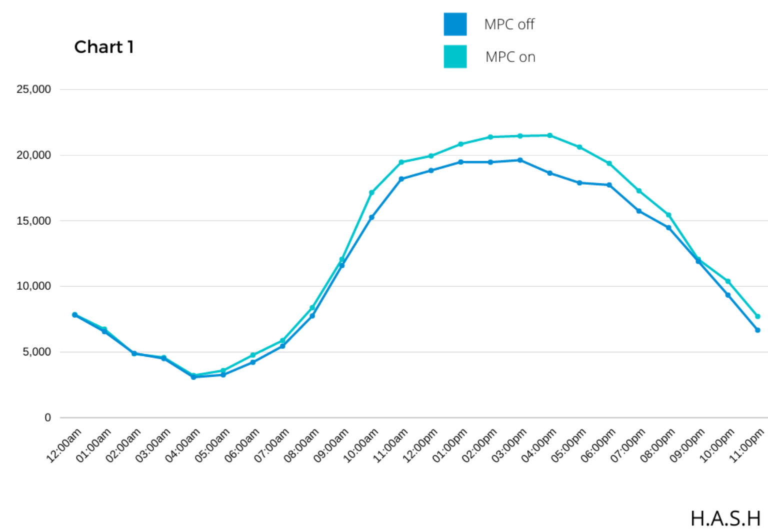

Figure 1: Chart of density of car at each time interval with and without MPC

The figures describe how many vehicles can go through a traffic point every hour in a day with and without Model Predictive Control. We assume the ideal condition where there are no crashes, no construction work, and people obey the speed limit while keeping lane changing to a minimum. The graph is the result from the simulation. The dark blue line represents the real data from Georgia Department of Transportation at one location in I-75 [6]. The line represents the density of traffic through that point every hour for a day. We picked up this intersection to simulate the situation due to its easy landscape and generally uncrowded details; then we can show the effectiveness of vehicles on and off interstate highway. The blue line is the data collect from the simulation with the identical condition. We had to bound the density of vehicles crossing the point at 2800 vehicles/hour. The reason is that when you compute a linear regression of optimization in any traffic flow, if the simulation perform significantly better than real data, it is hard to conclude that we can apply our method. If you can restrict simulated data so that vehicles, weather, road, traffic lights, etc. can behave like real time, then the result should be identical and less than 10% significant. Eventually, our goal is to let vehicles move away from shockwave point and reach to the damping point faster, not more vehicles can go through traffic. The regression line only show that at the same amount of time, there are more vehicles can escape traffics so that the traffic flow rate perform better with our on-ramp or off-ramp control system.

These are non-practical results since we can only run in a simulator. The margin of error of the scenarios would be much closer if we apply it to real traffic. The results still stands in favor of the MPC model since there was a significant improvement in traffic congestion.

Discussion

We use the METANET model to further improve on what the system lacks, combining with Traffic flow function and model prediction to make MPC even more efficient. Our contribution is by using the Rankie-Hugoniot condition to find the initial time and position of given density to determine when the shock wave will form to further control traffic by using improved Model Predictive Control, prevent or “stretch” the flow rate so that the congestion point can damp out faster.

We found that the MPC was able to solve traffic problem by reducing the time by approximately 20% to 25%. The system produces safety on the road by reducing the collision percentage since speed is moderated. In addition, it is efficiency such that high-density locations have already applied the equipment but only use very basic traffic sensors. Using this algorithm would improve our potential to control traffic. It is relatively cheaper and more affordable than other solutions. Due to low maintenance of the system, we can remotely control it by cloud base system. Finally, we can optimize or upgrade our algorithm, further can collect data, using AI to develop a better version of it.

The limitation we found is the METANET model itself. The METANET model does not describe the effect of speed limits, and the dynamic equations are nonlinear. If the constraints of the control variable are not properly set up, it causes problems for the numerical optimization process. The density and ramp meter rate through density dynamics are tightly and non-linearly coupled with the speed control variables complicates the control design. The model cannot quickly identify significant changes in fast transition phases of traffic dynamics [7]. Another limitation is that the system cannot be use in roundabout traffic. While running the algorithm in one roundabout, the meter cannot indicate how long one car should stop since they cannot predict the direction of one vehicle moving. It causes even more traffic in moderate traffic conditions. There are also human factor. As long as people disobey the time and the speed assigned for each vehicle, the traffic will still be there if we speed up and do not keep a good distance with other vehicles. It won't completely solve the traffic problem. We suppose that there will always be traffic by a random phenomenon. Despite all the crashes, high density grows much faster than traffic flow; hence, we can only reduce the time participated vehicles stuck in traffic rather than make traffic congestion disappear.

In the future, as more Electric Vehicles (EV) are mass produced and generally more available to the public, these self-driving cars play a major factor in solving traffic problems. We can implement the MPC model to the EV for them to follow the procedures to prevent congestion at choke points. The MPC model allows for the design of multivariable feedback controllers with a similar procedural complexity to that of single variable one. Additionally, it provides specification in the design phase of constraints on system inputs, states, and outputs, that are then enforced by the controller [13]. In the article by Diaz, et al., the MPC strategy are applied to control the power dispatch of an EV fleet in an EV charging station. This allows the MPC to compensate the generation disturbances and the deviations in the EVs arrival SoC, fulfilling the departure SoC desired by the owner [5].

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